Risultati recenti sulla violazione di CP nei decadimenti del B^0_s

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CPV nel B_s e Nuova Fisica

$$\begin{tikzcd}
B_s^0 \arrow[rr, "\phi_D"] \arrow[dr, bend right, "\phi_M"'] & & J/\psi\phi \\
& \bar{B}_s^0 \arrow[ur, bend right, "-\phi_D"'] &
\end{tikzcd}$$

$\phi_s = \Phi_M - 2\Phi_D$

- La differenza di fase tra il decadimento del B_s con e senza mixing, non conserva CP.
- Nel Modello Standard:

$\phi_s^{SM} \cong -2\beta_s$

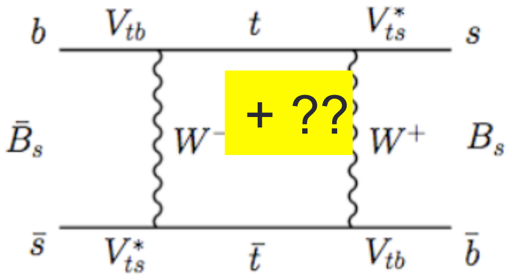
con $\beta_s = \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right)$

molto piccola e calcolata precisamente: $2\beta_s = 0.036 \pm 0.002$ rad

J.Charles et al PRD84(2011)

- Contributi aggiuntivi di diagrammi a pinguino in $B_s \rightarrow J/\psi\phi \sim 10^{-4}$ -10^{-3}
- La presenza di nuove particelle nel diagramma a box può dar luogo ad una fase aggiuntiva:

$\phi_s = \phi_s^{SM} + \phi^{NP}$

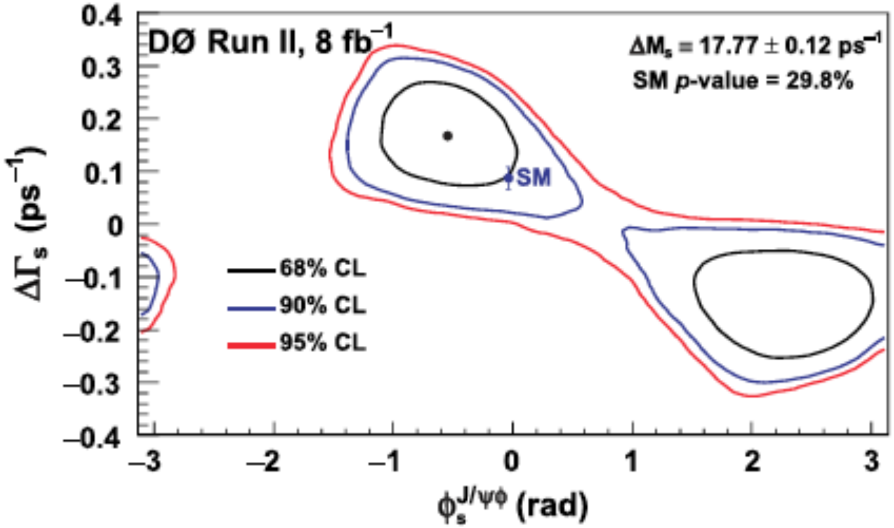


- Una misura di ϕ_s “grande” è segnale chiaro di nuova fisica

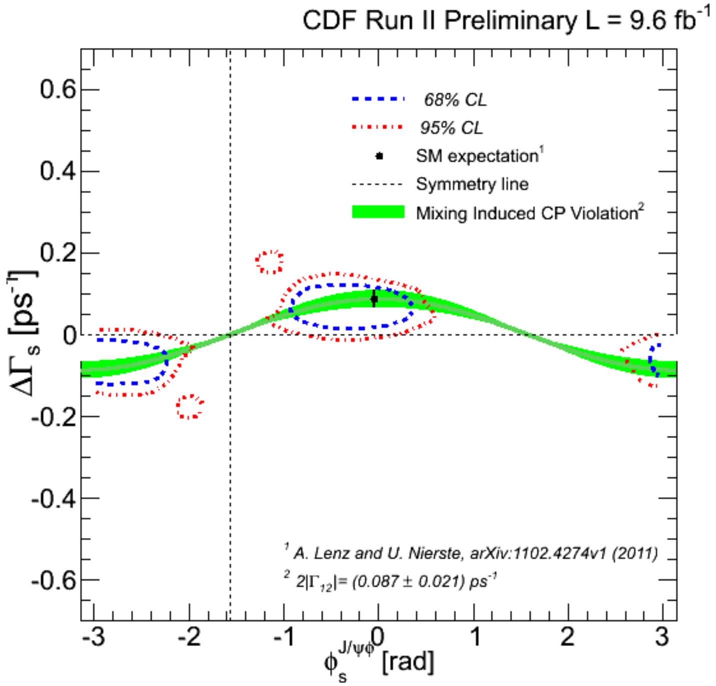
ϕ_s da $B_s \rightarrow J/\psi(\mu\mu)\phi(KK)$ al Tevatron

- Run II completato il 30/11/2011, ricchissima produzione di risultati nella Fisica del Flavour, analisi di tutti i dati in arrivo.

- Aggiornamento di D0 su ϕ_s a 8 fb^{-1} .
- Aggiornamento di CDF su ϕ_s a 9.6 fb^{-1} → talk di L. Sabato



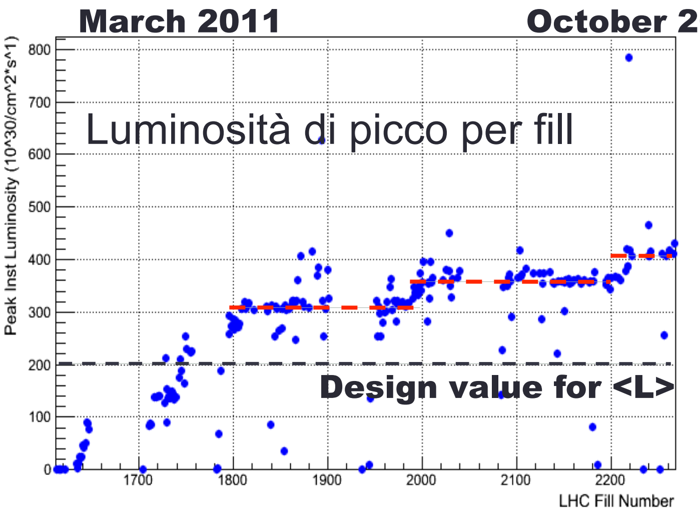
Phys Rev D 85, 032006 (2012)



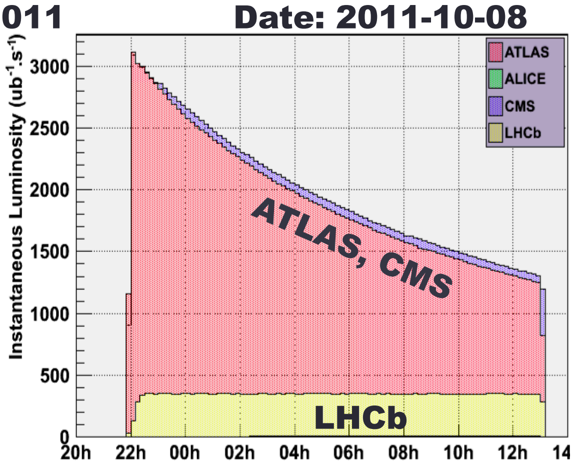
[1] A. Lenz and U. Nierste, arXiv:1102.4274v1 (2011)

[2] $2|\Gamma_{12}| = (0.087 \pm 0.021)$ ps^{-1}

LHCb: run 2011

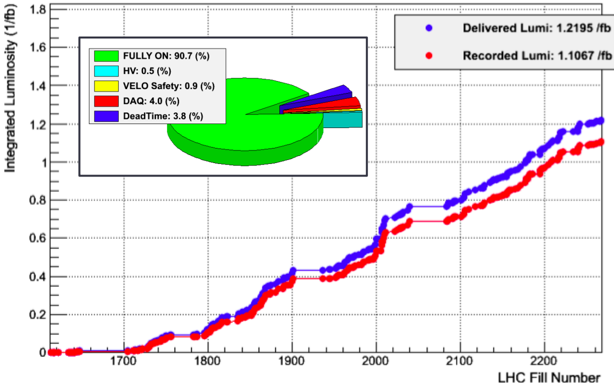


Luminosità di picco per fill



Delivered: 1.2 fb^{-1}

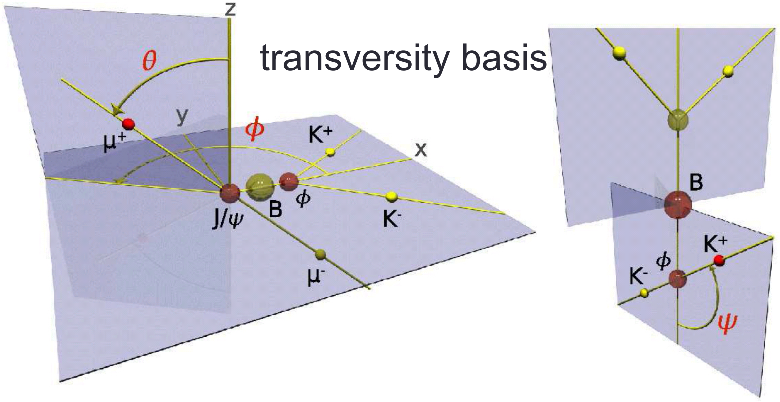
On tape: 1.1 fb^{-1}



- Livellamento della Luminosità istantaneo a LHCb tramite separazione verticale dei fasci.
- $<L> \sim 2.7 \times 10^{32}$ $cm^{-2}s^{-1}$ superiore al valore di disegno.
- Pile-up (numero interazioni pp visibili per bunch crossing) ~1.6
- Efficienza DAQ ~ 91%, efficienza del rivelatore ~99%.
- Output HLT: ~1kHz beauty, ~1kHz charm, ~1kHz altro

- LHC & LHCb ripartiti con successo nel 2012!

$B_s \rightarrow J/\psi\ \phi(KK)$



- $B_s \rightarrow J/\psi$ KK stato finale con componenti CP-pari e CP-dispari, richiede analisi angolare per poterle distinguere

KK in P-wave: ampiezze $A_\perp(t)$, $A_{//}(t)$, $A_0(t) \rightarrow$ CP-dispari o CP-pari
KK in S-wave: ampiezza $A_S(t)$ → CP-dispari

$$\frac{d^4\Gamma(B_s^0 \to J/\psi\,\phi)}{dt\ d\Omega} \propto \sum_{k=1}^{10} h_k(t)\ f_k(\Omega)$$

$$h_k(t) = N_k e^{-\Gamma_s t}\left[c_k \cos(\Delta m_s t) + d_k \sin(\Delta m_s t) + a_k \cosh\left(\tfrac{1}{2}\Delta\Gamma_s t\right) + b_k \sinh\left(\tfrac{1}{2}\Delta\Gamma_s t\right)\right]$$

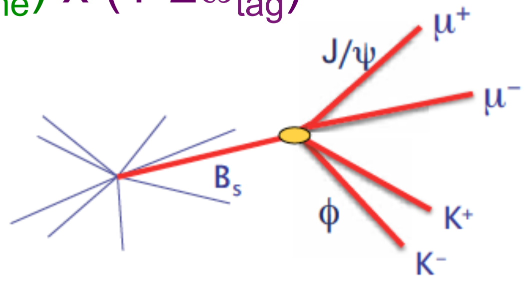
- 10 parameteri fisici
  - ϕ_s
  - $\Gamma_s = (\Gamma_L + \Gamma_H) / 2 = \tau_{Bs}^{-1}$
  - $\Delta\Gamma_s = \Gamma_L - \Gamma_H$
  - $\Delta m_s = M_H - M_L$
  - 3 rapporti di ampiezze
  - 3 differenze di fase forti

(per i due autostati di massa B_L e B_H)

ϕ_s da $B_s \rightarrow J/\psi\ \phi$ ad LHCb

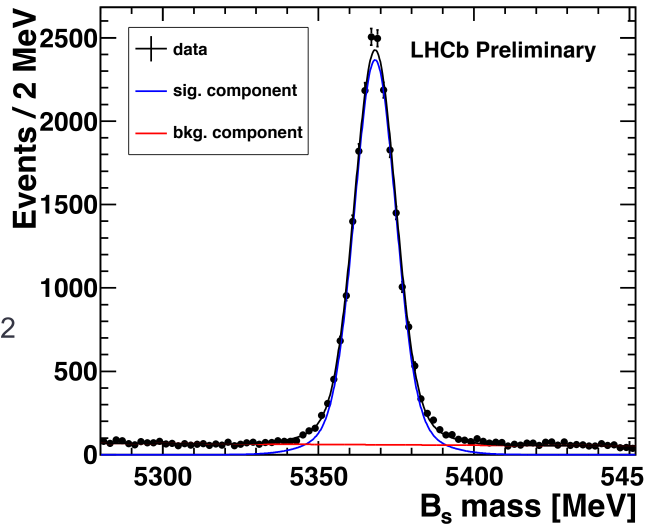
- Fondamentalmente si misura: $\sin(\phi_s)\sin(\Delta m_s\ t)$ x $D(\sigma_{time})$ x $(1\text{-}2\omega_{tag})$

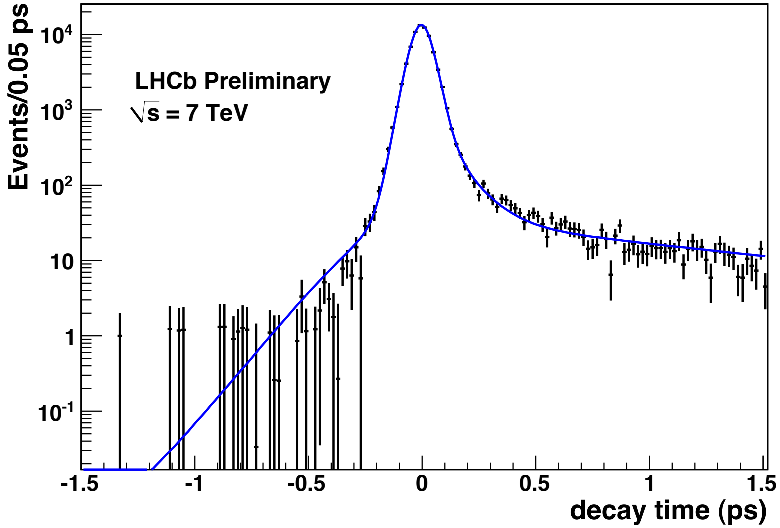
  → cruciali: risoluzione temporale, flavour tagging



- Nuova misura LHCb su 1fb^{-1} (tutto 2011)
- Selezione segnale "cut based": 21.200 eventi con t>0.3ps, molto puliti.
- Risoluzione temporale evento per evento, calibrata sui dati con $J/\psi \rightarrow \mu\mu$ risoluzione effettiva: σ_{time}~45 fs

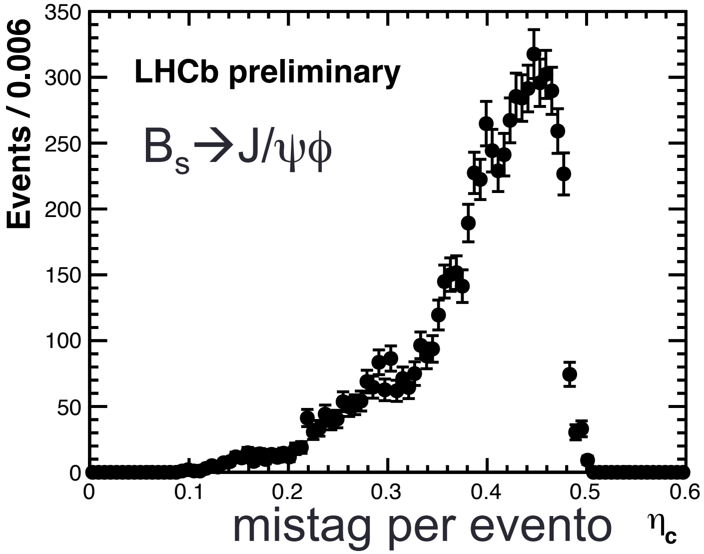
$\sigma(M_B)$ = 6 MeV/c^2

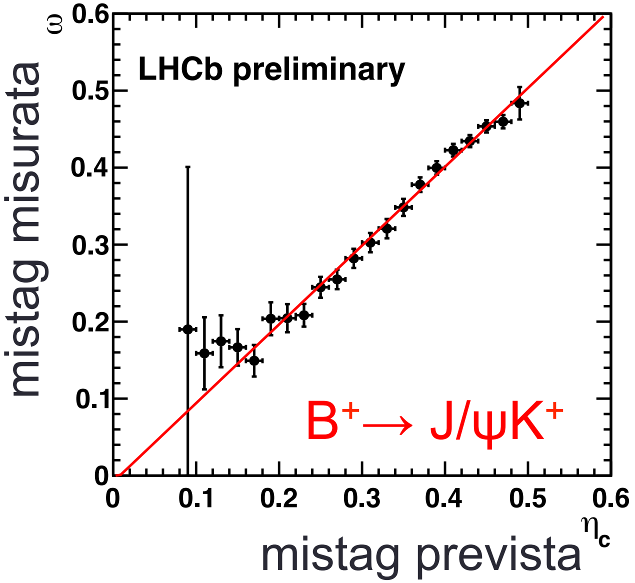




Flavour Tagging a LHCb

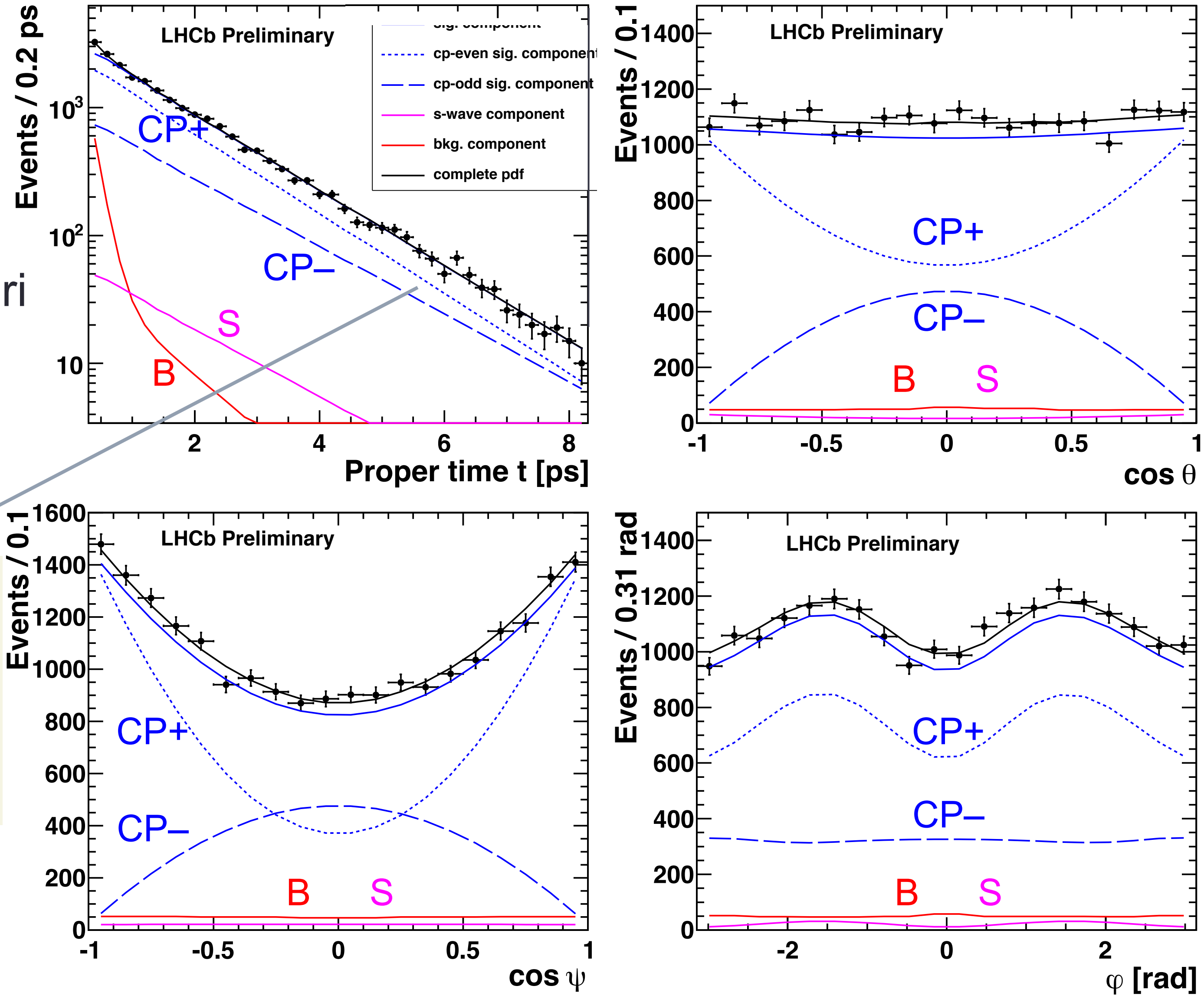
- Separazione di B^0 e $\overline{B}^0$ necessaria per sensibilità a ϕ_s
- "Opposite Side Tagging": sfrutta la produzione pp→$b\overline{b}$ e i decadimenti del b opposto: semileptonici, b→c→K, e ricostruzione inclusiva di vertici secondari
- Nel fit probabilità di mistag per evento, calcolata da NNet. Calibrata sui dati con diversi canali di controllo (→poster A.Falabella)
- Tagging power per $B_s \to J/\psi\phi$: $\varepsilon(1-2\omega_{tag})^2 = (2.29 \pm 0.07 \pm 0.26)\%$





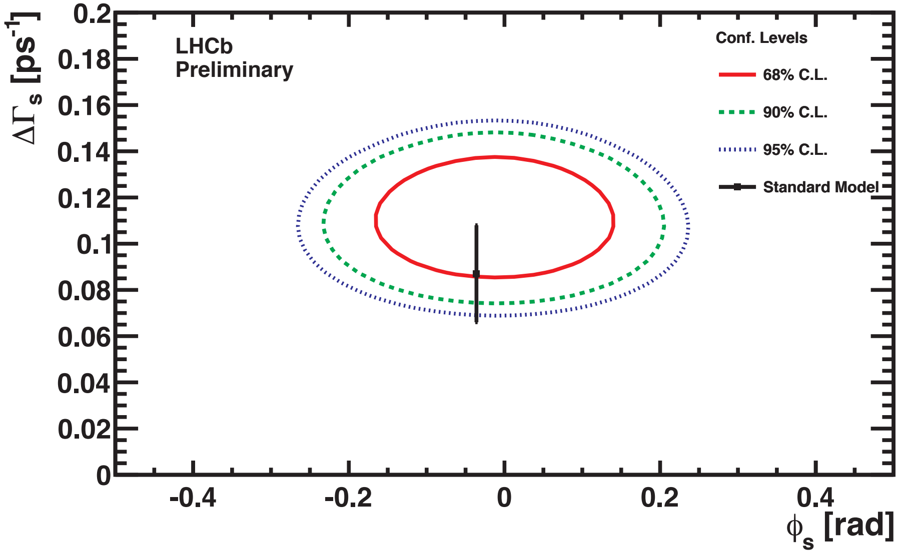
Fit con $1fb^{-1}$

Fit a
massa, tempo
distribuzioni angolari



Prima osservazione
>5σ di $\Delta\Gamma_s \neq 0$
"Gli stati CP– vivono
piu' a lungo che gli
stati CP+"

ϕ_s da $B_s \rightarrow J/\psi\phi$



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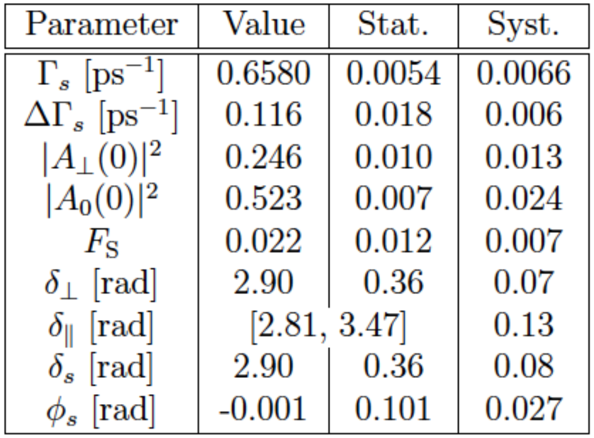
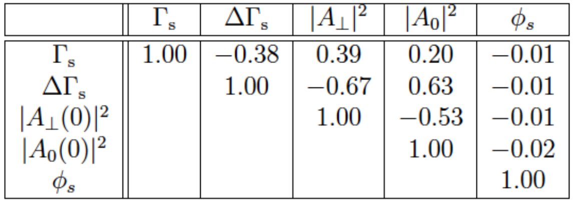


Parameter	Value	Stat.	Syst.
Γ_s [ps^{-1}]	0.6580	0.0054	0.0066
$\Delta\Gamma_s$ [ps^{-1}]	0.116	0.018	0.006
$\lvert A_\perp(0)\rvert^2$	0.246	0.010	0.013
$\lvert A_0(0)\rvert^2$	0.523	0.007	0.024
F_S	0.022	0.012	0.007
$\delta_\perp$ [rad]	2.90	0.36	0.07
$\delta_\parallel$ [rad]	[2.81, 3.47]		0.13
δ_s [rad]	2.90	0.36	0.08
ϕ_s [rad]	-0.001	0.101	0.027



	Γ_s	$\Delta\Gamma_s$	$\lvert A_\perp\rvert^2$	$\lvert A_0\rvert^2$	ϕ_s
Γ_s	1.00	−0.38	0.39	0.20	−0.01
$\Delta\Gamma_s$		1.00	−0.67	0.63	−0.01
$\lvert A_\perp(0)\rvert^2$			1.00	−0.53	−0.01
$\lvert A_0(0)\rvert^2$				1.00	−0.02
ϕ_s					1.00

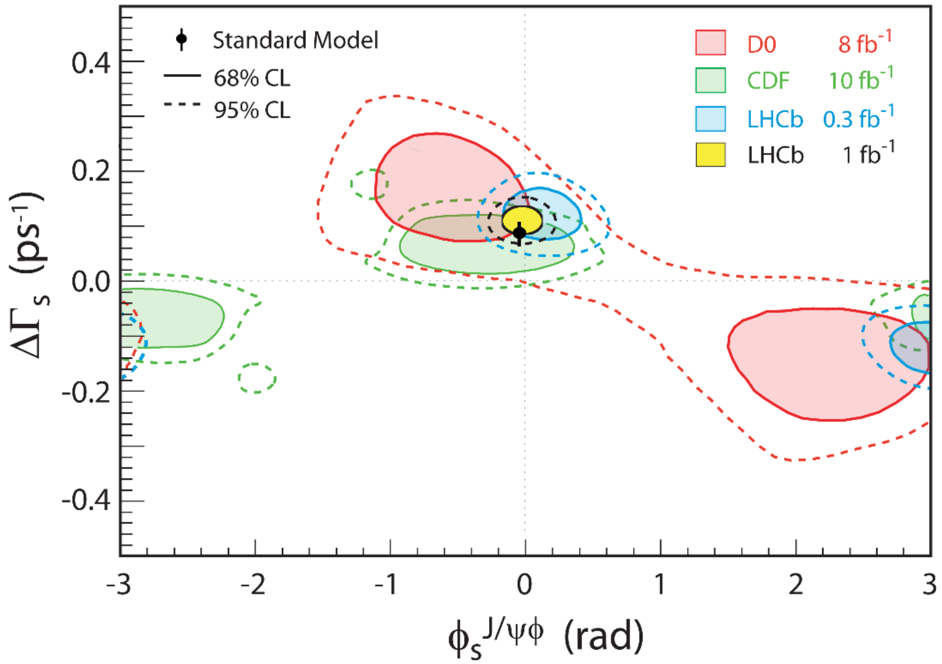
- $\Delta m_s = 17.63 \pm 0.11$ ps^{-1} vincolato alla misura di LHCb in $B_s \rightarrow D_s\pi$, ma è possibile misurare Δm_s anche in questo stesso fit.
- Soluzione con $\Delta\Gamma_s > 0$:

$$\Gamma_s = 0.658 \pm 0.005\ (\text{stat}) \pm 0.007\ (\text{syst})\ \text{ps}^{-1}$$

$$\Delta\Gamma_s = 0.116 \pm 0.018\ (\text{stat}) \pm 0.006\ (\text{syst})\ \text{ps}^{-1}$$

$$\phi_s = -0.001 \pm 0.101\ (\text{stat}) \pm 0.027\ (\text{syst})\ \text{rad}$$

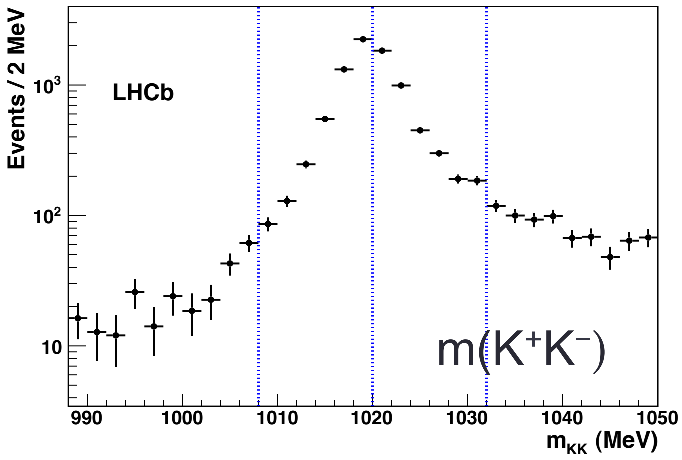
ϕ_s da $B_s \rightarrow J/\psi\phi$: visione globale



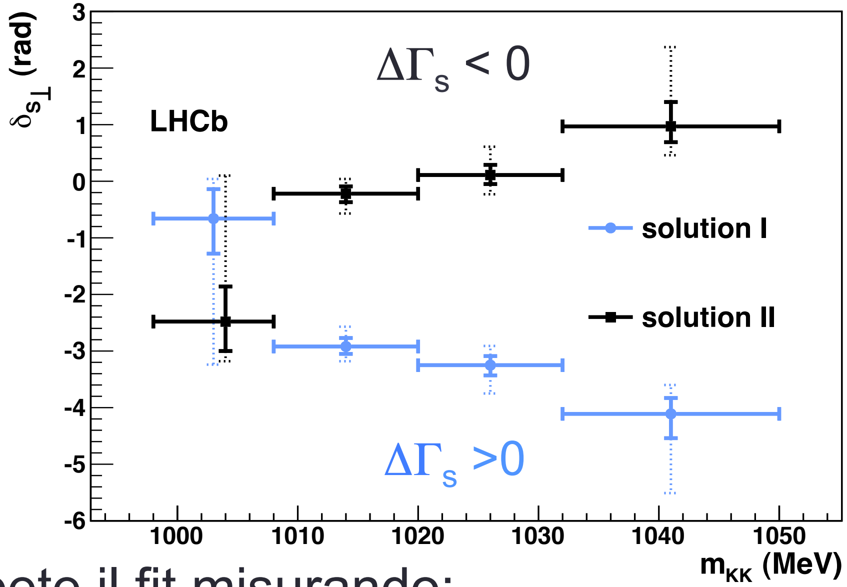
- Prossimo update LHCb:
  - Aumento event yield: selezione eventi con analisi multivariata, ampliamento linee di trigger usate.
  - Aggiunta “same-side kaon” tagging: ottimizzazione e calibrazione in corso con misura del mixing del B_s.
- Attesi altri ~1.5 fb^{-1} a $\sqrt{s}$=8TeV nel 2012.

Determinazione del segno di $\Delta\Gamma_s$

- Due soluzioni ai fit per l'ambiguità nelle decay rates: $(\phi_s,\Delta\Gamma_s) \rightarrow (\pi-\phi_s, -\Delta\Gamma_s)$
- L'ambiguità viene risolta usando l'interferenza tra onda-P e onda-S in range di $m(K^+K^-)$ esteso attorno la risonanza $\phi(1020)$



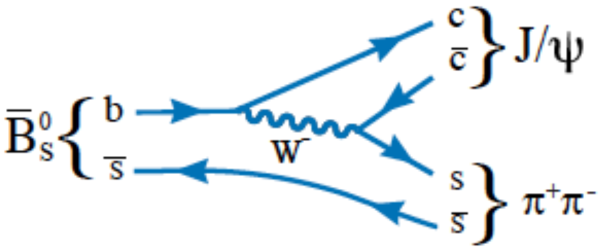
LHCb, arXiv:1202.4717



- Per ciascuno dei 4 bins di massa si ripete il fit misurando:
  - frazione onda-S
  - differenza relativa di fase forte: $\delta_{S\perp} = \delta_S - \delta_\perp$

$\delta_{S\perp}$ è prevista decrescere rapidamente → scelta la soluzione con $\Delta\Gamma_s > 0$

ϕ_s da $B_s \to J/\psi\, \pi^+\pi^-$



- $B_s \to J/\psi\, f_0$ osservato per la prima volta a LHCb.

  Con 0.4 fb^{-1} misurato ϕ_s in $B_s \to J/\psi\, f_0$.
- Misura aggiornata a 1 fb^{-1} usando più largo range di massa ππ.
- Analisi Dalitz dimostra che lo stato finale è 97.7% CP-dispari @95% C.L.

  → Non necessita di analisi angolare. Contributo di $f_0(980)$ ~70%.
- ~7400 eventi di segnale (selezione con BDT)

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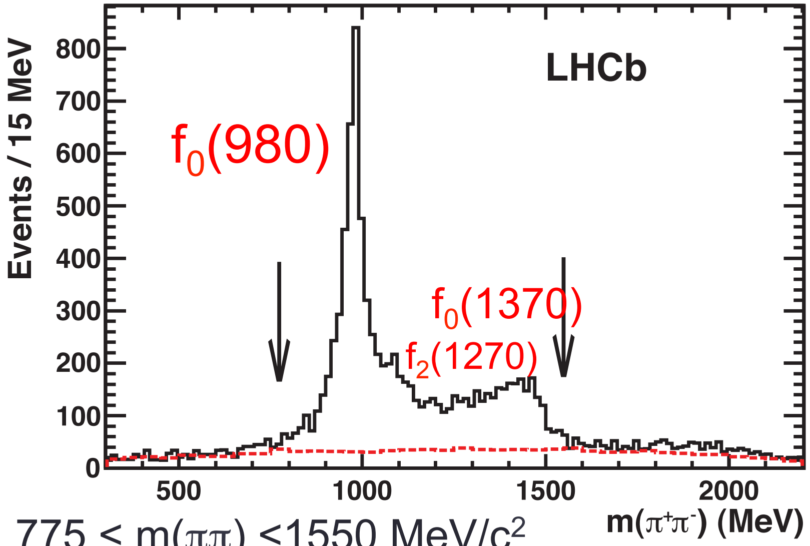
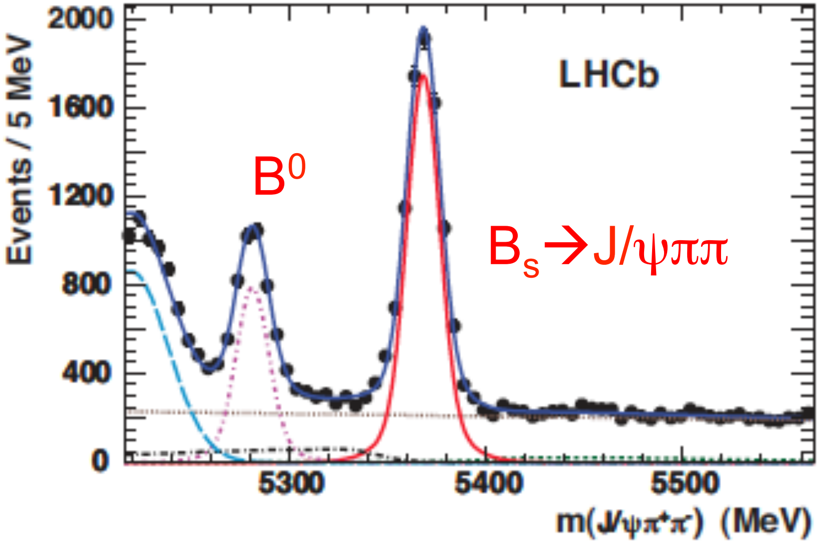


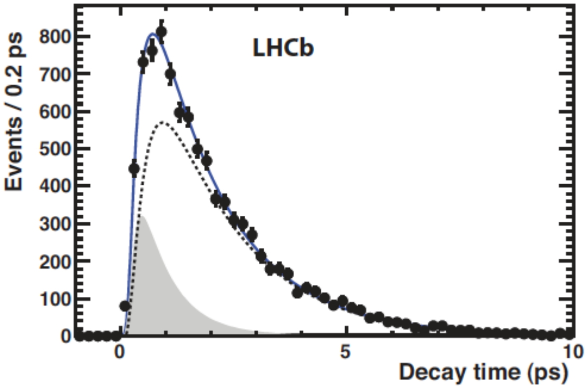
775 < m(ππ) <1550 MeV/c^2

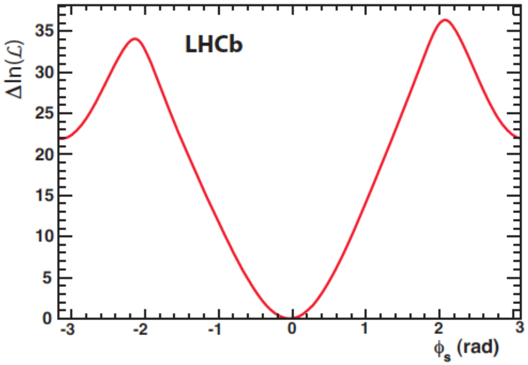


ϕ_s da $B_s \to J/\psi\ \pi^+\pi^-$

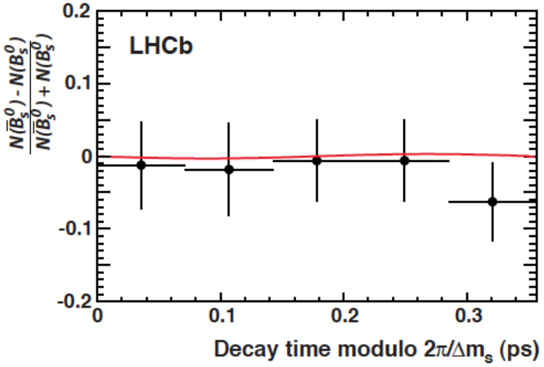
- Fit a massa e tempo.
- Input Γ_s e $\Delta\Gamma_s$ da analisi $B_s \to J/\psi\phi$.

Resonance	Normalized fraction (%)
$f_0(980)$	69.7 ± 2.3
$f_0(1370)$	21.2 ± 2.7
non-resonant $\pi^+\pi^-$	8.4 ± 1.5
$f_2(1270)$, $\Lambda = 0$	0.49 ± 0.16
$f_2(1270)$, $\lvert\Lambda\rvert = 1$	0.21 ± 0.65





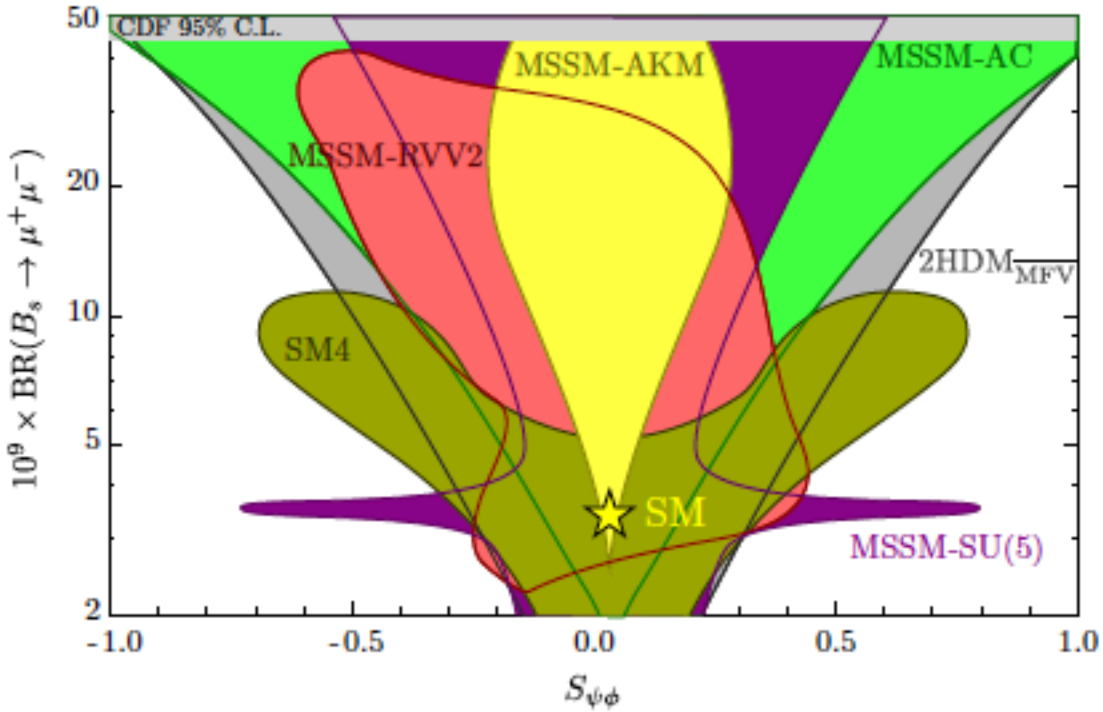
Asimmetria vs decay time



$B_s \to J/\psi\ \pi\pi$ ϕ_s = -0.02 ± 0.17 (stat) ± 0.02 (syst) rad

Combinato con $B_s \to J/\psi\phi$ ϕ_s = -0.002 ± 0.083 (stat) ± 0.027 (syst) rad

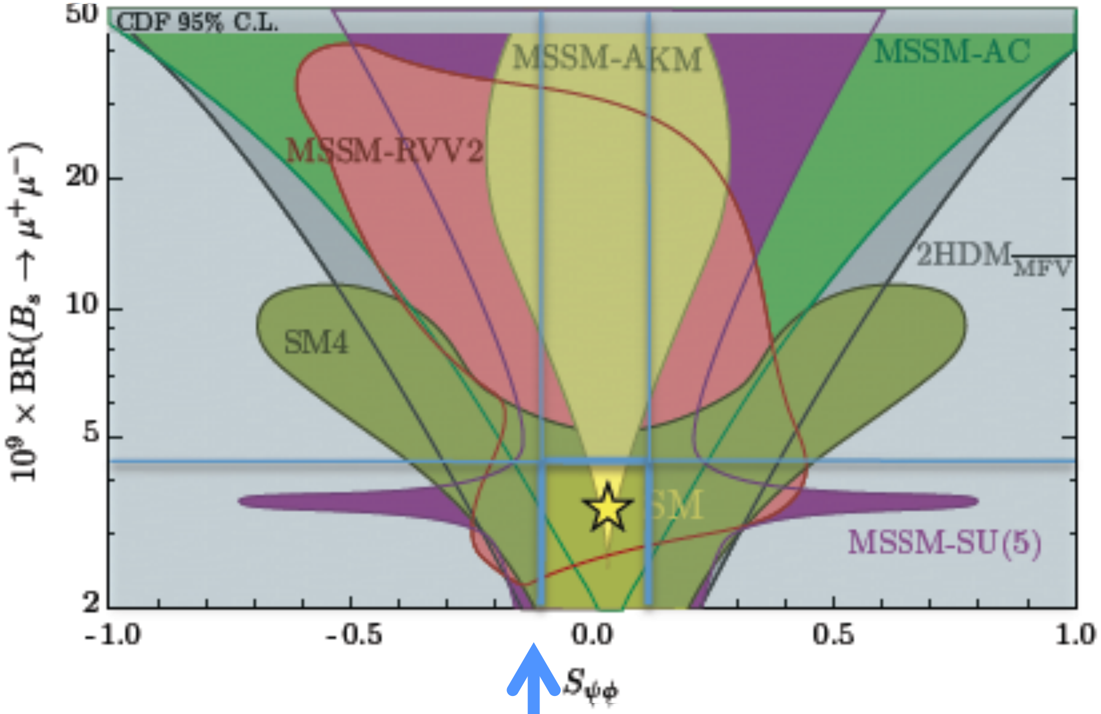
Implicazioni: un esempio



D.Straub [arXiv:1107.0266]. Correlazione tra BR($B_s \rightarrow \mu\mu$) e ϕ_s in diversi modelli: SM4, two-Higgs con fasi flavour-blinded e modelli SUSY.

Implicazioni: un esempio

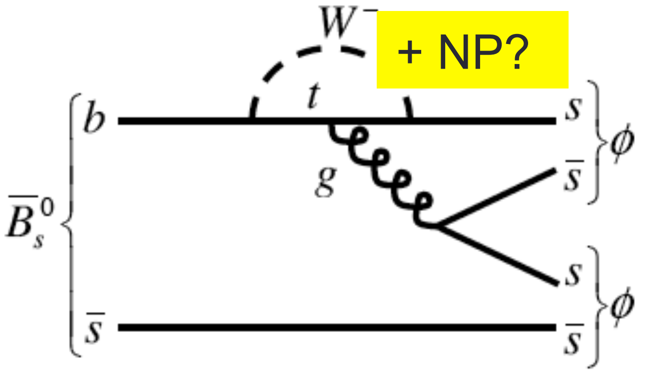
D. Straub [arXiv:1107.0266v1]



LHCb
$B_s \rightarrow \mu\mu$ 1fb^{-1}

LHCb $B_s \rightarrow J/\psi\phi$ 1fb^{-1}

$B_s \to \phi\phi$

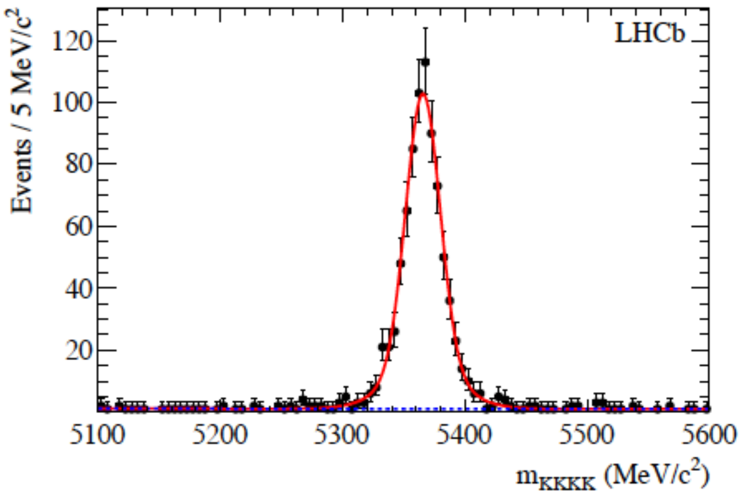


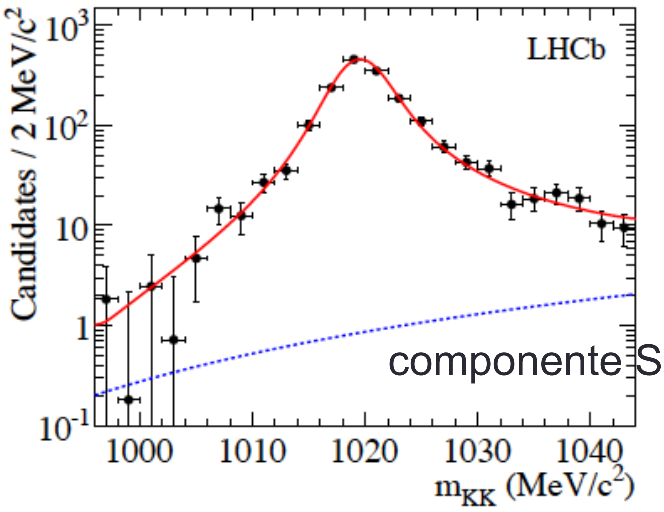
+ NP?

- Canale promettente per ricerca di NP.
- Nel MS la CPV ≈ 0

$$\phi_{B_s \to \phi\phi}^{SM} \approx 2 \arg(V_{ts}^* V_{tb}) - \arg(V_{ts} V_{tb}^*) = 0$$

- Segnale molto pulito ~ 800 eventi.



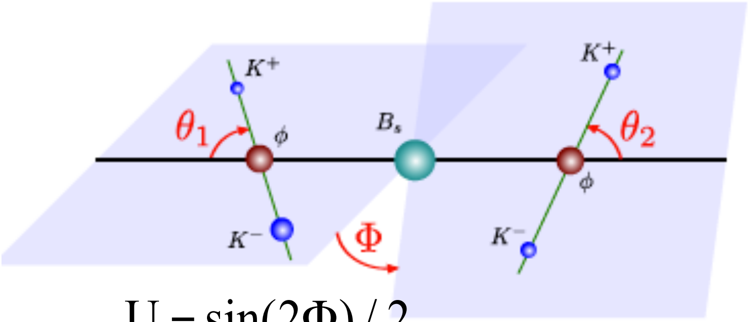


componente S-wave

- Per studio completo di CPV necessaria analisi angolare, con flavour tagging, dipendente dal tempo.
- Smoking gun: asimmetria nei tripli prodotti, funzioni che violano CP.

$B_s \rightarrow \phi\,\phi$

$$A_U = \frac{N(U>0) - N(U<0)}{N(U>0) + N(U<0)}$$ e simile per A_V



$$U = \sin(2\Phi)/2$$

$$V = \pm\sin\Phi \quad \cos\theta_1\cos\theta_2 \geq 0, < 0$$

- $A_{U,V} \neq 0$ → segnale di CPV da NP
- Primo risultato da CDF (PRL 107(2011) :

  $A_U = -0.007 \pm 0.064 \pm 0.018$ $A_V = -0.120 \pm 0.064 \pm 0.016$
- LHCb 1 fb^{-1}

$$\boxed{\begin{aligned} A_U &= -0.055 \pm 0.036\,(\text{stat}) \pm 0.018\,(\text{syst}) \\ A_V &= 0.010 \pm 0.036\,(\text{stat}) \pm 0.018\,(\text{syst}) \end{aligned}}$$

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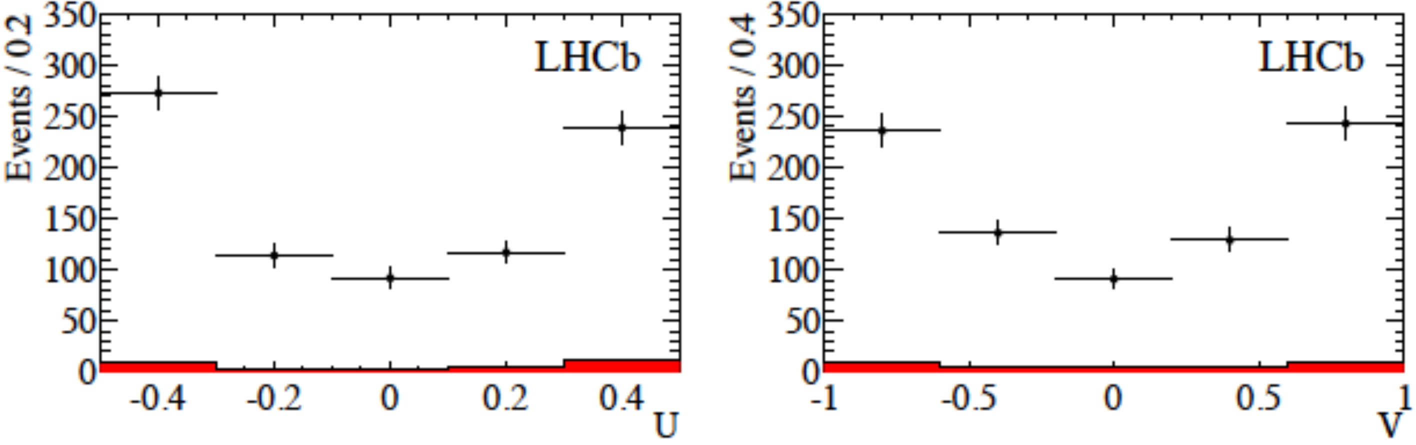


Vite medie in decadimenti in autostati di CP

- Altro modo per accedere a $\Delta\Gamma_s$ e ϕ_s: misura delle vite medie in decadimenti a stati finali puramente CP-pari o CP-dispari.

$$\Gamma(t) \propto (1 - \mathcal{A}_{\Delta\Gamma_s})\, e^{-\Gamma_L t} + (1 + \mathcal{A}_{\Delta\Gamma_s})\, e^{-\Gamma_H t}$$

$$A_{\Delta\Gamma_s} = \frac{-2\eta_f \cos\phi_s}{(1+|\lambda|^2)}$$

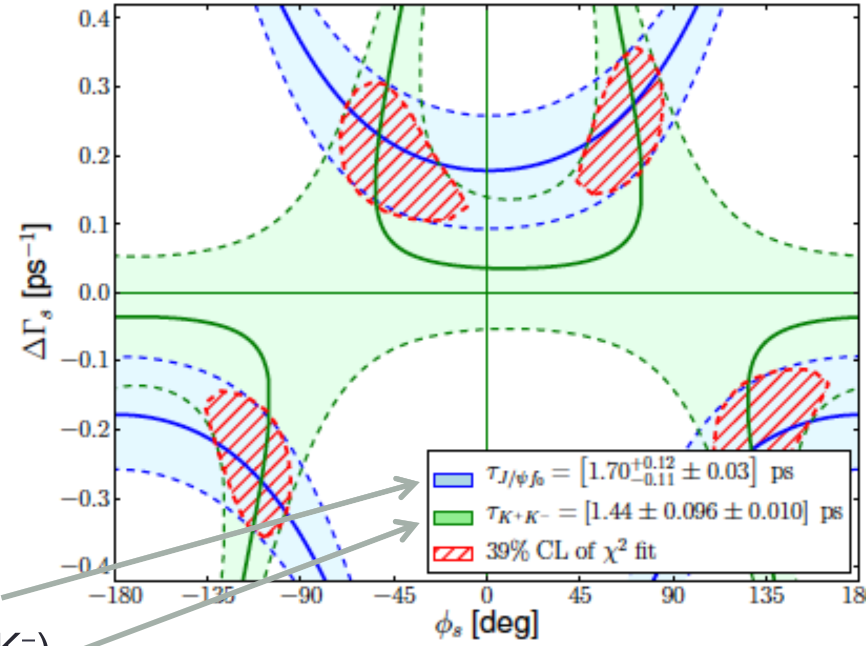
- In assenza di CPV: $\mathcal{A}_{\Delta\Gamma_s} = \pm 1$

$$\lambda = (q/p)(\overline{A}/A)$$

CP-pari:	Γ_L	$B_s \rightarrow K^+K^-$
CP-dispari:	Γ_H	$B_s \rightarrow J/\psi\, f_0$

- Misura con singolo esponziale dà vita media effettiva:

$$\tau_{KK} = \tau_{B_s^0}\left(1 + A_{\Delta\Gamma_s}\frac{\Delta\Gamma_s}{\Gamma_s} + O\left(\left(\frac{\Delta\Gamma_s}{\Gamma_s}\right)^2\right)\right)$$

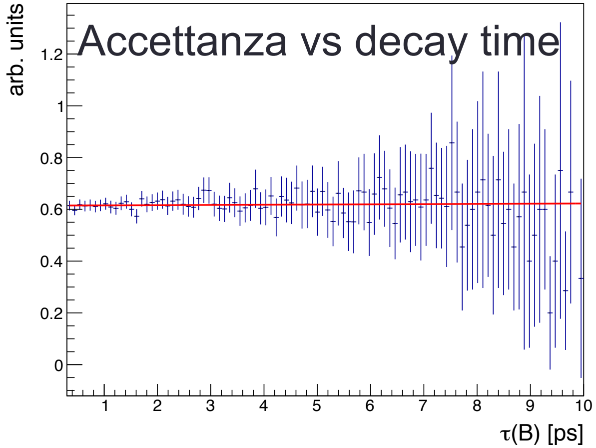


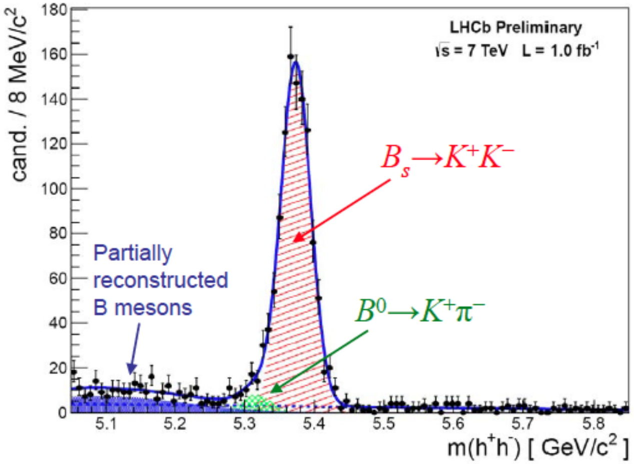
CDF 2011 $\tau(B_s \rightarrow J/\psi f_0)$
LHCb 2010 $\tau(B_s \rightarrow J/\psi K^+K^-)$

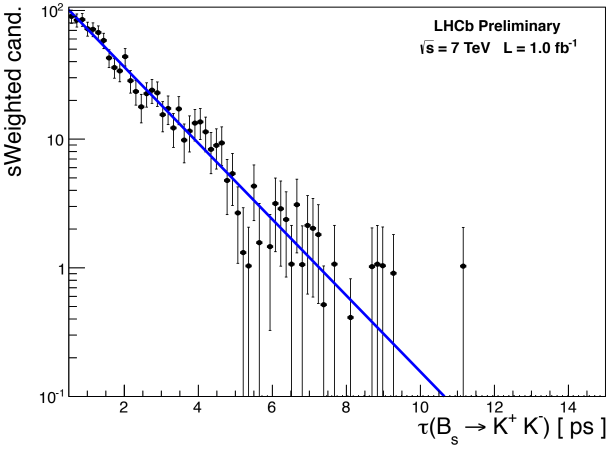
R.Fleischer,R.Knegjens arXiv:1109.5115

Lifetime $B_s \rightarrow K^+K^-$

- Due misure sui dati 2010 con approcci diversi per correggere bias sulla lunghezza di decadimento dovute a trigger e selezione.
- Nuova misura su 1 fb^{-1} minimizza le distorsioni in selezione, usa trigger dedicato.







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$$\tau_{KK} = 1.468 \pm 0.046\ (\text{stat.}) \pm 0.011\ (\text{syst.})\ \text{ps}$$

In accordo con misure precedenti e previsione del MS: $\tau_{KK}^{SM} = (1.390 \pm 0.032)$ ps

Conclusioni

- Grazie agli esperimenti ai collisori adronici la CPV incomincia ad essere misurata anche il settore del B_s. Accordo col MS, ma c' è ancora spazio per la ricerca di NP.
- Misura preliminare di LHCb con $B_s \rightarrow J/\psi\phi$ su 1 fb^{-1}:

$$\Gamma_s = 0.658 \pm 0.005\ (\text{stat}) \pm 0.007\ (\text{syst})\ \ ps^{-1}$$

$$\Delta\Gamma_s = 0.116 \pm 0.018\ (\text{stat}) \pm 0.006\ (\text{syst})\ \ ps^{-1}$$

$$\phi_s = -0.001 \pm 0.101\ (\text{stat}) \pm 0.027\ (\text{syst})\ \ \text{rad}$$

- Combinata con la misura in $B_s \rightarrow J/\psi\pi\pi$:

$$\phi_s = -0.002 \pm 0.083\ (\text{stat}) \pm 0.027\ (\text{syst})\ \ \text{rad}$$

- I primi risultati su $B_s \rightarrow \phi\phi$ aprono buone prospettive per le ricerche ad alta luminosità.
- La misura precisa delle vite medie in diversi canali aggiungerà ulteriori informazioni su $\Delta\Gamma_s$ e ϕ_s

Backup

ϕ_s da $B_s \to J/\psi\phi$ - Decay rate

$$\frac{\mathrm{d}^4\Gamma(B_s^0 \to J/\psi\,\phi)}{\mathrm{d}t\;\mathrm{d}\Omega} \propto \sum_{k=1}^{10} h_k(t)\; f_k(\Omega)$$

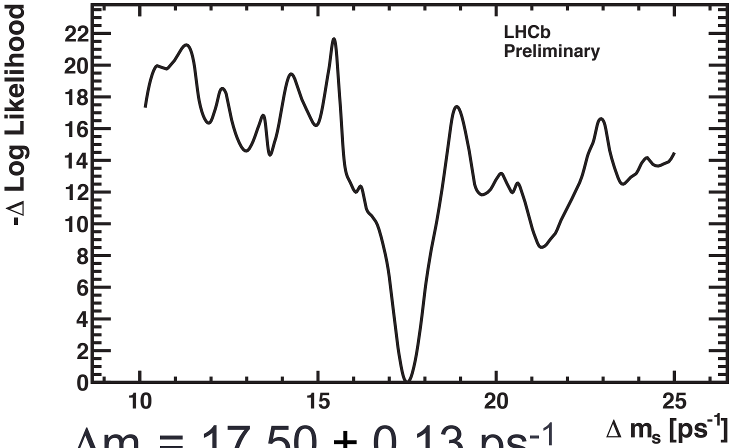
$$\begin{aligned} h_k(t) \;=\; & N_k e^{-\Gamma_s t}\,\left[c_k \cos(\Delta m_s t) + d_k \sin(\Delta m_s t)\right. \\ & \left. + a_k \cosh\left(\tfrac{1}{2}\Delta\Gamma_s t\right) + b_k \sinh\left(\tfrac{1}{2}\Delta\Gamma_s t\right)\right] \end{aligned}$$

k	$f_k(\theta,\psi,\varphi)$	N_k	a_k	b_k	c_k	d_k
1	$2\cos^2\psi\left(1-\sin^2\theta\cos^2\phi\right)$	$\|A_0(0)\|^2$	1	$-\cos\phi_s$	0	$\sin\phi_s$
2	$\sin^2\psi\left(1-\sin^2\theta\sin^2\phi\right)$	$\|A_\|(0)\|^2$	1	$-\cos\phi_s$	0	$\sin\phi_s$
3	$\sin^2\psi\sin^2\theta$	$\|A_\perp(0)\|^2$	1	$\cos\phi_s$	0	$-\sin\phi_s$
4	$-\sin^2\psi\sin 2\theta\sin\phi$	$\|A_\|(0)A_\perp(0)\|$	0	$-\cos(\delta_\perp-\delta_\|)\sin\phi_s$	$\sin(\delta_\perp-\delta_\|)$	$-\cos(\delta_\perp-\delta_\|)\cos\phi_s$
5	$\frac{1}{2}\sqrt{2}\sin 2\psi\sin^2\theta\sin 2\phi$	$\|A_0(0)A_\|(0)\|$	$\cos(\delta_\|-\delta_0)$	$-\cos(\delta_\|-\delta_0)\cos\phi_s$	0	$\cos(\delta_\|-\delta_0)\sin\phi_s$
6	$\frac{1}{2}\sqrt{2}\sin 2\psi\sin 2\theta\cos\phi$	$\|A_0(0)A_\perp(0)\|$	0	$-\cos(\delta_\perp-\delta_0)\sin\phi_s$	$\sin(\delta_\perp-\delta_0)$	$-\cos(\delta_\perp-\delta_0)\cos\phi_s$
7	$\frac{2}{3}(1-\sin^2\theta\cos^2\phi)$	$\|A_s(0)\|^2$	1	$\cos\phi_s$	0	$-\sin\phi_s$
8	$\frac{1}{3}\sqrt{6}\sin\psi\sin^2\theta\sin 2\phi$	$\|A_s(0)A_\|(0)\|$	0	$-\sin(\delta_\|-\delta_S)\sin\phi_s$	$\cos(\delta_\|-\delta_S)$	$-\sin(\delta_\|-\delta_S)\cos\phi_s$
9	$\frac{1}{3}\sqrt{6}\sin\psi\sin 2\theta\cos\phi$	$\|A_s(0)A_\perp(0)\|$	$\sin(\delta_\perp-\delta_S)$	$\sin(\delta_\perp-\delta_S)\cos\phi_s$	0	$-\sin(\delta_\perp-\delta_S)\sin\phi_s$
10	$\frac{4}{3}\sqrt{3}\cos\psi(1-\sin^2\theta\cos^2\phi)$	$\|A_s(0)A_0(0)\|$	0	$-\sin(\delta_0-\delta_S)\sin\phi_s$	$\cos(\delta_0-\delta_S)$	$-\sin(\delta_0-\delta_S)\cos\phi_s$

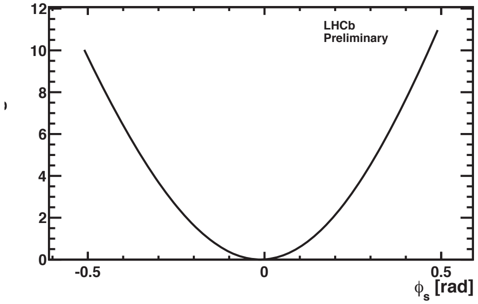
δ_0 set to 0

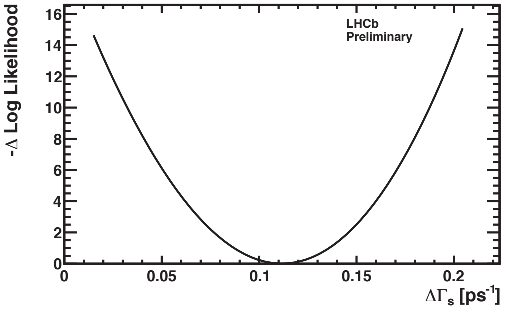
LHCb: ϕ_s da $B_s \rightarrow J/\psi\phi$

Source	Γ_s [ps^{-1}]	$\Delta\Gamma_s$ [ps^{-1}]	$A_\perp^2$	A_0^2	F_S	$\delta_\parallel$ [rad]	$\delta_\perp$ [rad]	δ_s [rad]	ϕ_s [rad]
Description of background	0.0010	0.004	-	0.002	0.005	0.04	0.04	0.06	0.011
Angular acceptances	0.0018	0.002	0.012	0.024	0.005	0.12	0.06	0.05	0.012
t acceptance model	0.0062	0.002	0.001	0.001	-	-	-	-	-
z and momentum scale	0.0009	-	-	-	-	-	-	-	-
Production asymmetry (± 10%)	0.0002	0.002	-	-	-	-	-	-	0.008
CPV mixing & decay (± 5%)	0.0003	0.002	-	-	-	-	-	-	0.020
Fit bias	-	0.001	0.003	-	0.001	0.02	0.02	0.01	0.005
Quadratic sum	0.0066	0.006	0.013	0.024	0.007	0.13	0.07	0.08	0.027



Δm_s = 17.50 ± 0.13 ps^{-1}



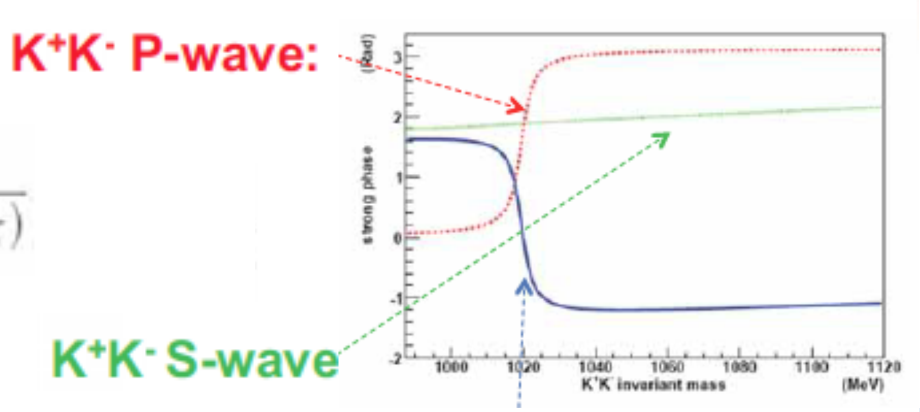


ϕ_s da $B_s \rightarrow J/\psi\phi$ - Risultati D0

$$
\begin{aligned}
\bar{\tau}_s &= 1.443^{+0.038}_{-0.035}\ \mathrm{ps}, & \Delta\Gamma_s &= 0.163^{+0.065}_{-0.064}\ \mathrm{ps}^{-1}, \\
\phi_s^{J/\psi\phi} &= -0.55^{+0.38}_{-0.36}, & |A_0|^2 &= 0.558^{+0.017}_{-0.019}, \\
|A_{\parallel}|^2 &= 0.231^{+0.024}_{-0.030}, & \delta_{\parallel} &= 3.15 \pm 0.22, \\
\cos(\delta_{\perp} - \delta_s) &= -0.11^{+0.27}_{-0.25}, & F_S &= 0.173 \pm 0.036,
\end{aligned}
\tag{13}
$$

LHCb: Determinazione del segno di $\Delta\Gamma_s$

$$BW(m_{KK}) = \frac{F_r F_D}{m_\phi^2 - m_{KK}^2 - i m_\phi \Gamma(m_{KK})}$$

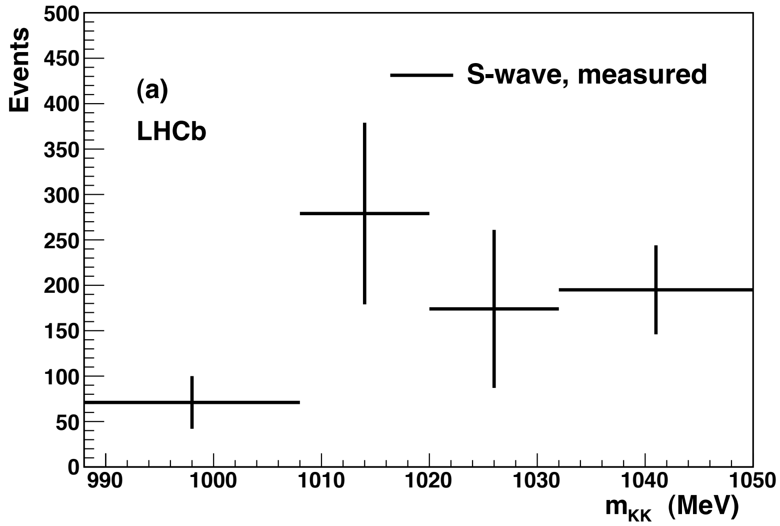


K^+K^- P-wave:

K^+K^- S-wave

Phase difference between S- and P-wave amplitudes

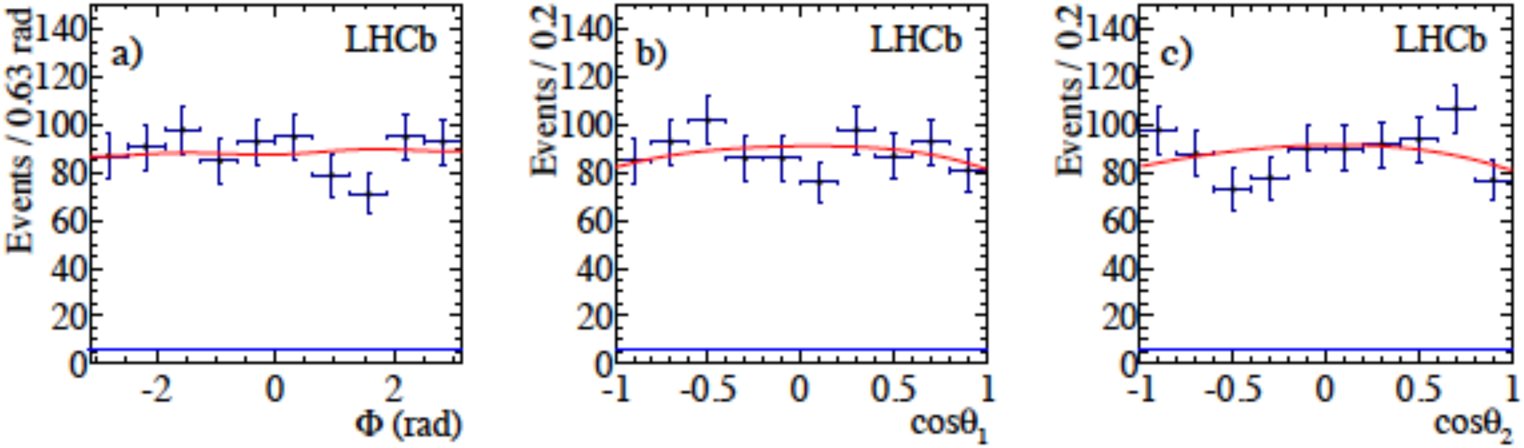
Frazione di onda S piccola ma misurabile



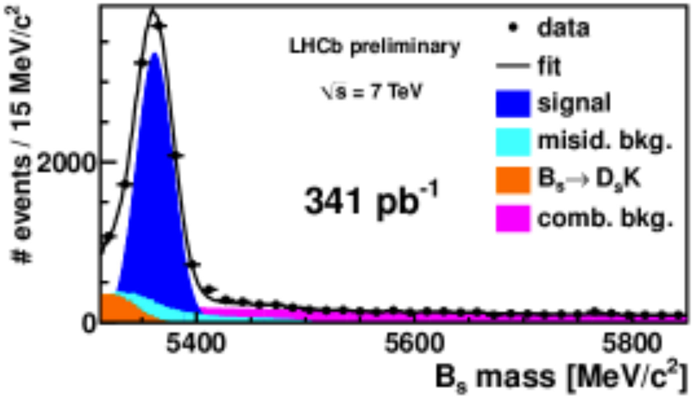
Parameter	Solution I	Solution II
ϕ_s (rad)	0.167 ± 0.175	2.975 ± 0.175
$\Delta\Gamma$ (ps^{-1})	0.120 ± 0.028	-0.120 ± 0.028
$F_{S;1}$	0.283 ± 0.113	0.283 ± 0.113
$F_{S;2}$	0.061 ± 0.022	0.061 ± 0.022
$F_{S;3}$	0.044 ± 0.022	0.044 ± 0.022
$F_{S;4}$	0.269 ± 0.067	0.269 ± 0.067
$\delta_{S\perp;1}$ (rad)	$-0.46\,^{+0.35}_{-0.42}$	$-2.68\,^{+0.42}_{-0.35}$
$\delta_{S\perp;2}$ (rad)	$-2.92\,^{+0.15}_{-0.13}$	$-0.22\,^{+0.13}_{-0.15}$
$\delta_{S\perp;3}$ (rad)	$-3.25\,^{+0.16}_{-0.18}$	$0.11\,^{+0.18}_{-0.16}$
$\delta_{S\perp;4}$ (rad)	$-4.11\,^{+0.28}_{-0.43}$	$0.97\,^{+0.43}_{-0.28}$

LHCb: $B_s \to \phi\phi$ Amplitude analysis

$$\begin{aligned}
|A_0|^2 &= 0.365 \pm 0.022\,(\mathrm{stat}) \pm 0.012\,(\mathrm{syst}) \\
|A_\perp|^2 &= 0.291 \pm 0.024\,(\mathrm{stat}) \pm 0.010\,(\mathrm{syst}) \\
|A_\parallel|^2 &= 0.344 \pm 0.024\,(\mathrm{stat}) \pm 0.014\,(\mathrm{syst}) \\
\cos(\delta_\parallel) &= -0.844 \pm 0.068\,(\mathrm{stat}) \pm 0.029\,(\mathrm{syst})
\end{aligned}$$

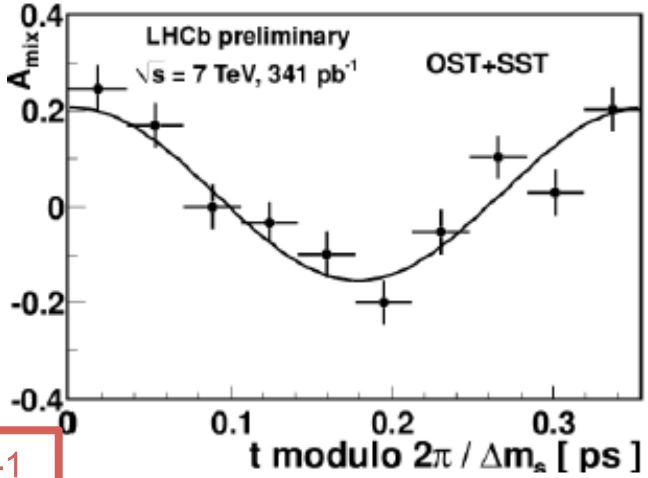


LHCb: B_s mixing in $B_s \rightarrow D_s^- \pi^+$



LHCb in 0.34 fb^{-1}

decay mode	# signal candidates
$B_s^0 \rightarrow D_s^-(\phi\pi^-)\pi^+$	4371 ± 91
$B_s^0 \rightarrow D_s^-(K^+K^-)\pi^+$	2910 ± 89
$B_s^0 \rightarrow D_s^-\pi^+$ non-resonant	1908 ± 74



- Average time resolution: ~44 fs
- Flavour tagging (SSK not yet optimized):
  εD^2_{OS} = 3.1 ± 0.8 %
  εD^2_{SSK}= 1.2 ± 0.4 %

Δm_s = 17.725 ± 0.041 (stat) ± 0.026 (syst) ps^{-1}

LHCb 2010 su 34 pb^{-1}
Δm_s =17.63+0.11+0.01 ps^{-1}

LHCb, PLB 709 (2012) 177

CDF 1 fb^{-1} :

$$\Delta m_s = 17.77 \pm 0.10\ (\text{stat.})\ \pm 0.07\ (\text{syst.})\ \text{ps}^{-1}$$