# Can burial dating benefit from muon tomography ?
# Considerations, discussions and questions (*)

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June 2024

First step is evidently to try to understand how burial dating works without muon tomography….

With the help slides and references from Tebogo I have tried to understand…..

Please be aware that there can be misunderstandings from my side …
and that is why I hope this discussion will be useful ……

(*) For those who listened to me a month ago….this will be extensions and clarifications of what I said at that time

Some concepts:

We must separate processes before and after burial

Before the burial there are two extremes which are considered at the surface:

- Constant exposure of the quartz grain -the surface is not eroding

or

- Steady erosion in which a quartz grain in a steadily eroding rock approaches the surface at a constant rate

The burial process is a "rapid" process in which the quartz grain for some reason is buried in a cave or similar

- If the grain is buried deep enough there is no post-burial production of $^{26}Al$ and $^{10}Be$.
- If the grain is not buried deep enough there will be post-burial production of $^{26}Al$ and $^{10}Be$

The inherited content of Be and Al in the grain is the content before the burial process

The burial time is the time from the burial process to today

# One more important concept: secular equilibrium

Basic equation to determine N (atoms/g) in a sample

$$dN_i/dt = P_i(t) - \lambda N_i$$

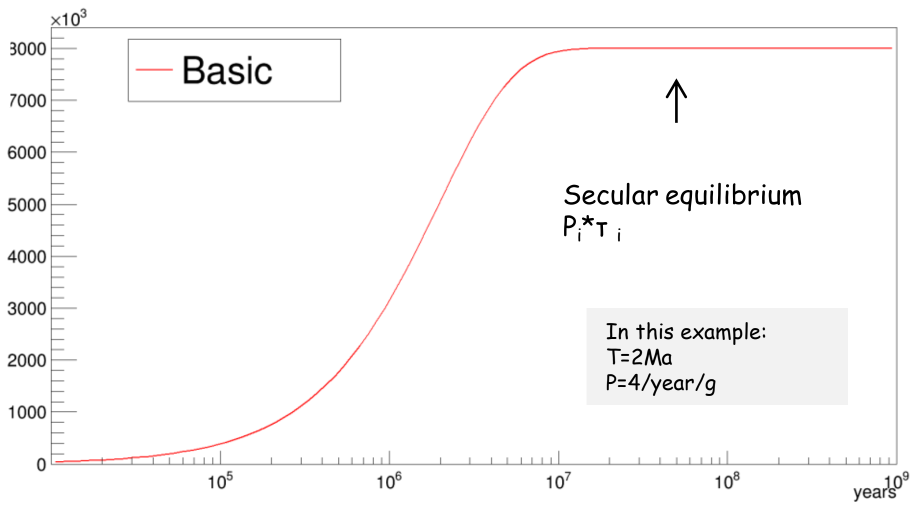
General solution

$\lambda = 1/\tau$

$$N_i = N_i(0)e^{-t/\tau_i} + P_i\tau_i\left(1 - e^{-t/\tau_i}\right)$$

Λ= the decay constant
T= mean life
P=production rate
( atoms per gram and year)
Indicis 10 or 26
for Be or Al

N (atoms/gr)



I will discuss the three methods for burial dating brought up by Granger 2014

## 14.7 Cosmogenic Nuclide Burial Dating in Archaeology and Paleoanthropology

**DE Granger**, Purdue University, West Lafayette, IN, USA



- Simple burial dating
- Min/max method
- Isochron dating

# Method 1:Simple burial dating

1. Simple burial method: does not account for post-burial production

$$t \approx \bar{\tau} ln \left[ -\frac{1}{2N_{10}^*} + \sqrt{\left(\frac{1}{2N_{10}^*}\right)^2 + \frac{2}{N_{26}^*}} \right]$$

t=burial time

$\tau$=average of $\tau_{10}$ and $\tau_{bur}$

$\tau_{bur} = 1/(1/\tau_{26} - 1/\tau_{10})$

* Means normalized to secular equilibrium at the surface

This formula is not derived in Granger 2014 but a set of equations which can be solved by iteration are derived. I have solved those equations by iteration and I have checked that the above formula gives the same answer as the equations given in Granger 2014.

Two things to note:

- The formula is derived under the steady erosion hypothesis ( I will come back to this)

- $N_{10}$ and $N_{26}$ must be normalized to the secular equilibrium at the **surface** i.e. $p_{10}$*$T_{10}$ or $p_{26}$*$T_{26}$

To calculate the simple burial time I need to know p10 and P26 at the **surface** i.e. the production of Be and Al…normally the units atoms/gr/year is used

At this point.. for illustrative purposes I will calculate p10 and p26 as a function of the depth/overburden

To start with I calculate $P_{10}$

$$P_i = \sum_j A_{i,j} e^{-z/L_j}$$

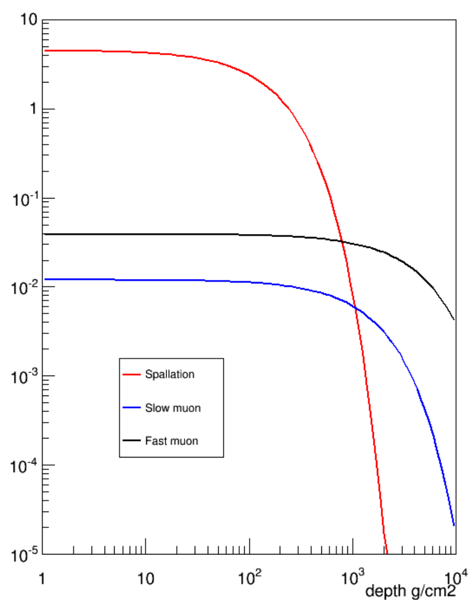
$L_j$ are attenuation lengths for spallation, slow muons and fast muons

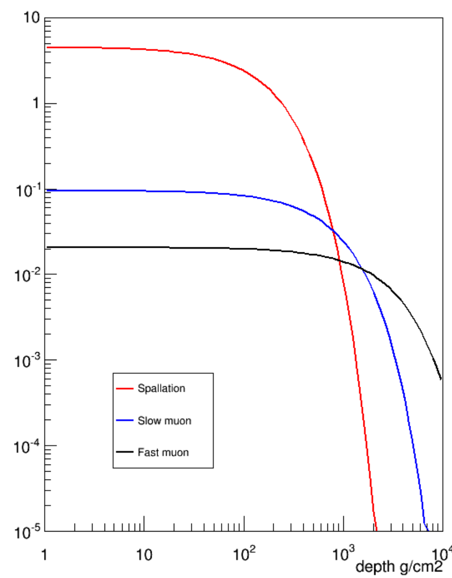
….. here I can use Granger 2014 table 2 which uses the reference Braucher 2011 normalized to sea level and high latitude(SLHL)

*Braucher et al. (2011)*

| $A_{10}$ | L |
|---|---|
| 4.5 | 160 |
| 0.012 | 1500 |
| 0.039 | 4320 |

$P_{10}$ Braucher 2011 (from Granger 2014)



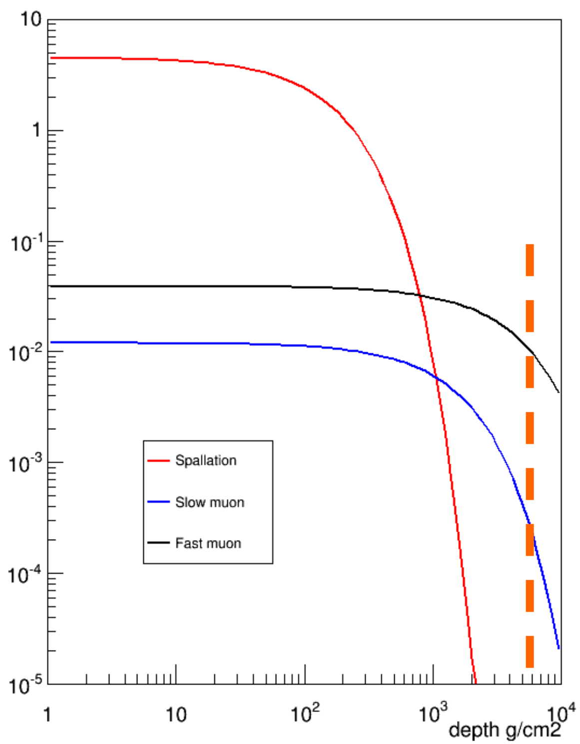



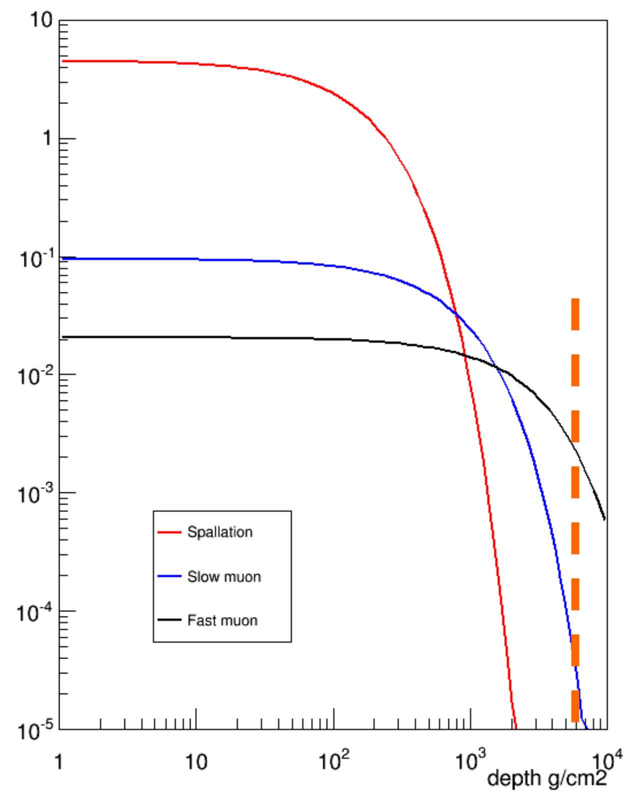

Cf with Granger Smith (2000) to get a feeling for the sensitivity

*Granger and Smith (2000)*

| $A_{10}$ | L |
|---|---|
| 4.5 | 160 |
| 0.0955 | 738.6 |
| 0.0206 | 2688 |
| 0.026 | 4360 |

I know from Tebogo that we have
vertical overburden about 30 m of density 2.1 g/cm3
thus we assume 6300 g/cm2 of overburden Carstic dolomite



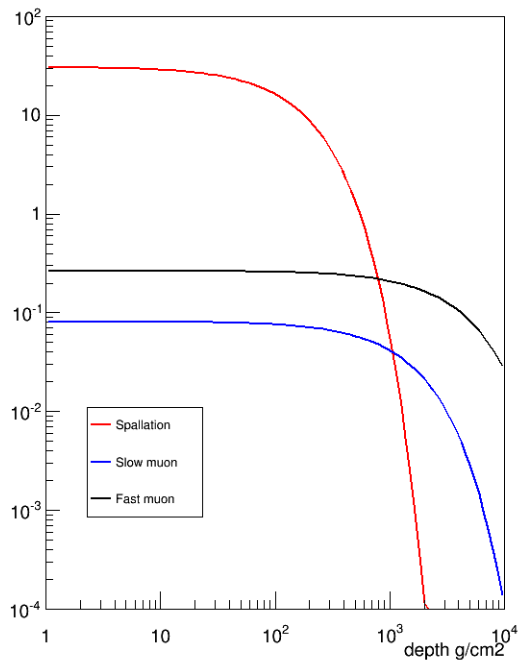




Consequence: We are in a region were spallation is of very little importance
and where fast and slow muons dominate....uncertainties are quite big
as seen above...we are also in a region where production rates are quite low...

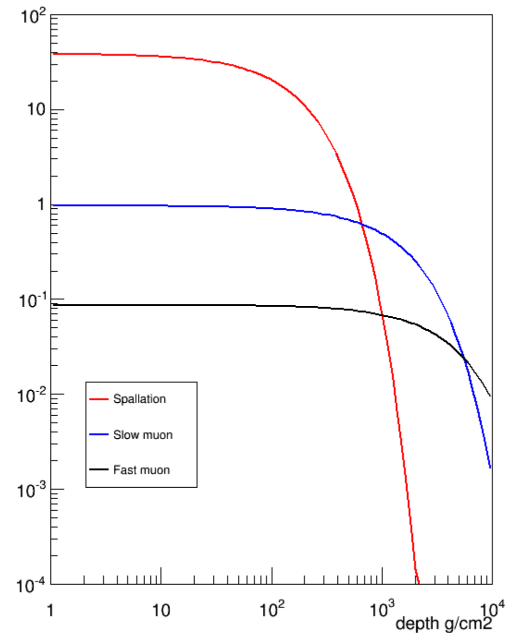
Now we need $P_{26}$ but… it is not given in the review Granger 2014…
But it is stated that in general P26 can to a good approximation be taken as 6.8*p10…

Braucher 2011 Table 4 gives values of $P_{26}$ for spallation, slow muons and fast muons … ( not normalized to SLHL)
I compare those with the result of multiplying all constants for $p_{10}$ with 6.8

$P_{26}$ from $p_{10}$*6.8




$p_{26}$ Braucher 2011



Equipped with values of P10 and P26 I can calculate simple burial using the Be and Al content from Tebogo's data sheet here below

| Sediment from blocks prepared for fossil recovery | | | | | | | | | Simple Burial | |
|---|---|---|---|---|---|---|---|---|---|---|
| Sample ID | True sample name | N10 | σN10 | N26 | σN26 | 26Al/10Be | σ26Al/10Be | | Age (Ma) | +/- (Ma) |
| TM-4 | UW105B008 | 6.39E+05 | 1.76E+04 | 1.72E+06 | 1.28E+05 | 2.69 | 0.08 | | 1.75 | 0.29 |
| TM-11 | UW105B005 | 5.81E+05 | 1.41E+04 | 8.81E+05 | 1.26E+05 | 1.52 | 0.14 | | 2.84 | 0.58 |
| TM-13 | UW105B002 | 5.20E+05 | 1.71E+04 | 3.95E+05 | 9.81E+04 | 0.76 | 0.25 | | 4.09 | 1.18 |
| TM-15 | UW105B007 | 6.40E+05 | 2.14E+04 | 1.01E+06 | 1.97E+05 | 1.58 | 0.20 | | 2.74 | 0.67 |
| TM-17 | UW105B004 | 5.52E+05 | 1.52E+04 | 1.03E+06 | 1.10E+05 | 1.87 | 0.11 | | 2.45 | 0.44 |

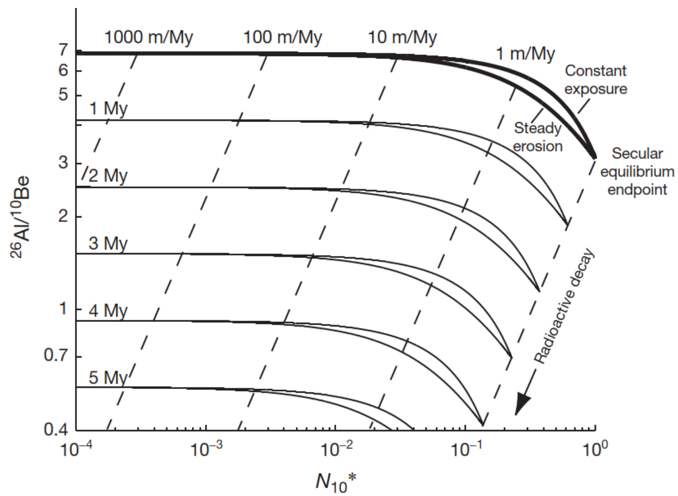
My results:

| Sample | Simple Burial Tebogo Sheet (Ma) | Uncer-Tainty (Ma) | | My attempt (Granger 2014 with p26= 6.8*p10) (Ma) | | My attempt (Braucher 2011) (Ma) |
|---|---|---|---|---|---|---|
| TM-4 | 1.75 | 0.29 | | 1.63 | | 1.67 |
| TM-11 | 2.84 | 0.58 | | 2.66 | | 2.73 |
| TM-13 | 4.09 | 1.18 | | 3.85 | | 3.94 |
| TM-15 | 2.74 | 0.67 | | 2.55 | | 2.62 |
| TM-17 | 2.45 | 0.44 | | 2.32 | | 2.37 |

Rather good agreement but my attempts always on the lower side

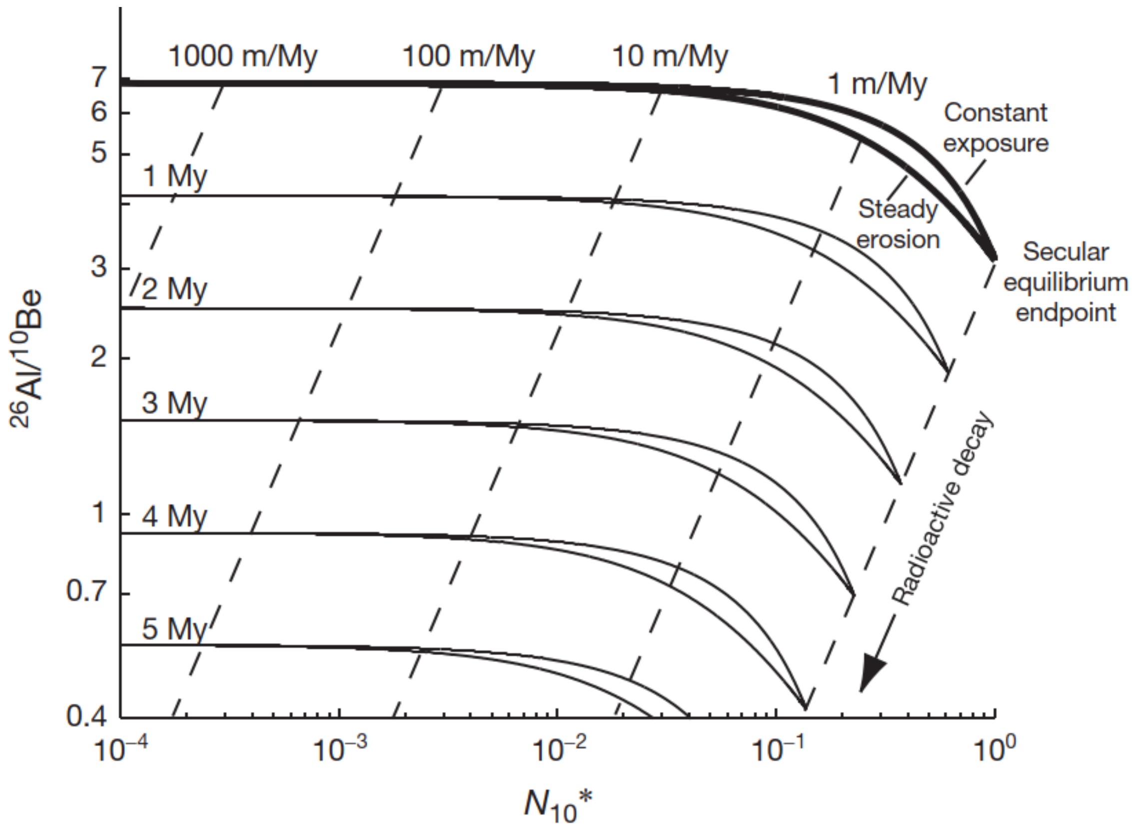
Question: How are the uncertainties estimated?

To try to understand better what is going on I decided to look at the “banana” plot




The plot is based upon a number of equations given in Granger 6, 9, 19,20,21…

# Lets have a look at the standard banana plot




One easy way to understand.
Consider burial time =0
If the surface has ben exposed for more than say 10 million years (seqular equilibrium: )
Then we are here.

If we have extremely fast erosion rates:
Then we are here

(It is not possible to distinguish between Beginning of exposure and long exposure But high ersoion rate for a long time)

If at the time when the burial is happening we are far away from the secular equilibrium at the surface then it is impossible to separate the scenario of constant exposure and a certain rate of “fast” erosion. (faster than 10m/year)

This distinction start to possible if the erosion rate is slower than about 10 m/My

Very important to observe !

The equations that are behind the banana plot describes
a situation where the sample have a “simple “ history .

This means either constant exposure without erosion or exposure with a steady erosion rate and then....

... suddenly followed by a “deep” rapid burial process.

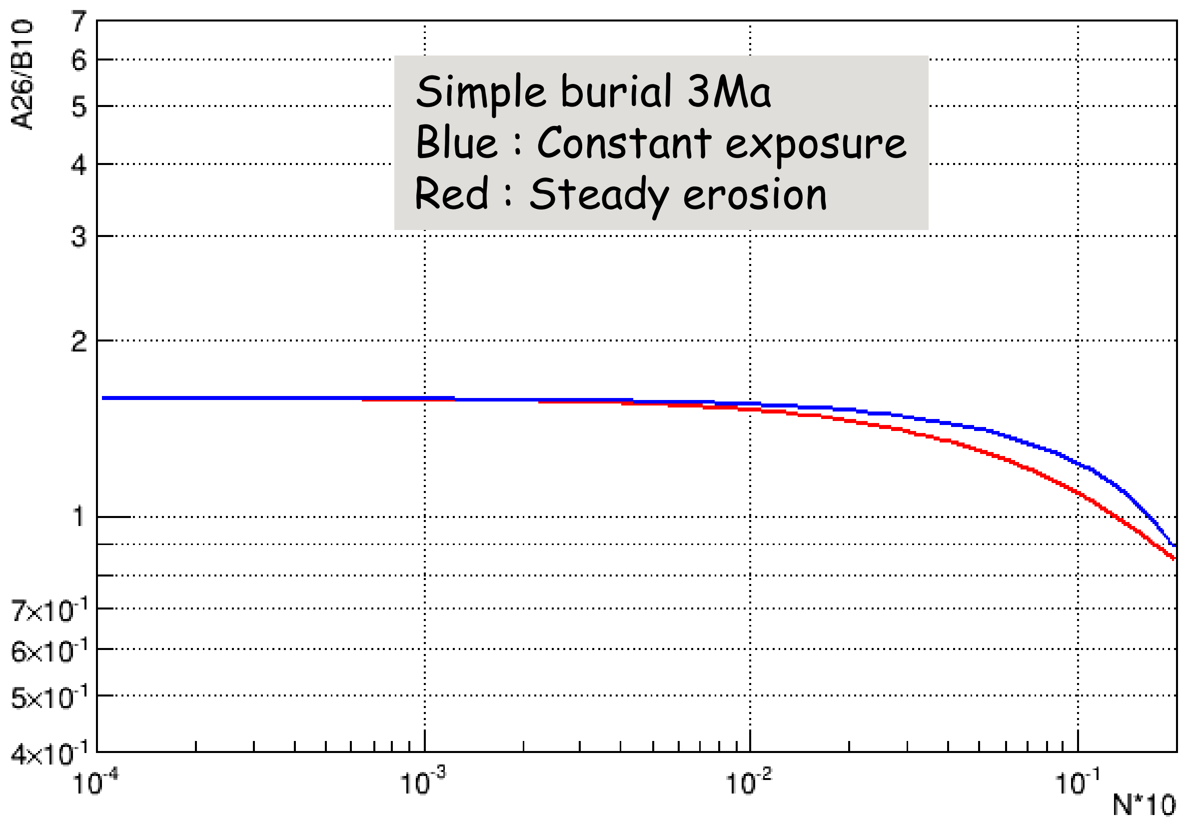
“Deep” means deep enough to have no post-burial exposure.

In reality the sample may have a much more complicated exposure-burial history

...i.e. repeated episodes of exposure, erosion, shallow burial or deep burial

To see where the rates from Tebogos samples are situated I made my own "banana" plot

Example: burial time 3Ma and using equations (19), (20) and (21) for erosion hypothesis and (19), (20) and (6) for constant exposure without erosion (Braucher 2011 used for $P_{10}$ and $p_{26}$)




Basic difference between the two hypothesis is what assumption one does concerning the inherited ratio $R_{inh}$ ..

Constant exposure

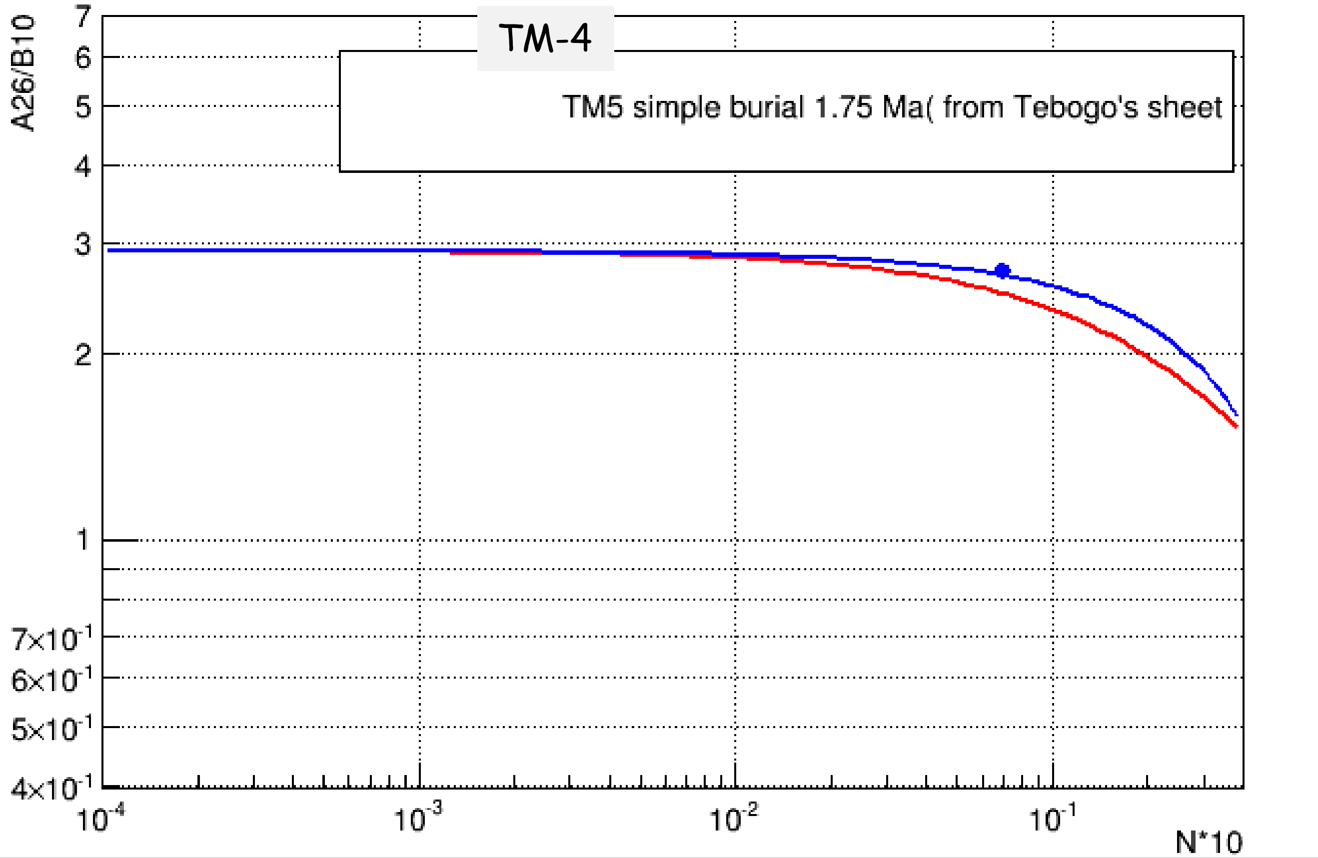
$$N_{26}/N_{10} = (P_{26}/P_{10})(1 - N^{*}_{10}/2)$$

Steady erosion

$$N_{26}/N_{10} = (P_{26}/P_{10})/(1 + N^{*}_{10})$$

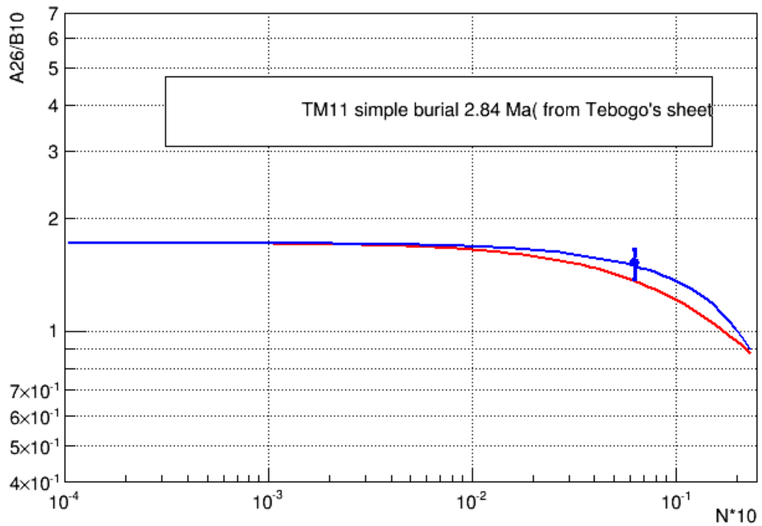
Where are the site 105 data points on the "banana"plot?

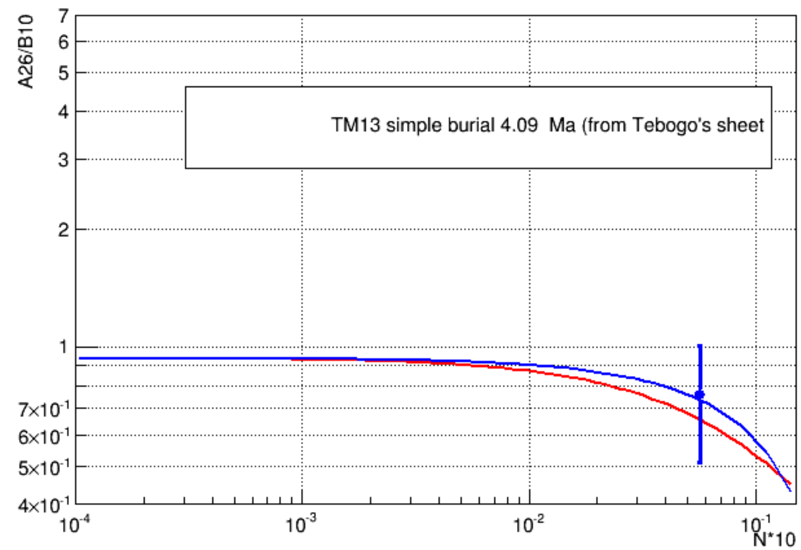
Take as an example TM-4

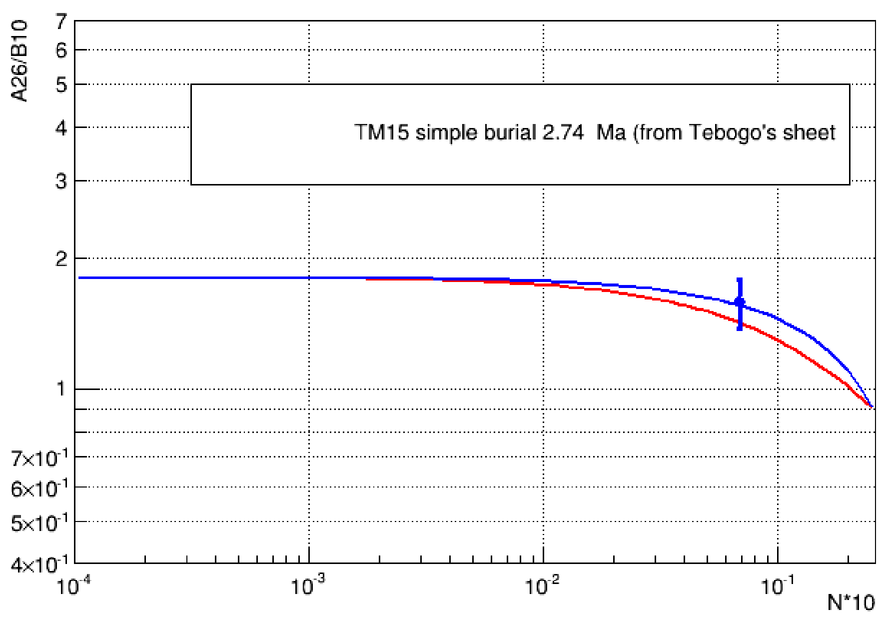


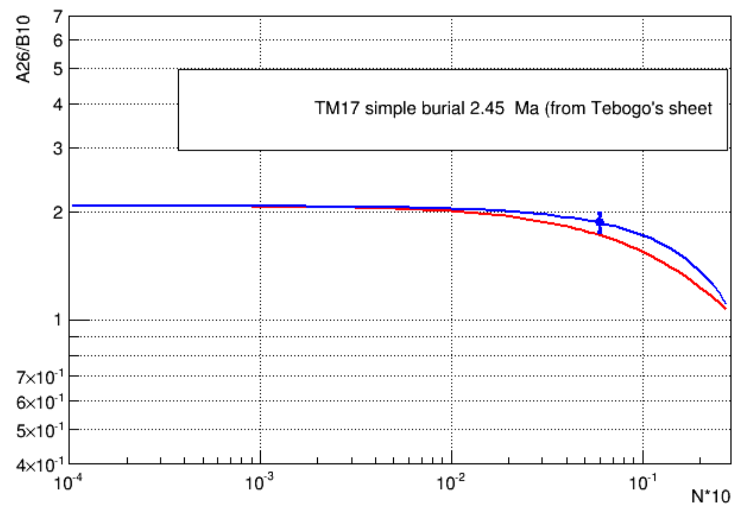

It looks like the data points on Tebogo's sheet have been calculated for the constant exposure hypothesis…no erosion …

The four other samples show the same thing…..



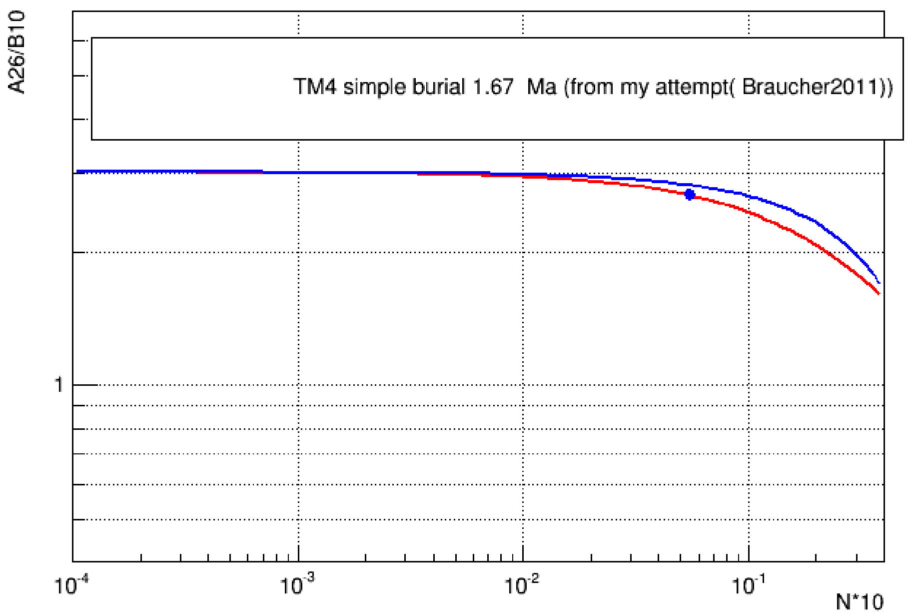



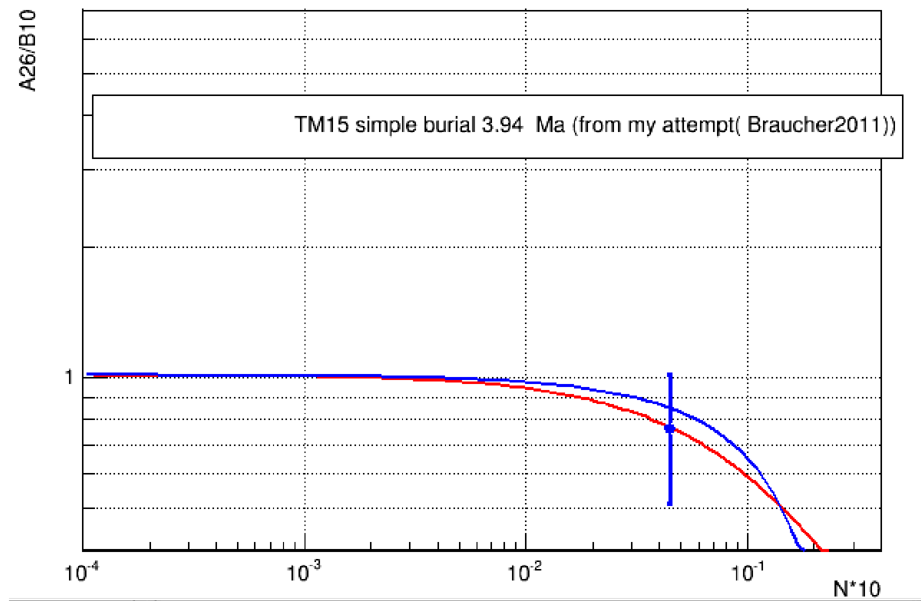



It looks like the difference between Tebogo's data sheet and my attempt is that I used the equation below which assumes steady erosion while Tebogo used the hypothesis of constant exposure

$$t \approx \tau ln\left[-\frac{1}{2N^*_{10}} + \sqrt{\left(\frac{1}{2N^*_{10}}\right)^2 + \frac{2}{N^*_{26}}}\right]$$

Confirmed by the banana plots below



From previous slide:

“It looks like the difference between Tebogo’s data sheet and my attempt is that I used the equation which assumes steady erosion while Tebogo used the hypothesis of constant exposure ”

Now I will show that my conclusion above turn out to be faulty:

I show this because it illustrates very well how sensitive we are to the fact if the p-values are normalized to the altitude and latitude of the cave or not.

For what I have shown you up to now I did not have the P’s adapted to altitude and latitude of the 105 Cave

Now I have got them from Tebogo !

From Tebogo for spallation;: **P10 = 8.8 +/- 0.88 at/g.yr** **and P26 = 59 +/- 5.9 at/g.yr.**
Combined muon production : **P10 = 0.12 +/- 0.012 at/g.yr** **and P26 = 1.136 +/- 0.1136 at/g.yr**

Question : The values are adapted to the height and latitude of the 105 site but where are the SLHL values coming from and how are the uncertainties estimated?

Using the steady erosion hypothesis and the values I got from Tebogo I get

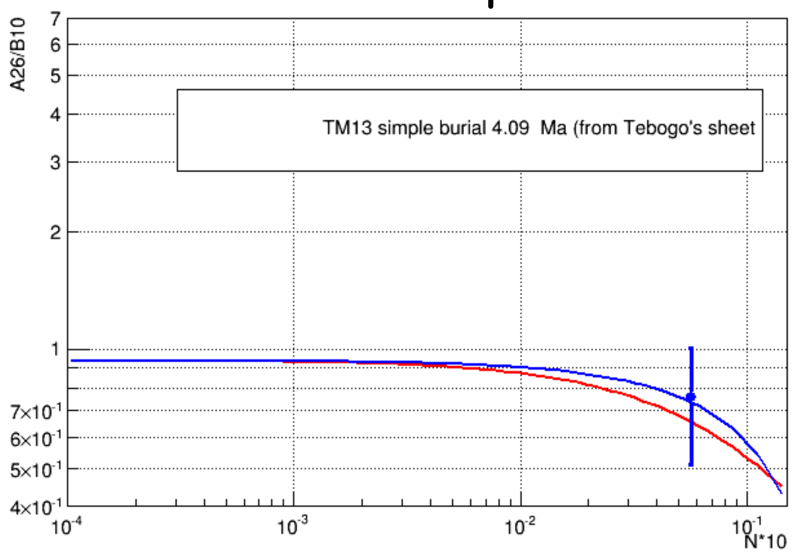
| Sample | Simple Burial Tebogo Sheet (Ma) | My calculation using P Values from Tebogo |
|---|---|---|
| TM-4 | 1.75 | 1.74 |
| TM-11 | 2.84 | 2.83 |
| TM-13 | 4.09 | 4.09 |
| TM-15 | 2.74 | 2.73 |
| TM-17 | 2.45 | 2.45 |

To be discussed:
…
To study how tomography helps
I need to separate the muons in fast and slow ( this is not needed for the simple burial dating here).
I need the different attenuation lengths for spallation, slow muons, fast muons ….

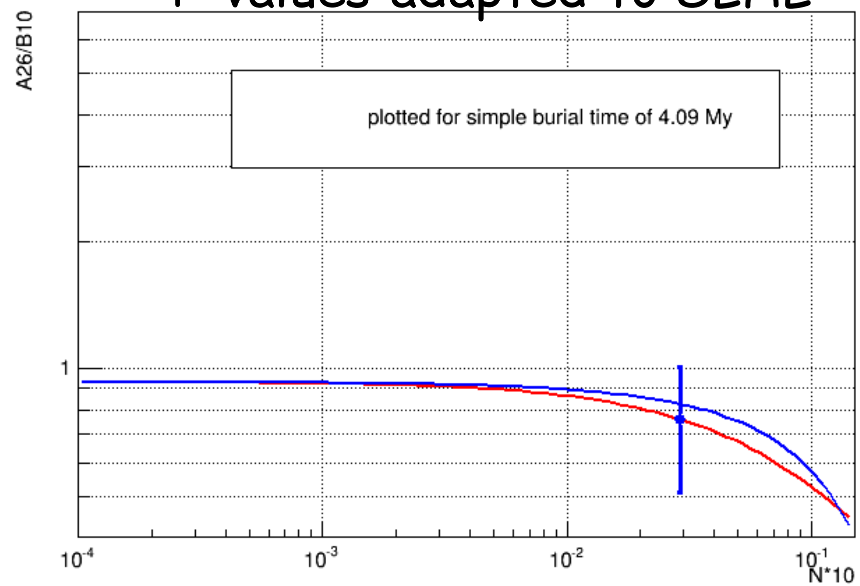
Excellent agreement

Going back to the banana plot with the p-values adapted to the 105 cave..

P-values not adapted SLHL




P-values adapted to SLHL




Thus my previous conclusion that Tebogo had used the constant exposure hypothesis was wrong because slightly wrong p-values changed the picture going from constant exposure to steady erosion by chance.

Or said differently

Tebogo has used the steady erosion hypothesis with p-values adapted to the cave while I used the constant exposure hypotheses with p-values not adapted to the cave and by chance this gave the same answer i.e. 4.09 Ma

What lesson can we draw from this?

Using method 1) i.e the simple burial dating we are very sensitive to p10 and p26 at the surface

| Sample | Simple Burial Tebogo Sheet (Ma) | Uncer-Tainty (Ma) | My attempt (Granger 2014 with p26= 6.8*p10) (Ma) | My attempt (Braucher 2011) (Ma) |
|---|---|---|---|---|
| TM-4 | 1.75 | 0.29 | 1.63 | 1.67 |
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| Sample | Simple Burial Tebogo Sheet (Ma) | My calculation using P Values from Tebogo |
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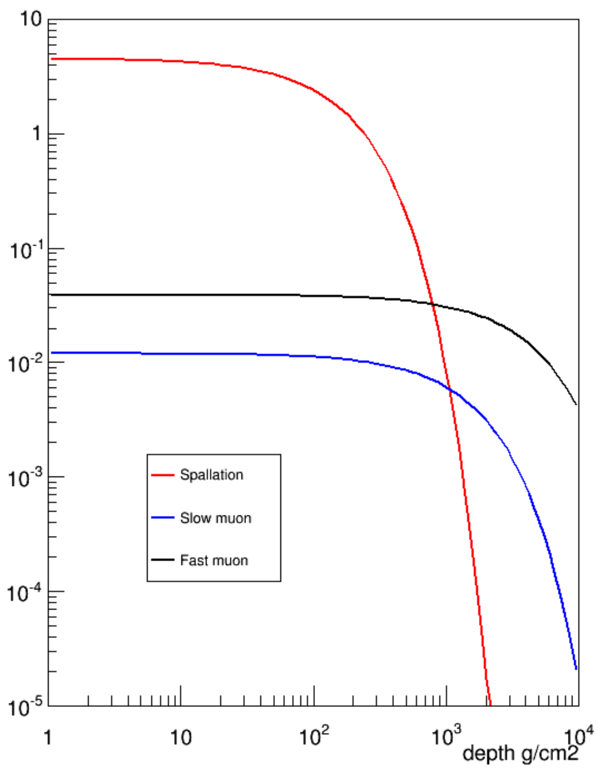
| TM-17 | 2.45 | 2.45 |

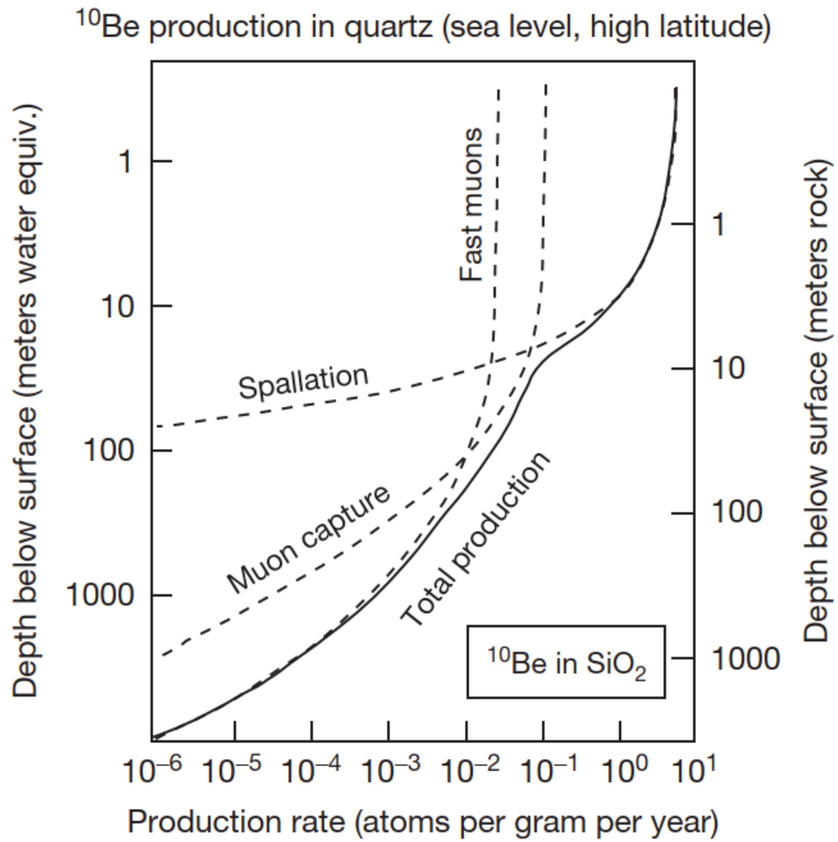
We see differences in the simple burial time of 5-10% depending on which p values used and to which extent they are adapted to the SLHL position of the cave....

P values at the surface are completely dominated by spallation ( originated by the cosmic neutrons)

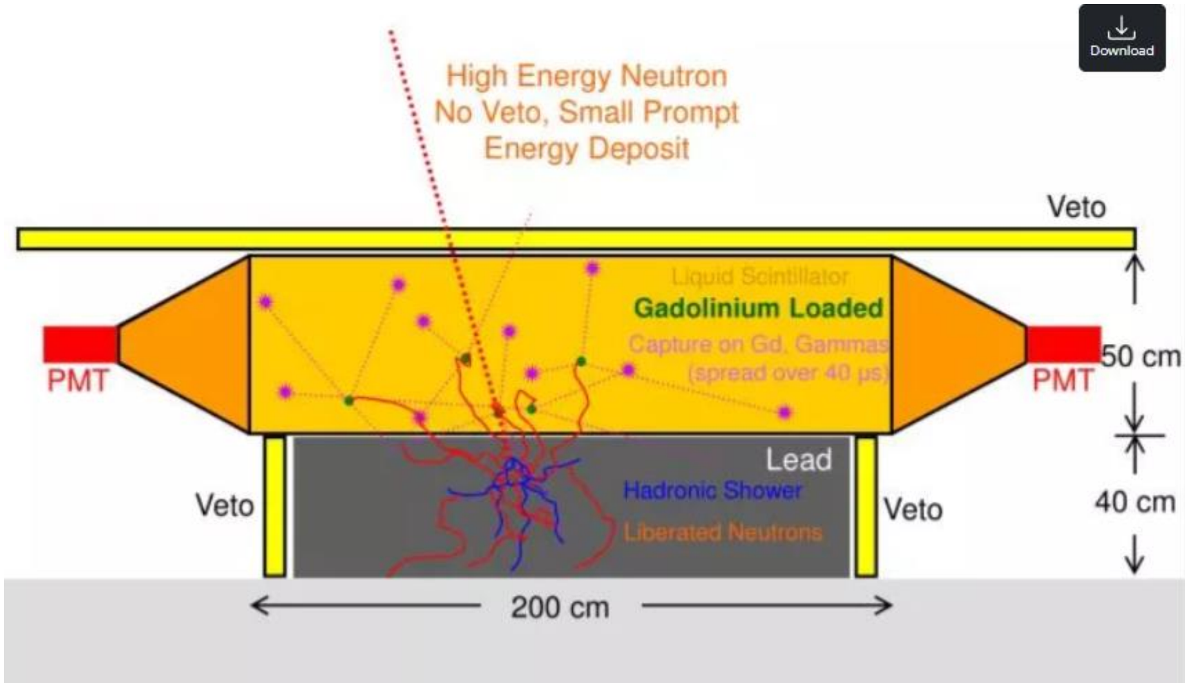
My curve




Granger 2014



To improve on burial dating with the simple burial method we need a neutron detector to be employed at the surface




( this is just an arbitrary picture.....
not likely the neutron counter one would need)

# Summary method one: Simple burial dating

- The simple burial method assumes that the burial is so deep that post burial production is irrelevant. This means that the knowledge of the overburden thickness is not needed.

- In this case muon tomography will not improve the dating …only if it is discovered that there are such big cavities above the cave that post-burial production can not be neglected any more.

- The simple burial method might give a more precise result if the neutron flux is measured at the surface…..
The importance of this and which type of detector to use may be something to be studied?

# Method 2: Min/Max method

…sample not buried deep enough to ignore post burial production

MIN determination straight forward….postburial production always increase Al/Be ratio

The simple method always indicates a minimum i.e no postburial production

Use the steady erosion hypothesis that always gives a lower value than constant exposure

| Sample | Min Tebogo data sheet (Ma) | My attempt (p26= 6.8*p10) |
|---|---|---|
| TM-4 | 1.68 | 1.63 |
| TM-11 | 2.64 | 2.66 |
| TM-13 | 3.52 | 3.85 |
| TM-15 | 2.58 | 2.55 |
| TM-17 | 2.31 | 2.32 |

| Sample | Simple Burial Tebogo Sheet (Ma) |
|---|---|
| TM-4 | 1.75 |
| TM-11 | 2.84 |
| TM-13 | 4.09 |
| TM-15 | 2.74 |
| TM-17 | 2.45 |

Question: Why is the values on Tebogos data sheet for minimum not identical to this for the simple burial dataing?

Now start with MAX rate which is more tricky!!).
One get the highest post burial production rate possible
by assuming that the sample has always been buried at present depth .....

Now calculate $P_i$ as a function of overburden

Use
$$P_i = \sum_j A_{i,j} e^{-z/L_j} \qquad [2]$$

Determine postburial production assuming initial value for burial t'

$$N_{i,\mathrm{pb}} = P_{i,\mathrm{pb}} \tau_i \left(1 - e^{-t/\tau_i}\right) \qquad [32]$$

Use equation (16) to calculate burial age

$$\left(N_{26} - N_{26,\mathrm{pb}}\right) / \left(N_{10} - N_{10,\mathrm{pb}}\right) = R_{\mathrm{inh}} e^{-t/\tau_{\mathrm{bur}}} \qquad [16]$$

where $\tau_{\mathrm{bur}}$ is a decay constant given by

$$\tau_{\mathrm{bur}} = 1/\left(1/\tau_{26} - 1/\tau_{10}\right) \qquad [17]$$

and

$$R_{\mathrm{inh}} = \left(N_{26,\mathrm{inh}} / N_{10,\mathrm{inh}}\right) \qquad [18]$$

Must keep in mind that the inherited concentration also has to be adjusted for postburial production

$$N_{i,\mathrm{inh}} = \left(N_i - N_{i,\mathrm{pb}}\right) e^{t'/\tau_i} \qquad [33]$$

Solve for the burial age by solving those equations by iteration !!

**Observe important**:

I use the steady erosion hypothesis to calculate $R_{inh}$. This gives a higher maximum than constant exposure

My understanding is that we calculate the max value by assuming steady erosion until burying and afterwards no erosion.

This might sound strange but it gives an absolute maximum possible…

Question: Is my understanding here correct ?

# My results for Max

Caveat: the correct application of the previous equations is a bit tricky and I might easily have made mistakes. At least the solution of the equations converge with the iteration method.

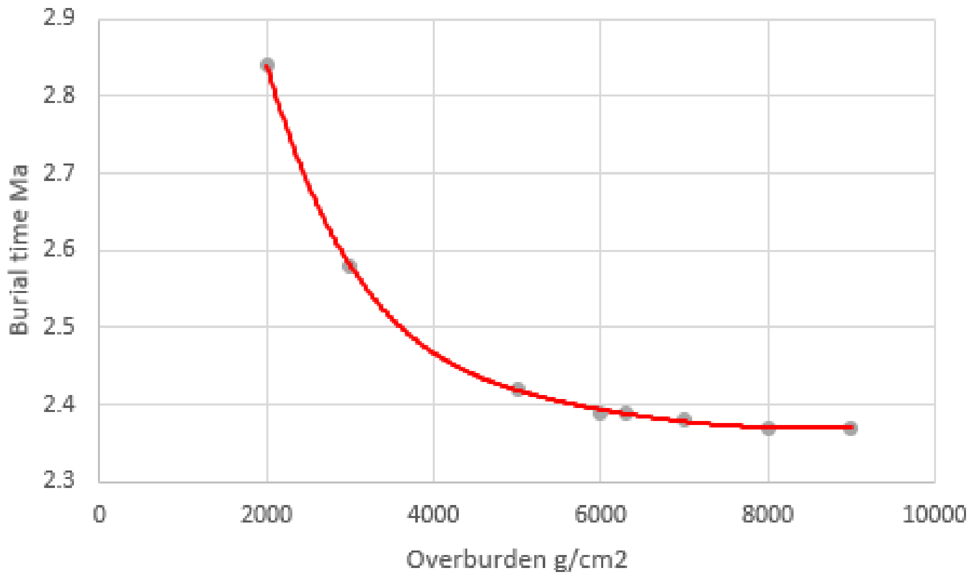
For this calculations I need to know the overburden and I use the approx. value from Tebogo of 6300 g/cm2 (observe spallation assumption does not matter here)

| Sample | Max Tebogo data sheet (Ma) | Braucher 2011 No SLHL corr | Granger 2014 P26 scaled 6.8*p10 |
|---|---|---|---|
| TM-4 | 1.71 | 1.67 | 1.64 |
| TM-11 | 2.73 | 2.76 | 2.75 |
| TM-13 | 3.80 | 4.06 | 4.09 |
| TM-15 | 2.65 | 2.66 | 2.63 |
| TM-17 | 2.37 | 2.39 | 2.38 |

Very good agreement with Tebogo except TM13....?
Tebogo: TM13 is in general problematic

However I might get even better agreement if I would have the attenuation length of fast and slow muons used by Tebogo

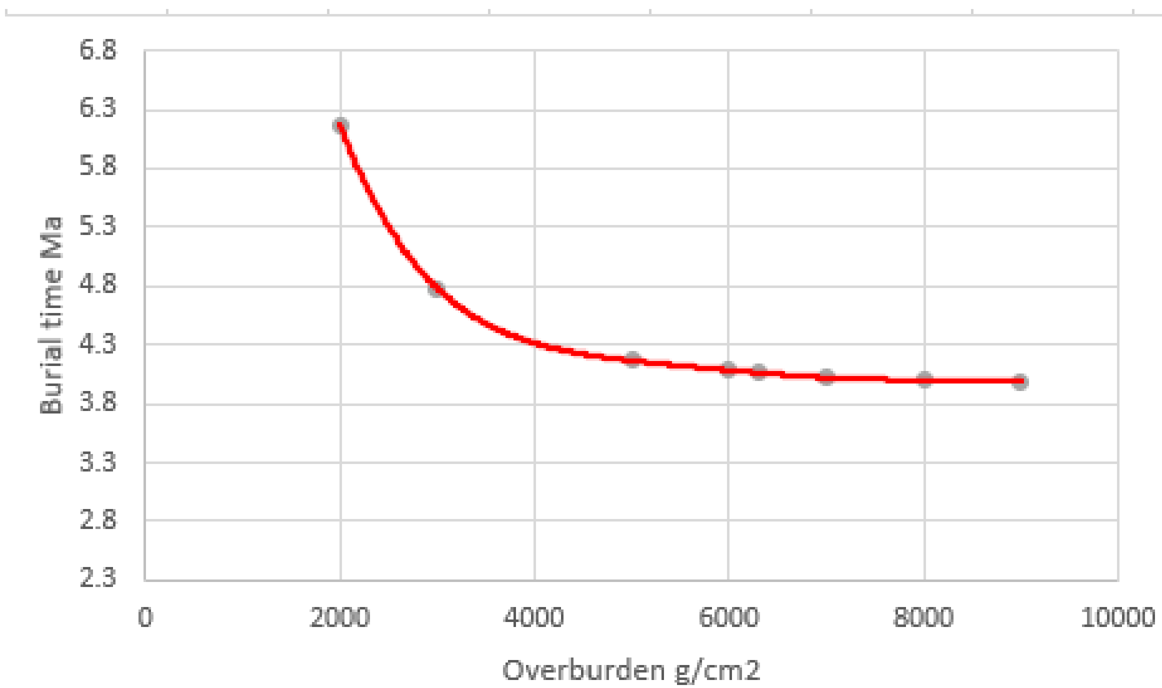
Now I am equipped to calculate the sensitivity to the overburden ( I take TM17 as an example and I use Braucher 2011)




We see that we are quite unsensitive to the overburden…..if the true overburden would be half of what we think the max time would change from 2.37 Ma to 2.68 Ma

To know to what extent muon tomography will improve dating we need to know what precision in the overburden thickness can be obtained
..and compare with the size of other uncertainties…..

Let’s see what happens for another sample….TM13 ( is somewhat extreme)




Also here we are quite insensitive to the overburden
if the true overburden would be half of what we think the max time
would change from 4.06 Ma to 4.78 Ma

Recently we have got from Tebogo shallow samples RS105 amphitheater-shallow samples (6m)…
Let's look at these samples before discussing method 3 ( isochron dating)

| Sediment from cave insitu samples | | | | | | | | Simple Burial | |
|---|---|---|---|---|---|---|---|---|---|
| Sample ID | True samp | N10 | σN10 | N26 | σN26 | 26Al/10Be | σ26Al/10Be | Age (Ma) | +/- (Ma) |
| TM-5 | AEW4C | 7.12E+05 | 1.86E+04 | 3.99E+06 | 2.31E+05 | 5.60 | 0.06 | 0.33 | 0.05 |
| TM-14 | AEW2C1 | 7.88E+05 | 2.09E+04 | 2.09E+06 | 2.64E+05 | 2.65 | 0.13 | 1.74 | 0.34 |
| TM-16 | AEW2C1 | 7.84E+05 | 1.99E+04 | 1.69E+06 | 1.40E+05 | 2.16 | 0.09 | 2.13 | 0.36 |
| TM-18 | AEW5C | 8.88E+05 | 2.41E+04 | 3.50E+06 | 1.74E+05 | 3.94 | 0.06 | 0.98 | 0.15 |

| Samples | Simple burial Tebogos data Sheet (Ma) | My calculation using p values from Teboo (Ma) |
|---|---|---|
| TM-5 | 0.33 | 0.31 |
| TM-14 | 1.74 | 1.73 |
| TM-16 | 2.13 | 2.12 |
| TM-18 | 0.98 | 0.97 |

Simple burial method…perfect agreement with Tebogo

To go further to estimate the sensitivity of the burial age to the depth
( using Method 2: Min/Max)
I need to separate slow and fast muons which have different attenuation length.....I don't have this information from Tebogos combined muons

Thus instead of using Tebogos combined I will see what happens if I use Braucher 2011 directly from their publication using the attenuation lengths which they use....

30 m sample

| Sample | Simple Burial Tebogo Sheet (Ma) | My calculation using P Values from Tebogo | Tebogo Spallation Muons Directly Braucher 2011 Erosion |
|---|---|---|---|
| TM-4 | 1.75 | 1.74 | 1.75 |
| TM-11 | 2.84 | 2.83 | 2.83 |
| TM-13 | 4.09 | 4.09 | 4.09 |
| TM-15 | 2.74 | 2.73 | 2.74 |
| TM-17 | 2.45 | 2.45 | 2.46 |

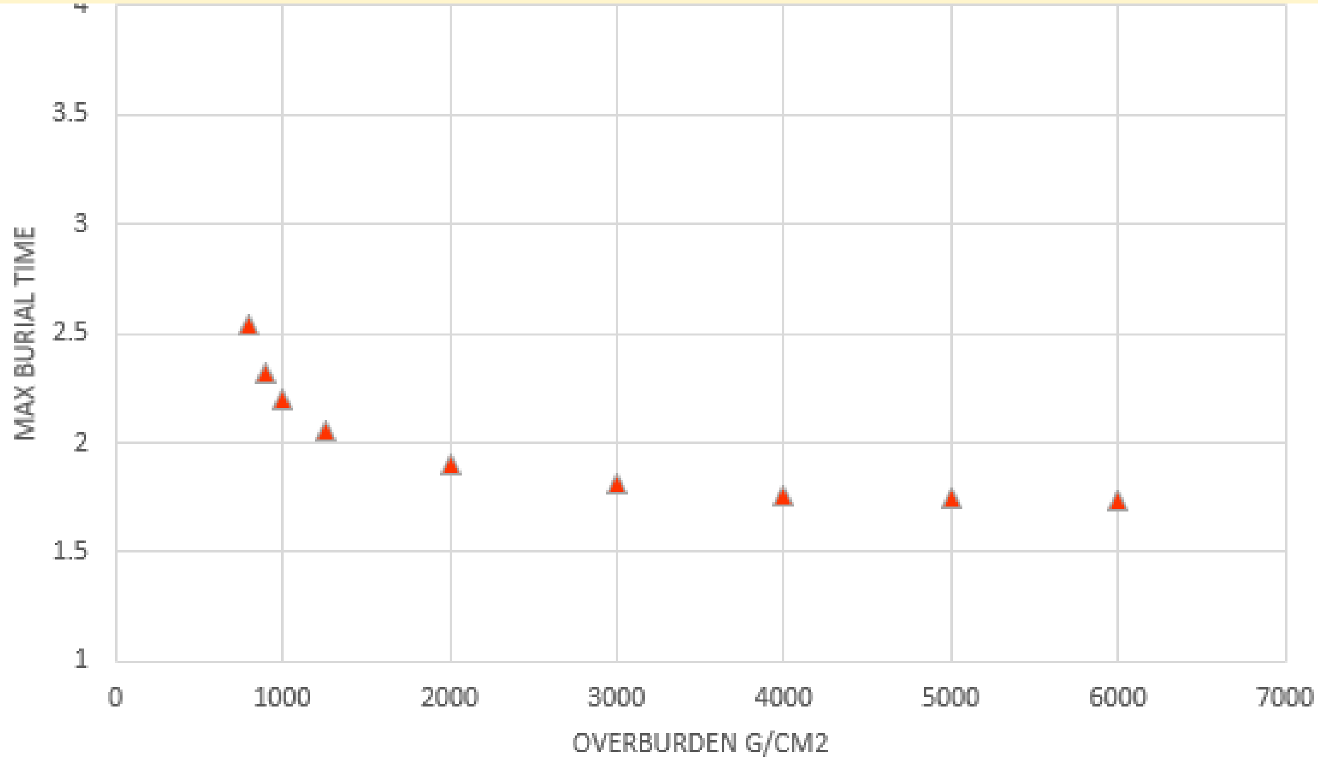
6 m sample

| Samples | Simple burial Tebogos data Sheet (Ma) | My calculation using p values from Teboo (Ma) | My calculation Spallation from Tebogo Muons from Braucher 2011 |
|---|---|---|---|
| TM-5 | 0.33 | 0.31 | 0.33 |
| TM-14 | 1.74 | 1.73 | 1.74 |
| TM-16 | 2.13 | 2.12 | 2.13 |
| TM-18 | 0.98 | 0.97 | 0.98 |

For simple burial method
Perfect agreement in both cases

⇒ I will use slow and fast muons from Braucher 2011

Shallow RS105 amphitheater
Sample TM-14. (observe…curve basically independent of spallation)
Max time sensitivity to knowledge of overburden
Nominal depth 6 m or 1260 g/cm2




…..if the true overburden would be 30 % less than we would change the max burial time with 15 %

Thus …as predicted much bigger sensitivity in the shallow cave

I also used the min/max method to determine the age of those samples
( min/max not available in Tebogos sheet for the shallow sample.

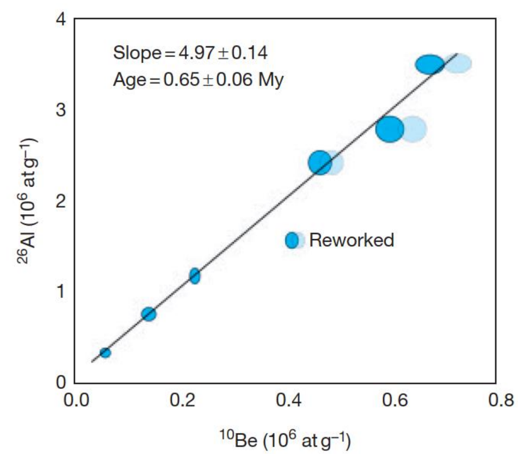
| Samples | Min Directly from Simple burial Method (Ma) | | Max 1260g/cm2 Muon from Braucher 2011 (Ma) |
|---|---|---|---|
| TM-5 | 0.33 | | 0.34 |
| TM-14 | 1.74 | | 2.06 |
| TM-16 | 2.13 | | 2.59 |
| TM-18 | 0.98 | | 1.08 |

# Third method –isochron dating

Necessary condition 1: the samples are taken from exactly the same location to be sure that they have exactly the same postburial history

Necessary condition 2: the samples have different inheritance i.e. different individual histories before burial

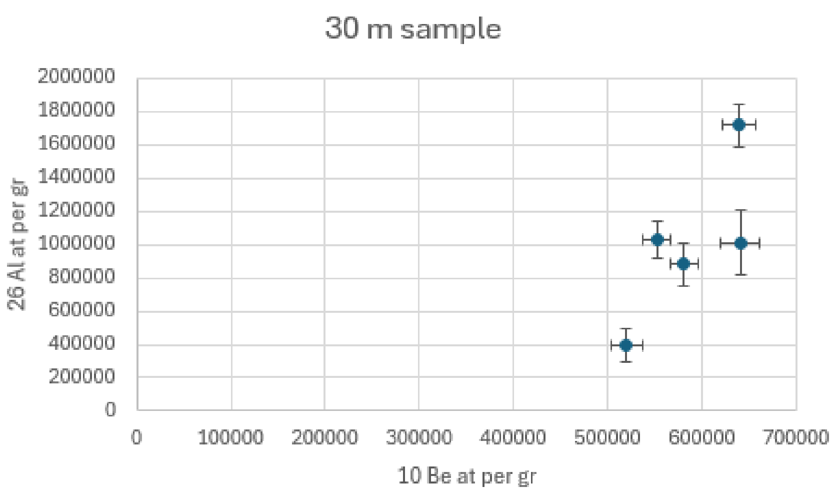
If 1) and 2) one can derive...

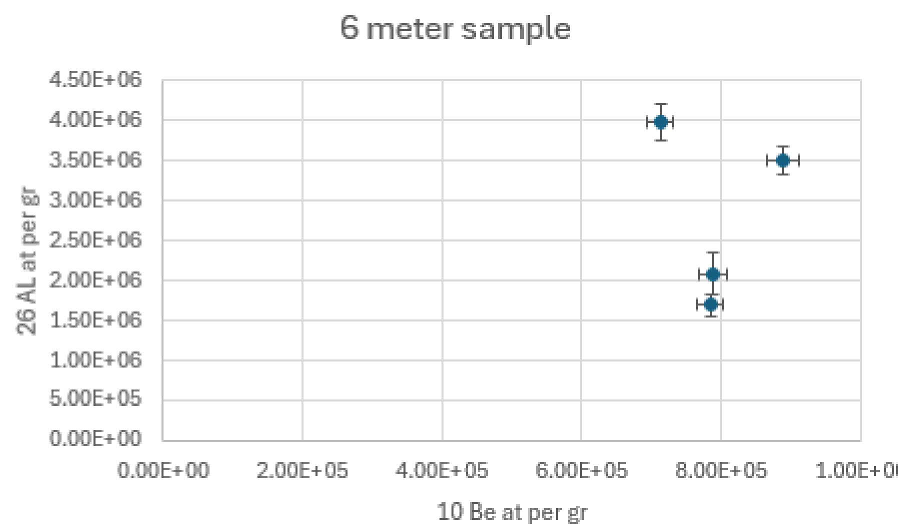


# …neither condition fulfilled in our case….

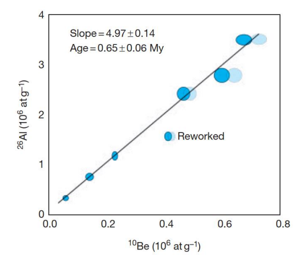
Necessary condition 1: the samples are taken from exactly the same location to be sure that they have exactly the same postburial history

Necessary condition 2: the samples have different inheritance i.e. different individual histories before burial




Compare good conditions



# Conclusion

I have looked at three methods used for dating of burial age
and tried to understand how muon tomography can improve

- The simple burial method is not sensitive to the overburden but could may be give a more accurate result from measuring the neutron flux at the surface

- The min/max method could give improved result using muon tomography in the ball park of 10% if if the samples are not deeper than say 10 meter

- Concerning the isochron method I do not see how we can use it for the two set of samples we have seen up to now

Important caveat to this conclusion-next slide

My consideration up to now have not addressed the important question how measurement of the muon flux at the surface and in the cave –possibly with momentum determination- can improve the p values which are important for the min/max method

Just one example…. Sample TM11 ( 30 m sample) gives a Max of 2.74Ma

If the P for fast muons would be 5x larger at 30 m we get Max 3.2 Ma

In order to understand to what extent measuring the muon flux in the cave helps we need to understand the relation between the muon flux and p values

Here I guess we need MC studies….

# Back up