Probing charged lepton flavor violation and quantum entanglement in muon on-target experiments

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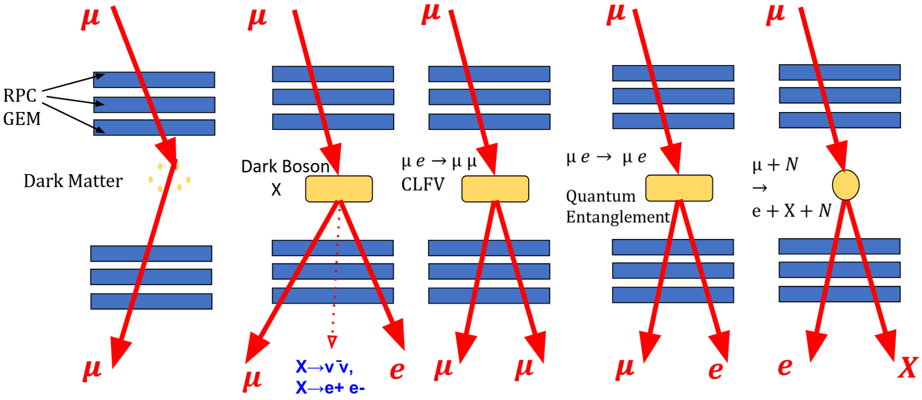
Based primarily on: arXiv:2410.20323, arXiv:2411.12518, arXiv:2502.07597, and arXiv:2503.22956

*https://lyazj.github.io/pkmuon-site/categories/activities/ and [1–5]

Outline

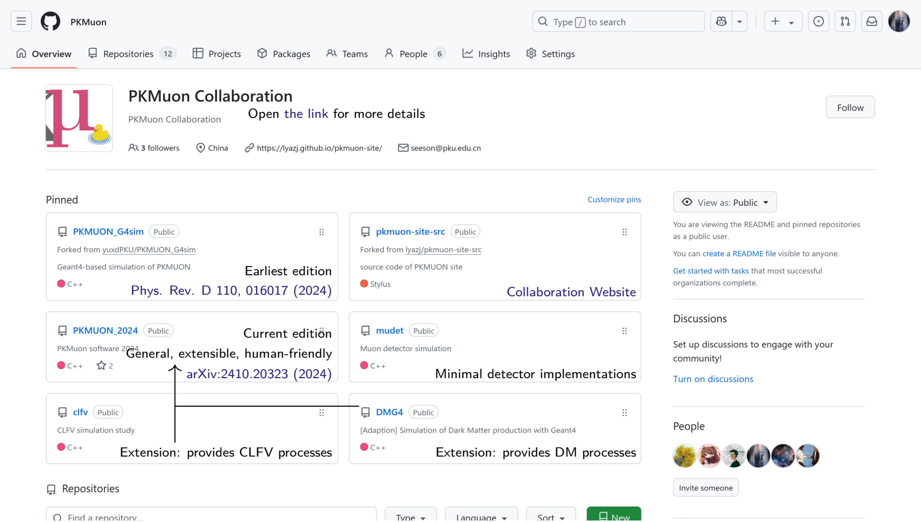
Outline

The PKMuon Experiment



This brief talk focuses on the third and fourth scenarios, while the second scenario will be discussed in detail in Zijian's talk on May 29, 2025.

PKMuon detector simulation framework



Outline

CLFV and recent studies

CLFV (Charged Lepton Flavor Violation):

- Strongly suppressed due to the tiny neutrino masses in the Standard Model (SM): $\mathrm{BR}(\mu \to e\gamma) \sim 10^{-54}$–$10^{-55}$
- Can be significantly enhanced in various BSM models

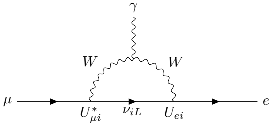


FIG. SM $\mu \to e\gamma$ decay process

Dedicated low-energy muon experiments, such as $\mu \to e\gamma$, set limits on $\lambda_{ee}\lambda_{e\mu}$ and $\lambda_{e\mu}\lambda_{\mu\mu}$ coherently according to Ref. [6] as

$$\Gamma(\mu \to e\gamma) = \frac{\alpha G_F^2 m_\mu^3 M_Z^4 \left(\sin^2\theta_W \left(\sin^2\theta_W - 1/2\right)\right)^2}{4\pi^4 M_{Z'}^4} \left(\lambda_{ee}\lambda_{e\mu}m_e + \lambda_{e\mu}\lambda_{\mu\mu}m_\mu + \lambda_{e\tau}\lambda_{\tau\mu}m_\tau + \ldots\right)^2,$$

The gray part involving τ and BSM contributions is not considered in previous works.

Strong interference and cancellation between the terms shown or omitted are possible, allowing the existence of terms with very large modulus, highlighting the necessity to probe each term individually.

Theoretical model and simulation setup

The $U(1)$ Z' CLFV model:

- Has the same gauge coupling and chiral strength as the SM Z boson except for allowing CLFV quantified by

$$\lambda = \begin{pmatrix} \lambda_{ee} & \lambda_{e\mu} & \lambda_{e\tau} \\ \lambda_{\mu e} & \lambda_{\mu\mu} & \lambda_{\mu\tau} \\ \lambda_{\tau e} & \lambda_{\tau\mu} & \lambda_{\tau\tau} \end{pmatrix},$$

  relative to SM, with the diagonal elements equal to 1 and the off-diagonal expected to be near 0

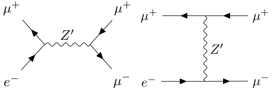


FIG. $U(1)$ Z'-mediated CLFV processes

The PKMuon detectors for $\mu^+ e^- \to \mu^+ \mu^-$ measurements:

- Applies to cosmic-ray and artificial muons
- Cost-effective: no magnetic/calorimeter, giving 2 average momentum directions for each track, without magnitudes
- 3 RPCs per group to suppress muon decay products
- A scintillator PID system downstream RPCs to reject (ionization/bremsstrahlung) electrons (not plotted)
- Generalizable (from DM detection) to other experiments

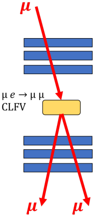
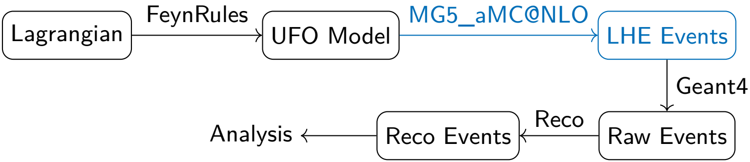


FIG. PKMuon RPC tracker for CLFV measurements

Efficient new physics modeling



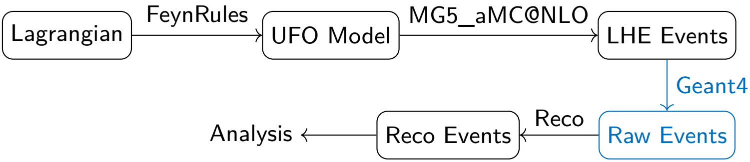
LHE events for arbitrary new physics can be efficiently and conveniently generated through FeynRules and MG5_aMC@NLO

- For various Z' masses and incoming muon energies (incoming muon energies should be varied for muons passing the first 3 RPC layers)
- However, efficient only for *bunch of* (at least $\sim 10^4$) events per mass/energy setup

Our speeding-up solution to avoid event-by-event launch of MG5_aMC@NLO

Pre-acquire the cross section and kinematic distributions for interpolation.

Detector simulation with background



The PKMuon RPC tracking detectors:

- 3 RPCs/group $\times$ 2 groups
- minor/major distance as 200/500 mm
- 2D pixel readout (0.7 mm Exp., 0.5 mm MC)
- trigger as 1 MeV (1/2 mean muon Edep.)

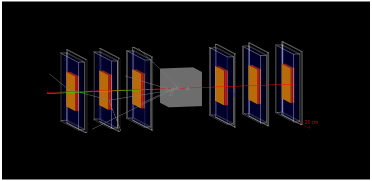
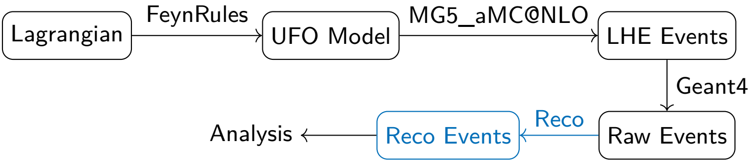


FIG. $\mu^+ e^- \to \mu^+ \mu^-$ event display

Event reconstruction and selection

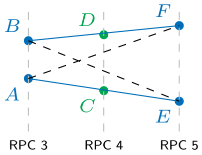


Pre-requirements:

- Comparing none/perfect electron rejection
- #hits on RPCs 0–5: [1, 1, 1, 2, 2, 2]

Reconstruction of 2 outgoing tracks:

- Step 1/3: match hits on RPC 3 and 5
  - expecting 2 tracks diverge from a single point
  - i.e. minimizing the sum of the 2 resulting track lengths
  - e.g. $BF + AE < BE + AF$
- Step 2/3: add hits on RPC 4, minimizing total χ^2 error
- Step 3/3: Recompute (fit) spatial lines BDF and ACE



Results and discussion

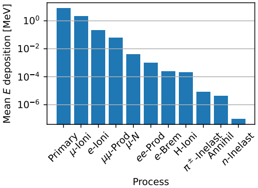
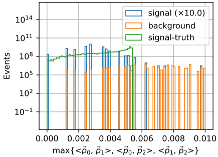
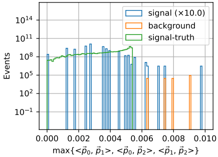


FIG. Typical background components (almost all e!)



(a) 3 cm Al



(b) 3 cm Al, with e-veto

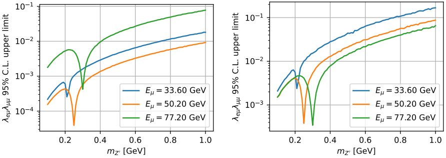
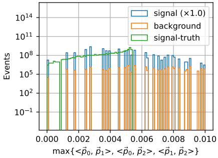
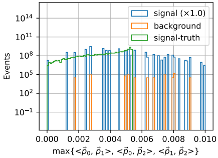


FIG. Prospected 95% ULs for 3 cm Al (L) and 8 cm Pb (R) targets, with e-veto, for a one-year run



(c) 8 cm Pb



(d) 8 cm Pb, with e-veto

FIG. Event distribution before final selection

Background events are vanishing, raising requirements on a higher statistics in the future. Finer selection can also be enabled in a higher statistics.

Outline

The current absence of free-traveling leptons in QE measurements

- The ATLAS and CMS Collaborations recently observed quantum entanglement involving **top quarks** at a center-of-mass energy of 13 TeV, marking the highest energy measurements of quantum entanglement to date [7–10]
- Most studies on charged lepton QE have concentrated on the decaying **tau** leptons [11–17], while less attention has been given to electrons and muons
- Solid-state quantum computation was established in 2005 with **electron pairs confined in semiconductor quantum dots** [18]: entangled states were prepared, coherently manipulated, and measured
- **No similar experiment has been done with free-traveling electrons** as measuring the spin of a single traveling electron poses a significant challenge due to interference from its orbital motion [19]

Our proposal

Conduct a **first** measurement of the polarization correlation between charged lepton beams through joint measurements of their individual polarization-sensitive scatterings off two separate targets.

Theory: Concurrence, CHSH inequality, and the kinematic approach

- Entanglement can be quantified by *concurrence* [20–22], defined as

$$\mathcal{C}(\rho_f) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \in [0, 1] \quad (1)$$

  for a two-qubit system, where λ_i ($\lambda_i \geq \lambda_j,\ \forall i < j$) are the square roots of the eigenvalues of the matrix $\rho_f(\sigma_2 \otimes \sigma_2)\rho_f^*(\sigma_2 \otimes \sigma_2)$. If $\mathcal{C} > 0$, the two-qubit system is entangled.

- The *CHSH inequality*, $I_2 \leq 2$ [23], is the Bell inequality for a two-qubit system. The optimal (maximal) I_2 [24] evaluates to

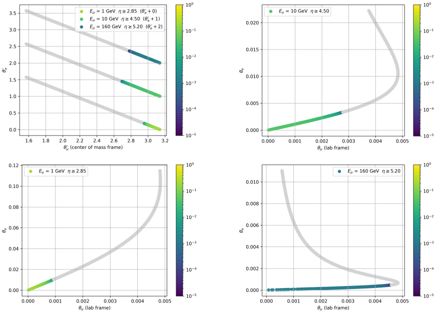
$$I_2 = 2\sqrt{\lambda_1 + \lambda_2}, \quad (2)$$

  where λ_1 and λ_2 are the two largest eigenvalues of the matrix $C^{\mathrm{T}}C$, and C is the correlation matrix calculated by $C_{ij} = \mathrm{Tr}\left(\rho_f\left(\sigma_i \otimes \sigma_j\right)\right)$. $I_2 = 2\sqrt{2}$ is the upper limit of the quantum mechanics.

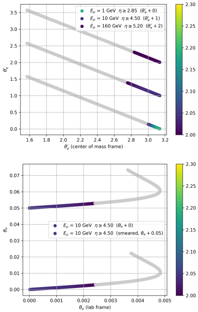
- In addition to the *decay approach* used for decaying particles, the *kinematic approach* [25, 26] can reconstruct quantum states from production kinematics, applicable to stable particles produced in simple QED scatterings.

Electron-muon entanglement sources via muon on-target experiments

Concurrence $\mathcal{C}(\rho_f)$:



Optimal I_2:

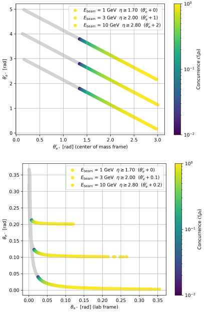


- Simulated by MG5_aMC@NLO 3.5.5 [27] in tree-level QED with non-zero lepton masses
- The light gray regions depict $\mathcal{C}(\rho_f) = 0$ or $I_2 \leq 2$
- Assuming a **1-day** run with a **10 GeV** muon beam of flux $\mathbf{10^5}$**/s** on $10\ \mathrm{cm}$ Al targets, the expected number of events with $\mathcal{C}(\rho_f) > 0$ is $\mathbf{2.6 \times 10^4}$

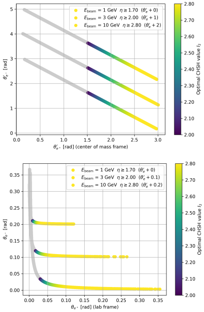
E_{beam}/GeV	E_{COM}/GeV	$\mathcal{C}(\rho_f)_{\max}$	$\theta_{\mu,\max}$/mrad	$\theta_{e,\max}$/mrad	$E_{\mu,\min}$/GeV	$E_{e,\min}$/GeV	σ_{E}/μb	$\sigma_{\mathrm{E},\,\theta \geq 0.5\,\mathrm{mrad}}$/μb
1	0.111	**0.22**	0.9	10.2	0.92	0.08	0.56	0.56
10	0.146	**0.044**	**2.8**	**3.3**	**5.2**	**4.5**	**0.39**	**0.39**
160	0.418	0.0014	4.6	0.5	10	145	0.027	0.022

Electron-positron entanglement sources via positron on-target experiments

Concurrence $\mathcal{C}(\rho_f)$:



Optimal I_2:



- The angular ranges exhibiting $\mathcal{C}(\rho_f) > 0$ in the center-of-mass frame are significantly broader
- The theoretical upper limits for both $\mathcal{C}(\rho_f)$ and I_2 in quantum mechanics are nearly reached as θ'_{e^+} approaches 3
- Assuming a **1 GeV** positron beam with a flux of $\mathbf{10^{12}}$**/s** directed at a 10 cm thick Al target, the expected entangled event rate is $\mathbf{1.9 \times 10^9/s}$
- A golden region for measurements:
  - $E_{\text{beam}} = \mathbf{1\ GeV}$, $0.05\ \text{rad} \leq \theta_{e^+} \leq 0.1\ \text{rad}$
  - 23.4% of all events with $\mathcal{C}(\rho_f) > 0$
  - $E \geq \mathbf{0.094\ GeV}$, $\theta \geq \mathbf{0.0103\ rad}$
  - $\mathcal{C}(\rho_f)$ reaching up to $\mathbf{0.953}$ and I_2 up to $\mathbf{2.8281}$

E_{beam}/GeV	E_{COM}/GeV	$\mathcal{C}^{\max}(\rho_f)$	$I_2^{\max}$	$E_{e^+}^{\min}$/GeV	$E_{e^-}^{\min}$/GeV	$\theta_{e^+}^{\min}$/rad	$\theta_{e^-}^{\min}$/rad	σ_{E}/μb
1	0.032	0.9996	2.8281	0.008	0.389	0.0255	0.0028	243.6
3	0.055	0.9997	2.8282	0.023	1.166	0.0147	0.0016	82.1
10	0.101	0.9997	2.8282	0.074	3.890	0.0081	0.0009	26.5

A first electron-positron beam correlation measurement proposal

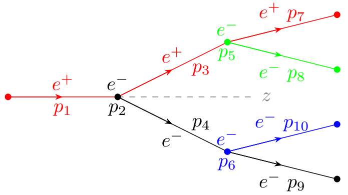


FIG. Proposed cascade experiment for measuring polarization correlations of the primary products

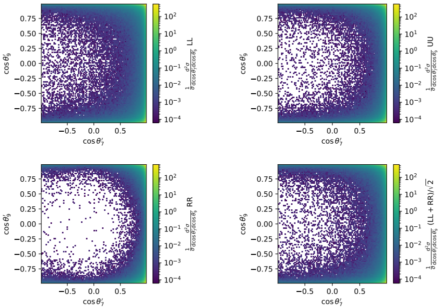


FIG. Joint angular distribution densities of the two secondary scattering processes

Simulation setup:

- $0.05\ \mathrm{rad} \leq \theta_3 \leq 0.1\ \mathrm{rad}$ in a 1 GeV positron on-target experiment
- The spins of target electrons 5 and 6 are aligned with the beam direction
- Consider the main component of the primary state, $(LL + RR)/\sqrt{2}$

Assuming the two secondary targets are $10\ \mathrm{cm}$ thick iron, the event rate in $\cos\theta_7' \leq 0.5 \wedge -0.75 \leq \theta_9' \leq 0.75$ is $\mathbf{1.4 \times 10^2}$**/s** for the state $(LL + RR)/\sqrt{2}$.

Future prospects: Scattering-based simplified state tomography

Take $0.05\ \mathrm{rad} \leq \theta_3 \leq 0.1\ \mathrm{rad}$ in a 1 GeV positron on-target experiment as an example:

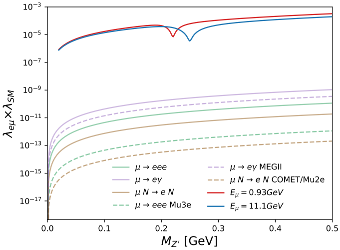
- The state of the primary products is approximately 1% $(RL + LR)/\sqrt{2}$, 1% $(RL - LR)/\sqrt{2}$, 7% $(RR - LL)/\sqrt{2}$, and 90% $(RR + LL)/\sqrt{2}$ in the lab frame
- The optimized ratio of the yields of $(LL + RR)/\sqrt{2}$ to UU is $1.29 \pm 0.03(\mathrm{MC})$, corresponding to 4.4×10^3 post-optimization efficient signal event counts and an expected signal yield over a **27-second** run; the result for $(LR + RL)/\sqrt{2}$ is $0.78 \pm 0.02(\mathrm{MC})$ in comparison
- Other uncertainties, such as those from process modeling and background suppression, may dominate the real experimental analysis
- For the 20% polarized targets, the ratios are 1.010 ± 0.009 and 0.986 ± 0.009 generated from 25 times the number of Monte Carlo events, corresponding to 2.5×10^4 efficient event counts accumulated in **680 seconds**
- The high event rate can help mitigate the decline in resolving power associated with low target polarization purities in real-world applications
- **A simplified state tomography can be performed assuming prior knowledge from the primary scattering**

Outline

Summary

PKMuon CLFV experiment:

- A novel and generalizable efficient event generation algorithm
- $\mu^+e^- \to \mu^+\mu^-$ simulation shows the expected 95% CL UL on $\lambda_{e\mu}\lambda_{\mu\mu} \sim 10^{-5}$ with Z' mass ~ 0.25 GeV for a one-year run
- Not competitive compared to $\lambda_{e\mu}\lambda_{\mu\mu} \leq 10^{-11}$ assuming $\lambda_{ee}\lambda_{e\mu} \sim 0$ and Z' mass ~ 0.25 GeV deducted from arXiv:2504.15711
- But exclusively sensitive to $\lambda_{e\mu}\lambda_{\mu\mu}$



PKMuon quantum entanglement experiment:

- GeV-scale muon and positron on-target experiments as **controllable entangled lepton pair sources**
- Quantum entanglement and the CHSH inequality violation are present
- A **first measurement of the correlation between entangled free-traveling lepton pairs** is proposed to verify the entanglement

Thanks for your attention!

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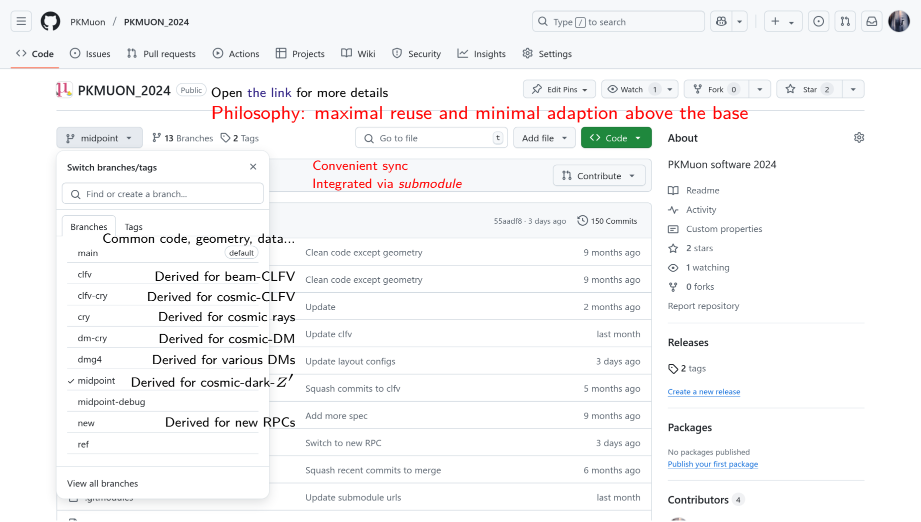
PKMuon detector simulation framework

Open the link for more details

Philosophy: maximal reuse and minimal adaption above the base

Convenient sync
Integrated via *submodule*

Common code, geometry, data...



clfv	Derived for beam-CLFV
clfv-cry	Derived for cosmic-CLFV
cry	Derived for cosmic rays
dm-cry	Derived for cosmic-DM
dmg4	Derived for various DMs
midpoint	Derived for cosmic-dark-Z'
new	Derived for new RPCs

PKMuon detector simulation framework

Files

main

Go to file

- DM_step1
- DM_step2
- analysis
- config
  - layout.yaml
  - layout_al.yaml
  - layout_pb.yaml
  - material_schema.yaml
  - rpc.yaml
  - rpc_material.yaml
  - rpc_readout.yaml
  - volume_schema.yaml
- include
- spec
- src
  - ActionInitialization.cc
  - DetectorConstruction.cc
  - EventAction.cc
  - GeometryConfig.cc
  - GpsPrimaryGeneratorAction.cc

PKMUON_2024 / config / rpc.yaml

Open the link for more details

Top

Code Blame 166 lines (143 loc) · 2.81 KB

Raw

```yaml
    - rpc_glass
  material: rpc_gas

rpc_electrode_pair:
  solid: bottom_up
  components:
    - rpc_electrode
    - rpc_electric
    - rpc_electrode
  material: rpc_gas

rpc_electrode_pair_0:
  solid: rotation
  components: [rpc_electrode_pair]

rpc_electrode_pair_1:
  solid: rotation
  components: [rpc_electrode_pair, [x, 180 deg]]

rpc_mainbody:
  solid: bottom_up
  components:
    - rpc_x_readout_board  # external
    - rpc_insulating_film
    - rpc_electrode_pair_0
    - rpc_insulating_film
    - rpc_gas
    - rpc_t_readout_board  # external
    - rpc_gas
    - rpc_insulating_film
    - rpc_electrode_pair_1
    - rpc_insulating_film
    - rpc_y_readout_board  # external
  material: rpc_gas
```

GEANT := GEometry ANd Tracking

Abstraction: Multiple geometrical operations supported

Modulization: Accepts external definitions (elsewhere)

Decorrelated from the code; human-friendly; error-repelling

Efficient new physics modeling

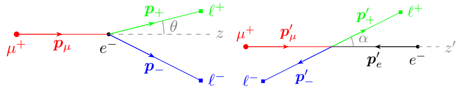


FIG. Lab and COM frame event display

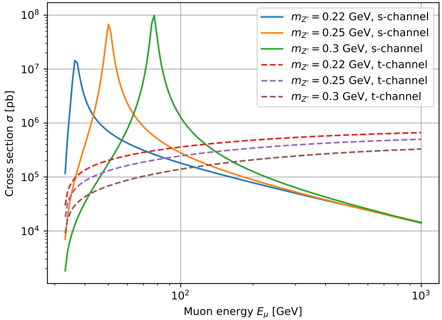
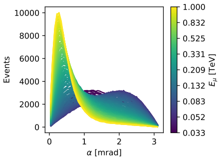
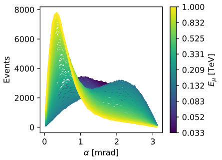


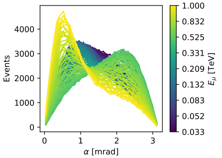
FIG. s- and t-channel cross sections



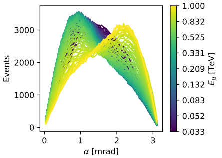
(e) $m_{Z'} = 0.22$ GeV



(f) $m_{Z'} = 0.30$ GeV



(g) $m_{Z'} = 0.50$ GeV



(h) $m_{Z'} = 0.80$ GeV

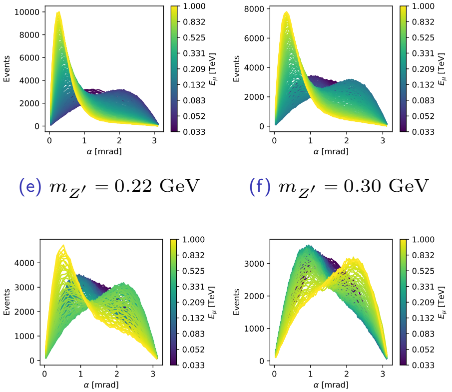
FIG. $\mu^+ e^- \to \mu^+ \mu^-$ event distributions in the COM frame with varying incident E_μ.

Efficient new physics modeling

Algorithm: Efficient $\mu^+e^- \to \ell^+\ell^-$ event generation

function $\textsc{GenerateMuELL}(E_\mu, \vec{p}_\mu, m_\ell)$
- $E_{\mu1}, E_{\mu2}, \sigma_1, \sigma_2, H_1, H_2 \leftarrow \textsc{AdjacentGridPoints}(E_\mu)$;
- **if** E_μ is out of the grid range **then**
  - **return** no $\mu^+e^- \to \ell^+\ell^-$ happens;
- **end if**
- $w_1 \leftarrow \frac{E_\mu - E_{\mu2}}{E_{\mu1} - E_{\mu2}}$, $w_2 \leftarrow \frac{E_{\mu1} - E_\mu}{E_{\mu1} - E_{\mu2}}$;
- $\sigma \leftarrow w_1\sigma_1 + w_2\sigma_2$; // cross section
- **if** $\textsc{Random}(0,1) < w_1$ **then**
  - $\alpha \leftarrow H_1.\textsc{Sample}()$; // polar angle
- **else**
  - $\alpha \leftarrow H_2.\textsc{Sample}()$;
- **end if**
- $\phi \leftarrow \textsc{Random}(0, 2\pi)$; // azimuthal angle
- $E', p', \gamma, \beta \leftarrow \textsc{Kinematics}(E_\mu, m_\ell)$;
- $p_{x+} \leftarrow p' \sin\alpha\cos\phi$, $p_{y+} \leftarrow p' \sin\alpha \sin\phi$;
- $p_{z+} \leftarrow \gamma\,(p' \cos\alpha + \beta E'/2)$;
- $\vec{p}_+ \leftarrow \textsc{ThreeVector}(p_{x+}, p_{y+}, p_{z+})$;
- $\vec{p}_+ \leftarrow \vec{p}_+.\textsc{RotateZAxisTo}(\hat{p}_\mu)$;
- $\vec{p}_- \leftarrow \vec{p}_\mu - \vec{p}_+$;
- **return** $\sigma, \vec{p}_+, \vec{p}_-$;

end function



(e) $m_{Z'} = 0.22$ GeV (f) $m_{Z'} = 0.30$ GeV

(g) $m_{Z'} = 0.50$ GeV (h) $m_{Z'} = 0.80$ GeV

FIG. $\mu^+e^- \to \mu^+\mu^-$ event distributions in the COM frame with varying incident E_μ.

Detector simulation with background

$$\begin{tikzcd}
\boxed{\text{Lagrangian}} \arrow[r, "\text{FeynRules}"] & \boxed{\text{UFO Model}} \arrow[r, "\text{MG5_aMC@NLO}"] & \boxed{\text{LHE Events}} \arrow[d, "\text{Geant4}"] \\
\text{Analysis} & \boxed{\text{Reco Events}} \arrow[l] & \boxed{\text{Raw Events}} \arrow[l, "\text{Reco}"']
\end{tikzcd}$$

Background-included signal detection simulated by Geant4 11.2.2:

- $\mu^+e^- \to \ell^+\ell^-$: subclasses `G4VDiscreteProcess`
- Simulated upon physics list `FTFP_BERT`
- Signal (or background) definition: whether (or not) CLFV happens
- Signal rate is scaled to 10^{-3} to 10^{-2} to balance precision and efficiency
- Allows a step length up to 10^{-3} times MFP assuring granularity

Detector simulation with background

The PKMuon RPC tracking detectors:

- 3 RPCs/group $\times$ 2 groups
- minor/major distance as 200/500 mm
- 2D pixel readout (0.7 mm Exp., 0.5 mm MC)
- trigger as 1 MeV (1/2 mean muon Edep.)

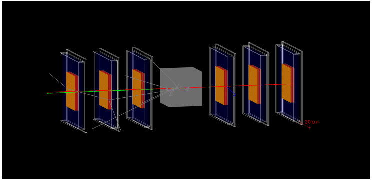


FIG. $\mu^+ e^- \rightarrow \mu^+ \mu^-$ event display

CLFV *can* happen outside the target, but those events are unintended for measurements and will be filtered out by event selection.

The parameters here and the target selection are to be optimized.

Since the measurement is insensitive to the focusing of the incoming muon beam, the beam is considered to be monochromatic, impinging perpendicularly to the detector and distributed uniformly over the area of the RPC module.

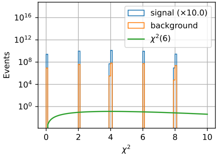
Event reconstruction and selection

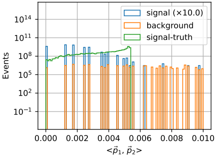
Event distribution:

- key signal feature: 3 highly collimated tracks (in: 0; out: 1, 2)
- distributed discretely due to limited detector granularity
- signal and background remain largely separated

Event selection:

- $\chi^2 \leq 6$
- $\max_{i \neq j} \langle \vec{p}_i, \vec{p}_j \rangle \leq 0.003$





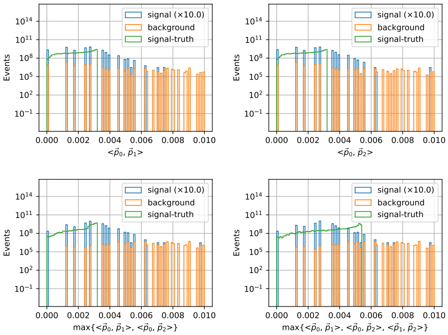


FIG. Event distributions for $E_\mu = 50.2$ GeV, $m_{Z'} = 0.25$ GeV, and $\lambda_{e\mu}$ scaled to 10. The target is a 30 mm thick aluminum block. The yields are normalized to 3×10^{13} muons on target, corresponding to a one-year run.

Quantum entanglement measurements — history and today

- As reviewed by C. N. Yang [28], the first experiment on quantum entanglement is the Wu-Shaknov Experiment published in 1950 [29] in which the angular correlation of two Compton-scattered **photons** arising from e^+e^- annihilation are measured
- The violation of Bell inequality was demonstrated in 1970s using entangled **photons** [30–32], confirming the non-locality of our universe
- Alain Aspect, John Clauser and Anton Zeilinger won the Nobel Prize in Physics in 2022 for demonstrating the potential to investigate and control particles (**photons**) that are in entangled states [33]

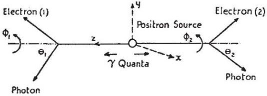


FIG. Angular correlation effects [34] demonstrated by the Wu-Shaknov Experiment

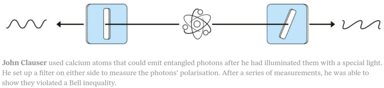


FIG. Clauser's photon entanglement experiment [33]

PKMuon quantum entanglement experiment prospects

- GeV-scale muon and positron on-target experiments are examined as **controllable entangled lepton pair sources** through the kinematic approach
- Quantum entanglement and the CHSH inequality violation are present in the primary scattering products
- A **first measurement of the correlation between entangled free-traveling lepton pairs** is proposed to verify the entanglement
- The electron-positron beam polarization correlation measurement can be conducted with **a high event rate at many domestic positron beam facilities**

Process	Incident flux	Primary event rate	Secondary coincidence rate
$\mu^- e^- \to \mu^- e^-$	$10^5/\mathrm{s}$	$2.6 \times 10^4/\mathrm{d}$	(not estimated)
$e^+ e^- \to e^+ e^-$	$10^5/\mathrm{s}$	$1.9 \times 10^2/\mathrm{s}$	$4.4 \times 10^2/\mathrm{y}$
$e^+ e^- \to e^+ e^-$	$10^{12}/\mathrm{s}^*$	$\mathbf{1.9 \times 10^9/s}$	$\mathbf{1.4 \times 10^2/s}$

*Possibly from the beam dump of the STCF.

Relevant activities

- 20'+10' oral report in *Quantum Observables for Collider Physics 2025*
- 20' oral report in *Workshop on Quantum Entanglement at the Energy Frontier*
- 8' brief oral report in *Muon4Future 2025* (today)