Testing flavour/modular symmetry theories in neutrino experiments

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ESS neutrino Super Beam plus

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Outline

- Neutrino Experiments: with a focus on ESSnuSB

  Alekou, *et al.* [ESSnuSB] Eur. Phys. J. ST **231** (2022), 3779-3955

- Testing a set of flavour symmetry models

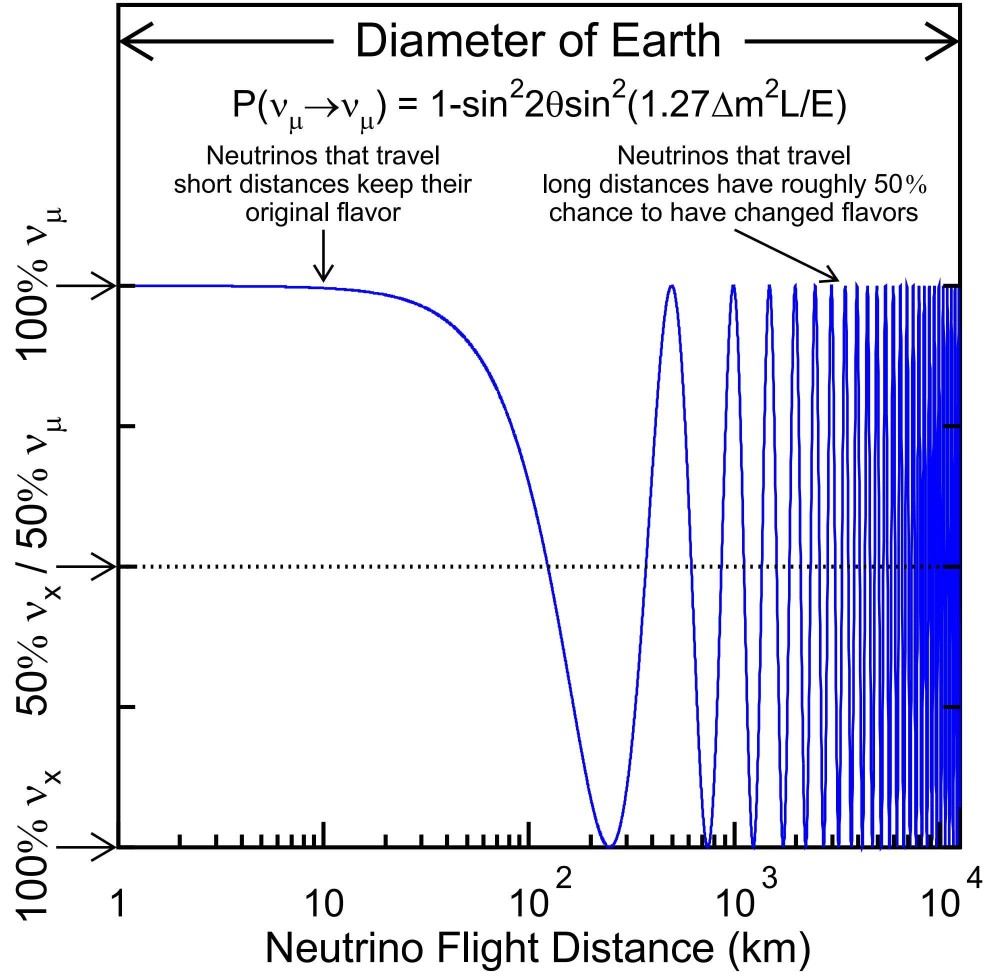
Blennow, MG, Ohlsson and Titov, JHEP **07** (2020), 014

Blennow, MG, Ohlsson and Titov, Phys. Rev. D **102** (2020), 115004

- Testing a set of modular symmetry models

Mishra, Behera, Panda, MG and Mohanta, JHEP **09** (2023), 144

Neutrino Oscillation



- Oscillation from one flavour to another
- Because flavour states and mass states are not same

$$|\nu_\alpha\rangle = U_{\alpha i}\ |\nu_i\rangle$$

U is the PMNS matrix

- The transition probability is given by

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2$$

3 mixing angles and 1 phase: $U^{3\nu}$ = $R_{23}(\theta_{23})$ $R_{13}(\theta_{13}, \delta_{CP})$ $R_{12}(\theta_{12})$

2 mass squared difference: $\Delta m^2_{21} = m^2_2 - m^2_1$, $\Delta m^2_{31} = m^2_3 - m^2_1$

Two instrinsic parameters: baseline L and energy E

Global fit status

... while fractional accuracy of **known** parameters improved.

In particular, Δm^2 formally determined at the subpercent level, $1\sigma = 0.8\%$

TABLE I: Global 3ν oscillation analysis: best-fit values and allowed ranges at $N_\sigma = 1, 2, 3$, for either NO or IO. The last column shows the formal "1σ parameter accuracy," defined as 1/6 of the 3σ range, divided by the best-fit value (in percent). We recall that $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$ and that δ/π is cyclic (mod 2). Last row: $\Delta\chi^2$ offset between IO and NO.

Parameter	Ordering	Best fit	1σ range	2σ range	3σ range	"1σ" (%)
$\delta m^2/10^{-5}$ eV^2	NO, IO	7.37	7.21 – 7.52	7.06 – 7.71	6.93 – 7.93	2.3
$\sin^2\theta_{12}/10^{-1}$	NO, IO	3.03	2.91 – 3.17	2.77 – 3.31	2.64 – 3.45	4.5
$\lvert\Delta m^2\rvert/10^{-3}$ eV^2	NO	2.495	2.475 – 2.515	2.454 – 2.536	2.433 – 2.558	0.8
	IO	2.465	2.444 – 2.485	2.423 – 2.506	2.403 – 2.527	0.8
$\sin^2\theta_{13}/10^{-2}$	NO	2.23	2.17 – 2.27	2.11 – 2.33	2.06 – 2.38	2.4
	IO	2.23	2.19 – 2.30	2.14 – 2.35	2.08 – 2.41	2.4
$\sin^2\theta_{23}/10^{-1}$	NO	4.73	4.60 – 4.96	4.47 – 5.68	4.37 – 5.81	5.1
	IO	5.45	5.28 – 5.60	4.58 – 5.73	4.43 – 5.83	4.3
δ/π	NO	1.20	1.07 – 1.37	0.88 – 1.81	0.73 – 2.03	18
	IO	1.48	1.36 – 1.61	1.24 – 1.72	1.12 – 1.83	8
$\Delta\chi^2_{\text{IO-NO}}$	IO–NO	+5.0				

The prediction of the flavour/symmetry models must be in agreement with experimental data

But there are reasons to be cautious about subpercent accuracy levels...

E.g., correlated effects of ν interaction uncertainties in different expts need improvement

Talk by Eligio

Motivation

- At present parameters with large 1 sigma uncertainties:

$$\theta_{23} : 5.1\%$$
$$\theta_{12} : 4.5\%$$
$$\delta_{CP} : 18\%$$

- Future experiments expected to measure these parameters within subpercent level

- Given this fact: we ask two questions:

(i) The models that are allowed now, will they be still allowed in future ?

(ii) Can the future experiments distinguish different models ?

Experiments

The experimts we consider to test the models are

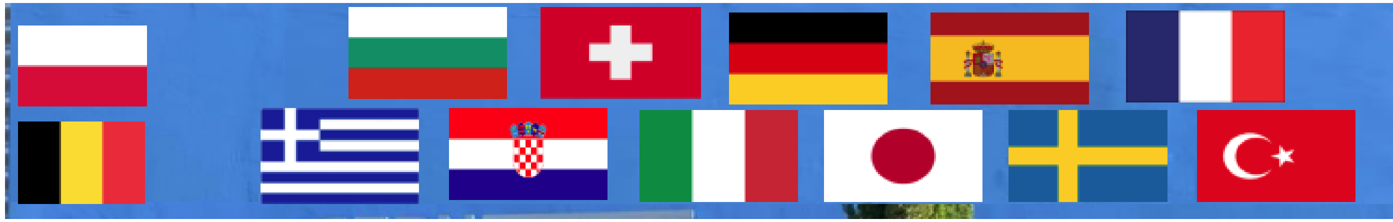
ESSnuSB in Europe: Will have the best precision of δ_{CP}
-I and Davide are part of this project

T2HK in Japan: Will have excellent sensitivity to θ_{23} and δ_{CP}
DUNE in USA: Will have excellent sensitivity to θ_{23} , δ_{CP}
JUNO in China: Will have the best sensitivity to θ_{12}

The ESSnuSB Experiment

ESSnuSB - Horizon (2018 - 2022) – 3 M€
ESSnuSB+ -Horizon EU (2023 - 2026) - 3 M€

13 countries
23 Instititutes



Water Cherenkov Far Detector- 540 kt
5 MW Proton beam, 2 Gev proton energy

Main Goal: Precision measurement of the CP Violation phase with a long baseline setup

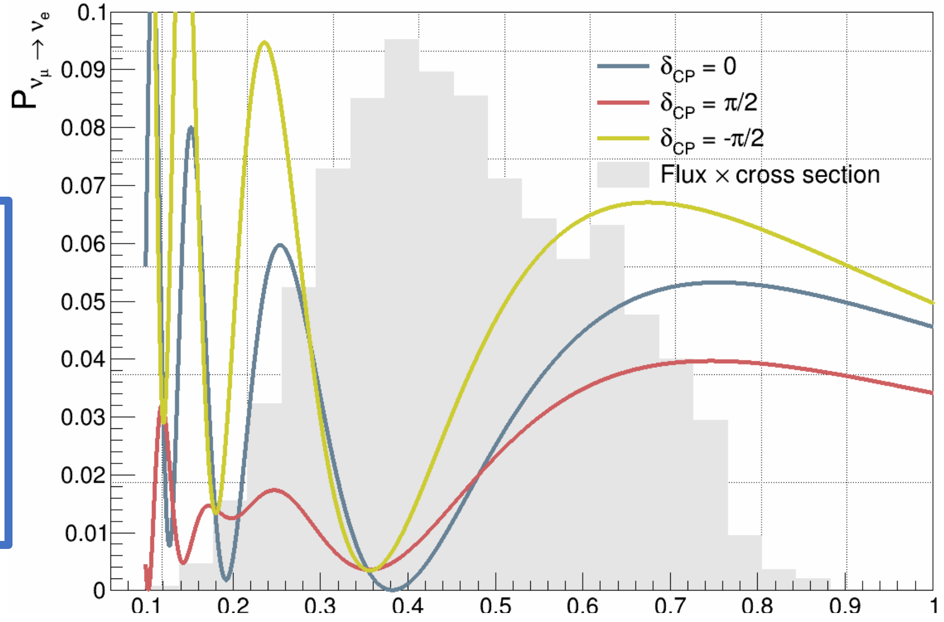
Other goals: Neutrino Cross-section measurements with the near detectors, Sterile neutrino searches with a short baseline setup, etc.

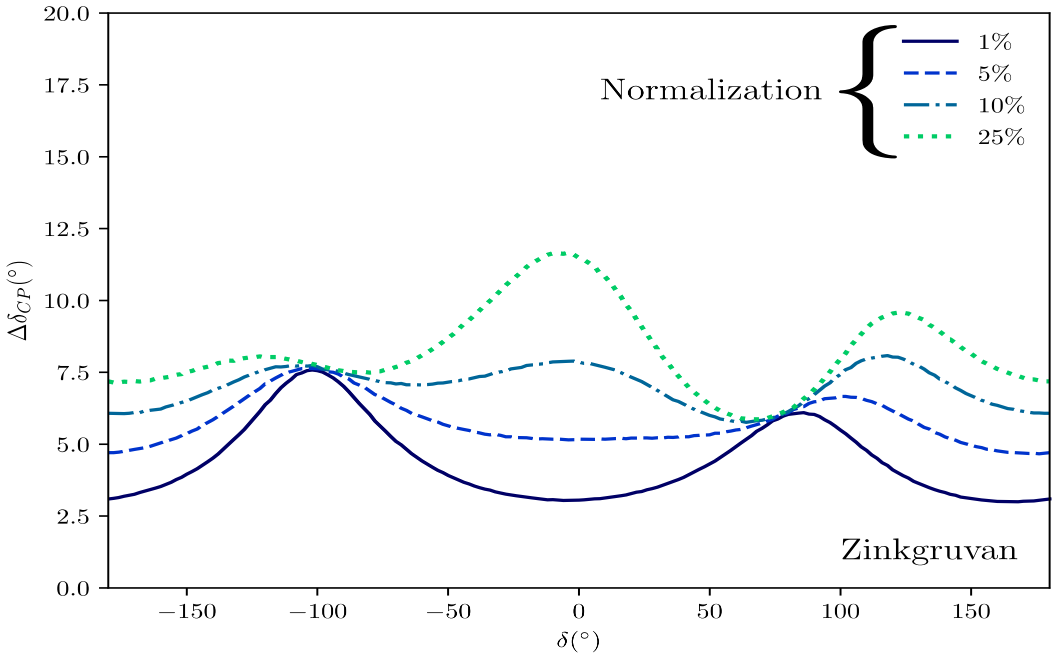
Sensitivity of ESSnuSB

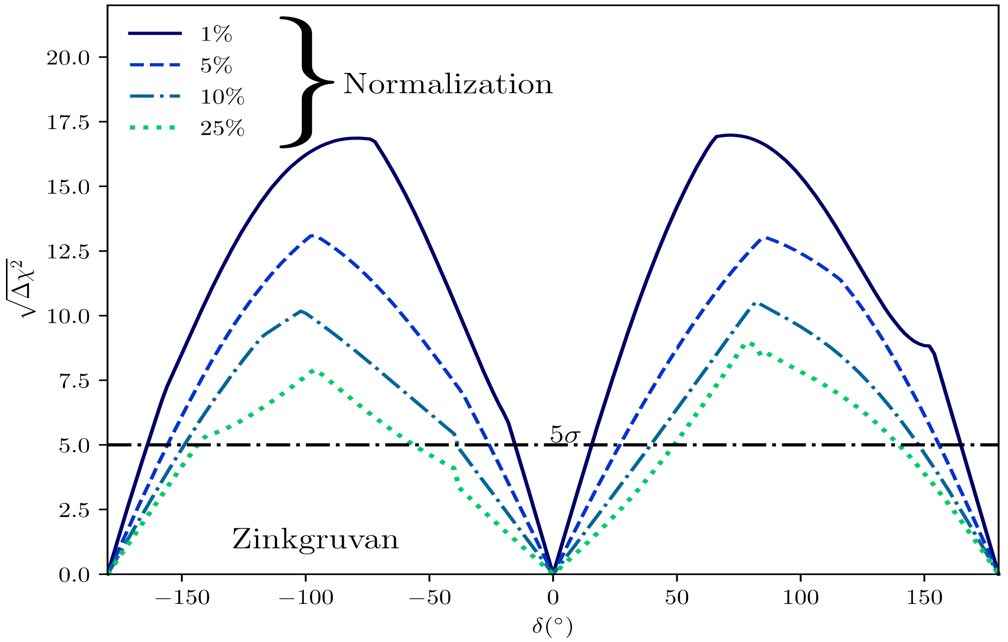
Designed to probe second oscillation maximum

12 sigma sensitivity to discover CP violation for maximal values

Will measure δ_{CP} with less than 8° precision for all values



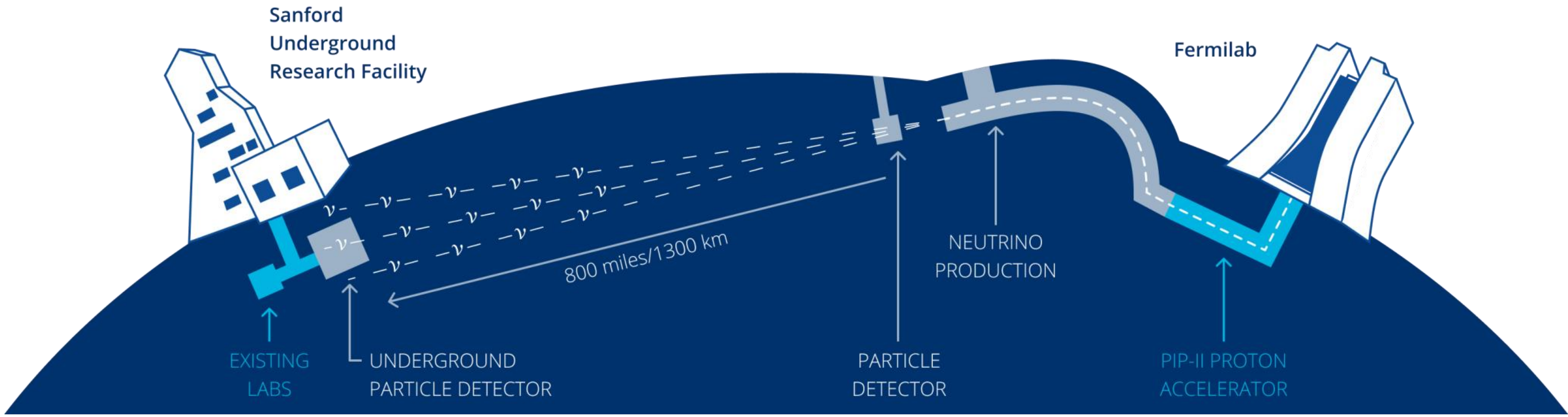


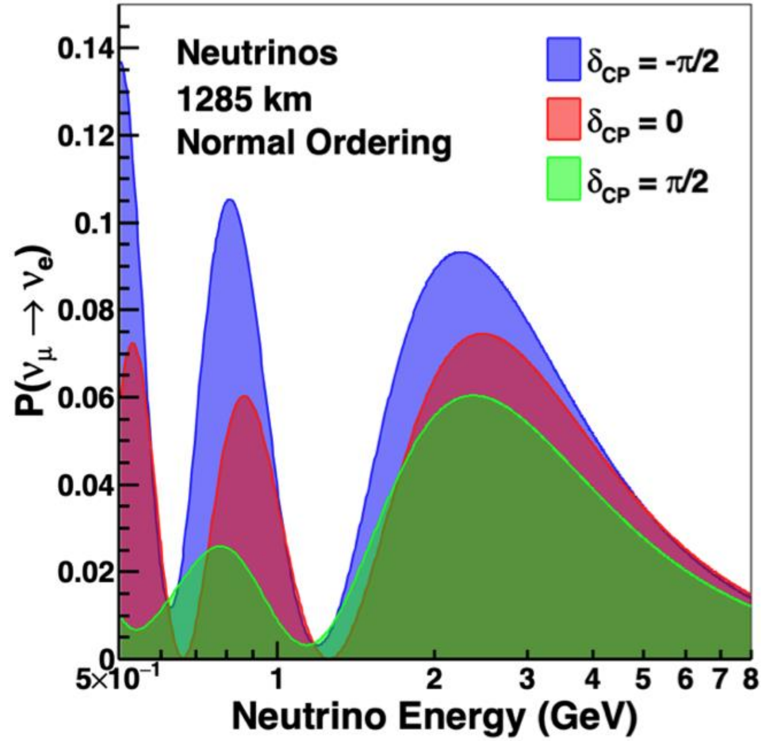


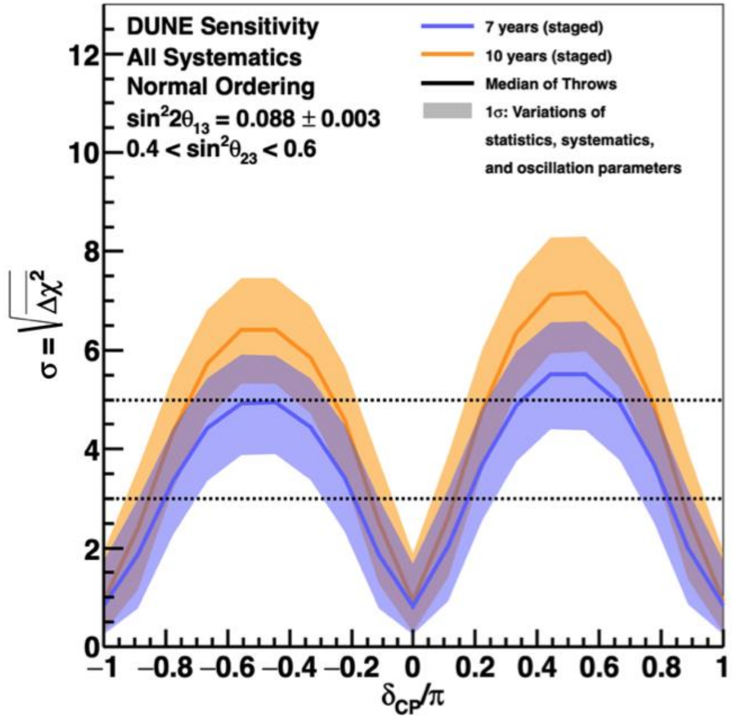
Alekou, *et al.* [ESSnuSB] Eur. Phys. J. ST **231** (2022), 3779-3955

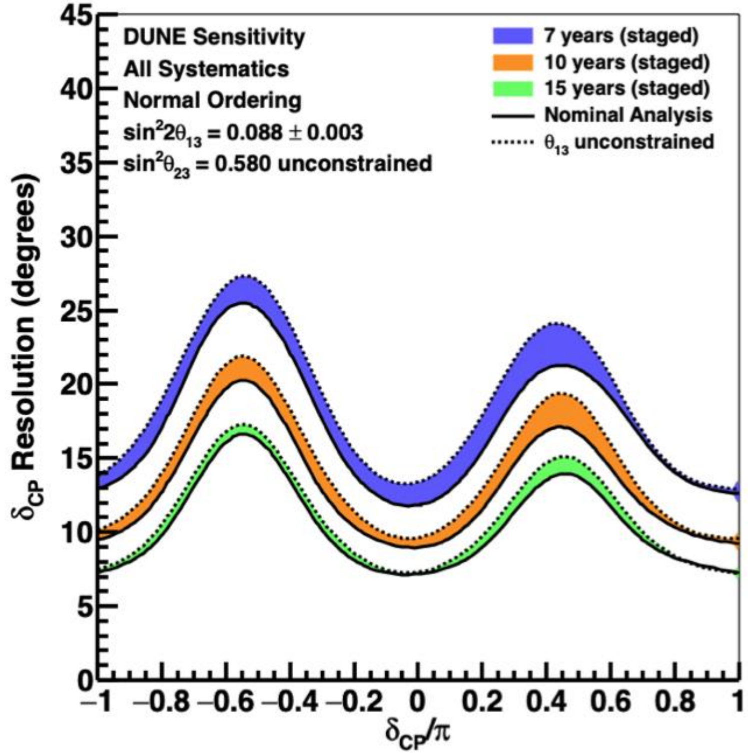
DUNE

Liquid Argon TPC of 40 kt
L = 1300 km





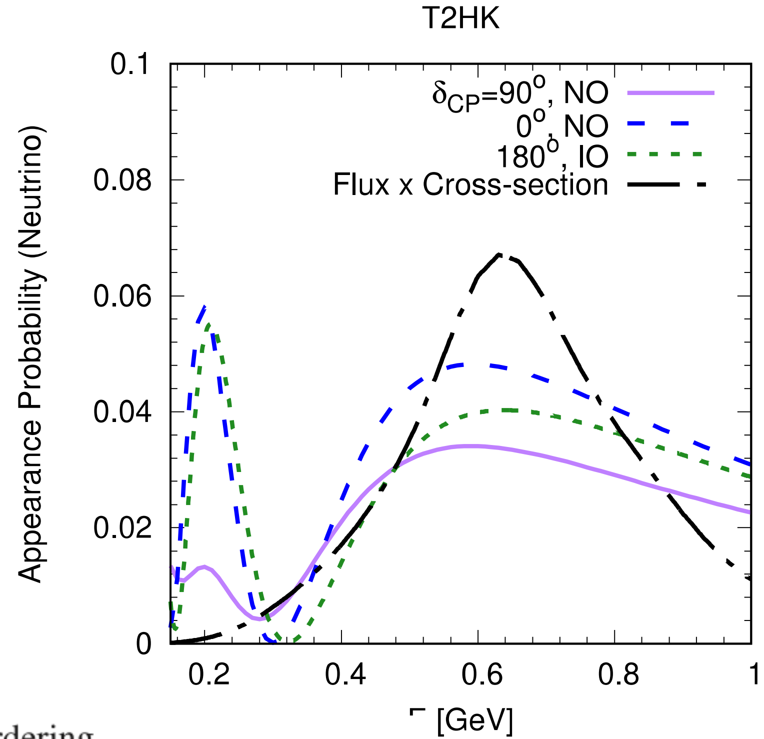


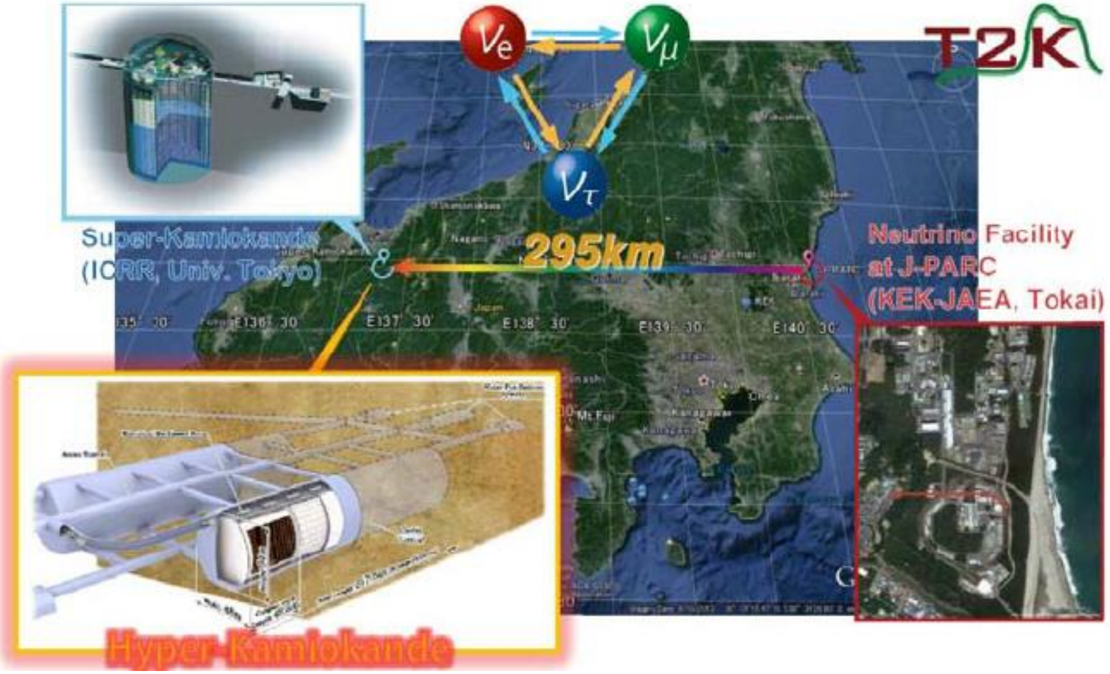


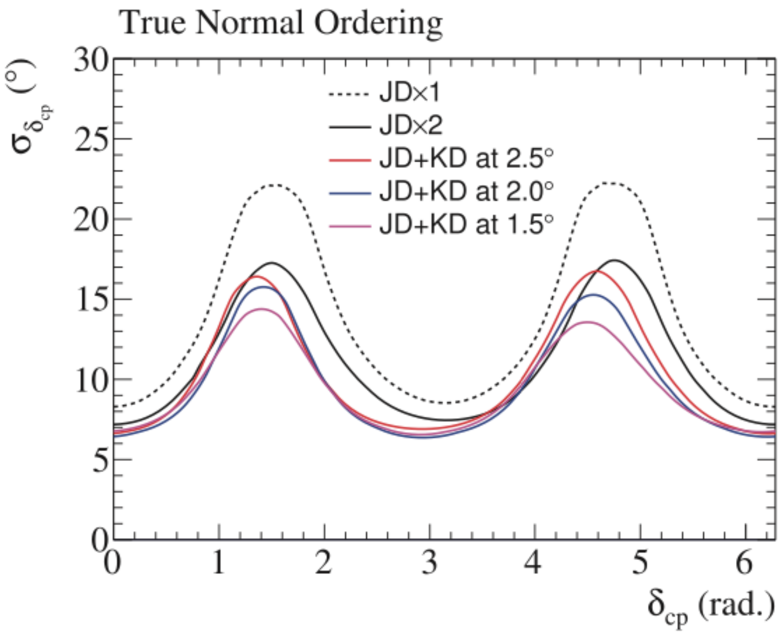
Abi et al. [DUNE], Eur. Phys. J. C 80 (2020), 978

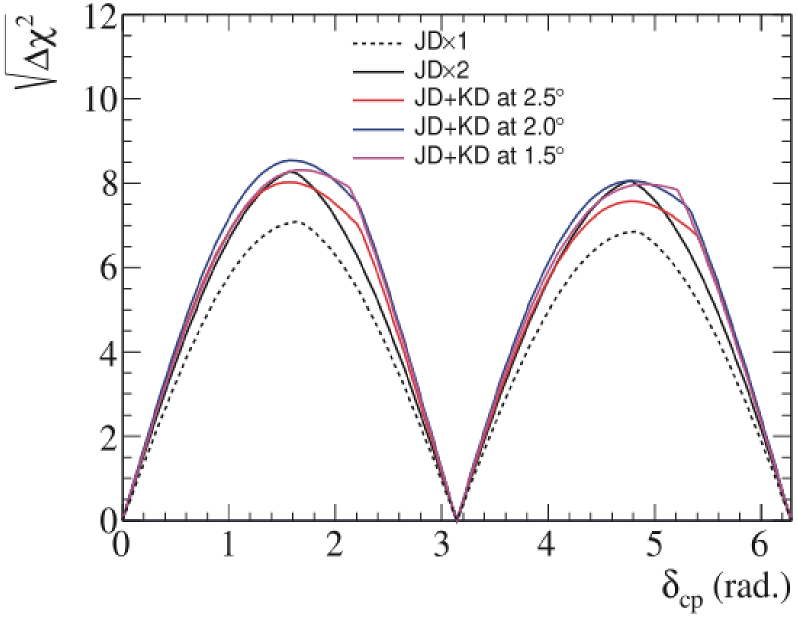
T2HK

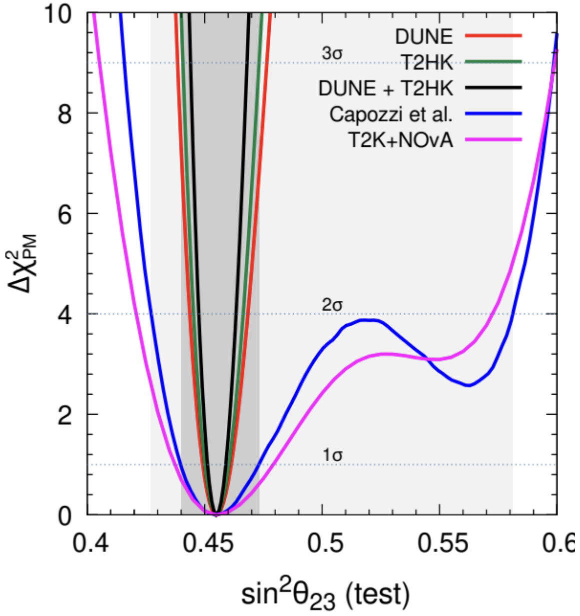
WC 374 kt
295 km







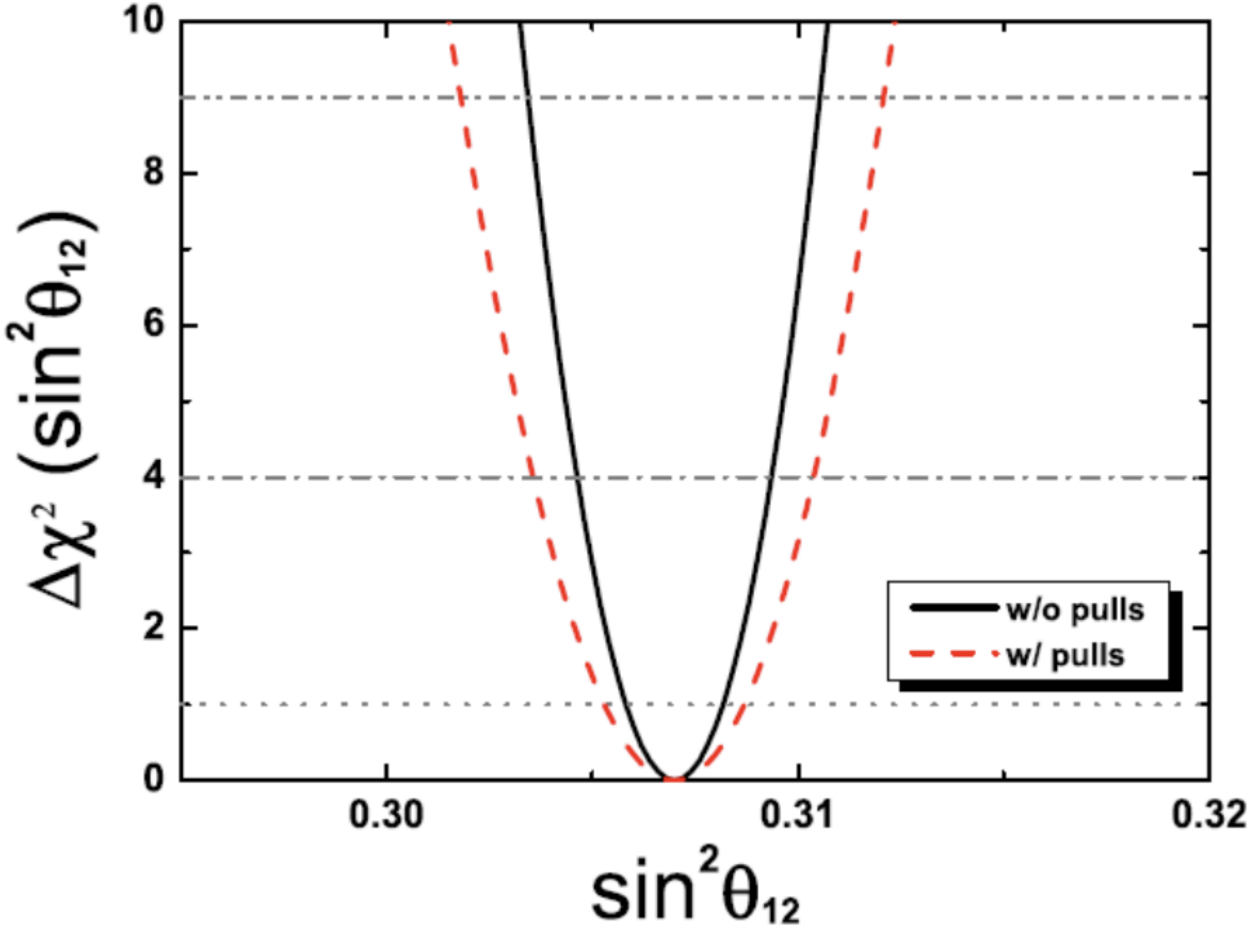


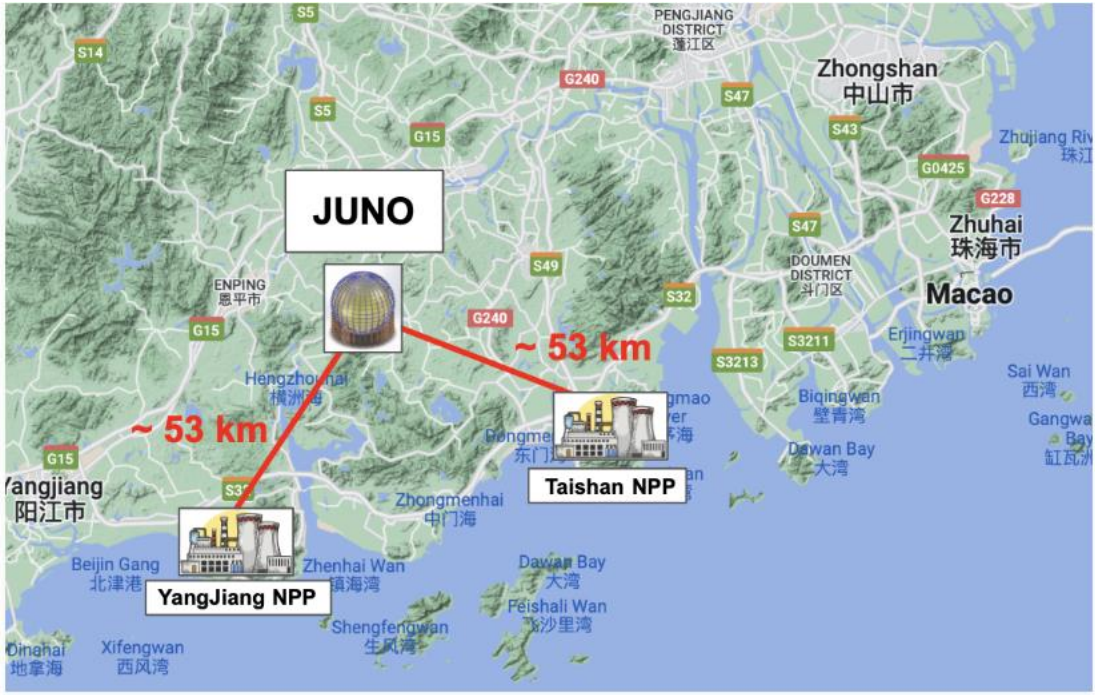


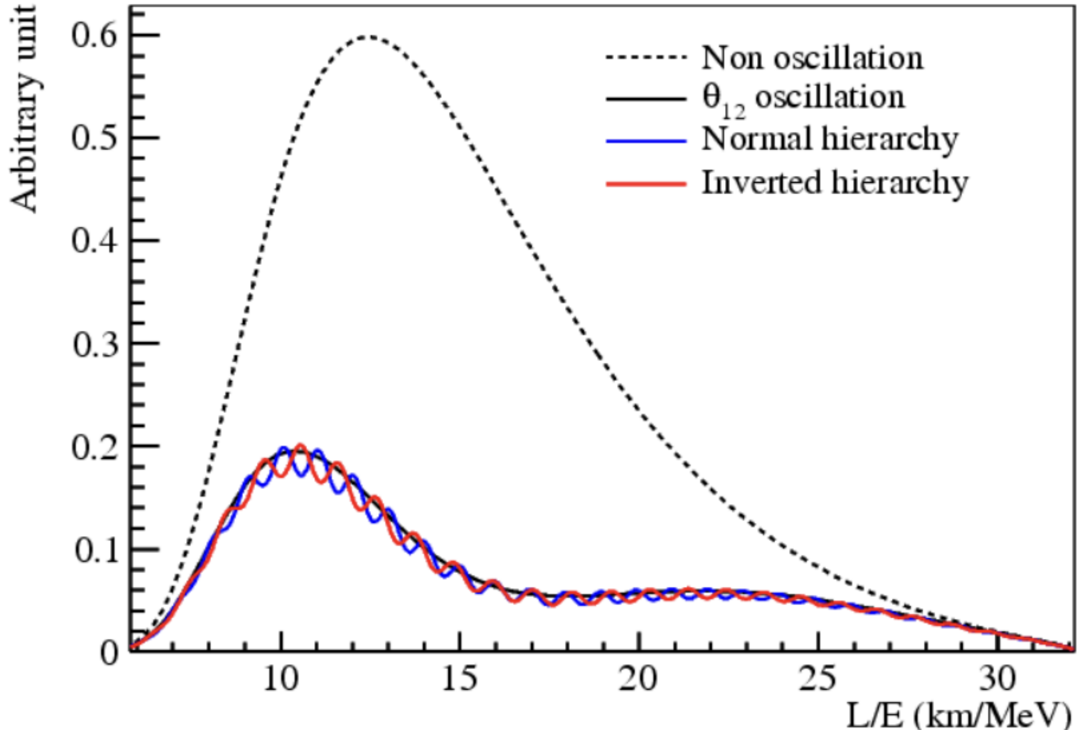
Abe et al. [Hyper-Kamiokande](2018), Ghosh and Ohlsson (2020), Agarwalla, Kundu and Singh (2024)

JUNO

Liquid Scintillator, 20 kt
53 km







An *et al.* [JUNO], J. Phys. G **43** (2016) no.3, 030401

Testing a flavour symmetry model

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Let's consider a model which predicts the values of mixing parameters in the form of a sum rule
And the sum rule depends only upon one variable: One parameter model

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For example: Feruglio, Hagedorn and Ziegler JHEP 07 (2013) 027

$$G_f = S_4 \rtimes CP$$

broken to $G_e = Z_3$ and $G_\nu = Z_2 \times CP$

	I	II	IV	V
$\sin^2\theta_{13}$	$\frac{2}{3}\sin^2\theta$	$\frac{2}{3}\sin^2\theta$	$\frac{1}{3}\sin^2\theta$	$\frac{1}{3}\sin^2\theta$
$\sin^2\theta_{12}$	$\frac{1}{2+\cos 2\theta}$	$\frac{1}{2+\cos 2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right)$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{2\sqrt{6}\sin 2\theta}{5+\cos 2\theta}\right)$
$\lvert\sin\delta\rvert$	1	0	1	0

Testing a flavour symmetry model

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Step1: Find a value of θ which gives the values of mixing parameters Consistent with their experimental values

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$\sin^2\theta_{12}$	$\frac{1}{2+\cos 2\theta}$	$\frac{1}{2+\cos 2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right)$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{2\sqrt{6}\sin 2\theta}{5+\cos 2\theta}\right)$
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$\sin^2\theta_{12}$	$\frac{1}{2+\cos 2\theta}$	$\frac{1}{2+\cos 2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$	$\frac{\cos^2\theta}{2+\cos^2\theta}$
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{\sqrt{3}\sin 2\theta}{2+\cos 2\theta}\right)$	$\frac{1}{2}$	$\frac{1}{2}\left(1-\frac{2\sqrt{6}\sin 2\theta}{5+\cos 2\theta}\right)$
$\lvert\sin\delta\rvert$	1	0	1	0

Step1: Find a value of θ which gives the values of mixing parameters Consistent with their experimental values

To do that define a χ^2 = (data – model)$^2/\sigma$

Model	θ^o	χ^2
I	10.5	12.6
II	169.5	8.91
IV	15.0	7.28
V	165.2	36.8

Full set of one parameter models

$$G_f = S_4 \rtimes CP$$

broken to $G_e = Z_4$ (or $Z_2 \times Z_2$) and $G_\nu = Z_2 \times CP$

	VI-a	VI-b
$\sin^2\theta_{13}$	$\frac{1}{4}\left(\sqrt{2}\cos\theta + \sin\theta\right)^2$	
$\sin^2\theta_{12}$	$\frac{2}{5-\cos 2\theta - 2\sqrt{2}\sin 2\theta}$	
$\sin^2\theta_{23}$	$\frac{4\sin^2\theta}{5-\cos 2\theta - 2\sqrt{2}\sin 2\theta}$	$1 - \frac{4\sin^2\theta}{5-\cos 2\theta - 2\sqrt{2}\sin 2\theta}$
$\sin\delta$	0	

$$G_f = A_5 \rtimes CP$$

broken to $G_e = Z_3$ $(Z_2 \times Z_2)$ and $G_\nu = Z_2 \times CP$

	V	VII-a	VII-b
$\sin^2\theta_{13}$	$\frac{1-\sin 2\theta}{3}$	$\frac{(\cos\theta - \varphi\sin\theta)^2}{4\varphi^2}$	
$\sin^2\theta_{12}$	$\frac{1}{2+\sin 2\theta}$	$\frac{(\varphi\cos\theta + \sin\theta)^2}{4\varphi^2 - (\cos\theta - \varphi\sin\theta)^2}$	
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{(\varphi^2\cos\theta - \sin\theta)^2}{4\varphi^2 - (\cos\theta - \varphi\sin\theta)^2}$	$\frac{\varphi^2(\cos\theta + \varphi\sin\theta)^2}{4\varphi^2 - (\cos\theta - \varphi\sin\theta)^2}$
$\lvert\sin\delta\rvert$	1	0	

broken to $G_e = Z_5$ and $G_\nu = Z_2 \times CP$

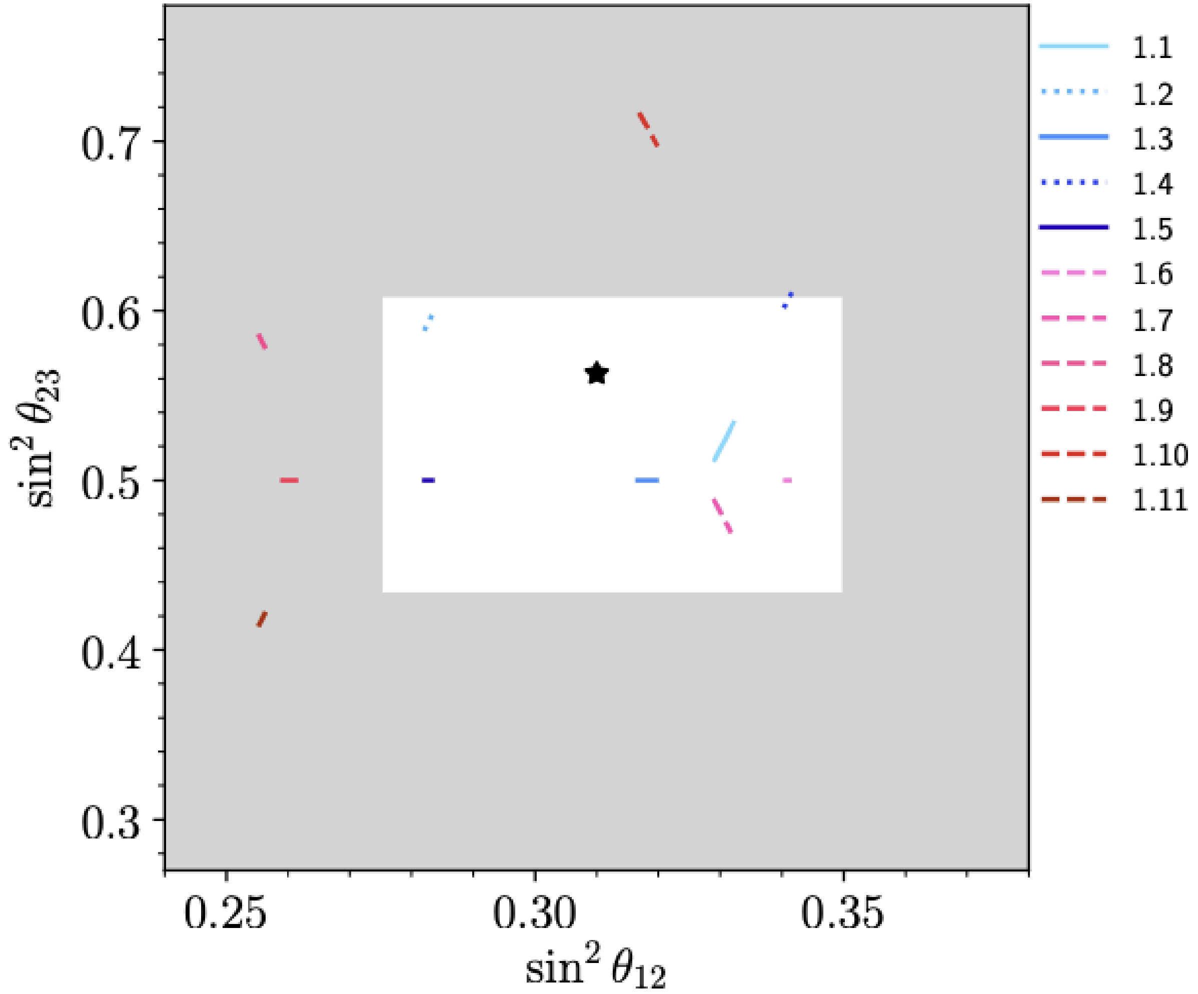
	II	III	IV
$\sin^2\theta_{13}$	$\frac{3-\varphi}{5}\sin^2\theta$	$\frac{\varphi}{\sqrt{5}}\sin^2\theta$	$\frac{\varphi}{\sqrt{5}}\sin^2\theta$
$\sin^2\theta_{12}$	$\frac{2\cos^2\theta}{3+2\varphi+\cos 2\theta}$	$\frac{4-2\varphi}{5-2\varphi+\cos 2\theta}$	$\frac{4-2\varphi}{5-2\varphi+\cos 2\theta}$
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2} - \frac{\sqrt{3-\varphi}\sin 2\theta}{3\varphi - 2 + \varphi\cos 2\theta}$	$\frac{1}{2}$
$\lvert\sin\delta\rvert$	1	0	1

Feruglio, Hagedorn and Ziegler JHEP 07 (2013) 027, Li and Ding, JHEP 05 (2015)

Step 2

Step2: List all the models according to the value of the χ^2 and reject the ones which have very high values of χ^2 : This excludes the models with respect to the current data

Model	Case	$\chi^2_{\rm min}$	$\theta_{\rm bf}$	$\theta_{3\sigma}$
1.1	VII-b (A_5)	5.37	17.0°	(16.3°, 17.7°)
1.2	III (A_5)	5.97	169.9°	(169.4°, 170.4°)
1.3	IV (S_4)	7.28	15.0°	(14.3°, 15.7°)
			165.0°	(164.3°, 165.7°)
1.4	II (S_4)	8.91	169.5°	(169.0°, 170.0°)
1.5	IV (A_5)	11.3	10.1°	(9.6°, 10.6°)
			169.9°	(169.4°, 170.4°)
1.6	I (S_4)	12.6	10.5°	(10.0°, 11.1°)
			169.5°	(168.9°, 170.0°)
1.7	VII-a (A_5)	14.8	16.9°	(16.2°, 17.6°)
1.8	VI-b (S_4)	18.1	115.3°	(114.8°, 115.8°)
1.9	II (A_5)	21.8	16.5°	(15.7°, 17.3°)
			163.5°	(162.7°, 164.3°)
1.10	V (S_4)	36.8	165.2°	(164.4°, 165.9°)
1.11	VI-a (S_4)	53.8	115.3°	(114.8°, 115.8°)



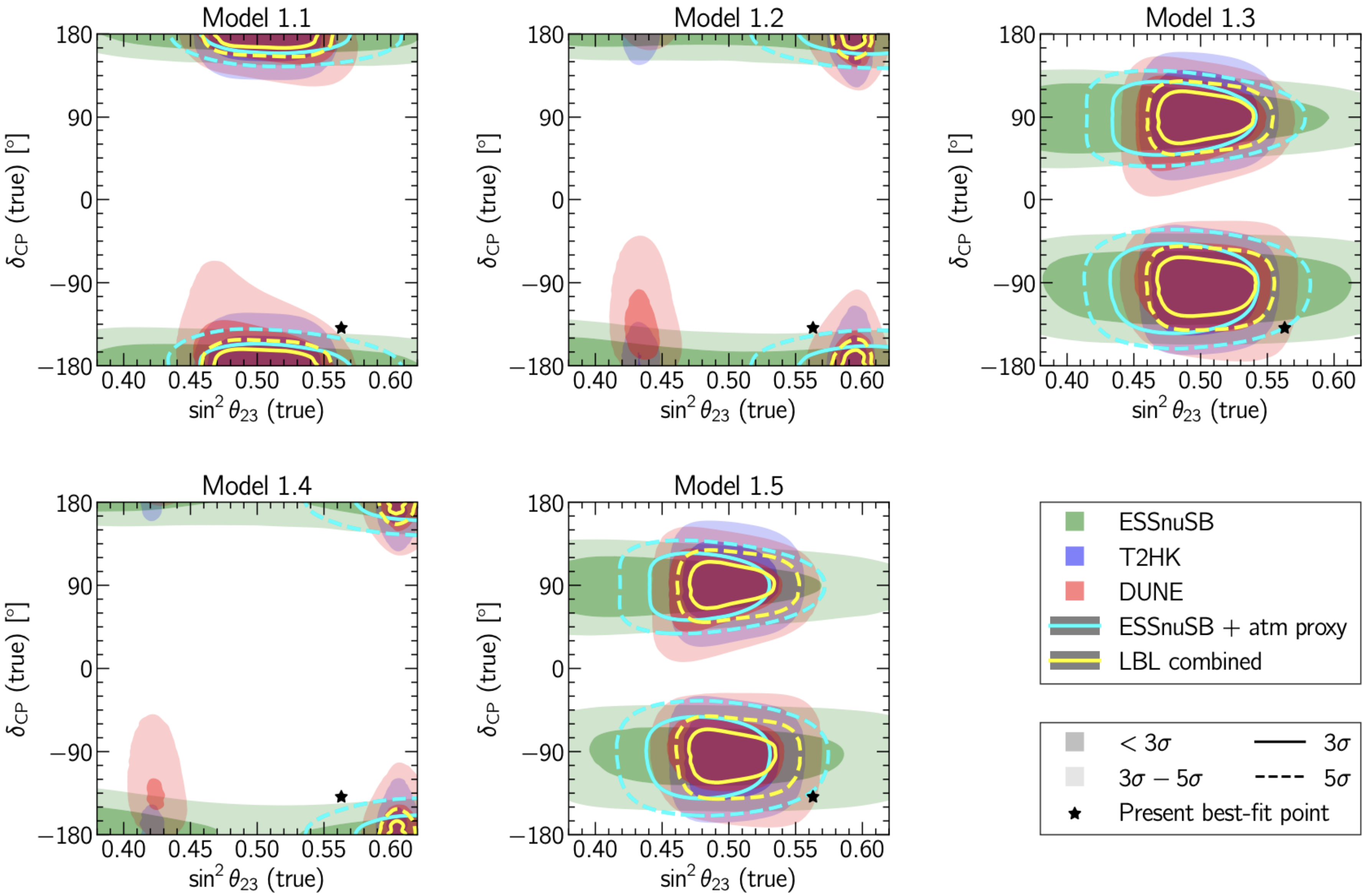
The Models 1.6 – 1.11 are excluded for further analysis

Blennow, MG, Ohlsson and Titov, JHEP **07** (2020), 014

Final step

Final step: Test the survived models in future experiments

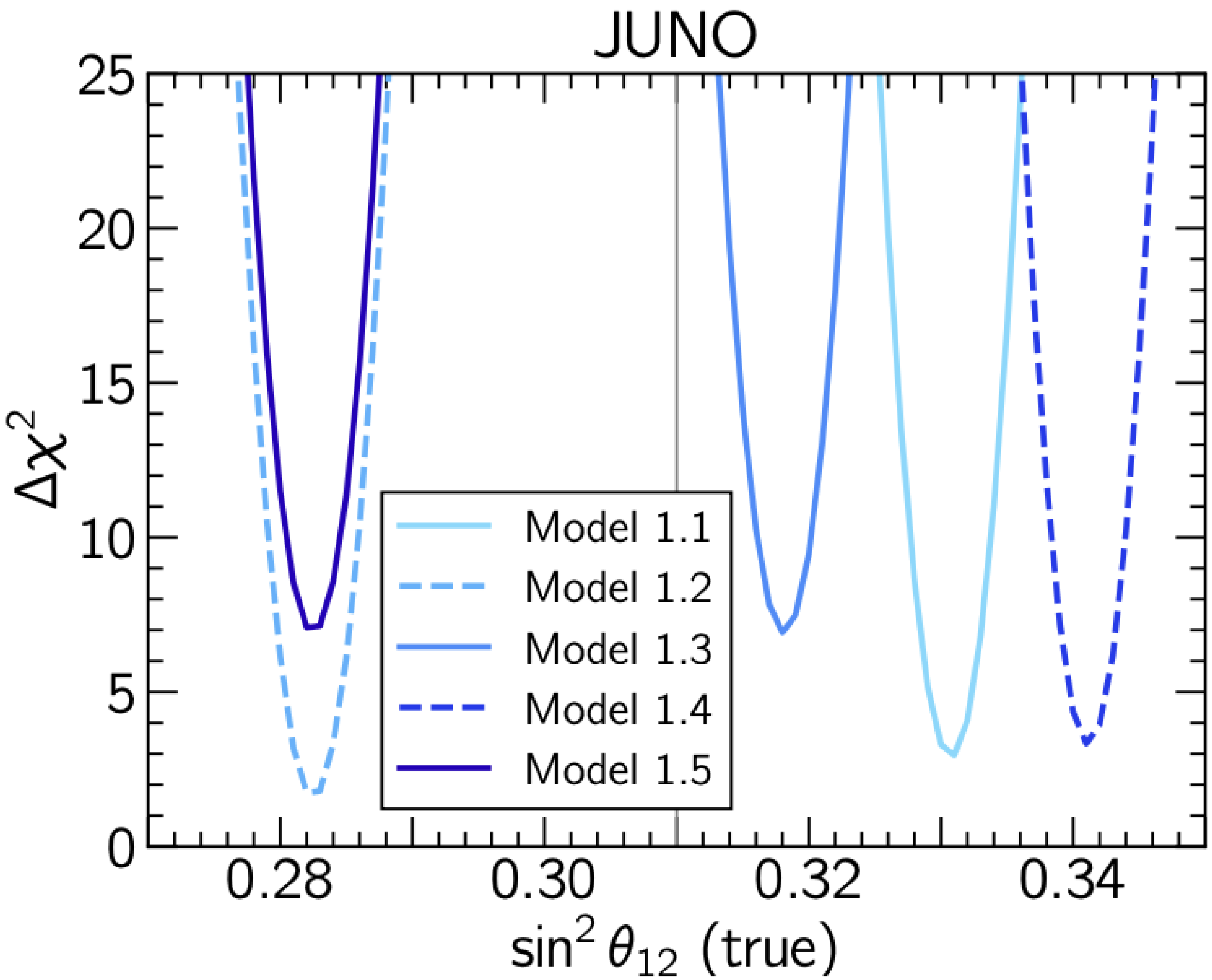
Model	$sin^2\theta_{23}$	δ_{CP}°
1.1	0.523	180
1.2	0.593	180
1.3	0.5	+-90
1.4	0.606	180
1.5	0.5	+-90



Depending on the precision, LBL can verify their viability

Blennow, MG, Ohlsson and Titov, Phys. Rev. D **102** (2020), 115004

Test in JUNO



Model	$sin^2\theta_{12}$
1.1	0.331
1.2	0.283
1.3	0.318
1.4	0.341
1.5	0.283

Depending on the precision,
JUNO can exclude all the models

Blennow, MG, Ohlsson and Titov, Phys. Rev. D **102** (2020), 115004

Two parameter models

Sum rule depends only upon two variables

$$G_f = S_4 \text{ or } A_5$$
$$\text{broken to } G_e = Z_k,\ k>2 \text{ or } Z_m \times Z_n,\ m,n \geq 2 \text{ and } G_\nu = Z_2,$$

Case B1. $\sin^2\theta_{13} = \cos^2\theta^\circ_{12}\sin^2\theta,$

$$\sin^2\theta_{23} = \frac{\cos^2\theta^\circ_{23}\sin^2\theta\sin^2\theta^\circ_{12} + \cos^2\theta\sin^2\theta^\circ_{23} - \frac{1}{2}\sin 2\theta\sin 2\theta^\circ_{23}\sin\theta^\circ_{12}\cos\phi}{1-\sin^2\theta_{13}},\quad \sin^2\theta_{12} = \frac{\sin^2\theta^\circ_{12}}{1-\sin^2\theta_{13}}$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\cos^2\theta_{23} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23}) - \sin^2\theta^\circ_{12}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\sin\theta^\circ_{12}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{12})^{\frac{1}{2}}}$$

Case B2. $\sin^2\theta_{13} = \cos^2\theta^\circ_{13}\sin^2\theta^\circ_{12}\sin^2\theta + \sin^2\theta^\circ_{13}\cos^2\theta + \frac{1}{2}\sin 2\theta\sin 2\theta^\circ_{13}\sin\theta^\circ_{12}\cos\phi$

$$\sin^2\theta_{23} = \frac{\cos^2\theta^\circ_{12}\sin^2\theta}{1-\sin^2\theta_{13}},\quad \sin^2\theta_{12} = \frac{\cos^2\theta_{13} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}}{1-\sin^2\theta_{13}}$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{12} - \cos^2\theta_{23}) + \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13}(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23}\sin\theta_{13}|\cos\theta^\circ_{12}\cos\theta^\circ_{13}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{13})^{\frac{1}{2}}}$$

G_e	G_ν	Case	$\sin^2\theta^\circ_{12}$	$\sin^2\theta^\circ_{13}$	$\sin^2\theta^\circ_{23}$
Z_3	Z_2	$B1S_4$	$1/3$	-	$1/2$
Z_3	Z_2	$B2S_4$	$1/6$	$1/5$	-
Z_5	Z_2	$B1A_5$	0.276	-	$1/2$
$Z_2 \times Z_2$	Z_2	$B2A_5$	0.095	0.276	-
$Z_2 \times Z_2$	Z_2	$B2A_5II$	$1/4$	0.127	-

$$G_f = A_5$$
$$\text{broken to } G_e = Z_2 \text{ and } G_\nu = Z_k,\ k>2 \text{ or } Z_m \times Z_n,\ m,n \geq 2,$$

Case A1. $\sin^2\theta_{13} = \cos^2\theta\sin^2\theta^\circ_{13} + \cos^2\theta^\circ_{13}\sin^2\theta\sin^2\theta^\circ_{23} + \frac{1}{2}\sin 2\theta\sin 2\theta^\circ_{13}\sin\theta^\circ_{23}\cos\phi$

$$\sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{13} - \sin^2\theta_{13} + \cos^2\theta^\circ_{13}\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}},\quad \sin^2\theta_{12} = \frac{\cos^2\theta^\circ_{23}\sin^2\theta}{1-\sin^2\theta_{13}}$$

$$\cos\delta = \frac{\cos^2\theta_{13}(\sin^2\theta^\circ_{23} - \cos^2\theta_{12}) + \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\circ_{13}\cos\theta^\circ_{23}|(\cos^2\theta_{13} - \cos^2\theta^\circ_{13}\cos^2\theta^\circ_{23})^{\frac{1}{2}}}.$$

Case A2.

$$\sin^2\theta_{13} = \sin^2\theta\cos^2\theta^\circ_{23},\quad \sin^2\theta_{23} = \frac{\sin^2\theta^\circ_{23}}{1-\sin^2\theta_{13}}$$

$$\sin^2\theta_{12} = \frac{\cos^2\theta\sin^2\theta^\circ_{12} + \cos^2\theta^\circ_{12}\sin^2\theta\sin^2\theta^\circ_{23} - \frac{1}{2}\sin 2\theta\sin 2\theta^\circ_{12}\sin\theta^\circ_{23}\cos\phi}{1-\sin^2\theta_{13}}$$

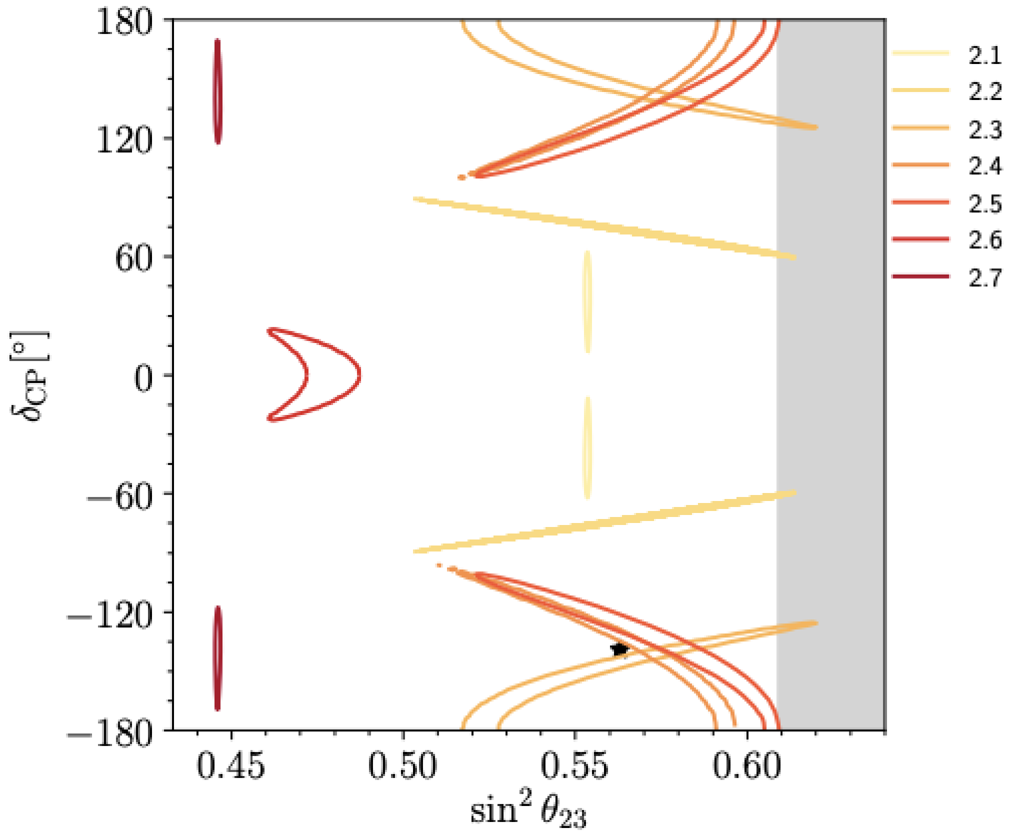
$$\cos\delta = \frac{\cos^2\theta_{13}(\cos^2\theta_{12} - \cos^2\theta^\circ_{12}\cos^2\theta^\circ_{23}) - \sin^2\theta^\circ_{23}(\cos^2\theta_{12} - \sin^2\theta_{12}\sin^2\theta_{13})}{\sin 2\theta_{12}\sin\theta_{13}|\sin\theta^\circ_{23}|(\cos^2\theta_{13} - \sin^2\theta^\circ_{23})^{\frac{1}{2}}}$$

G_e	G_ν	Case	$\sin^2\theta^\circ_{12}$	$\sin^2\theta^\circ_{13}$	$\sin^2\theta^\circ_{23}$
Z_2	Z_3	$A1A_5$	-	0.226	0.436
Z_2	Z_3	$A2A_5$	0.226	-	0.436

Girardi, Petcov, Stuart and Titov , Nucl. Phys. B902 (2016) 1

Model predictions

Model	Case	$\chi^2_{\min}$	$\theta_{\rm bf}$	$\theta_{3\sigma}$	$\phi_{\rm bf}$	$\phi_{3\sigma}$
2.1	A1 (A_5)	0.151	47.2°	(43.2°, 50.9°)	163.2°	(158.0°, 180°]
			132.8°	(129.1°, 136.8°)	16.8°	[0, 22.0°)
2.2	B2 (S_4)	0.386	54.4°	(49.3°, 59.7°)	149.7°	(148.0°, 154.3°)
			125.6°	(120.3°, 130.7°)	30.3°	(25.7°, 32.0°)
2.3	B2 (A_5)	2.49	51.3°	(48.2°, 56.0°)	161.4°	(150.4°, 180°]
			128.7°	(124.0°, 131.8°)	18.6°	[0, 29.6°)
2.4	B1 (A_5)	4.40	10.1°	(9.6°, 10.6°)	132.6°	(84.4°, 180°]
			169.9°	(169.4°, 170.4°)	47.4°	[0, 95.6°)
2.5	B1	5.67	10.5°	(10.0°, 11.0°)	126.4°	(85.1°, 180°]
			169.5°	(169.0°, 170.0°)	53.6°	[0, 94.9°)
2.6	B2 (A_5 II)	14.8	52.2°	(50.1°, 52.9°)	180°	(164.7°, 180°]
			127.8°	(127.1°, 129.9°)	0	[0, 15.3°)
2.7	A2 (A_5)	23.6	11.5°	(10.9°, 12.0°)	132.4°	(108.6°, 180°]
			168.5°	(168.0°, 169.1°)	47.6°	[0, 71.4°)

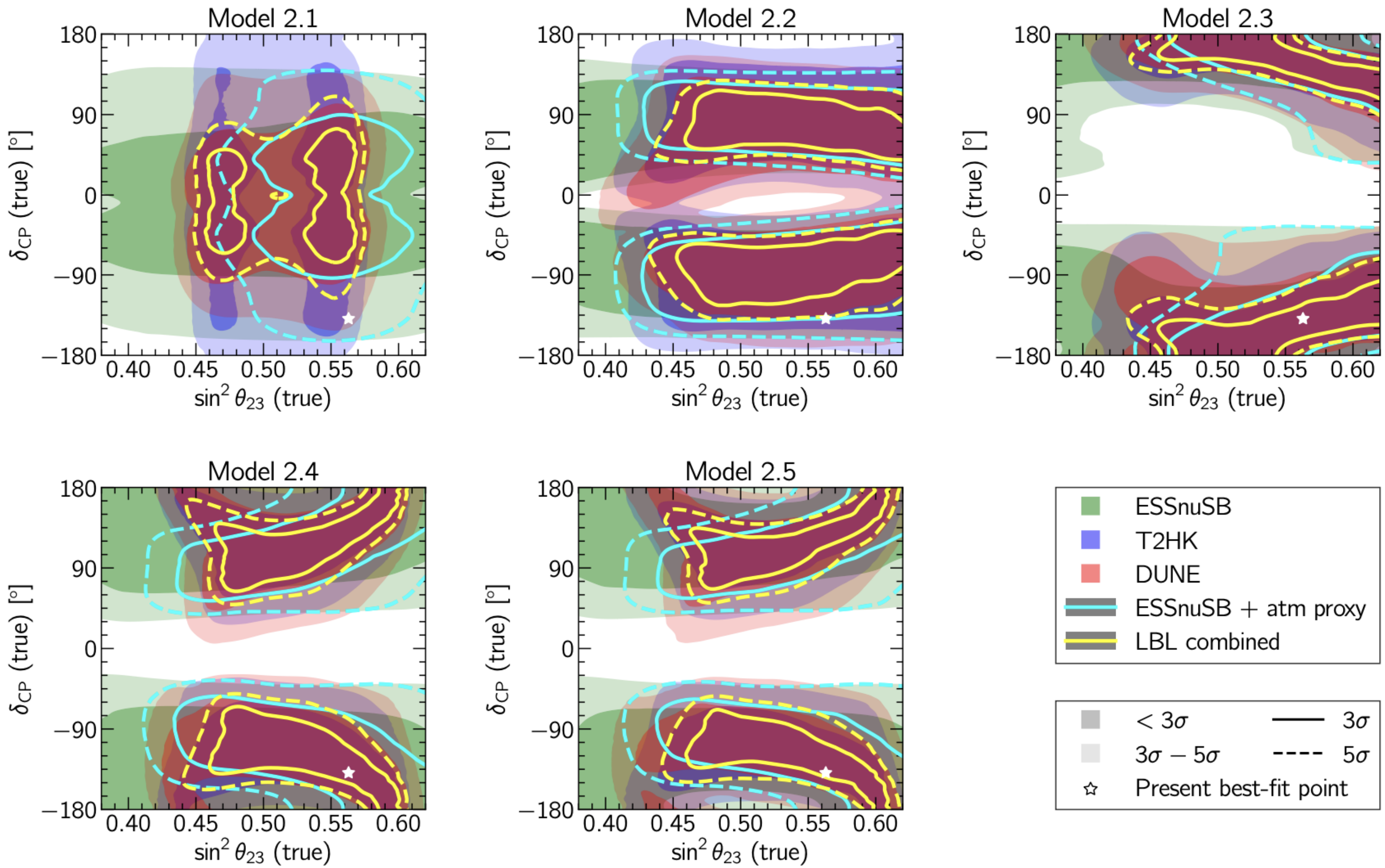


The Models 2.6 – 2.7 are excluded for further analysis

Blennow, MG, Ohlsson and Titov, JHEP **07** (2020), 014

Test in LBL

Difficult to exclude by LBL



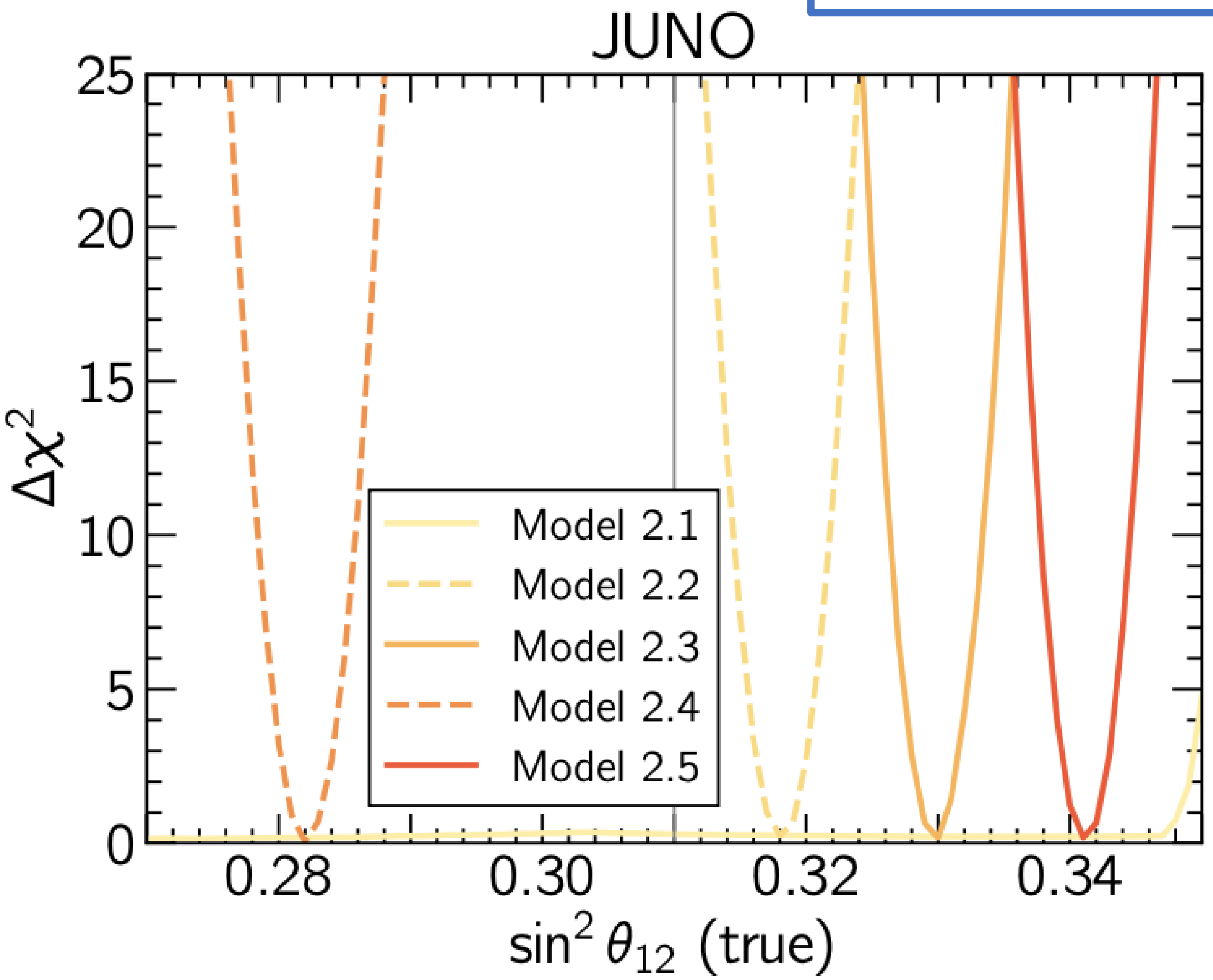
Model	$\sin^2\theta_{23}$	δ_{CP}°
2.1	0.554	{−41.9, 41.9}
2.2	0.563	{74.0, −74.0}
2.3	0.563	{145.2, −145.2}
2.4	0.563	{−132.1, 132.1}
2.5	0.563	{−125.7, 125.7}

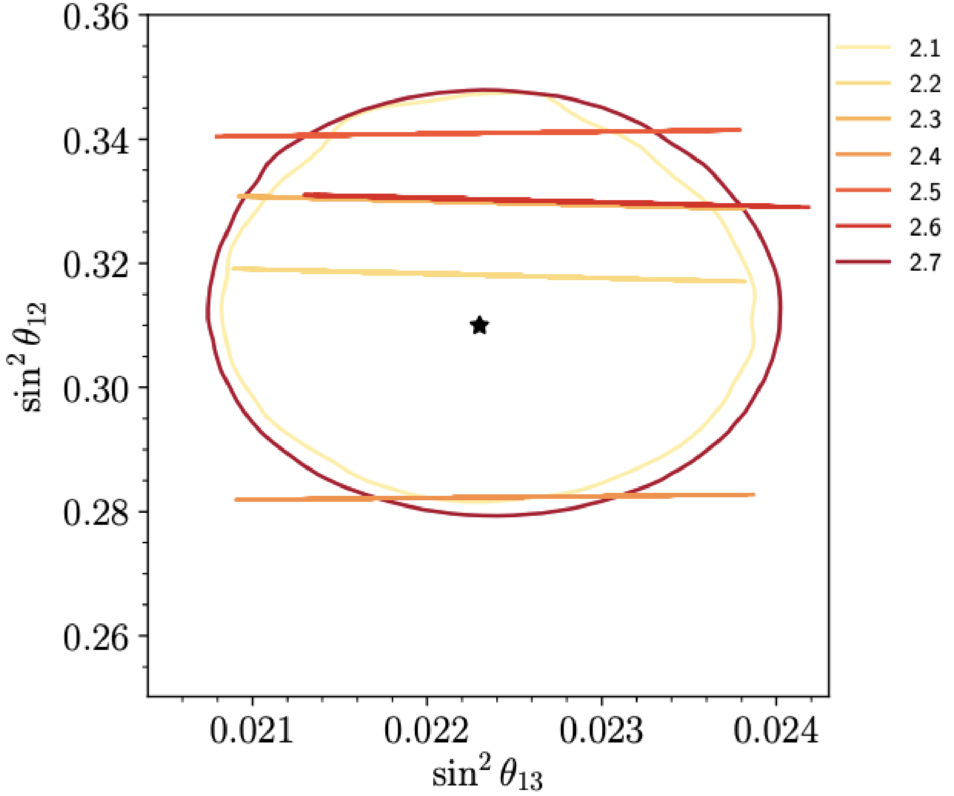
Blennow, MG, Ohlsson and Titov, Phys. Rev. D **102** (2020), 115004

Test in JUNO

Depending on the precision,
JUNO can exclude all the models
Except 2.1 and 2.2 (may be)

Model	$sin^2\theta_{12}$
2.1	0.310
2.2	0.318
2.3	0.330
2.4	0.283
2.5	0.341





Blennow, MG, Ohlsson and Titov, JHEP **07** (2020), 014, Blennow, MG, Ohlsson and Titov, Phys. Rev. D **102** (2020), 115004

Testing a modular symmetry theory

They need different treatment as generally they don't give a sum rule

Model A: With linear seesaw

Behera, S. Mishra, Singirala and Mohanta, Phys. Dark Univ. 36 (2022) 101027

	Fermions						Scalars		Yukawa couplings
Fields	e_R^c	μ_R^c	τ_R^c	$L_{\wp L}$	N_R	S_L^c	$H_{u,d}$	ρ_a	$\mathbf{Y} = (y_1, y_2, y_3)$
$SU(2)_L$	1	1	1	2	1	1	2	1	–
$U(1)_Y$	1	1	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}, -\frac{1}{2}$	0	–
A_4	1	$1'$	$1''$	$1, 1'', 1'$	3	3	1	1	3
k_I	1	1	1	-1	-1	-1	0	0	2

3 theories based on A_4 modular symmetry

Model B: With type I seesaw

F. Feruglio, 1706.08749

	Fermions					Scalars	Yukawa couplings
Fields	E_{1R}^c	E_{2R}^c	E_{3R}^c	L	N_R^c	$H_{u,d}$	$\mathbf{Y} = (y_1, y_2, y_3)$
$SU(2)_L$	1	1	1	2	1	2	–
$U(1)_Y$	1	1	1	$-\frac{1}{2}$	0	$\frac{1}{2}, -\frac{1}{2}$	–
A_4	1	$1''$	$1'$	3	3	1	3
k_I	-1	-1	-1	-1	-1	0	2

Model C: With type III seesaw

P. Mishra, Behera, Panda and Mohanta, Eur. Phys. J. C 82 (2022) 1115

	Fermions					Scalars		Yukawa couplings
Fields	E_{1R}^c	E_{2R}^c	E_{3R}^c	L	$\Sigma_{R_i}^c$	$H_{u,d}$	ρ_c	$\mathbf{Y} = (y_1, y_2, y_3)$
$SU(2)_L$	1	1	1	2	3	2	1	–
$U(1)_Y$	1	1	1	$-\frac{1}{2}$	0	$\frac{1}{2}, -\frac{1}{2}$	0	–
A_4	1	$1'$	$1''$	$1, 1'', 1'$	3	1	1	3
k_I	0	0	0	0	-2	0	2	2

Model predictions

$$\mathcal{W}_\nu = G_D L_{\wp_L} H_u\, (\mathbf{Y} N_R)_{1,1'',1'} + G'_D\, [L_{\wp_L} H_u\, (\mathbf{Y} S^c_L)_{1,1',1''}]\, \frac{\rho_a^3}{\Lambda_a^3}$$
$$+\ [\alpha_{NS} \mathbf{Y} (S^c_L N_R)_{\text{sym}} + \beta_{NS} \mathbf{Y} (S^c_L N_R)_{\text{Anti-sym}}]\rho_a$$

$$G_D = \text{diag}(\alpha_D,\ \beta_D,\ \gamma_D) \text{ and } G'_D = \text{diag}(\alpha'_D,\ \beta'_D,\ \gamma'_D)$$

$$\mathcal{W}_\nu = g(N^c_R H_u L Y)_1 + M_b(N^c_R N^c_R Y)_1$$

g term gives two invariant singlets $g_1$and g_2

$$\mathcal{W}_\nu = (\alpha_\Sigma)_{ij} \left[H_u \Sigma^c_{R_i} \sqrt{2} \mathbf{Y} L_j\right] + \frac{M'_\Sigma}{2} \left(\beta_\Sigma \mathrm{Tr}\left[\mathbf{\Sigma^c_{R_i}} \mathbf{Y} \mathbf{\Sigma^c_{R_i}}\right]_s + \gamma'_\Sigma \mathrm{Tr}\left[\mathbf{\Sigma^c_{R_i}} \mathbf{Y} \mathbf{\Sigma^c_{R_i}}\right]_a\right) \frac{\rho_c}{\Lambda_c}$$

$$\alpha_\Sigma = \text{diag}\ (a_1,\ a_2,\ a_3),\ \beta_\Sigma = \text{diag}\ (b_1,\ b_2,\ b_3) \text{ and } \gamma'_\Sigma = diag(\gamma_1, \gamma_2, \gamma_3)$$

Model A and C: predict normal mass ordering

Model B: Predicts inverted mass ordering

Parameter ranges for Model A

Parameter	Range	Parameter	range
$\mathrm{Re}[\tau]$	$[-0.5, 0.5]$	β'_D	$[6.0 - 9.9]\times10^{-3}$
$\mathrm{Im}[\tau]$	$[1.2, 1.8]$	γ'_D	$[3.2 - 8.3]\times10^{-3}$
α_D	$[6.7 - 9.7]\times10^{-6}$	α_{NS}	$[0.1 - 0.24]$
β_D	$[4.4 - 4.8]\times10^{-6}$	β_{NS}	$[1 - 2.2]\times10^{-5}$
γ_D	$[5.2 - 8.8]\times10^{-6}$	Λ_a	$[10^5, 10^6]$ GeV
α'_D	$[2.3 - 6.3]\times10^{-3}$	v_{ρ_a}	$[10^4, 10^5]$ GeV

Parameter ranges for Model B

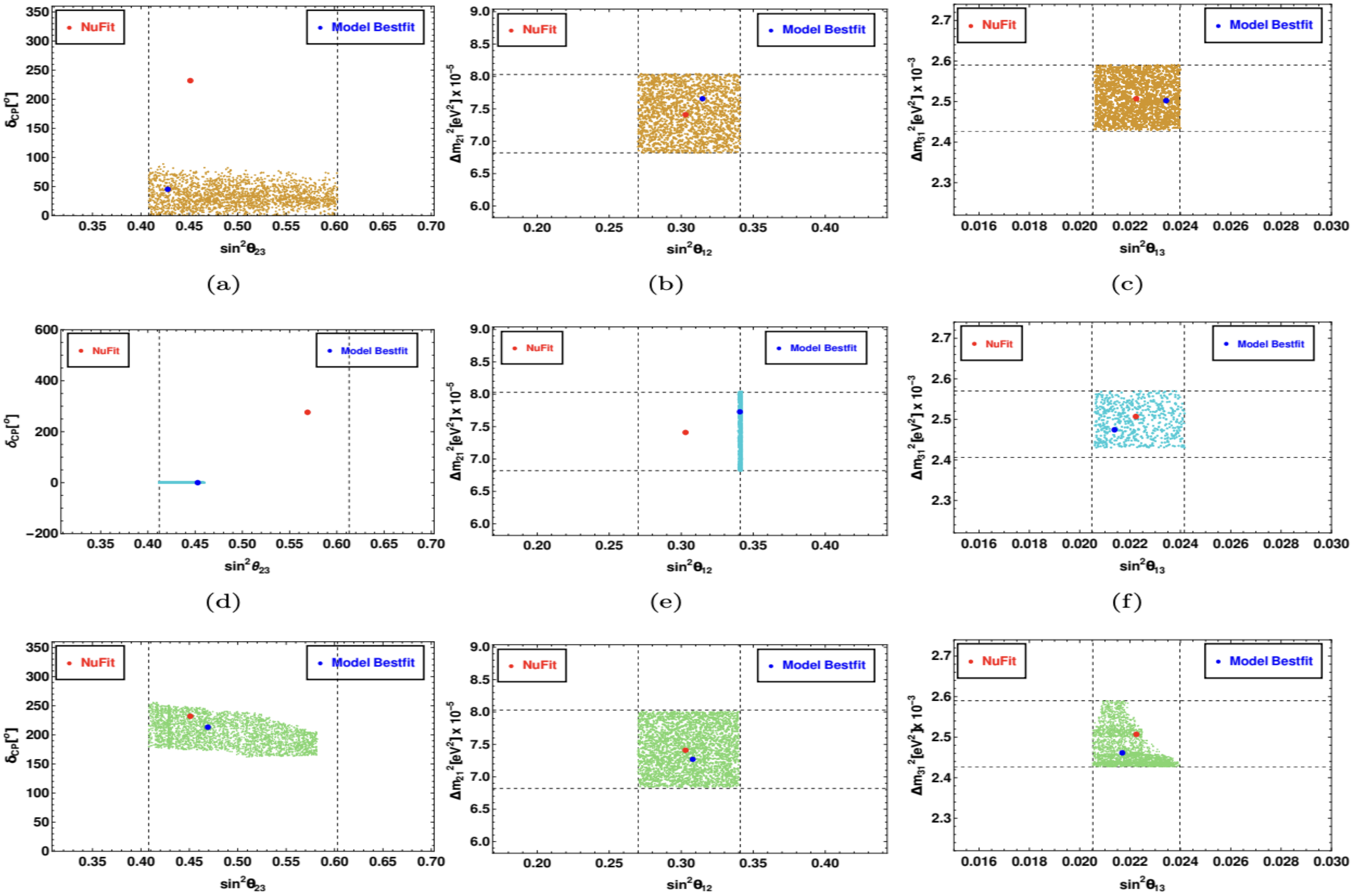
$\mathrm{Re}(\tau)$	$\mathrm{Im}(\tau)$	g_1	g_2	M_b [GeV]
$[-0.1, 0.1]$	$[1.5, 2.0]$	$[1, 3]\times10^{-5}$	$[2, 5]\times10^{-5}$	$[10^6, 10^7]$

Parameter ranges for Model C

Parameter	Range	Parameter	range
$\mathrm{Re}[\tau]$	$[-0.5, 0.5]$	b_2	$[0.8 - 8]\times10^{-1}$
$\mathrm{Im}[\tau]$	$[0.75, 2]$	b_3	$[1 - 7]\times10^{-3}$
a_1	$[5 - 10]\times10^{-7}$	γ_Σ	$[0.1, 1]\times10^{-9}$
a_2	$[4.5 - 10]\times10^{-6}$	M'_Σ	$[10^7, 10^8]$ GeV
a_3	$[0.5 - 5]\times10^{-7}$	Λ_c	$[10^7, 10^8]$ GeV
b_1	$[0.7 - 5]\times10^{-2}$	v_{ρ_c}	$[10^6, 10^7]$ GeV

Mishra, Behera, Panda, MG and Mohanta, JHEP **09** (2023), 144

Allowed parameter space



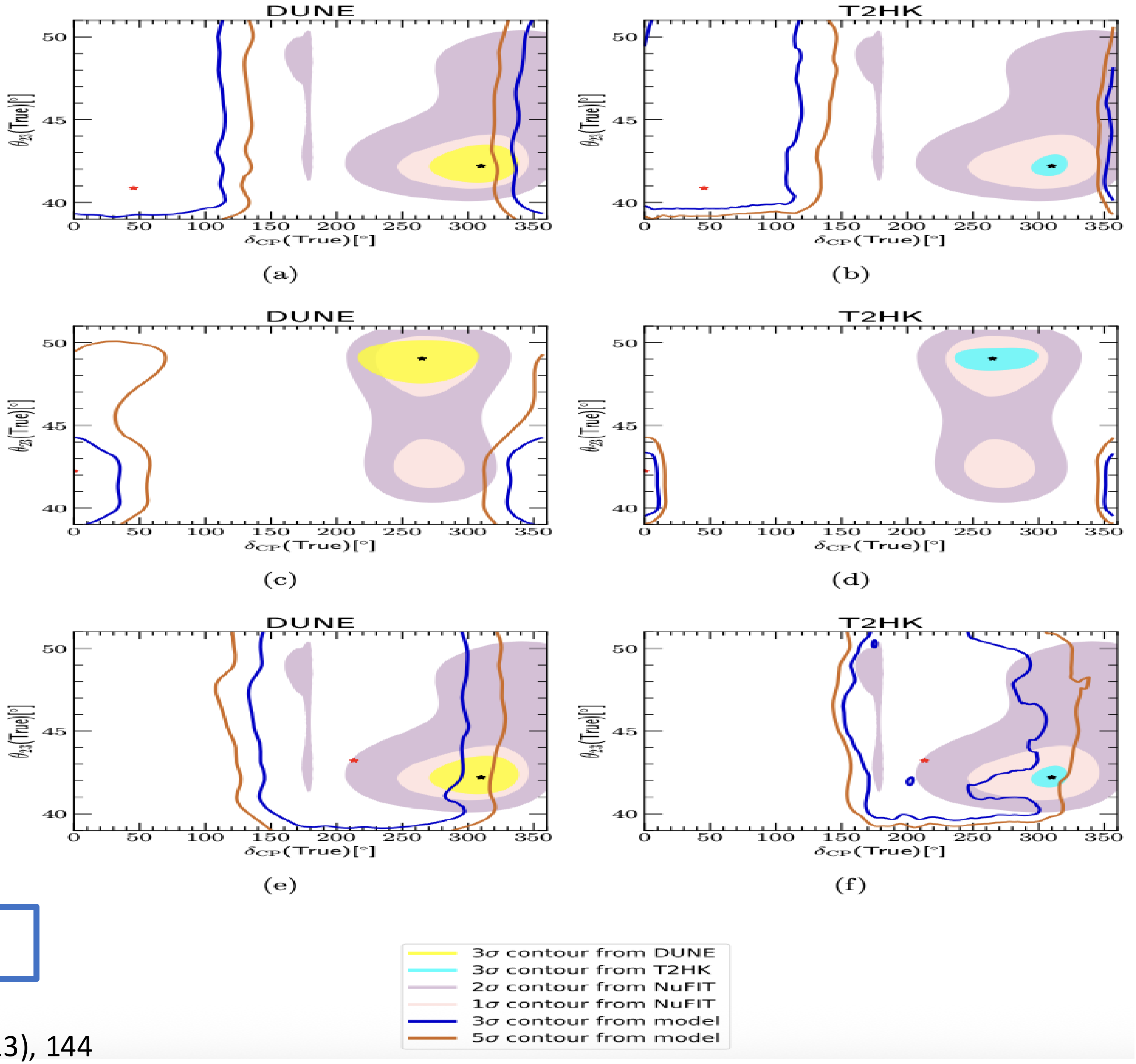
Model A

Model B

Model C

Tests in LBL

Model	$sin^2\theta_{23}$	δ_{CP}°
A	0.428	45.3
B	0.452	0
C	0.469	213.18

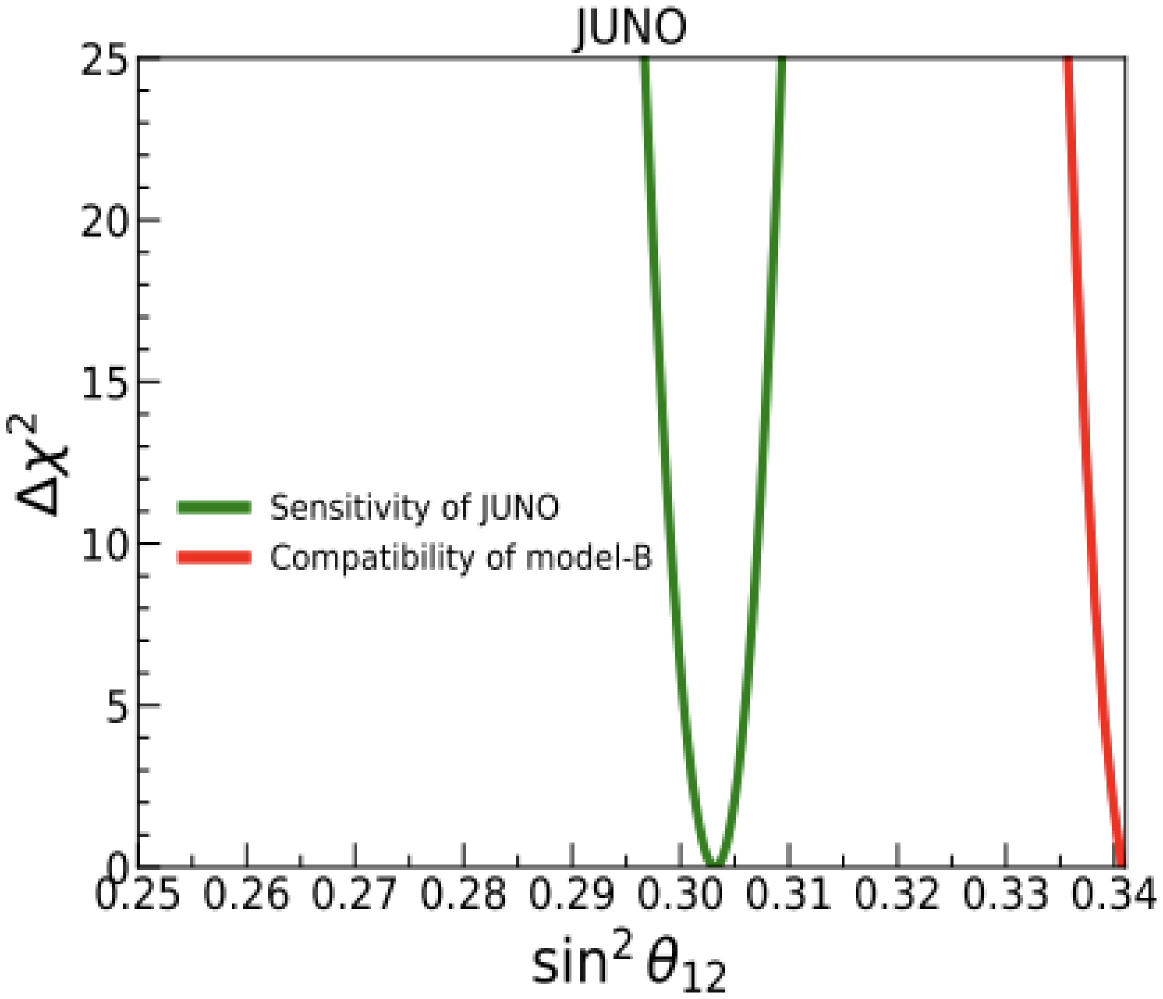


Models can be severely constrained

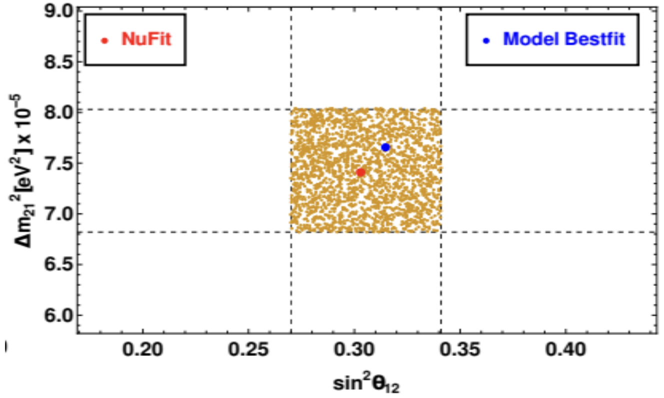
Mishra, Behera, Panda, MG and Mohanta, JHEP **09** (2023), 144

Test in JUNO

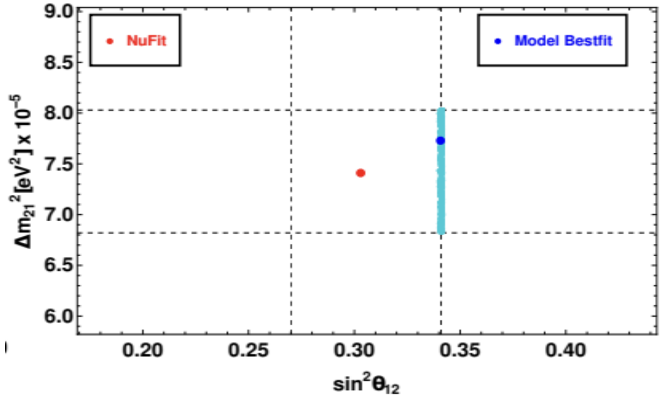
JUNO can exclude Model B



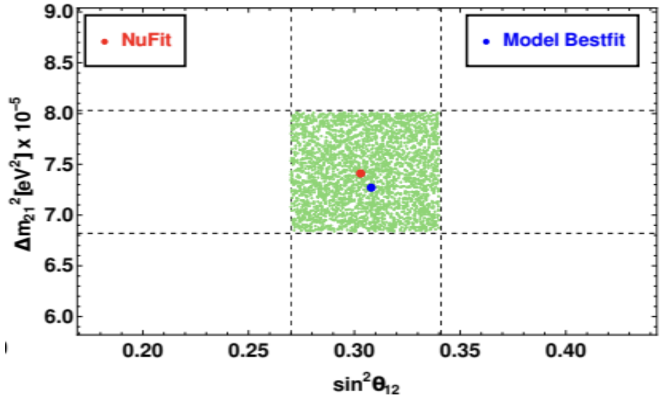
Model	$sin^2\theta_{12}$
A	0.315
B	0.340
C	0.308



(b)



(e)



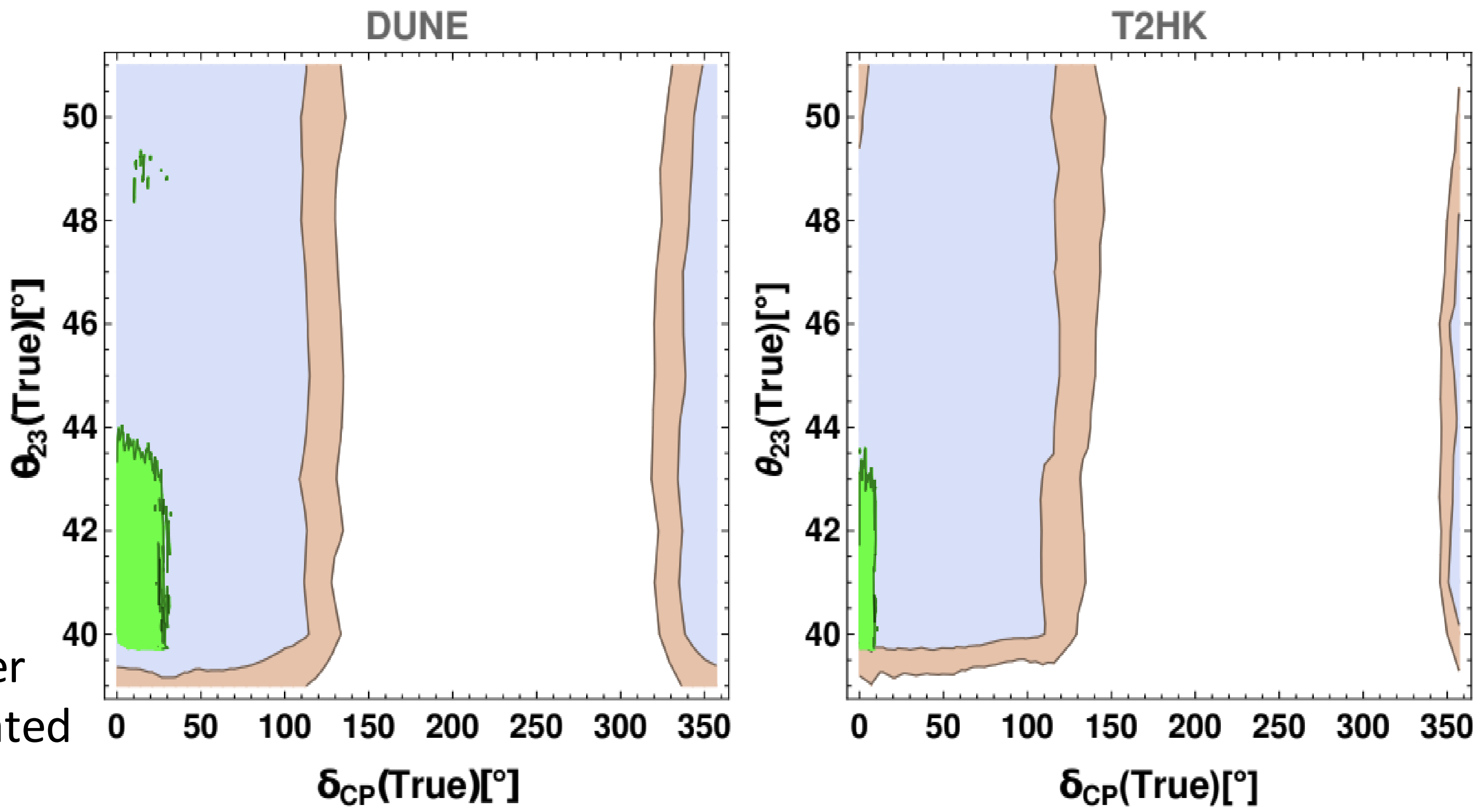
Mishra, Behera, Panda, MG and Mohanta, JHEP **09** (2023), 144

Further test: Seprating Model-A from Model-B

Model A has assumed to be true model and Model B is under test

Blue and brown regions are the allowed region for Model-A at 3σ C.L and 5 σ C.L.

Green region corresponds to the parameter space for which Model-B cannot be separated from Model-A at 3σ C.L



Mishra, Behera, Panda, MG and Mohanta, JHEP **09** (2023), 144

Summary

- Flavour/modular symmetry theories predict values of the mixing parameters which are allowed by the data
- Future experiments will measure the mixing parameters with sub percent precision
- Therefore, these models can be tested in these experiments
- ESSnuSB, T2HK, DUNE and JUNO will play a major role in validating these models.

Summary

- Flavour/modular symmetry theories predict values of the mixing parameters which are allowed by the data
- Future experiments will measure the mixing parameters with sub percent precision
- Therefore, these models can be tested in these experiments
- ESSnuSB, T2HK, DUNE and JUNO will play a major role in validating these models.

Thank You