Goofy Symmetries

Andreas Trautner

based on:

arXiv:2505.00099

FLASY 2025, Roma

04.07.25

Outline

- The original GOOFy transformation
- Definition of Goofy transformations / *regular* vs. *goofy*
- RG stability of regular / goofy transformations in general
- Additional goofy transformations in 2HDM
- Applications of goofy transformations
- Conclusions

Disclaimer

This is a very young topic, all input is welcome and important.

Goofy transformations are new, understanding them for yourself may be rewarding.

Disclaimer

This is a very young topic, all input is welcome and important.
Goofy transformations are new, understanding them for yourself may be rewarding.

Complete list of existing literature on Goofy transformations:

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO → GOOFy**
  Original goofy transformation in 2HDM.
- [Haber and Ferreira 2502.11011]
  Goofy transformations in two-scalar toy model of [Grzadkowski @ Multi-Higgs, Lisbon, 2024]
- [AT 2505.00099]
  My take on the matter, this talk
- [Ferreira, Grzadkowski, Ogreid 2506.21145]
  "imaginary scaling" goofy transformations in 2HDM and two-scalar toy model

Very brief remarks on the original Goofy transformation in:

- [Cao, Cheng, Xu 2305.12764v2 (of 3/2024)]
- [Pilaftsis 2408.04511]

Disclaimer

This is a very young topic, all input is welcome and important.
Goofy transformations are new, understanding them for yourself may be rewarding.

Complete list of existing literature on Goofy transformations:

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO $\rightarrow$ GOOFy**
  Original goofy transformation in 2HDM.
- [Haber and Ferreira 2502.11011]
  Goofy transformations in two-scalar toy model of [Grzadkowski @ Multi-Higgs, Lisbon, 2024]
- [AT 2505.00099]
  My take on the matter, this talk
- [Ferreira, Grzadkowski, Ogreid 2506.21145]
  "imaginary scaling" goofy transformations in 2HDM and two-scalar toy model

Very brief remarks on the original Goofy transformation in:

- [Cao, Cheng, Xu 2305.12764v2 (of 3/2024)]
- [Pilaftsis 2408.04511]

Coincidentially, German "Youth word of the year" 2023: **Goofy** [Langenscheidt], [Wikipedia]

Disclaimer

This is a very young topic, all input is welcome and important.
Goofy transformations are new, understanding them for yourself may be rewarding.

Complete list of existing literature on Goofy transformations:

- [Ferreira, Grzadkowski, Ogreid, Osland 2306.02410] **FGOO $\to$ GOOFy**
  Original goofy transformation in 2HDM.
- [Haber and Ferreira 2502.11011]
  Goofy transformations in two-scalar toy model of [Grzadkowski @ Multi-Higgs, Lisbon, 2024]
- [AT 2505.00099]
  My take on the matter, this talk
- [Ferreira, Grzadkowski, Ogreid 2506.21145]
  "imaginary scaling" goofy transformations in 2HDM and two-scalar toy model

Very brief remarks on the original Goofy transformation in:

- [Cao, Cheng, Xu 2305.12764v2 (of 3/2024)]
- [Pilaftsis 2408.04511]

Coincidentially, German "Youth word of the year" 2023: **Goofy** [Langenscheidt], [Wikipedia]

e.g. 2012: YOLO, ..., 2020: lost, 2021: cringe, ..., 2023: goofy

The first Goofy transformation

2HDM: Scalar doublets $\Phi_{a=1,2}(x)$ in $(\mathbf{2}, -1/2)$ of $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$.

The invariant scalar potential conventionally written as

$m_{11,22},\ \lambda_{1,2,3,4} \in \mathbb{R}$
$m_{12},\ \lambda_{5,6,7} \in \mathbb{C}$

$$V(\Phi^*, \Phi) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$
$$- \left\{ m_{12}^2 \Phi_1^\dagger \Phi_2 \right\} + \left\{ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 \right\} + \text{h.c.}.$$

Community agrees, all possible (exact global) symmetries of 2HDM are known:
[Ivanov '06, '07], [Ferreira, Haber, Maniatis, Nachtmann, Silva '11], [Deshpande, Ma '78], [Ginzburg, Krawczyk '05], [Nishi '11], [Pilaftsis '12], ...

$$\mathrm{CP1}\,, \quad \mathbb{Z}_2\,, \quad \mathrm{U}(1)\,, \quad \mathrm{CP2}\,, \quad \mathrm{CP3}\,, \quad \mathrm{SU}(2)\,.$$

However, **FGOO**≡[Ferreira, Grzadkowski, Ogreid, Osland '23] found the relations

$$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,.$$

- These are renormalization group (RG) stable *to all orders*.
- Do **not** correspond to any of the known *regular* 2HDM symmetries.

The first Goofy transformation

2HDM: Scalar doublets $\Phi_{a=1,2}(x)$ in $(\mathbf{2},-1/2)$ of $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$.

The invariant scalar potential conventionally written as

$m_{11,22},\ \lambda_{1,2,3,4}\in\mathbb{R}$
$m_{12},\ \lambda_{5,6,7}\in\mathbb{C}$

$$\begin{aligned}V(\Phi^*,\Phi) \;=\; & m_{11}^2\Phi_1^\dagger\Phi_1+m_{22}^2\Phi_2^\dagger\Phi_2+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\ & -\left\{m_{12}^2\Phi_1^\dagger\Phi_2\right\}+\left\{\lambda_5(\Phi_1^\dagger\Phi_2)^2+\left[\lambda_6(\Phi_1^\dagger\Phi_1)+\lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2\right\}+\mathrm{h.c.}.\end{aligned}$$

Community agrees, all possible (exact global) symmetries of 2HDM are known:
[Ivanov '06, '07], [Ferreira, Haber, Maniatis, Nachtmann, Silva '11], [Deshpande, Ma '78], [Ginzburg, Krawczyk '05], [Nishi '11], [Pilaftsis '12], ...

$$\mathrm{CP1}\,,\quad \mathbb{Z}_2\,,\quad \mathrm{U}(1)\,,\quad \mathrm{CP2}\,,\quad \mathrm{CP3}\,,\quad \mathrm{SU}(2)\,.$$

However, **FGOO**≡[Ferreira, Grzadkowski, Ogreid, Osland '23] found the relations

$$m_{11}^2=-m_{22}^2\,,\qquad \lambda_1=\lambda_2\,,\qquad \lambda_6=-\lambda_7\,.$$

- These are renormalization group (RG) stable *to all orders*.
- Do **not** correspond to any of the known *regular* 2HDM symmetries.

The first Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland ’23]

$$\Phi_1 \mapsto -\Phi_2^*\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \Phi_1^* \mapsto \Phi_2\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

The first Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:
[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\Phi_1 \mapsto -\Phi_2^*\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \Phi_1^* \mapsto \Phi_2\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

The first Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\underline{\boldsymbol{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\boldsymbol{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

The first Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring invariance of $V(\Phi^*, \Phi)$ under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\underline{\boldsymbol{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\boldsymbol{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

Consider canonical (gauge-)kinetic terms

$$\mathcal{K} \;=\; (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2)\;\;,$$

Applied to the (gauge-)kinetic terms the transformation maps

$$\mathcal{K} \;\mapsto\; -\mathcal{K}\;,$$

If imposed as an exact symmetry this would enforce $\mathcal{K} = 0$.

The first Goofy transformation

The relations $$m_{11}^2 = -m_{22}^2\,, \qquad \lambda_1 = \lambda_2\,, \qquad \lambda_6 = -\lambda_7\,, \tag{1}$$

can be obtained by requiring <u>invariance of $V(\Phi^*, \Phi)$</u> under the transformation:

[Ferreira, Grzadkowski, Ogreid, Osland '23]

$$\underline{\mathbf{\Phi_1 \mapsto -\Phi_2^*}}\,, \quad \Phi_2 \mapsto \Phi_1^*\,, \quad \underline{\mathbf{\Phi_1^* \mapsto \Phi_2}}\,, \quad \Phi_2^* \mapsto -\Phi_1\,.$$

$$\Longleftrightarrow \quad m_{11}^2 \longleftrightarrow -m_{22}^2\,, \quad \lambda_1 \longleftrightarrow \lambda_2\,, \quad \lambda_6 \longleftrightarrow -\lambda_7\,.$$

Crucial: Independent transformation of Φ_i and Φ_i^*.

Consider canonical (gauge-)kinetic terms

$$\mathcal{K} \;=\; (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)\;\;,$$

Applied to the (gauge-)kinetic terms the transformation maps

$$\mathcal{K} \;\mapsto\; -\mathcal{K}\,,$$

If imposed as an exact symmetry this would enforce $\mathcal{K} = 0$.

$\Rightarrow$ This transformation cannot be an *exact* symmetry in a *dynamical* theory, $\mathcal{K} \neq 0$.

But how can relations (1) then be RG stable?

Definition of Goofy transformations

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(kin.)}[\phi] \;-\; \mathcal{V}_{(pot.)}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...

Definition of Goofy transformations

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(kin.)}[\phi] \;-\; \mathcal{V}_{(pot.)}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...

  ...but this is just wrong!

Definition of Goofy transformations

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(kin.)}[\phi] \;-\; \mathcal{V}_{(pot.)}[\phi] \;.$$

- How come, when talking about symmetry transformations, we usually only consider constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...

  . . . but this is just wrong!

Defining criterion:

Regular transformations act **trivially** on kinetic term.	$\longleftrightarrow$	*Goofy* transformations act **non-trivially** on kinetic term.

Definition of Goofy transformations

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(kin.)}[\phi] \;-\; \mathcal{V}_{(pot.)}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...

  ...but this is just wrong!

Defining criterion:

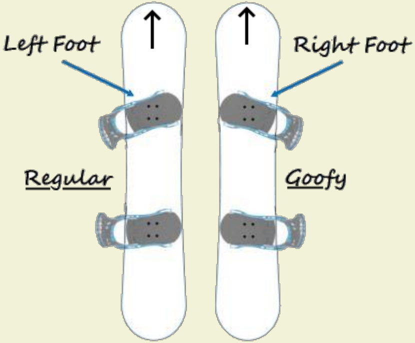
Regular transformations act **trivially** on kinetic term.	$\longleftrightarrow$	***Goofy*** transformations act **non-trivially** on kinetic term.

Crucial: **Transformation $\neq$ Symmetry.**

Transformations (regular/goofy) can be physically important, even if explicitly broken.

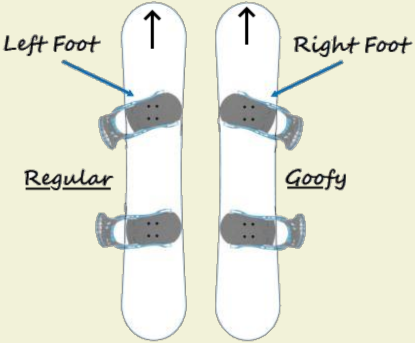
Regular vs. Goofy

in Surfing, Skateboarding, Snowboarding, . . .



Regular vs. Goofy

in Surfing, Skateboarding, Snowboarding, . . .



RG stability of Goofy parameter relations

There are parameter relations derived from a transformation acting on $V(\Phi^*, \Phi)$.

Central question:

How can these parameter relations be RG stable, even if the corresponding transformation is explicitly broken by the (gauge-)kinetic terms $\mathcal{K}$?

RG stability of ~~Goofy~~ parameter relations

There are parameter relations derived from a transformation acting on $V(\Phi^*, \Phi)$.

Central question:

How can these parameter relations be RG stable, even if the corresponding transformation is explicitly broken by the (gauge-)kinetic terms $\mathcal{K}$?

Why in the first place do symmetries lead to RG stable parameter relations?

RG stable parameter relations and symmetry

Why in the first place do symmetries lead to RG stable parameter relations?

Preserved symmetry $\Longleftrightarrow$ RG stable parameter relations.

RG stable parameter relations and symmetry

Why in the first place do symmetries lead to RG stable parameter relations?

Preserved symmetry $\Longleftrightarrow$ RG stable parameter relations.

This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \frac{\mathrm{d}\,g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

Proof of this (for all possible symmetries) has never been given to the best of my knowledge.

RG stable parameter relations and symmetry

Why in the first place do symmetries lead to RG stable parameter relations?

Preserved symmetry $\Longleftrightarrow$ RG stable parameter relations.

This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \frac{\mathrm{d}\, g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

Proof of this (for all possible symmetries) has never been given to the best of my knowledge.

We will present a general (non-perturbative) formal argument in [De Boer, AT to appear].

It is based on the fact that *prospective* symmetries can be viewed as (linear) maps in the parameter space of a theory because they act as *outer automorphisms*.
[Fallbacher, AT '15]

In short: Couplings transform covariantly $\Rightarrow$ β-functions trafo covariantly.
Symmetries of β-functions $\geq$ symmetries of theory.
[De Boer, AT to appear]

Computing RG fixed points

[AT '25], [De Boer, AT to appear]

Consider theory with fields $\phi_a,\ \phi^*_{a=1,\dots,N} \in \mathbb{C}$ and couplings $\lambda_{i=1,\dots,K}$.
If there is a transformation that acts on the fields as (A, B unitary)

$$T: \qquad \vec{\phi} \mapsto A\vec{\phi}\,, \qquad \vec{\phi}^* \mapsto B^*\vec{\phi}^*\,,$$

($A = B$: regular, $A \neq B$: goofy)

that can *equivalently* be represented as a mapping in the space of couplings*

$$T: \qquad \vec{\lambda} \mapsto \mathcal{O}\vec{\lambda}\,,$$

($\mathcal{O}$ can be derived from A and B)

Computing RG fixed points

[AT '25], [De Boer, AT to appear]

Consider theory with fields $\phi_a,\ \phi^*_{a=1,\dots,N} \in \mathbb{C}$ and couplings $\lambda_{i=1,\dots,K}$.
If there is a transformation that acts on the fields as (A, B unitary)

$$T: \qquad \vec{\phi} \mapsto A\vec{\phi}\,, \qquad \vec{\phi}^* \mapsto B^*\vec{\phi}^*\,, \qquad (A = B\text{: regular, } A \neq B\text{: goofy})$$

that can *equivalently* be represented as a mapping in the space of couplings*

$$T: \qquad \vec{\lambda} \mapsto \mathcal{O}\vec{\lambda}\,, \qquad (\mathcal{O}\text{ can be derived from } A \text{ and } B)$$

then the full coupled system of (non-linear) beta functions

$$\beta_{\vec{\lambda}} \equiv \mu\frac{\mathrm{d}\,\vec{\lambda}}{\mathrm{d}\mu} = \vec{f}(\lambda_1, \lambda_2, \dots)\,,$$

transforms covariantly, and **in the same irreps as the couplings themselves.**
This poses non-perturbative, all-order constraints on $\beta_{\vec{\lambda}}$ of the form $\mathcal{O}\beta_{\vec{\lambda}} = \vec{f}\Big|_{\vec{\lambda}\to\mathcal{O}\vec{\lambda}}$.

Computing RG fixed points

- The existence of such transformations* imposes strong all order exact constraints on system of β functions.

  Namely, the covariant β functions are spanned by covariant combination of couplings.
- This argument does not require such transformations to be conserved, the mere existence is enough.
- This argument holds at the non-perturbative level.
- The more of such possible transformations exist, the more restricted is overall system of β functions.

- **If** the transformations are *imposed* to be conserved as symmetries
  $\Longrightarrow$ Non-trivally transforming covariant combination of λ_i's must vanish,
  $\Longrightarrow$ Beta functions of nontrivially transforming λ_i's are forced to $\beta_{\lambda_i} = 0$.

This is the completed version of 't Hooft's argument.

[AT '25], [De Boer, AT to appear]

*All *outer automorphism* transformations are of this kind.

Example for a regular transformation in 2HDM

- *In absence of other symmetries*, most general $\mathrm{SU}(2)$ Higgs-basis change is outer automorphism $\Phi' = U\Phi$, $\Phi'^* = U^*\Phi^*$. Couplings transform covariantly.
  see e.g. [Ferreira, Haber, et al. '10], [AT '18], [Bednyakov '18],...

Example for a regular transformation in 2HDM

- *In absence of other symmetries*, most general $\mathrm{SU}(2)$ Higgs-basis change is outer automorphism $\Phi' = U\Phi$, $\Phi'^* = U^*\Phi^*$. Couplings transform covariantly.
  see e.g. [Ferreira, Haber, et al. '10], [AT '18], [Bednyakov '18],...

  E.g. the following combination transforms as $\mathrm{SU}(2)$ triplet (vector)

$$\vec{\Lambda} := const. \times \left(\mathrm{Re}(\lambda_6+\lambda_7), -\mathrm{Im}(\lambda_6+\lambda_7), \frac{1}{2}(\lambda_1-\lambda_2)\right)^{\mathrm{T}} .$$

  Our argument then implies that
  explicitly known to six loops [Bednyakov '18, '24]

$$\beta_{\vec{\Lambda}} \propto \vec{\Lambda} .$$

- This is because contributions from other vectors do not contribute to β_Λ:
  - $\vec{M}(m_{11}^2, m_{22}^2, m_{12}^2)$ wrong mass dimension. (our argument for scaling outer automorphism)
  - $\mathbf{5}$-plet $\tilde{\Lambda}(\lambda_3, \lambda_4, \dots)$, $\mathbf{3} \subset (\mathbf{5} \otimes \mathbf{5} \otimes \dots)$, but these contractions vanish for single $\mathbf{5}$.

$\Rightarrow$ This shows $\vec{\Lambda} = 0$ is RG fixed point to all orders in scalar+gauge corrections.

Example for a regular transformation in 2HDM

- *In absence of other symmetries*, most general $\mathrm{SU}(2)$ Higgs-basis change is outer automorphism $\Phi' = U\Phi$, $\Phi'^* = U^*\Phi^*$. Couplings transform covariantly.

  see e.g. [Ferreira, Haber, et al. '10], [AT '18], [Bednyakov '18],...

  E.g. the following combination transforms as $\mathrm{SU}(2)$ triplet (vector)

$$\vec{\Lambda} \;:=\; const. \times \left(\mathrm{Re}(\lambda_6+\lambda_7), -\mathrm{Im}(\lambda_6+\lambda_7), \frac{1}{2}(\lambda_1-\lambda_2)\right)^{\mathrm{T}} \;.$$

  Our argument then implies that

  explicitly known to six loops [Bednyakov '18, '24]

$$\beta_{\vec{\Lambda}} \;\propto\; \vec{\Lambda}\,.$$

- This is because contributions from other vectors do not contribute to β_{Λ}:
  - $\vec{M}(m_{11}^2, m_{22}^2, m_{12}^2)$ wrong mass dimension. (our argument for scaling outer automorphism)
  - $\mathbf{5}$-plet $\tilde{\Lambda}(\lambda_3, \lambda_4, \dots)$, $\mathbf{3} \subset (\mathbf{5}\otimes\mathbf{5}\otimes\dots)$, but these contractions vanish for single $\mathbf{5}$.

$\Rightarrow$ This shows $\vec{\Lambda} = 0$ is RG fixed point to all orders in scalar+gauge corrections.

- Fixed point $\vec{\Lambda} = 0$, implied by any trafo that requires $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$.
- E.g. $\mathrm{CP2}$ implies $\vec{\Lambda} = 0$. And this survives **soft** breaking ($m_{11}^2 \neq \pm m_{22}^2$).

What is different for Goofy transformations?

- Most general basis for (gauge-)kinetic terms

$$\mathcal{K} = (D_\mu \Phi_i)^\dagger K_{ij} (D^\mu \Phi_j) \qquad \text{with} \qquad K_{ij} = K_{ji}^* \,.$$

- Wave function renormalization coeffs. K_{ij} trafo *covariantly* under goofy trafos.
- In view of RGEs, K_{ij} should be treated exactly like couplings of the potential.
- Unfortunately, however, β functions are generally computed starting from canonical basis (not wrong, but not good here). $\longrightarrow$ workaround:

What is different for Goofy transformations?

- Most general basis for (gauge-)kinetic terms

$$\mathcal{K} = (D_\mu\Phi_i)^\dagger\, K_{ij}\, (D^\mu\Phi_j) \qquad \text{with} \qquad K_{ij} = K_{ji}^*\ .$$

- Wave function renormalization coeffs. K_{ij} trafo *covariantly* under goofy trafos.
- In view of RGEs, K_{ij} should be treated exactly like couplings of the potential.
- Unfortunately, however, β functions are generally computed starting from canonical basis (not wrong, but not good here). $\longrightarrow$ workaround:
- Starting from canonical kinetic terms $K_{ij} = \delta_{ij}$, goofy transformations

$$\Phi \mapsto A\Phi\ , \qquad \Phi^\dagger \mapsto \Phi^\dagger B^\dagger\ . \qquad \text{Here,}\quad (B^\dagger A)\quad \text{is unitary + must be hermitean.}$$

$\Rightarrow$ There exists a basis where effect of the most general transformation on (gauge-)kinetic terms is given by

$$\mathcal{K}\ \mapsto\ \kappa_1\,(D_\mu\Phi_1)^\dagger(D^\mu\Phi_1)+\kappa_2\,(D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) \qquad \text{with} \qquad \kappa_1=\pm1,\ \kappa_2=\pm1\ .$$

$\curvearrowright$ Can use std. RGEs + propagator / gauge vertex counting to **restore** covariants κ_i.

Back to original question: RG stability of goofy parameter relations

FGOO 2HDM parameter relations: $m_{11}^2 = -m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7.$

- Consider e.g. $\beta_{\lambda_1 - \lambda_2}$. Under FGOO trafo, must trafo like $(\lambda_1 - \lambda_2)$, i.e. as a $\mathbf{1}'$.
- Other $\mathbf{1}'$ covariants are $(\lambda_6 + \lambda_7)$ and κ_1, κ_2. ($m_{11}^2 + m_{22}^2$ has wrong mass dimension)

$\Rightarrow$ Beta function $\beta_{\lambda_1 - \lambda_2}$ to all orders(!) *can only* be given by

$$\beta_{\lambda_1 - \lambda_2} \; = \; (\lambda_1 - \lambda_2)\, f_+(\lambda_i) \; + \; (\lambda_6 + \lambda_7)\, g_+(\lambda_i) \; + \; \overbrace{\kappa_1^n \kappa_2^m}^{n+m=\text{odd}}\, h_+(\lambda_i) \; ,$$

where f_+, g_+, h_+ are functions of all λ_i that transform trivially.

Back to original question: RG stability of goofy parameter relations

FGOO 2HDM parameter relations: $m_{11}^2 = -m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7.$

- Consider e.g. $\beta_{\lambda_1-\lambda_2}$. Under FGOO trafo, must trafo like $(\lambda_1 - \lambda_2)$, i.e. as a $\mathbf{1}'$.
- Other $\mathbf{1}'$ covariants are $(\lambda_6 + \lambda_7)$ and κ_1, κ_2. ($m_{11}^2 + m_{22}^2$ has wrong mass dimension)

$\Rightarrow$ Beta function $\beta_{\lambda_1-\lambda_2}$ to all orders(!) *can only* be given by

$$\beta_{\lambda_1-\lambda_2} \;=\; (\lambda_1 - \lambda_2)\, f_+(\lambda_i) \;+\; (\lambda_6 + \lambda_7)\, g_+(\lambda_i) \;+\; \overbrace{\kappa_1^n \kappa_2^m}^{n+m=\mathrm{odd}}\, h_+(\lambda_i)\;,$$

where f_+, g_+, h_+ are functions of all λ_i that transform trivially.

- Turns out for 2HDM:
  The global sign of the kinetic term does not enter the β functions $\Leftrightarrow h_+ = 0$.
- Analogous arguments hold for $\beta_{m_{11}^2+m_{22}^2}$ and $\beta_{\lambda_6+\lambda_7}$.
- This shows that **if** FGOO relation is imposed, it is **not** violated in the RG flow.

$\curvearrowright$ Goofy symmetries are *explicitly* broken by $\mathcal{K} \neq 0$, but this breaking is **soft**!

New goofy transformations in 2HDM

[AT '25]

Explicit action of flavor-/CP-type global-sign-flipping **Goofy** transformations

$$\vec{\Phi} \equiv \left(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*\right)^{\mathrm{T}} , \qquad \vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & -S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ -X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Goofy trafo.	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	accidental regular sym.
	parameter relations										
$\mathcal{P}_G$ $(\equiv \mathbb{1} \oplus -\mathbb{1})$	0	0	0								$-$
$\mathbb{Z}_{2,G}$ $(\equiv \sigma_3 \oplus -\sigma_3)$	0	0							0	0	$-$
$\mathrm{CP1}_G$	0	0	${-m_{12}^2}^*$					λ_5^*	λ_6^*	λ_7^*	$-$
$\mathrm{CP2}_G$ (FGOO)		$-m_{11}^2$			λ_1					$-\lambda_6$	$-$
$\mathrm{U}(1)_G$	0	0	0					0	0	0	$\mathrm{U}(1)$
$\mathrm{CP3}_G$	0	0	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$	0	0	$\mathrm{CP2}$
$\mathrm{SU}(2)_G$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0	$\mathrm{SU}(2)$
$\mathrm{CP2}_G^{\mathrm{soft}}$					λ_1					$-\lambda_6$	$-$

Parameter relations for goofy transformations, RG stable to all orders in scalar and gauge quantum corrections.

These are *genuinely new* goofy transformations. FGOO discussed: $\mathrm{CP2}_G$ + all regular.

Applications of Goofy transformations

- An entirely new class of possible transformations and associated RG stable fixed points have been missed so far in *all* QFTs, all models, ...
- Bare scalar mass terms $\phi^\dagger\phi$ explicitly break some goofy transformations (non-zero mass gaps are possible).
  $\Rightarrow$ Goofy symmetries can be instrumental in solving EW hierarchy problem.
- Relative-kinetic-term-sign-flipping goofy trafos are **not** RG stable but expose RG sensitivity to *generation dependent* sign flips $\longrightarrow$ connection to flavor.
- Goofy transformations can constrain non-canonical kinetic terms.

  In particular, the non-trivial Kähler potential of SUSY theories. $\Rightarrow$ Possibility to remove a major roadblock for predictivity of many classes of flavor models (discrete symmetries, modular symmetries, ...)
  [Chen, Fallbacher, (Omura), Ratz, Staudt '12,'13], [Chen, Ramos-Sánchez, Ratz '19]

“Dynamical classicalization” and a request

- If goofy relations are imposed only on V, the “couplings” of the kinetic terms (WFR) K_{ij} still run under RG.
- In this case, vanishing kinetic term(s) are RG fixed points (symmetry is enhanced there).
- Approaching these points in RG flow, the theory approaches a regime where one or more of the fields become non-propagating
  “quasi-classical background” or “auxiliary” fields.

$\Rightarrow$ RG evolution *dynamically* approaches a quasi-classical regime!

- To fully explore this regime, and systematically explore the all-order constraints on WFR (anomalous dimensions), RGEs should be formulated starting from the most general possible basis.

$\curvearrowright$ Could track effect of goofy trafos on K_{ij} and get all-order constraints on WFR (anomalous dimensions), just as for the other couplings.

Working exclusively in the canonical basis one is blind to both of these effects!

Conclusions

- *Goofy* transformations by definition do not leave invariant the kinetic terms.
- Explicit breaking of goofy trafos in kinetic terms can be soft, and parameter relations enforced by goofy trafos can be stable to all orders in RG evolution.
- Symmetry of β functions $\geq$ symmetry of action. [AT '25], [De Boer, AT to appear]
- It is *mandatory* to include goofy transformations to understand all (partial) RG fixed points [RG fixed hyperplanes] of *any* QFT. [AT '25], [De Boer, AT to appear]
- Many of the most important puzzles in our theoretical understanding of Nature may be related to goofy transformations (e.g. EW hierarchy, flavor, . . .)
- Parameter regions of exact goofy symmetry are points where a QFT dynamically approaches a quasi-classical regime for some of the fields.
- There are many goofy avenues worth exploring...

. . . I cannot think about QFT *without* goofy transformations anymore.

Thank You!

Image credits: PNGaaa.com, Walt Disney "Hawaiian Holiday" 1937

Backup slides

2HDM – Regular and global-sign-flipping goofy

Explicit action of global **regular** flavor- and CP-type transformations: [AT '25]

$$\vec{\Phi} \equiv \left(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*\right)^{\mathrm{T}} , \qquad \vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Flavor- and CP-type **Goofy versions** of these transformations:

$$\vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & -S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ -X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Note: all of these are "global-sign-flipping goofy transformations" $\mathcal{K} \mapsto -\mathcal{K}$.

Explicit choices for matrix generators of 2HDM transformations ($\xi, \psi, \theta \in \mathbb{R}$)

$$\mathbb{Z}_{2,(G)} : \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \sigma_3 , \qquad \mathrm{CP1}_{(G)} : \quad X = \mathbb{1}_2 ,$$

$$\mathrm{U(1)}_{(G)} : \quad S = \begin{pmatrix} e^{-i\xi} & 0 \\ 0 & e^{i\xi} \end{pmatrix} , \qquad \mathrm{CP2}_{(G)} : \quad X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv \varepsilon ,$$

$$\mathrm{SU(2)}_{(G)} : \quad S = \begin{pmatrix} e^{-i\xi}\cos\theta & -e^{-i\psi}\sin\theta \\ e^{i\psi}\sin\theta & e^{i\xi}\cos\theta \end{pmatrix} , \qquad \mathrm{CP3}_{(G)} : \quad X = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} .$$

2HDM – Relative-sign-flipping goofy transformations

We can also construct transformations that act with $\kappa_1 = -\kappa_2 = \pm 1$.
Action of goofy transformations with relative sign flip of the 2HDM kinetic terms in flavor- or CP-type subvariants:

$$\vec{\Phi} \mapsto \begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & B^* \end{pmatrix} \vec{\Phi}\,, \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & C \\ D^* & \mathbf{0} \end{pmatrix} \vec{\Phi}\,.$$

With examples of explicit matrix generators given by

$$\mathbb{Z}_{2,G}^{-}: \qquad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ B^* = \mathbb{1}, \qquad C = D^* = 0\,,$$

$$\mathrm{CP4}_{G}^{-}: \qquad A = B^* = 0, \qquad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ D^* = \mathbb{1}\,.$$

$$\mathbb{Z}_{4,G}^{-}: \qquad A = B^* = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \qquad C = D^* = 0\,.$$

It turns out that none of these transformations is radiatively stable.
Reason: *The <u>relative</u> sign of the kinetic terms <u>does</u> enter the β functions of the 2HDM*.

Goofy transformations 101 – Goofy basis change

Consider multiplet of complex scalars $\phi_{a=1,\ldots,N} \in \mathbb{C}$ and hermitean conjugate fields $\phi_a^\dagger$.

Standard "canonical" kinetic term

$$\mathcal{K}[\phi,\phi^\dagger] \;=\; \partial_\mu\phi^\dagger\,\partial^\mu\phi \;=\; \partial_\mu\phi_a^\dagger\,\delta^{ab}\,\partial^\mu\phi_b\;.$$

Field space metric $\kappa^{ab}\phi_a^\dagger\phi_b$ is only unity, $\kappa^{ab}=\delta^{ab}$, in the **canonical basis.**
In any other basis $\kappa^{ab}\neq\delta^{ab}$. Recall that ϕ and $\phi^\dagger$ are independent degrees of freedom. In particular, we can always do a general *passive* **field redefinition**

$$\phi' \equiv V\phi\;, \qquad \phi'^\dagger \equiv \phi^\dagger U^\dagger\;.$$

Regular field redefinition: $U=V$, Goofy redefinition: $U\neq V$.
The kinetic term then reads

$$\mathcal{K}[\phi,\phi^\dagger] = \mathcal{K}[V^\dagger\phi',\phi'^\dagger U] = \partial_\mu\phi'^\dagger\left(UV^\dagger\right)\partial^\mu\phi'\;, \quad \Rightarrow\; \kappa = UV^\dagger\;.$$

This is nothing else than rotating the kinetic term **to** / **from** a non-canonical basis.

Note: this discussion holds for any term $(\phi^\dagger\phi)_{\mathbf{1}_0}$, (gauge-covariant) derivatives play no role here.

Goofy transformations 101 – Goofy basis change

We actually do this all the time in going to the canonical basis in the first place!
In the most general basis, the kinetic terms are

$$\mathcal{K} = \kappa^{ab}\,(\partial_\mu \phi_a)^\dagger (\partial^\mu \phi_b) \qquad \text{with} \qquad \kappa^{ab} = \kappa^{ba\,*}\ .$$

Wave function renormalization coefficients $\kappa^{ab} \equiv$ “couplings” of kinetic term.
Any hermitean matrix κ can be written as

$$\kappa \ = \ U^\dagger \begin{pmatrix} k_1 & 0 & \\ 0 & k_2 & \\ & & \ddots \end{pmatrix} U\,, \qquad (U \text{ unitary, eigenvalues } k_a \in \mathbb{R})$$

U’s can be absorbed in $\phi^\dagger$, ϕ by a *regular* field redefinition.
Canonical diagonal entries are obtained by subsequent rescaling (no sum)

$$\phi_a = \sqrt{k_a}\,\phi'_a\,, \qquad \text{and} \qquad \phi^\dagger_a = \sqrt{k_a}\,\phi'^\dagger_a\ .$$

Note: if any one of the $k_a < 0$, this “rescaling” <u>is</u> a *goofy* transformation.

Goofy transformations 101 – Active goofy transformations

Goofy transformations as *active* transformations (we assume unitary A, and B)

$$\phi \mapsto A\phi\,, \qquad \phi^\dagger \mapsto \phi^\dagger B^\dagger\,.$$

For canonical kinetic term this corresponds to a mapping

$$\phi_a^\dagger\,\delta^{ab}\,\phi_b \mapsto \phi_a^\dagger (B^\dagger A)^{ab}\phi_b\,.$$

For the action to stay real valued, $(B^\dagger A)$ is required to be hermitean and unitary. Therefore, $(B^\dagger A)^2 = \mathbb{1}$ is of order 2. This means there is a basis in which

$$\sum_a \partial_\mu\phi_a^\dagger\,\partial^\mu\phi_a \quad\longmapsto\quad \sum_a \kappa_a\,\partial_\mu\phi_a^\dagger\,\partial^\mu\phi_a \qquad \text{with} \qquad \kappa_a = \pm 1 \quad \text{(uncorrelated signs)}\,.$$

In this basis goofy transformation corresponds to pure sign flips of kinetic terms.