Muon catalyzed fusion

Recent theoretical progress and Recent experiments at J-PARC

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& HEATES collaboration
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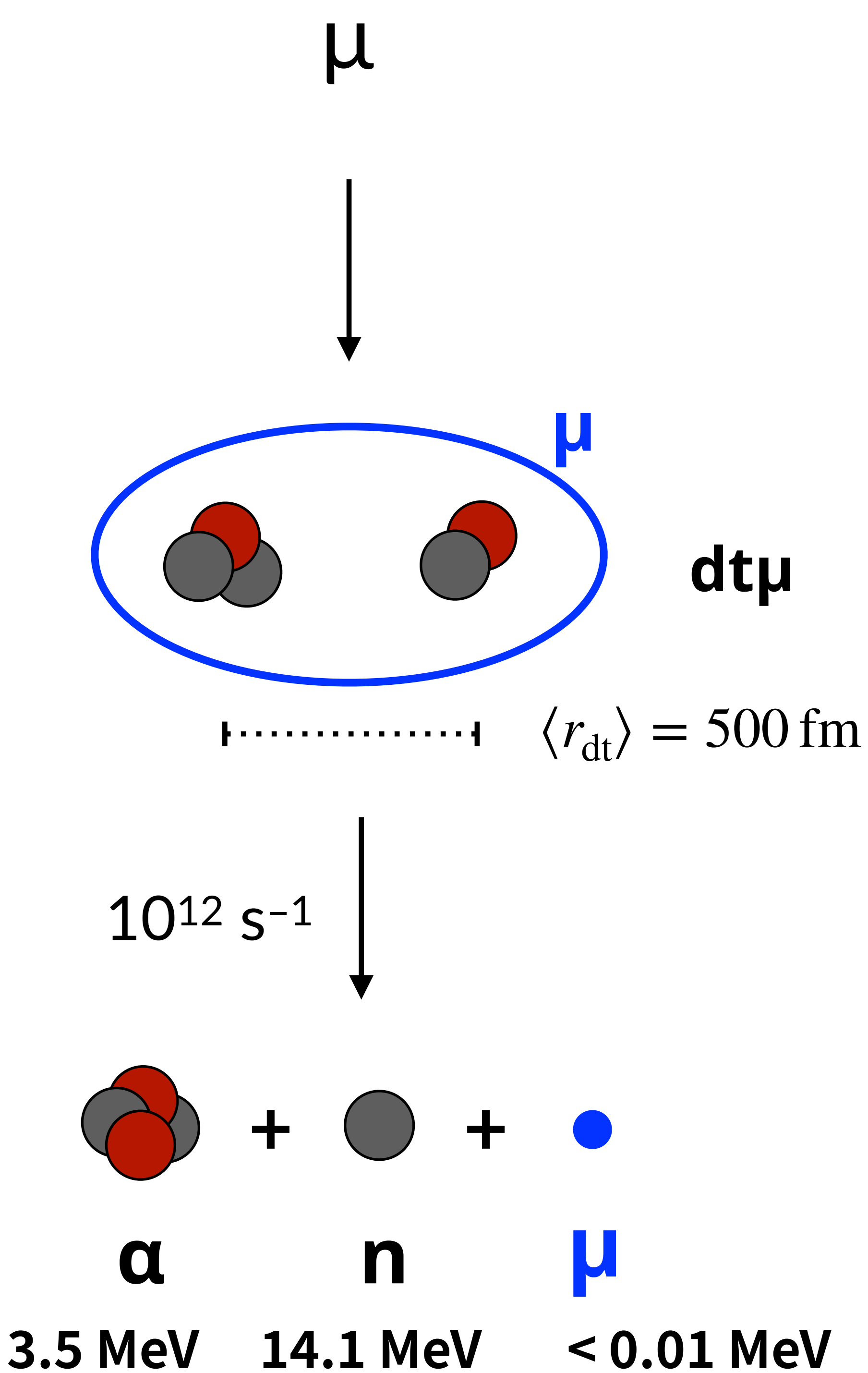
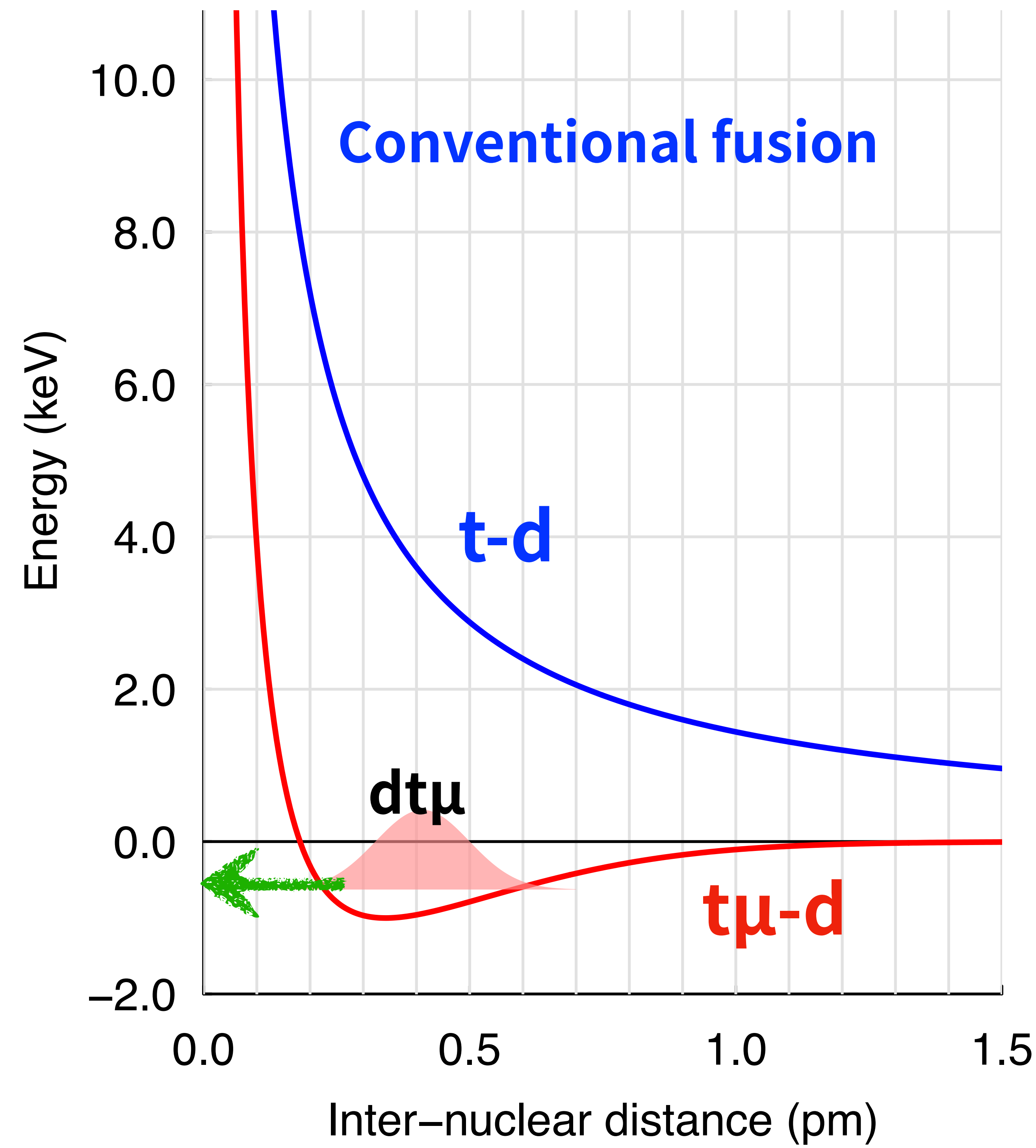
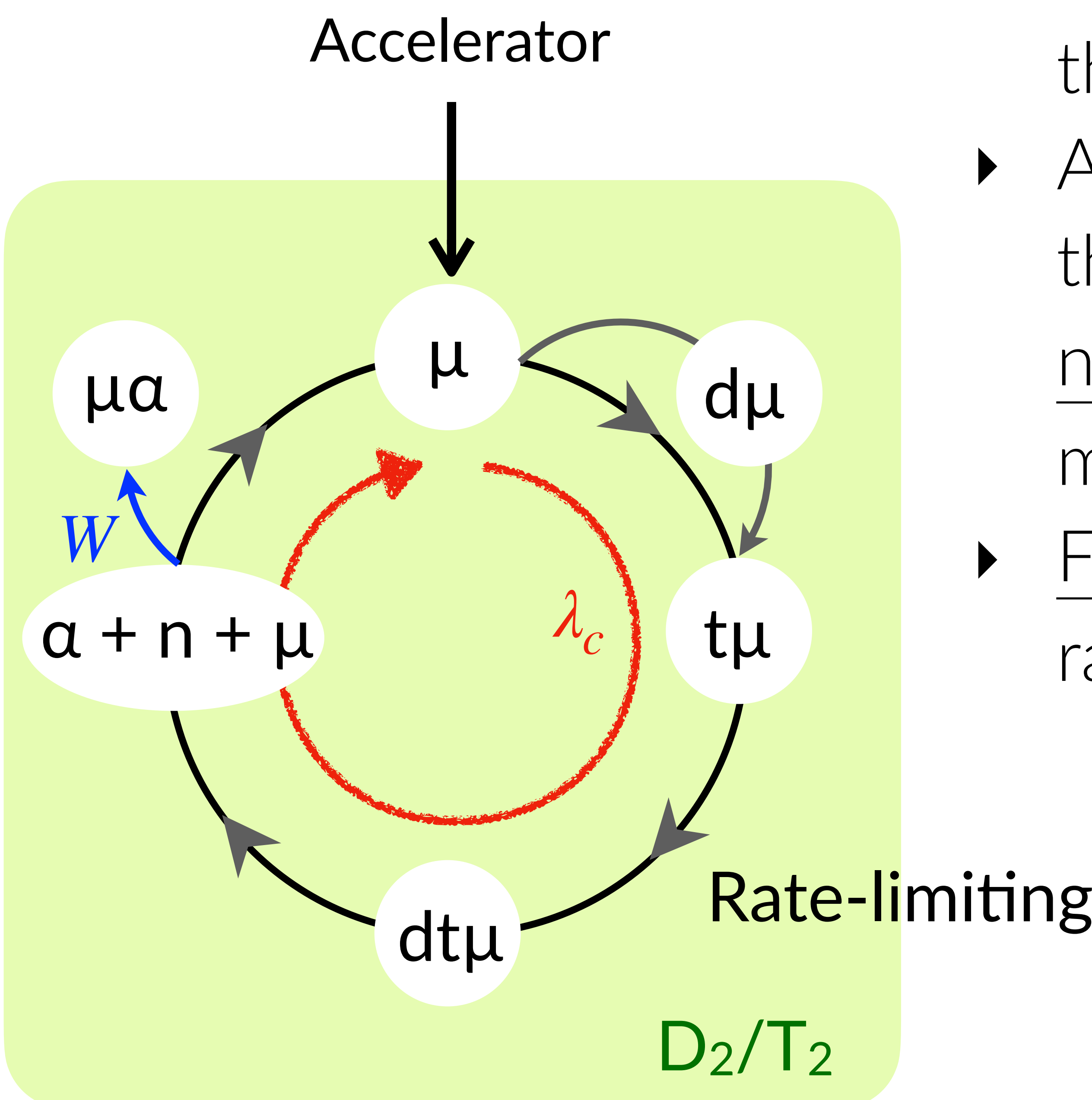
Muon catalyzed fusion (μCF)

μ^- confines d and t in a molecule and facilitates fusion reaction





Muon catalyzed fusion "cycle"



- Muons first form muonic atoms and then form the muonic molecules
- After nuclear fusion, small part of the muons sticks to the helium nucleus, which results in a loss of muons.
- Fusion yield depends on the cycle rate and the sticking probability.

$$W \sim 0.5\,\%\,,\, \lambda_c \sim 10^8\,\mathrm{s}^{-1}$$

@Liquid hydrogen density

$$\longrightarrow \quad Y_{\mathrm{fusion}} \approx 10^2$$

$$Y_{\mathrm{fusion}} \simeq \left(W + \lambda_0/\lambda_c\phi\right)^{-1}$$

300 fusions/μ = scientific breakeven
150 fusions/μ is the current world record.

Muonic molecule

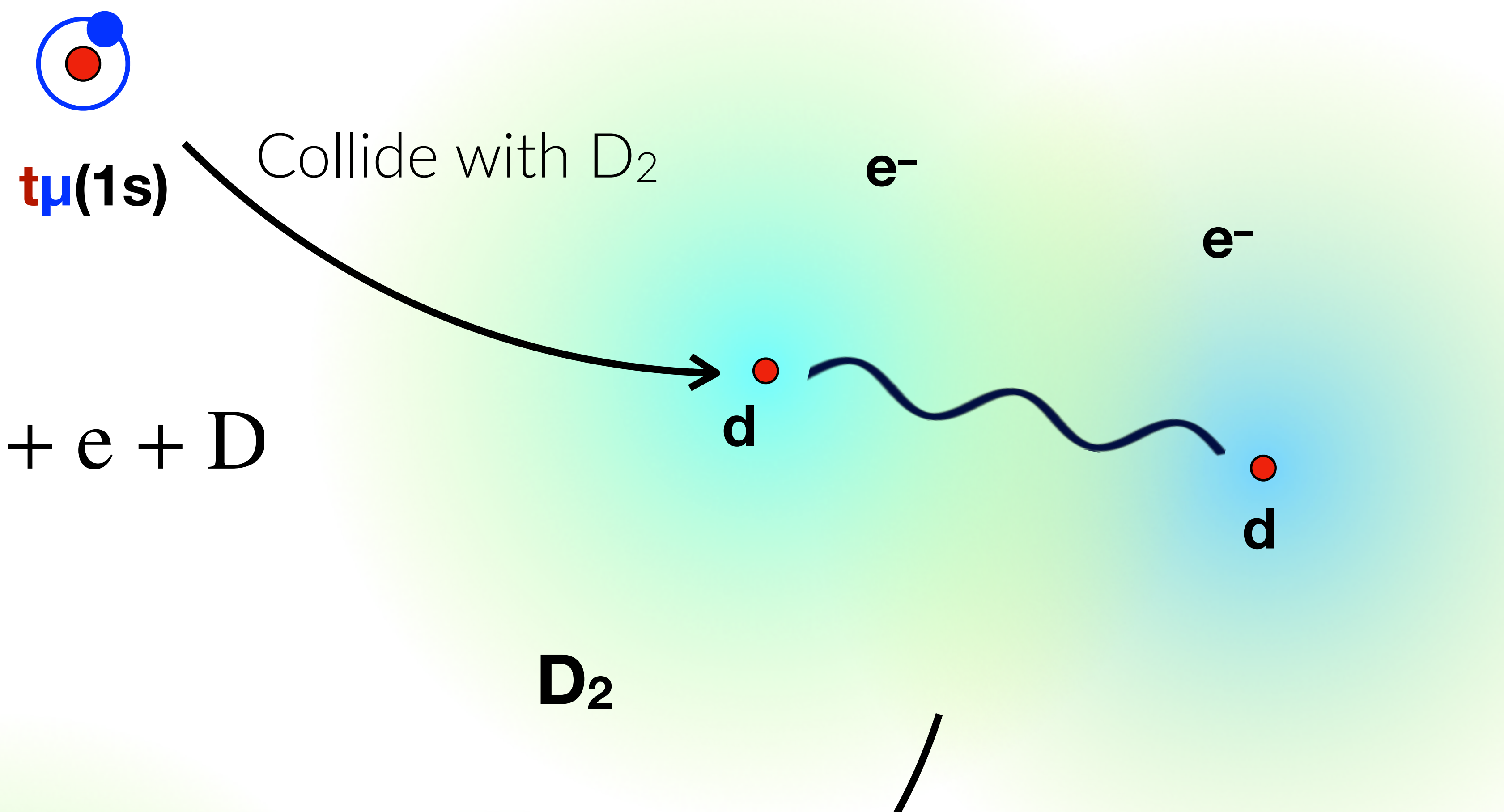
- Typically have **200-300 eV binding energy**
- **Sparse** energy levels
- **ddμ, dtμ** have an excited state with **small binding energy**

Table 2 Coulomb molecular binding energies[a] (in eV)

J, v	ppμ	pdμ	ptμ	ddμ	dtμ	ttμ
0, 0	253.15	221.55	213.84	325.07	319.14	362.91
0, 1				35.84	34.83	83.77
1, 0	107.27	97.50	99.13	226.68	232.47	289.14
1, 1	—	—	—	1.97[b]	0.66[c]	45.21
2, 0	--	--	—	86.45	102.65[d]	172.65
3, 0						48.70

W. H. Breunlich et al., Annu. Rev. Nucl. Part. Sci. 39, 311 (1989).

Vesman mechanism: an efficient dtμ/ddμ formation

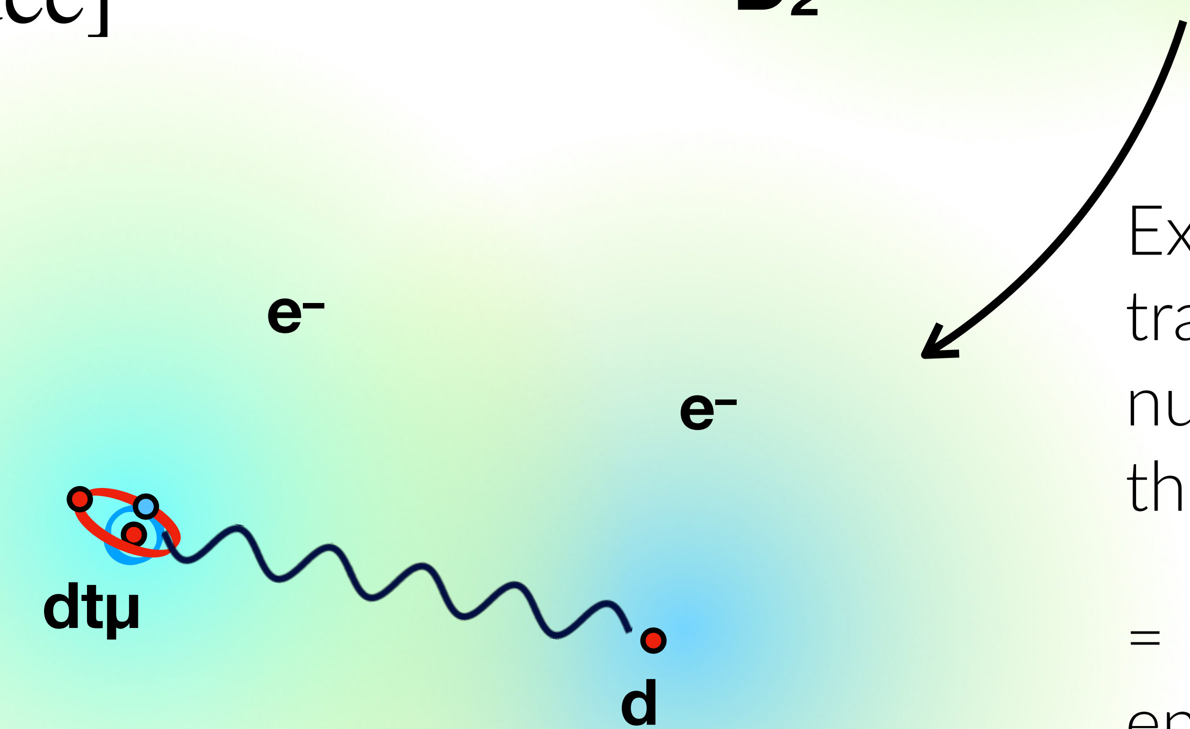


Non-resonant formation (slow)

$$t\mu(1s) + D_2 \rightarrow dt\mu + D^+ + e + D$$

Resonant formation (fast)

$$t\mu(1s) + D_2 \rightarrow [(dt\mu)dee]$$



Excess energy is transferred to inter-nuclear excitation of the host molecule

= Resonance process, enhancement at the matching energy

dtμ becomes a quasi-nucleus of the new molecule

A brief history of μCF

1947, 1948 Theoretical prediction of μCF (Frank, Sakharov)

1956 Observation of pdμ fusion by cosmic muons (Alvarez)

1967 Theory of resonant ddμ formation mechanism (Vesman)

1977 Prediction of dtμ reaction rate (Gerstein&Ponomarev)

1987 Observation of X-rays from dt-μCF (Nagamine)

1987 Precise calculation of dtμ binding energy (Kamimura)

1987-2003 Dedicated experiments at PSI, RAL, TRIUMF, JINR, etc

(Fusion yield reached 100-150 per muon)

Review article	W. H. Breunlich et al., Ann. Rev. Nucl. Part. Phys. 1989 L. I. Ponomarev, Contemp. Phys. 1990 P. Froelich, Adv. Phys. 1992
Review of JINR, PSI experiments	V. R. Bom et al., J. Expt. Theo. Phys. 2005 D. V. Balin et al., Phys. Part. Nucl. 2011
New insights	M. C. Fujiwara et al., PRL 2000 N. Kawamura et al., PRL 2003

Revisit μCF

Upgrade of experimental techniques

- Intensity of slow muon beams
- New X-ray detectors

Motivations

- **Future energy source**

  No plasma confinement; **150 fusions/muon** has been achieved so far

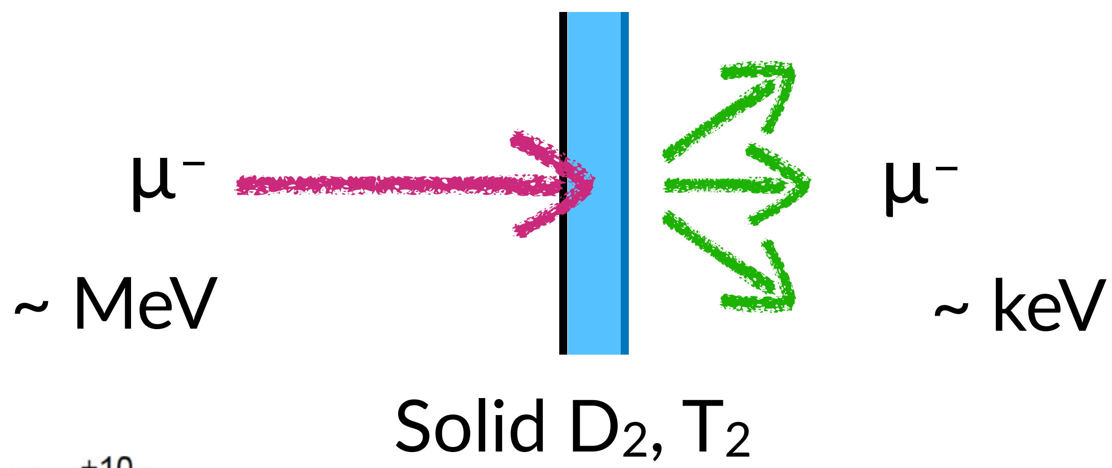
- **Mono-energetic neutron source**

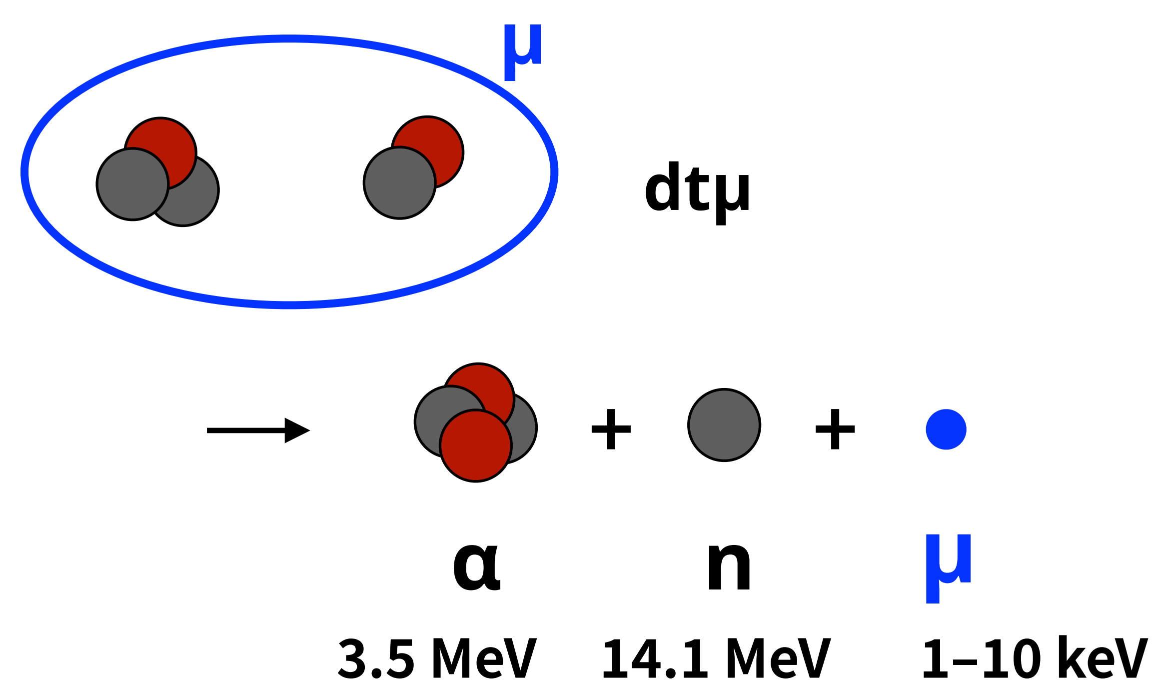
  Material analysis, transmutation of LLFPs, etc

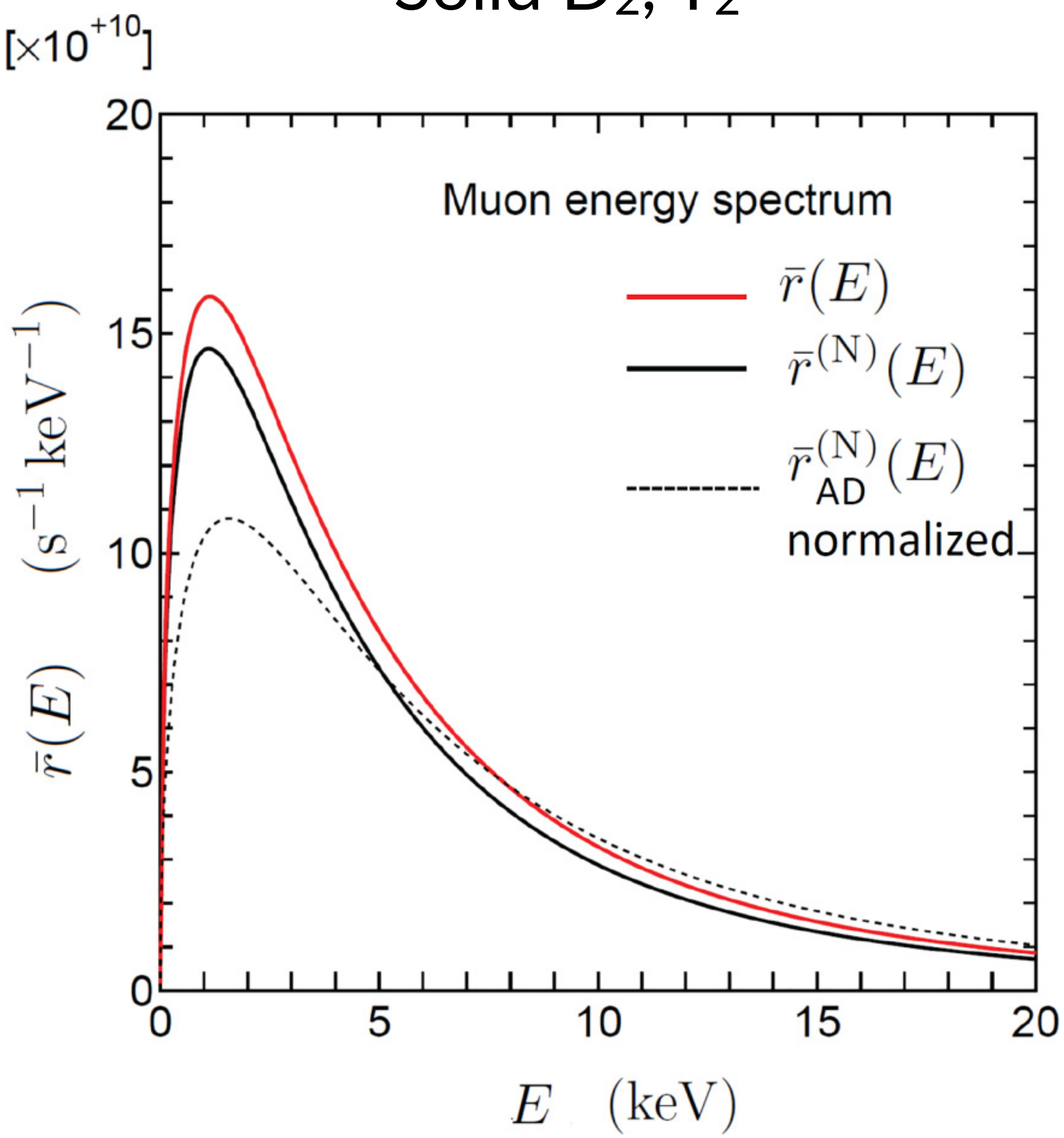
- **Cooling of μ^- beam**

  Muon microscope, 3D-elemental analysis, Muon collider, etc

μCF as a moderator of μ⁻

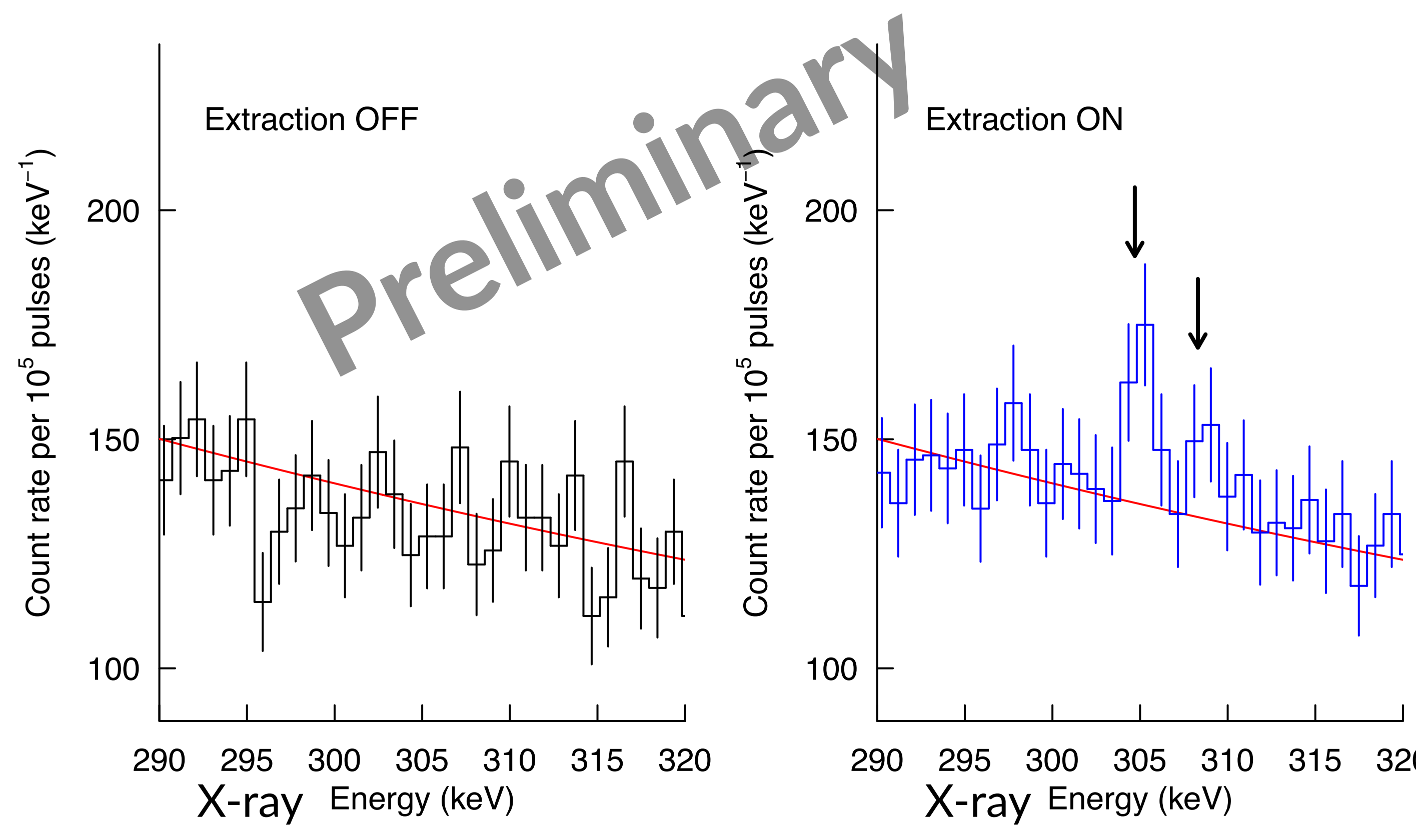






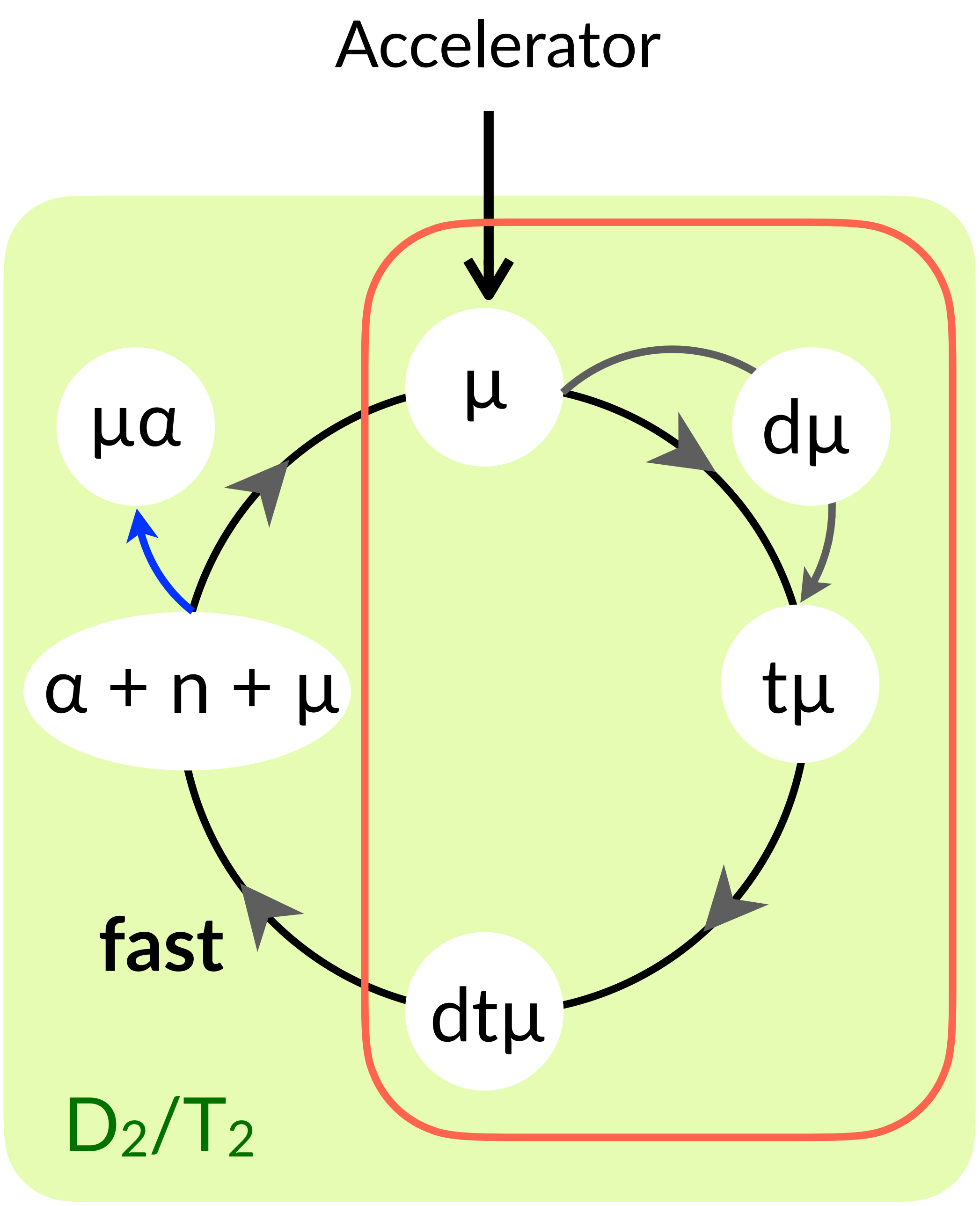
M. Kamimura, Y. Kino, T. Yamashita, PRC (2023)

Our experiments at J-PARC



K. Okutsu, TY, et al., (in preparation)

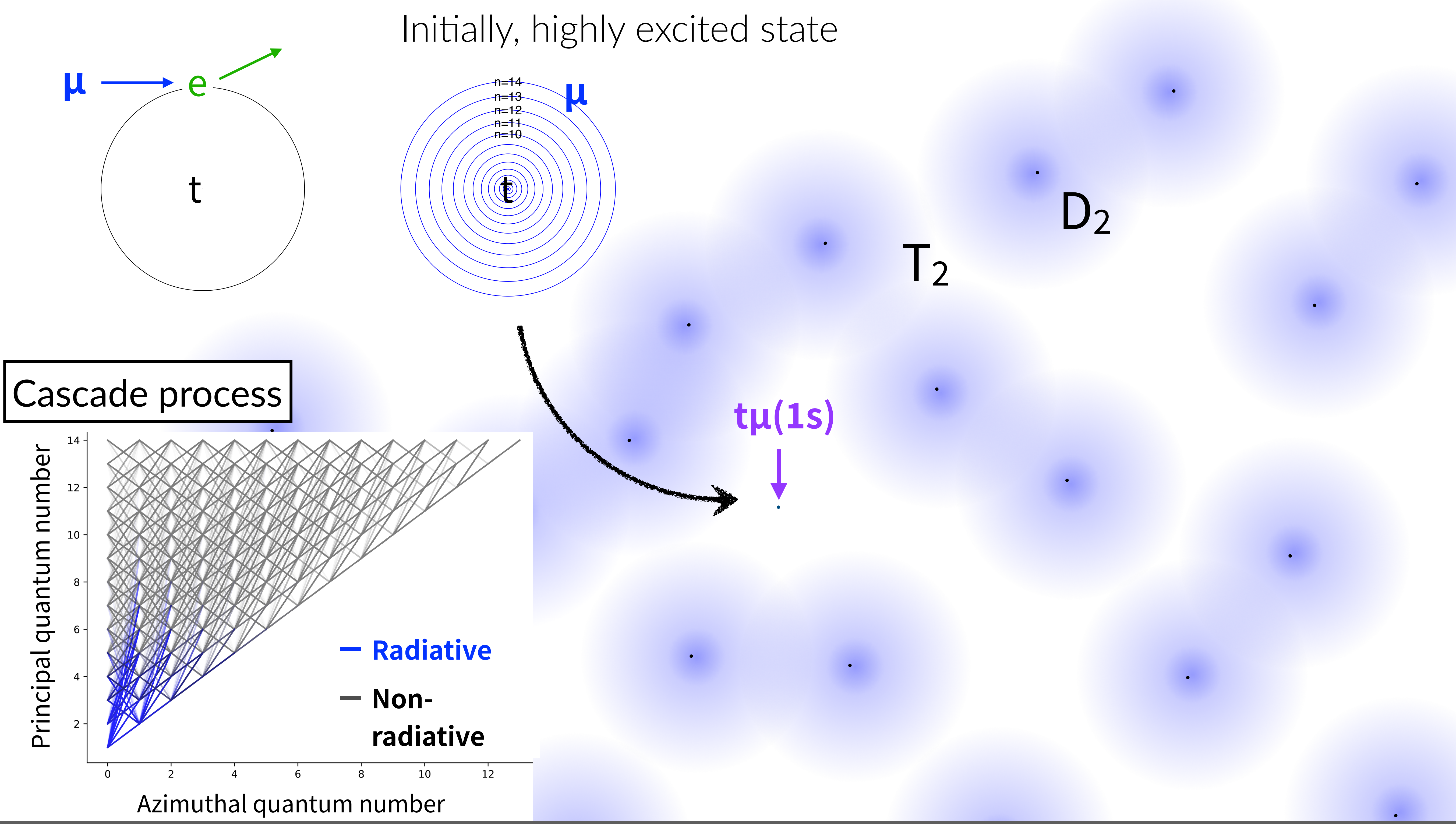
Toward achieving a high fusion yield



Dynamics resulting in the dtμ molecule from μ is crucial.

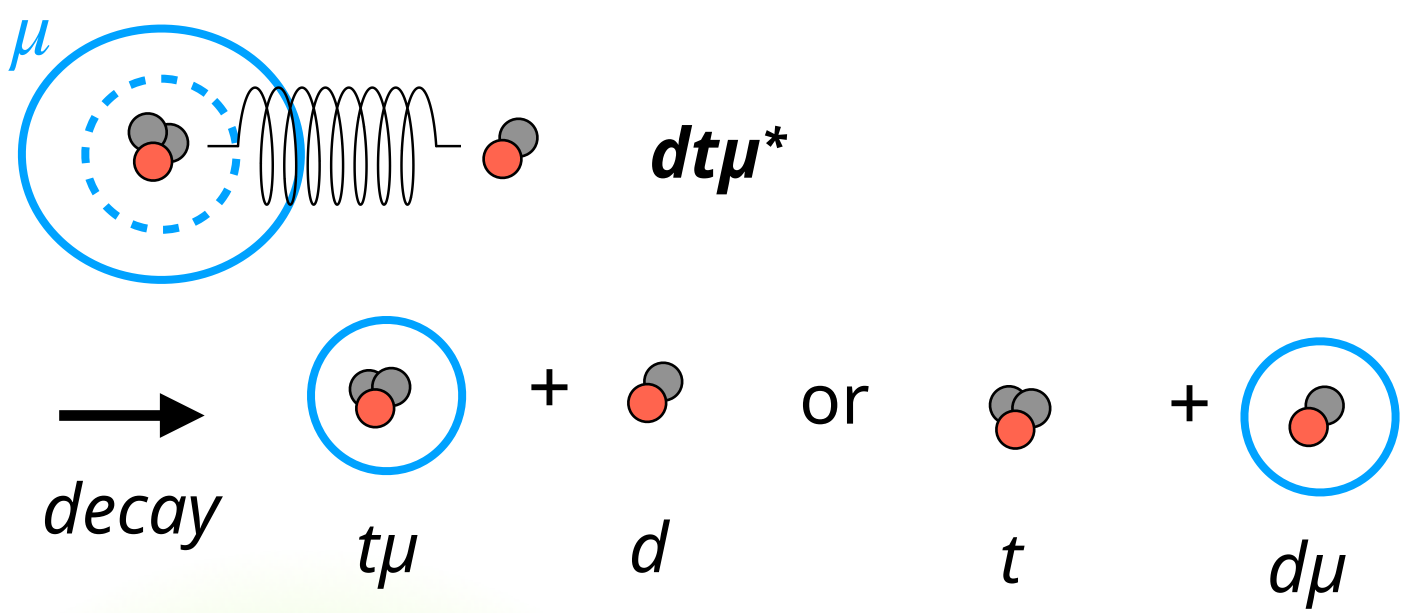
Cascade process of the muonic atoms in a dense target

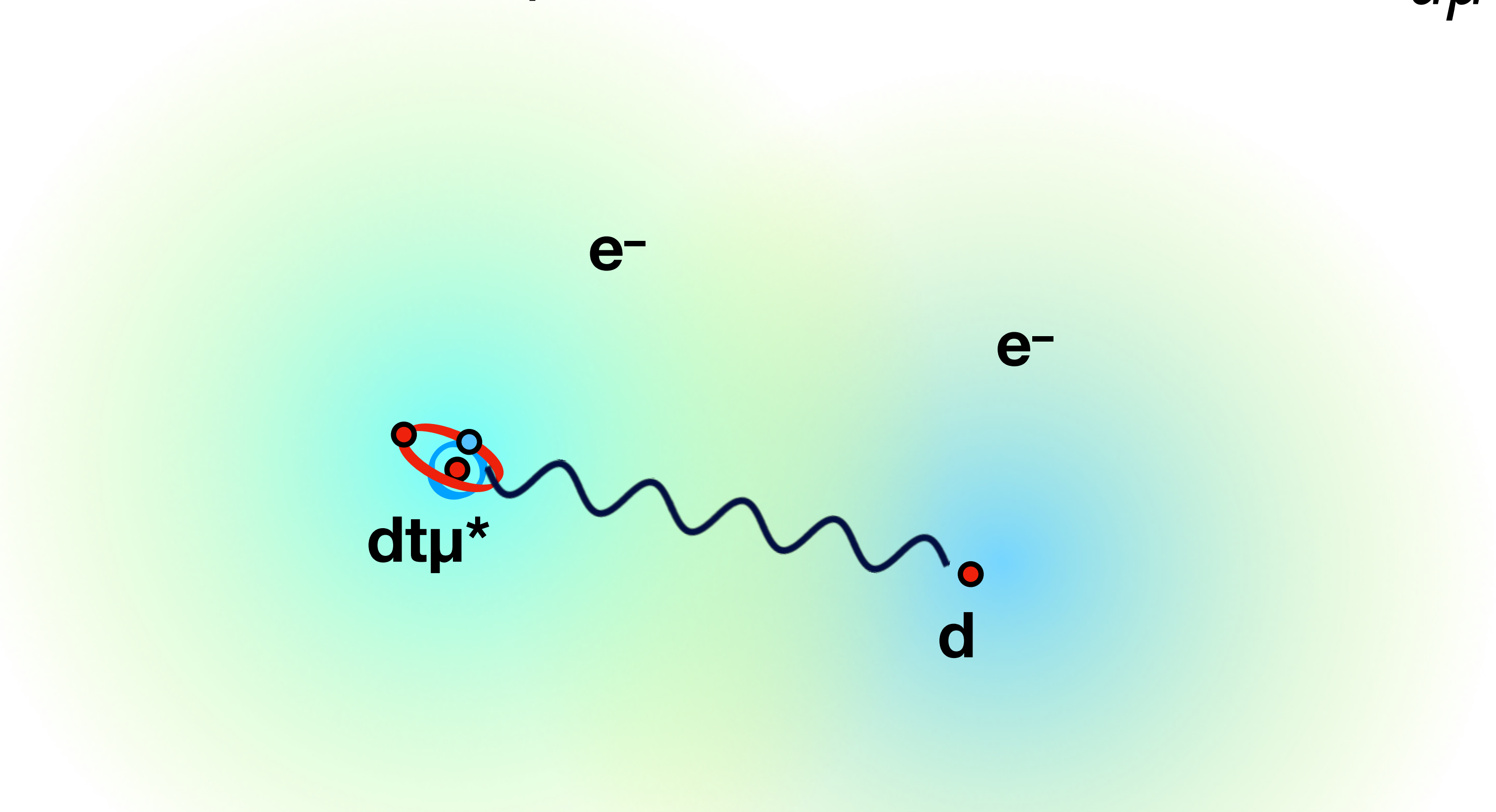
Muonic atoms de-excite during collisions with the other molecules

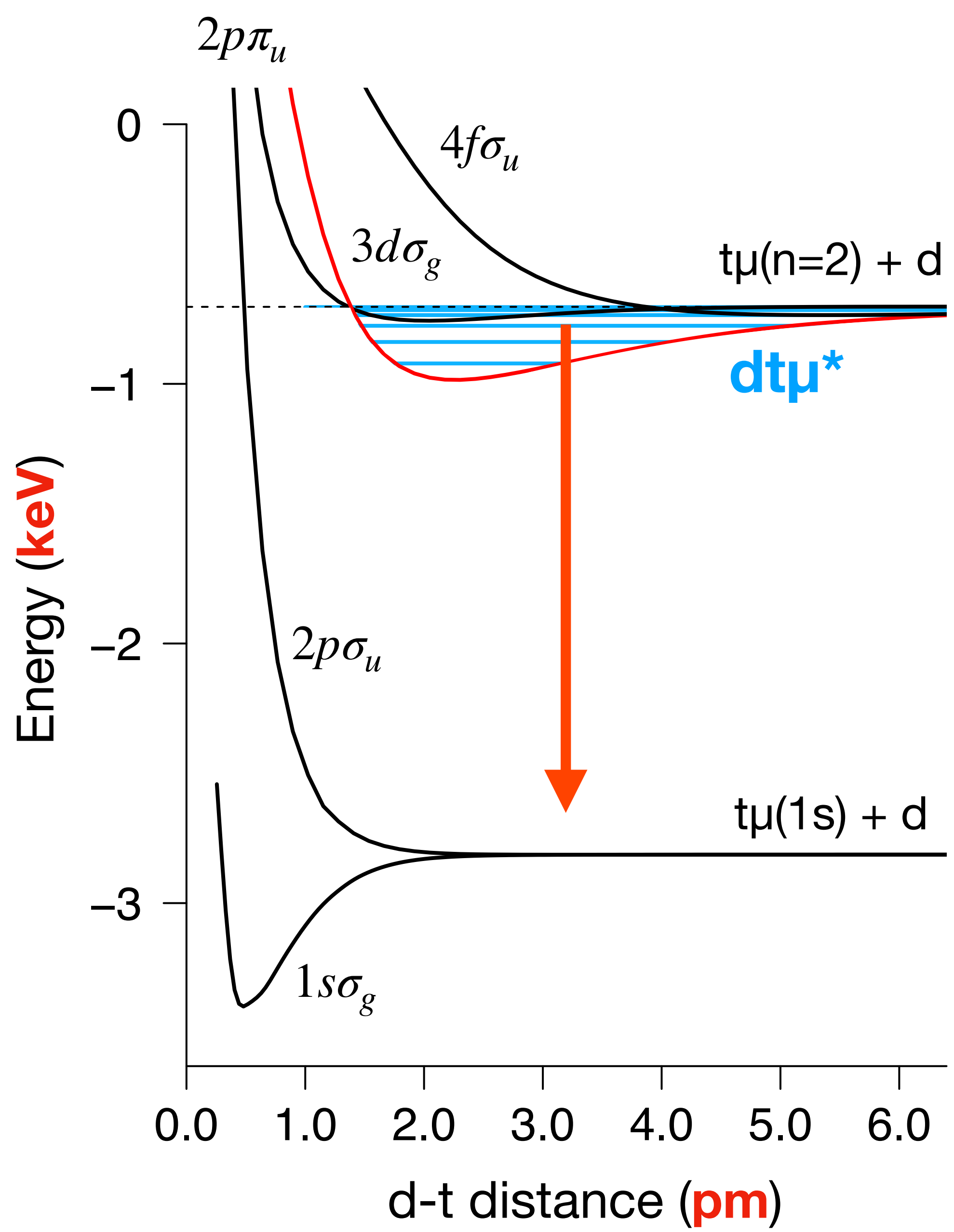


A long-standing problem in μCF kinetics: Resonance state

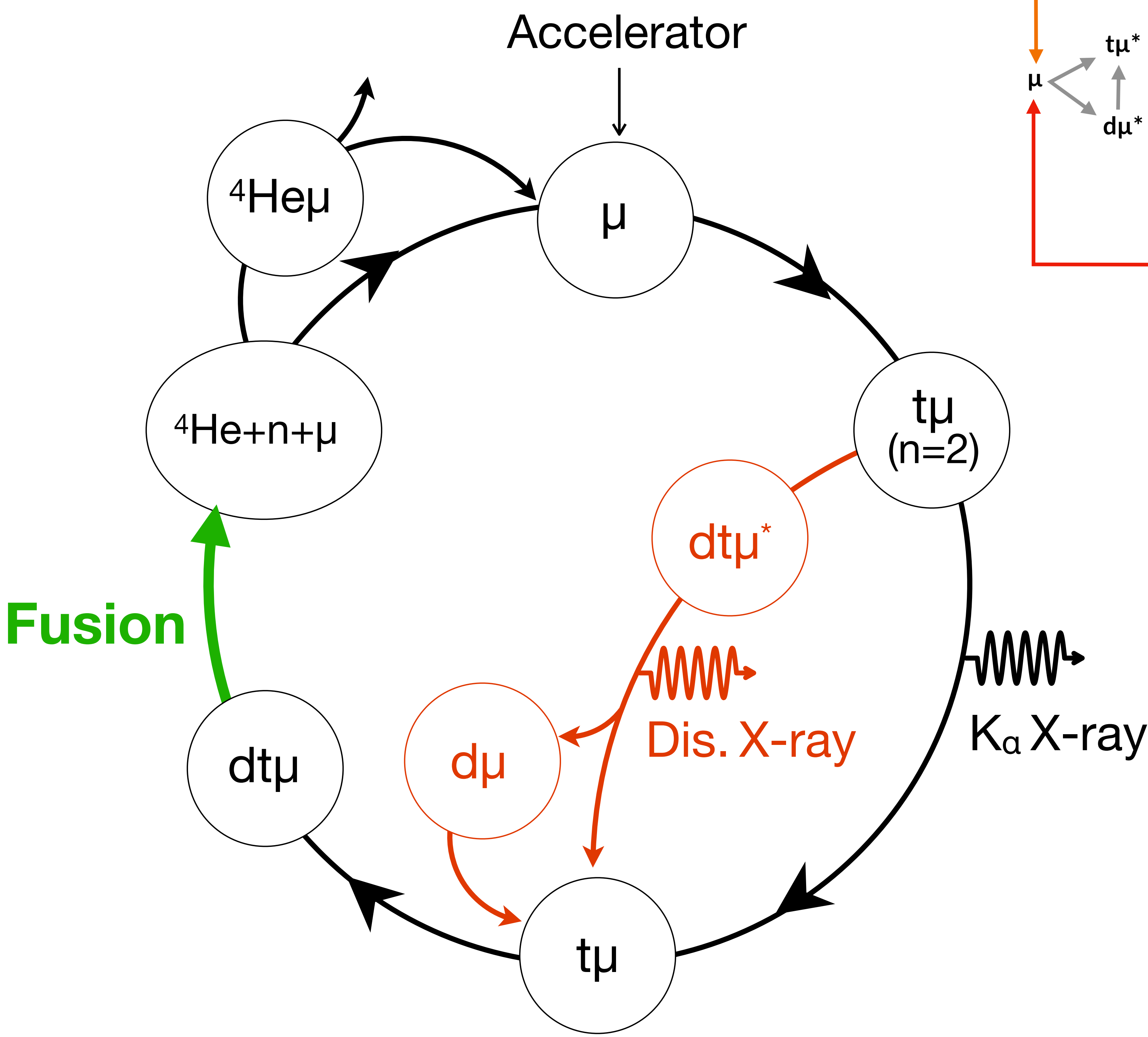
- Feshbach resonance state between tμ(n=2) and d.
- Decay with lifetime ~ 10 ps.
- **Spontaneously formed during cascade?**

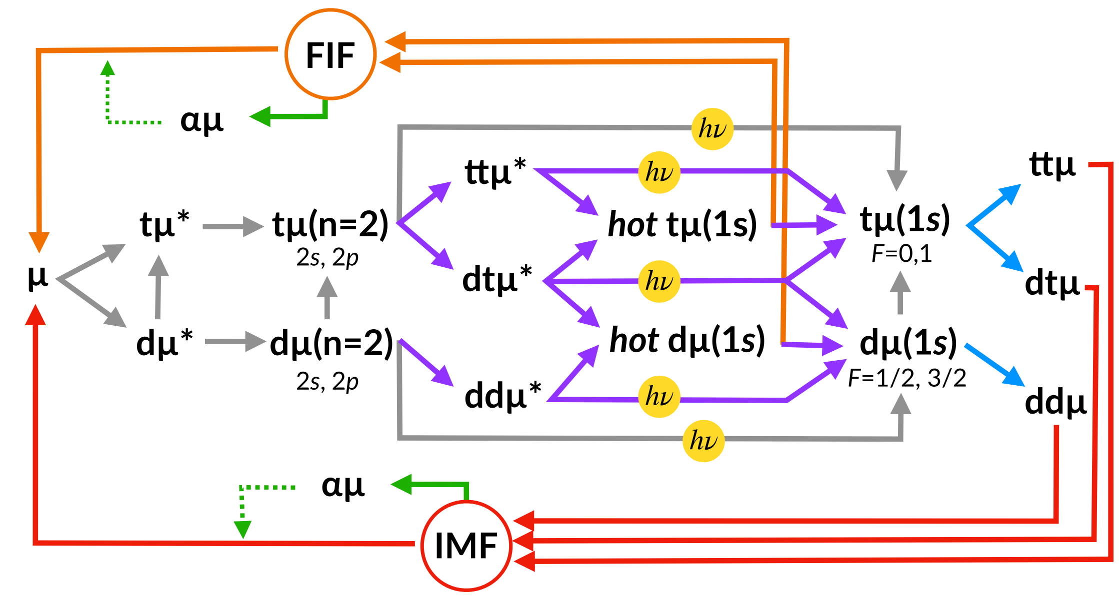






Impact of the resonance states on μCF cycles





More detailed cycle
with resonances of
all isotopologues

Kinetic model with resonance states may explain the previous experimental results

TY *et al.*, *Scientific Reports* **12**, 6393 (2022).

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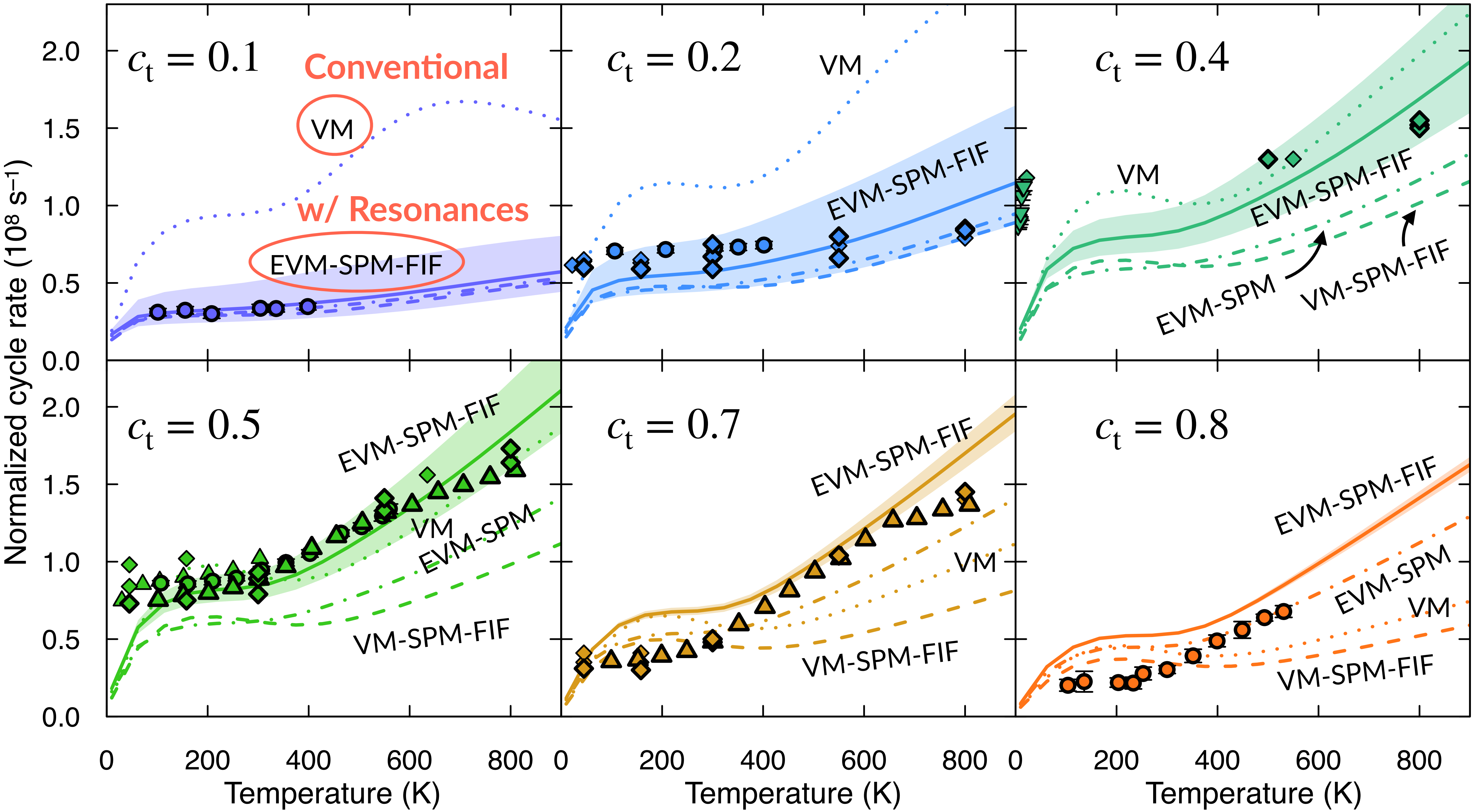
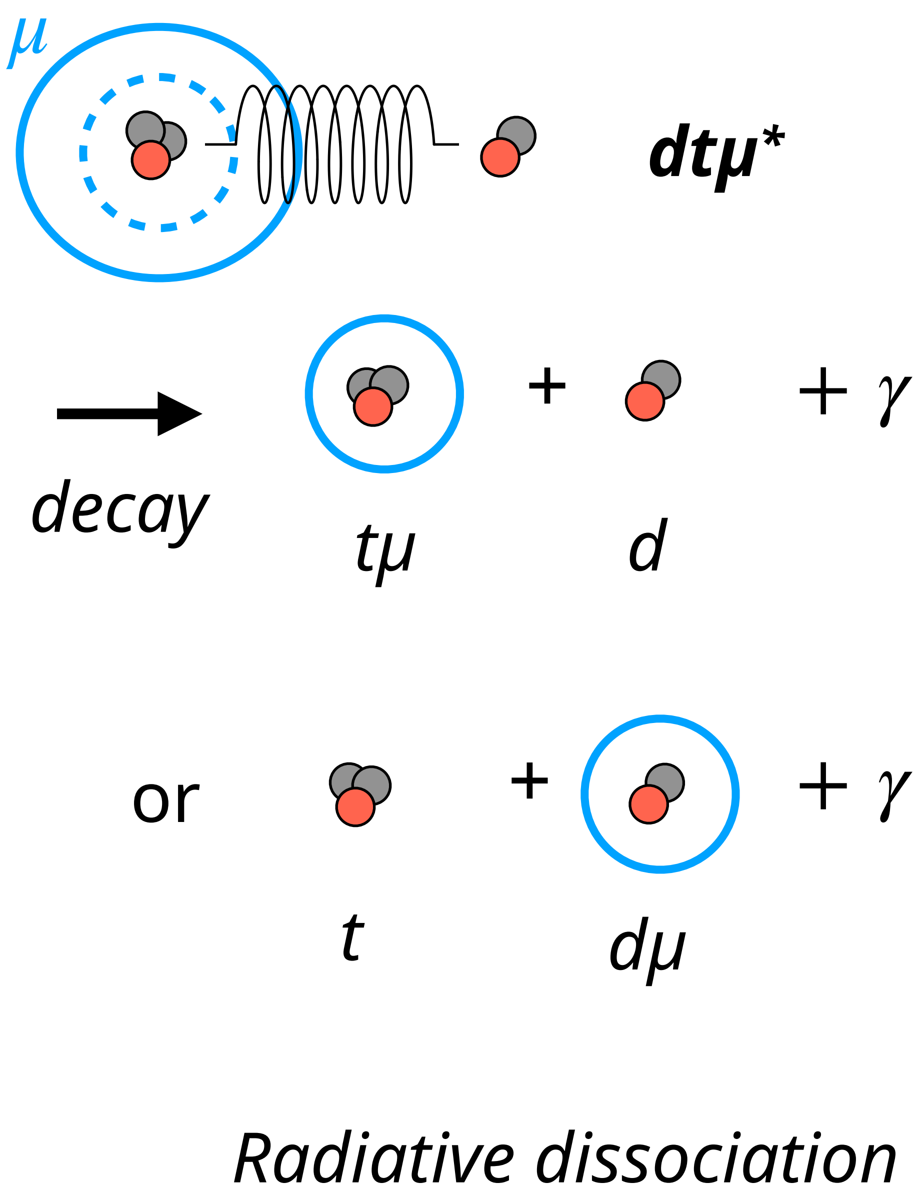
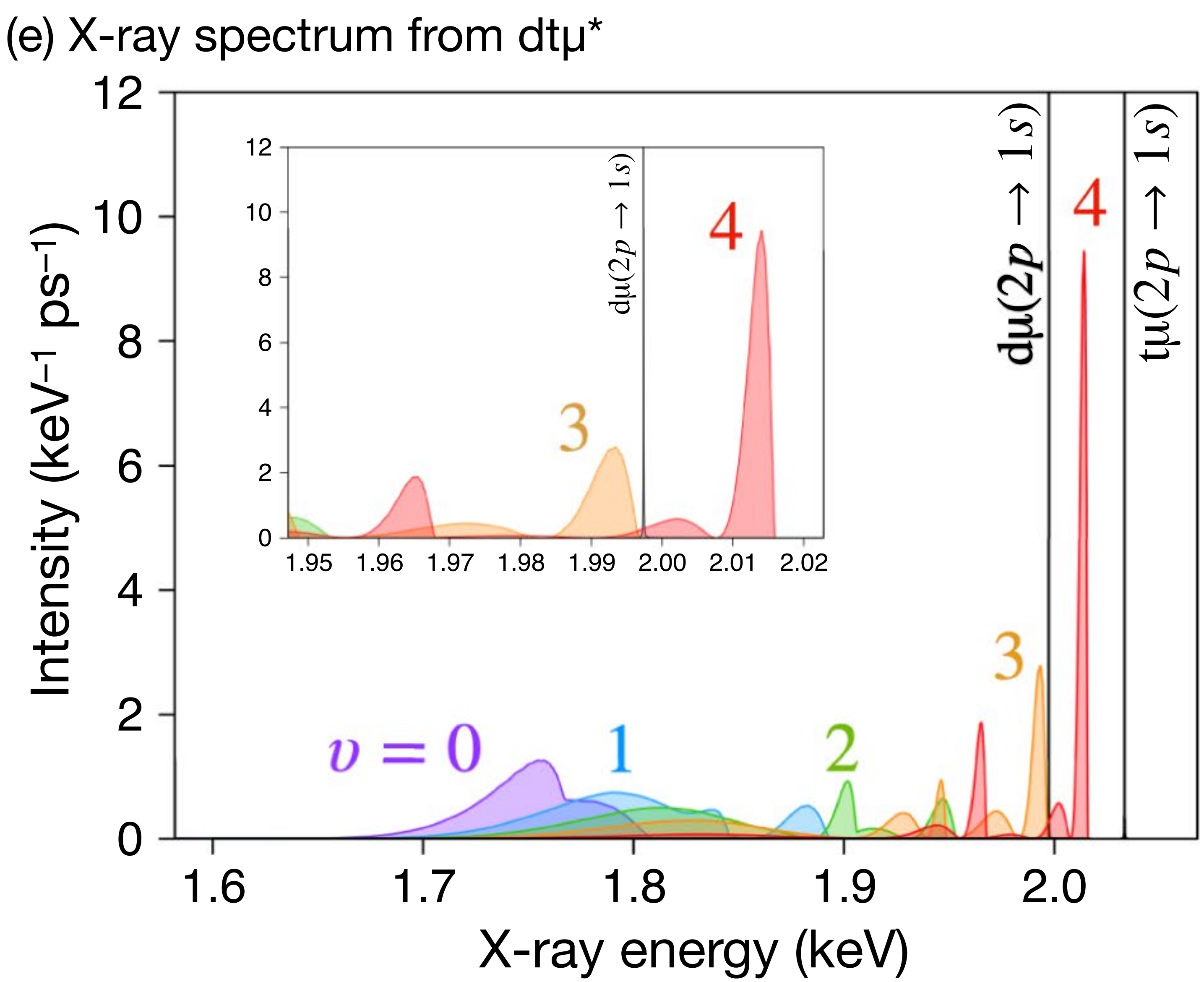


X-rays can be an evidence of dtμ* and ddμ*

The X-ray spectrum has a state-specific energy structure and becomes their footprints.

However, K X-rays from the muonic atoms are also nearby…

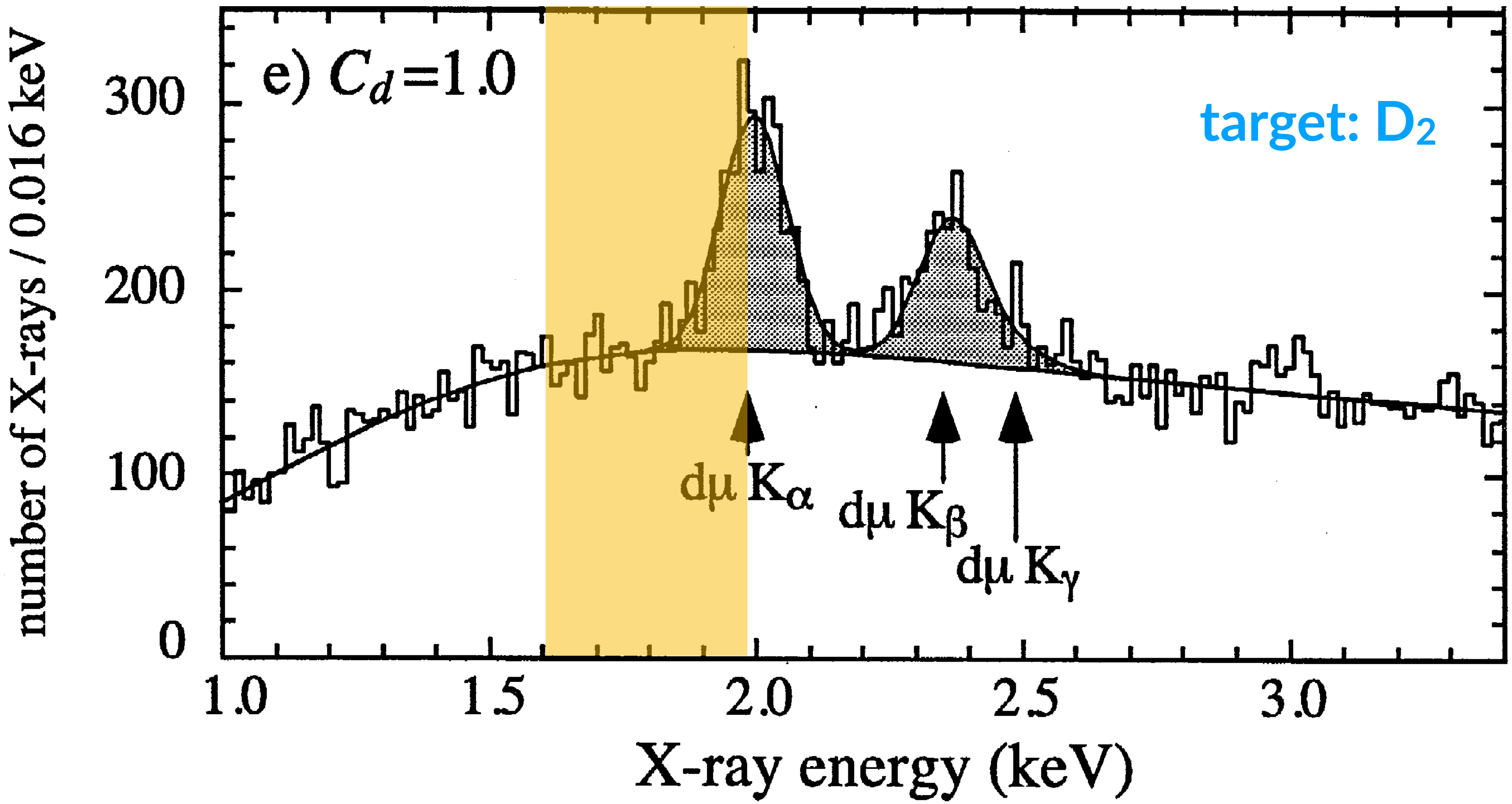




TY *et al.*, *Scientific Reports* **12**, 6393 (2022).

How difficult to observe the X-rays from resonance molecules?

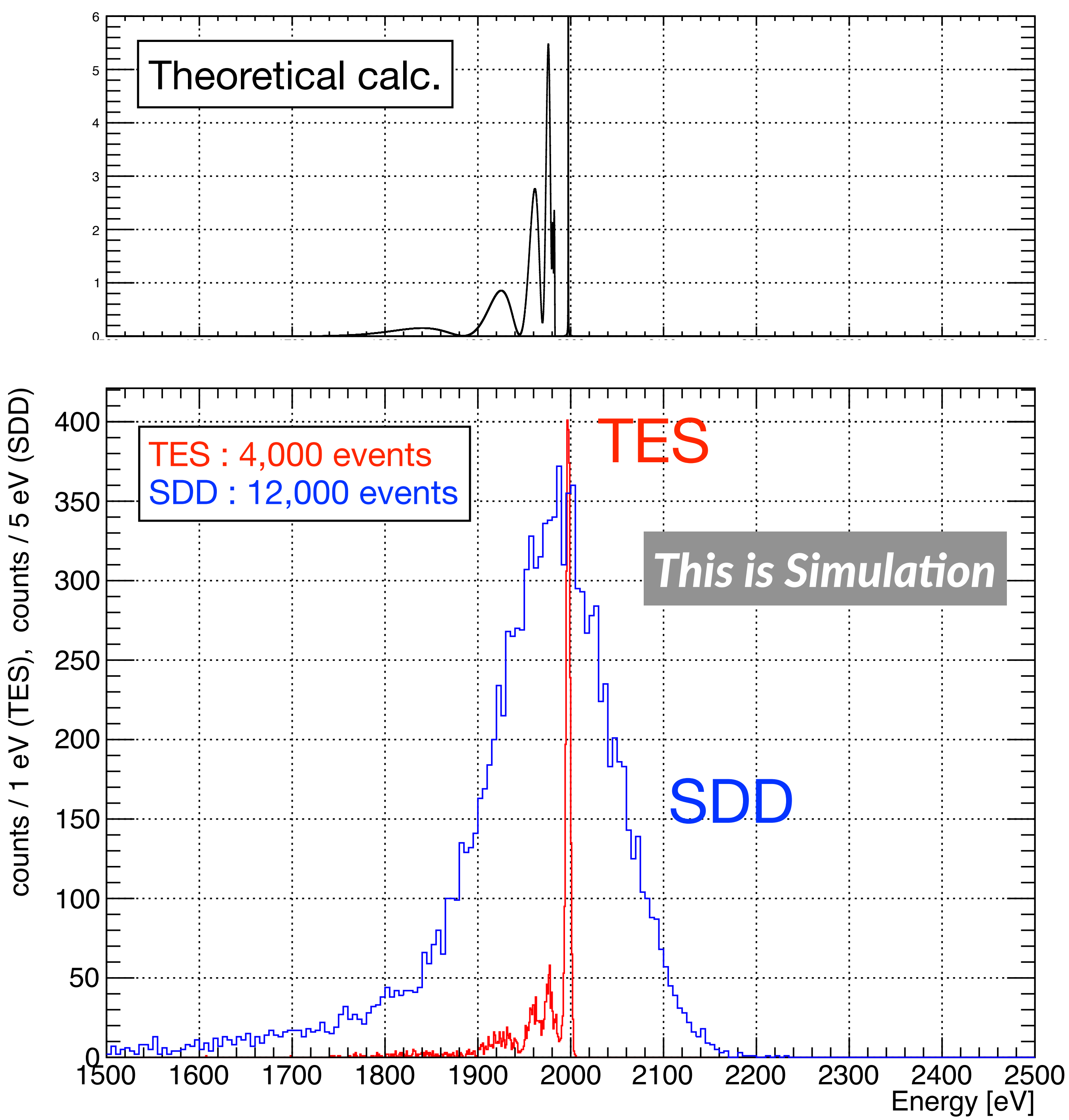
- We need to distinguish dμ(2p->1s) at 2.0 keV from the molecular X-ray signal spread in 1.6-2.0 keV.
- Typical energy resolution of SDD: >100 eV



S. Sakamoto, K. Ishida, K. Nagamine, Phys. Lett. A **260**, 253 (1999).

Transition-edge sensor (TES) for precise X-ray spectrometry

High energy resolution may distinguish the molecular X-ray from the dμ X-ray!



5.5 eV FWHM at 6 keV

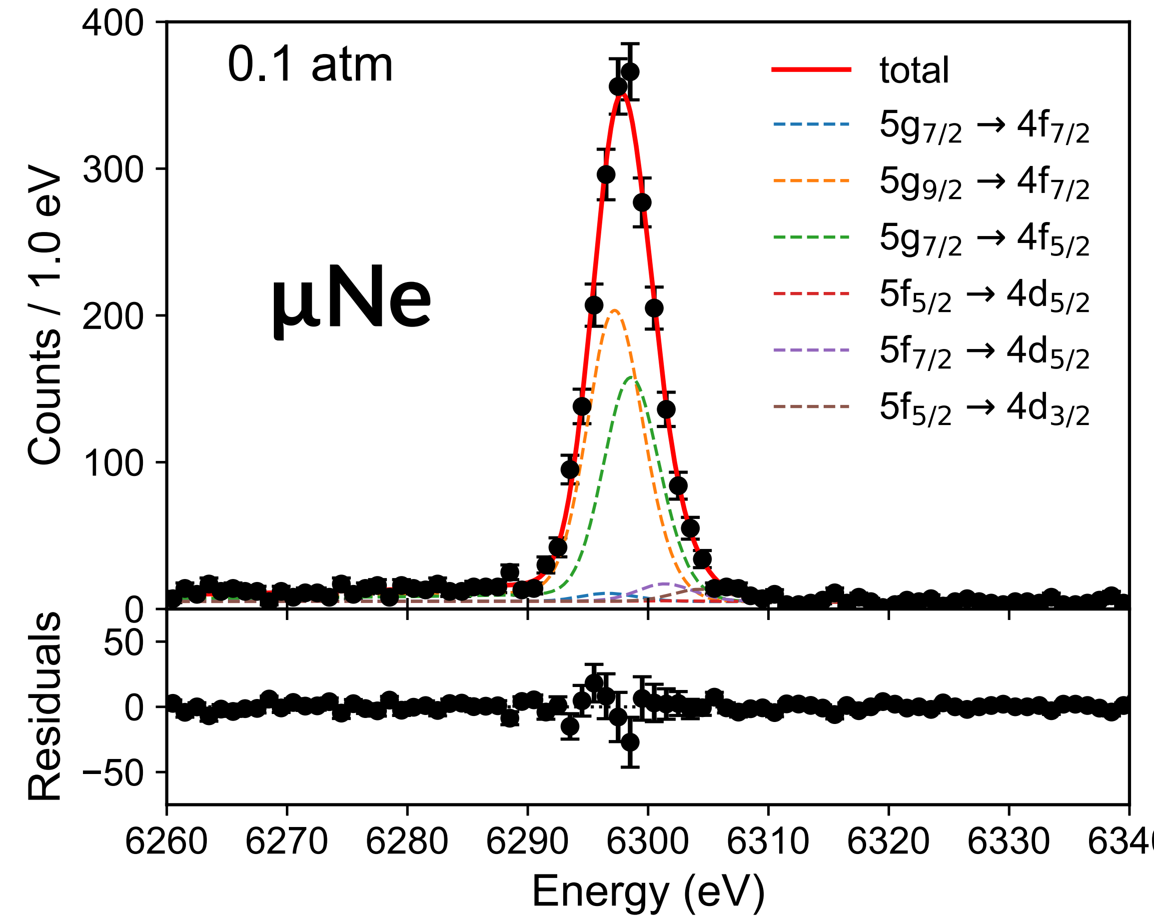


FIG. 2. X-ray spectra from 5–4 transitions of μNe at a pressure of 0.1 atm. The fitted profiles obtained by summing up μ^{20}Ne and μ^{22}Ne contributions are also shown with residual errors. The fitting is carried out by using three spectra at pressures of 0.1, 0.4, and 0.9 atm simultaneously, and the reduced χ^2 (number of degrees of freedom, 284) for the total fitting is evaluated to be 1.26.

T. Okumura et al., PRL 130, 173001 (2023).

Theoretical studies on the X-ray spectra from ddμ*

So far…

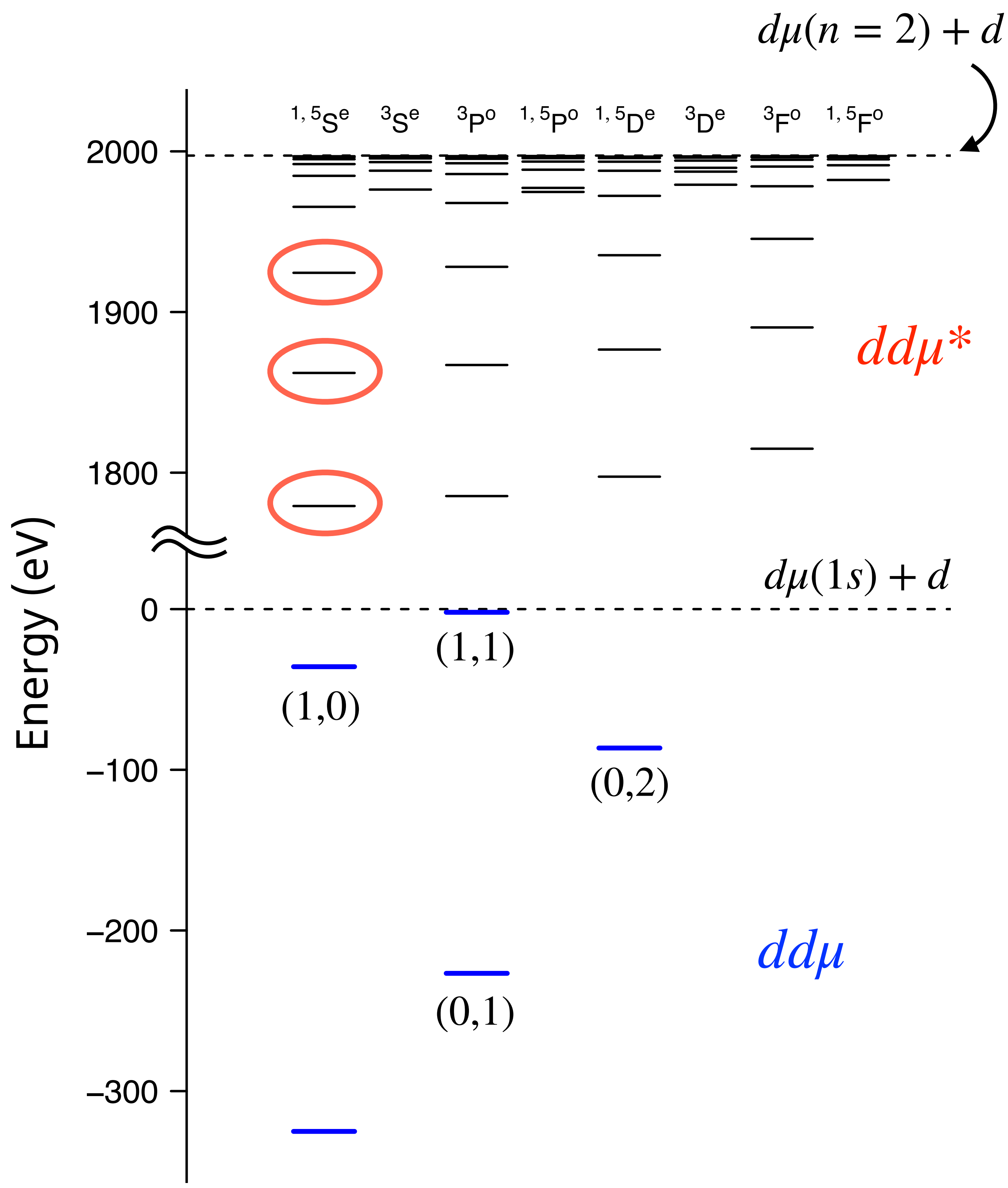
✓ **Only 3 states** were reported

Challenges

✳ **Quantum chemical approach breaks down** (large m_μ)
✳ Transition to continuum state

This work

★ Calculate **precisely** the X-ray spectra from **all possible states**
★ By our **in-house** few-body calculation code



Resonance state wavefunction

Gaussian expansion method

E. Hiyama et al., Prog. Part. Nucl. Phys. **51**, 223 (2003).

$$\hat{H} = \sum_{i=1}^{3} -\frac{1}{2m_i}\nabla^2_{r_i} + \frac{1}{2m_G}\nabla^2_{r_G} + \frac{1}{r_{12}} - \frac{1}{r_{13}} - \frac{1}{r_{23}}$$

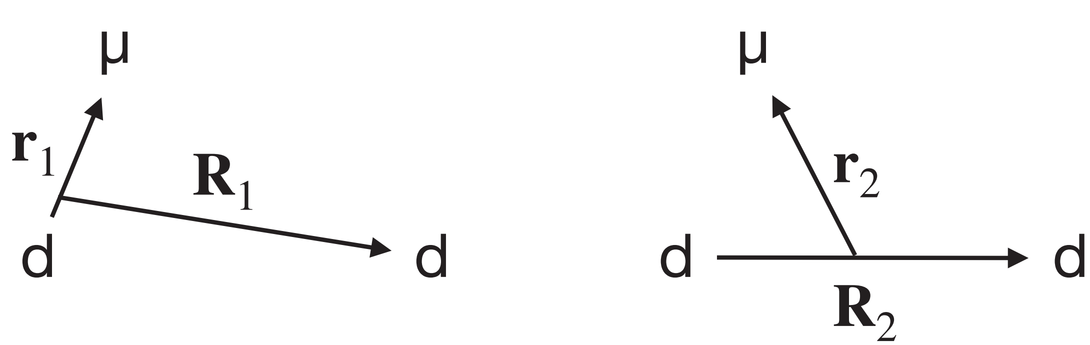
Gaussian function

$$\psi_{J,v}(\theta) = \sum_{c=1}^{2}\sum_{l_{ci},L_{cj}}\sum_{ij}(1 \pm P_{\rm dd})\Big\{C^{(v)}_{cijl_{ci}L_{cj}}(\theta)\, r_c^{l_{ci}} R_c^{L_{cj}} \exp\left(-a_i r_c^2 - A_j R_c^2\right) \sin\left(\beta A_j R_c^2\right)$$

$$+ D^{(v)}_{cijl_{ci}L_{cj}}(\theta)\, r_c^{l_{ci}} R_c^{L_{cj}} \exp\left(-a_i r_c^2 - A_j R_c^2\right) \cos\left(\beta A_j R_c^2\right)\Big\}\left[Y_{l_{ci}}(\hat{\mathbf{r}}_c) \otimes Y_{L_{cj}}(\hat{\mathbf{R}}_c)\right]_{JM}$$

Coupled channelsfor faster convergence

Bipolar spherical harmonics



$c = 1$ $c = 2$

Solve generalized eigenvalue problem

$$\langle \bar{\psi}_n(\theta) \,|\, H(\theta) \,|\, \psi_{n'}(\theta) \rangle = E_n(\theta)\delta_{nn'}$$

$$r \to re^{i\theta} : \quad E_n(\theta) = E_r - i\Gamma/2$$

Symmetry	$v_{\rm r}$	This work	Ref. [24]	Ref. [23]	Ref. [19]
$^{1,5}S^e$	0	218.1111	218.111 567	218.112	218.113
$^{1,5}S^e$	1	135.2785	135.279 003	135.279	135.278
$^{1,5}S^e$	2	72.9662	72.967 058	72.697	72.962
$^{1,5}S^e$	3	31.9011	31.901 769	31.902	31.884
$^{1,5}S^e$	4	12.6165	12.616 688	12.617	12.606
$^{1,5}S^e$	5	5.3112	5.311 346	5.311	5.304
$^{1,5}S^e$	6	2.2750	2.275 273		2.210
$^{1,5}S^e$	7	0.9810	0.981 232		
$^{1,5}S^e$	8	0.4241			
$^{3}S^e$	0	21.1551			21.156
$^{3}S^e$	1	9.4149			9.415
$^{3}S^e$	2	4.0801			4.080
$^{3}S^e$	3	1.7656			1.603
$^{3}S^e$	4	0.7645			
$^{3}S^e$	5	0.3311			

X-ray spectrum calculation

Dipole approximation for resonance-continuum transition

$$\frac{\mathrm{d}\Gamma_\gamma}{\mathrm{d}E_\gamma} = \frac{4}{3}\alpha^3 E_\gamma^3 \left| \langle \Psi_E | \mathbf{d} | \Psi_\mathrm{i} \rangle \right|^2$$

$$|\Psi_E\rangle\langle\Psi_E| = \frac{1}{2i\pi}\left(G^-(E) - G^+(E)\right) \qquad G^\pm(E) = \frac{1}{E \pm i\epsilon - H}$$

$\Big\downarrow$ Analytic continuation

Complex coordinate rotation

$$H(\theta) = R(\theta) H R(-\theta)$$

$$(r \to e^{i\theta} r)$$

$$G^\pm(E) = R(-\theta)\frac{1}{E - H(\theta)}R(\theta)$$

$$G^+(E) : \theta > 0, \quad G^-(E) : \theta < 0$$

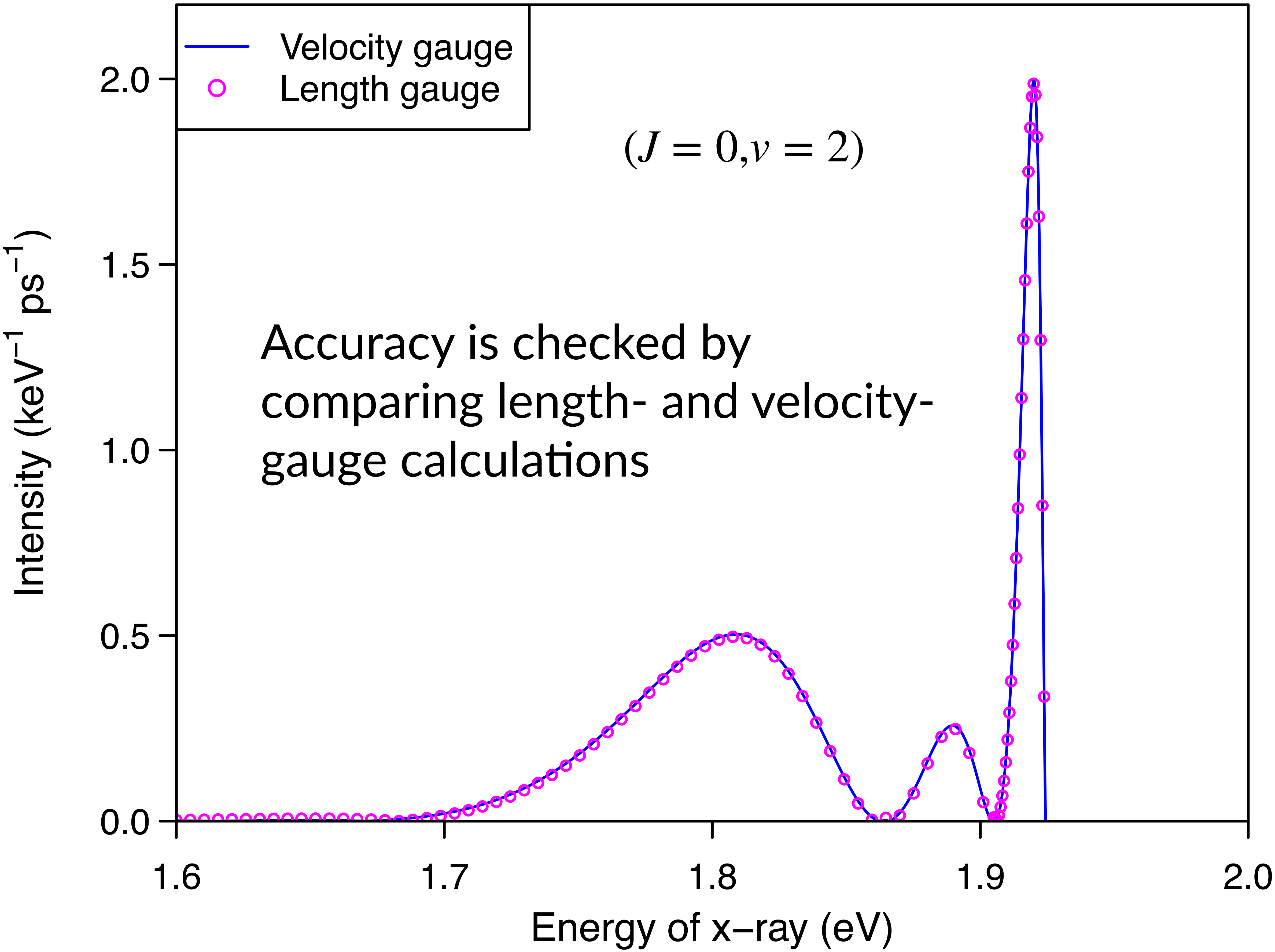
$$|\Psi_E\rangle\langle\Psi_E| = \frac{1}{2i\pi}\left(R(\theta)\frac{1}{E - H(-\theta)}R(-\theta) - R(-\theta)\frac{1}{E - H(\theta)}R(\theta)\right)$$

A Buchleitner et al 1994 J. Phys. B: At. Mol. Opt. Phys. 27 2663

Basis function expansion

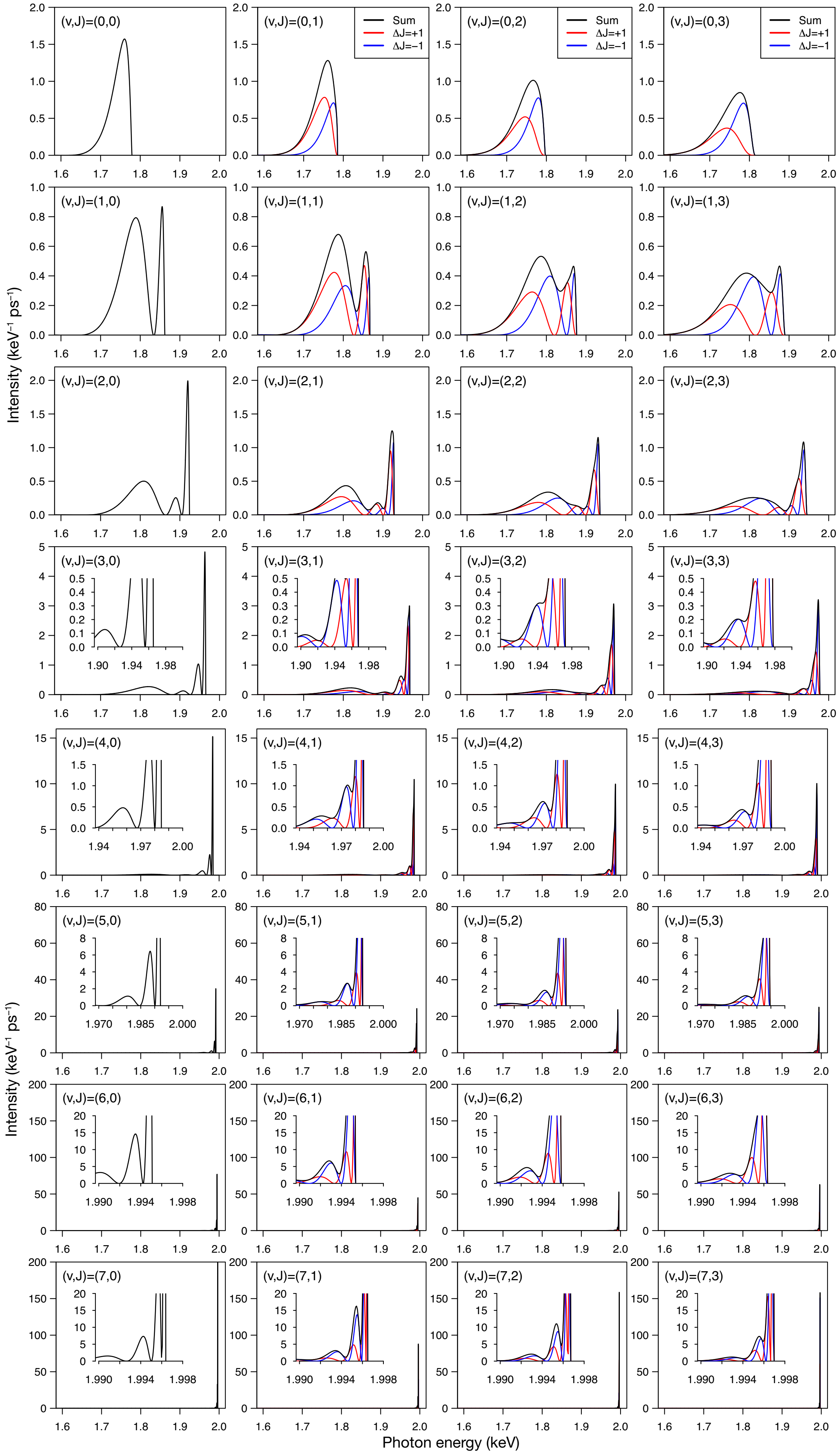
$$\langle \bar{\psi}_n(\theta) | H(\theta) | \psi_m(\theta) \rangle = E_n(\theta)\delta_{nm}$$

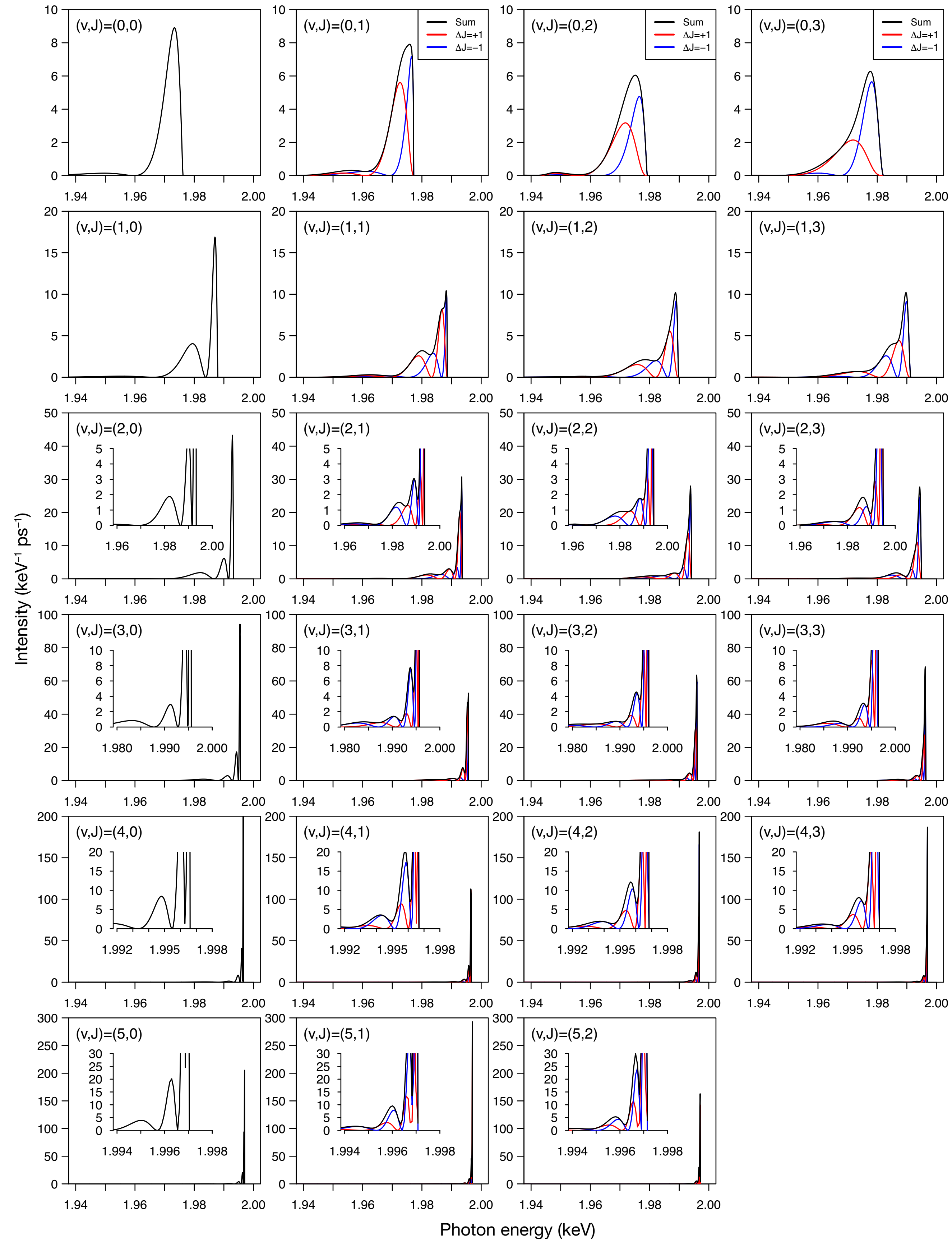
$$\frac{\mathrm{d}\Gamma_\gamma}{\mathrm{d}E_\gamma} = \frac{4}{3}\alpha^3 E_\gamma^3 \frac{1}{\pi} \mathrm{Im} \sum_n \left[\frac{\langle \bar{\psi}_n(\theta) | \mathbf{d}(\theta) | \Psi_\mathrm{i}(\theta) \rangle^2}{E_n(\theta) - E_\mathrm{f}} \right]$$


Velocity gauge
Length gauge
$(J = 0, v = 2)$
Accuracy is checked by comparing length- and velocity-gauge calculations
2.0
1.5
1.0
0.5
0.0
Intensity (keV^{-1} ps^{-1})
1.6
1.7
1.8
1.9
2.0
Energy of x–ray (eV)

X-ray spectra from v=0-8, J=0-3

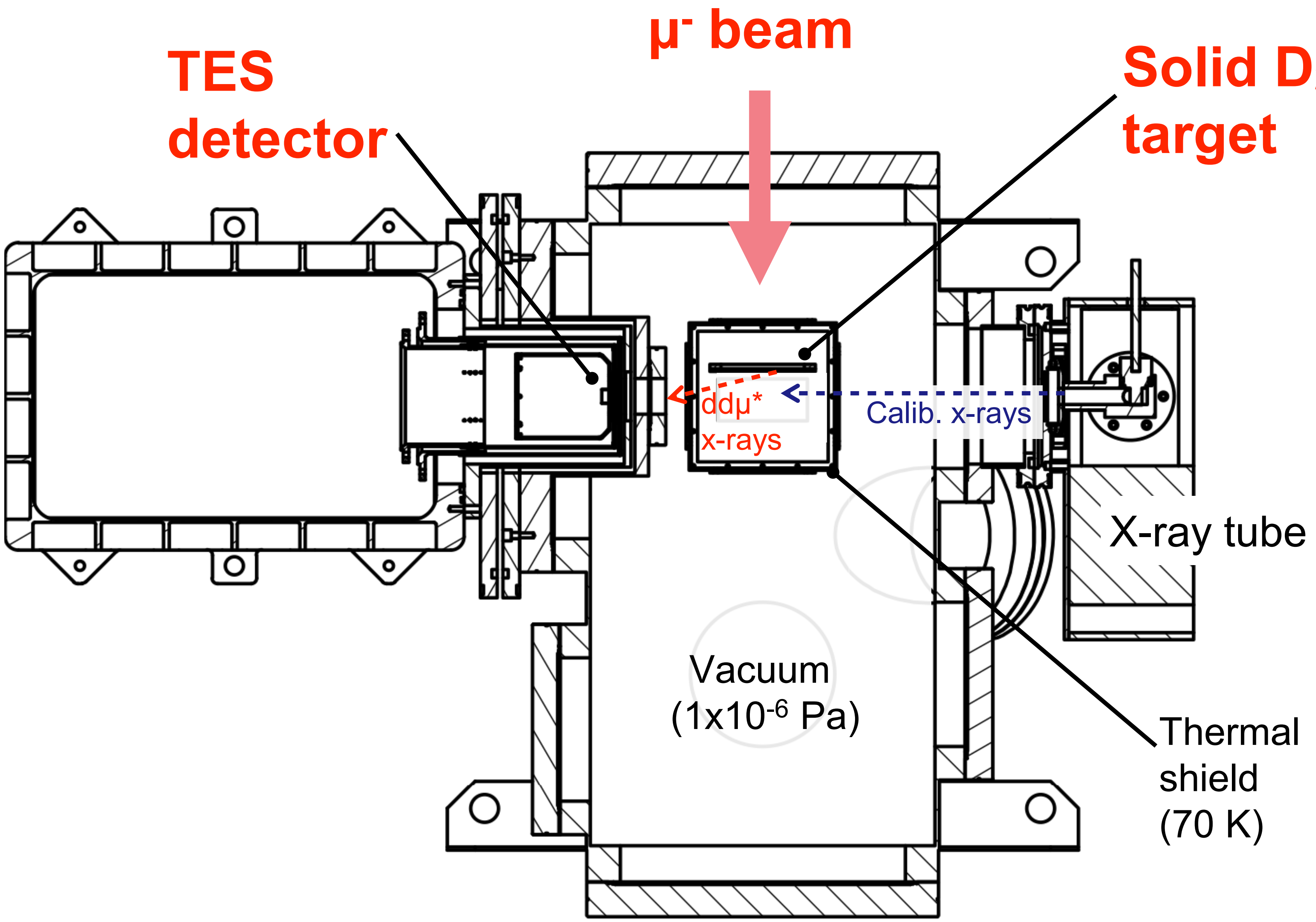
TY et al., Phys. Rev. A **111**, 012811 (2025).





Experiment at J-PARC with TES detector & solid D_2 target

Note: We used pure D_2 instead of D_2-T_2 mixture as the first step



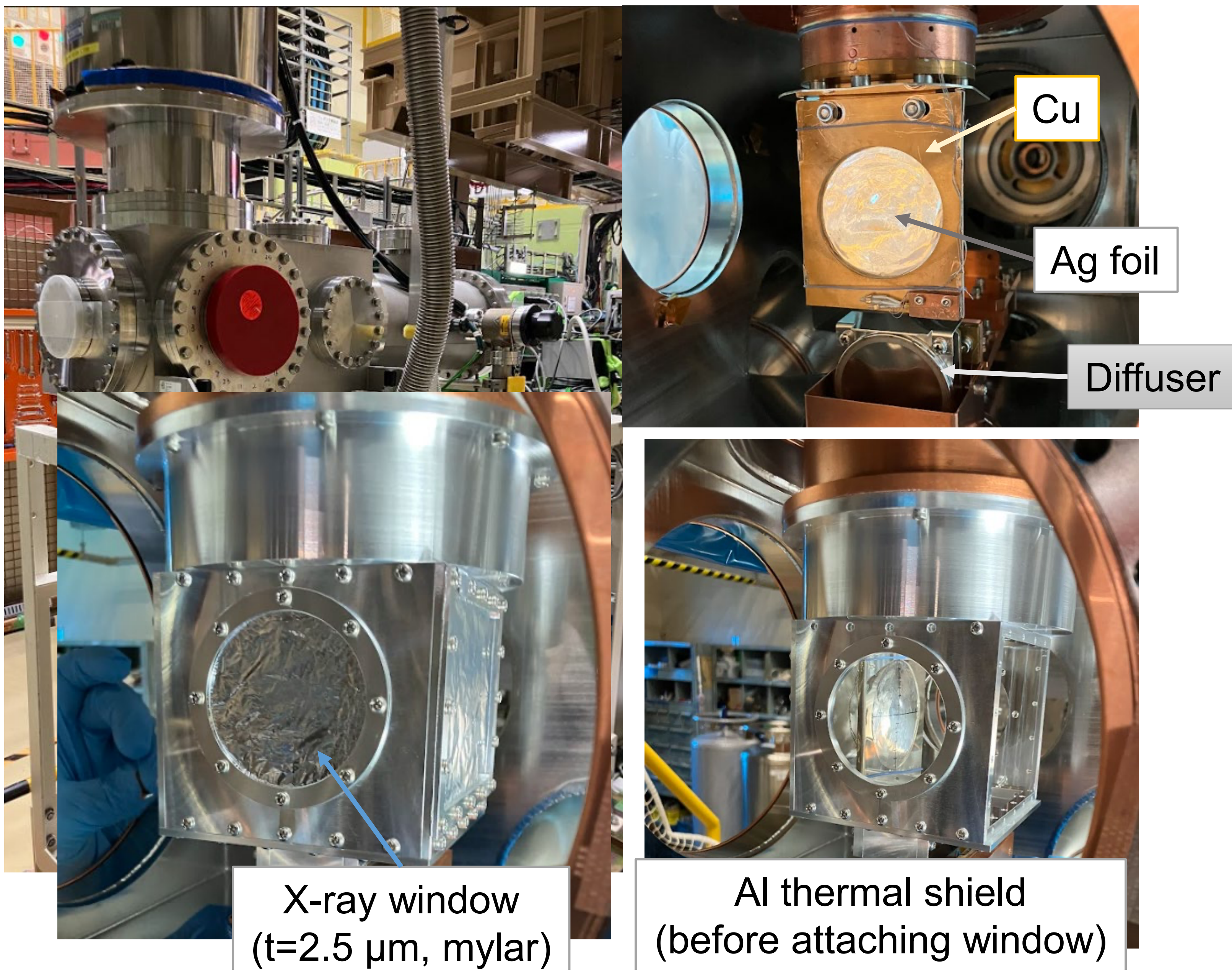
Feb 5 - 11, 2023

- μ^- beam was irradiated to the solid deuterium target.
- ddμ* was produced in the target
- Dissociative X-rays of ddμ* were detected by TES detector
  - Transmittance: ~40%@2 keV
- Calibration X-rays from X-ray tube were measured simultaneously.

- Other detectors
  - Plastic scintillator + PMT (for decay e^-)
  - SDD (for reference)
  - MiniPIX (for proton, scattered μ^-)

Y. Toyama et al., EXA conference (2024).

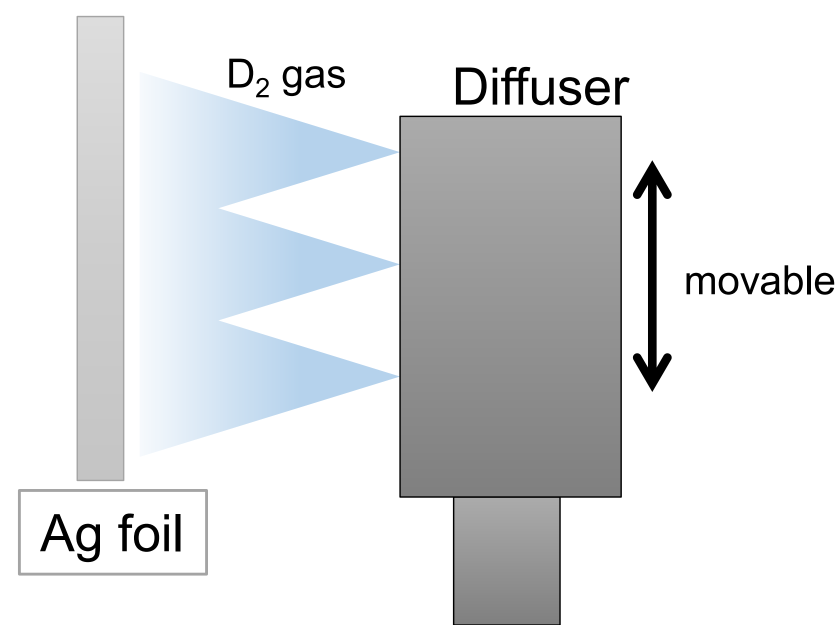
Solid deuterium target



Solid deuterium target system

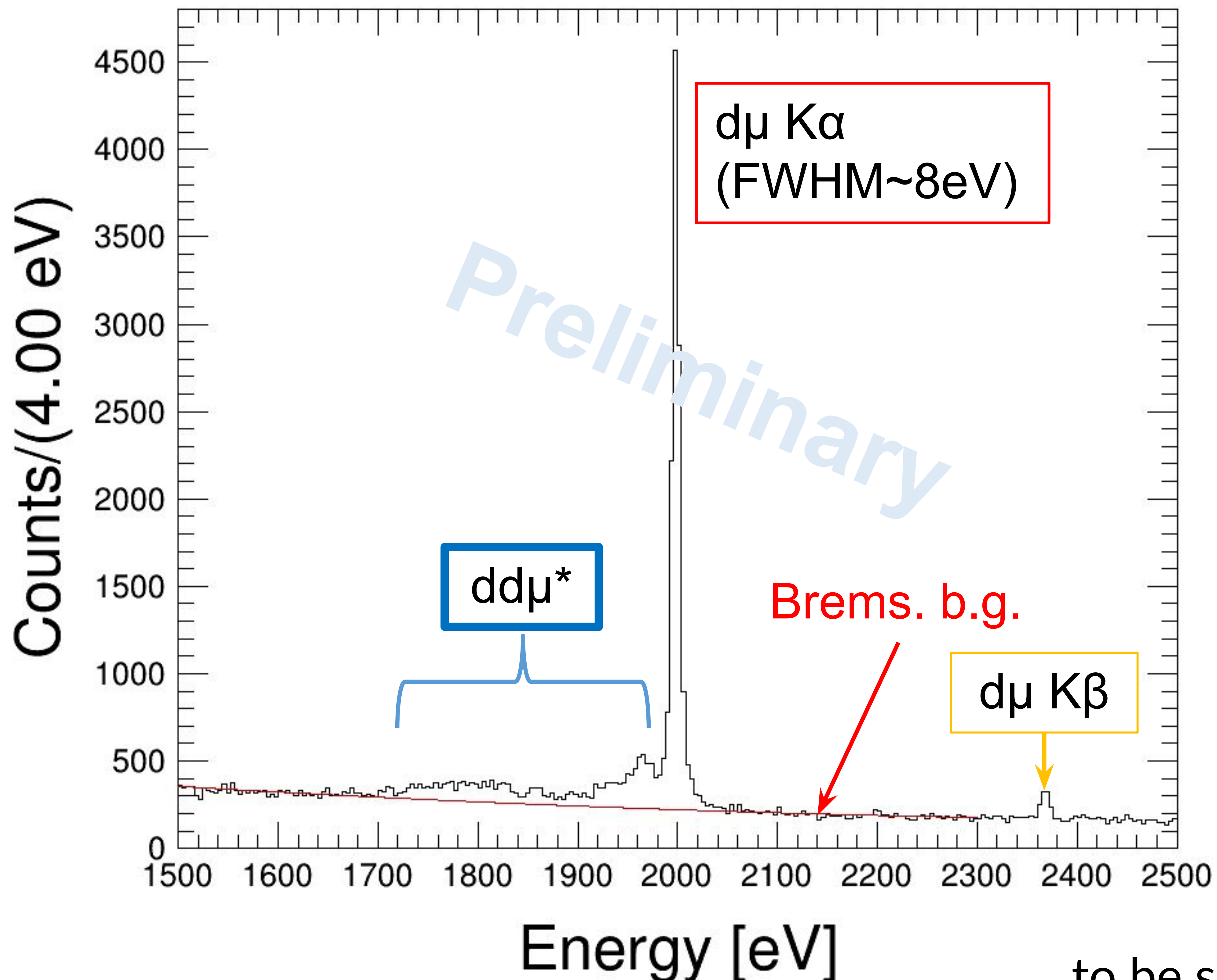
- Base: Ag foil (100 μm)
- Target size: Φ60
- Thickness: 1 mm
- Temperature: ~3 K (Liq. He coolant)
- Chamber pressure: 1×10^{-6} Pa

✓ Stable operation during the measurement.



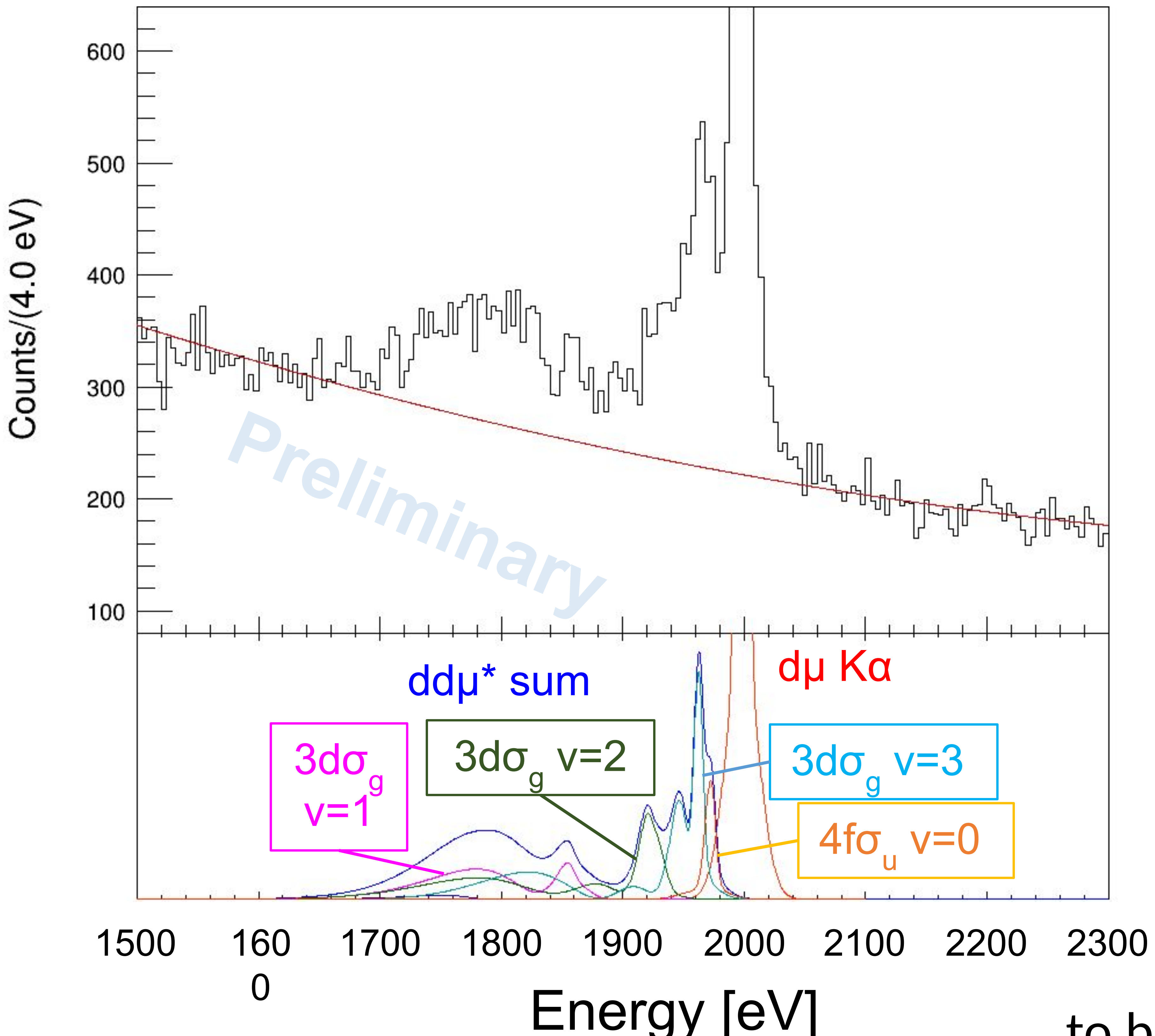
Y. Toyama et al., EXA conference (2024).

Results of the experiment



to be submitted

Results of the experiment: Close-up view



to be submitted

How important is this discovery?

First Observation of muonic molecules in resonance state

- **μCF kinetics including $dt\mu^*$ was first proposed in 1995.**
- **Several X-ray spectra were first calculated in 2003.**
- **However, it has never been proved experimentally.**
- **Our $dd\mu^*$ evidence is a breakthrough in μCF studies.**

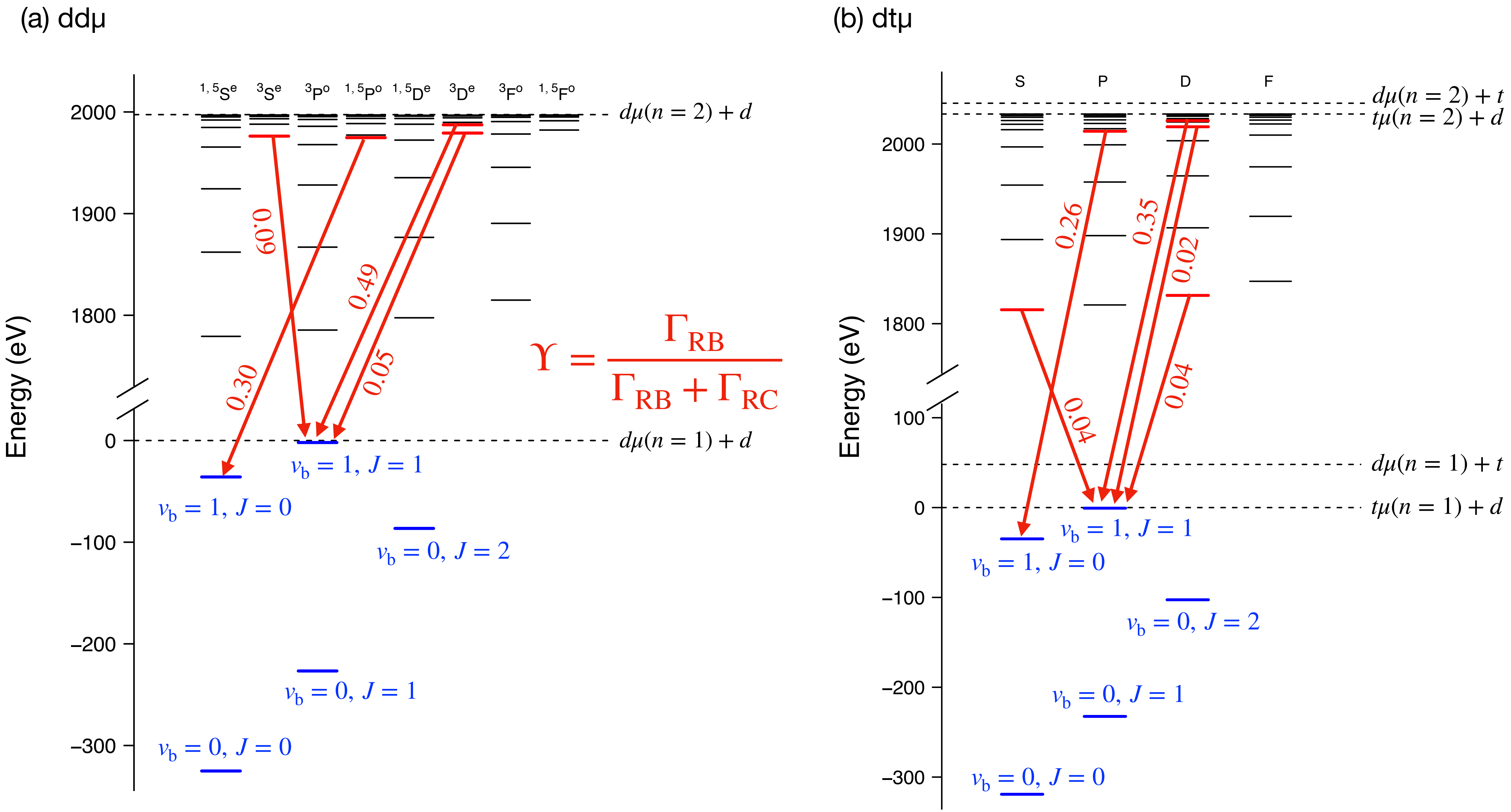
Providing a NEW probe for μCF

- **μCF has been studied by fusion neutrons & unresolved X-rays.**
- **The resolved X-ray spectrum contains much more rich information about the cascade processes.**
  e.g., Intensity ratio of the molecular X-ray to the atomic X-ray provides reaction rates/temperature of excited muonic atoms.

Novel transition from resonance to bound state

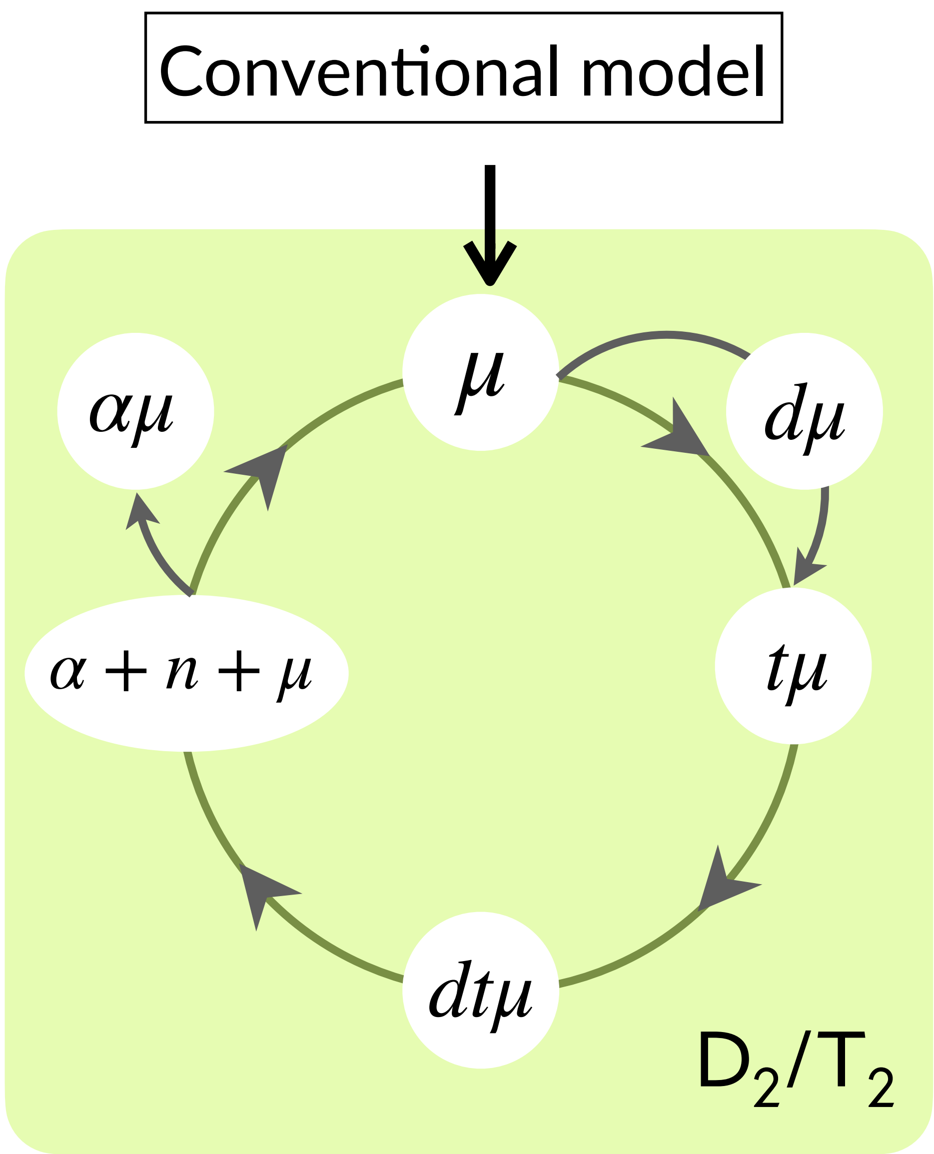
We theoretically discovered that some resonance states show a high branching ratio directly resulting in a bound state molecule.

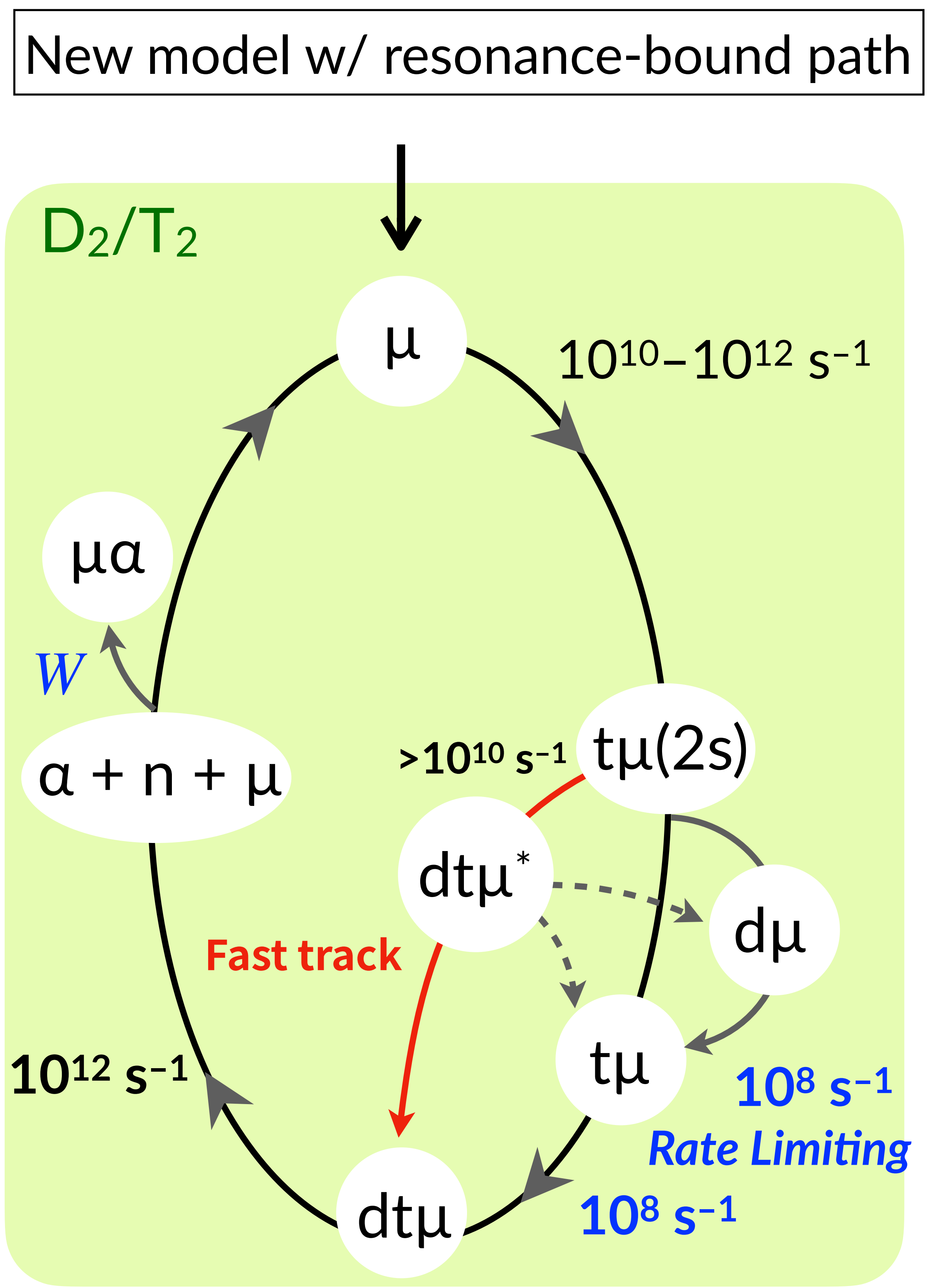
(immediately undergoes fusion)



TY et al., Phys. Rev. A **111**, 012811 (2025).

Possible application of RB transition: Fast track





De-excitation rate of tμ(2s) via dtμ* into dtμ (*if selected*) is faster than ordinary Vesman formation of dtμ from tμ(1s), rate-limiting process.

Summary

- We are reactivating μCF studies in these years.
- μCF could be used as neutron source, energy production, and a negative muon moderator.
- Theoretical predictions stimulated new experiments.
- Experiments at J-PARC MUSE with a new X-ray detector answered a long-standing question in μCF kinetics.

Future experiments

We want to …

- Probe **dtμ*** X-rays using D_2-T_2 mixture target
- **Control** the spontaneous formation of the ddμ* and dtμ*
- Reveal the **target dependency** of the X-ray spectrum

(Solid, Liquid, Dense gas, etc…)