Computational modelling of the semi-classical quantum vacuum in 3D

Z. Zhang[1,*], R. Aboushelbaya[1], I. Ouatu[1], E. Denis[1], A. James[1], R. J. L. Timmis[1], M. W. von der Leyen[1], R. Torres[2], T. Grismayer[2], L. O. Silva[2], P.A. Norreys[1,3]

[1] University of Oxford, Clarendon Laboratory, UK,

[2] GoLP/Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, Portugal,

[3] John Adams Institute for Accelerator Science, University of Oxford, UK,

[*] zixin.zhang@physics.ox.ac.uk

Abstract

The commissioning of multi-Petawatt laser systems is gathering pace around the world, promising unparalleled access to ultra-high electromagnetic fields for probing the quantum vacuum. Here, we present the first real-time three-dimensional simulation results of two quantum vacuum effects using a semi-classical numerical solver for the Heisenberg-Euler Lagrangian. The simulation model is benchmarked against vacuum birefringence analytical results using counter-propagating probe and pump pulses. Simulations of both plane-wave and Gaussian pulses show results consistent with theoretical predictions. The solver is then applied to four-wave mixing using three Gaussian pulses with real-time information on the evolution of the harmonics produced. The time-resolved feature of the solver provides quantitative explanations for the astigmatism in the output pulse in terms of the asymmetry in the interaction region, and produces precise estimates of the interaction time and size. Results of the output electric field, power and photon number are compared with the plane-wave model and benchmarked against previous numerical results.

Solver algorithm

Weak-field QED in vacuum can be approximated by the HE Lagrangian:

$$\mathcal{L} = \frac{1}{8\pi}\left(E^2 - B^2\right) + \frac{\xi}{8\pi}\left[\left(E^2 - B^2\right)^2 + 7\left(\mathbf{E}\cdot\mathbf{B}\right)^2\right]. \tag{1}$$

From which a set of modified Maxwell's equations can be derived, containing cubic non-linearities. The solver is based on a modified Yee scheme, with a predictor-corrector approach where the classical Maxwell's equations are solved as the zeroth order solution to the fields. The solver is implemented into OSIRIS, a widely used and massively parallel PIC code framework.

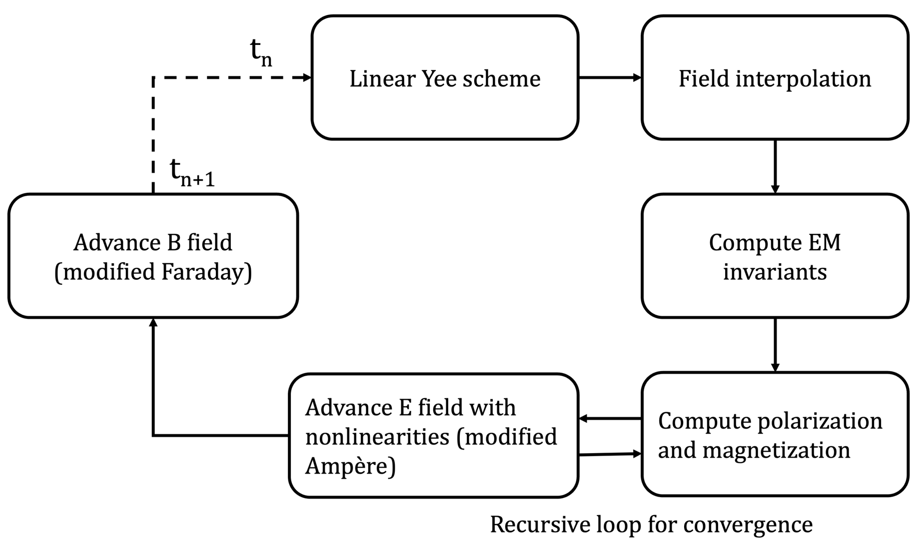
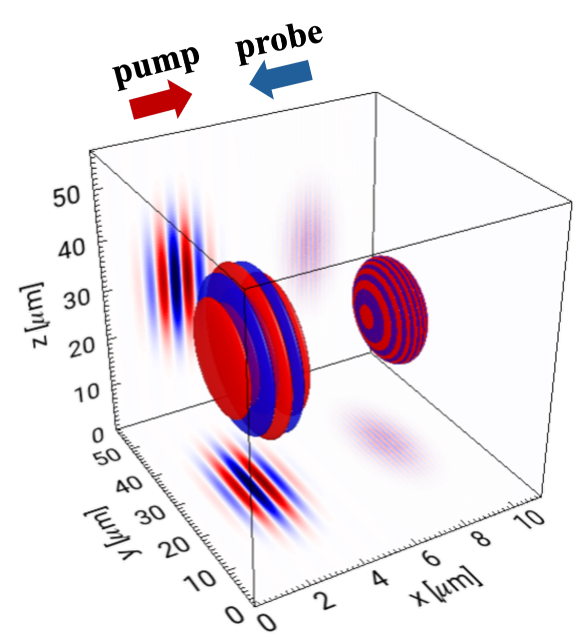


Figure 1. Iterative loop in the modified Yee scheme.

Vacuum birefringence

Figure 2. A schematic view of vacuum birefringence, using a counter-propagating probe and pump setup in 3D



The solver was benchmarked against analytical results of vacuum birefringence. A linearly polarised probe pulse will develop an ellipticity after its interaction with a strong pump pulse. The ellipticity is expected to mirror the Gaussian profile of the pump pulse across the $\hat{\mathbf{y}}$-$\hat{\mathbf{z}}$ plane, as shown in Fig. 3. Comparison with theory is shown in slices along the central $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ axes. The deviation from theory at the peak of the Gaussian is 2.1%, and the slices show good alignment with theory.

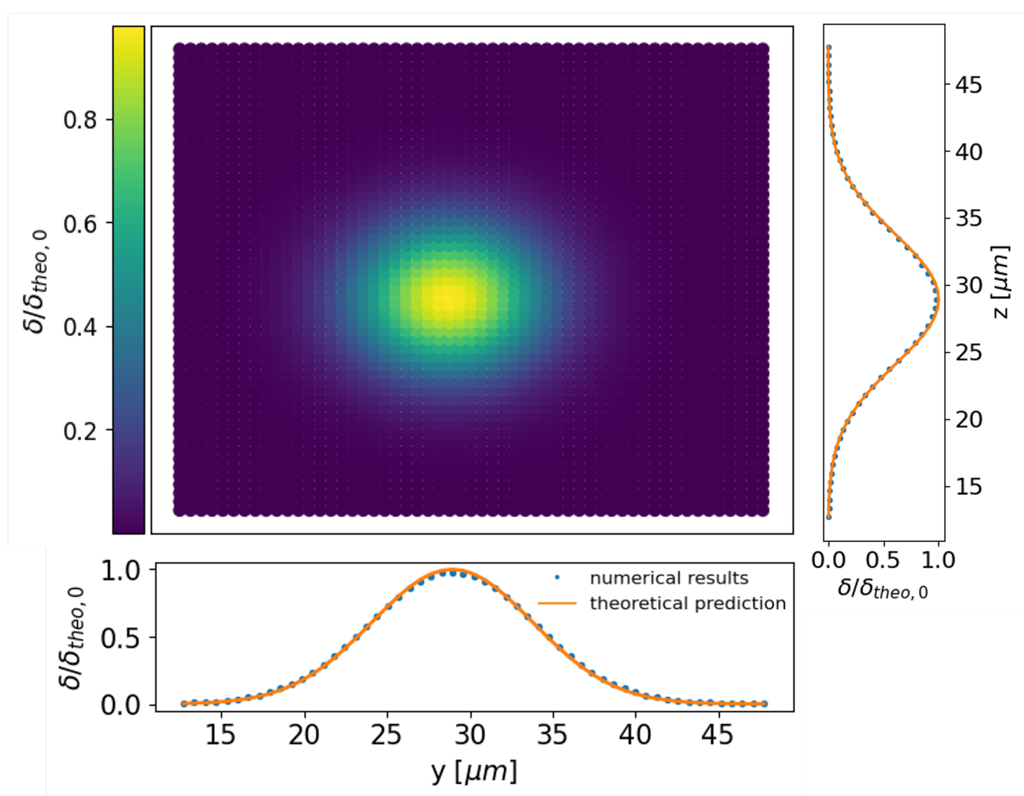


Figure 3. Normalised ellipticity across transverse plane for the Gaussian scenario, where the ellipticity echos the Gaussian profile of the pump pulse, consistent with theory.

Four-wave mixing simulation

- In vacuum four-wave mixing, three input beams interact to form a fourth output going beam at a direction and frequency governed by phase-matching conditions.
- The geometry shown in Fig. 4 produces a third harmonic output photon.
- The third harmonic is visible in the Fourier Transforms throughout the simulation as predicted.
- Multiple other harmonics are present at the midpoint of the simulation. Most of these are off-shell modes, hence exhibit evanescent behaviour.

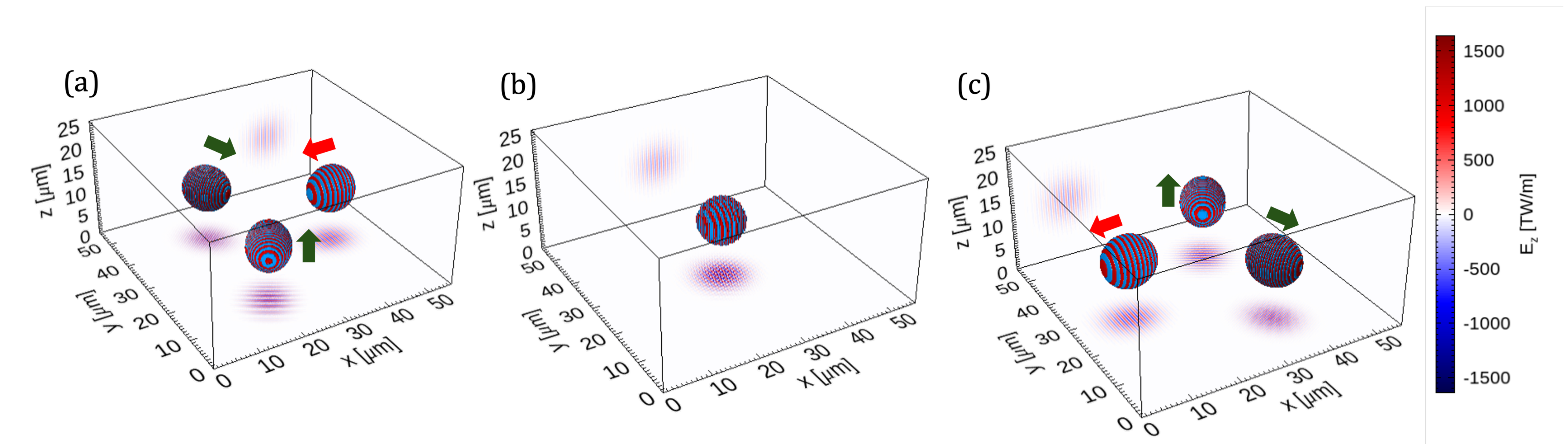


Figure 4. Three-dimensional setup of four-wave mixing at three stages, ωt= (a) 0, (b) 100 and (c) 200.

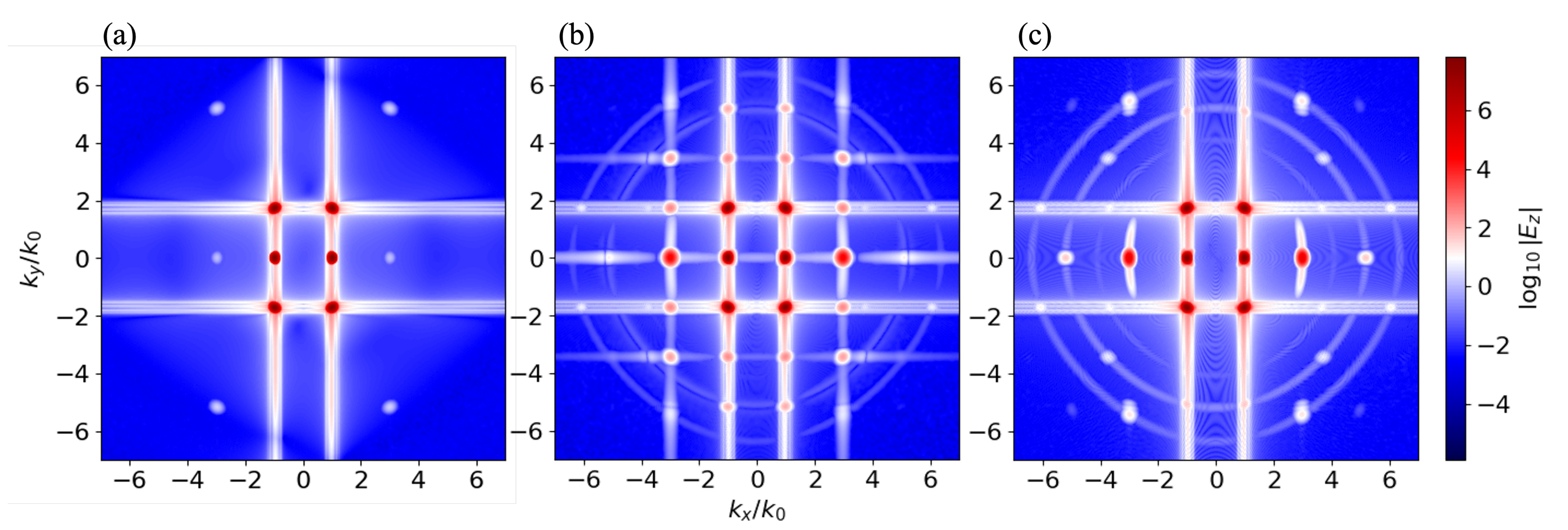


Figure 5. FFT in the x-y plane at ωt= (a) 1, (b) 100 and (c) 200, where ω is the fundamental frequency. k values are normalised by the fundamental frequency.

Time-resolved outputs of four-wave mixing

- Key characteristics of the output pulse can be resolved accurately in the near field and traced with time.
- The asymmetry in the output pulse as shown by Fig. 6(c) can be linked to the same asymmetry in the interaction region, which resembles an ellipsoid (Fig. 4(b)), whose radii differ depending on the axis.
- An interaction time can be clearly identified from Fig. 6(a) and (b). This helps narrow down the detection window for time-gated photon detectors. Temporal evolution is inaccessible by previous analytical and numerical works.
- The total photon number obtained is in excellent agreement with previous numerical results.
- Comparing with the plane-wave model, with the same input energy, Gaussian beams produce 8 times higher photon count.

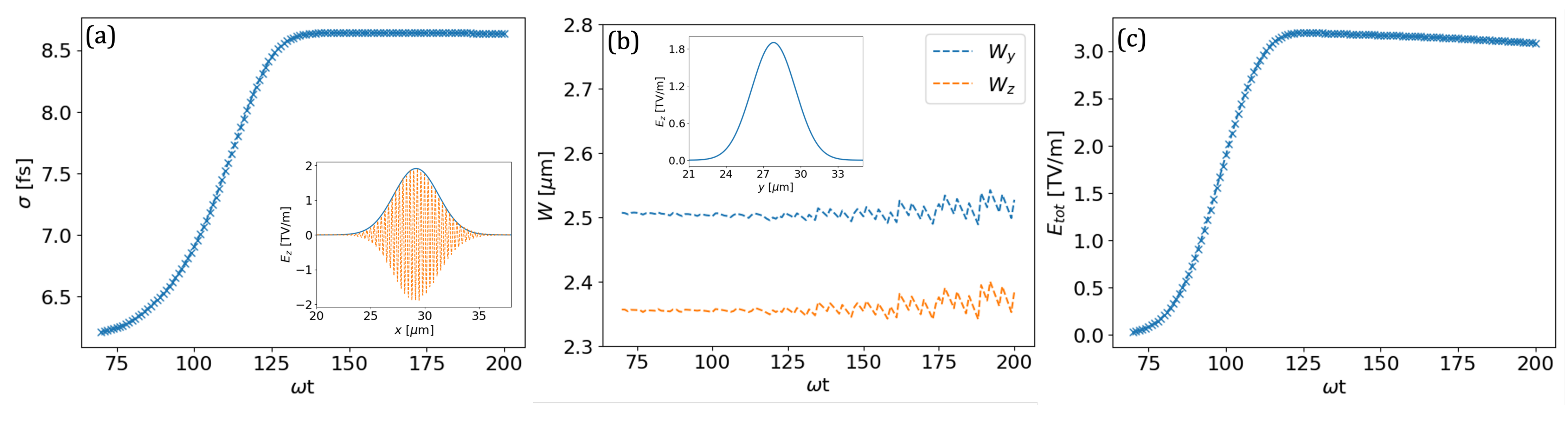


Figure 6. Temporal evolution of (a) the longitudinal duration; (b) the transverse widths along $\hat{y}$ and $\hat{z}$ and (c) the total electric field strength.

Conclusion and outlook

- The QED solver is a full-scale real-time three-dimensional simulation that models any generic laser setup.
- The time-resolved capability of the solver is able to uncover deeper and richer physical insights than alternative numerical methods.
- The implementation into OSIRIS opens up the potential of integrating massive particles, which will provide the first estimates of future noise levels.