pp cross-sections: a QCD model compared with LHC data

+ checking asymptotia in elastic scattering

Giulia Pancheri-INFN Frascati

With A. Grau, S. Pacetti and Y.N. Srivastava

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Total cross section data before 2011

\[ \sigma_{\text{tot}}(\text{mb}) \]

\[ \times \text{ad hoc factor} \]

\[ \propto \frac{1}{\alpha_{QED}} \]

Soon new data from Auger

LHC

Our model
TOTEM measurements in 2011

\[ \sigma_{\text{total}} = 98.3 \pm 3 \text{mb} \]

\[ \frac{d\sigma}{dt} \]


Interesting?

- Is asymptotia reached? i.e. is the Froissart bound (FB) for sigma total saturated? Why would this be interesting?

1. Because saturation of FB could exclude power-like behaviour as from hidden extra dimensions [Block Halzen 2012, Srivastava et al, 2011]

2. Or data could hint to new baryonic interactions at 10-100 TeV and thus solve problems with cosmic rays composition based on current \( \sigma_{total} \) extrapolations [Piran, april 2012]

3. Because there is a connection between Froissart bound and confinement which the total cross-section can investigate

- Why the dip in pp elastic differential cross-section?
The total cross-section: **confinement** and **deconfinement** at work

\[ \sigma_{total} = \sigma_{elastic} + \sigma_{inelastic} \]

A **confined** system: quarks and gluons remain inside the original hadrons even at high energy

**Central production:** quarks and gluons scatter away and then hadronize

**Fully deconfined**

**deconfined**

**Single and double diffractive Production:** quarks and gluons remain “close” to original hadrons and then hadronize
Our QCD model: a formalism to study confinement in total cross-section

We have developed a model green band in which connects to the study of ultra soft gluon coupling

Where one can expect confinement effects to arise
In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering (mini-jets) and restores the Froissart bound.

\[ \sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 \left[ 1 - e^{-C(s)}e^{-(b\Lambda)^{2p}} \right] \]

\[ \sigma_{tot}(s) \rightarrow [\varepsilon \ln(s)]^{(1/p)} \]

\[ \frac{1}{2} < p < 1 \]
The model

- Start with eikonal representation
  \[ \sigma_{\text{tot}}(s) = 2 \int (d^2 b) [1 - e^{-\tilde{n}(b,s)/2}] \quad \text{Re} \chi \approx 0 \]

- Low and high energy component
  \[ \tilde{n}(b, s) = \tilde{n}_{\text{low}}(b, s) + \tilde{n}_{\text{high}}(b, s) \]

- Low energy component is parametrized with No rising term

- High energy (rising) component is from PQCD
  \[ \tilde{n}_{\text{high}} = A(b, s) \sigma_{\text{jet}}(s) \]
  Minijets to get the rise

\[ p_{t,\text{parton-out}} \geq p_{t,\text{min}} \approx 1 \text{ GeV} \]

- To tame the rise \( A(b, s) \) is obtained from with integration down into the infrared with an ansatz for infrared behaviour
  \[ \alpha_{\text{eff}}(k_t \to 0) \sim k_t^{-2p} \]

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Cartoon view of the model for $\sigma_{\text{total}}$

- QCD minijets with PDFs from CERNLIB to drive the rise
- Soft Gluon $k_t$-resummation (ISR) in the infrared main original ingredient of our model
- Multiple scattering (in Eikonal representation to implement unitarity)
Issues in a QCD mini-jet description

What generates the rise? Low-x parton collisions

A cut off obtained by [embedding into the eikonal] the acollinearity induced by IR kt-emission

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What tames the rise into to a Froissart-like behavior?

A cut off obtained by [embedding into the eikonal] the acollinearity induced by IR kt-emission

Soft gluon emission introduces acollinearity

Acollinearity reduces the collision cross-section as partons do not scatter head-on any more, i.e. the gluon cloud is too thick for partons to see each other: gluon saturation
We model the impact parameter distribution as the Fourier-transform of ISR soft $k_t$ distribution and thus obtain a cut-off at large distances: Froissart bound?

$$A_{BN}(b, s) = N \int d^2 K_\perp e^{-iK_\perp \cdot b} \frac{d^2 P(K_\perp)}{d^2 K_\perp} \frac{e^{-h(b, q_{max})}}{\int d^2 b \ e^{-h(b, q_{max})}}$$

$$h(b, E) = \frac{16}{3\pi} \int_0^{q_{max}} \frac{dk_t}{k_t} \alpha_{eff}(k_t) \ln\left(\frac{2q_{max}}{k_t}\right)[1 - J_0(bk_t)]$$

$$\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}$$

$$A_{BN}(b, s) \sim e^{-(b \Lambda)^{2p}}$$

\(q_{max}\) Fixed by single gluon emission kinematics\(^\dagger\)
the large-s limit

\[
\sigma_{total} \to 2\pi \int db^2 [1 - e^{-C(s)e^{-(bq)^2p}}]
\]

\[C(s) = (s/s_0)^\varepsilon \sigma_1\]

\[A(b, s) \propto e^{-(bq)^2p}\]

Mini-jets Ultra-soft gluons effects

\[\sigma_T \approx \frac{2\pi}{\Lambda^2} [\varepsilon \ln \frac{s}{s_0}]^{1/p}\]

\[\sim \ln^2 s \quad p = 1/2\]

\[\sim \ln s \quad p = 1\]
The eikonal 2-component formulation has problems

- Ok for the \textit{sigma total} but

Sigma \textit{elastic} and sigma \textit{inelastic} get mixed up: diffraction, single and double, goes into the elastic [GP et al PRD84]

- Need for a different formalism [Lipari\&Lusignoli 2009]

- Or further understanding

- Turn to the elastic differential to see what happens
The return of the dip: $pp$ vs $p\bar{p}$

A simpler system

Complicated

By resonances
A very simple model from Barger and Phillips in 1973

\[ A(s, t) = i\sqrt{A(s)}e^{\frac{1}{2}B(s)t} + \sqrt{C(s)}e^{i\phi(s)}e^{\frac{1}{2}D(s)t} \]

\[ \frac{d\sigma}{dt} = A(s)e^{B(s)t} + C(s)e^{D(s)t} + 2\sqrt{A(s)}\sqrt{C(s)}e^{\frac{(B(s)+D(s))t}{2}} \cos \phi \]

five s-dependent real parameters, A B C D \[ \phi \]

How does it work with LHC TOTEM data?
How to describe both the diffraction peak and the tail of TOTEM data: models for the tail

- Model 1: two exponentials
- TOTEM $|t|^{-n}$ with $n = 7.8 \pm 0.3^{\text{stat}} \pm 0.1^{\text{syst}}$
- Donnachie and Landshoff (1996) $|t|^8$

- Model 2: $A(s, t) = i\left[\sqrt{A(s)}e^{Bt/2} + \frac{\sqrt{C(s)}}{(-t + t_0)^4} e^{i\phi}\right]$

The two exponential is still the best
How to check asymptotia?

$$\mathcal{F}(s, t) = i \int_{0}^{\infty} (b b b) J_0(b \sqrt{-t}) \left[ 1 - e^{2i\delta_R(b, s)} e^{-2\delta_I(b, s)} \right]$$

$$\sigma_{total}(s) = 4\pi \Im m \mathcal{F}(s, 0)$$

• Two asymptotic sum rules in impact parameter space [EPJC 2005]

\[
\left( \frac{1}{2} \right) \int_{-\infty}^{0} (dt) \Im m \mathcal{F}(s, t) \to 1; \quad as \ s \to \infty. \quad S_1
\]

\[
\int_{-\infty}^{0} (dt) \Re e \mathcal{F}(s, t) \to 0; \quad as \ s \to \infty. \quad S_0
\]
BP model allows easy check of the sum rules

- With parameters from fit

\[ s_1 = \sqrt{\frac{A}{1 + \rho^2}} \frac{1}{\sqrt{\pi}B} + \frac{\sqrt{C}}{\sqrt{\pi}D} \cos \phi = 0.94 \]  
  at LHC7

- At ISR 53 GeV  
  \[ s_1 = 0.75 \]
To satisfy both sum rules, add a real part to the first term

\[ s \leftrightarrow u \]  Use our minijet model with soft gluon resummation with $0.66<p<0.77$ PLB08

\[ A(s, 0) \rightarrow i \left[ \ln\left(\frac{s}{s_0}e^{-i\pi/2}\right) \right]^{1/p} \]

\[ = i\left(\left(\ln\left(\frac{s}{s_0}\right) - i\pi/2\right)\right)^{1/p} \]

\[ \frac{\Re A(s, 0)}{\Im m A(s, 0)} \rightarrow \frac{\pi}{2\ln\left(\frac{s}{s_0}\right)} = 0.134 \div 0.115 \]

\[ s_0 \sim 0.05 \text{ LHC7} \]
TOTEM: the slope of forward peak

\[ \frac{d\sigma}{dt} = \frac{d\sigma}{dt}\bigg|_{t=0} e^{B_{exp}t} \]

- The slope actually changes as one measures away from \( t=0 \) to the dip region

- \( \sim 20 \text{ GeV}^{-2} \) at small \( 0.02<-t<0.33 \)

- \( \sim 23 \text{ GeV}^{-2} \) at \(-t\) before the dip

Fig. 3: The measured differential cross-section \( d\sigma/dt \). The superimposed fits and their parameter values are discussed in the text.
Slope from data

Ryskin 2012 : log²s behaviour?
How about the slope in the two exponential model

\[ B_{eff}(s, t) = \frac{d \ln \frac{d\sigma}{dt}}{dt} \]

\[ B_{eff}(s, t) = \frac{ABe^{Bt} + CD e^{Dt} + \sqrt{A} \sqrt{C} (B + D) e^{(B+D)t/2} \cos \phi}{A e^{Bt} + C e^{Dt} + \sqrt{A} \sqrt{C} e^{(B+D)t/2} \cos \phi} \]
Conclusion

• A model with minijets and soft gluon resummation is able to describe the total cross-section from 5 GeV to cosmic rays energies

• A model with two exponential and a phase is well suited to describe the dip structure at LHC as well as the forward diffraction peak and should be used to parametrize future data at 8 TeV or beyond

• The connections between these two models is still under study
Dip or no dip?

• Before and after the dip the two processes $pp$ and $pp$ should be described by the same physics.

• At the dip the basic amplitude is almost zero (5 orders of magnitude lower in the cross-section) so the leftovers from Regge exchange, present only in $pp$, fill the dip.
R.M. Godbole, A. Grau, G.P. Y.N. Srivastava, +A. Achilli, +A. Corsetti + O. Shekhovtsova

- Phys. Rev D 2011
- Phys. Lett. 2010
Some details

Mini-jets

\[
\sigma_{jet}^{AB}(s; p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_is)}^{1} dx_2 \\
\sum_{i,j,k,l} f_i|_A(x_1, p_t^2)f_j|_B(x_2, p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}.
\]

DGLAP evolved

Which value of \( p_{tmin} \)?
Which densities?

Parametrize data choosing PDF and \( p_{tmin} \) to catch the early rise of \( \sigma_{total} \)
Mini-jets drive the rise of $\sigma_{total}$

$$\sigma_{jet}^{AB}(s, p_{tmin}) = \int_{p_{tmin}}^{\sqrt{s}/2} dp_t \int_0^1 dx_1 \int_0^1 dx_2 \times \sum_{i,j,k,l} f_{i|A}(x_1, p_t^2) f_{j|B}(x_2, p^2_t) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}$$

$p_{tmin} \sim 1 \div 2$ GeV

DGLAP evolved PDF

Parton-parton x-sections: $\text{parton}_i + \text{parton}_j \rightarrow \text{parton}_k(p_t) + \text{parton}_i(-p_t)$
Building $\sigma_{\text{total}}$ total

$$\sigma_{\text{total}} = 2 \int d^2 b [1 - e^{-\Im m \chi(b, s)} \cos \Re e \chi(b, s)]$$

$$\bar{n}(b, s) = 2 \Im m \chi(b, s) \sim A(b) \sigma(s)$$

$\Re e \chi(b, s) \approx 0$

Two component simplest model

$$\bar{n}(b, s) = \bar{n}_{\text{soft}}(b, s) + \bar{n}_{\text{hard}}(b, s)$$

$$\bar{n}_{\text{soft/hard}}(b, s) = A_{\text{soft/hard}}(b, s) \sigma_{\text{soft/hard}}(s)$$

Overlap function
What makes the cross-section rise?

Mini-jets are responsible for the rise of the total cross-section

DGLAP Parton densities

\[ \sqrt{s} \]

\[ \sqrt{s_{\text{jet-jet}}} \]
One component **missing** in the mini-jet picture is **soft gluon emission** from the initial state to break the collinearity and reduce the parton-parton cross-section.
Eikonal models: b-distribution can quench the rise

\[ n_{\text{hard-minijets}}(b) \approx A(b, s) \sigma_{\text{jet}}(s, p_{t\text{min}}) \]

How to choose it: Form factors?
Choice of **densities** for mini-jet x-section

Because we use resummation to access large distance behaviour

- LO PDFs are used, to avoid double counting the most important contribution (small $kt$) to observables like $\sigma_{tot}$
- LO: GRV, MRST, CTEQ
- For illustration purposes: GRV
- Bands are also presented with GRV and MRST
- We are working to include other densities
The single soft gluon integration limit can be obtained from kinematics

\[ q_{max} = \frac{\sqrt{\hat{s}}}{2} \left( 1 - \frac{Q^2}{\hat{s}} \right) \]
The model at work

- minijets
- Soft gluon upper limit
- b-distribution from Soft gluons
- Eikonalized expression
and the large-s limit

\[2 \sum m \chi = n_{soft} + n_{hard-minijets}\]

\[\sigma_{total} = 2 \int d^2 \vec{b} \left[ 1 - e^{-n_{soft}-n_{hard-minijets}} \right]\]

\[n_{hard-minijets}(b) \approx A(b, s) \sigma_{jet}(s, p_{tmin}) \gg n_{soft}\]

\[\sigma_{total} \rightarrow 2\pi \int db^2 \left[ 1 - e^{-C(s)e^{-\left(bq\right)^2}} \right]\]

\[C(s) = (s/s_0)\varepsilon \sigma_1\]

Ultra-soft gluons effects
At very large energy: from power law to log behaviour

\[ \sigma_T(s) \approx \frac{2\pi}{p} \frac{1}{\Lambda^2} \int_0^\infty du u^{1/p-1} \left[1 - e^{-C(s)e^{-u}}\right] \]

\[ u = (\Lambda b)^{2p} \]

\[ I(u, s) = 1 - e^{-C(s)e^{-u}} \] has the limits

\[ I(u, s) \to 1 \text{ at } u = 0 \]

\[ I(u, s) \to 0 \text{ as } u = \infty \]

\[ \sigma_T \approx \frac{2\pi}{\Lambda^2} \left[ \varepsilon \ln \frac{s}{s_0} \right]^{1/p} \]

- \[ \sim \ln^2 s \quad p = 1/2 \]
- \[ \sim \ln s \quad p = 1 \]
A general scheme for various processes

- Start with PDF for the chosen process
  - Proton-proton, pion-proton, pion-pion, photons (nuclear matter, heavy ions)
  - Calculate mini-jet basic cross-section, quark-antiquark, gluon-gluon (dominant), quark-gluon
  - Calculate $q_{\text{max}}(s)$ for soft emission

- Fix $p$ (singularity) for one process, say proton-proton
- Calculate $A(b, q_{\text{max}}(s))$
- Parametrize $\tilde{n}_{\text{soft}}(b, s)$
- Eikonalize and integrate