Modular Flavour Symmetries

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Summary of data: quark mixing

Wolfenstein parametrisation

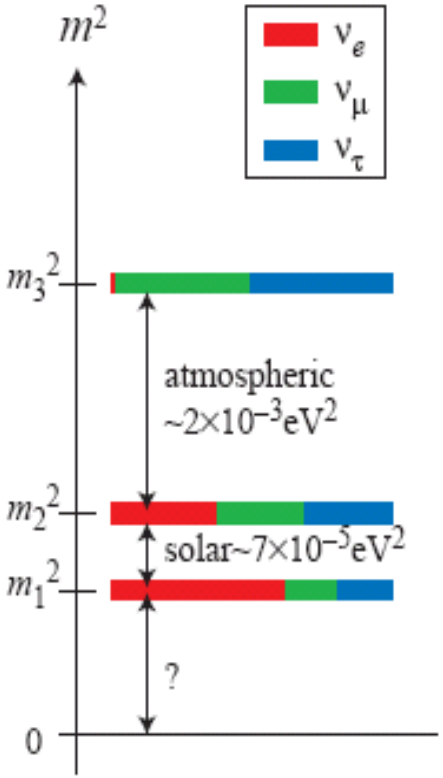
$$V_{CKM} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

$\lambda \simeq 0.23$ (Sine of the Cabibbo angle)

Summary of data: lepton mixing

Tri-bi-maximal (TBM) mixing

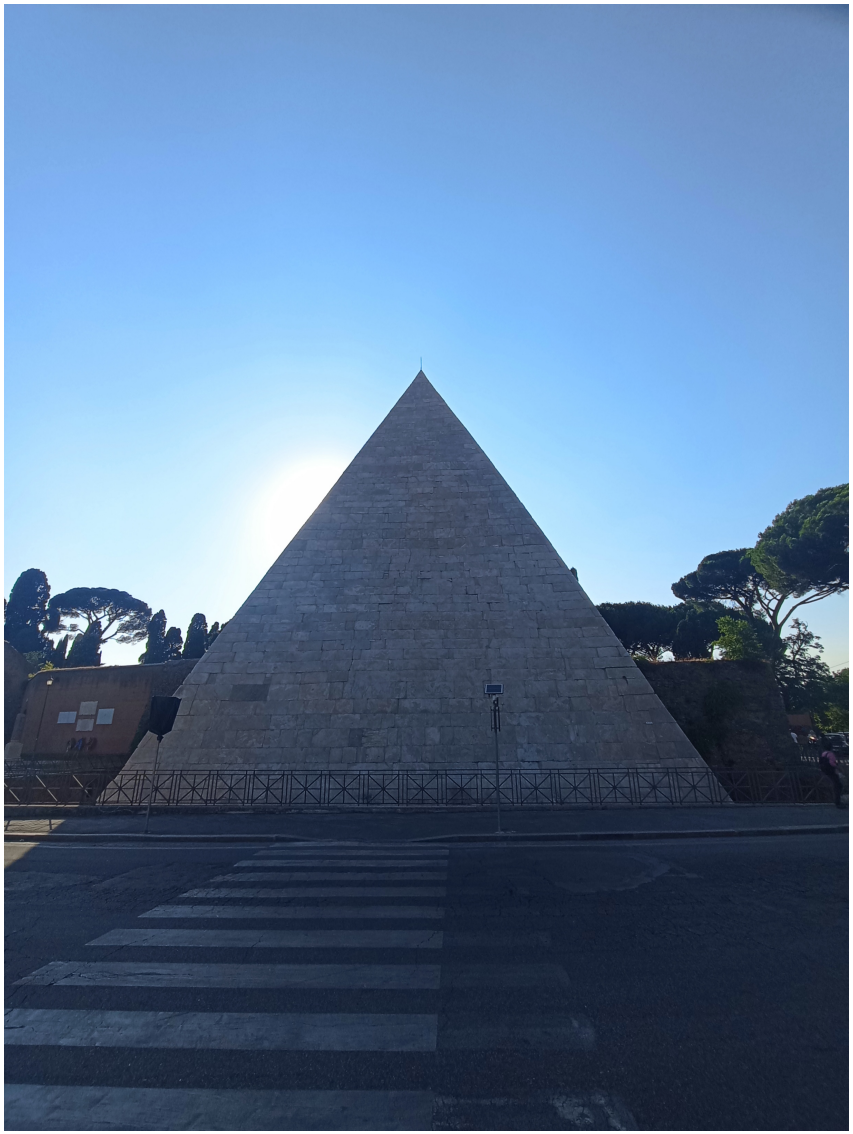
$$V_{PMNS} \simeq \begin{pmatrix} -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}$$



TBM history

See talk by Harrison
https://agenda.infn.it/event/42270/contributions/263811/

Non-Abelian Discrete Symmetries (NADS)



NADS


PACK INCLUDES:
x1 NAD'S BRAZILIAN & BIKINI WAX
SPATULAS x2
PRECISION SHAPING STICK x1
NAD'S POST WAX CALMING OIL WIPES x3
Nad's BRAZILIAN AND BIKINI WAX
Nad's POST WAX CALMING OIL WIPES
x4 CHEEKY BIKINI SHAPES

Some NADS

$\Delta(6)$, $\Delta(12)$, $\Delta(24)$, $\Delta(27)$, $\Delta(54)$, ...

Serving a Flavour triplet

Multiple Modular?

Multiple Modulus enables

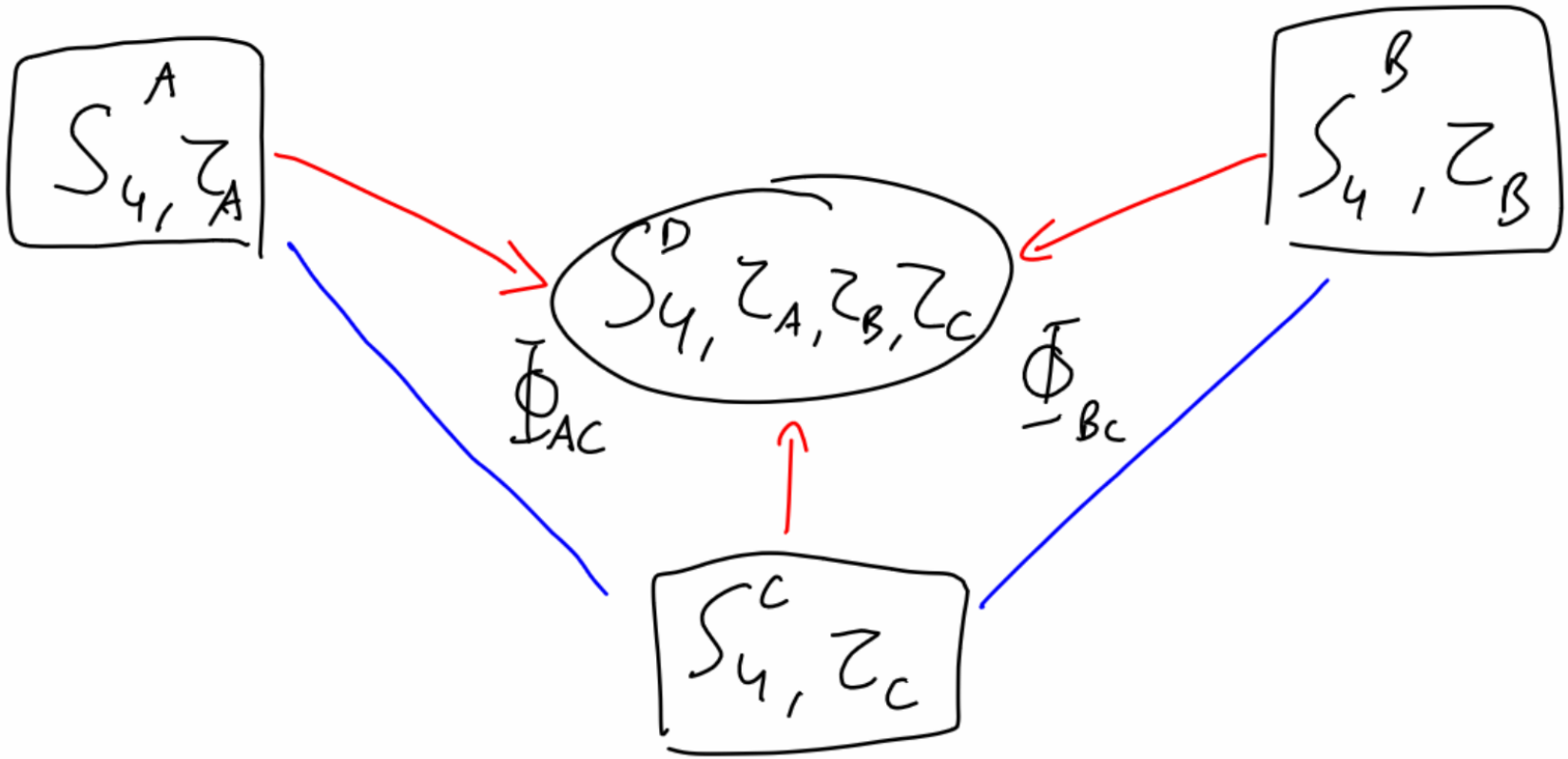
- Multiple stabilizers / fixed points
- Stabilizers outside fundamental domain

Modular Models with residual symmetries!

The framework

IdMV, King, Zhou

https://arxiv.org/abs/1906.02208



The framework (paper version)

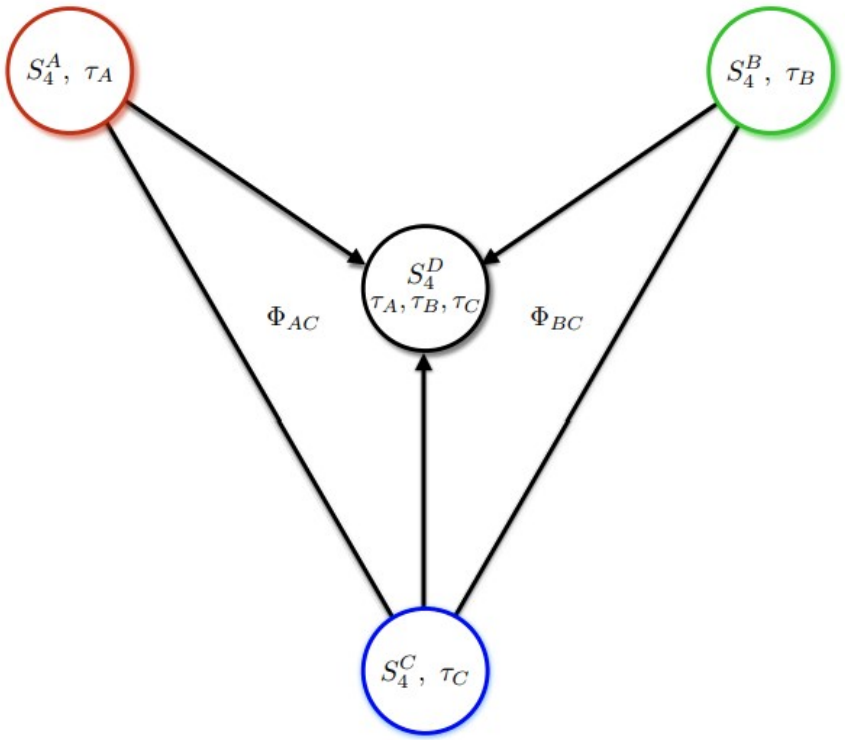


Figure 1: Illustration of the breaking of $S_4^A \times S_4^B \times S_4^C \to S_4^D$, identified as the diagonal subgroup, via the VEVs of Φ_{AC} and Φ_{BC}.

Example model with 3 S_4

Field	S_4^A	S_4^B	S_4^C	$2k_A$	$2k_B$	$2k_C$
L	**1**	**1**	**3**	0	0	0
e^c	**1**	**1**	**1**	0	0	-6
μ^c	**1**	**1**	**1**	0	0	-4
τ^c	**1**	**1**	**1**	0	0	-2
N_A^c	**1**	**1**	**1**	-6	0	0
N_B^c	**1**	**1**	**1**	0	-4	0
Φ_{AC}	**3**	**1**	**3**	0	0	0
Φ_{BC}	**1**	**3**	**3**	0	0	0

Example superpotential

$$
\begin{aligned}
w_\ell \;=\; & \frac{1}{\Lambda}\left[L\Phi_{AC}Y_A(\tau_A)N_A^c + L\Phi_{BC}Y_B(\tau_B)N_B^c\right]H_u \\
& + \left[LY_e(\tau_C)e^c + LY_\mu(\tau_C)\mu^c + LY_\tau(\tau_C)\tau^c\right]H_d \\
& + \frac{1}{2}M_A(\tau_A)N_A^cN_A^c + \frac{1}{2}M_B(\tau_B)N_B^cN_B^c + M_{AB}(\tau_A,\tau_B)N_A^cN_B^c
\end{aligned}
$$

Example effective superpotential

$$w_\ell^{\text{eff}} = \left[\frac{v_{AC}}{\Lambda} L Y_A(\tau_A) N_A^c + \frac{v_{BC}}{\Lambda} L Y_B(\tau_B) N_B^c\right] H_u + (\ldots)$$

Multiple Multiple Modular Models

King, Zhou
https://arxiv.org/abs/1908.02770 (2 S_4)
King, Zhou
https://arxiv.org/abs/2103.02633 ($SU(5)$)
IdMV, Lourenço
https://arxiv.org/abs/2107.04042 (2 A_4)
IdMV, Lourenço
https://arxiv.org/abs/2206.14869 (2 A_5)
IdMV, King, Levy
https://arxiv.org/abs/2211.00654 (Littlest Modular)
IdMV, King, Levy
https://arxiv.org/abs/2309.15901 ($SU(5)$ Littlest Modular)

Littlest Modular Seesaw

Ding, King, Liu, Lu
https://arxiv.org/abs/1910.03460
Ding, King, Yao
https://arxiv.org/abs/2103.16311

Littlest Modular Seesaw

$$\tau_A = \frac{1}{2} + \frac{i}{2} : \qquad Y^{(4)}_{\mathbf{3'}}(\tau_A) = (0, -1, 1)\,, \tag{1}$$

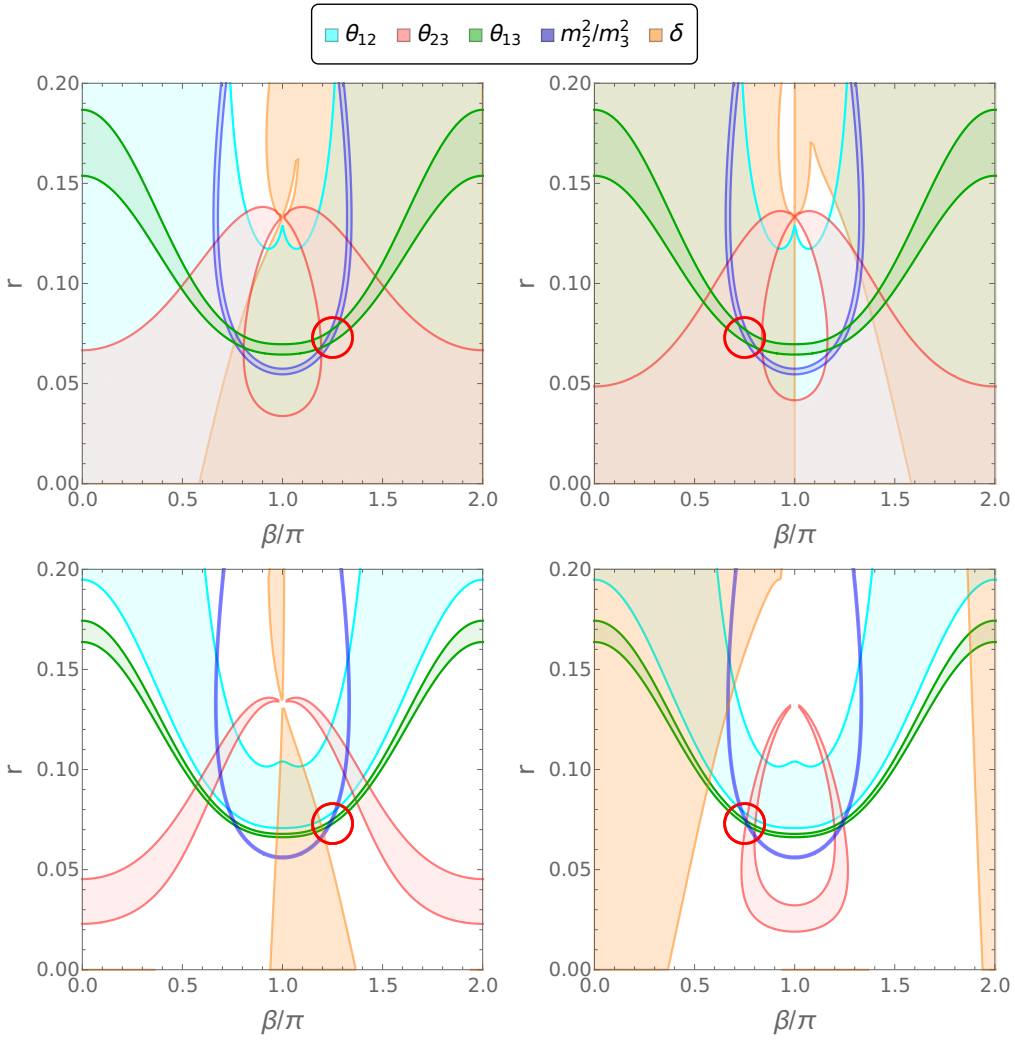
for one of the Dirac mass matrix columns, and

$$\tau_B = \frac{3}{2} + \frac{i}{2} : \qquad Y^{(2)}_{\mathbf{3'}}(\tau_B) = (1, 1-\sqrt{6}, 1+\sqrt{6})\,, \tag{2}$$

or

$$\tau_B = -\frac{1}{2} + \frac{i}{2} : \qquad Y^{(2)}_{\mathbf{3'}}(\tau_B) = (1, 1+\sqrt{6}, 1-\sqrt{6})\,, \tag{3}$$

LMS Results


θ_{12} θ_{23} θ_{13} m_2^2/m_3^2 δ
r
β/π
0.00
0.05
0.10
0.15
0.20
0.0
0.5
1.0
1.5
2.0

Automorphic Forms (multiple moduli)

Ding, Feruglio, Liu

https://arxiv.org/abs/2010.07952

Modular Hierarchies

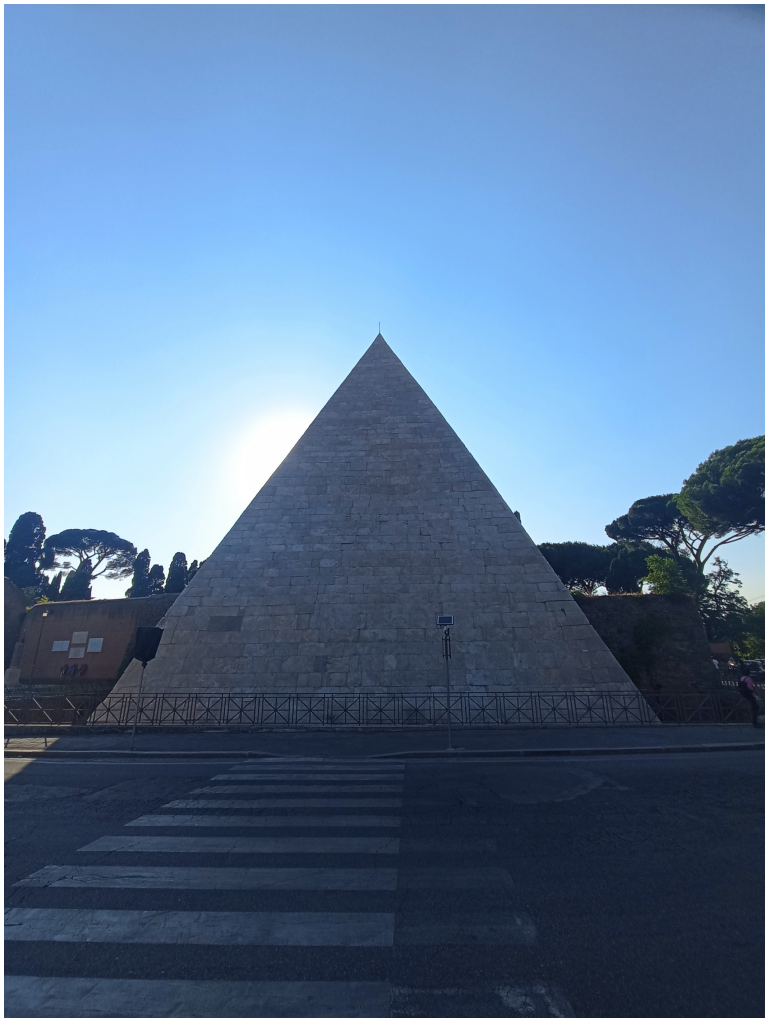
See talk by King
https://agenda.infn.it/event/42270/contributions/267075/

King, King
https://arxiv.org/abs/2002.00969
IdMV, King, Levy
https://arxiv.org/abs/2211.00654
Hierarchies with weightons

Novichkov, Penedo, Petcov
https://arxiv.org/abs/2102.07488
IdMV, Levy, Penedo, Petcov
https://arxiv.org/abs/2307.14410
Hierarchies near stabilisers

Grand Unified Theories (GUTs)

GUT $\times$ NADS



$SU(5)$ with Littlest Modular

IdMV, King, Levy
`https://arxiv.org/abs/2309.15901` ($SU(5)$ Littlest Modular)

$$Y_u = \begin{pmatrix} y_{uu}\epsilon_T^4 + y'_{uu}\epsilon_F^3\epsilon_T & y_{uc}\epsilon_T^3 + y'_{uc}\epsilon_F^3 & y_{ut}\epsilon_T^2 \\ . & y_{cc}\epsilon_T^2 & y_{ct}\epsilon_T \\ . & . & y_{tt} \end{pmatrix},$$

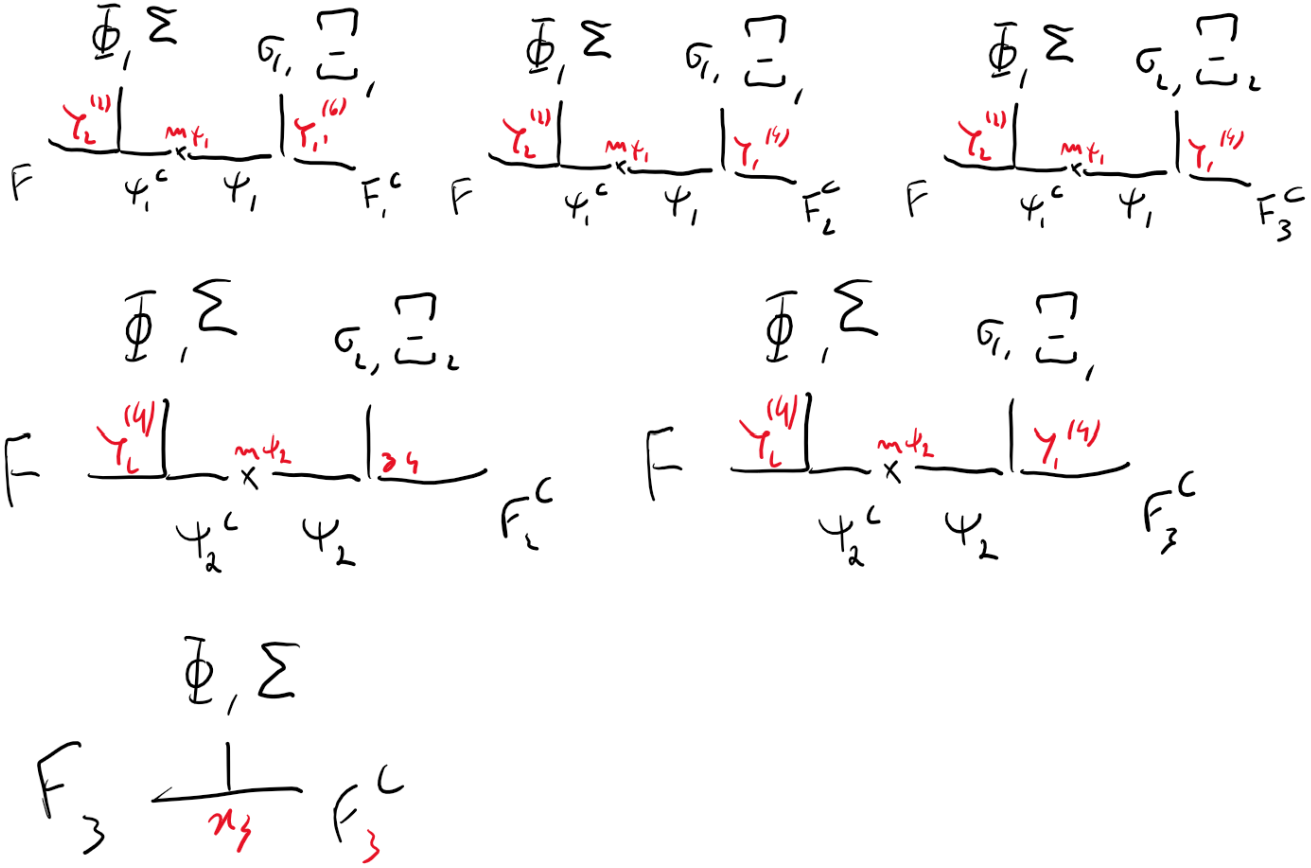
$$Y_d = \begin{pmatrix} y_{dd}\epsilon_F^3 & y_{ds}\epsilon_F^2\epsilon_T & y_{db}\epsilon_F\epsilon_T^2 \\ 0 & y_{ss}\epsilon_F^2 & y_{sb}\epsilon_F\epsilon_T \\ 0 & 0 & y_{bb}\epsilon_F \end{pmatrix} \qquad Y_\ell = \begin{pmatrix} y_{ee}\epsilon_F^3 & 0 & 0 \\ y_{\mu e}\epsilon_F^2\epsilon_T & y_{\mu\mu}\epsilon_F^2 & 0 \\ y_{\tau e}\epsilon_F\epsilon_T^2 & y_{\tau\mu}\epsilon_F\epsilon_T & y_{\tau\tau}\epsilon_F \end{pmatrix}.$$

Pati-Salam GUT with Modular S_3

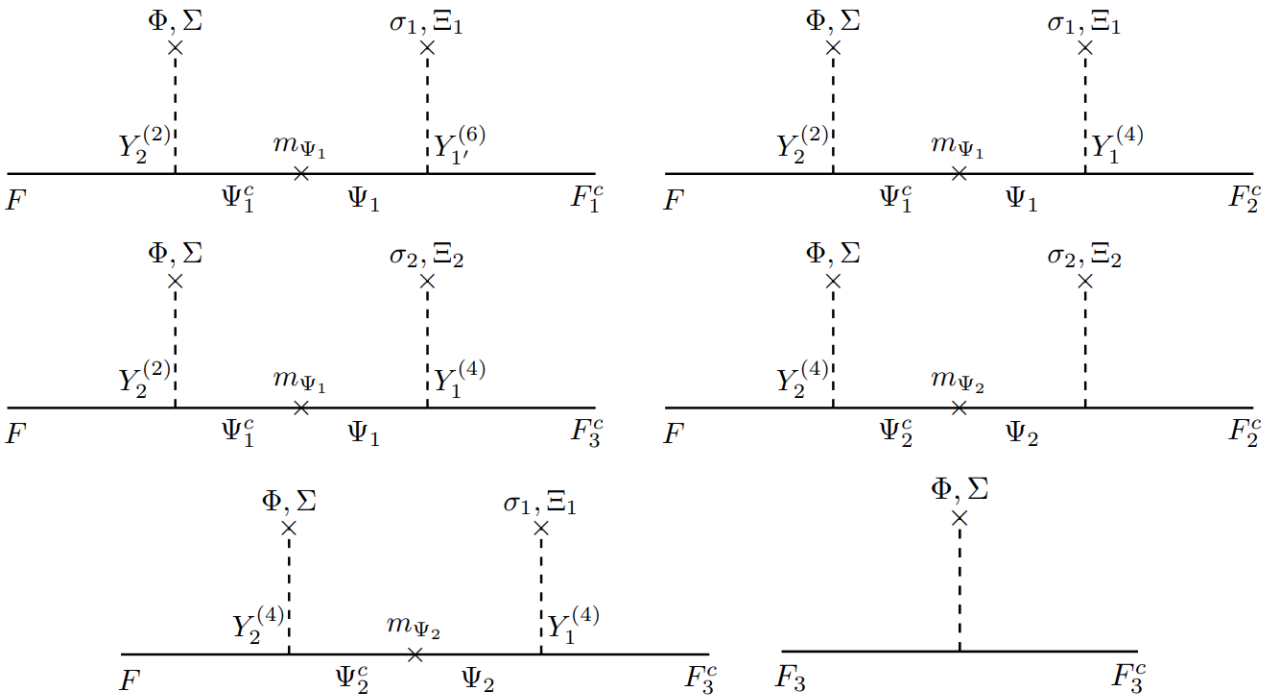
Cárcamo Hernández, IdMV, King, Vishnudath

https://arxiv.org/abs/2503.14610

PS S3 diagrams



PS S3 diagrams (paper version)



Modular Flavour Symmetries

- Interesting and ambitious to build models GUT $\times$ NADS.
- Modular symmetries can play key roles.
- Viable and predictive models.

For the Portuguese participants

Figure: Tripeiro, Tripperia - Roma, 2025/07