$\mu \to e$ conversion in nuclei:
Summary of a recent ECT* workshop

Frederic Noël, Universität Bern, Institute for Theoretical Physics

Muon4Future 2025

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Workshop on LFV change in nuclei at ECT*

Workshop about $\mu \to e$ conversion in nuclei:

organized by Karim Bennaceur & Sacha Davidson

Goal:

Bring together lepton, χPT/nucleon and nuclear theorists, in order to improve the multi-scale theoretical rate calculations [...] to the accuracy required by upcoming experiments.

Specific Focus:

- Overlap integrals
- Necessary/Possible precision & uncertainties

participants:

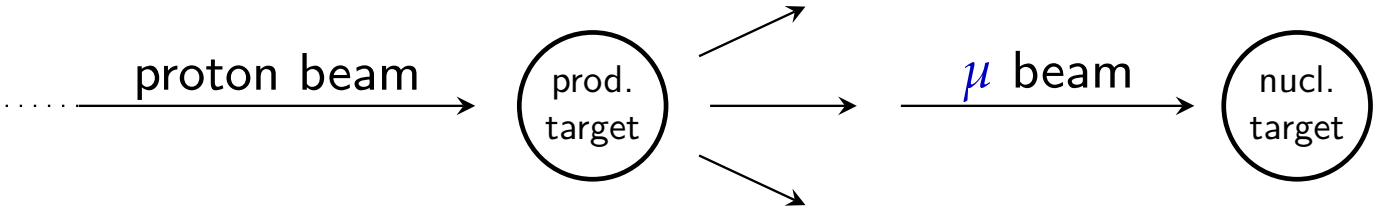
Pierre	Arthuis	IJCLab Orsay, CNRS & U. Paris-Saclay
Karim	Bennaceur	Université Claude Bernard Lyon 1, IP2I
Cristina	Carloganu	LPCA/IN2P3/CNRS
Vincenzo	Cirigliano	Institute for Nuclear Theory, University of Washington
Andrzej	Czarnecki	University of Alberta
Sacha	Davidson	IN2P3, CNRS
Jacek	Dobaczewski	University of York
Matthias	Heinz	Oak Ridge National Laboratory
Lotta	Jokiniemi	TRIUMF / TU Darmstadt
Markus	Kortelainen	University of Jyväskylä
Yoshitaka	KUNO	Osaka University
Kilian	Möhling	TU Dresden
Frederic	Noël	ITP, AEC, Uni Bern
Paride	Paradisi	University of Padua and INFN
Joe	Sato	Yokohama National University
Dominik	Stöckinger	TU Dresden
Yuichi	Uesaka	Dokkyo Medical University
Ubirajara	van Kolck	ECT*

Disclaimer

Biased summary with respect to my background and interests

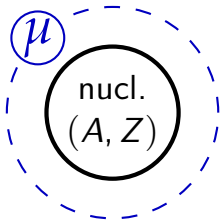
What is $\mu \to e$ conversion? (theorist's perspective)

- Experimental Setup/Preparation:



- Conversion process:

  (within Coulomb field of the nucleus)



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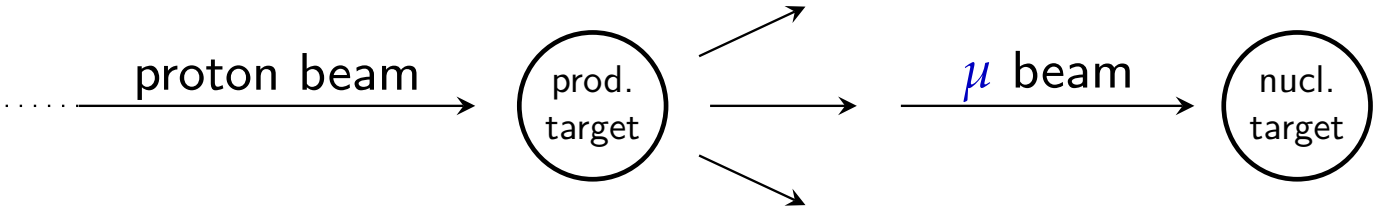

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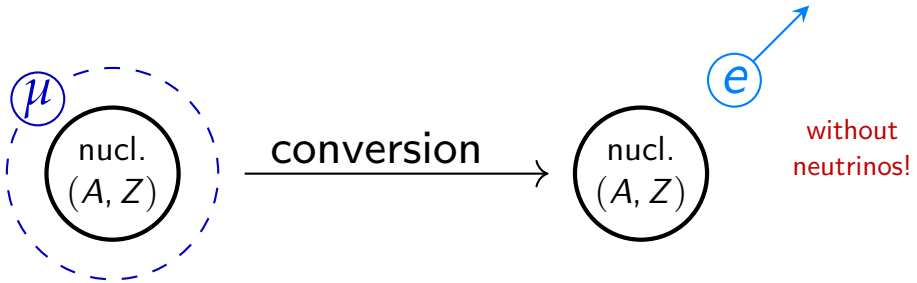

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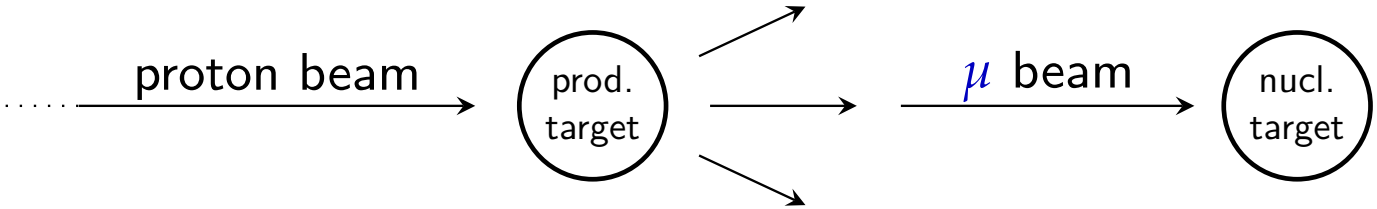

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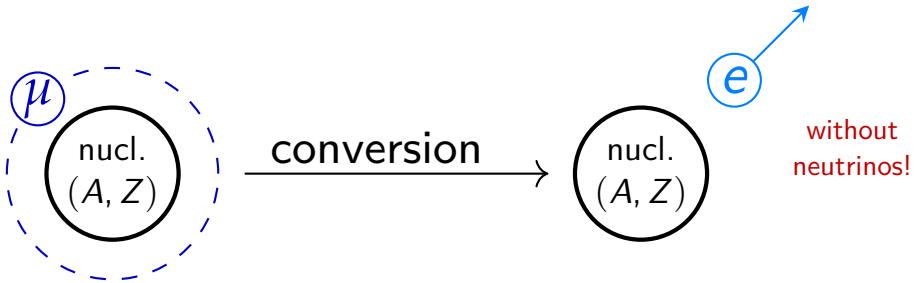

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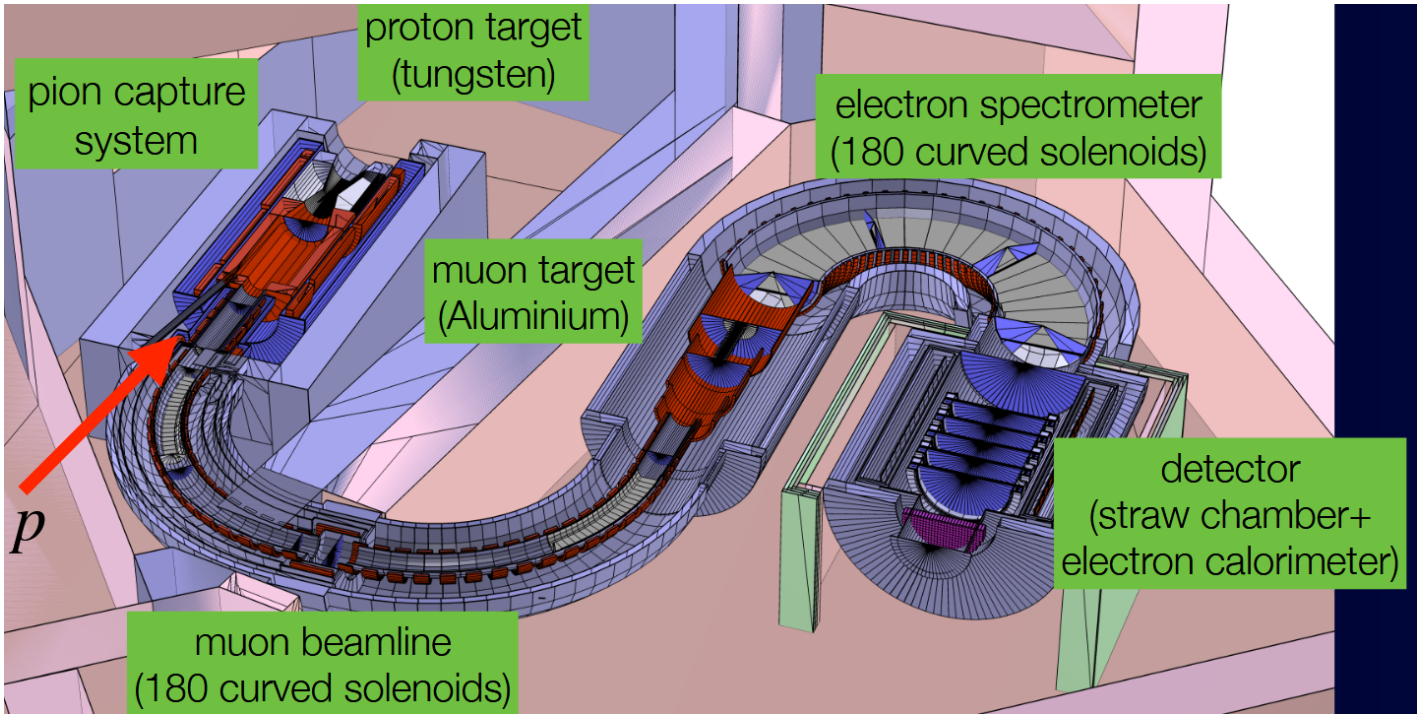
- Current best limits on Gold and Titanium [SINDRUM II]:
  $\mathrm{Br}(\mathrm{Au}\,\mu^- \to \mathrm{Au}\,e^-) < 7 \cdot 10^{-13}$
  $\mathrm{Br}(\mathrm{Ti}\,\mu^- \to \mathrm{Ti}\,e^-) < 6.1 \cdot 10^{-13}$
- Next generation of experiments measuring on Aluminum:
  Mu2e & COMET (+ Upgrades) [Talks: tomorrow morning]

Experimental Setup (COMET as an example)

- COMET (Phase II) [Workshop talks: Yoshitaka Kuno & Cristina Carloganu]



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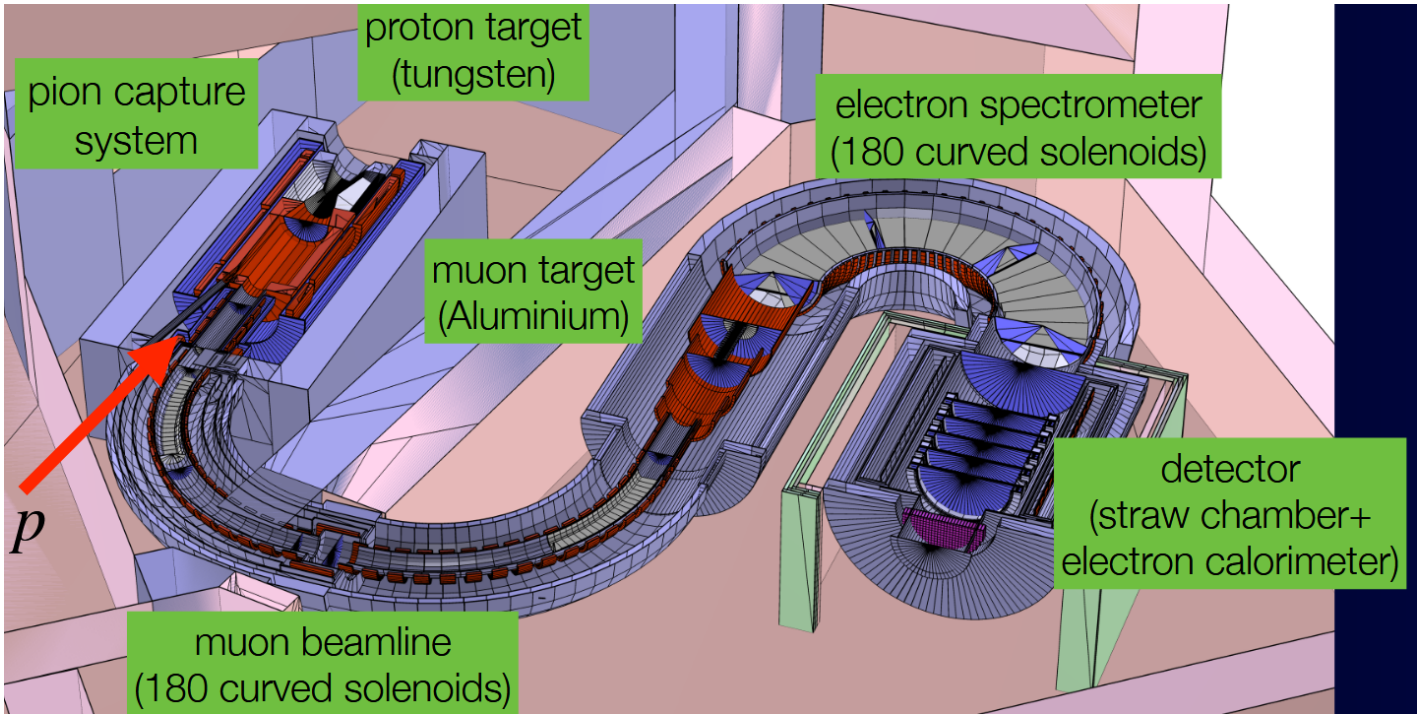
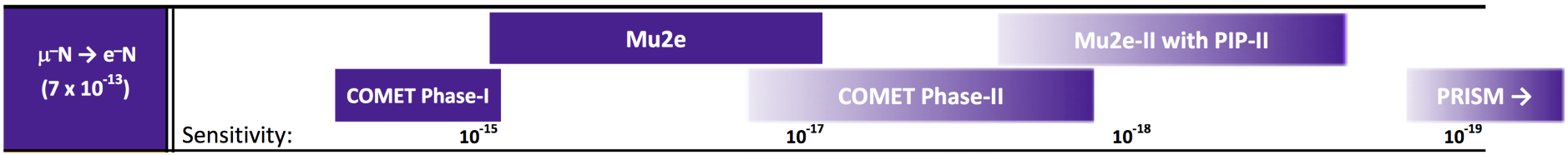


Major experimental advances upcoming:



- To access heavy nuclei: PRISM necessary to remove pions

Backgrounds and related processes

While backgrounds are extremely small, we are reaching precision levels where they start to become relevant

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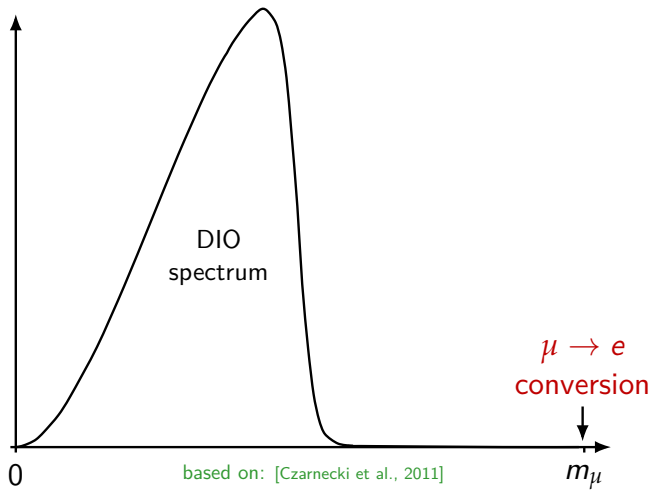
- Backgrounds:
  - decay in orbit: $\mu^- \to \nu_\mu \bar{\nu}_e e^-$
  - (radiative) muon capture
  - (radiative) pion capture
  - cosmic radiation
- historically muon capture used as normalization for $\mu \to e$



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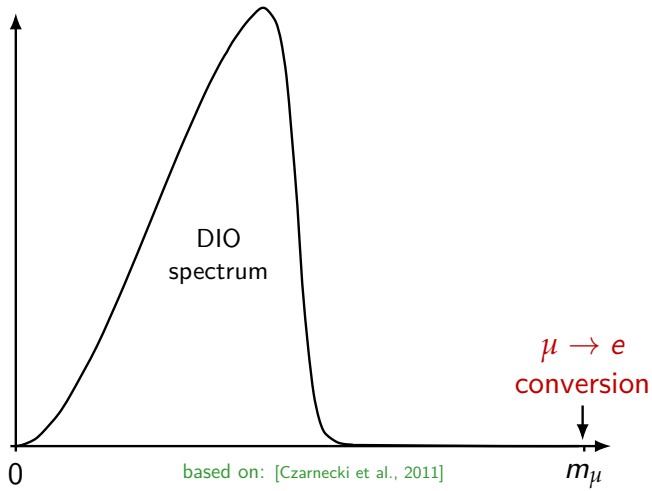

- Processes from muonic atoms require reliable inputs for ...
  - ... nuclear structure (ab-initio) [Workshop talk: Lotta Jokiniemi]
  - ... muon/electron wave functions [Workshop talk: Yuichi Uesaka]
- Same for $\mu \to e$ conversion and other nuclear processes like $0\nu\beta\beta$

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Experimental advances motivate improvements of the theoretical description

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- Testing specific BSM models for CLFV limits:
  - Study of various models (LQ, ν_R, ...)
    [Workshop talk on vector-like leptons: Kilian Möhring]
  - RG evolution tools now extend to CLFV
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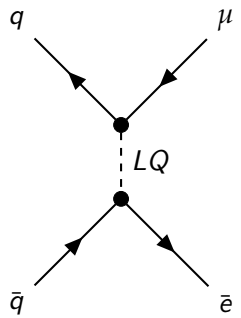

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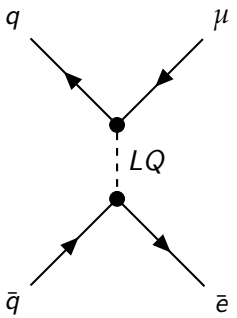
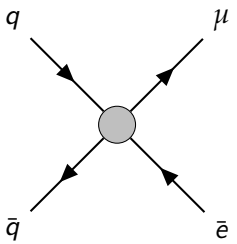
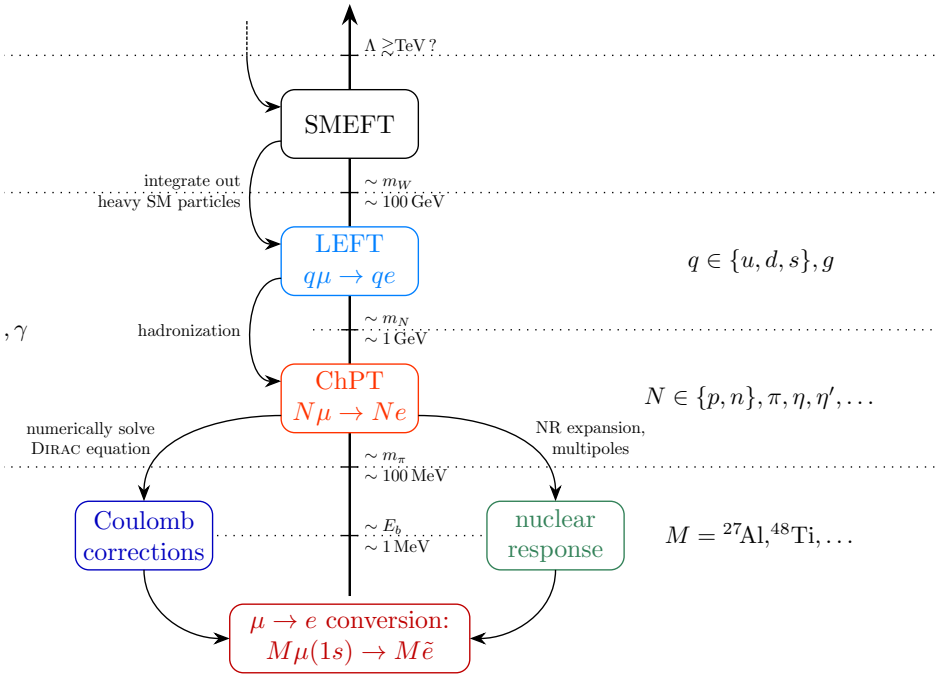


- "Model-independent" studies using EFTs
  [Workshop talk on (nuclear) EFTs: Bira van Kolck]
  - Many LFV effective operators can contribute
  - RG evolution and interference of SMEFT operators studied until EW scale [Crivellin et al., 2017,...]
  - Tower of EFTs necessary to reach low-energy scale



$\mu \to e$ conversion framework components

Many different scales matter:



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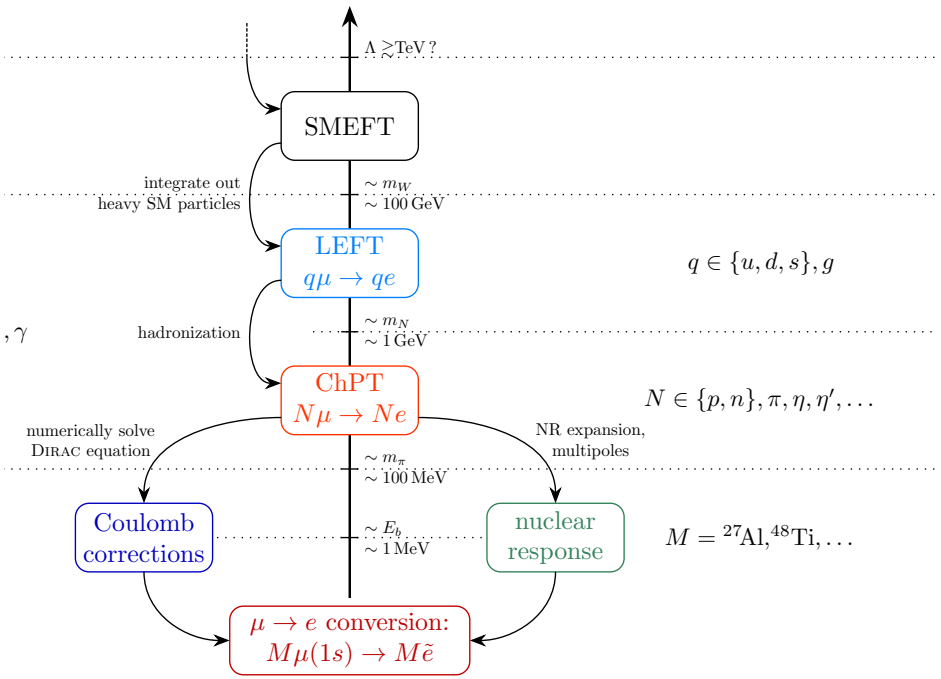

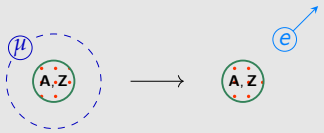

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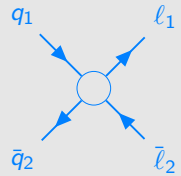

(short distance) EFT operators

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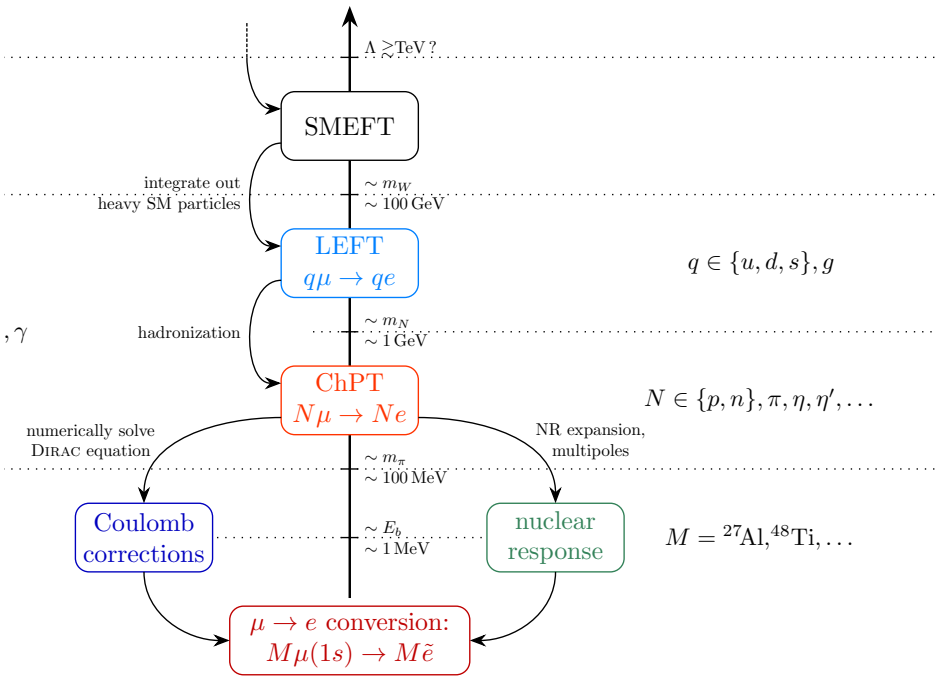

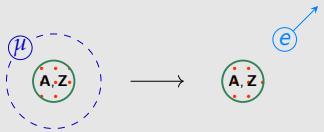

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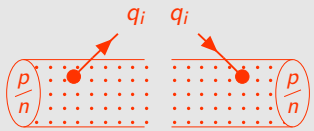
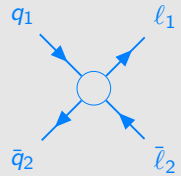


hadronic matrix elements

$\otimes$



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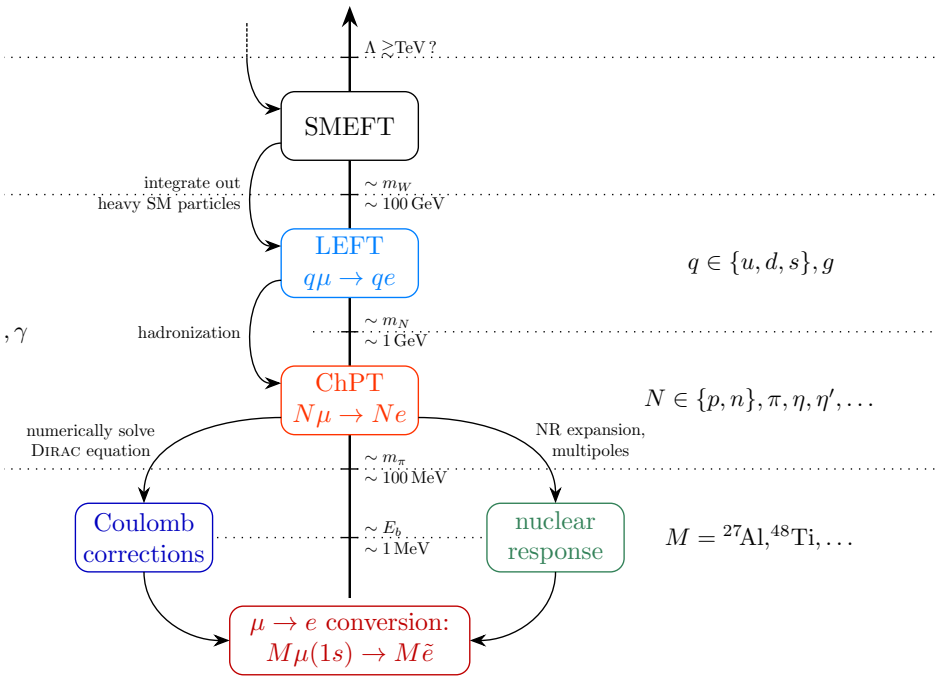

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nuclear response

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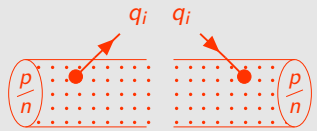

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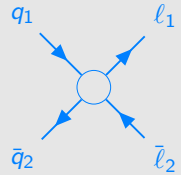

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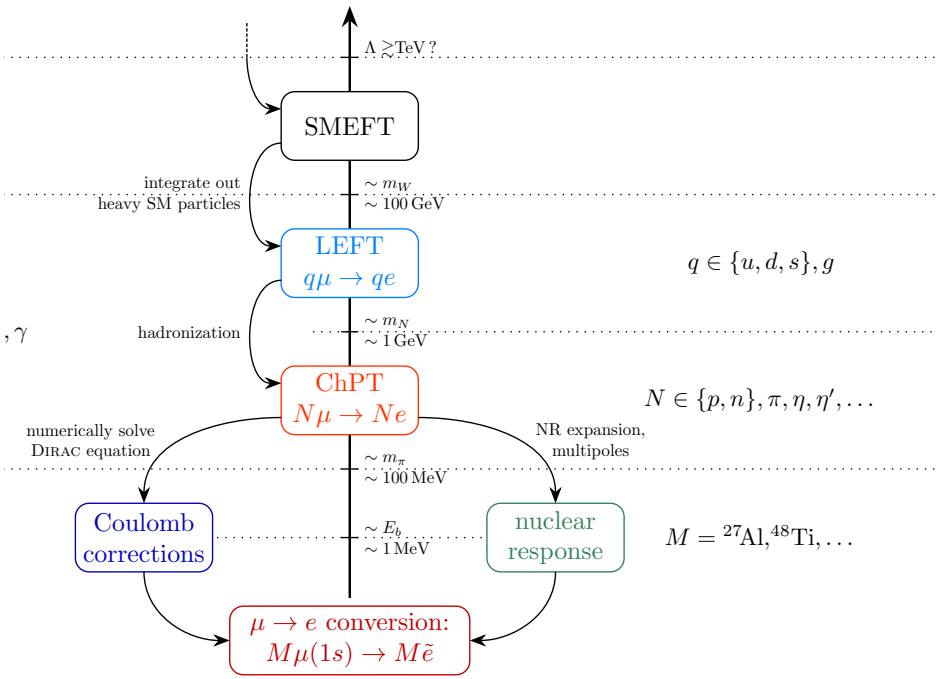

$\mu \to e$ conversion | Coulomb corrections | nuclear response | hadronic matrix elements | (short distance) EFT operators

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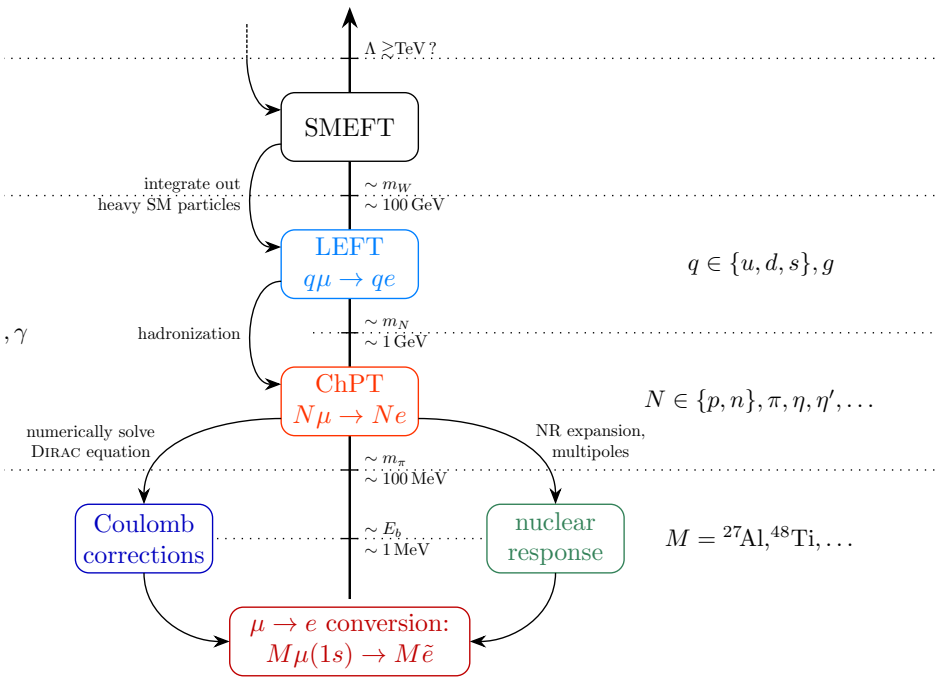

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- Compare different LFV probes
- Discriminate BSM operators
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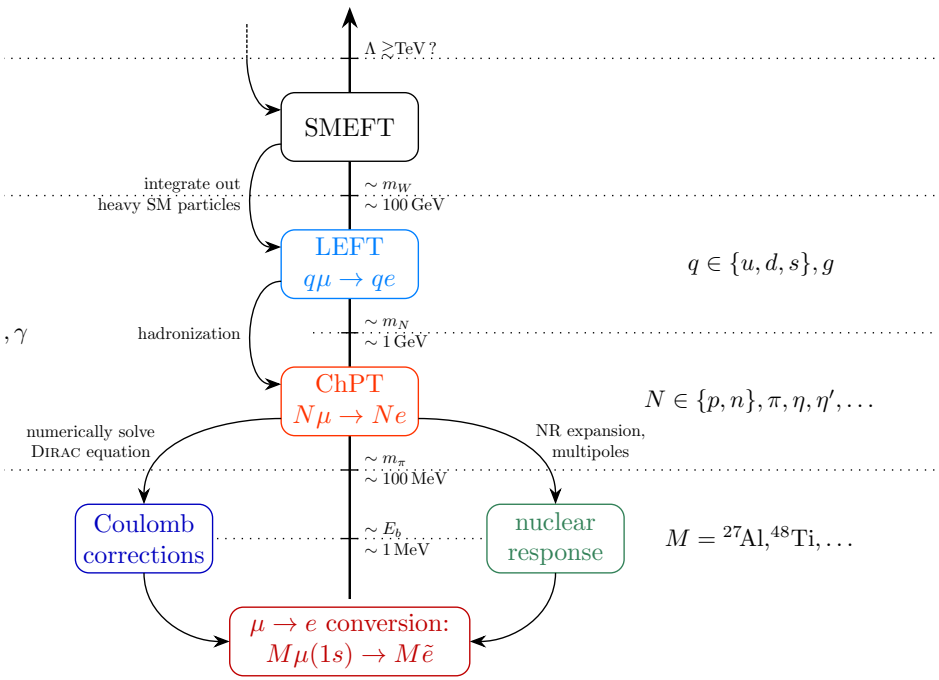

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- Compare different LFV probes
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- Control theory uncertainties:
  - Hadronic matrix elements
  - Nuclear response
  - Coulomb corrections

At all steps uncertainties need to be controlled!

Components and Challenges

- Hadronic matrix elements: from LatticeQCD & Phenomenology
  - Relevance of momentum dependence? (up to 5% ?)
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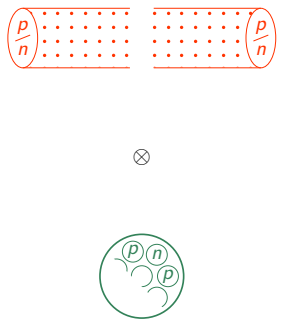

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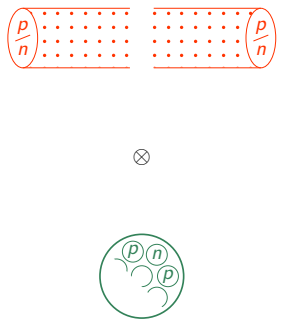

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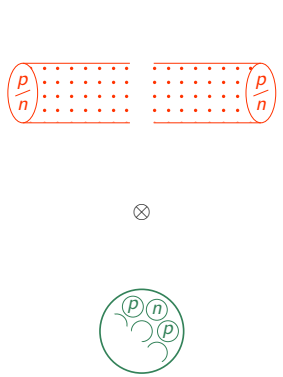

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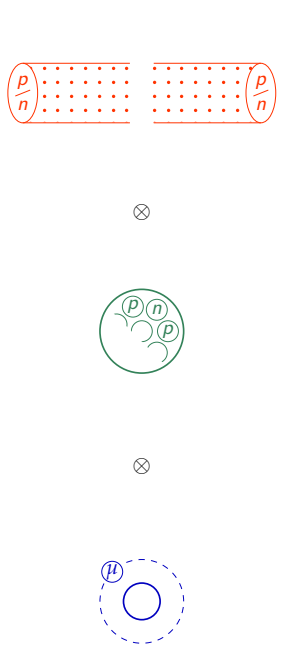
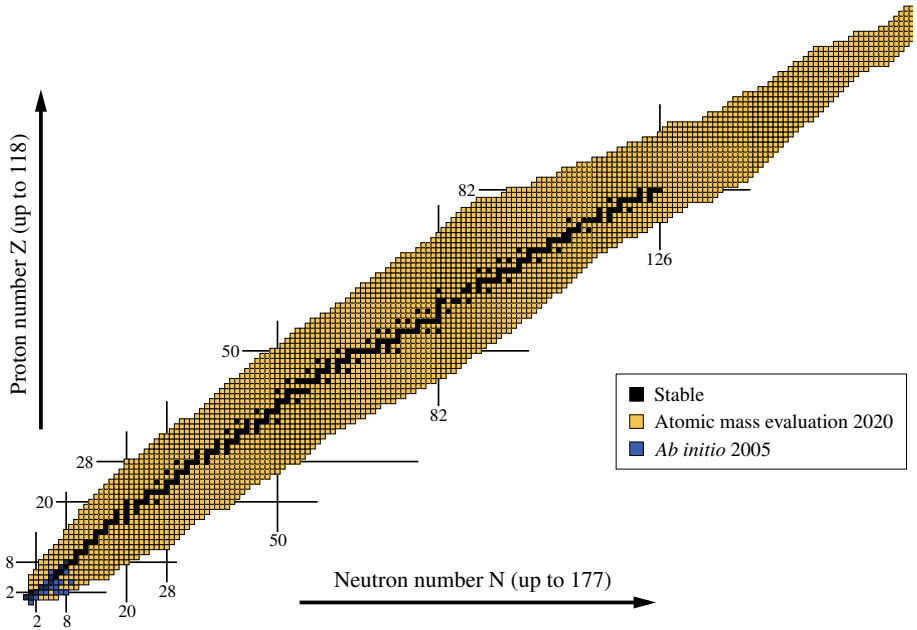
- Coulomb corrections: Solving Dirac equation numerically (using ρ_{ch})
  [Workshop Talks: Yuichi Uesaka & FN]
  - Recent (re)extraction of charge densities including uncertainties (see below)



Ab-initio calculations

- Tremendous improvements over the last 20 years:



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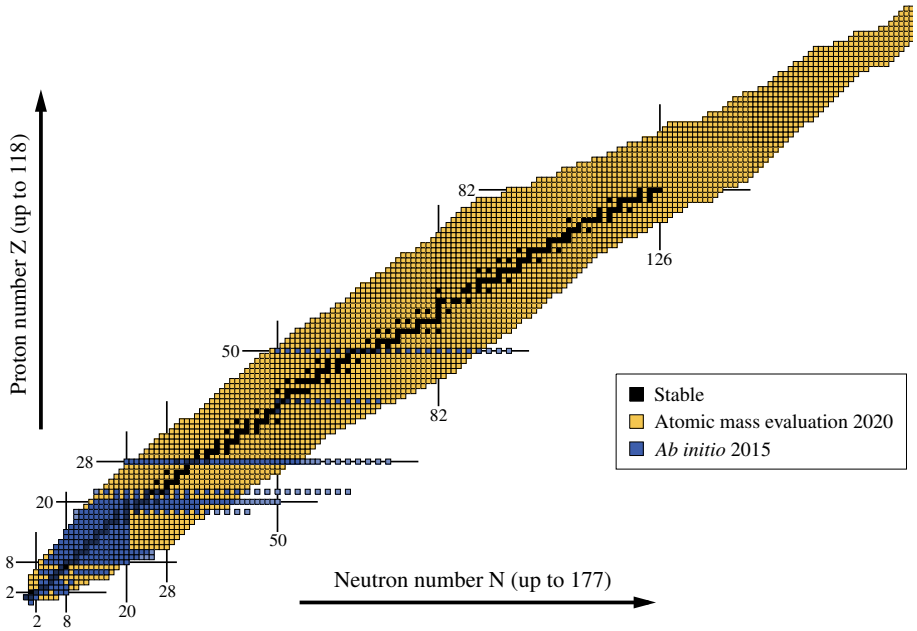

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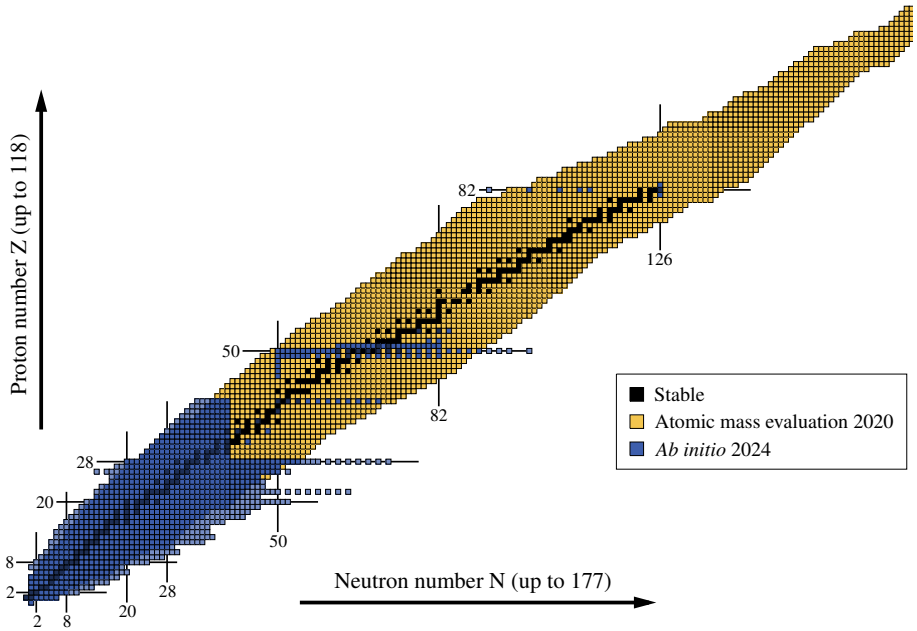

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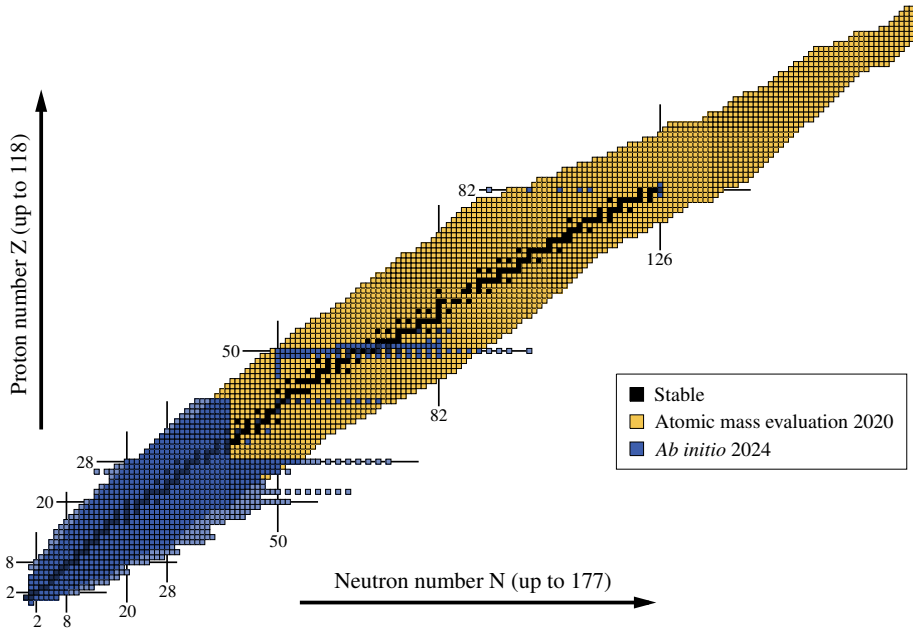

- Diverse and complementary set of approaches [Workshop talk: P. Arthuis]
- For nuclei of $\mu \rightarrow e$ conversion: (VS-)IMSRG [Workshop talk: M. Heinz]
- Uncertainties dominated by chiral Hamiltonians,
  not by many-body solutions $\rightarrow$ often stable correlations
- Utilization of correlations requires references (e.g. charge density)

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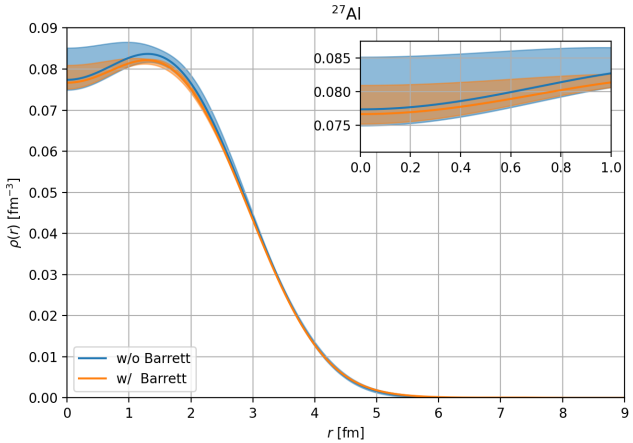

[FN, Hoferichter, 2024]

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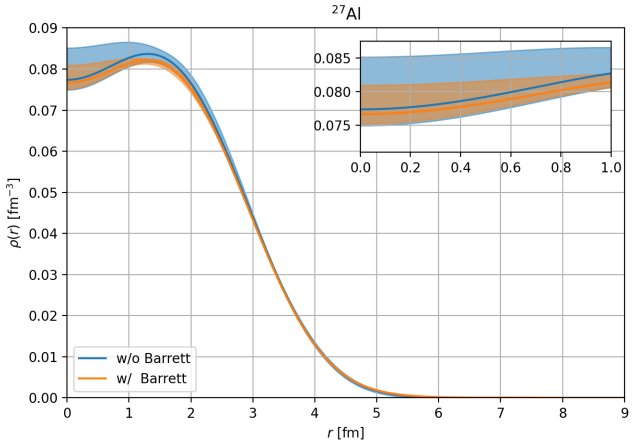
- Practical challenges:
  - Most data at least 50 years old
    (hard/impossible to find, limited uncertainties)
  - Computationally intensive (w.r.t. uncertainties)



[FN, Hoferichter, 2024]

New charge densities with uncertainties [Workshop talk: FN]

- Renew interest in charge distributions
  [Workshop talk on mean field calc: Karim Bennaceur]
- Historical results are without uncertainties
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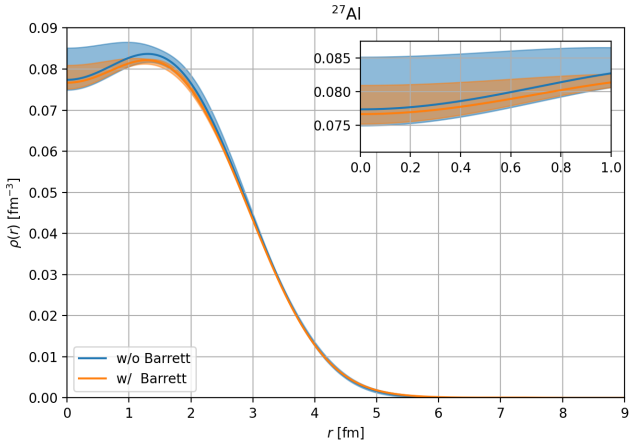

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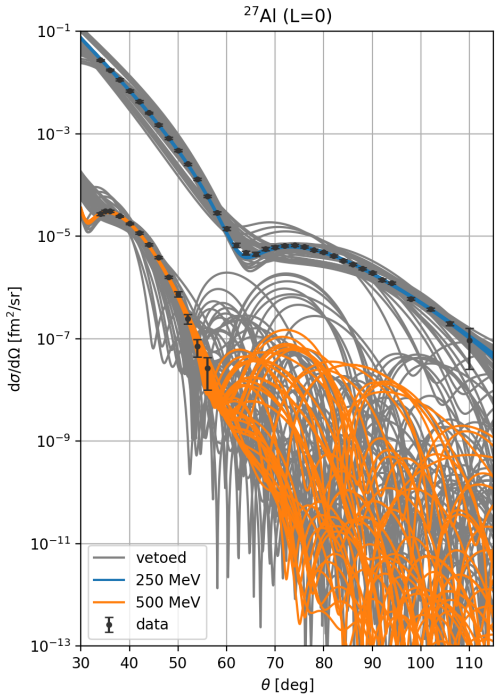

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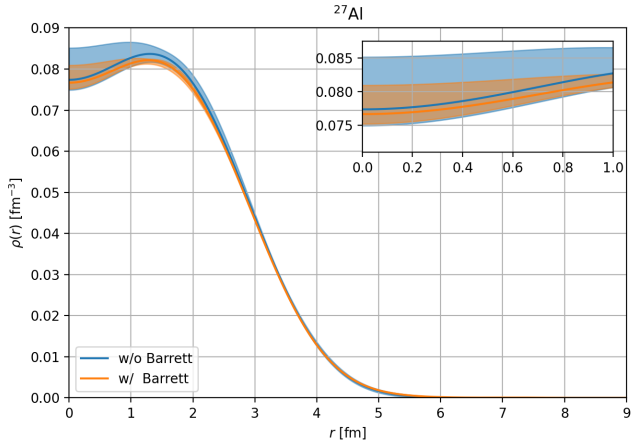

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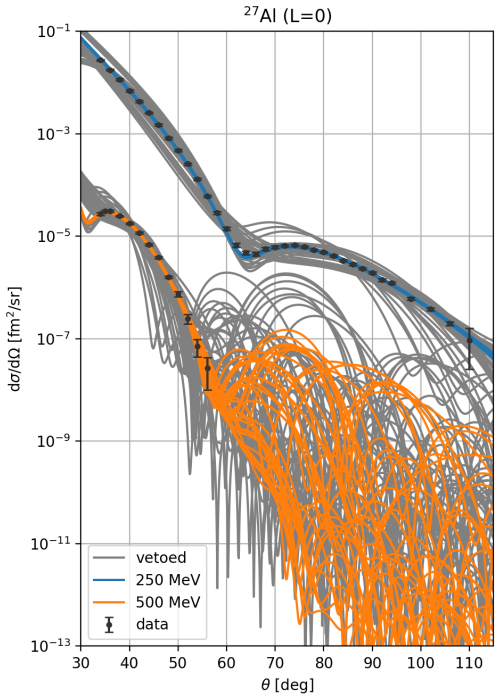
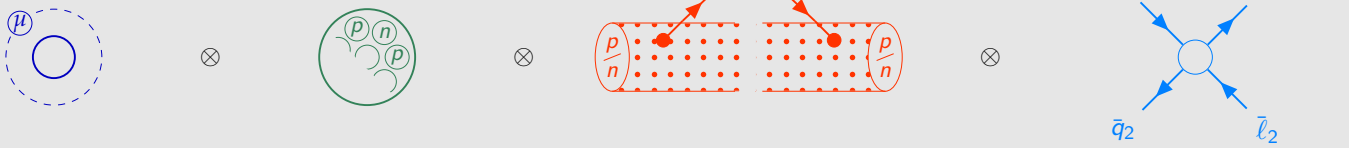


Carried out for ^{27}Al, ^{40,48}Ca, ^{48,50}Ti

Results available in `python` notebook [2406.06677]

Overlap Integrals

- Overlap integrals combine nuclear responses and Coulomb corrections

Conversion Rate $=$ 

Coulomb corrections | nuclear response | hadronic matrix elements | (short distance) EFT operator

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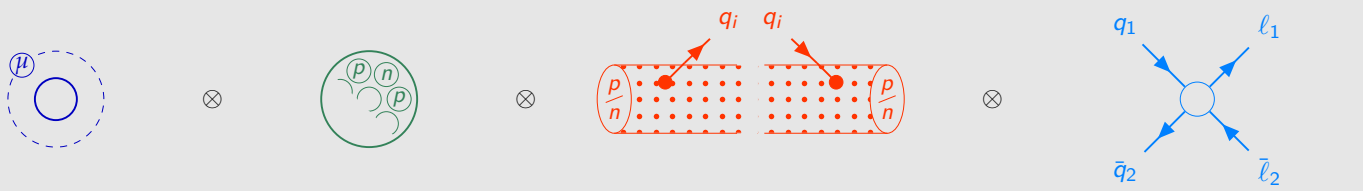
Conversion Rate $=$ 

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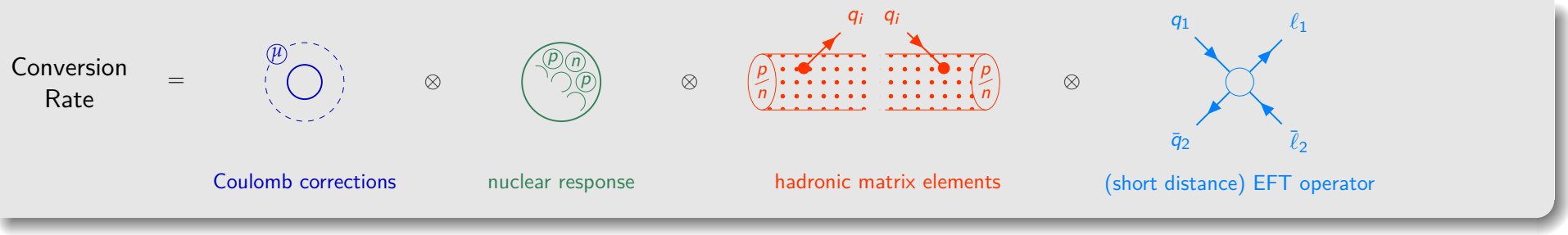


[Kitano et al., 2002]

Leading / SI overlap integrals:

$$\text{scalar:} \quad S^{(N)} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r \; (\#N)\rho_N(r) \left[g_{-1}^{(e)}(r)\, g_{-1}^{(\mu)}(r) - f_{-1}^{(e)}(r)\, f_{-1}^{(\mu)}(r) \right]$$

$$\text{vector:} \quad V^{(N)} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r \; (\#N)\rho_N(r) \left[g_{-1}^{(e)}(r)\, g_{-1}^{(\mu)}(r) + f_{-1}^{(e)}(r)\, f_{-1}^{(\mu)}(r) \right]$$

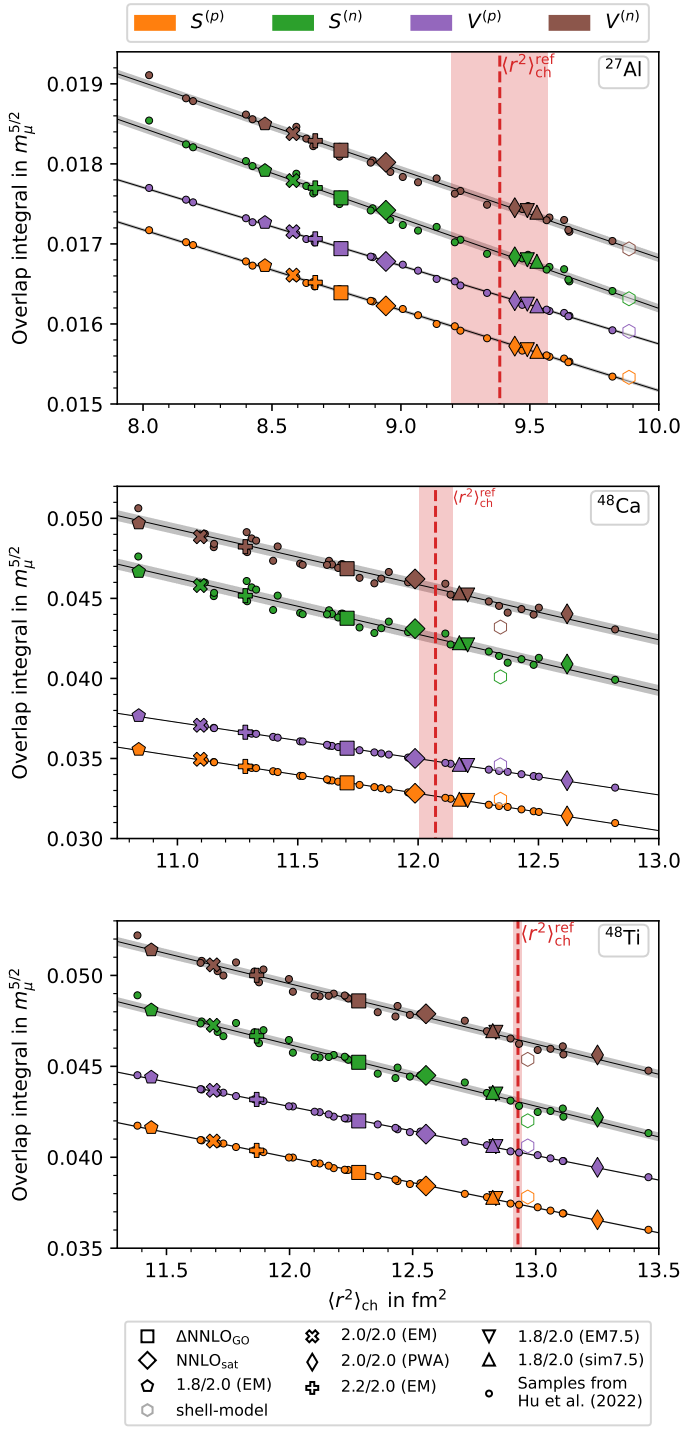
$$\text{dipole:} \quad D = -\frac{4m_\mu}{\sqrt{2}} \int_0^\infty \mathrm{d}r \quad E(r) \quad \underbrace{\left[g_{-1}^{(e)}(r)\, f_{-1}^{(\mu)}(r) + f_{-1}^{(e)}(r)\, g_{-1}^{(\mu)}(r) \right]}_{\text{electron and muon wave functions}}$$

- Development of subleading overlap integrals is currently in process

Correlation Analysis

[Workshop talks: M. Heinz & FN; Heinz et al., 2024]

- Establish correlations from IMSRG over an ensemble of 42 Hamiltonians
- Observed tight correlations between overlap integrals and $\langle r^2 \rangle_{\text{ch}}$



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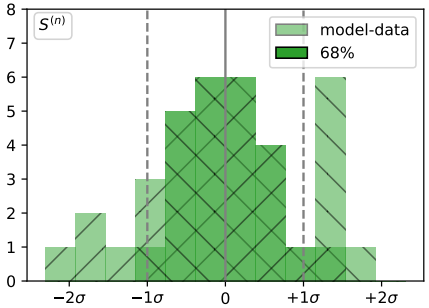

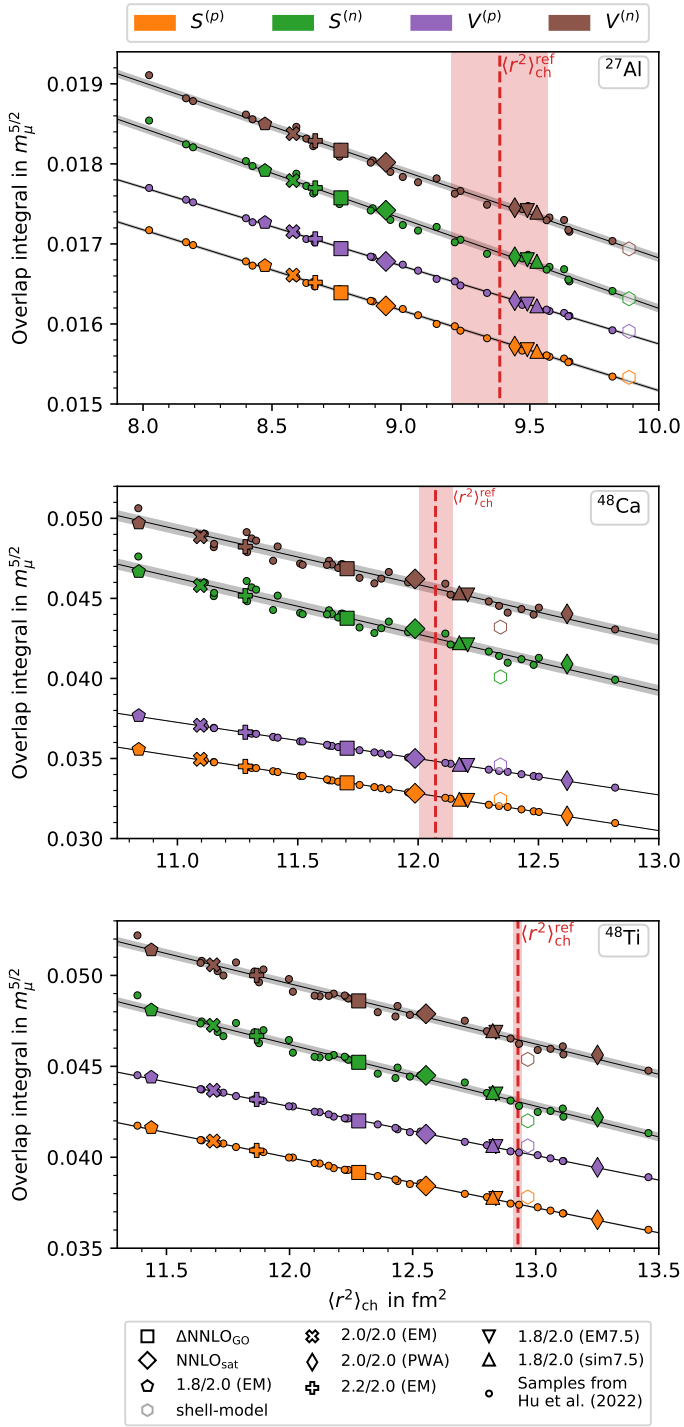

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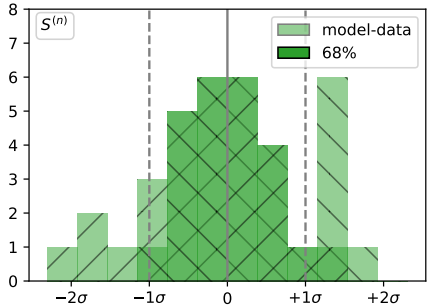

For the first time:

Quantitative Overlap integrals with fully quantified uncertainties

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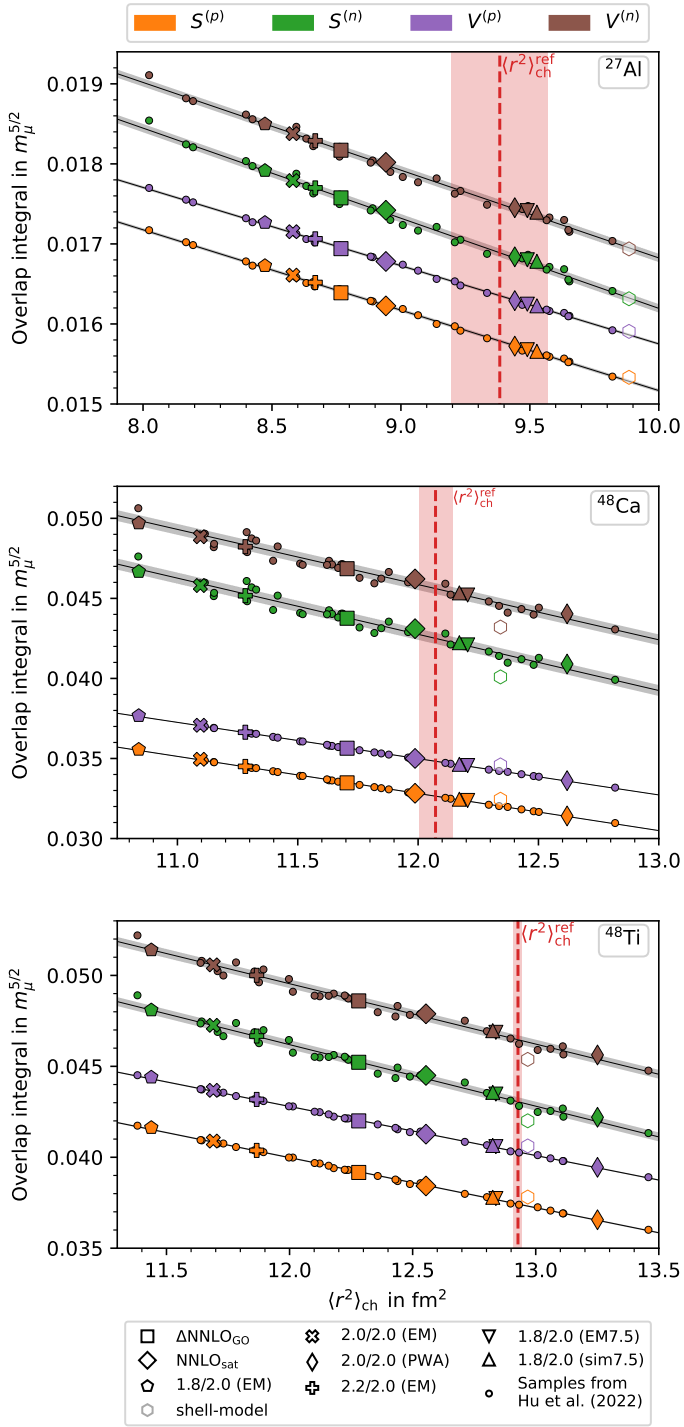

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More generally: nuclear structure calculations are very versatile

- parity-violating electron scattering (PVES)
- neutrinoless double beta decay ($0\nu\beta\beta$)
- neutrino-nucleus scattering
- DM-nucleus scattering

$\to$ lots of work happening (past/present/future)

Results of the Workshop

- Goal to discriminate BSM operators
  - What are nuclei with the most complementarity?
    $\rightarrow$ Make nuclei/isotope suggestions for experiments
  - Can we control uncertainties sufficiently?
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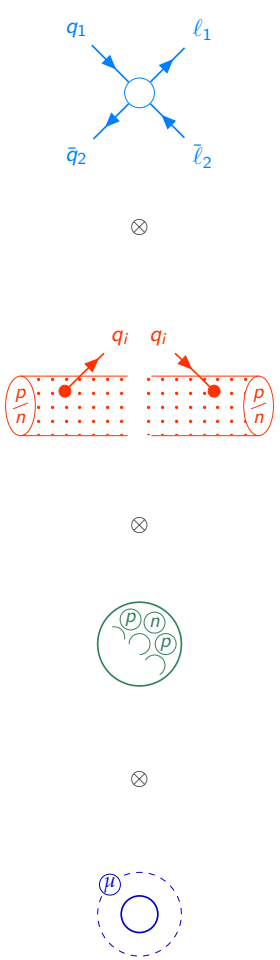
Hopefully, a lot of these points can be addressed in the near future

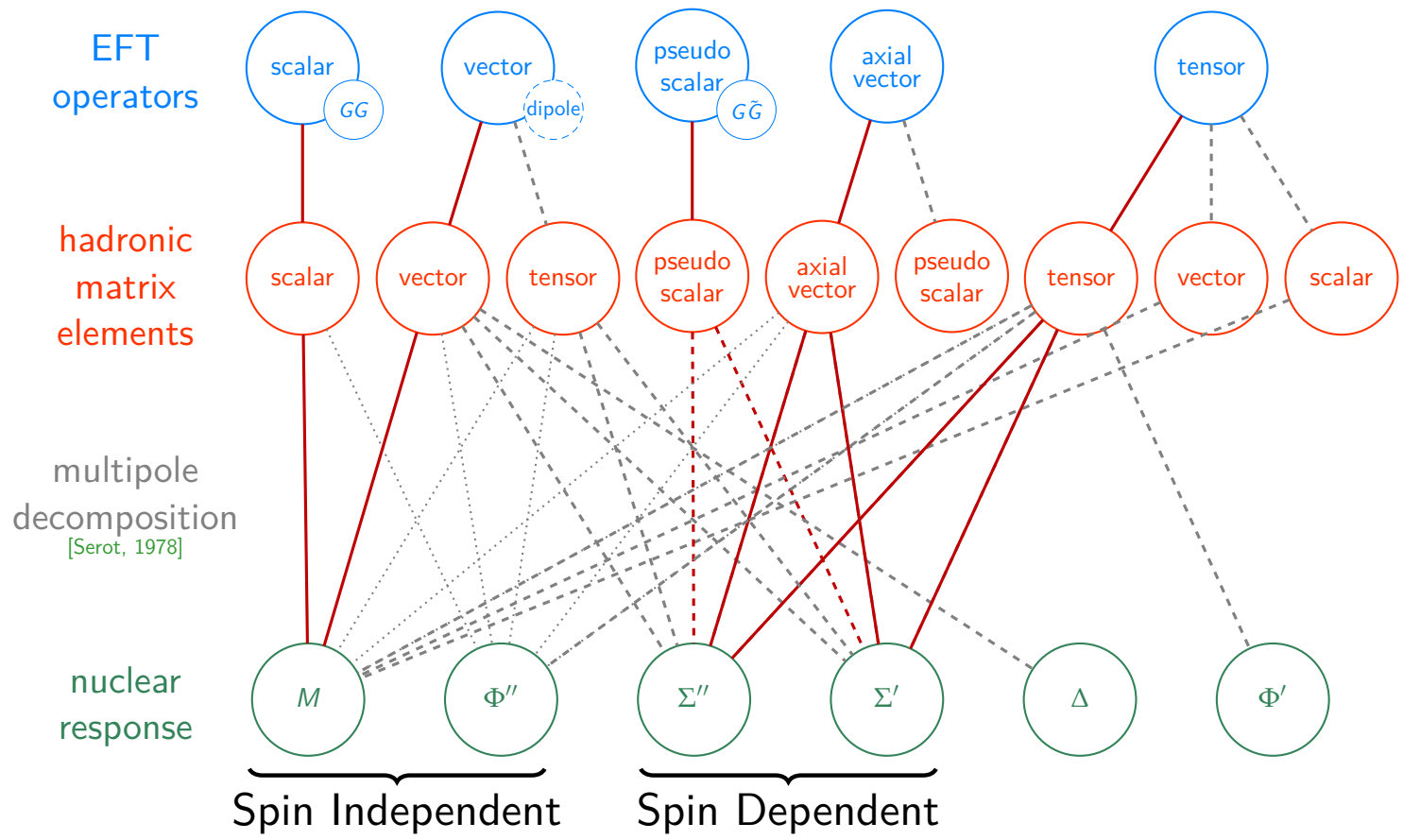
Thank you for your attention!

Thanks to Sacha and Karim for organizing the workshop!
(as well as to Bira and the ECT* staff)

Backup Slides

Backup: Decomposition of the hadronic side





\+ Combine with Coulomb corrections on the leptonic side

- SI: coherently enhanced; $\Gamma_{\mathrm{SI}} \sim \#N^2$; e.g. [Kitano et al., 2002,...]
- SD: not coherently enhanced; only for $J > 0$; e.g. [Davidson et al., 2018,...]