Analytical 3D Reconstruction from Simulations with Lenses

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Italia

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GENERAL PLAN

MULTIPLE VIEW PROJECTIVE GEOMETRY applied to

Previously

LIGHT POINTS RECONSTRUCTION ALGORITHM

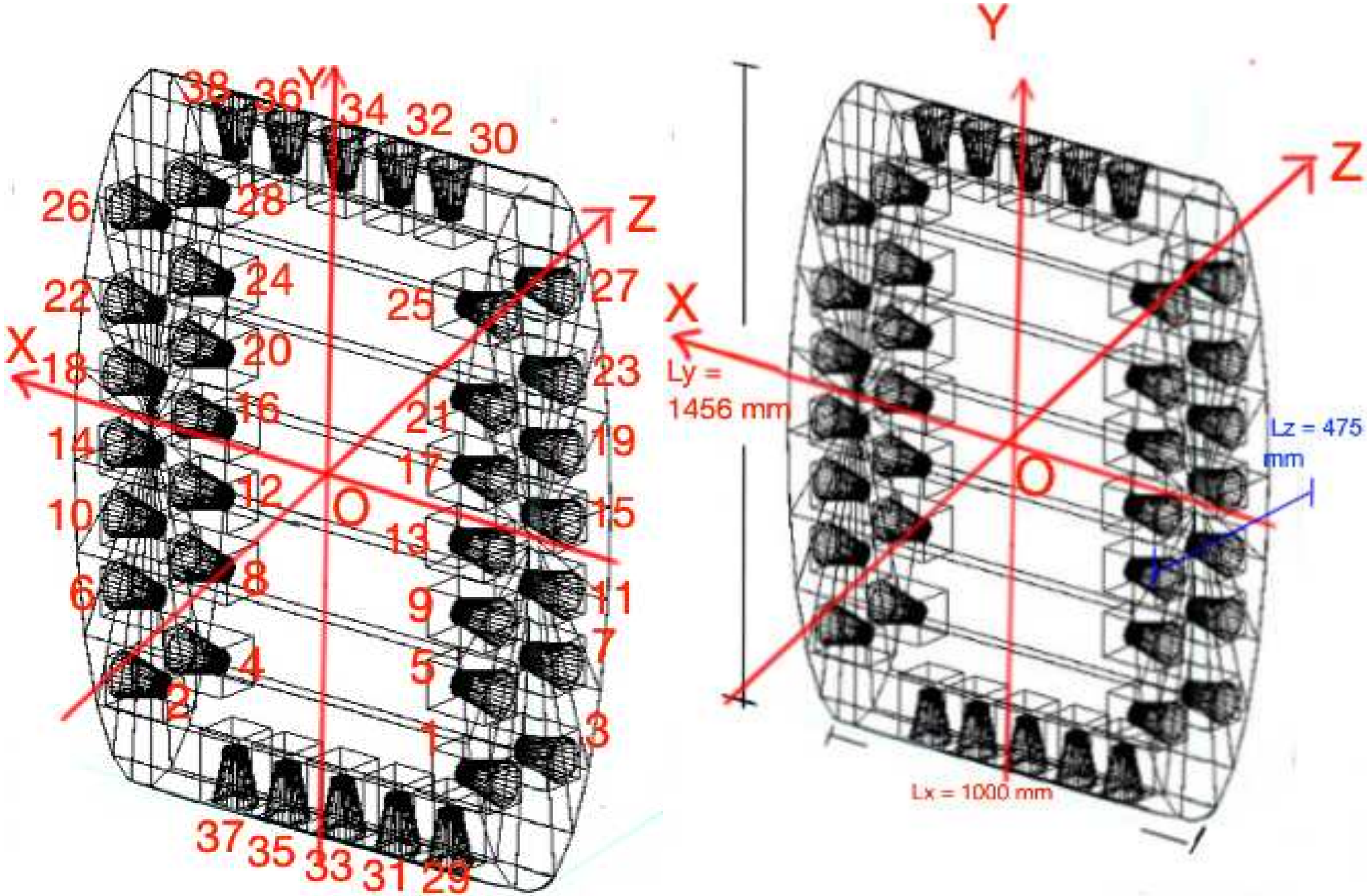
Now

SINGLE and DOUBLE TRACK RECONSTRUCTION ALGORITHM

Then

MULTIPLE TRACK EVENTS RECONSTRUCTION ALGORITHM

GRAIN as a multiple lens system



Light Point Space Resolutions

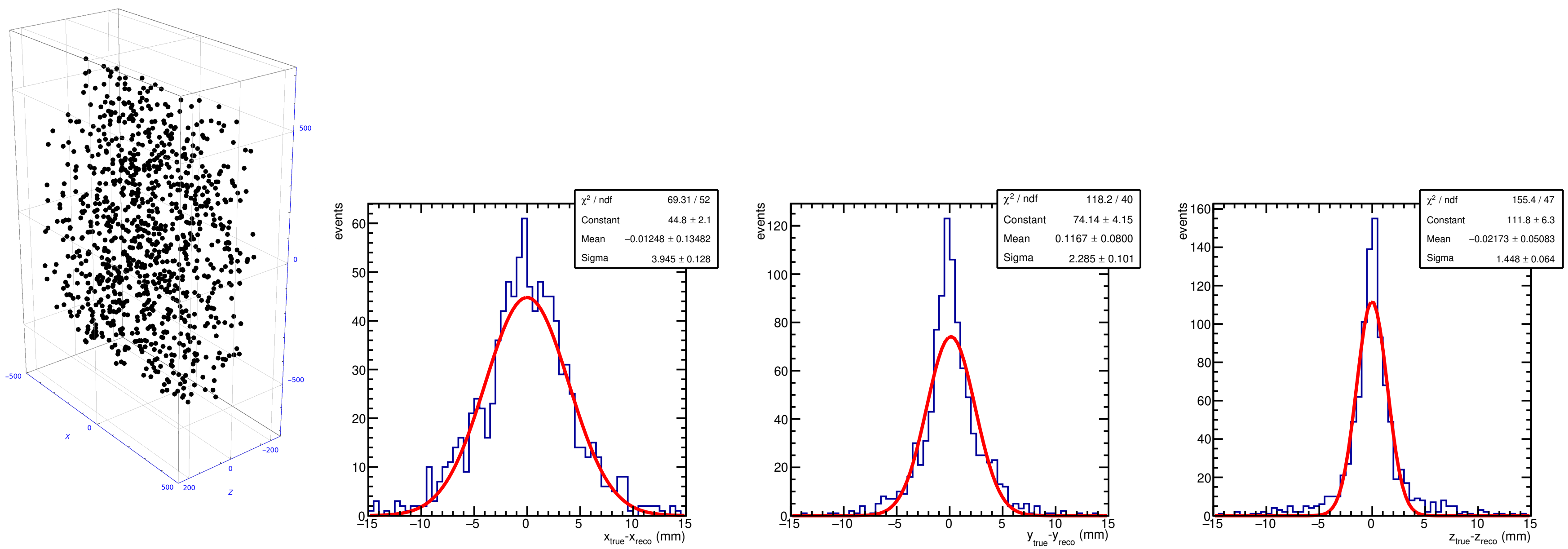
Case Study: 1k light source points (randomly/uniformly)

(meant to be candidates for interaction vertices)

energy release: 100 MeV per point

GRAIN volume $1000 \times 1456 \times 475$

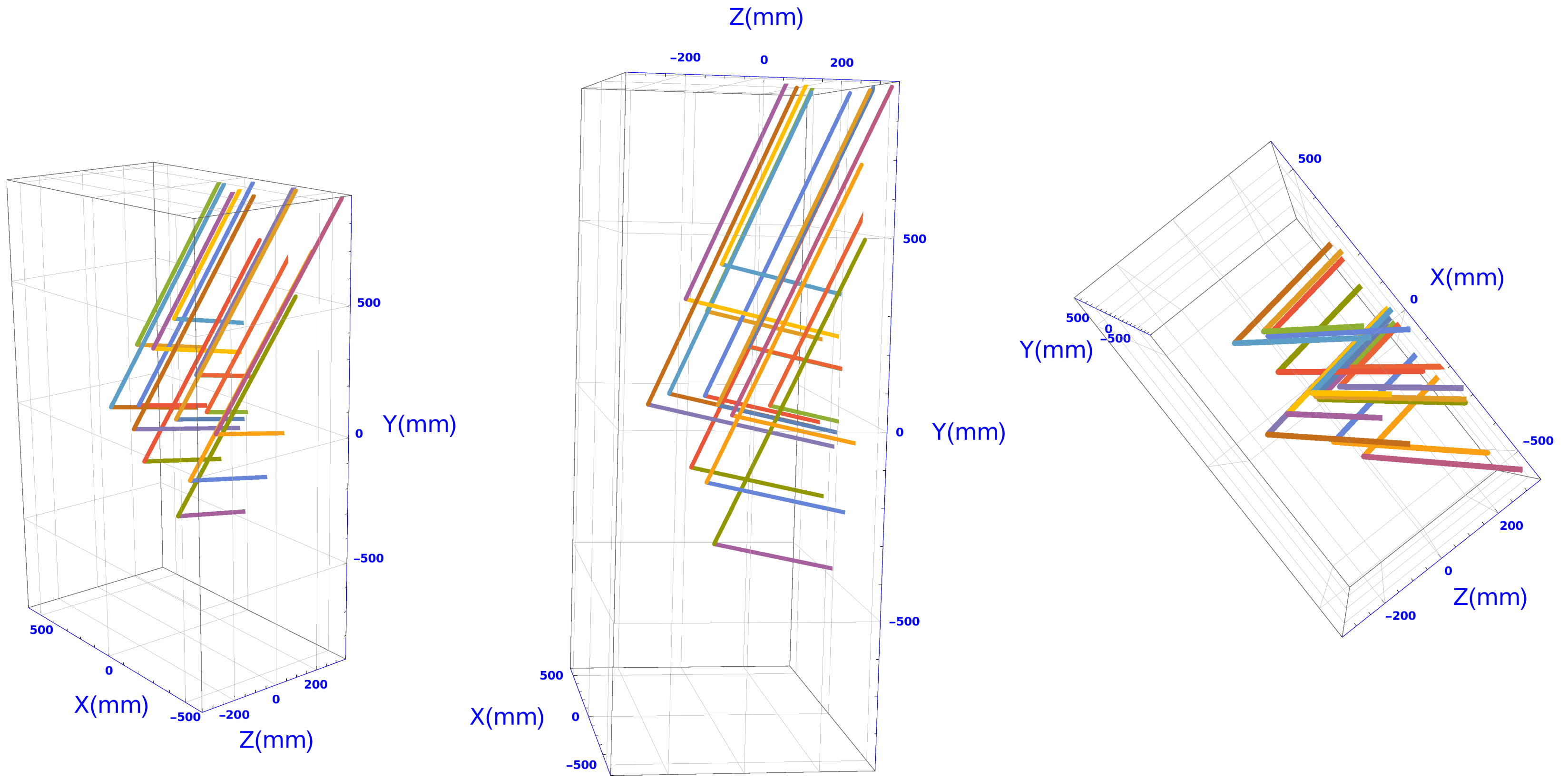
Point volume $800 \times 1200 \times 460$ **units:** mm



A small sample of Events

Track 0: muon Track 1: proton

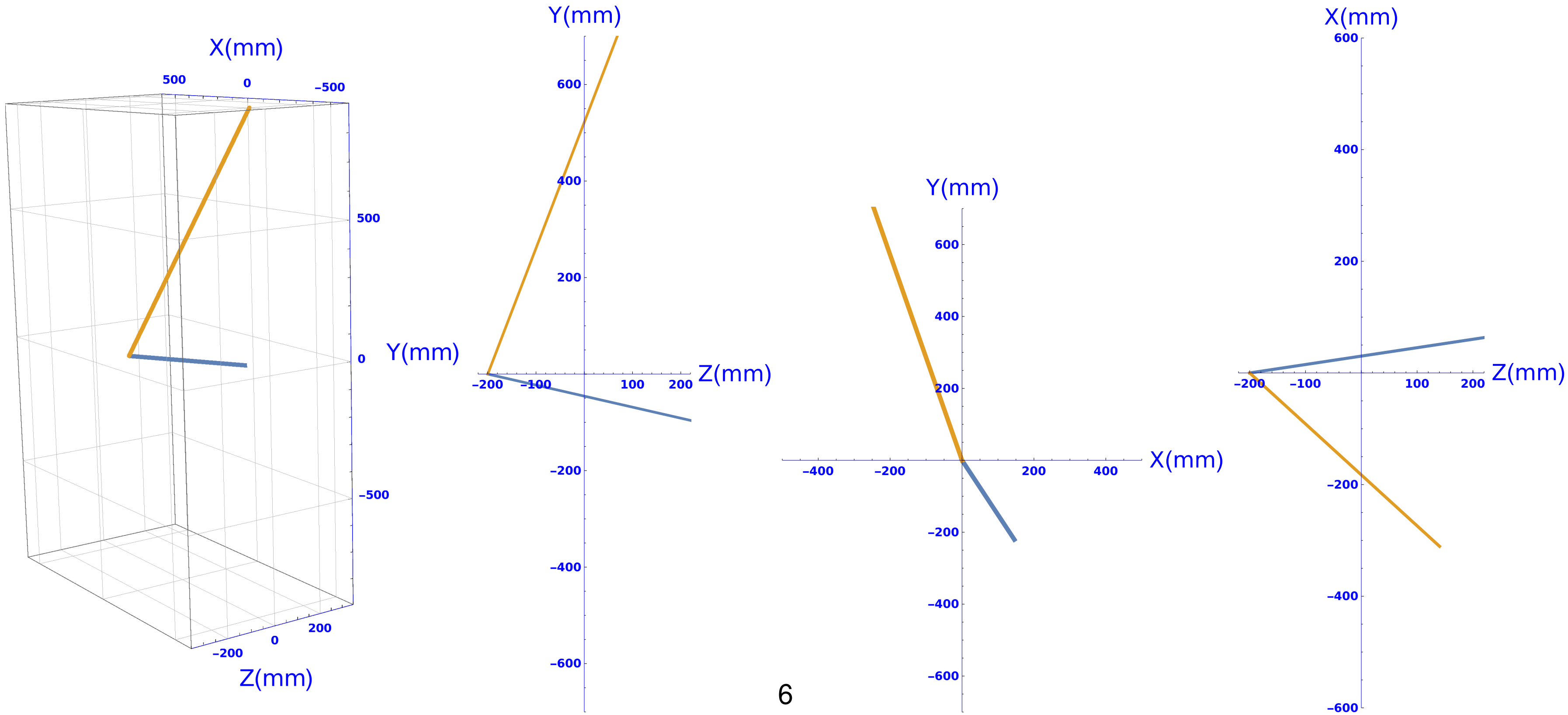
Track 0 (degrees)	θ_X	θ_Y	θ_Z	Track 1 (degrees)	θ_X	θ_Y	θ_Z
True	81.64	102.73	15.31	True	108.10	27.43	70.11



Event 1

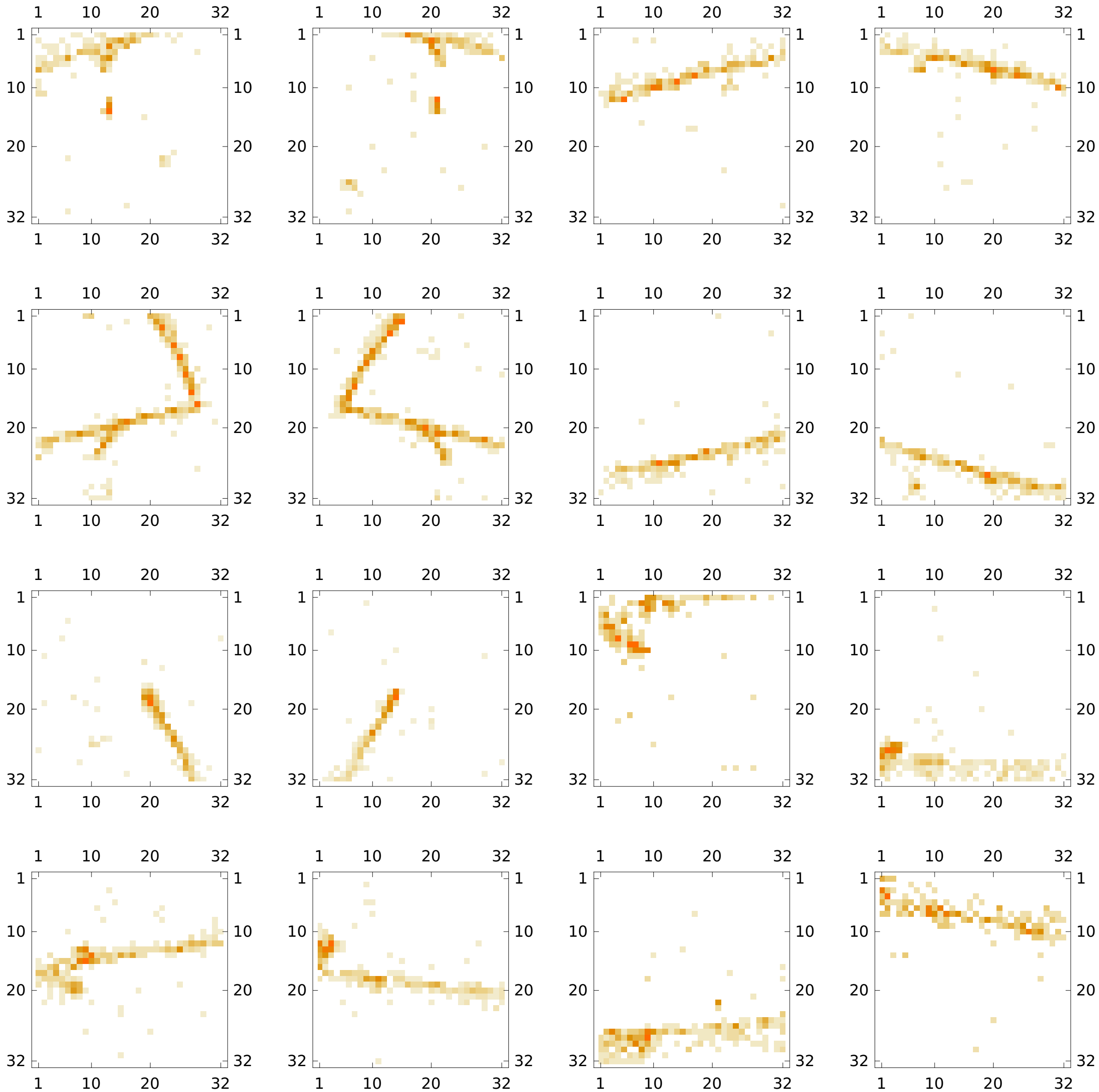
Vertex (mm)	V_X	V_Y	V_Z
True	0	0	-200

Track 0 (degrees)	θ_X	θ_Y	θ_Z	Track 1 (degrees)	θ_X	θ_Y	θ_Z
True	81.64	102.73	15.31	True	108.10	27.43	70.11



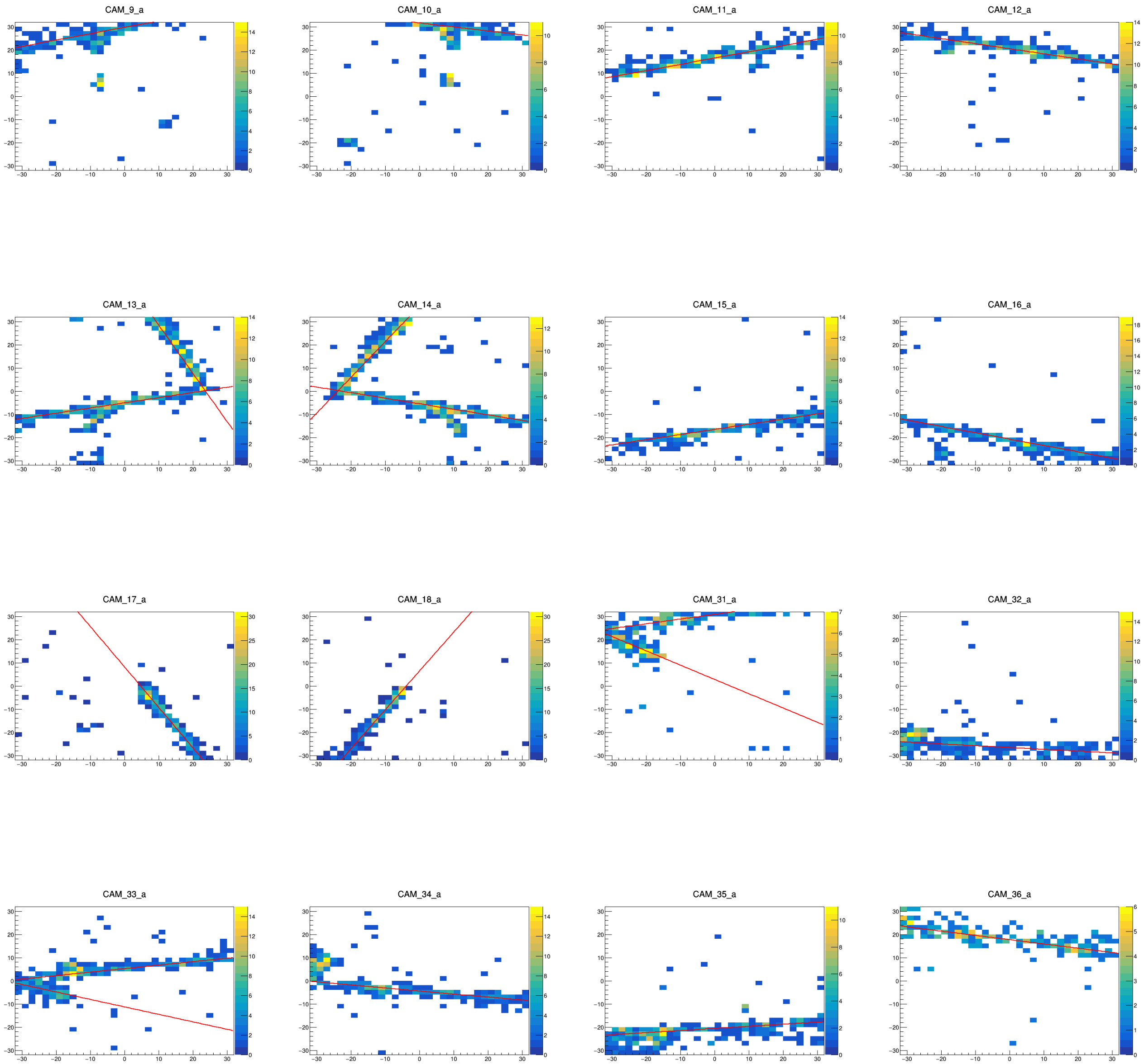
Images on Fired Cameras

Out[ ]=

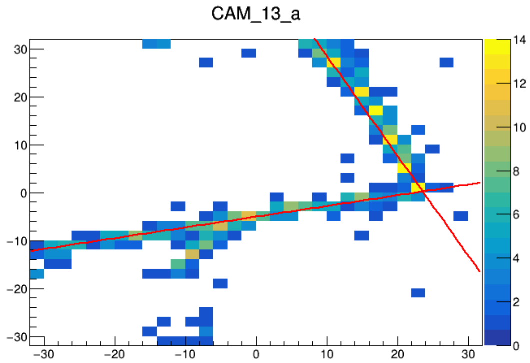


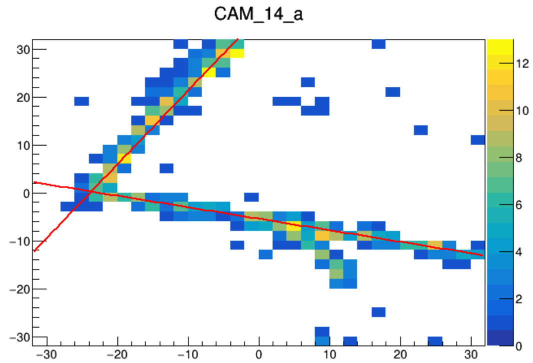
2D Image Fitting

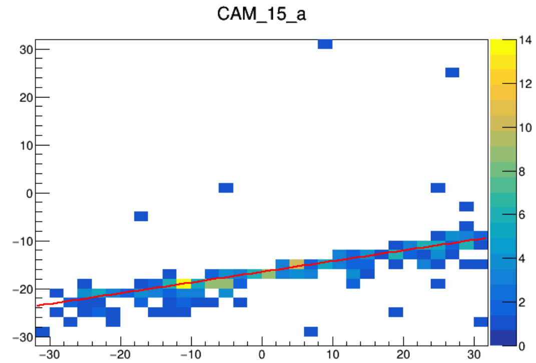
Out[∘]=

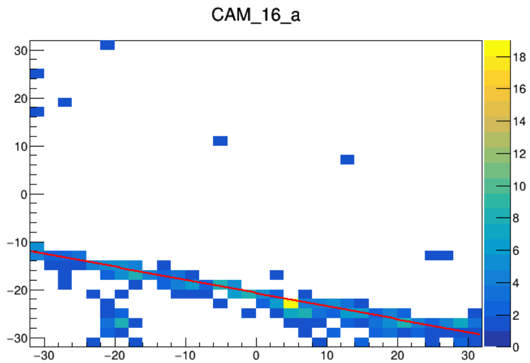


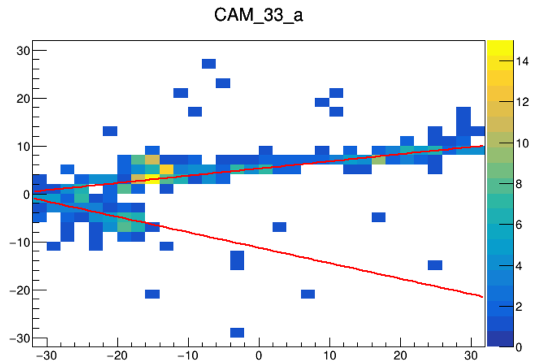
Track Matching and 2D Recognition

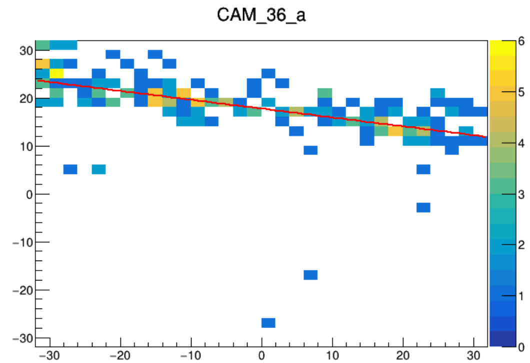








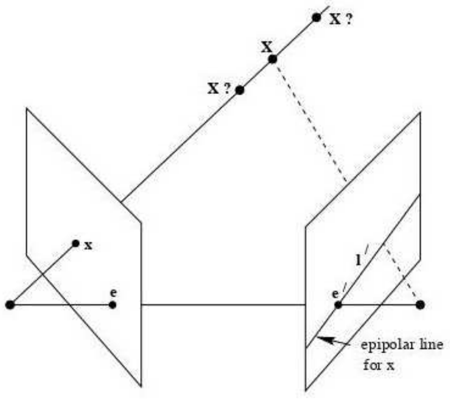


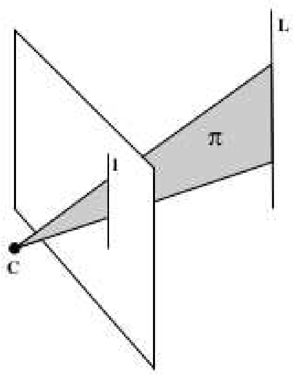


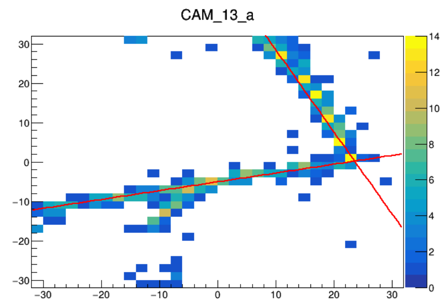
FROM POINTS TO TRACKS

Track reconstruction: theoretical preliminaries

Back-projection of lines and points



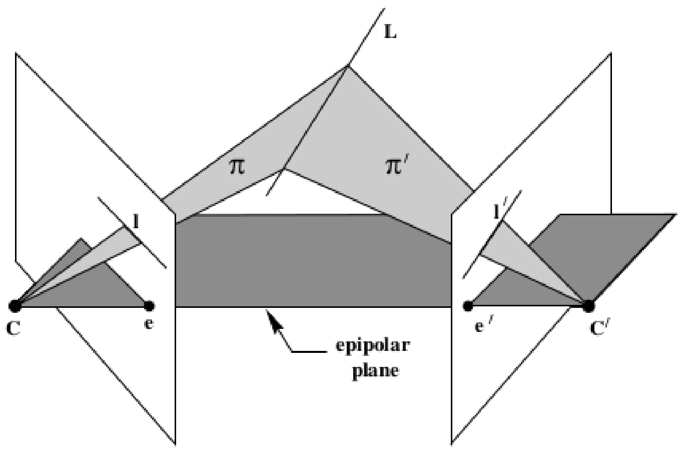




$$\mathbf{x} = \mathbf{P}\mathbf{X} \qquad\qquad \pi = \mathbf{P}^T\mathbf{l}$$

$\mathbf{x}$ image point, $\mathbf{X}$ 3D light source point, π vector of plane parameters in 3D space, $\mathbf{P}$ camera matrix, $\mathbf{l}$ vector of line parameters on the sensor, $\mathbf{L}$ infinite line in 3D space to be reconstructed

Line Reconstruction



$$\mathbf{L} = \begin{pmatrix} \mathbf{l}^T\mathbf{P} \\ \mathbf{l}'^T\mathbf{P}' \end{pmatrix}$$

$\mathbf{l}^T\mathbf{P}$ vector of plane π parameters in 3D space,$\mathbf{l}'^T\mathbf{P}'$ vector of plane π' parameters in 3D space, $\mathbf{P}, \mathbf{P}'$ camera matrices, $\mathbf{l}, \mathbf{l}'$ vectors of lines l, l' parameters on the sensor, $\mathbf{L}$ infinite line in 3D space (to be reconstructed)

Reconstruction Formula

$$\boxed{\mathbf{L}\mathbf{X} = \mathbf{0}} \tag{1}$$

$\mathbf{X}$: generic point on the 3D line

$\mathbf{L}$: 2×4 matrix of plane parameters

From Lenses to P-matrices

A $\mathbf{P}$-matrix is associated to each of the 38 camera-lenses of GRAIN

$$\text{GRAIN} \Longleftrightarrow \{\mathbf{P}_j\}_{j=1,\ldots,38} \tag{2}$$

$$\mathbf{P}_j = \mathbf{K}\left[\mathbf{R}| - \mathbf{R}C_j\right], \quad j = 1, \ldots, 38 \tag{3}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

C_j coordinates of lens centers in GRAIN, $\mathbf{R}$ a rotation matrix (placement in GRAIN)

3D reconstruction of Points and Tracks

$$i, j = 1, ...38$$

$$\pi_i = \mathbf{P}_i{}^T \mathbf{l}_i$$

π_i vector of plane parameters in 3D space, $\mathbf{P}_i$ camera matrix i, $\mathbf{l}_i$ vector of line parameters on the sensor i,

$$\boxed{\mathbf{L}_{ij} = \begin{pmatrix} \mathbf{l}_i^T \mathbf{P}_i \\ \mathbf{l}_j^T \mathbf{P}_j \end{pmatrix}}$$

$\mathbf{L}_{ij}$ infinite line in 3D space to be reconstructed $\mathbf{l}_i^T \mathbf{P}$ vector of plane π_i parameters in 3D space,$\mathbf{l}_j^T \mathbf{P}_j$ vector of plane π_j parameters in 3D space, $\mathbf{P}_i, \mathbf{P}_j$ camera matrices, $\mathbf{l}_i, \mathbf{l}_j$ vectors of lines l_i, l_j parameters on the sensor, $\mathbf{L}_{ij}$ infinite line in 3D space (to be reconstructed)

$$\boxed{\mathbf{L}_{ij}\mathbf{X} = \mathbf{0}} \tag{5}$$

$\mathbf{X}$: generic point on the 3D line

$\mathbf{L}_{ij}$: 2×4 matrix of plane parameters

Points: reconstruction formula

$$\mathbf{0} = \mathbf{x}_i \times \mathbf{x}_i = \mathbf{x}_i \times \mathbf{P}_i \mathbf{X}_{ij} \tag{6}$$

$$\mathbf{0} = \mathbf{x}_j \times \mathbf{x}_j = \mathbf{x}_j \times \mathbf{P}_j \mathbf{X}_{ij} \tag{7}$$

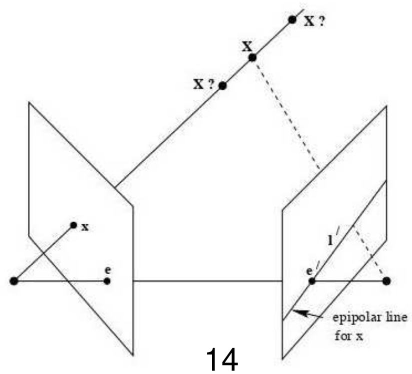
$$\boxed{\mathbf{X}_{ij} = \mathbf{P}_i{}^{+} \mathbf{x}_i + \left[\frac{(\mathbf{P}_j \mathbf{P}_i^{+} \mathbf{x}_i \times \mathbf{x}_j) \cdot (\mathbf{x}_j \times \mathbf{P}_j C_i)}{(\mathbf{x}_j \times \mathbf{P}_j C_i) \cdot (\mathbf{x}_j \times \mathbf{P}_j C_i)} \right] C_i} \tag{8}$$

$$\mathbf{P}_i^{+} = \mathbf{P}_i^{T} \left(\mathbf{P}_i \mathbf{P}_i^{T} \right)^{-1} \tag{9}$$

$\mathbf{P}_i$ camera matrix i

$\mathbf{X}_{ij}$ reconstructed 3D point using cameras i and j, $i \neq j$.

$\mathbf{x}_i$ image point on camera i, C_i camera center i



Vertex Reconstruction (POINT ALGORITHM)

- Vertex in GRAIN
- Vertex is detected/seen by N cameras: **as image vertex we take the intersection point on cameras with two track images**
- There are $M = \frac{N!}{2!(N-2)!}$ possible double-view reconstructions for the vertex
- We perform M reconstructions
- We take the mean value of the M possible reconstructions for each coordinate

$$\boxed{V_X = \frac{\sum_{i<j}^{N} V_{X,ij}}{M}} \boxed{V_Y = \frac{\sum_{i<j}^{N} V_{Y,ij}}{M}} \boxed{V_Z = \frac{\sum_{i<j}^{N} V_{Z,ij}}{M}} \qquad (10)$$

i, j camera indices

Global Multiple View Reconstruction of a Track (TRACK ALGORITHM)

- The track is detected/seen by N cameras
- There are $M = \frac{N!}{2!(N-2)!}$ possible double-view reconstructions for the track
- We perform M reconstructions: direction cosines, slopes and intercepts of the line orthogonal projections onto GRAIN coordinate planes XY, XZ, YZ
- We take the mean value of the M possible reconstructions for each line parameter (director cosines $(\cos\theta_X, \cos\theta_Y, \cos\theta_Z)$)

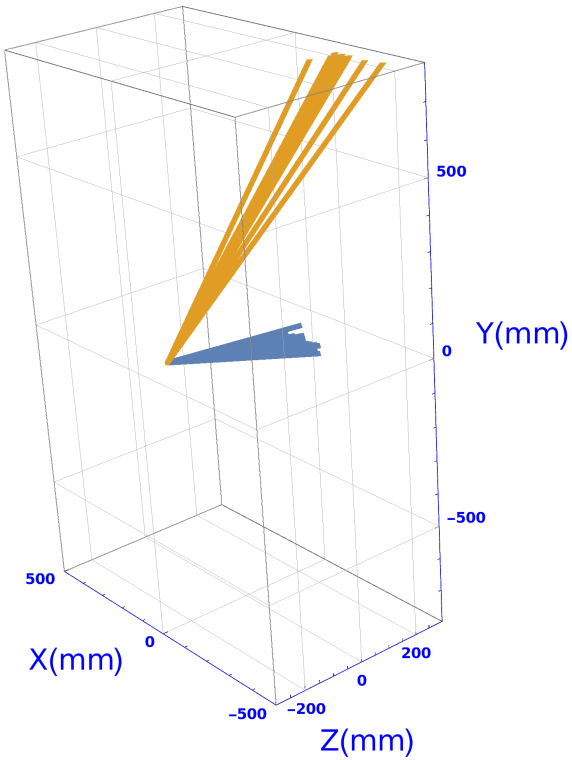
$$\boxed{\cos\theta_X = \frac{\sum_{i<j}^{N} \cos\theta_{X,ij}}{M}} \boxed{\cos\theta_Y = \frac{\sum_{i<j}^{N} \cos\theta_{Y,ij}}{M}} \boxed{\cos\theta_Z = \frac{\sum_{i<j}^{N} \cos\theta_{Z,ij}}{M}} \tag{11}$$

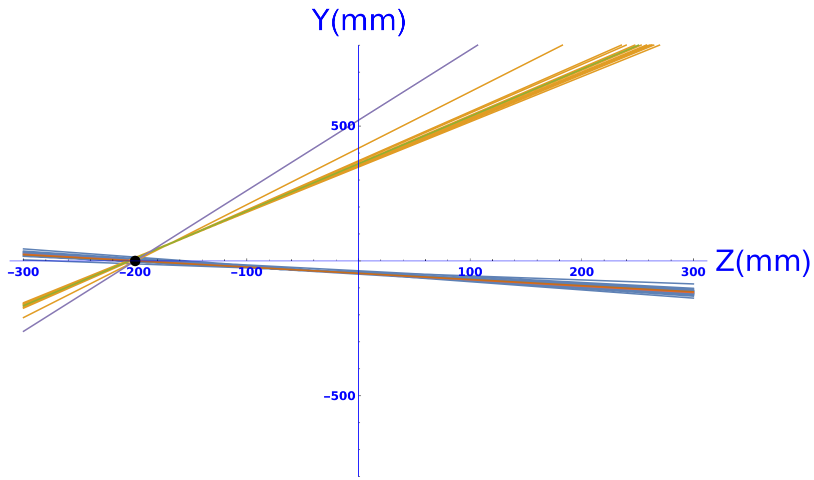
i, j camera indices

- Averaging of intercepts and slopes of lines orthogonally projected onto GRAIN coordinate planes XY, XZ, YZ of the M reconstructions
- Intersection of reconstructed lines in 3D: additional Vertex Determination

Event 1: Vertex and track distribution

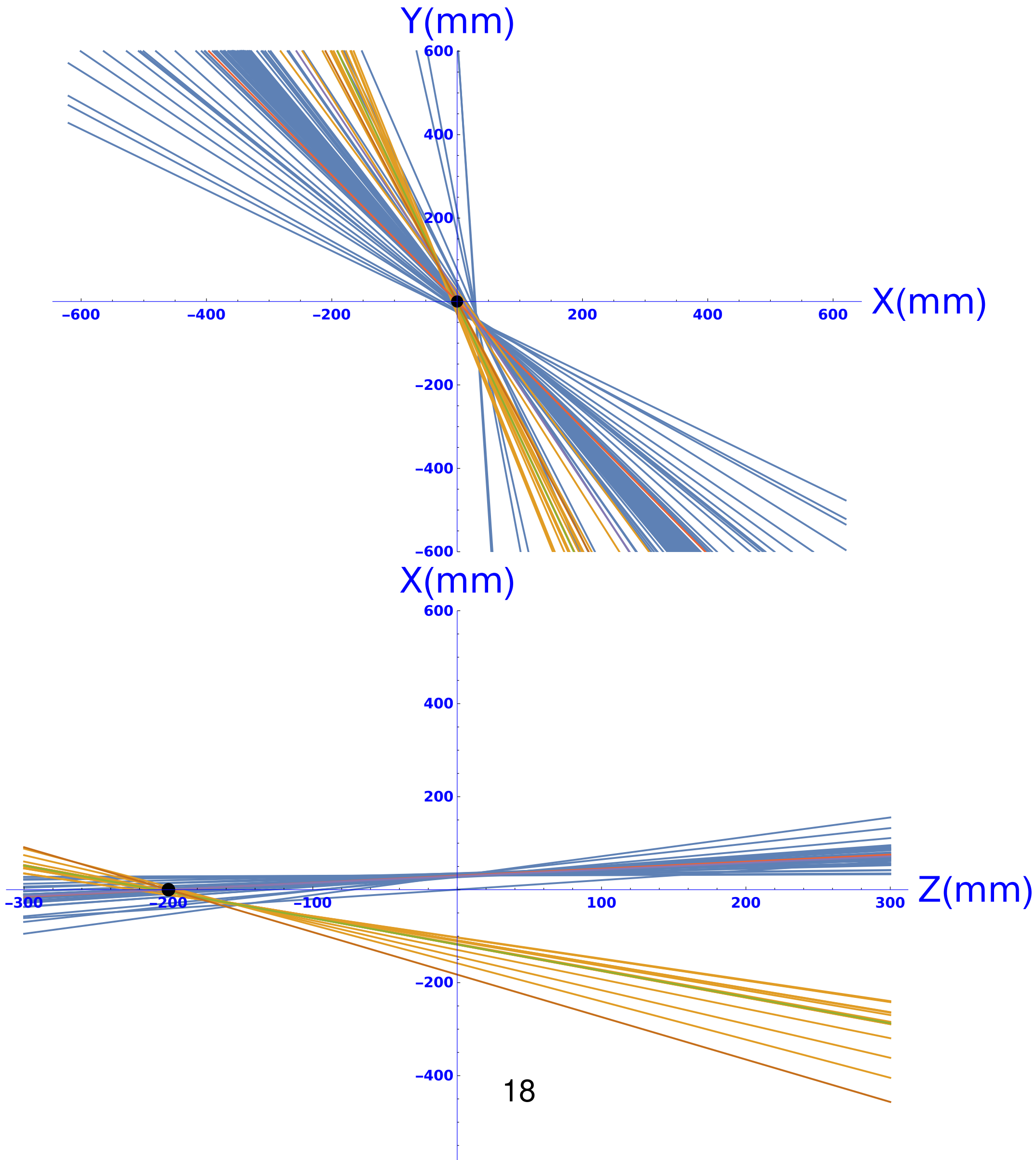
Black Spot ⇒ True Vertex



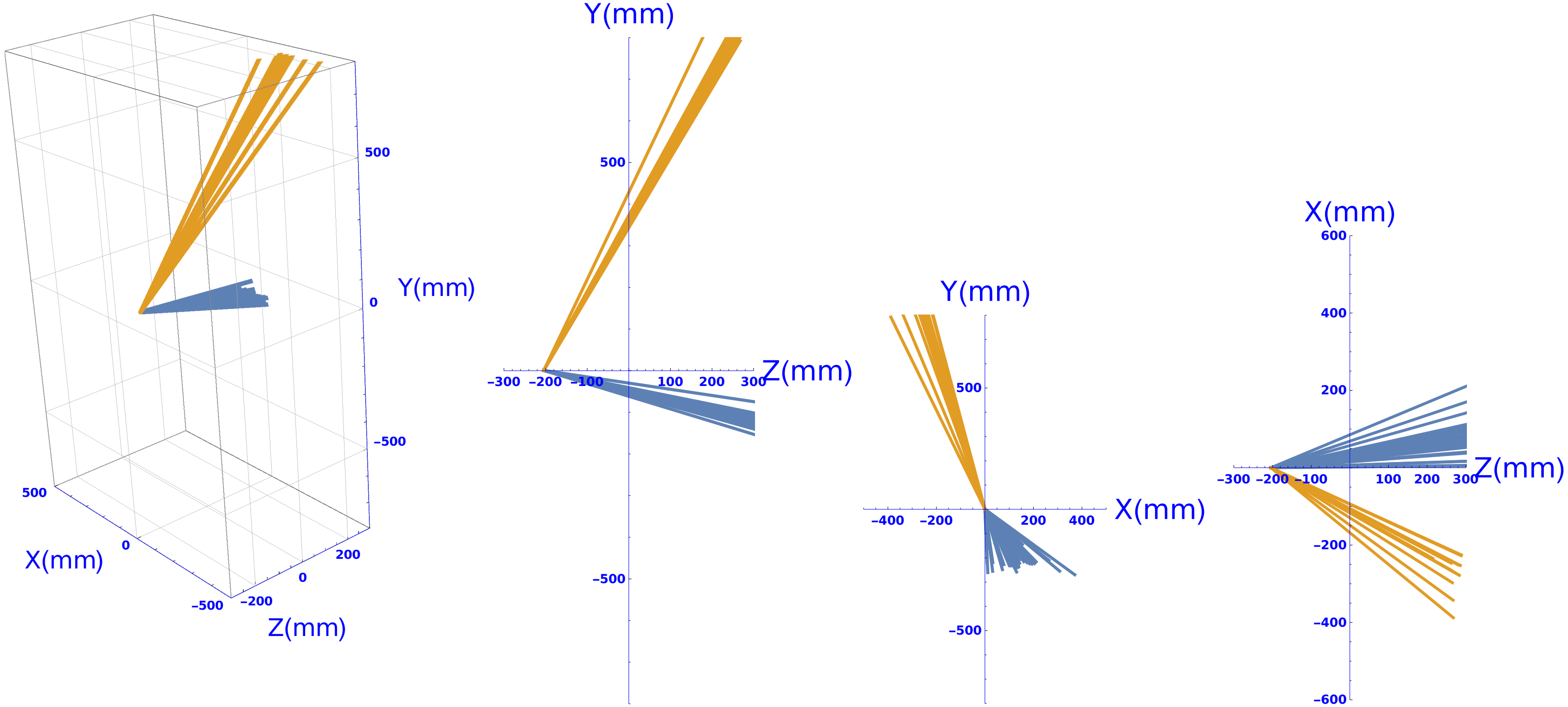


Event 1: Vertex and track distribution

Black Spot $\Rightarrow$ True Vertex

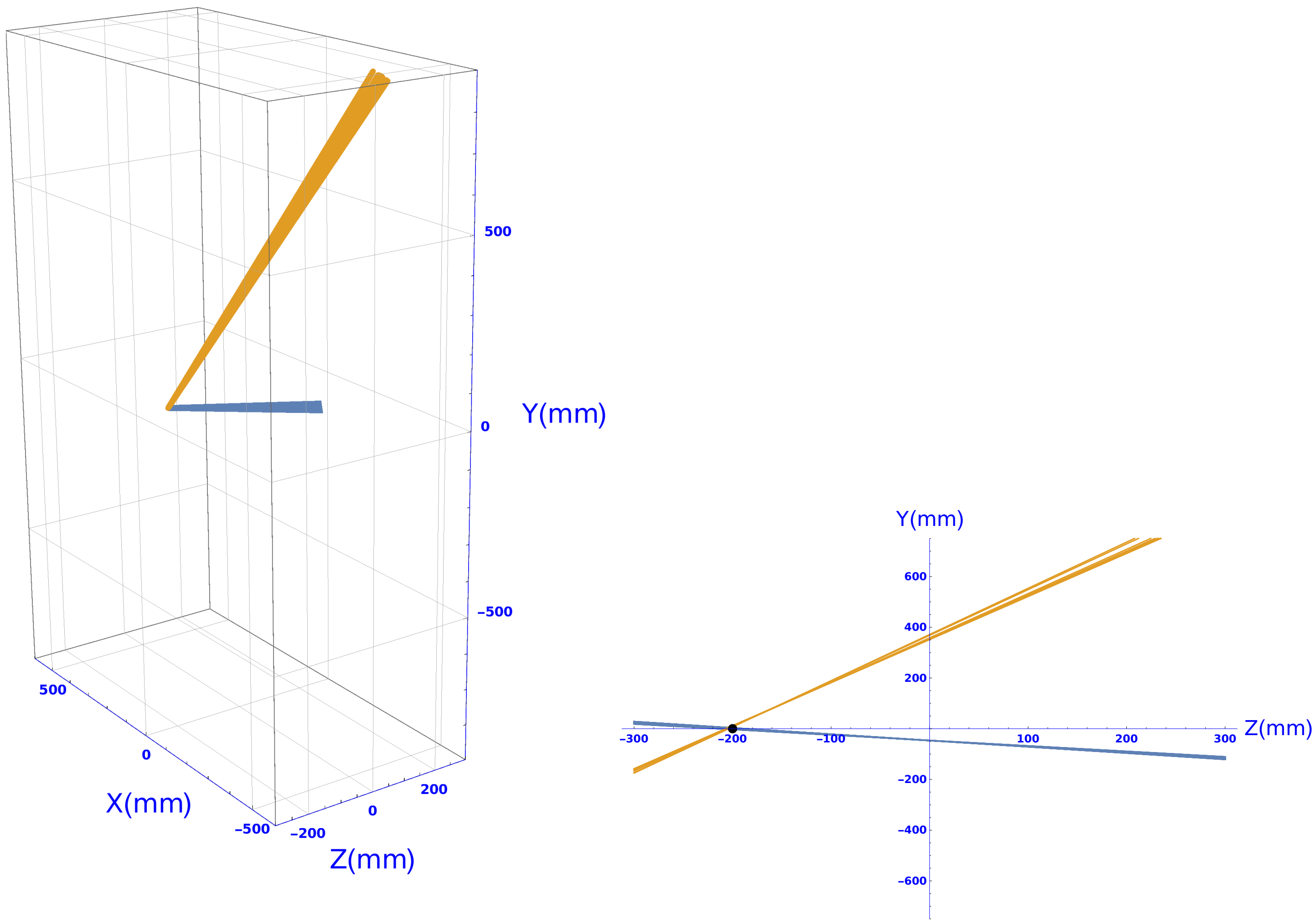


Event 1: "Best" Vertex and track distribution in 3D



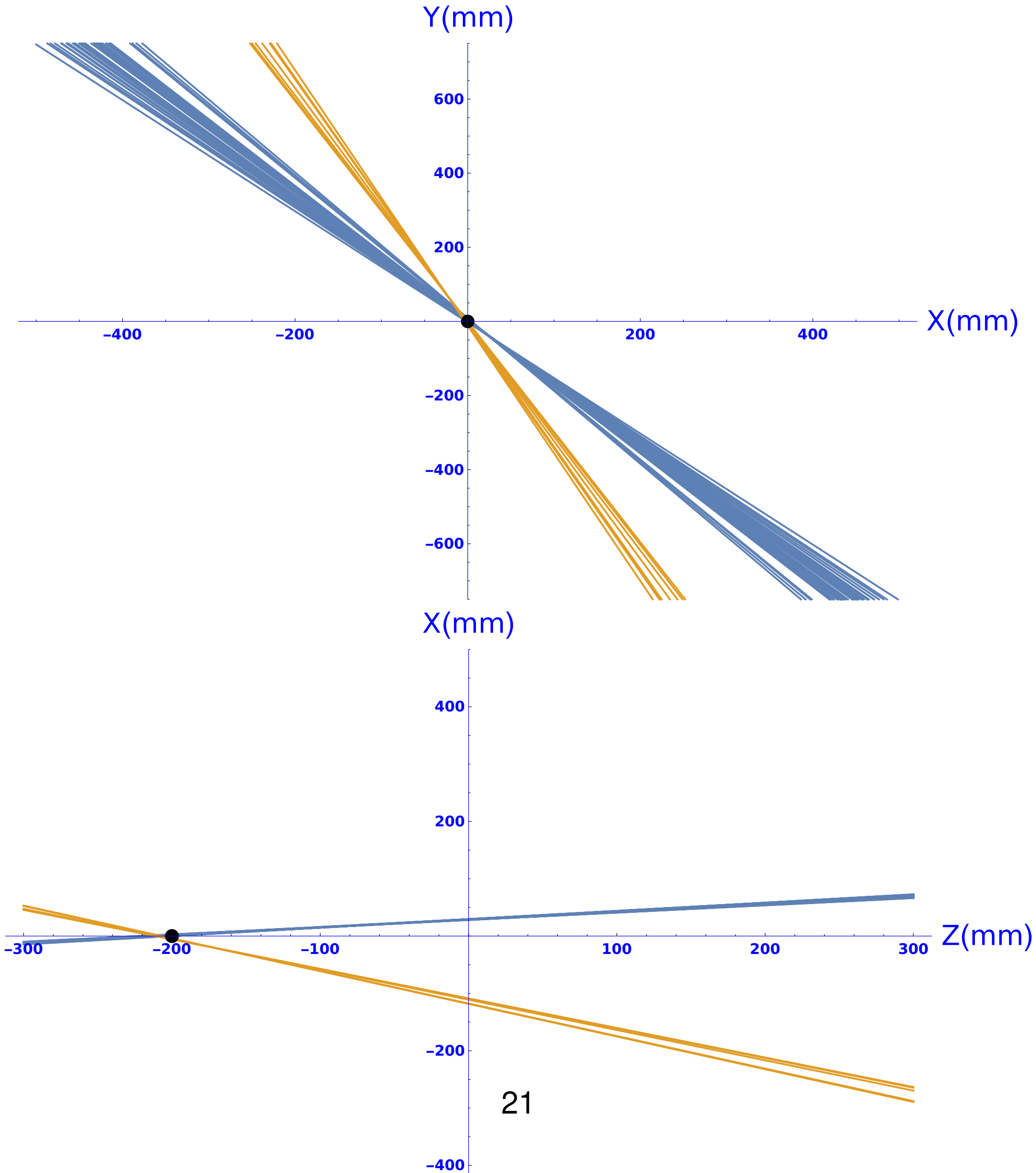
Event 1: Mutually Double–Orthogonal Cameras

Black Spot ⇒ True Vertex

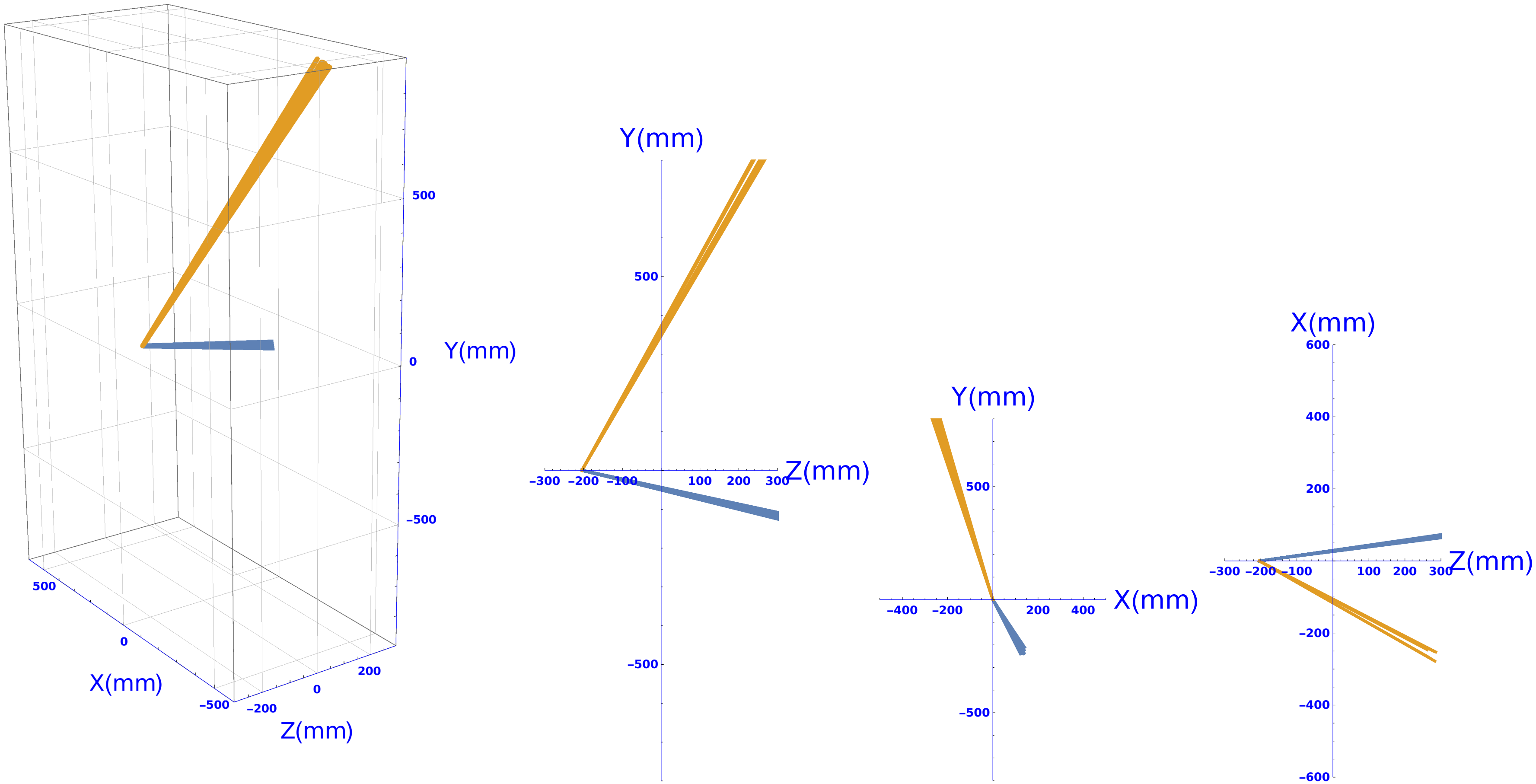


Event 1: Mutually Double–Orthogonal Cameras Projection

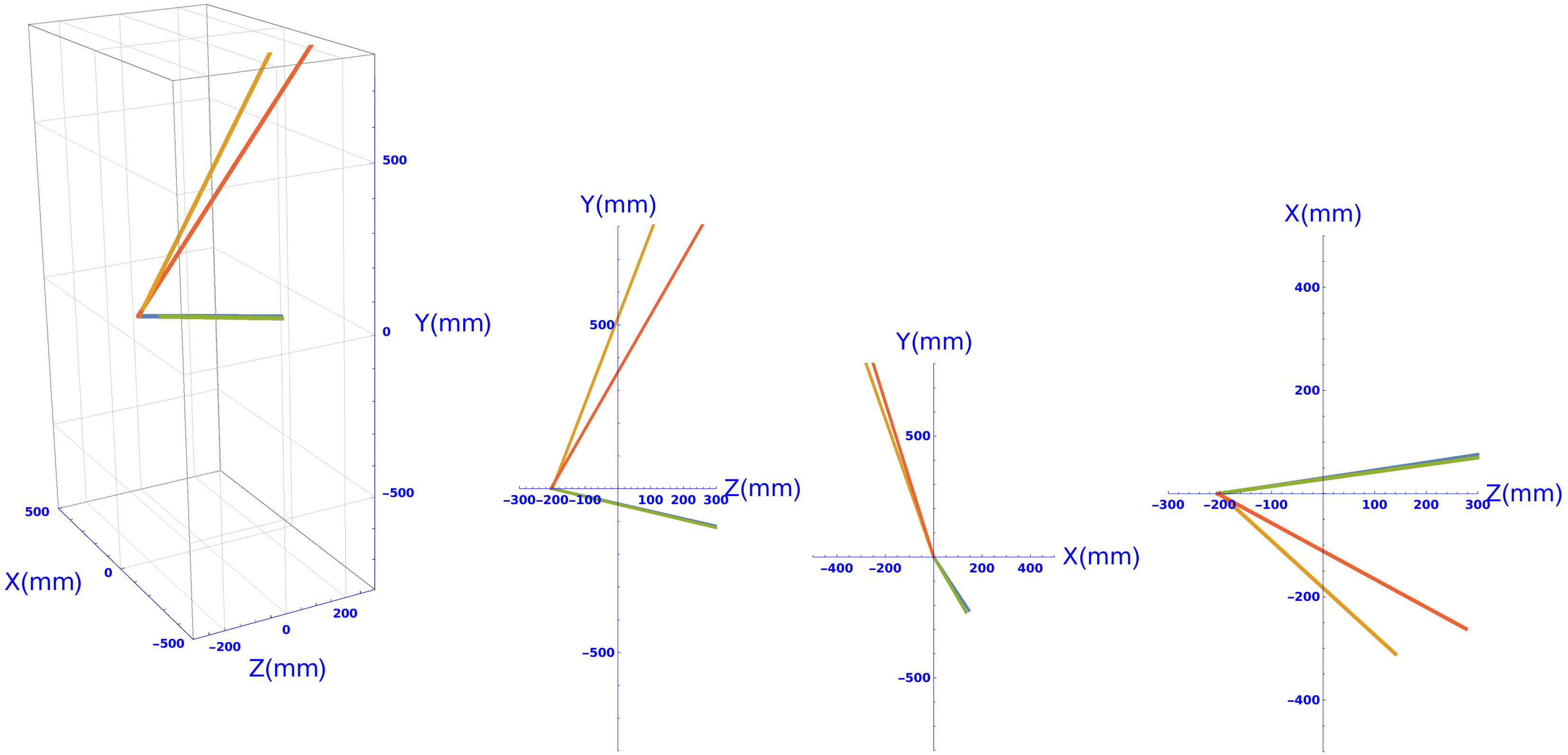
Black Spot $\Rightarrow$ True Vertex



Event 1: Mutually Double–Orthogonal Cameras Reconstruction



Event 1: averaging



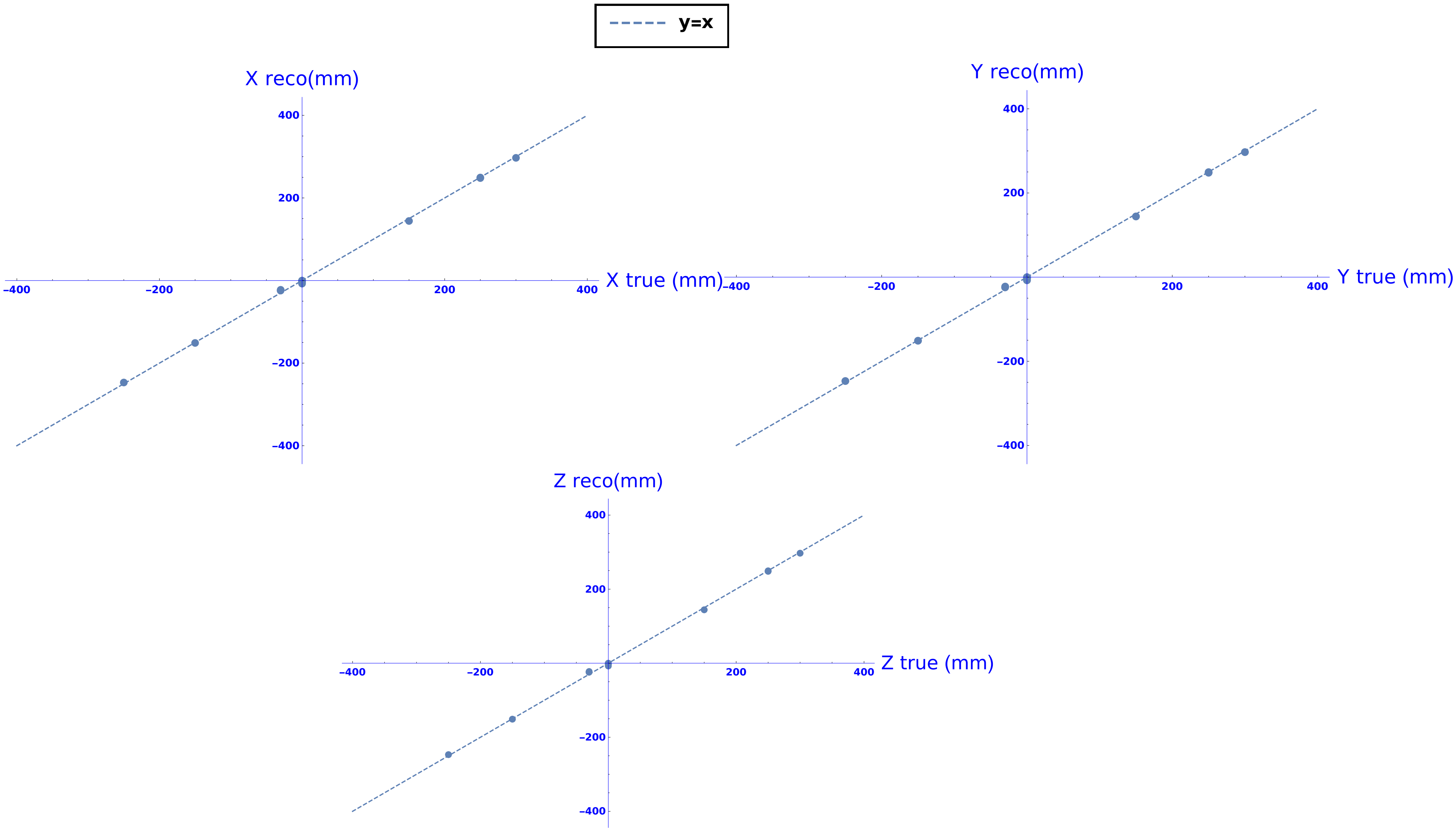
Numerics Event 1

Vertex (mm)	V_X	V_Y	V_Z
True	0	0	-200
Reco	-0.04	0.62	-204.34

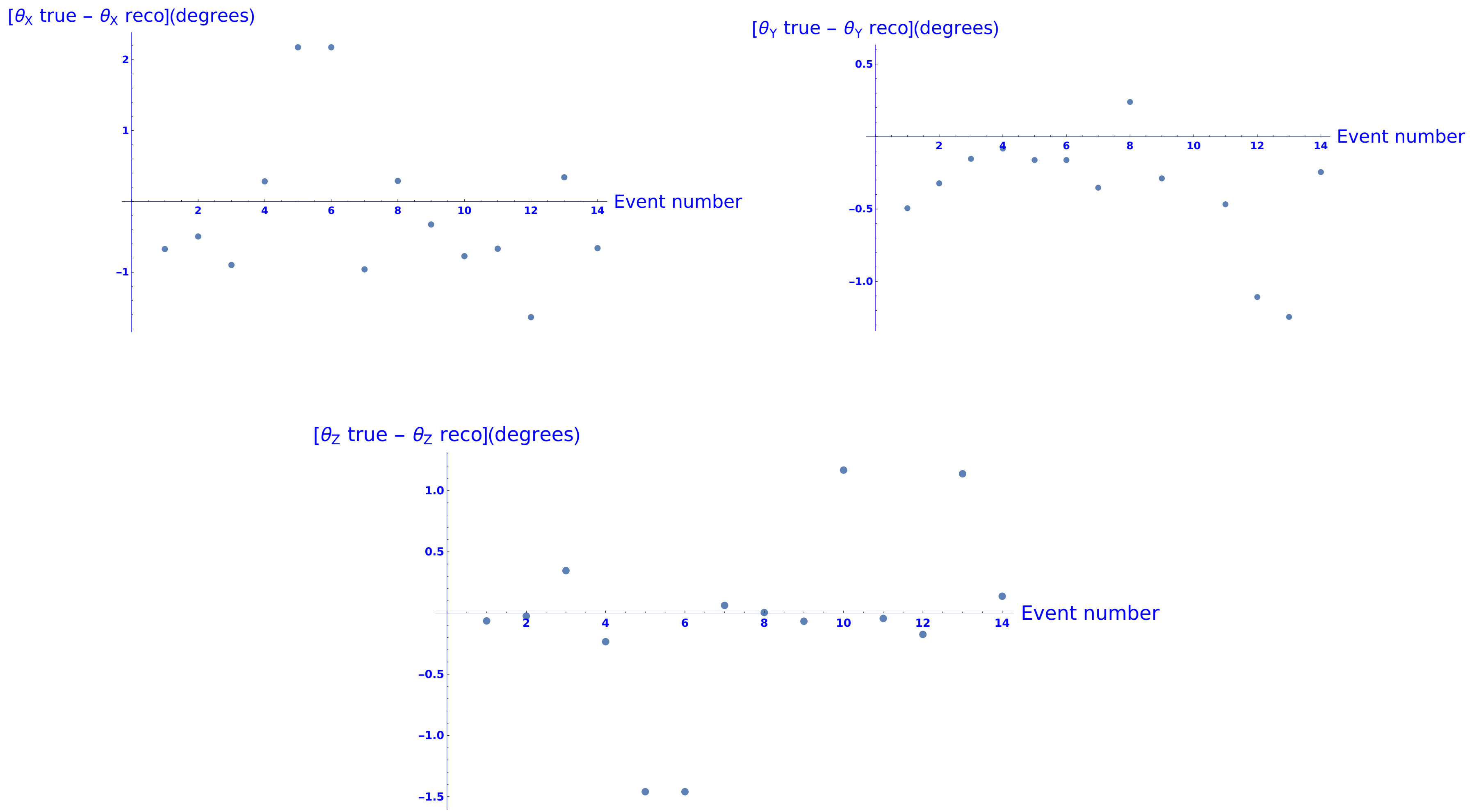
Track 0 (degrees)	θ_X	θ_Y	θ_Z	Track 1 (degrees)	θ_X	θ_Y	θ_Z
True	81.64	102.73	15.31	True	108.10	27.43	70.11
Reco	82.31	103.26	15.37	Reco	105.18	33.22	61.25

Track 0	$\cos\theta_X$	$\cos\theta_Y$	$\cos\theta_Z$	Track 1	$\cos\theta_X$	$\cos\theta_Y$	$\cos\theta_Z$
True	0.145	-0.221	0.965	True	-0.311	0.888	0.340
Reco	0.134	-0.229	0.964	Reco	-0.262	0.837	0.481

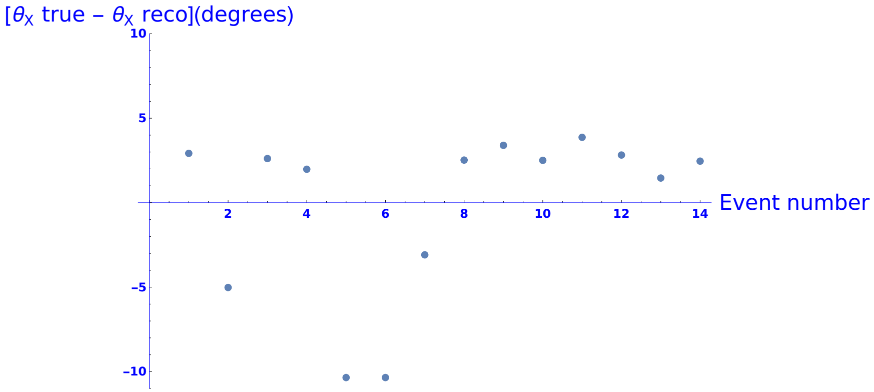
A small sample of Events: vertex coordinates

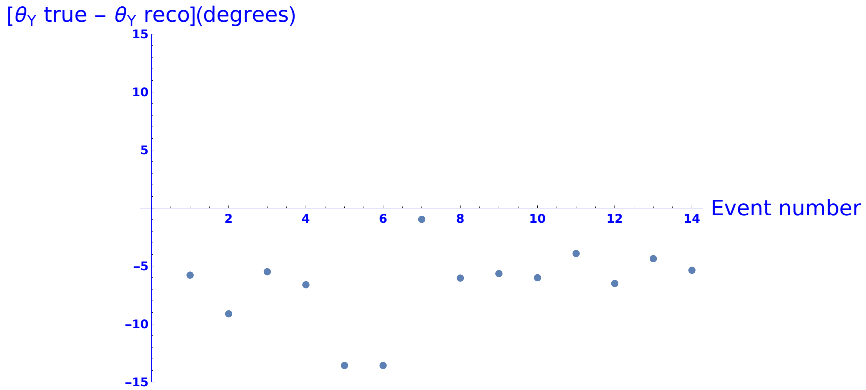


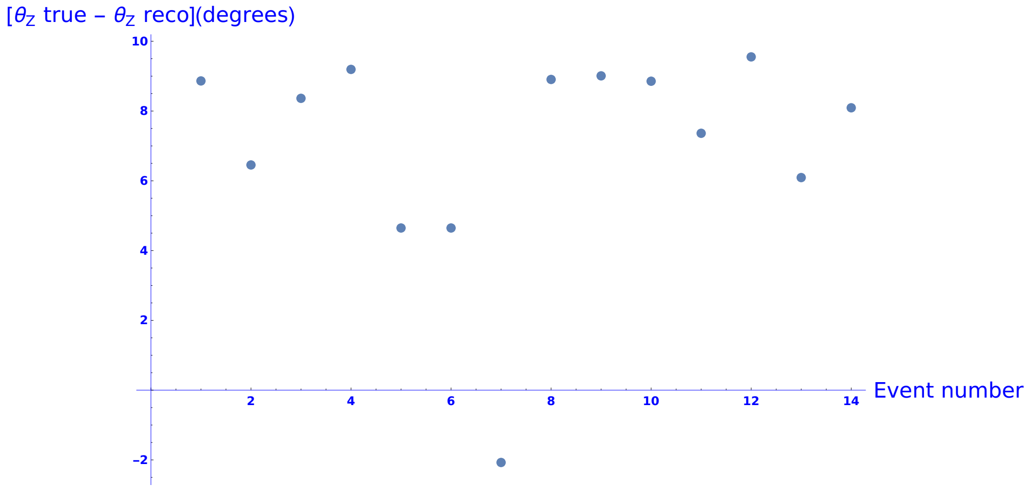
A small sample of Events: $\theta_X, \theta_Y, \theta_Z$ residuals for track 0



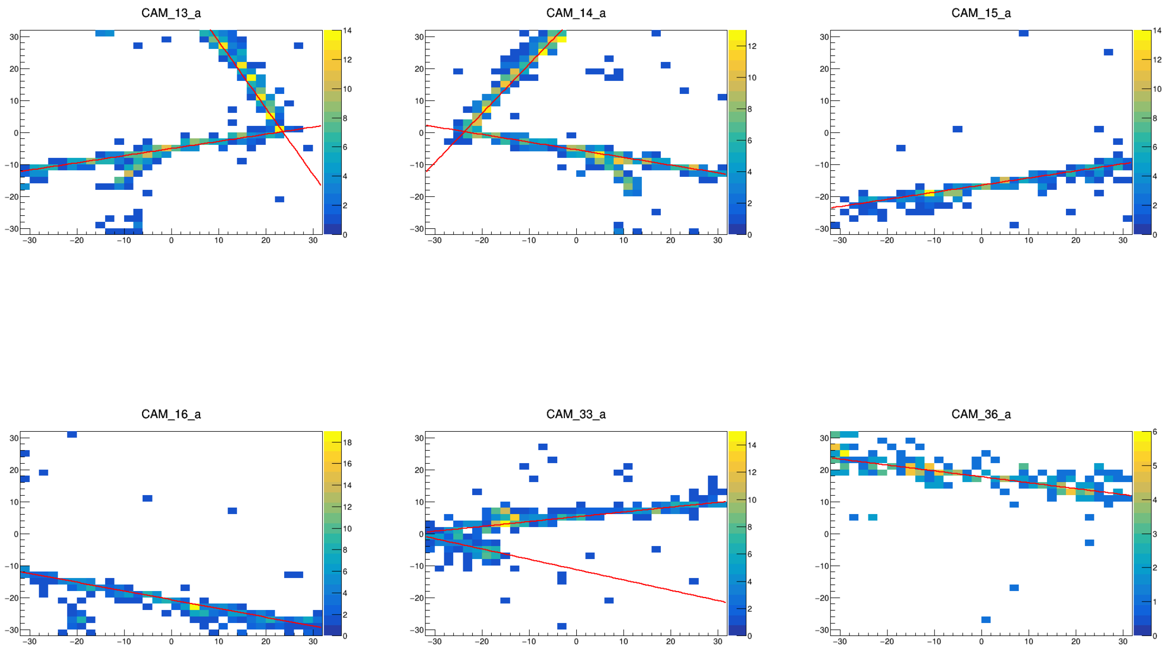
A small sample of Events: $\theta_X, \theta_Y, \theta_Z$ residuals for track 1

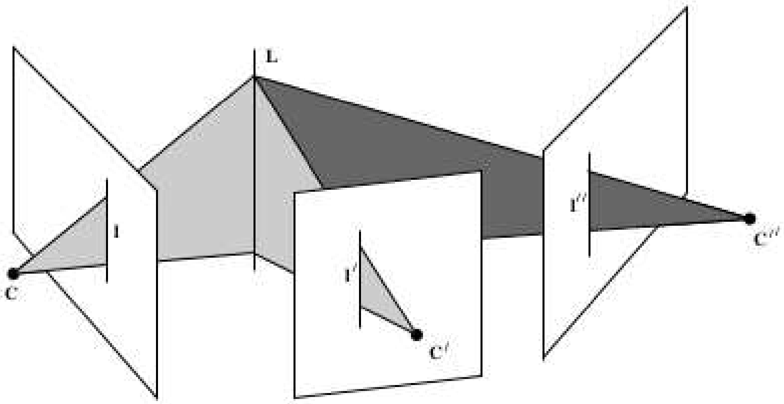






Track Matching problem, 2D Recognition and the Trifocal Tensor





TRIFOCAL TENSOR CONDITION and TRANSFER

$$\boxed{\mathcal{T}_i^{qr} = f_i^{qr}(\mathbf{P}, \mathbf{P}', \mathbf{P}'')}$$

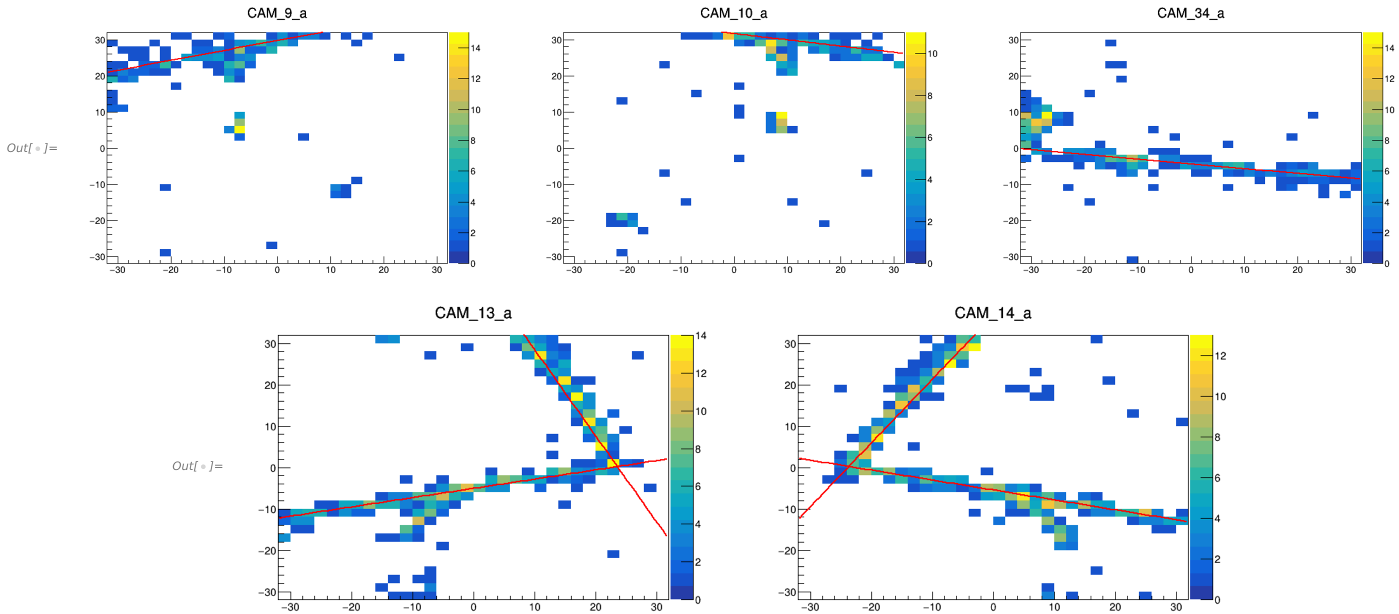
$$\mathcal{T}_i^{qr} = \frac{1}{2}\epsilon_{ilm} \det \begin{pmatrix} \mathbf{P}_{l1} & \mathbf{P}_{l2} & \mathbf{P}_{l3} & \mathbf{P}_{l4} \\ \mathbf{P}_{m1} & \mathbf{P}_{m2} & \mathbf{P}_{m3} & \mathbf{P}_{m4} \\ \mathbf{P}'_{q1} & \mathbf{P}'_{q2} & \mathbf{P}'_{q3} & \mathbf{P}'_{q4} \\ \mathbf{P}''_{r1} & \mathbf{P}''_{r2} & \mathbf{P}''_{r3} & \mathbf{P}''_{r4} \end{pmatrix}$$

Track Matching Condition

$$\boxed{l_p l'_q l''_r \epsilon^{piw} \mathcal{T}_i^{qr} = 0^w}$$

Current simulations: positive matching $\approx 10^{-2}$**, no matching** $\approx 10^3$

Image Transfer Equation



$$\boxed{l_p = l'_q l''_r \mathcal{T}_p^{qr}}$$

Conclusions

So far

- **Reconstruction of Light Points in GRAIN via Multiple View Projective Geometry**
- **Reconstruction of Tracks in GRAIN via Multiple View Projective Geometry**
- **Method improved by points and directions correspondence and vertex finding**
- **Triple view geometry for Track Recognition and IMAGE TRANSFER: Trifocal Tensor**

TO BE DONE

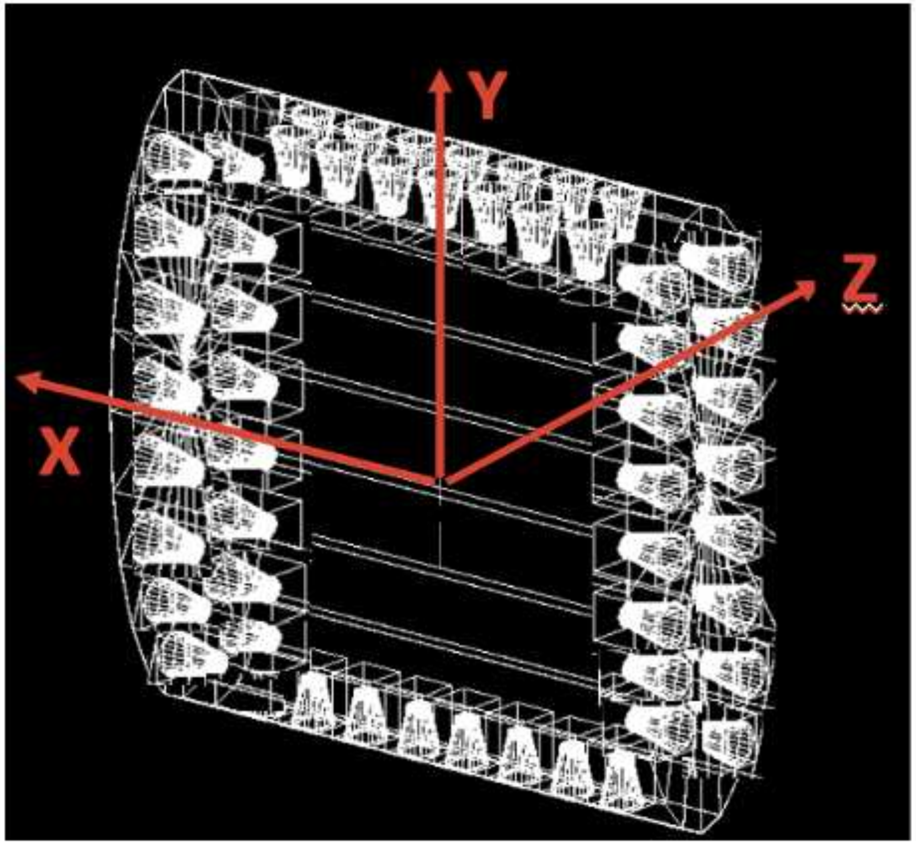
- **Study of a sample of 1K events distributed over GRAIN volume: single track and double track with vertex**
- **Extension to events with multiple vertices and multiple tracks**
- P **camera matrix calibration**
- **Software**

and ...

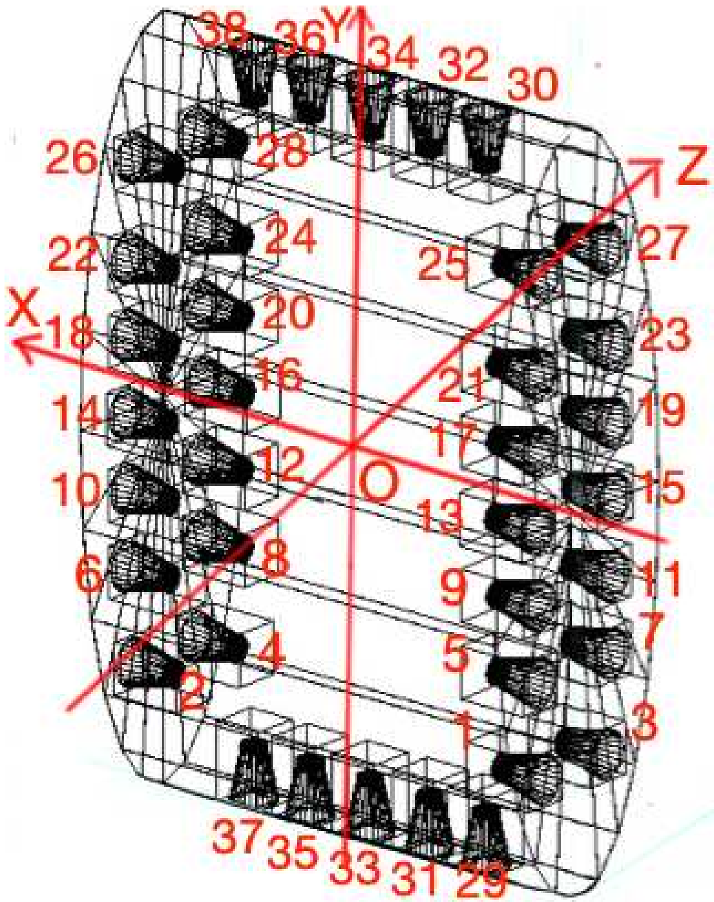
THANK YOU

For Your Attention!

Back-up



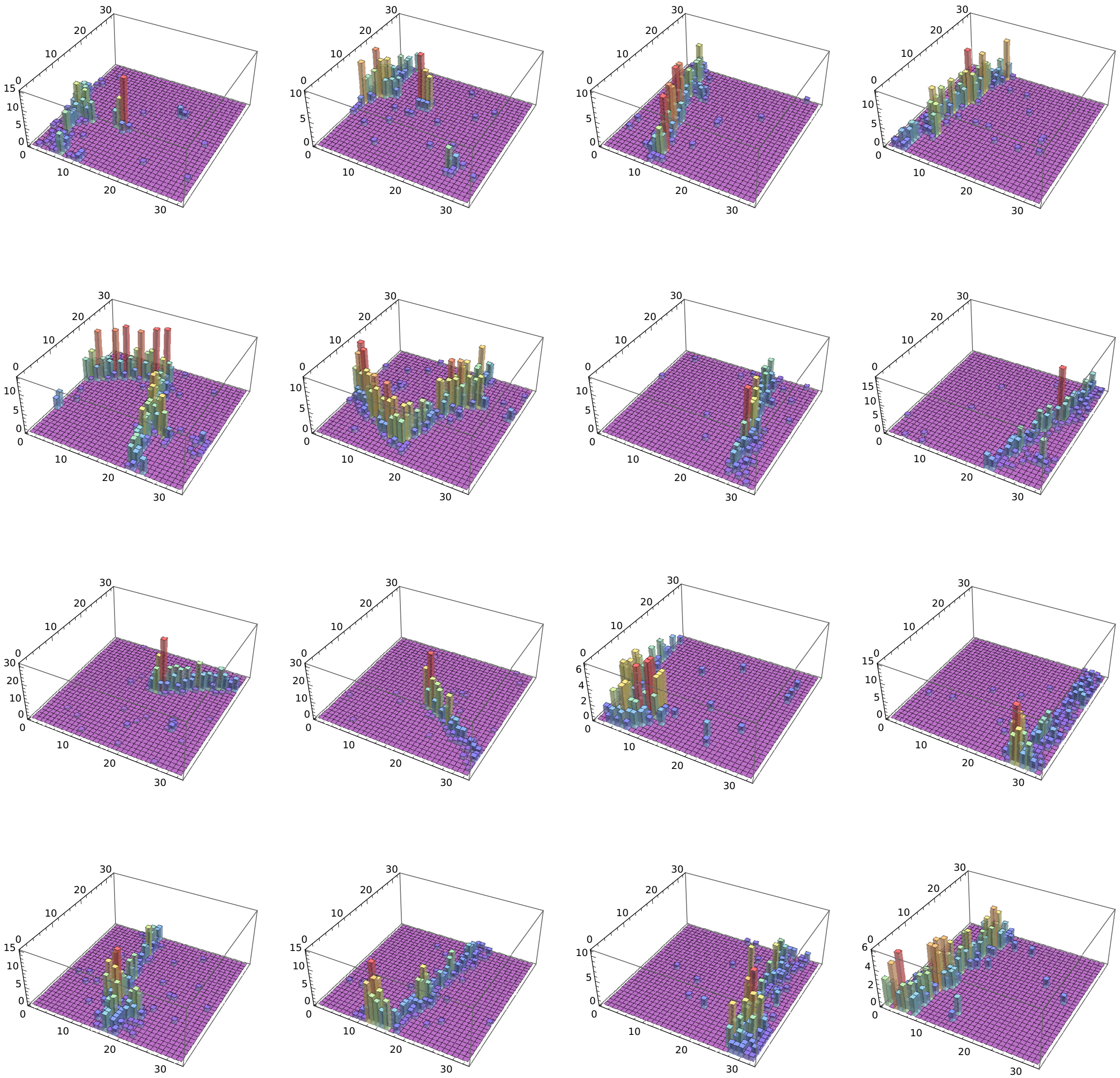
Y
Z
X



38 36 Y 34 32 30
26 28
22 24 27
25
X 18 20 Z 23
14 16 21 19
10 12 O 17 15
6 8 13 11
2 4 9 7
5 3
1
37 35 33 31 29

Photon Distribution on Fired Cameras

Out[ ]=



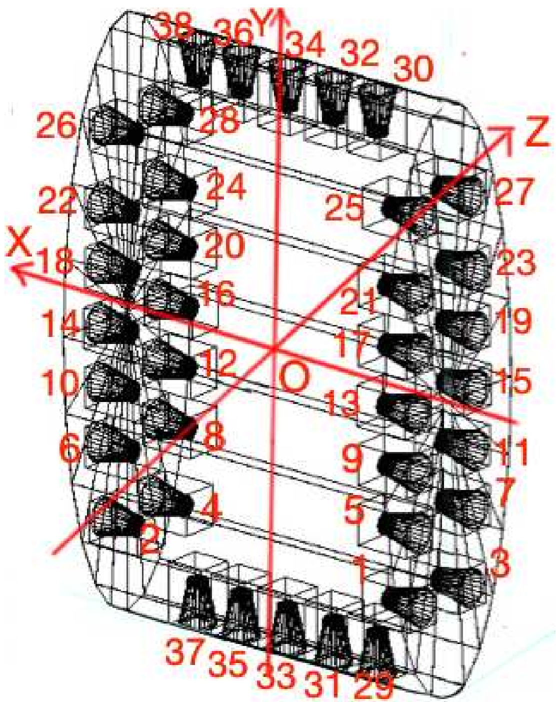
Details on Simulations

- 1000 light point sources
- 4×10^6 photons per point
- light-yield 4×10^4 photons/MeV
- energy release: 100 MeV per point
- processes: scattering, absorption, refractive indices depending on the wavelength

From Lenses to P-matrices

A $\mathbf{P}$-matrix is associated to each of the 38 camera-lenses of GRAIN

$$\text{GRAIN} \Longleftrightarrow \{\mathbf{P}_j\}_{j=1,\ldots,38} \tag{12}$$



Cameras on the 4 sides

$$\text{elliptic side 1, even numbers} \quad \mathbf{P}_{2j}, \quad j = 1, ..., 14 \tag{13}$$

$$\text{elliptic side 2, odd numbers} \quad \mathbf{P}_{2j+1}, \quad j = 0, ..., 13 \tag{14}$$

$$\text{top side} \quad \mathbf{P}_{2j}, \quad j = 15, ..., 19 \tag{15}$$

$$\text{bottom side} \quad \mathbf{P}_{2j+1}, \quad j = 14, ..., 18 \tag{16}$$

Elliptic side 1, even numbers

$$\mathbf{P}_{2j} = \mathbf{K}\left[\mathbf{R}| - \mathbf{R}C_{2j}\right], \quad j = 1, ..., 14 \tag{17}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \tag{18}$$

$$C_{2j}, \quad j = 1, ...14 \qquad \text{lens centers in GRAIN} \tag{19}$$

parameters: $c = 399, f_x = f_y = f = 100, p_z = 110, q_z = 105,$

$r_z = 90, p_y = 145, q_y = 290, r_y = 475, s = x_0 = y_0 = 0$

units: mm

Further parameterization of P-matrices for general Calibration

In fact, each of the 38 $\mathbf{P}$ matrices can be further parameterized by including **radial distorsion**

$$\mathbf{P}_{2j} = \mathbf{L}_{2j}\mathbf{K}\left[\mathbf{R}| - \mathbf{R}C_{2j}\right], \quad j = 1, ..., 14 \tag{20}$$

$$\mathbf{L}_{2j} = \begin{pmatrix} L_{2j}(r) & 0 & x_c \\ 0 & L_{2j}(r) & y_c \\ 0 & 0 & 1 \end{pmatrix} \tag{21}$$

$$L_{2j}(r) = 1 + k_1^{2j} r + k_2^{2j} r^2 + k_3^{2j} r^3 + ..., \qquad r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \tag{22}$$

(x, y) image coordinates on the sensor coordinate local frame, (x_c, y_c) distortion center on he sensor coordinate local frame

Elliptic side 2, even numbers

$$\mathbf{P}_{2j+1} = \mathbf{K}\left[\mathbf{R}_1 | -\mathbf{R}_1 C_{2j+1}\right], \quad j = 0, ..., 13 \tag{23}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R}_1 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{24}$$

$$C_{2j+1}, \quad j = 0, ...13 \qquad \text{lens centers in GRAIN} \tag{25}$$

parameters: $c = 399, f = 100, p_z = 110, q_z = 105,$

$r_z = 90, p_y = 145, q_y = 290, r_y = 475, s = x_0 = y_0 = 0$

units: mm

Top side, even numbers

$$\mathbf{P}_{2j} = \mathbf{K}\left[\mathbf{R}_2 | - \mathbf{R}_2 C_{2j}\right], \quad j = 15, ..., 19 \tag{26}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R}_2 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \tag{27}$$

$$C_{2j}, \quad j = 15, ...19 \qquad \text{lens centers in GRAIN} \tag{28}$$

parameters: $d = 609, f = 100, w_x = 145, v_x = 280, s = x_0 = y_0 = 0$

units: mm

Bottom side, odd numbers

$$\mathbf{P}_{2j+1} = \mathbf{K}\left[\mathbf{R}_3 | -\mathbf{R}_3 C_{2j+1}\right], \quad j = 14, ..., 18 \tag{29}$$

$$\mathbf{K} = \begin{pmatrix} -f_x & s & x_0 \\ 0 & -f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{R}_3 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{30}$$

$$C_{2j+1}, \quad j = 14, ...18 \qquad \text{lens centers in GRAIN} \tag{31}$$

parameters: $d = 609, f = 100, w_x = 145, v_x = 280, s = x_0 = y_0 = 0$

units: mm

Further parameterization of P-matrices for general Calibration

In fact, each of the 38 $\mathbf{P}$ matrices can be further parameterized by including **radial distorsion**

$$\mathbf{P}_j = \mathbf{L}_j \mathbf{K} \left[\mathbf{R}_j | - \mathbf{R}_j C_j \right], \quad j = 1, ..., 38 \tag{32}$$

$$\mathbf{L}_j = \begin{pmatrix} L_j(r) & 0 & x_c \\ 0 & L_j(r) & y_c \\ 0 & 0 & 1 \end{pmatrix} \tag{33}$$

$$L_j(r) = 1 + k_1^j r + k_2^j r^2 + k_3^j r^3 + ..., \qquad r = \sqrt{(x - x_c)^2 + (y - y_c)^2} \tag{34}$$

(x, y) image coordinates on the sensor local coordinate frame, (x_c, y_c) distortion center on the sensor local coordinate frame