Calcoli di precisione ed incertezze teoriche per le osservabili elettrodeboli ai collisori adronici

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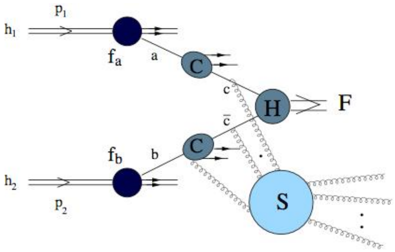
IFAE 2012
Ferrara, 12.04.2012

Outline

1. Introduction
2. NNLO calculations: methods and associated uncertainties
3. Numerical resummation: methods and associated uncertainties
4. Analytical resummation: methods and associated uncertainties
5. Impact of PDF uncertainties on W mass measurements

Hadronic cross sections in perturbative QCD



- h_1, h_2 = initial state hadrons (with momenta p_1, p_2)
- f_a, f_b = parton distribution functions
- C = coefficient functions (partonic splitting)
- H = perturbatively computed partonic event
- F = final state particle(s)
- S = resummation of soft radiation from incoming partons

→ Precise predictions depend on good knowledge of f,C,H *and* S!

K-factor

- LO cross sections suffer from large scale uncertainties
  - → σ^{part} does not depend on μ_R, μ_F
  - → pdf and α_S dependence are not balanced
  - → LO gives just the order of magnitude
- Reliable *central values* start at NLO

$$K = \frac{\sigma_{HO}(pp \to H + X)}{\sigma_{LO}(pp \to H + X)}$$

  - → α_S and pdfs have to be consistently evaluated at HO and LO (otherwise K artificially large, since α_S(NLO) < α_S(LO))
  - → NLO error *not reliable*
- NNLO can give a realistic estimate of theoretical uncertainty

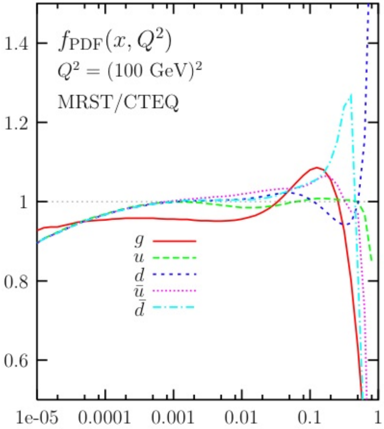
Scale dependence

- Usually one fixes a ”natural” scale μ_0 (typically the one that allows to absorb large logarithms...)
- Then μ_R, μ_F are independently or collectively varied within

$$\frac{\mu_0}{a} \leq \mu_F, \mu_R \leq \mu_0 a$$

- Dependence on μ_R, μ_F → evaluation of theoretical uncertainty?
  - → The narrower the uncertainty band is, the smaller the HO corrections are expected to be (not always true!)
  - → In principle the scale uncertainty should be reduced when going to higher orders (not always true!)
  - → BUT remember that all this is unphysical and there is no rigorous way to estimate the theoretical uncertainty other than performing the higher-order calculation!

Parton Distribution Functions



- Differences between pdfs arise from
  - → choice of data points
  - → theoretical assumptions made for the fit
  - → choice of tolerance used to define the error in the fit
- Low-x ($x<10^{-3}$) and high-x ($x>0.7$) regions are critical: uncertainties of a few tens of %
- Intermediate-x region more reliable: uncertainties of a few %
- No clear separation between regions in the gluon case

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A NNLO calculation

- For a general $2 \to n$ process we need
  - Two-loop amplitude for $2 \to n$
  - One-loop amplitude for $2 \to n+1$
  - Tree-level amplitude for $2 \to n+2$
- Each term has its own singularities
  - Ultraviolet (removed by renormalization)
  - Infrared (have to cancel among each other)

→ Much more difficult than NLO cancellation!

1. Fully inclusive quantities
   - analytical computation of contributions is possible
   - explicit cancellation of singularities
2. Fully exclusive quantities (real world!)
   - IR singularity structure at NNLO understood
     [Catani,Grazzini;Campbell,Glover;Bern,DelDuca,Kilgore,Schmidt;
     Kosower,Uwer;Sterman,Tejeda-Yeomans]
   - numerical integration still very difficult

   → **Sector Decomposition**

   → **Subtraction Method**

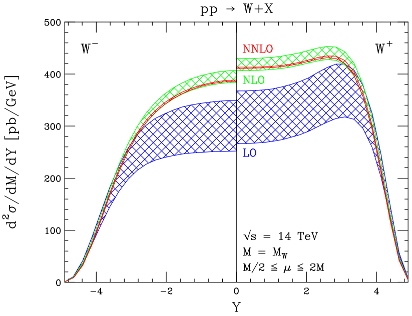
Sector Decomposition

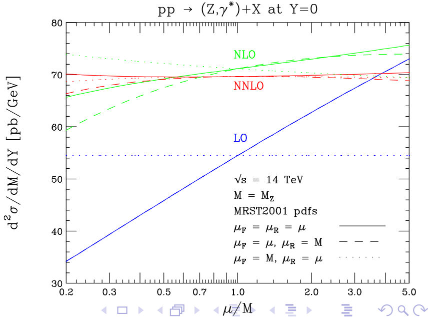
"Split the integration region into sectors, each containing a single singularity, and explicit the pole by expanding it into distributions"

`Binoth,Heinrich[00,04];Anastasiou,Melnikov,Petriello[04]`

AMP developed a fully automated procedure to compute pole coefficients and finite terms and applied it to

Higgs (*FEHiP*, 2005), W/Z (*FEWZ*, 2006)





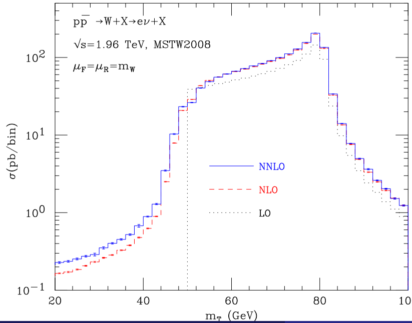
Subtraction Method

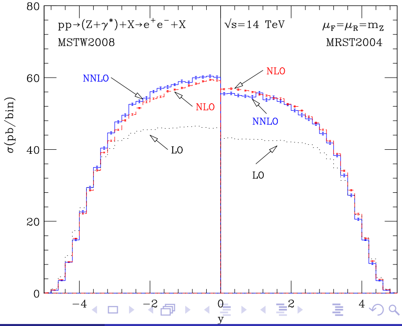
"Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton"

`(NNLO):Kosower[03,05];Weinzierl[03];Frixione,Grazzini[04];`
`Gehrmann,Glover[05];Somogyi,Trocsanyi,DelDuca[05,07]`

Applications: *HNNLO* (2007), *DYNNLO* (2009), *2γNNLO* (2011)

H:Catani,Grazzini[07];W,Z,$\gamma\gamma$:Catani,Cieri,DeFlorian,Ferrera,Grazzini[09,11]

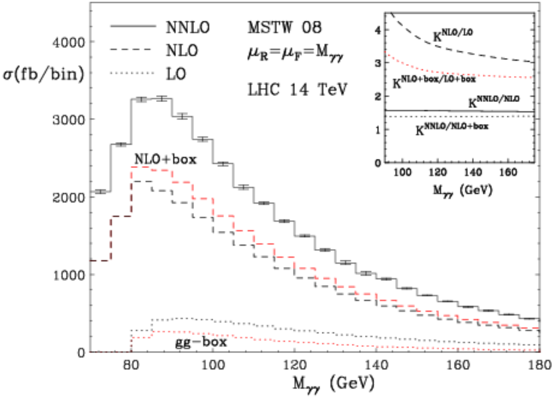




NNLO uncertainty

Differences between the two prescriptions:
at the level of statistical precision

Theoretical uncertainty = PDF and scale variation, BUT be careful!



Catani,DeFlorian,Ferrera,Grazzini[11]

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The need for resummation

Partonic cross section as a perturbative series

$$\begin{aligned}\sigma_{ab}^{part}(p_1,p_2,Q,Q_i,\mu_R,\mu_F) &= \alpha_s^k(\mu_R)[\sigma_{LO}(p_1,p_2,Q,Q_i)\\ &+ \alpha_s(\mu_R)\sigma_{NLO}(p_1,p_2,Q,Q_i,\mu_R,\mu_F)\\ &+ \alpha_s^2(\mu_R)\sigma_{NNLO}(p_1,p_2,Q,Q_i,\mu_R,\mu_F)+\ldots]\end{aligned}$$

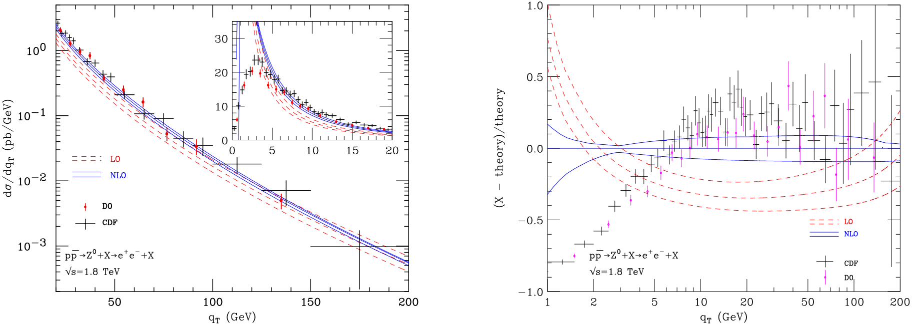
- The fixed-order result gives reliable result only when all the scales are of the same order of magnitude
- If $Q_i \gg Q$ or $Q_i \ll Q$, the appearance of $\alpha_s log(Qi/Q)$ terms could spoil the perturbative result: they need to be resummed!

An example: the small-q_T region ($q_T \ll Q$)

- Bulk of the events in the region $q_T \ll Q$
- Kinematical unbalance between real and virtual contributions

→ perturbative coefficients enhanced by $\alpha_S^n \log^m(\frac{Q^2}{q_T^2})$

→ convergence of perturbative result $\underline{\text{completely spoiled}}$



Bozzi,Catani,DeFlorian,Ferrera,Grazzini[09]

→ need for resummation!

Parton Shower vs. Matrix Elements

Parton Shower Generator	**Matrix Element Generator**
Resums leading logs to all orders	Only go up to NLO
High multiplicity *hadrons* in final state	Low multiplicity *partons* in final state
Good for regions of low relative p_T	Good for regions of high relative p_T
Total rate accurate to LO	Total rate accurate to NLO

The perfect matching

- generates total rates accurate at NLO
- treats hard emission as in Matrix Element Generators
- treats soft/collinear emission as in Parton Shower Generators
- generates a set of fully exclusive events which can be interfaced with a hadronization model

NLO matching

- **MC@NLO** [Frixione,Webber(02)]
  - add difference between exact(ME) NLO and approx.(PS) NLO
  - automatization (aMC@NLO) based on FKS subtraction @ NLO

    [Frederix,Frixione,Maltoni,Stelzer(09)]

  → dependent on the shower details

  → difference may be negative
- **POWHEG** [Nason(04)]
  - Generate the hardest emission at NLO accuracy (mod. Sudakov)
  - Angular-ordered showers: add truncated shower from hard scale

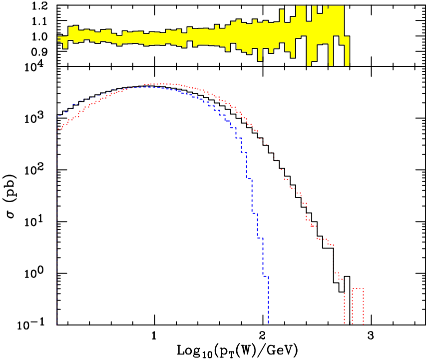
  → always positive weights

  → discrepancies with respect to MC@NLO thoroughly analysed in several publications

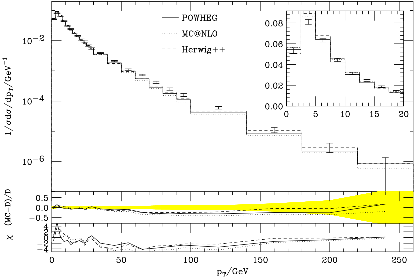
NLO matching uncertainties

Differences between matching procedures

MC@NLO/HW *vs.* MC@NLO/PY *vs.* PY [Frixione,Torrielli(10)]



POWHEG *vs.* MC@NLO *vs.* HERWIG *vs.* DATA
`[Hamilton,Richardson,Tully(08)]`



Theoretical uncertainty = PDF, choice of method/shower

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Analytical Resummation: the main idea

$\alpha_s L^2$	$\alpha_s L$	$\dots$	$\dots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\dots$	$\mathcal{O}(\alpha_s^n)$	(N^nLO)
LL	NLL	NNLL	$\dots$	$\dots$	

- Ratio of two successive rows: $\mathcal{O}(\alpha_s L^2)$
- improved expansion
  - *reorganization* of the terms into *towers of logs*
  - *all-order summation* of the terms in each class
- key-point: *exponentiation in a conjugate space (Fourier, Mellin)*

$$\sigma^{res} \sim \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right] \quad (L = \log(Qb/b_0))$$

- Ratio of two successive columns: $\mathcal{O}(1/L)$

Going back to the physical space

Problem:

Resummation involves integration over b from 0 to ∞:
$\alpha_s(1/b)$ large when $b \to 1/\Lambda_{QCD}$, how to go back?

- Proposed solutions
  - return to p_T space (expansion of the exponent + inverse transformation performed analytically) [Ellis,Veseli(97);Frixione,Nason,Ridolfi(99);Kulesza,Stirling(99-03)]
  - integration over a complex b-plane to avoid singularities [Laenen,Sterman,Vogelsan(00);Kulesza,Sterman,Vogelsang(02)Bozzi,Catani,DeFlorian,Grazzini(05-09)]
  - extrapolation of perturbative results into large-b region [Qiu,Zhang(01)]
  - using Borel resummation [Bonvini,Forte,Ridolfi(08)]
- Improved matching [Bozzi,Catani,DeFlorian,Grazzini(05,07,09)]

$$\tilde{L} = \log(\frac{bQ}{b_0} + 1) \to \int dp_T \frac{d\sigma_{NLO}}{dp_T} = \sigma_{NNLO}$$

$\longrightarrow$introduction of resummation scale$\longleftarrow$

Drell-Yan at NNLL+NLO [Bozzi,Catani,deFlorian,Ferrera,Grazzini(10)]

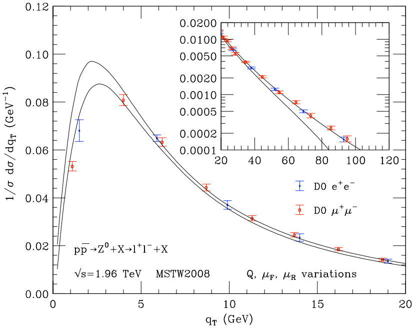
- Normalized q_T distribution
- Scales fixed to Z mass

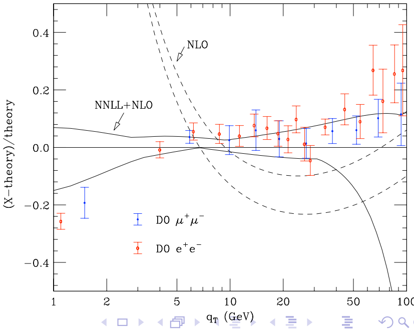
→ Uncertainty dominated by Q variation

→ Good agreement with Run II D0 data

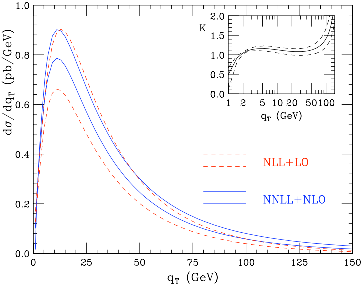
→ Experimental errors are smaller than theoretical uncertainty

- most accurate QCD perturbative prediction for W and Z





Higgs @ NNLL+NLO [Bozzi,Catani,DeFlorian,Grazzini(03,05,07)]



- NNLL+NLO uncertainty band overlaps with NLL+LO one
  - → very good convergence of the resummed perturbative result
- q_T-dependent K-factor

$$K(q_T) = \frac{d\sigma_{NNLL+NLO}(\mu_F, \mu_R)}{d\sigma_{NLL+LO}(\mu_F = \mu_R = M_H)}$$

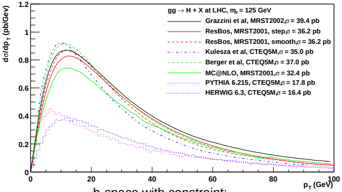
  - $\sim$ 1.1-1.2 in the central region
  - increase (decrease) drastically for $q_T > 50$ ($q_T < 2$)
  - → no simple rescaling of NLL+LO
- similar features when including rapidity dependence

Analytical resummation uncertainties

Differences between resummation prescriptions: work in progress!

Higgs production via gluon fusion at the LHC

[*Balazs, Grazzini, Huston, AK, Puljak'04*]



NNLL+NLO	b-space with constraint: $\int dp_{\mathrm{T}} \frac{d\sigma^{\mathrm{NLO}}}{dp_{\mathrm{T}}} = \sigma^{\mathrm{NNLO}}$	[*Bozzi et al.'03'05*]
"Sudakov" NNLL + LO	b-space	[*Berger, Qiu'02*]
"Sudakov" NNLL + LO	joint	[*A.K., Sterman, Vogelsang'03*]
"Sudakov" NNLL + (N)LO	b-space	[*Balazs, Yuan'00*]
MC@NLO	LO p_{T}-distribution + parton shower	[*Frixione, Webber'02*]
PYTHIA	with hard matrix el. corrections	
HERWIG	without hard matrix el. corrections	

A. Kulesza, p_T resummation for colour-singlet hadronic production – p. 24/28

Theoretical uncertainty = PDF, choice of prescription

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Normalized lepton pair transverse mass

$$\mathcal{O}\left(M_\perp^W\right) = \frac{d\sigma}{dM_\perp^W}\left(M_\perp^W\right), \qquad M_\perp^W = \sqrt{2p_t^l p_t^\nu \left(1 - \cos\left(\phi^l - \phi^\nu\right)\right)}$$

- QCD corrections quite moderate with respect to lepton p_T
- small QCD effects on the shape of the distribution
- PDF uncertainties induce similar effects w.r.t. other observables

$$\widetilde{\mathcal{O}}\left(M_\perp^W\right) = \frac{1}{\sigma^{\rm fit}}\frac{d\sigma}{dM_\perp^W}\left(M_\perp^W\right), \qquad \sigma^{\rm fit} = \int_{M_\perp^{W,\rm min}}^{M_\perp^{W,\rm max}} dM \frac{d\sigma}{dM_\perp^W}(M)$$

$$(M_\perp^{W,\rm min} = 50 \text{ GeV}, \; M_\perp^{W,\rm max} = 100 \text{ GeV})$$

- normalization greatly reduces the effect of PDF uncertainty

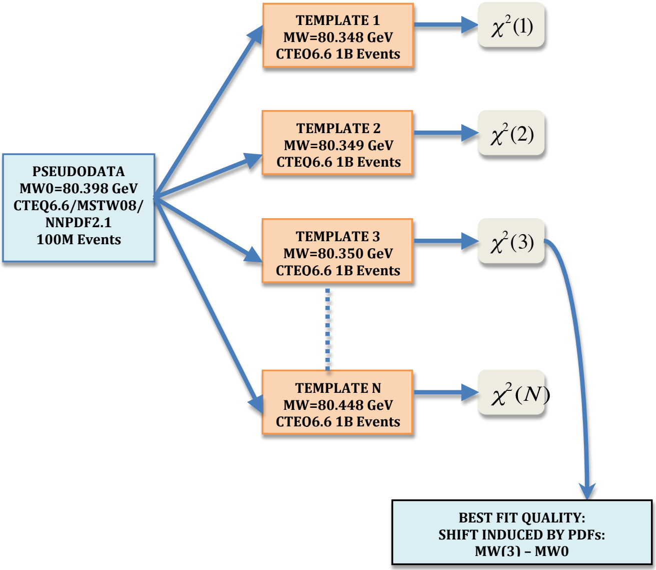
The fitting strategy [Bozzi,Rojo,Vicini(11)]

1. generate templates for a given fixed PDF set and for different values of m_W with very high statistics (1B events)
2. for each member of the PDF sets considered, generate pseudo-data with fixed $m_W^0 = 80.398$ GeV with lower statistics (100M events)
3. compute the χ^2 between the pseudo-data and each of the templates

$$\chi_j^2 = \frac{1}{N_{\rm bins}} \sum_{i=1}^{N_{\rm bins}} \frac{\left(O_i^j - O_i^{\rm data}\right)^2}{(\sigma_i^{\rm data})^2 + (\sigma_i^j)^2} \qquad j = 1, \dots, N_{\rm templates}$$

4. the template with best χ^2 provides the information on Δm_W induced by this particular PDF set

The fitting strategy [Bozzi,Rojo,Vicini(11)]



PSEUDODATA
MW0=80.398 GeV
CTEQ6.6/MSTW08/
NNPDF2.1
100M Events

TEMPLATE 1
MW=80.348 GeV
CTEQ6.6 1B Events

$\chi^2(1)$

TEMPLATE 2
MW=80.349 GeV
CTEQ6.6 1B Events

$\chi^2(2)$

TEMPLATE 3
MW=80.350 GeV
CTEQ6.6 1B Events

$\chi^2(3)$

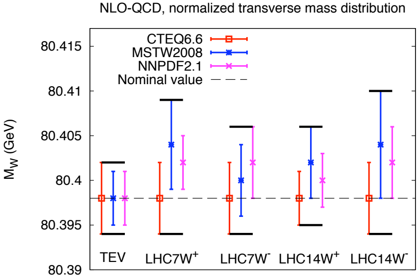
TEMPLATE N
MW=80.448 GeV
CTEQ6.6 1B Events

$\chi^2(N)$

BEST FIT QUALITY:
SHIFT INDUCED BY PDFs:
MW(3) – MW0

NLO-QCD results [Bozzi,Rojo,Vicini(11)]

m_W (GeV)	CTEQ6.6		MSTW2008		NNPDF2.1		
	$m_W \pm \delta_{\rm pdf}$	$\langle\chi^2\rangle$	$m_W \pm \delta_{\rm pdf}$	$\langle\chi^2\rangle$	$m_W \pm \delta_{\rm pdf}$	$\langle\chi^2\rangle$	$\delta^{\rm tot}_{\rm pdf}$
Tevatron, $W^\pm$	80.398 ± 0.004	1.42	80.398 ± 0.003	1.42	80.398 ± 0.003	1.30	4
LHC 7 TeV W^+	80.398 ± 0.004	1.22	80.404 ± 0.005	1.55	80.402 ± 0.003	1.35	8
LHC 7 TeV W^-	80.398 ± 0.004	1.22	80.400 ± 0.004	1.19	80.402 ± 0.004	1.78	6
LHC 14 TeV W^+	80.398 ± 0.003	1.34	80.402 ± 0.004	1.48	80.400 ± 0.003	1.41	6
LHC 14 TeV W^-	80.398 ± 0.004	1.44	80.404 ± 0.006	1.38	80.402 ± 0.004	1.57	8



total (envelope) error at most 8 MeV + excellent agreement at Tevatron

Conclusions

There are MANY sources of theoretical uncertainties!

- factorization scale
- renormalization scale
- resummation scale
- type of resummation (shower vs. analytical)
- non-perturbative contributions
- different PDF parametrizations

→ a detailed investigation is essential