Prospects for True Muonium $(\mu^+\mu^-)$ observation at existing beamlines and colliders

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Leptonic -onium states & True Muonium

• "Onium" $(f\bar{f})$ purely leptonic states: positronium (e^+e^-) , true muonium $(\mu^+\mu^-)$, true tauonium $(\tau^+\tau^-)$

 $\rightarrow\,$ Positronium is extensively studied - also produced every time a e^+ source (Na-22) is used

- True muonium (TM) and true tauonium never observed
 - \rightarrow Same properties of dark photons (spin 1) and ALPs (spin 0)
- Possible to search for new physics and, in parallel, discover TM, "cross-motivating" both studies
- "Dream" experiment: Precision TM spectroscopy (as for muonium) → access to vacuum polarization and new-physics, à la g-2 [1]

Production methods: classification

- Tried to classify proposed production methods in terms of feasibility: inspirehep.net/dimuonium|true muonium
- Employs an existing beamline or collider?
 - $\rightarrow\,$ Yes, with relatively large significance (~ 1 paper)

 $\circ~\eta \rightarrow T M \gamma$ at LHCb with Run3 data [..]

- \rightarrow Yes, but with small significance (~ 12 papers)
- \rightarrow No, needs new facilities (\sim 8 papers)
- Tried to bridge the gap in the green section by proposing two new methods
 - \rightarrow Resonant $e^+e^- \rightarrow TM \rightarrow e^+e^-$ at H4 (+1 paper)
 - $\rightarrow \gamma \gamma$ fusion $e^+e^- \rightarrow TM \rightarrow e^+e^-$ at Belle-II (+1 paper)



Prospects in the near future

- $\eta \rightarrow TM\gamma$ ($BR \sim 5 \times 10^{-10}$) @ LHCb with Run3 data [2]
 - $\rightarrow\,$ Work by Vidal et al. in 2019: Possibility of observation in next years ?

• $e^+e^- \rightarrow TM \rightarrow e^+e^-$ at SPS-H4 (our work): PRD 110, 092015

 \rightarrow Rare events **displaced-vertex** search with ~ 0 bkg: requires **3 months** dedicated **positron** beam time with a 12m long multi-target assembly

 \rightarrow Design of target assembly, detectors, background rejection strategy, complete simulation, after preliminary work by P. Crivelli et al. [3]

 \rightarrow Our target optimization inserted in the NA64 e^+ phase-2 prospects, in Physics Beyond Colliders report for ESPPU [..]

ightarrow In the same report a TM factory at future FCC-ee injectors is proposed

 \circ Large ($10^3 - 10^4$ TM/day) rates thanks to excellent σ_E^{beam} , allowing spectroscopy

• TM at Belle-II with 2020-22 data (our work): see next slides

True Muonium with photon-photon fusion

- Belle-II can produce TM via $\gamma\gamma$ fusion: $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-{\rm TM}$
- para-TM (spin-0) is created: decays to two photons: TM $ightarrow \gamma\gamma$
- Collinear photon scattering \rightarrow leptons at low angles
- Signature: no leptons but two isolated photons with $m_{\gamma\gamma} \sim 0.211~{\rm GeV}$



- In order to apply the collinear photon approximation in the analysis we cut $Q^2_{max} < m^2_{TM}/10$ (not necessary in the data)
- Complementary phase-space with visible leptons also sensitive to TM (see next slides)

Background and generator-level simulation

- Dominant background: light-by-light (LBL) scattering $\gamma\gamma \rightarrow \gamma\gamma$
 - \rightarrow Subdominant: $e^+e^- \rightarrow \gamma\gamma\gamma\gamma$, double radiative Bhabha ($e^+e^- \rightarrow e^+e^-\gamma\gamma$)
- Photon-coupled ALPs and para-TM have same quantum numbers \rightarrow In narrow-width approximation they are equivalent if $\Gamma_{ALP} = \Gamma_{TM}$
- Simulation using SUPERCHIC generator: LbL bkg & ALP signal
- Analytical and SUPERCHIC XS values match at 3%
- + σ_S ~30.6 fb, σ_B ~ 2000 fb in a 50 MeV signal window



Reco-level simulation

- Acceptance, resolution, efficiencies, trigger and isolation cuts included
 - ightarrow Threshold-like shapes due to competing trigger cuts and Q^2 cuts
- With 363 fb⁻¹ (2020-22), mass-cuts significance: $S(\sim 300)/\sqrt{B(\sim 13k)} \sim 2.7\sigma$
- S/B \sim 2% \rightarrow Systematic effects degrade significance
 - ightarrow For both reasons further discrimination required, see next slides



Background suppression

- ExtraTrees classifier (BDT-like but simpler and more randomized) trained on ALP samples with flat mass distribution as **signal** to avoid mass sculpting + half of **bkg** sample
- Large signal/background separation using kinematical features



- Performances tested on original m_{TM} signal sample + other half of bkg sample
- Cut on the classifier score to be optimized (see next slide)

Significance

- Discovery significance $>5\sigma$ achieved for systematics on bkg \leq 3%
- Total background in the signal region to be well under control
- Employed conservative Q^2_{max} cut to ensure collinearity \rightarrow could go better in data if also the remaining phase-space is simulated



Summary and other possibilities

- TM observable with Belle-II 2020-22 data with collinear $\gamma\gamma$ fusion :)
 - \rightarrow arXiv:2501.17753, recently accepted in Physics Review D



- Other channel: visible $e^+e^- + \gamma\gamma$ final states (see doi.org/10.1007/JHEP12(2024)099 for ALPs, lumi-scaled by us)
- Discovery level significance also in this channel \rightarrow possibility to combine both



Atoms & other QED bound states

- Most common bound states: nuclei, atoms & gravitation-bound systems
- Atoms are bound by quantum electrodynamics (QED)
 - \rightarrow precise predictions available
- Many other QED bound states, can be divided in:
 - ightarrow purely leptonic & semi-leptonic: e^+e^- , μ^+e^- , μ^-p , ...
 - \rightarrow purely hadronic: K^-p , π^-p , p^-p^+ , ...
- Purely leptonic states allow high-precision spectroscopy: \rightarrow muonium (μ^+e^-) at PSI





True Muonium levels

- Ortho-TM: Spin 1: $e^+e^- \leftrightarrow TM$
- Para-TM: Spin 0: $\gamma\gamma \leftrightarrow TM$
- Lifetimes scaling as n^3 with the energy level n
- Ion. energy: 1.4 keV

$$\tau(nS_{s=1} \to e^+e^-) = \frac{6\hbar n^3}{\alpha^5 m_\mu c^2} \sim n^3 \times 1.8 \,\mathrm{ps}$$

$$\tau(nS_{s=0} \to \gamma\gamma) = \frac{1}{3}\tau(nS_{s=1} \to e^+e^-) \,.$$



• Lifetimes in ps region, like B/D mesons

Production methods: $e^+e^- \rightarrow TM(X)$

- $e^+e^- \rightarrow TM \rightarrow e^+e^-$ on resonance (displaced vertices search)
 - $\rightarrow\,$ at new dedicated colliders:

 $\rightarrow n = 1$ boosted TM with $\theta_{coll} = 30^{\circ}$ and O(1) GeV beams [4] [5] $\rightarrow n > 1$ TM with 105 MeV e^+/e^- beams (our previous work) [6]

- → SPS-H4 with $\sim 43.7 \ e^+$ beams $\sqrt{s} \approx 2m_\mu$, available at CERN → **Our work**, after preliminary studies by Crivelli et al. [3]
- Out-of-resonance production at existing e^+e^- colliders:

 $\rightarrow e^+e^- \rightarrow TM\gamma$ at $\sqrt{s} = O(1)$ GeV $\rightarrow \sigma \sim O(10^{-1})$ fb [7]

→ Photon-photon fusion at $\sqrt{s} = O(10)$ GeV → $\sigma \sim O(50)$ fb → Belle-II with already collected dataset (our work) [m]

Other production methods

From meson decays:

 $\rightarrow \eta \rightarrow T M \gamma \ (BR \sim 5 \times 10^{-10})$ @ LHCb [8] [9] [10] [11]

 $\rightarrow K_L \rightarrow TM\gamma \ (BR \sim 7 \times 10^{-13})$ @ neutral kaons beamlines [12]

- Other possibilities:
 - → Bremmstrhalung-like and triplet-like processes $eZ \rightarrow eTMZ$ with O(10) GeV beams, $\sigma \sim O(10^{-2})$ fb [13] [14]
 - \rightarrow Photon-photon fusion in relativistic heavy ion collisions [15]
 - $\rightarrow\,$ Interactions of ultra-slow μ^+ and μ^-
 - $\rightarrow \mu^{-}$ beam on $\mu^{+}e^{-}$ / μ^{+} beam on $\mu^{-}p$ [16], maybe at J-PARC (Only method allowing spectroscopy)

True muonium at SPS-H4: Pillars

- $e^+e^- \rightarrow TM \rightarrow e^+e^-$ with 43.6 GeV e^+ beam $\rightarrow \sqrt{s} = \sqrt{2m_eE} = 2m_\mu$
- TM dissociates in matter with huge XS \rightarrow multiple thin lithium targets
- · Displaced vertex search, rejecting Bhabha scattering background
- Our work: arXiv:2409.11342, accepted by Physics Review D



True muonium resonant production: Theory

- Peak cross-section: $\sigma_P = \frac{3}{2}\pi\alpha \cdot \sigma^{rel}_{e^+e^- \to \mu^+\mu^-} = \frac{2\pi^2\alpha^3}{s}$ = 66.6 nb
- Probability to produce the spin 1 n-th state: $n^{-3} \rightarrow p_{1S}$ = 83 %
- Need $\sqrt{s} \in [2m_{\mu}, 2m_{\mu} \delta E]$ where $\delta E = 1.4$ keV (ion. energy)
 - $\rightarrow\,$ XS reduced by integrating the \sqrt{s} distribution in the energy window
 - \rightarrow At H4 energy spread in a $\pm 1.2\%$ window. Including ISR: $\sigma_{eff} = 29$ pb



 \rightarrow Similar to dark photon production

$$\rightarrow \Gamma_{TM}^{1S} = 3.6 \times 10^{-10} \text{ MeV} = \frac{1}{3} \alpha \epsilon^2 m_{TM}$$

$$\rightarrow$$
 Coupling: $\epsilon = 2.6 \times 10^{-5}$

Limits for dark photons in the TM region



• Similar to dark photon production

•
$$\Gamma_{TM}^{1S} = 3.6 \times 10^{-10} \text{ MeV} =$$

= $\frac{1}{3} \alpha \epsilon^2 m_{TM}$

- Coupling: $\epsilon = 2.6 \times 10^{-5}$
- TM / dark photon differences due to TM behaviour in matter

Dissociation in matter

- Due to nuclei electrical fields TM can be ionized very easily: $TM \rightarrow \mu^+\mu^-$
- Huge dissociation cross-section: $\sigma_D = 13 Z^2 \; {\rm b}$
- Most important limit to TM discovery so far and biggest difference with dark photons
- Low probability to flip spin instead of dissociating: $\sigma_{flip} = O(1) \text{ mb}$



Expected yield on target: theory

- Defining an inverse dissociation length: $\mu_d = \rho N_A / A \cdot 13 Z^2 b$
- Evaluating the expected yield per e^+ on a target with thickness L
- TM at a depth z must survive for a length (L-z):

$$\rightarrow \ \frac{dTM}{de^+ dN_{\text{target}}} = \rho N_A Z / A \sigma_{eff} \int_0^L dz e^{-\mu_d (L-z)} = \frac{\sigma_{eff}}{13Z \,\text{b}} (1 - e^{-\mu_d L})$$

• $L = 2\mu_d^{-1} \rightarrow > 80\%$ of the maximum yield is achieved



Target choice

- Yield proportional to $1/Z \rightarrow \text{Low } Z$ target
- TM yields saturates for high target thickness
 - $\rightarrow~$ Thick targets only increase backgrounds
 - \rightarrow Need very thin target $L \sim 2\mu_d^{-1}$
- A single thin target limits discovery potential
 - \rightarrow Target assembly made by **multiple** 4mm ($\sim 2\mu_d^{-1}$) lithium foils (Z=3)
- · Very reactive with air and moisture
 - \rightarrow Challenging to handle but feasible





Evaluating σ_{eff} : Electron motion

- Electron motion can be important at fixed target e^+e^- [17]
- Full \sqrt{s} formula: $\sqrt{2m_e^2 + 2E_-E_+ 2E_+k_-\cos\theta_-}$
- Polycrystalline material $\rightarrow \cos \theta_{-}$ uniform
- Electron momentum (k) distribution from Compton profile data [18]
- Resulting spread in \sqrt{s} : ~ 200 keV $\rightarrow \sigma_{\sqrt{s}}^{ele}/m_{TM} \sim 10^{-3}$ (very small \checkmark)



Evaluating σ_{eff} : Beam energy spread (BES)

- H4 momentum selection: $\pm 1.2\%$ (uniform spread) [19]
- · BES convoluted with electron motion and fitted in the uniform region
- At m_{TM} the \sqrt{s} distribution is uniform $\rightarrow \delta \sqrt{s}_{eff} = 2.54 \text{ MeV}$
 - $ightarrow \sigma_{eff} \sim \delta E/\delta \sqrt{s}_{eff} \cdot \sigma_P = 1.4 \text{ keV}/2.54 \text{ MeV} \cdot 66.6 \text{ nb} = 36.7 \text{ pb}$
- One effect missing \rightarrow Initial State Radiation \rightarrow Final XS: 29 pb



Evaluating σ_{eff} : Initial State Radiation (ISR)

- Evaluating the combined effect of ISR and BES is not trivial
 - ightarrow A fraction x of initial \sqrt{s} is kept after ISR, but \sqrt{s} fluctuates
- Representing $\sigma_{TM}(\sqrt{s})$ as a rectangle of width δE and height σ_P
- Ingredients: $f_{ISR}(x,\sqrt{s})$ (QED radiator) & $G(\sqrt{s})$ (BES p.d.f.)



Expected yield and target assembly

- Inputs: σ_{eff} = 29 pb, L = 4 mm, $\mu_d = 1.86$ mm, Z = 3
- Result: $\frac{dTM}{de^+ dN_{\rm target}} = \frac{\sigma_{eff}}{13Z\,{\rm b}}(1-e^{-\mu_d L})$ = 6.6×10^{-13}
- Goal: ~ 5 events for $10^{12}e^+$ with \sim 20% eff. \rightarrow 40 targets (=10 cells)
- Displaced vertex search ($\beta\gamma c\tau = 11.3 \text{ cm}$) \rightarrow 20 cm distance between targets to avoid dissociations + 2-layer silicon trackers every 4 targets



- \rightarrow 1 cell = 4 targets + 2 Si trackers
- → #targets per cell limited by multiple scattering (all in vacuum)
- → #cells limited by space before Goliath (checked from a LEMMA TB [20])
- ightarrow Energy loss in the whole target: ~20 MeV ightarrow negligible \sqrt{s} fluctuation

Background: preliminary discussion

- e^+ on target interact e.m. with e^-, p and weakly with e^-, p, n
- e^+e^- Bhabha, e^+p Moller scattering (~ $1/s = m_e/m_p \sim 1/2000$ smaller)
- Weak XS of the same order of $\bar{\nu}N \rightarrow \mu^+X$: 3 fb / E_{e+} [GeV] \rightarrow O(0.1) pb
- Bhabha ($e^+e^- \rightarrow e^+e^-$), except for the displaced vertices and θ_{cm} distribution, shows identical features as $TM \rightarrow e^+e^-$
 - $\rightarrow\,$ Used to estimate signal acceptance and design the detector
 - $\rightarrow\,$ Dominant background, but minor ones are anyway included in MC
- Strategy: identify Bhabha/TM events \rightarrow cut in $\theta_{CM} \rightarrow$ look at vtx z

Angular acceptance





• Maximize $S/\sqrt{B} \rightarrow \bar{\theta} = 53^{\circ}$. Increase very low TM yield $\rightarrow \bar{\theta} = 45^{\circ}$



Detectors

- Goal: 100 % acceptance for $TM \rightarrow e^+e^-$ after θ_{CM} cut
- Employing the Goliath magnet (1.2T) as a key element of the setup
- 1. Gas Cherenchov to reject hadron contamination
- 2. Targets + Si TRK, measuring #part., θ_{lab} , vtx of each cell
- 3. Trackers ($\sigma_x \sim 5$ mm) before/after Goliath for γ -rejection + measuring #part., charge from all cells
- 4. ECAL to reject photons and measuring energies



Silicon detectors requirements

- $4.5 \times 4.5 \text{ cm}^2$ area taking into account beam spot and tracks angle
- Requirements (from MC): $\sigma_{x,y} = 5 \ \mu m$ and 0.3% X_0 per-layer

 Very thin monolithic pixel sensors like ALICE ITS-3

 405 cm² in total → scaling **naively** cost with area → ~ 20 kCHF



Tracking planes

- Goliath magnet: Vertical B=1.2 T over Δz = 2 m, with geometrical apertures Δx ~ 2 m, Δy ~ 1 m and 4.5m total external length [21]
- Beam-momentum particles curve by 16.4 mrad $\rightarrow \Delta x = 7.4$ cm
- Trackers before/after Goliath (σ_x = 5 mm) $\rightarrow > 20 \sigma e^+/e^-$ separation
- Low-p particles instead curve by 0.5 m (fitting magnet aperture \checkmark)
- Tracker areas: $40 \times 40 \text{ cm}^2$ (1st), $55 \times 156 \text{ cm}^2$ (2nd)
- Cost-effective solution: scintillating bars (at least for the 2nd tracker)

Photon rejection & ECAL

- Hard Bremmstrhalung photons could be dangerous $\rightarrow \gamma$ tagging
 - $ightarrow ~\gamma$ conversions in 1st tracker $ightarrow e^+/e^-$ pair vetoed by 2nd tracker
 - $ightarrow \ \gamma$ conversions in 2st tracker ightarrow no track matching
 - $\rightarrow~\gamma$ only in $\mathbf{ECAL} \rightarrow$ no track matching
- · ECAL also needed to constrain total energy to beam momentum
 - \rightarrow ECAL resolution: $\sigma_E/E = 5\%/\sqrt{E[\text{GeV}]} \oplus 10\%/E[\text{GeV}] \oplus 1\%$
- Large ECAL area: $> 55 \times 156$ cm² (at least matching 2nd tracker)
 - Cost-effective solution: 16×6 NA62 LAV-like lead glass blocks [22]



Simulation and trigger

- Geant4 proof-of-concept simulation to evaluate bkg rejection
- 10¹⁴ POT simulated with mono-energetic pencil beam
- Only 1 cell simulated (4 targets + 2 Si detectors), acceptable because:
 - $\rightarrow\,$ Analysis cuts designed to select clean 2-body processes + displaced vtx
 - $ightarrow \, e^+/e^-$ from Bhabha and TM decays only cross a few cells
 - \rightarrow Each cell has a 1% X_0 material budget \rightarrow small effect on signal efficiency
 - $ightarrow\,$ Interactions of bkg e^+/e^- in next cells don't spoil vtx ightarrow small bkg increase
- Virtual detector (VD) downstream to simulate trackers+calorimeter
- Trigger applied during simulation:
 - $\rightarrow = 1$ positive and = 1 negative tracks on VD + any number of neutrals
 - ightarrow Both tracks with 2 < $heta_{lab}$ <20 mrad and 3 < E <42 GeV

Smearing, reconstruction and kinematics selection

- · Energies and silicon detector positions smeared
- θ_{lab} angles reconstructed using Si trackers positions
- Extrapolating z at the target as $\sqrt{x^2+y^2}/\tan(\theta_{lab})$
- Kinematics-based selection applied:
 - ightarrow = 2 tracks in each silicon detector (clean Bhaha/TM event)
 - $\rightarrow |E_+ + E_- E_{beam}| < 2 \ {
 m GeV}$ (no energy loss)
 - ightarrow Combined mass of track pair within 15 MeV of m_{TM} (no energy loss)
 - $\rightarrow ~|p_{+}^{x}-p_{-}^{x}|<8~{\rm MeV}$ & $|p_{+}^{y}-p_{-}^{y}|<8~{\rm MeV}$ (limits mult. scattering)
 - $ightarrow | heta_{cm} \pi/4| < \pi/2$ (Bhabha scattering reduction)
- Vertex z evaluated very simply as $z = (z_+ + z_-)/2$

Efficiency and vertex selection

- Efficiency of reconstruction and kinematics-based selection for $|\theta_{cm} \pi/4| < \pi/2$ Bhabha events : $\epsilon_{reco} = 77.4\%$
- Angular acceptance of TM in the $| heta_{cm} \pi/4| < \pi/2$ region: $\epsilon_{ heta_{cm}} = 61\%$
- Select regions in z with 0 bkg in MC $\rightarrow \frac{\#BKG}{\#e^+} = N_{cells}POT_{MC}^{-1} = 10^{-13}$
- z-selection efficiency: $\epsilon_z = 42.5\% \rightarrow \text{Total eff.:} \ \epsilon_{1S}\epsilon_{reco}\epsilon_{\theta_{cm}}\epsilon_z = 16.2\%$



Target $z \text{ [mm]}$	$z_{min} [mm]$	z_{max} [mm]	Partial ϵ_v
0	70	150	27.3%
200	250	356	39.1%
400	438	571	49.4%
600	631	782	56.0%

- Last peak due to hits in the 1st silicon detector \rightarrow fake vtx

Vertex selection, efficiency and significance

- $10^{14}e^+$ simulated in GEANT4 on 1 cell to estimate bkg rejection
- Quality + Angular cuts to reduce Bhabha $(e^+e^- \rightarrow e^+e^-)$ + other bkgs
- Select regions in z with 0 bkg in MC $\rightarrow \frac{\#BKG}{\#e^+} = N_{\text{cells}}POT_{MC}^{-1} = 10^{-13}$
- Efficiency estimated with Bhahba + exp. integral in z regions: 16.2%



- With 3k spills/day, 3 months run and 10⁷ e⁺/spill:
 - S=12, B=0.3 \rightarrow 8 σ significance
- Good start, clearly if needed more refined simulations should be made

Significance and positron rates

- Including total. eff. $\epsilon = 16.2\% \rightarrow \text{TM}$ yield $\frac{\#TM}{\#e^+} = 4.35 \times 10^{-12}$
- Average SPS spill rate: 3000/day. Rate $#e^+$ /spill to be discussed
 - ightarrow At LEMMA TB $5 imes 10^6 \, e^+$ /spill at 44 GeV without exploiting max. intensity
 - ightarrow NA64 quoted $5-7 imes 10^6$ at 100 GeV, but at 44 GeV we expect higher rates
 - \rightarrow Two scenarios for e^+ /spill: conservative (5 imes 10⁶), optimistic (10⁷)
 - \rightarrow In 3 months run: $\#e^+ = 1.35(2.7) \times 10^{12}$ being conservative (optimistic)
- Signal ev.: 5.8 (11.6), Bkg ev.: 0.13 (0.26), Significance: 5.8 (8.2) σ
- Expected bkg could be overestimated, due to limited MC statistics
- · Good start, clearly if needed more refined simulations should be made

TM from 43.6 GeV positron beam: outlook

At SPS-H4 with current rates:

- Target made by 40 lithium foils (4mm) with Si trk
- 2 scintillating tracker planes + ECAL after target
- Space needed: 12m before Goliath + ~ 2 m after
- Discovery potential in a 3 months run
- · In parallel dark photon searches could be made

In the future:

• Our target optimization inserted in the NA64 e^+ phase-2 prospects

 $\rightarrow\,$ See Physics Beyond Colliders report for ESPPU [arXiv:2505.00947v1]

- In the same report a TM factory at FCC-ee injectors is proposed, exploiting the excellent energy resolution
 - $\rightarrow\,$ Large (10^3-10^4 TM/day) rates, potentially allowing spectroscopy