# Revisiting a Flavor Model with Dark Matter and Leptogenesis

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References: JHEP 09 (2024) 036 and arXiv:2505.19924 (submitting to PTEP)

11th Workshop FLASY2025@Rome

# Motivation

- **Standard Model (SM)** is incomplete: no explanation for **neutrino masses**, **dark matter (DM)**, or **baryon asymmetry of the universe (BAU)**.
- **$A_4$ flavor symmetry** explains **lepton mixing angles** and **mass hierarchies**.
  G. Altarelli and F. Feruglio, Nucl. Phys. B 720 (2005); Nucl. Phys. B 741 (2006).
  Y. S., M. Tanimoto and A. Watanabe, Prog. Theor. Phys. 126 (2011), 81-90.
  T. Morozumi, H. Okane, H. Sakamoto, Y. S., K. Takagi and H. Umeeda, Chin. Phys.C 42 (2018) no.2, 023102.
- SUSY introduces new **dark matter** candidates: **flavino.** T. Nomura, Y. S., and T. Takahashi, JHEP 09 (2024) 036.
- **Leptogenesis** explains **BAU** via the right-handed neutrino decays. M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986), 45-47.

# Model Overview

- Symmetry: $SU(2)_L \times A_4 \times Z_3 \times U(1)_R$

| | $\Phi_\ell$ | $\Phi_{e_R^c}$ | $\Phi_{\mu_R^c}$ | $\Phi_{\tau_R^c}$ | $\Phi_N$ | $\Phi_{u,d}$ | $\Phi_T$ | $\Phi_S$ | $\Phi_\xi$ | $\Phi_{\xi'}$ | $\Phi_0^T$ | $\Phi_0^S$ | $\Phi_0^\xi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $SU(2)_L$ | 2 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $A_4$ | **3** | **1** | **1''** | **1'** | **3** | **1** | **3** | **3** | **1** | **1'** | **3** | **3** | **1** |
| $Z_3$ | $\omega$ | $\omega^2$ | $\omega^2$ | $\omega^2$ | $\omega^2$ | 1 | 1 | $\omega^2$ | $\omega^2$ | $\omega^2$ | 1 | $\omega^2$ | $\omega^2$ |
| $U(1)_R$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 |

- Chiral superfields: $\Phi_i = \phi_i + \sqrt{2}\theta\psi_i + \theta\theta F_i$
- Superpotential:

$$
\begin{aligned}
w &= w_Y + w_d,\\
w_Y &= w_\ell + w_D + w_N,\\
w_\ell &= y_e \Phi_T \Phi_\ell \Phi_{e_R^c} \Phi_d/\Lambda + y_\mu \Phi_T \Phi_\ell \Phi_{\mu_R^c} \Phi_d/\Lambda + y_\tau \Phi_T \Phi_\ell \Phi_{\tau_R^c} \Phi_d/\Lambda,\\
w_D &= y_D \Phi_\ell \Phi_N \Phi_u,\\
w_N &= y_{\Phi_S} \Phi_N \Phi_N \Phi_S + y_\xi \Phi_N \Phi_N \Phi_\xi + y_{\Phi_{\xi'}} \Phi_N \Phi_N \Phi_{\xi'},\\
w_d &= w_d^T + w_d^S,\\
w_d^T &= -M \Phi_0^T \Phi_T + g \Phi_0^T \Phi_T \Phi_T,\\
w_d^S &= g_1 \Phi_0^S \Phi_S \Phi_S + g_2 \Phi_0^S \Phi_S \Phi_\xi + g_2' \Phi_0^S \Phi_S \Phi_{\xi'} + g_3 \Phi_0^\xi \Phi_S \Phi_S - g_4 \Phi_0^\xi \Phi_\xi \Phi_\xi
\end{aligned}
$$

# Model Overview

- Lagrangian: $\mathcal{L} = \mathcal{L}_Y + \mathcal{L}_d - V$

$$\mathcal{L}_Y = \int d^2\theta w_Y + \int d^2\bar{\theta}\bar{w}_Y, \;\; \mathcal{L}_d = \int d^2\theta w_d + \int d^2\bar{\theta}\bar{w}_d, \;\; V = V_Y + V_d.$$

- Scalar potential: $V_d = V_T + V_S$

$$V_T = \sum_X \left| \frac{\partial w_d^T}{\partial X} \right|^2 \qquad V_S = \sum_Y \left| \frac{\partial w_d^S}{\partial Y} \right|^2$$

# Model Overview

- Scalar potential: $V_T = \sum_X \left|\frac{\partial w_d^T}{\partial X}\right|^2$ $\left(X = \phi_{T1}, \phi_{T2}, \phi_{T3}, \phi_{01}^T, \phi_{02}^T, \text{ and } \phi_{03}^T\right)$

$$
\begin{aligned}
&= \left|-M\phi_{01}^T + \frac{2}{3}g\left(2\phi_{01}^T\phi_{T1} - \phi_{02}^T\phi_{T3} - \phi_{03}^T\phi_{T2}\right)\right|^2 \\
&+ \left|-M\phi_{03}^T + \frac{2}{3}g\left(-\phi_{01}^T\phi_{T3} + 2\phi_{02}^T\phi_{T2} - \phi_{03}^T\phi_{T1}\right)\right|^2 \\
&+ \left|-M\phi_{02}^T + \frac{2}{3}g\left(-\phi_{01}^T\phi_{T2} - \phi_{02}^T\phi_{T1} + 2\phi_{03}^T\phi_{T3}\right)\right|^2 \\
&+ \left|-M\phi_{T1} + \frac{2}{3}g\left(\phi_{T1}\phi_{T1} - \phi_{T2}\phi_{T3}\right)\right|^2 \\
&+ \left|-M\phi_{T3} + \frac{2}{3}g\left(\phi_{T2}\phi_{T2} - \phi_{T3}\phi_{T1}\right)\right|^2 \\
&+ \left|-M\phi_{T2} + \frac{2}{3}g\left(\phi_{T3}\phi_{T3} - \phi_{T1}\phi_{T2}\right)\right|^2,
\end{aligned}
$$

# Model Overview

- Scalar potential: $V_S = \sum_Y \left|\frac{\partial w_d^S}{\partial Y}\right|^2$ $\left(Y = \phi_{S1}, \phi_{S2}, \phi_{S3}, \phi_{01}^S, \phi_{02}^S, \phi_{03}^S, \phi_\xi, \phi_{\xi'}, \text{ and } \phi_0^\xi\right)$

$$
\begin{aligned}
&= \left|\frac{2}{3}g_1\left(2\phi_{01}^S\phi_{S1} - \phi_{02}^S\phi_{S3} - \phi_{03}^S\phi_{S2}\right) + g_2\phi_{01}^S\phi_\xi + g_2'\phi_{03}^S\phi_{\xi'} + 2g_3\phi_0^\xi\phi_{S1}\right|^2 \\
&+ \left|\frac{2}{3}g_1\left(-\phi_{01}^S\phi_{S3} + 2\phi_{02}^S\phi_{S2} - \phi_{03}^S\phi_{S1}\right) + g_2\phi_{03}^S\phi_\xi + g_2'\phi_{02}^S\phi_{\xi'} + 2g_3\phi_0^\xi\phi_{S3}\right|^2 \\
&+ \left|\frac{2}{3}g_1\left(-\phi_{01}^S\phi_{S2} - \phi_{02}^S\phi_{S1} + 2\phi_{03}^S\phi_{S3}\right) + g_2\phi_{02}^S\phi_\xi + g_2'\phi_{01}^S\phi_{\xi'} + 2g_3\phi_0^\xi\phi_{S2}\right|^2 \\
&+ \left|\frac{2}{3}g_1\left(\phi_{S1}\phi_{S1} - \phi_{S2}\phi_{S3}\right) + g_2\phi_{S1}\phi_\xi + g_2'\phi_{S3}\phi_{\xi'}\right|^2 \\
&+ \left|\frac{2}{3}g_1\left(\phi_{S2}\phi_{S2} - \phi_{S3}\phi_{S1}\right) + g_2\phi_{S3}\phi_\xi + g_2'\phi_{S2}\phi_{\xi'}\right|^2 \\
&+ \left|\frac{2}{3}g_1\left(\phi_{S3}\phi_{S3} - \phi_{S1}\phi_{S2}\right) + g_2\phi_{S2}\phi_\xi + g_2'\phi_{S1}\phi_{\xi'}\right|^2 \\
&+ \left|g_2\left(\phi_{01}^S\phi_{S1} + \phi_{02}^S\phi_{S3} + \phi_{03}^S\phi_{S2}\right) - 2g_4\phi_0^\xi\phi_\xi\right|^2 \\
&+ \left|g_2'\left(\phi_{02}^S\phi_{S2} + \phi_{03}^S\phi_{S1} + \phi_{01}^S\phi_{S3}\right)\right|^2 \\
&+ \left|g_3\left(\phi_{S1}\phi_{S1} + 2\phi_{S2}\phi_{S3}\right) - g_4\phi_\xi\phi_\xi\right|^2 .
\end{aligned}
$$

# Model Overview

- Potential minimal conditions:

$V_T = 0$

$$\langle \phi_T \rangle = v_T(1,0,0), \qquad v_T = \frac{3M}{2g}, \qquad \langle \phi_0^T \rangle = (0,0,0).$$

$V_S = 0$

$$\langle \phi_S \rangle = v_S(1,1,1), \quad v_S = \sqrt{\frac{g_4}{3g_3}}u, \quad \langle \phi_0^S \rangle = (0,0,0), \quad \langle \phi_{\xi'} \rangle = u' = \frac{g_2}{g_2'}u, \quad \langle \phi_\xi \rangle = u, \quad \langle \phi_0^\xi \rangle = 0.$$

# Model Overview

- F-term contribution:

$$
\begin{aligned}
\int d^2\theta w_{\Phi_T} =\; & y_e(\phi_{T1}\ell_1 + \phi_{T2}\ell_3 + \phi_{T3}\ell_2)e^c_R h_d/\Lambda \\
& + y_\mu(\phi_{T3}\ell_3 + \phi_{T1}\ell_2 + \phi_{T2}\ell_1)\mu^c_R h_d/\Lambda \\
& + y_\tau(\phi_{T2}\ell_2 + \phi_{T3}\ell_1 + \phi_{T1}\ell_3)\tau^c_R h_d/\Lambda \\
& + y_e(\psi_{\phi_{T1}}\phi_{\ell_1} + \psi_{\phi_{T2}}\phi_{\ell_3} + \psi_{\phi_{T3}}\phi_{\ell_2})e^c_R h_d/\Lambda \\
& + y_\mu(\bar\psi_{\phi_{T3}}\bar\phi_{\ell_3} + \bar\psi_{\phi_{T1}}\bar\phi_{\ell_2} + \bar\psi_{\phi_{T2}}\bar\phi_{\ell_1})\mu^c_R h_d/\Lambda \\
& + y_\tau(\bar\psi_{\phi_{T2}}\bar\phi_{\ell_2} + \bar\psi_{\phi_{T3}}\bar\phi_{\ell_1} + \bar\psi_{\phi_{T1}}\bar\phi_{\ell_3})\tau^c_R h_d/\Lambda \\
& + y_e(\bar\psi_{\phi_{T1}}\ell_1 + \psi_{\phi_{T2}}\ell_3 + \psi_{\phi_{T3}}\ell_2)\bar\phi_{e^c_R} h_d/\Lambda \\
& + y_\mu(\bar\psi_{\phi_{T3}}\ell_3 + \psi_{\phi_{T1}}\ell_2 + \psi_{\phi_{T2}}\ell_1)\bar\phi_{\mu^c_R} h_d/\Lambda \\
& + y_\tau(\bar\psi_{\phi_{T2}}\ell_2 + \psi_{\phi_{T3}}\ell_1 + \psi_{\phi_{T1}}\ell_3)\bar\phi_{\tau^c_R} h_d/\Lambda \\
& + y_e(\bar\psi_{\phi T1}\bar\phi_{\ell_1} + \bar\psi_{\phi T2}\bar\phi_{\ell_3} + \bar\psi_{\phi T3}\bar\phi_{\ell_2})\bar\phi_{e^c_R}\bar\psi_{h_d}/\Lambda \\
& + y_\mu(\bar\psi_{\phi T3}\bar\phi_{\ell_3} + \bar\psi_{\phi T1}\bar\phi_{\ell_2} + \bar\psi_{\phi T2}\bar\phi_{\ell_1})\bar\phi_{\mu^c_R}\bar\psi_{h_d}/\Lambda \\
& + y_\tau(\bar\psi_{\phi T2}\bar\phi_{\ell_2} + \bar\psi_{\phi T3}\bar\phi_{\ell_1} + \bar\psi_{\phi T1}\bar\phi_{\ell_3})\bar\phi_{\tau^c_R}\bar\psi_{h_d}/\Lambda \\
& + y_e(\phi_{T1}\ell_1 + \phi_{T2}\ell_3 + \phi_{T3}\ell_2)\bar\phi_{e^c_R}\bar\psi_{h_d}/\Lambda \\
& + y_\mu(\phi_{T3}\ell_3 + \phi_{T1}\ell_2 + \phi_{T2}\ell_1)\bar\phi_{\mu^c_R}\bar\psi_{h_d}/\Lambda \\
& + y_\tau(\phi_{T2}\ell_2 + \phi_{T3}\ell_1 + \phi_{T1}\ell_3)\bar\phi_{\tau^c_R}\bar\psi_{h_d}/\Lambda \\
& + y_e(\phi_{T1}\bar\phi_{\ell_1} + \phi_{T2}\bar\phi_{\ell_3} + \phi_{T3}\bar\phi_{\ell_2})e^c_R\bar\psi_{h_d}/\Lambda \\
& + y_\mu(\phi_{T3}\bar\phi_{\ell_3} + \phi_{T1}\bar\phi_{\ell_2} + \phi_{T2}\bar\phi_{\ell_1})\mu^c_R\bar\psi_{h_d}/\Lambda \\
& + y_\tau(\phi_{T2}\bar\phi_{\ell_2} + \phi_{T3}\bar\phi_{\ell_1} + \phi_{T1}\bar\phi_{\ell_3})\tau^c_R\bar\psi_{h_d}/\Lambda \\
& - M(\bar\psi_{\phi^T_{01}}\bar\psi_{\phi_{T1}} + \bar\psi_{\phi^T_{02}}\bar\psi_{\phi_{T3}} + \bar\psi_{\phi^T_{03}}\bar\psi_{\phi_{T2}}) \\
& + \frac{2}{3}g\Big[\phi^T_{01}(\bar\psi_{\phi_{T1}}\bar\psi_{\phi_{T1}} - \bar\psi_{\Phi_{T2}}\bar\psi_{\phi_{T3}}) + \phi^T_{02}(\bar\psi_{\phi_{T2}}\bar\psi_{\phi_{T2}} - \bar\psi_{\phi_{T3}}\bar\psi_{\phi_{T1}}) \\
& + \phi^T_{03}(\bar\psi_{\phi_{T3}}\bar\psi_{\phi_{T3}} - \bar\psi_{\phi_{T1}}\bar\psi_{\phi_{T2}}) \\
& + \bar\psi_{\phi^T_{01}}(\phi T1\bar\psi_{\phi_{T1}} - \phi T2\bar\psi_{\phi_{T3}}) + \bar\psi_{\phi^T_{02}}(\phi T2\bar\psi_{\phi_{T2}} - \phi T3\bar\psi_{\phi_{T1}}) \\
& + \bar\psi_{\phi^T_{03}}(\phi T3\bar\psi_{\phi_{T3}} - \phi T1\bar\psi_{\phi_{T2}}) + \bar\psi_{\phi^T_{01}}(\bar\psi_{\phi_{T1}}\phi T1 - \bar\psi_{\phi_{T2}}\phi T3) \\
& + \bar\psi_{\phi^T_{02}}(\bar\psi_{\phi_{T2}}\phi T2 - \bar\psi_{\phi_{T3}}\phi T1) + \bar\psi_{\phi^T_{03}}(\bar\psi_{\phi_{T3}}\phi T3 - \bar\psi_{\phi_{T1}}\phi T2)\Big],
\end{aligned}
$$

# Model Overview

- Charged lepton mass matrix:

$$M_\ell = \frac{v_d v_T}{\Lambda}\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}.$$

- Dirac neutrino mass matrix:

$$M_D = y_D v_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

# Model Overview

- Right-handed Majorana neutrino mass matrix:

$$M_R = \frac{1}{3} y_{\phi_S} v_S \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + y_\xi u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + y_{\xi'} u' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} .$$

- Left-handed Majorana neutrino mass matrix:

$$M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

※a, b, c, and d are combinations for Yukawa couplings and VEVs.

Y. S., M. Tanimoto and A. Watanabe, Prog. Theor. Phys. 126 (2011), 81-90.

# Numerical analysis

NuFIT 6.0 (2024)

| IC24 with SK atmospheric data | | Normal Ordering (best fit) | | Inverted Ordering ($\Delta\chi^2 = 6.1$) | |
|---|---|---|---|---|---|
| | | bfp $\pm 1\sigma$ | $3\sigma$ range | bfp $\pm 1\sigma$ | $3\sigma$ range |
| | $\sin^2\theta_{12}$ | $0.308^{+0.012}_{-0.011}$ | $0.275 \to 0.345$ | $0.308^{+0.012}_{-0.011}$ | $0.275 \to 0.345$ |
| | $\theta_{12}/^\circ$ | $33.68^{+0.73}_{-0.70}$ | $31.63 \to 35.95$ | $33.68^{+0.73}_{-0.70}$ | $31.63 \to 35.95$ |
| | $\sin^2\theta_{23}$ | $0.470^{+0.017}_{-0.013}$ | $0.435 \to 0.585$ | $0.550^{+0.012}_{-0.015}$ | $0.440 \to 0.584$ |
| | $\theta_{23}/^\circ$ | $43.3^{+1.0}_{-0.8}$ | $41.3 \to 49.9$ | $47.9^{+0.7}_{-0.9}$ | $41.5 \to 49.8$ |
| | $\sin^2\theta_{13}$ | $0.02215^{+0.00056}_{-0.00058}$ | $0.02030 \to 0.02388$ | $0.02231^{+0.00056}_{-0.00056}$ | $0.02060 \to 0.02409$ |
| | $\theta_{13}/^\circ$ | $8.56^{+0.11}_{-0.11}$ | $8.19 \to 8.89$ | $8.59^{+0.11}_{-0.11}$ | $8.25 \to 8.93$ |
| | $\delta_{\rm CP}/^\circ$ | $212^{+26}_{-41}$ | $124 \to 364$ | $274^{+22}_{-25}$ | $201 \to 335$ |
| | $\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$ | $7.49^{+0.19}_{-0.19}$ | $6.92 \to 8.05$ | $7.49^{+0.19}_{-0.19}$ | $6.92 \to 8.05$ |
| | $\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$ | $+2.513^{+0.021}_{-0.019}$ | $+2.451 \to +2.578$ | $-2.484^{+0.020}_{-0.020}$ | $-2.547 \to -2.421$ |

# Numerical analysis

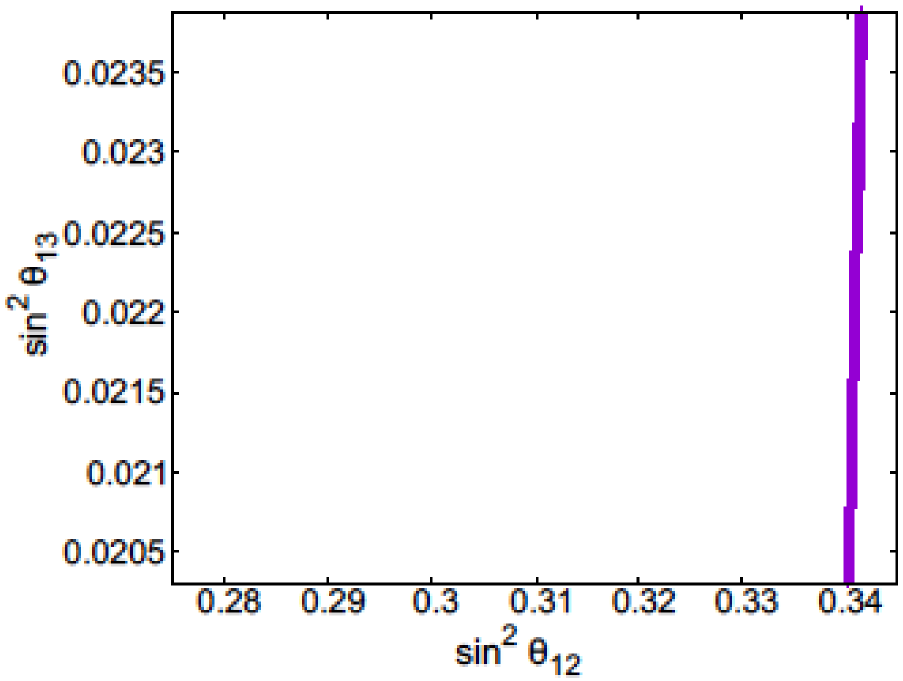



Figure 1: The allowed regions of the lepton mixing anlges and Dirac CP phase. Horizontal axis:$\sin\theta_{12}$, Vertical axis:$\sin\theta_{13}$.

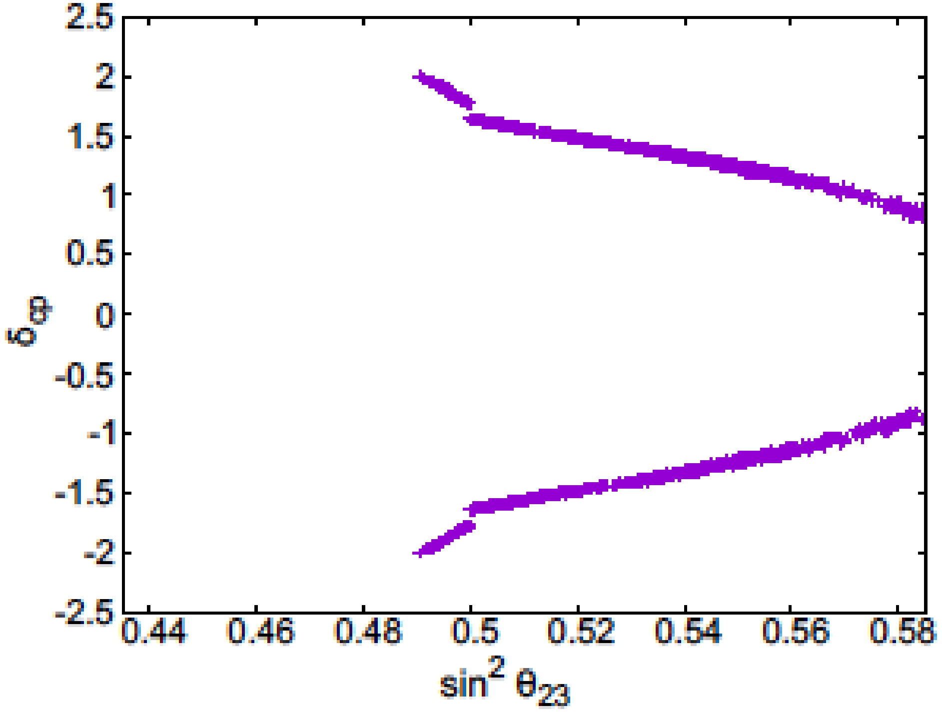
# Numerical analysis

- Hereafter, we consider $y_{\xi'}$ is only complex parameter in the right-handed Majorana neutrino mass matrix.

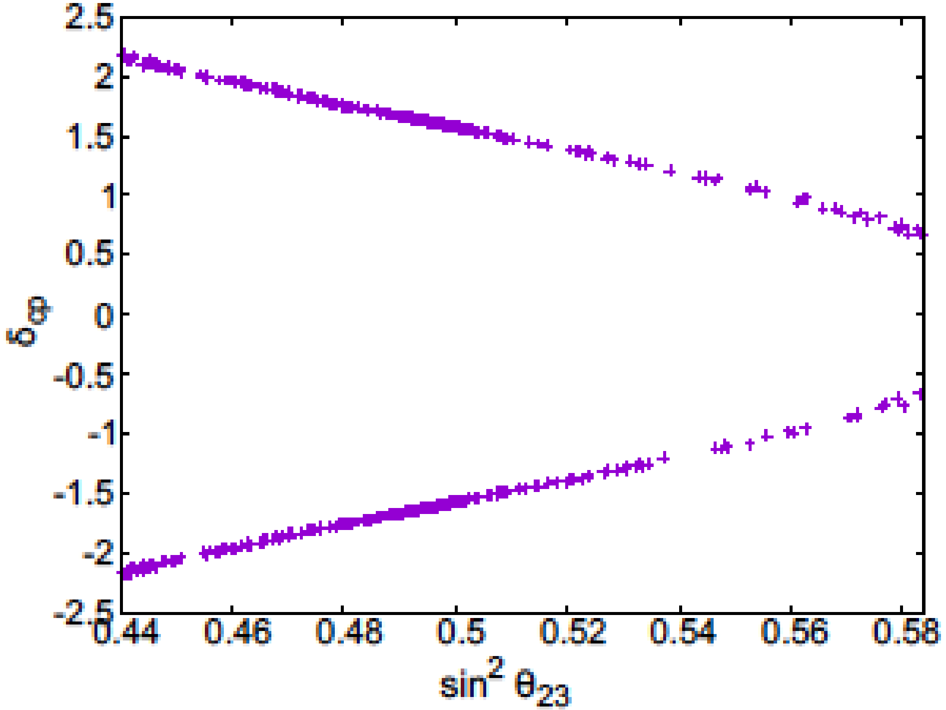
- Right-handed Majorana neutrino mass matrix:

$$M_R = \frac{1}{3} y_{\phi_S} v_S \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + y_\xi u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + y_{\xi'} u' \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} .$$

# Numerical analysis



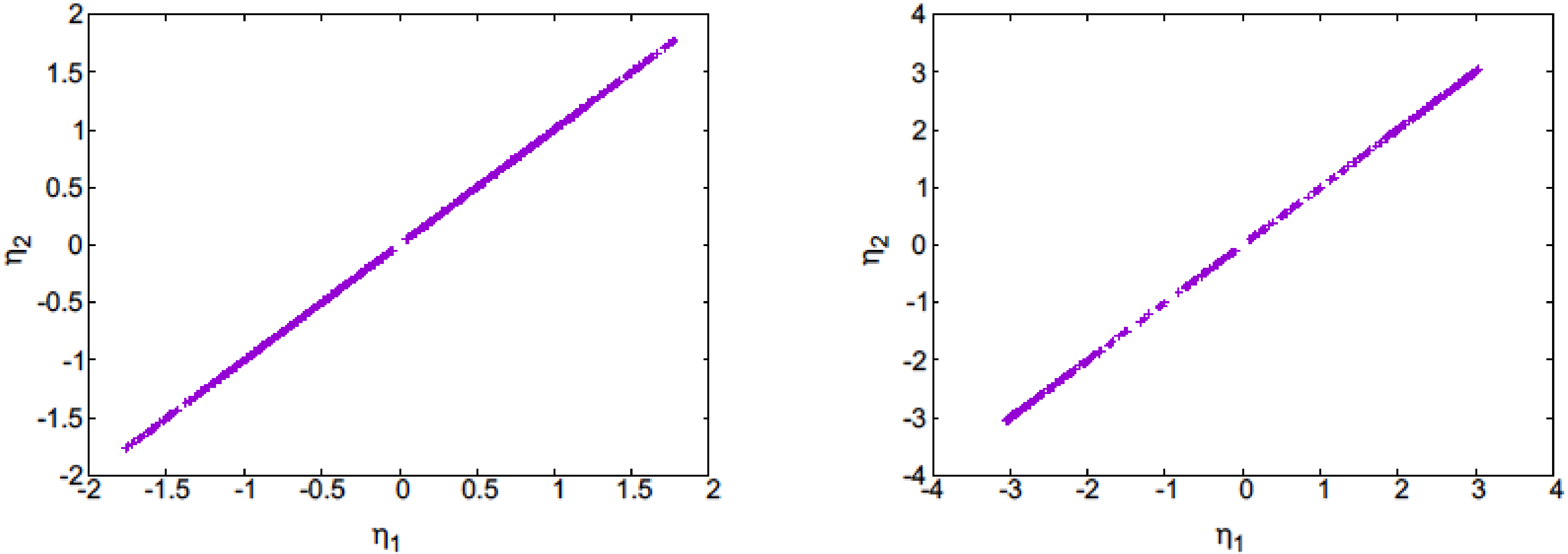

(a) The $\sin \theta_{23}$-$\delta_{CP}$ in the NH case.




(b) The $\sin \theta_{23}$-$\delta_{CP}$ in the IH case.

Figure 2: The allowed regions of the lepton mixing anlge and Dirac CP phase.

# Numerical analysis

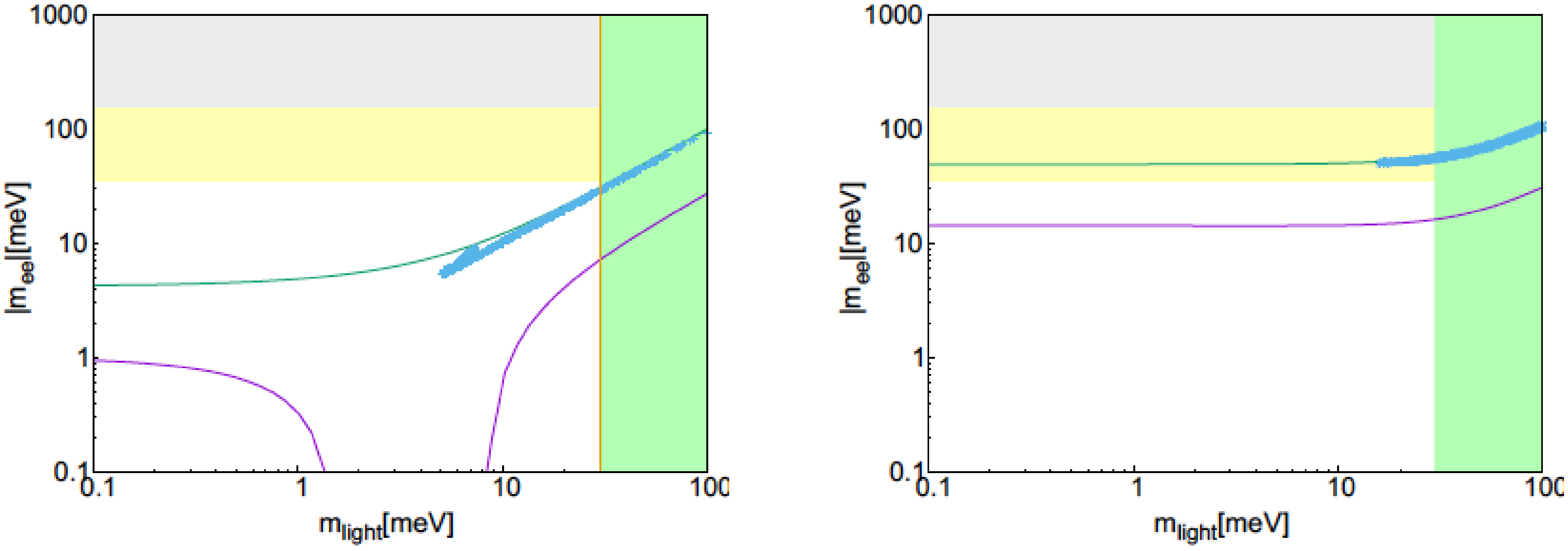



(a) The $\eta_1$-$\eta_2$ plane in the NH.

(b) The $\eta_1$-$\eta_2$ plane in the IH.

Figure 3: Majorana phases.

# Numerical analysis




(a) The $m_{\text{light}}$-$|m_{ee}|$ plane in the NH.

(b) The $m_{\text{light}}$-$|m_{ee}|$ plane in the IH.

Figure 4: The effective mass for the $0\nu\beta\beta$ decay.

# Dark Matter

- F-term contribution:

$$\int d^2\theta w_{\Phi_T} = y_e(\phi_{T1}\ell_1 + \phi_{T2}\ell_3 + \phi_{T3}\ell_2)e_R^c h_d/\Lambda$$
$$+ y_\mu(\phi_{T3}\ell_3 + \phi_{T1}\ell_2 + \phi_{T2}\ell_1)\mu_R^c h_d/\Lambda$$
$$+ y_\tau(\phi_{T2}\ell_2 + \phi_{T3}\ell_1 + \phi_{T1}\ell_3)\tau_R^c h_d/\Lambda$$
$$+ y_e(\bar{\psi}_{\phi_{T1}}\bar{\phi}_{\ell_1} + \bar{\psi}_{\phi_{T2}}\bar{\phi}_{\ell_3} + \bar{\psi}_{\phi_{T3}}\bar{\phi}_{\ell_2})e_R^c h_d/\Lambda$$
$$+ y_\mu(\bar{\psi}_{\phi_{T3}}\bar{\phi}_{\ell_3} + \bar{\psi}_{\phi_{T1}}\bar{\phi}_{\ell_2} + \bar{\psi}_{\phi_{T2}}\bar{\phi}_{\ell_1})\mu_R^c h_d/\Lambda$$
$$+ y_\tau(\bar{\psi}_{\phi_{T2}}\bar{\phi}_{\ell_2} + \bar{\psi}_{\phi_{T3}}\bar{\phi}_{\ell_1} + \bar{\psi}_{\phi_{T1}}\bar{\phi}_{\ell_3})\tau_R^c h_d/\Lambda$$
$$+ y_e(\bar{\psi}_{\phi_{T1}}\ell_1 + \psi_{\phi_{T2}}\ell_3 + \psi_{\phi_{T3}}\ell_2)\bar{\phi}_{e_R^c} h_d/\Lambda$$
$$+ y_\mu(\bar{\psi}_{\phi_{T3}}\ell_3 + \psi_{\phi_{T1}}\ell_2 + \psi_{\phi_{T2}}\ell_1)\bar{\phi}_{\mu_R^c} h_d/\Lambda$$
$$+ y_\tau(\bar{\psi}_{\phi_{T2}}\ell_2 + \psi_{\phi_{T3}}\ell_1 + \psi_{\phi_{T1}}\ell_3)\bar{\phi}_{\tau_R^c} h_d/\Lambda$$
$$+ y_e(\bar{\psi}_{\phi T1}\bar{\phi}_{\ell_1} + \bar{\psi}_{\phi T2}\bar{\phi}_{\ell_3} + \bar{\psi}_{\phi T3}\bar{\phi}_{\ell_2})\bar{\phi}_{e_R^c}\bar{\psi}_{h_d}/\Lambda$$
$$+ y_\mu(\bar{\psi}_{\phi T3}\bar{\phi}_{\ell_3} + \bar{\psi}_{\phi T1}\bar{\phi}_{\ell_2} + \bar{\psi}_{\phi T2}\bar{\phi}_{\ell_1})\bar{\phi}_{\mu_R^c}\bar{\psi}_{h_d}/\Lambda$$
$$+ y_\tau(\bar{\psi}_{\phi T2}\bar{\phi}_{\ell_2} + \bar{\psi}_{\phi T3}\bar{\phi}_{\ell_1} + \bar{\psi}_{\phi T1}\bar{\phi}_{\ell_3})\bar{\phi}_{\tau_R^c}\bar{\psi}_{h_d}/\Lambda$$
$$+ y_e(\phi_{T1}\ell_1 + \phi_{T2}\ell_3 + \phi_{T3}\ell_2)\bar{\phi}_{e_R^c}\bar{\psi}_{h_d}/\Lambda$$
$$+ y_\mu(\phi_{T3}\ell_3 + \phi_{T1}\ell_2 + \phi_{T2}\ell_1)\bar{\phi}_{\mu_R^c}\bar{\psi}_{h_d}/\Lambda$$
$$+ y_\tau(\phi_{T2}\ell_2 + \phi_{T3}\ell_1 + \phi_{T1}\ell_3)\bar{\phi}_{\tau_R^c}\bar{\psi}_{h_d}/\Lambda$$
$$+ y_e(\phi_{T1}\bar{\phi}_{\ell_1} + \phi_{T2}\bar{\phi}_{\ell_3} + \phi_{T3}\bar{\phi}_{\ell_2})e_R^c\bar{\psi}_{h_d}/\Lambda$$
$$+ y_\mu(\phi_{T3}\bar{\phi}_{\ell_3} + \phi_{T1}\bar{\phi}_{\ell_2} + \phi_{T2}\bar{\phi}_{\ell_1})\mu_R^c\bar{\psi}_{h_d}/\Lambda$$
$$+ y_\tau(\phi_{T2}\bar{\phi}_{\ell_2} + \phi_{T3}\bar{\phi}_{\ell_1} + \phi_{T1}\bar{\phi}_{\ell_3})\tau_R^c\bar{\psi}_{h_d}/\Lambda$$
$$- M(\bar{\psi}_{\phi_{01}^T}\bar{\psi}_{\phi_{T1}} + \bar{\psi}_{\phi_{02}^T}\bar{\psi}_{\phi_{T3}} + \bar{\psi}_{\phi_{03}^T}\bar{\psi}_{\phi_{T2}})$$
$$+ \frac{2}{3}g\Big[\phi_{01}^T(\bar{\psi}_{\phi_{T1}}\bar{\psi}_{\phi_{T1}} - \bar{\psi}_{\Phi_{T2}}\bar{\psi}_{\phi_{T3}}) + \phi_{02}^T(\bar{\psi}_{\phi_{T2}}\bar{\psi}_{\phi_{T2}} - \bar{\psi}_{\phi_{T3}}\bar{\psi}_{\phi_{T1}})$$
$$+ \phi_{03}^T(\bar{\psi}_{\phi_{T3}}\bar{\psi}_{\phi_{T3}} - \bar{\psi}_{\phi_{T1}}\bar{\psi}_{\phi_{T2}})$$
$$+ \bar{\psi}_{\phi_{01}^T}(\phi_{T1}\bar{\psi}_{\phi_{T1}} - \phi_{T2}\bar{\psi}_{\phi_{T3}}) + \bar{\psi}_{\phi_{02}^T}(\phi_{T2}\bar{\psi}_{\phi_{T2}} - \phi_{T3}\bar{\psi}_{\phi_{T1}})$$
$$+ \bar{\psi}_{\phi_{03}^T}(\phi_{T3}\bar{\psi}_{\phi_{T3}} - \phi_{T1}\bar{\psi}_{\phi_{T2}}) + \bar{\psi}_{\phi_{01}^T}(\bar{\psi}_{\phi_{T1}}\phi_{T1} - \bar{\psi}_{\phi_{T2}}\phi_{T3})$$
$$+ \bar{\psi}_{\phi_{02}^T}(\bar{\psi}_{\phi_{T2}}\phi_{T2} - \bar{\psi}_{\phi_{T3}}\phi_{T1}) + \bar{\psi}_{\phi_{03}^T}(\bar{\psi}_{\phi_{T3}}\phi_{T3} - \bar{\psi}_{\phi_{T1}}\phi_{T2})\Big],$$

# Dark Matter

- The mass terms of flavons and flavinos:

$$\mathcal{L}_{\Phi_T} \supset M^2|\varphi_{T1}|^2 + 4M^2|\phi_{T2}|^2 + 4M^2|\phi_{T3}|^2 + M^2|\phi^T_{01}|^2 + 4M^2|\phi^T_{02}|^2 + 4M^2|\phi^T_{03}|^2 - M\tilde{\psi}_{\phi^T_{01}}\tilde{\psi}_{\phi_{T1}} - 2M(\tilde{\psi}_{\phi^T_{02}}\tilde{\psi}_{\phi_{T3}} + \tilde{\psi}_{\phi^T_{03}}\tilde{\psi}_{\phi_{T2}}) + \mathrm{h.c.}\,.$$

- The relevant interactions: $v_T = \dfrac{3M}{2g}$

$$\mathcal{L}_{\Phi_T} \supset \frac{M}{v_T}\left[2\varphi_{T1}\overline{X_R}X_L + \phi^T_{01}\overline{X^c_L}X_L + \mathrm{h.c.}\right] + \frac{M^2}{v_T}\left[\varphi_{T1}\varphi^*_{T1}\varphi^*_{T1} + \mathrm{c.c.}\right] - \frac{2M^2}{v_T}\left[\phi^T_{01}\phi^{T*}_{01}\varphi^*_{T1} + \mathrm{c.c.}\right]$$

$$X_R \equiv \tilde{\psi}^c_{\phi^T_{01}}\,, \qquad X_L \equiv \tilde{\psi}_{\phi_{T1}}\,,$$

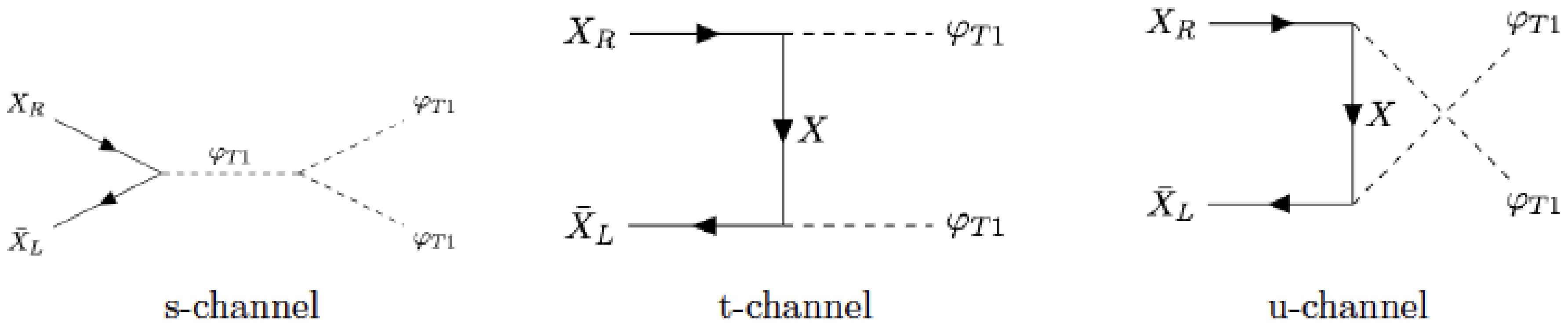
# Dark Matter

- DM annihilation processes:

$$\overline{X}X \to \{\varphi_{T1}\varphi_{T1}, \overline{\varphi_{T1}}\ \overline{\varphi_{T1}}, \overline{\phi_{01}^T}\phi_{01}^T, \overline{\varphi_{T1}}\varphi_{T1}\},$$
$$XX \to \overline{\phi_{01}^T}\ \overline{\varphi_{T1}}, \quad \overline{X}\ \overline{X} \to \phi_{01}^T\varphi_{T1}.$$

# Dark Matter

- DM annihilation processes: $X\bar{X} \longrightarrow \varphi_{T1}\varphi_{T1}$




$$\mathcal{M}_s = -\left(\frac{2M}{v_T}\right)\left(\frac{2M^2}{v_T}\right)\frac{1}{s-M^2}\bar{v}_{(p_2)}P_R u_{(p_1)}$$

$$\mathcal{M}_t \sim \left(\frac{2M}{v_T}\right)^2\frac{1}{M}\bar{v}_{(p_2)}P_R u_{(p_1)}$$

$$\mathcal{M}_u \sim \left(\frac{2M}{v_T}\right)^2\frac{1}{M}\bar{v}_{(p_2)}P_R u_{(p_1)},$$

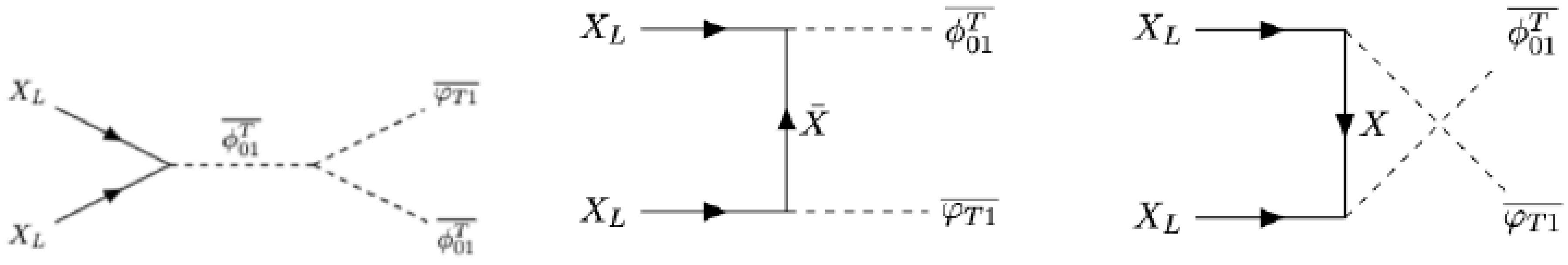
where $s = (p_1 + p_2)^2$ with $p_{1(2)}$ being flavino $X(\bar{X})$ momentum in the initial state. Here, we ignored other Mandelstam variables assuming $t, u \ll M^2$. Since we consider non-retivistic limit, we approximate $s \sim 4M^2$. Thus, the matrix element of the process is

$$\mathcal{M} \sim \frac{20M}{3{v_T}^2}\bar{v}_{(p_2)}P_R u_{(p_1)}.$$

The cross section: $\sigma_{X\bar{X}\to\varphi_{T1}\varphi_{T1}} \sim \frac{25}{144\pi}\frac{M^2}{{v_T}^4}.$

# Dark Matter

- DM annihilation processes: $XX \longrightarrow \overline{\phi_{01}^{T}}\,\overline{\phi_{T1}}$




s-channel t-channel u-channel

$$\mathcal{M}_s = \left(\frac{M}{v_T}\right)\left(\frac{2M^2}{v_T}\right)\frac{1}{s-M^2}\bar{v}_{(p_2)}P_L u_{(p_1)}$$

$$\mathcal{M}_t \sim \mathcal{M}_u \sim \left(\frac{2M}{{v_T}^2}\right)\bar{v}_{(p_2)}P_L u_{(p_1)}$$

$$\mathcal{M} \sim \left(\frac{14M}{3{v_T}^2}\right)\bar{v}_{(p_2)}P_L u_{(p_1)}.$$

The cross section:

$$\sigma_{XX\rightarrow\overline{\phi^T_{01}}\,\overline{\phi_{T1}}} \sim \left(\frac{49}{288\pi}\right)\left(\frac{M^2}{{v_T}^4}\right).$$

# Dark Matter

- DM annihilation processes:

$X\bar{X} \longrightarrow \overline{\varphi_{T1}}\,\overline{\varphi_{T1}}$:

This process are just conjugate of $X\bar{X} \longrightarrow \varphi_{T1}\varphi_{T1}$. Thus, cross section is

$$\sigma_{X\bar{X}\rightarrow\overline{\varphi_{T1}}\,\overline{\varphi_{T1}}} = \sigma_{X\bar{X}\rightarrow\varphi_{T1}\varphi_{T1}} \sim \frac{25}{144\pi}\frac{M^2}{{\upsilon_T}^4}.$$

$\bar{X}\bar{X} \longrightarrow \phi^T_{01}\varphi_{T1}$:

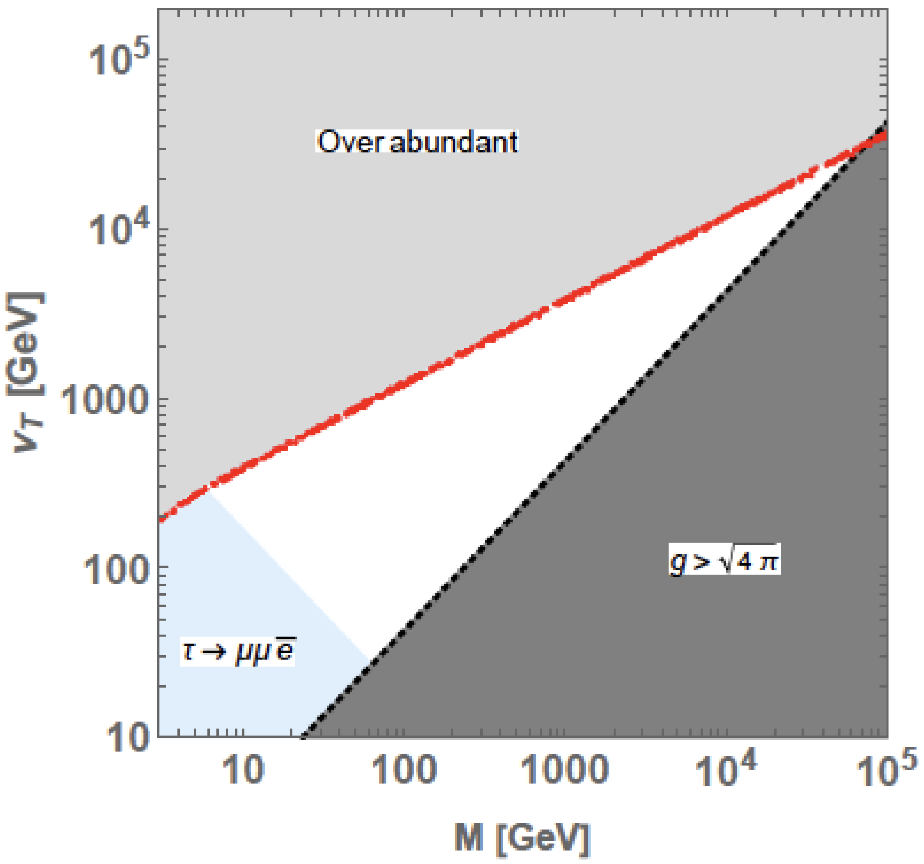
This process are just conjugate of $XX \longrightarrow \overline{\phi^T_{01}}\,\overline{\varphi_{T1}}$. Thus, cross section is

$$\sigma_{\bar{X}\bar{X}\rightarrow\phi^T_{01}\varphi_{T1}} = \sigma_{XX\rightarrow\overline{\varphi^T_{01}}\,\overline{\varphi_{T1}}} \sim \left(\frac{49}{288\pi}\right)\left(\frac{M^2}{{\upsilon_T}^4}\right).$$

We apply these cross sections in DM relic density calculation.

# Dark Matter

- Allowed region:




- Three body decay:

$$\mathrm{BR}(\tau \to \mu\mu\bar{e}) = \tau_\tau \frac{m_\tau^5}{3027\pi^3}\left(\left|\frac{m_\tau m_\mu}{v_T^2 m_{\phi_{T2}}^2}\right|^2 + \left|\frac{m_\mu m_e}{v_T^2 m_{\phi_{T3}}^2}\right|^2\right)$$

$$\simeq \frac{2.9\times 10^6\ \mathrm{GeV}^8}{v_T^4 (2M)^4}$$

Y. Muramatsu, T. Nomura and Y. S., JHEP 03 (2016) 192.

$$\mathrm{BR}(\tau \to \mu\mu\bar{e}) < 1.7\times 10^{-8}$$

K. Hayasaka et al., Phys. Lett. B 687 (2010) 139.

In charged lepton sector, the $A_4$ symmetry is broken down to the residual $Z_3$ symmetry. The $Z_3$ assignments are $(e,\mu,\tau) = (1,\omega,\omega^2)$. In charged lepton flavor violating processes, the $Z_3$ symmetry must be kept. Thus, $\tau \to ee\bar{e}$, $\tau \to \mu\mu\bar{\mu}$, $\tau \to \mu e\bar{e}$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ processes are forbidden.

# Leptogenesis

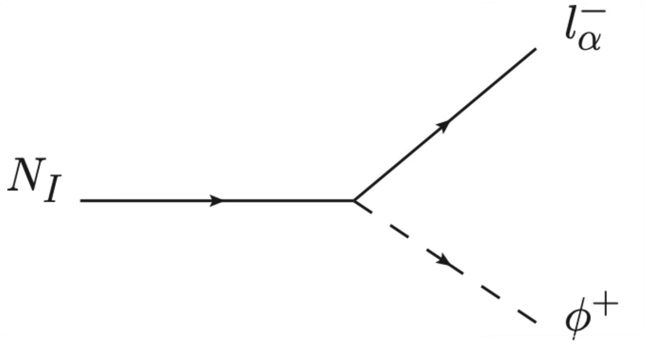
- CP asymmetry parameter:

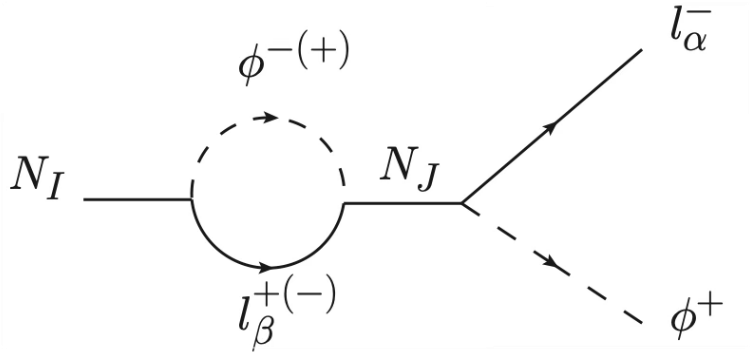
$$\epsilon_I = \frac{\Gamma\left(N_I \to \ell + \bar{h}_u\right) - \Gamma\left(N_I \to \bar{\ell} + h_u\right)}{\Gamma\left(N_I \to \ell + \bar{h}_u\right) + \Gamma\left(N_I \to \bar{\ell} + h_u\right)}$$

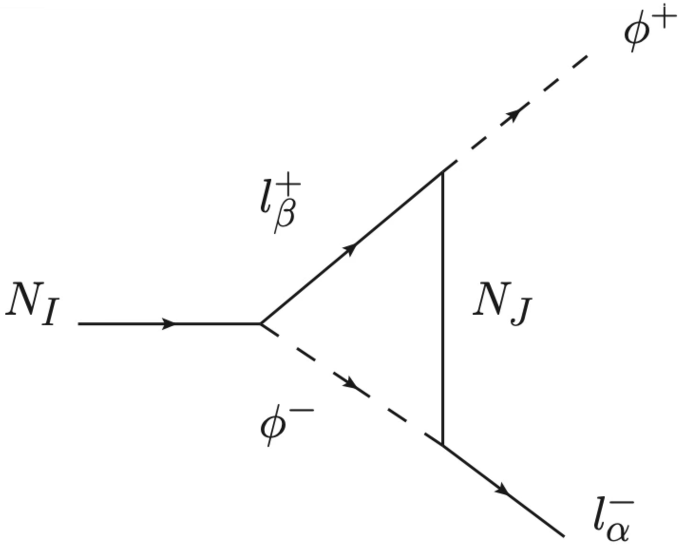

L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996), 169-174.

W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A 15 (2000), 5047-5086.

G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685 (2004), 89-149.

# Leptogenesis

- CP asymmetry parameter:

$$\epsilon_I = \frac{\Gamma\left(N_I \to \ell + \bar{h_u}\right) - \Gamma\left(N_I \to \bar{\ell} + h_u\right)}{\Gamma\left(N_I \to \ell + \bar{h_u}\right) + \Gamma\left(N_I \to \bar{\ell} + h_u\right)}$$

$$= -\frac{1}{8\pi}\sum_{J\neq I}\frac{\mathrm{Im}\left[\left\{\left(Y_D^\dagger Y_D\right)_{JI}\right\}^2\right]}{\left(Y_D^\dagger Y_D\right)_{II}}\left[f^V\left(\frac{M_J^2}{M_I^2}\right) + f^S\left(\frac{M_J^2}{M_I^2}\right)\right],$$

$$f^V(x) = \sqrt{x}\left[(x+1)\ln\left(1+\frac{1}{x}\right) - 1\right], \quad f^S(x) = \frac{\sqrt{x}}{x-1},$$

$$\epsilon_I \propto \sum_{J\neq I}\mathrm{Im}\left[\left\{\left(Y_D^\dagger Y_D\right)_{JI}\right\}^2\right]$$

L. Covi, E. Roulet and F. Vissani, Phys. Lett. B 384 (1996), 169-174.

W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A 15 (2000), 5047-5086.

G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685 (2004), 89-149.

# Leptogenesis

- Dirac Yukawa matrix in the real diagonal base for the right-handed Majorana neutrino mass matrix:

$$Y_D^{\mathrm{L}} = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R^T P_R^T,$$

$$\epsilon_I \propto \sum_{J \neq I} \mathrm{Im}\left[\left\{\left(Y_D^\dagger Y_D\right)_{JI}\right\}^2\right]$$

$$Y_D^{\mathrm{L}\dagger} Y_D^{\mathrm{L}} = |y_D|^2 P_R^* U_R^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_R^T P_R^T$$

$$= |y_D|^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Leptogenesis does not work...

# Leptogenesis

- We consider the next-to-leading order (NLO):

$$
\begin{aligned}
w_D^{\rm NL} &= y_D^{\rm NL}\Phi_\ell\Phi_N\Phi_u\Phi_T/\Lambda \\
&= \frac{1}{3}y_D^{\rm NLS}\big[(2\Phi_{\ell1}\Phi_{N1}-\Phi_{\ell2}\Phi_{N3}-\Phi_{\ell3}\Phi_{N2})\Phi_{T1}+(2\Phi_{\ell2}\Phi_{N2}-\Phi_{\ell3}\Phi_{N1}-\Phi_{\ell1}\Phi_{N3})\Phi_{T2} \\
&+(2\Phi_{\ell3}\Phi_{N3}-\Phi_{\ell1}\Phi_{N2}-\Phi_{\ell2}\Phi_{N1})\Phi_{T3}\big]\Phi_u/\Lambda \\
&+\frac{1}{2}y_D^{\rm NLA}\big[(\Phi_{\ell2}\Phi_{N3}-\Phi_{\ell3}\Phi_{N2})\Phi_{T1}+(\Phi_{\ell3}\Phi_{N1}-\Phi_{\ell1}\Phi_{N3})\Phi_{T2}+(\Phi_{\ell1}\Phi_{N2}-\Phi_{\ell2}\Phi_{N1})\Phi_{T3}\big]\Phi_u/\Lambda
\end{aligned}
$$

$$
Y_D^{\rm L+NL} = y_D\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + \frac{1}{3}y_D^{\rm NLS}\frac{v_T}{\Lambda}\begin{pmatrix}2&0&0\\0&0&-1\\0&-1&0\end{pmatrix} + \frac{1}{2}y_D^{\rm NLA}\frac{v_T}{\Lambda}\begin{pmatrix}0&0&0\\0&0&1\\0&-1&0\end{pmatrix}
$$

$$
Y_D^{L+NL\,\dagger}Y_D^{L+NL} \neq \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}
$$

Leptogenesis works!!

# Summary

- **$A_4$ SUSY flavor model** explains:
  - **Lepton mixing angles** and **mass hierarchies**
  - **Dark matter (flavino)**
  - **BAU (via NLO leptogenesis)**
- **$A_4$ SUSY flavor model** is consistent with **current experimental data**.

# Summary

- $A_4$ **SUSY flavor model** explains:
  - **Lepton mixing angles** and **mass hierarchies**
  - **Dark matter (flavino)**
  - **BAU (via NLO leptogenesis)**
- $A_4$ **SUSY flavor model** is consistent with **current experimental data**.

**Thank you for your attention!!**

# Back up

# Multiplication rule of $A_4$ group

H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. S., and M. Tanimoto, Non-Abelian Discrete Symmetries in Particle Physics, Prog. Theor. Phys. Suppl. 183 (2010) 1; Lect. Notes Phys. 858 (2012) 1, Springer.

S. F. King, A. Merle, S. Morisi, Y. S., and M. Tanimoto, New J. Phys. 16 (2014), 045018.

T. Kobayashi, H. Ohki, H. Okada, Y. S., and M. Tanimoto, Lect. Notes Phys. 995 (2022), 1-353, Springer.

$$S^2 = T^3 = (ST)^3 = 1.$$

$$\begin{aligned}
\mathbf{1}: &\quad S = 1, \quad T = 1, \\
\mathbf{1}': &\quad S = 1, \quad T = e^{4\pi i/3} \equiv \omega^2, \\
\mathbf{1}'': &\quad S = 1, \quad T = e^{2\pi i/3} \equiv \omega.
\end{aligned}$$

$$\mathbf{3}: \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}.$$

$$\begin{aligned}
\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} &= (a_1b_1 + a_2b_3 + a_3b_2)_{\mathbf{1}} \oplus (a_3b_3 + a_1b_2 + a_2b_1)_{\mathbf{1}'} \\
&\oplus (a_2b_2 + a_1b_3 + a_3b_1)_{\mathbf{1}''} \\
&\oplus \frac{1}{3}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_3b_1 - a_1b_3 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_3b_1 - a_1b_3 \end{pmatrix}_{\mathbf{3}} .
\end{aligned}$$

# Relic density

Relic density of flavino DM is obtained by solving Boltzmann equation for number density $n_X$ of DM $X$,

$$\dot{n}_X + 3Hn_X = \langle\sigma v\rangle(n_{X_{\rm eq}}^2 - n_X^2),$$

where $H$ is Hubble parameter, $\langle\sigma v\rangle$ is thermal average of DM annihilation cross section and $n_{X_{eq}}$ is density of $X$ in equilibrium. The annihilation cross section is the sum of cross sections discussed in the previous subsection. Relic density $\Omega_X h^2$ can be approximately estimated as

$$\Omega_X h^2 \simeq \frac{1.07\times 10^9}{\sqrt{g^*(x_f)}M_{Pl}J(x_f)\ [\mathrm{GeV}]},$$

where $x_f = M/T_f$ with $T_f$ being freeze out temperature, $g^*(x_f)$ is effective relativistic degrees of freedom at $T_f$, and $M_{Pl} \simeq 1.22\times 10^{19}$ is the Planck mass. The factor $J(x_f) \equiv \int_{x_f}^{\infty} dx \frac{\langle\sigma v\rangle}{x^2}$ is written by

$$J(x_f) = \int_{x_f}^{\infty}\left[\frac{\int_{4M^2}^{\infty} ds\sqrt{s-4M^2}(\sigma v)K_1\left(\frac{\sqrt{s}}{M}x\right)}{16M^5 x[K_2(x)]^2}\right],$$

where $K_{1,2}$ denote the modified Bessel functions of the second kind of order 1 and 2.

# Comments on other flavino DM physics

Here we provide brief discussion regarding some phenomenology of flavino DM.

**Direct detection.** The interactions from superpotential do not contribute to flavino-nucleon scattering at tree and one-loop level since they only have coupling terms among flavino, flavon, Higgs(Higgsinos) and leptons(sleptons); there is no flavino-flavino-Higgs interaction. Thus these interactions are not constrained by the direct detection experiments; flavino can also interact with electron via flavon $\varphi_{T1}$ exchange but cross section is tiny since flavon-electron coupling is proportional to electron mass. In fact flavino-nucleon scattering is possible via Higgs portal interactions when flavon and Higgs bosons mix through SUSY-breaking terms. In this work we assume effect of SUSY breaking is small for flavon sector to avoid current experimental bounds, e.g. XENON1T , PandaX-4T and LUX-ZEPLIN .

# Comments on other flavino DM physics

**Indirect detection.** In the scenario cross sections of flavino annihilation into flavons are suppressed at current universe due to mass degeneracy between the lightest flavino and flavon. Flavino can also annihilate into charged leptons via flavon exchanging processes, $X\bar{X} \to \varphi_{T1} \to \ell^+\ell^-$, but cross section of the processes are much smaller than order of $\sim 10^{-26}\,\mathrm{cm}^3/\mathrm{s}$, since flavon-lepton coupling is small as $m_\ell/v_T$. Thus the model is safe from indirect detection constraints where the strongest bound on the annihilation cross section is given by Fermi-LAT data

**Collider search.** Flavino DM can be searched for at collider experiments such as the LHC as the preferred scale of flavino mass is $\mathcal{O}(10)$ GeV to $\mathcal{O}(10)$ TeV. One possible process is slepton pair production followed by slepton decay $\tilde{\ell} \to \ell X$ when a slepton is the next to lightest SUSY particle. The signal of this case is the same as slepton decaying into neutralino DM and it is difficult to distinguish. We can expect more specific signals of the model when heavier flavinos and flavons are lighter than sleptons. In such a case we would have cascade decay of slepton, e.g. $\tilde{\ell} \to \ell\overline{\tilde{\psi}_{T2}} \to \ell\overline{X}\phi_{T3}(\to \bar{\ell}'\ell')$, inducing multi-leptons with missing-energy signal. For these signals we may be able to reconstruct flavon mass and it helps us to confirm the model. Detailed analysis of collider signals is beyond the scope of this paper and we left it in future work.