Theory overview of charged Lepton Flavour Violation

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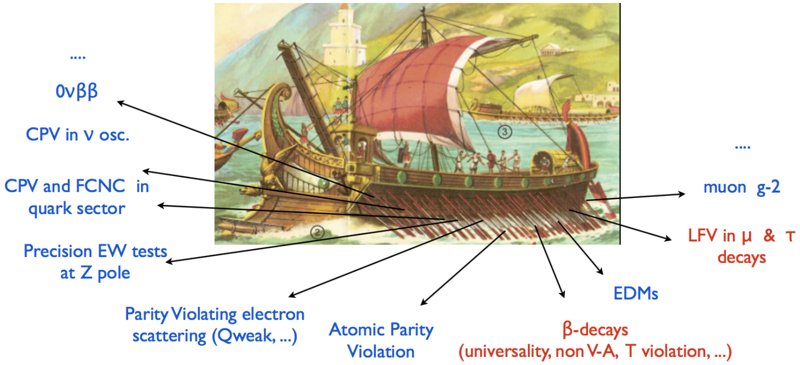
Plan of the talk

1. **Strategies to look for New Physics at low-energy**
2. **Current status of LFV**
3. **EDMs, g-2 and LFV interrelationship**
4. **LFV @ FCC-ee & Muon Collider (MuC)**
5. **Conclusions and future prospects**

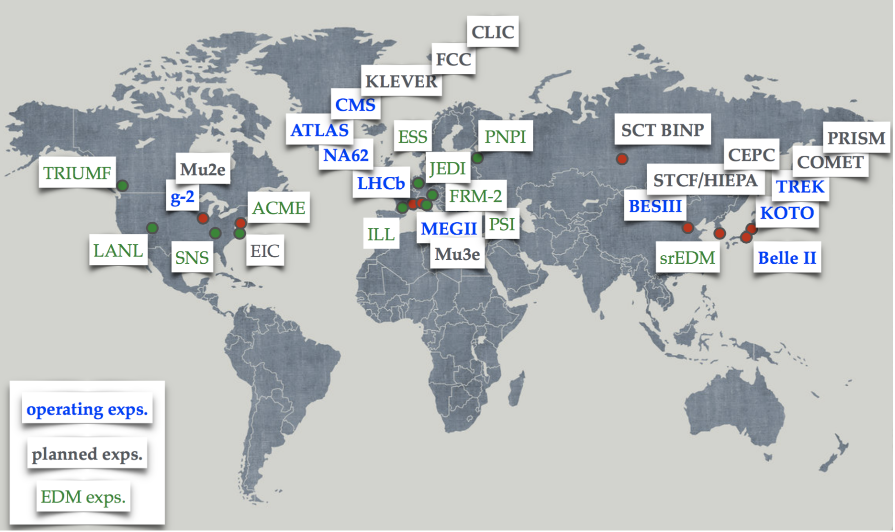
Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
- **Processes predicted with high precision in the SM**



High-intensity frontier: A collective effort to determine the NP dynamics

Experimental status



Experimental bounds

Process	Present	Experiment	Future	Experiment
$\mu \to e\gamma$	1.5×10^{-13}	MEG	$\approx 6 \times 10^{-14}$	MEG II
$\mu \to 3e$	1.0×10^{-12}	SINDRUM	$\approx 10^{-16}$	Mu3e
μ^- Au $\to e^-$ Au	7.0×10^{-13}	SINDRUM II	?	
μ^- Ti $\to e^-$ Ti	4.3×10^{-12}	SINDRUM II	?	
μ^- Al $\to e^-$ Al	–		$\approx 10^{-16}$	COMET, MU2e
$\tau \to e\gamma$	3.3×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to \mu\gamma$	4.4×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to 3e$	2.7×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$\tau \to 3\mu$	2.1×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$d_e(\mathrm{e\ cm})$	1.1×10^{-29}	ACME	$\sim 3 \times 10^{-31}$	ACME III
$d_\mu(\mathrm{e\ cm})$	1.8×10^{-19}	Muon (g-2)	$\sim 10^{-22}$	PSI

Table: Present and future experimental sensitivities for relevant low-energy observables.

- So far, only upper bounds. Still excellent prospects for exp. improvements.
- We can expect a NP signal in all above observables below the current bounds.

Charged LFV in the SM

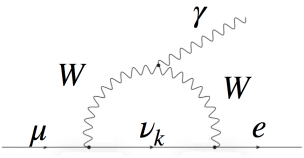
- GIM mechanism very effective in LFV transitions
- amplitude proportional to $A(\mu \to e\gamma) \propto m_\nu^2$

Very small !!!

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U^*_{ek} m^2_{\nu_k}}{M_W^2} \right|^2 .$$

$$\mathrm{BR}(\mu \to e\gamma) = 10^{-55} \div 10^{-54}$$

- similar suppressions for $\mu \to 3e, \tau \to 3\mu, \mu \to e, \ldots$

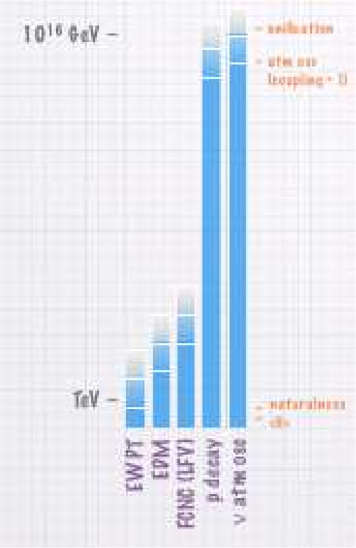


Why flavor violation is visible in neutrino oscillation while it's not in charged LFV?
The uncertainty principle sets the oscillation time for $\mu \to e\gamma$ to be $t \sim h/M_W$!

Message: Any evidence for LFV would be an unambiguous signal of NP!

Why do we need New Physics (NP)?

- **Gravity** $\Longrightarrow$ $\Lambda_{\text{Planck}} \sim 10^{18-19}$ GeV
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}$ GeV
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{NP} \lesssim$ TeV
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}$ GeV
- **Dark Matter (WIMP)** $\Longrightarrow$ $\Lambda_{NP} \lesssim$ TeV
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{NP} \lesssim$ TeV



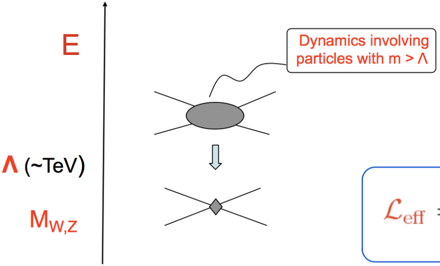
SM = effective theory at the EW scale

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}}\, O_{ij}^{(d)}$$

- $\mathcal{L}^{d=5}_{\rm eff} = \frac{y_\nu^{ij}}{\Lambda_{\rm see-saw}} L_i L_j \phi\phi,$
- $\mathcal{L}^{d=6}_{\rm eff}$ generates FCNC operators

EFT approach to NP

- **Dynamics below the scale Λ [$\sim$ mass of new particles] is described by $L_{\rm eff}$**



$$\mathcal{L}_{SM+\rm Heavy}$$
$$\Downarrow$$
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \sum_{d\geq 5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)}[\phi_{SM}]$$

- $L_{\rm eff}$ is built out of relevant low-energy degrees of freedom (SM fields)
- $L_{\rm eff}$ respects the SM gauge symmetries $G_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
- $L_{\rm eff}$ is organized in inverse powers of Λ (amplitudes suppressed by powers of E/Λ)

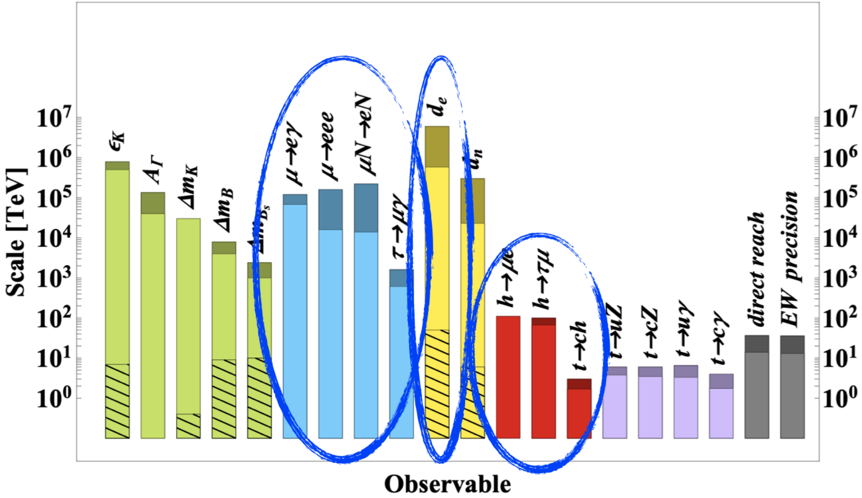
- **Experiments at the precision frontier probe energy scale Λ and symmetries of the new interactions (coeff. & structure of $\hat{O}_n^{(d)}$)**

EFT approach to LFV

4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

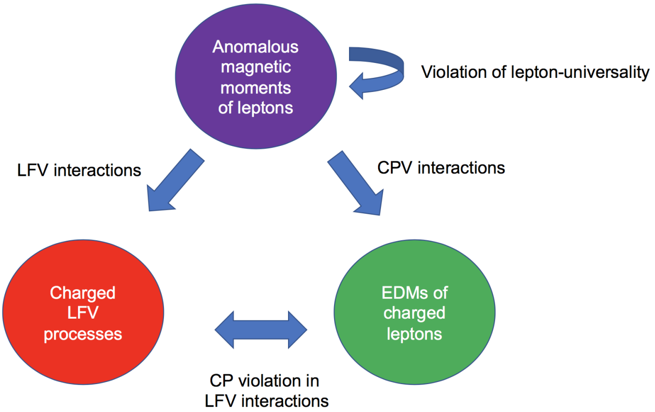
$\mu\to e\gamma$ $\mu\to 3e$ $\mu\to e$

Bounds on the NP scale



[Physics Briefing Book, 1910.11775]

Probing NP in the leptonic sector



New Physics for the muon $g-2$: at which scale?

- **Δa_μ discrepancy at $\sim 4.2\,\sigma$ level:**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.*
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.

*Favoured by the *hierarchy problem* and by a WIMP DM candidate but disfavoured by the LEP and LHC bounds (supersymmetry being the most prominent example).

On leptonic dipoles: $\ell \to \ell'\gamma$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^\star_{\ell\ell'}\ell_R\right)F^{\mu\nu} \qquad \ell,\ell' = e,\mu,\tau\,,$$

  - **Branching ratios of $\ell \to \ell'\gamma$**

$$\frac{\mathrm{BR}(\ell\to\ell'\gamma)}{\mathrm{BR}(\ell\to\ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell})\,.$$

  - **"Naive scaling":** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}\,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j\gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}}\right)^2$$

- **EDMs vs.** $(g-2)_\mu$

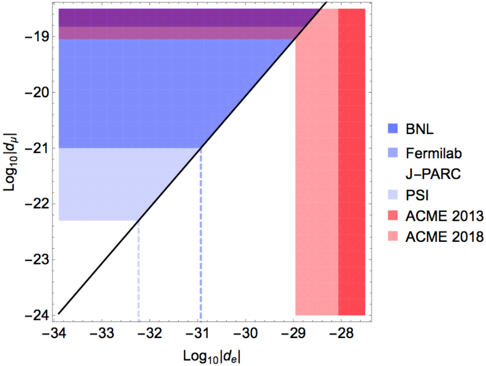
$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-29} \left(\frac{\phi_e^{CPV}}{10^{-5}}\right) e\,\mathrm{cm}\,,$$

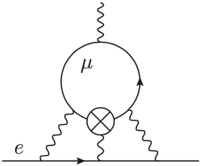
$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 2 \times 10^{-22}\, \phi_\mu^{CPV}\; e\,\mathrm{cm}\,,$$

- **Main messages:**
  - $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\, e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12]

Experimental status of the muon EDM





$$d_\mu \lesssim 10^{-21}\, e\,\mathrm{cm} \left(\frac{d_e}{10^{-31}\, e\,\mathrm{cm}} \right)$$

[Crivellin, Hoferichter & Schmidt-Wellenburg, '18]

$$d_\mu \quad \simeq \quad \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22}\, \phi_\mu^{CPV} \quad e\,\mathrm{cm}\,,$$

[Giudice, PP & Passera, '12]

Correlating LFV signals

- **LFV** operators **@ dim-6**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2_{\textbf{LFV}}}\,\mathcal{O}^{\textbf{dim}-\textbf{6}} + \ldots\,.$$

$$\mathcal{O}^{\rm dim-6} \ni \;\; \bar\mu_R\,\sigma^{\mu\nu}\,H\,e_L\,F_{\mu\nu}\,,\;\; (\bar\mu_L\gamma^\mu e_L)\left(\bar f_L\gamma^\mu f_L\right)\,,\;\; (\bar\mu_R e_L)\left(\bar f_R f_L\right)\,,\; f = e,u,d$$

- $\ell \to \ell'\gamma$ probe ONLY the dipole-operator (at tree level)
- $\ell_i \to \ell_j\bar\ell_k\ell_k$ and $\mu \to e$ in Nuclei probe dipole and 4-fermion operators
- When the dipole-operator is dominant:

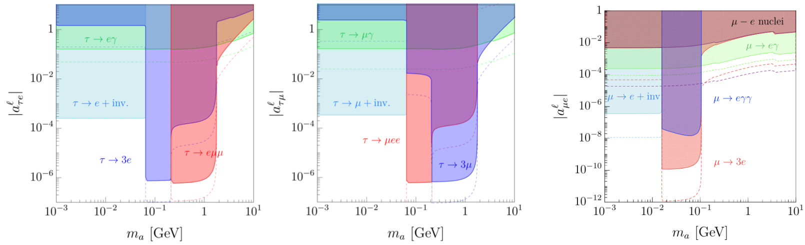
$$\mathrm{BR}(\ell_i \to \ell_j\ell_k\bar\ell_k) \approx \alpha \times \mathrm{BR}(\ell_i \to \ell_j\gamma)$$
$$\mathrm{CR}(\mu \to e \text{ in N}) \approx \alpha \times \mathrm{BR}(\mu \to e\gamma)$$

$$\frac{\mathrm{BR}(\mu \to \mathbf{3e})}{\mathbf{3 \times 10^{-15}}} \approx \frac{\mathrm{BR}(\mu \to \mathbf{e}\gamma)}{\mathbf{5 \times 10^{-13}}} \approx \frac{\mathrm{CR}(\mu \to \mathbf{e} \text{ in N})}{\mathbf{3 \times 10^{-15}}}$$

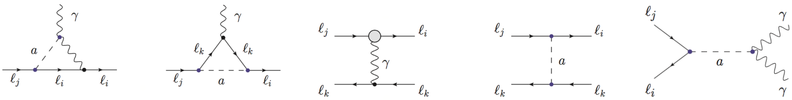
- **Ratios like** $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$ **probe the NP flavor structure**
- **Ratios like** $Br(\mu \to e\gamma)/Br(\mu \to eee)$ **probe the NP operator at work**

LFV from light NP: axionlike particles (ALP)

$$\mathcal{L}_{\rm eff}^{d\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2 a^2}{2} + e^2\, c_{\gamma\gamma}\frac{a}{\Lambda}F_{\mu\nu}\widetilde{F}^{\mu\nu} + g_s^2\, c_{gg}\frac{a}{\Lambda}G_{\mu\nu}\widetilde{G}^{\mu\nu} - \frac{\partial_\mu a}{\Lambda}\sum_{f,i,j}\bar{f}_i\gamma^\mu(v_{ij}^f - a_{ij}^f\gamma_5)f_j$$



[Cornella, P.P. & Sumensari, '19. See also Diego's talk.]



$$\mathcal{B}(\ell_i \to \ell_j\gamma\gamma) \approx \mathcal{B}(\ell_i \to \ell_j a) \times \mathcal{B}(a \to \gamma\gamma) \quad \Rightarrow \quad \text{BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k) \not\approx \alpha \times \text{BR}(\ell_i \to \ell_j\gamma)$$

Message: correlations among LFV signals discriminate heavy vs. light NP!

Low-energy vs high-energy LFV

Purely leptonic contact interations:
- $e^+e^- \to \mu\tau$,
- $\mu^+\mu^- \to \mu e$,
- $\mu^- \to eee$, … .

4-leptons operators

$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$

Dipole operators

Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$

Loop generated dipoles:
- $\mu^- \to e\gamma$,
- $\tau^- \to e\gamma$,
- $\tau^- \to \mu\gamma$.

2-lepton 2-quark operators

$Q_{\ell q}^{(1)}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q_{\ell q}^{(3)}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q_{\ell equ}^{(1)}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q_{\ell equ}^{(3)}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$

Semi-leptonic operators:
- $\mu N \to e N$,
- $pp \to l_i l_k$,
- $B \to \pi e\mu$,
- $B \to K\mu\tau$, … .

Lepton-Higgs operators

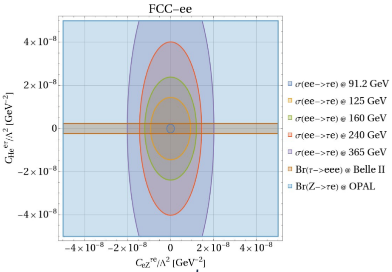
$Q_{\Phi\ell}^{(1)}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q_{\Phi\ell}^{(3)}$	$(\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

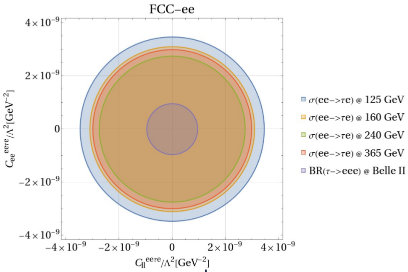
Lepton-Higgs Interactions:
- $h \to \tau\mu$,
- $Z \to \tau\mu$,
- $e^+e^- \to e\tau$,
- $\tau^- \to \mu ee$.

Energy dependence of LFV signals

$$\Gamma(\tau \to 3e) \sim \frac{m_\tau^5}{\Lambda^4} \qquad \Gamma(Z \to \tau e) \sim \frac{m_Z^5}{\Lambda^4} \qquad \sigma(ee \to \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

- # of Z events @ FCC-ee: $N_Z(e^+e^- \to Z) \sim 10^{11}$
- # of Z events @ MuC: $N_Z(\mu^+\mu^- \to \bar{\nu}\nu Z) \sim 10^{11}\left(\frac{\sqrt{s}}{30\ TeV}\right)^4$





$(\bar{L}_L \sigma^{\mu\nu} e_R)\Phi Z_{\mu\nu}$
vs
$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R \gamma^\mu e_R)$

FCC-ee constraints on 2-dimensional parameter space:

- LFV Z couplings better probed by FCC-ee than BelleII,
- LFV 4f operators better probed by Belle II than FCC-ee.

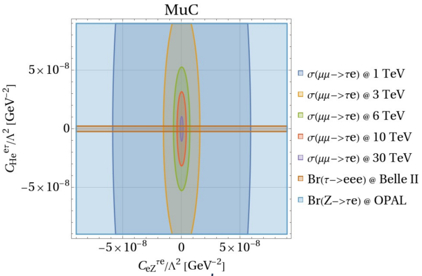
$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L)$
vs
$(\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$

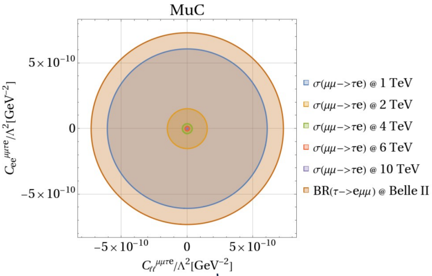
Energy dependence of LFV signals

$$\Gamma(\tau \to 3e) \sim \frac{m_\tau^5}{\Lambda^4} \qquad \Gamma(Z \to \tau e) \sim \frac{m_Z^5}{\Lambda^4} \qquad \sigma(ee \to \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

Message: The high luminosity FCC-ee is competitive with Belle II to probe LFV

[Bartocci, Pagani, PP & Scantamburlo, to appear. See also Calibbi et al., ’21 & Altmannshofer et al. ’23]





$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi Z_{\mu\nu}$
vs
$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$

MuC constraints on 2-dimensional parameter space:

- LFV Z couplings probed by both FCC-ee and BelleII,
- LFV 4f operators better probed by MuC than Belle II.

$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$
vs
$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$

Energy dependence of LFV signals

$$\Gamma(\tau\to 3e)\sim\frac{m_\tau^5}{\Lambda^4}\qquad\Gamma(Z\to\tau e)\sim\frac{m_Z^5}{\Lambda^4}\qquad\sigma(\mu\mu\to\tau e)\sim\frac{E_{CM}^2}{\Lambda^4}$$

Message: Energy helps accuracy!

[Bartocci, Pagani, PP & Scantamburlo, to appear.]

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - We can expect any deviation from the SM expectations below the current bounds.
  - LFV processes and leptonic EDMs do not suffer from theoretical limitations and there are still excellent prospects for experimental improvements.
  - If the muon $g-2$ anomaly will survive, we expect relevant enhancements in leptonic EDMs (especially in the muon EDM) and LFV decays $\mu \to e\gamma,\ \mu \to eee$, ...
  - FCC-ee and a high-energy Muon Collider are complementary with each other and also with Belle II and LHCb to probe LFV.

Message: an exciting program is in progress at the Intensity Frontier which would greatly benefit from the next high-energy frontier program (FCC & MuC)!