Topological Defects from Non-Abelian Discrete Symmetries

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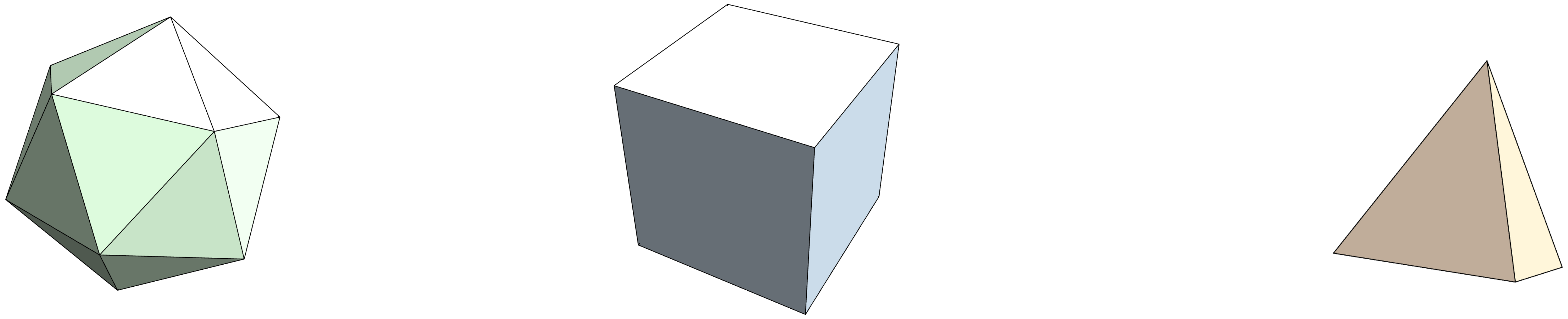
Talk based on work with B. Fu, S. King, L. Marsili, S. Pascoli & Y.L. Zhou

Based on 2409.16359

Models of Flavour

Discrete non-Abelian symmetries succesfully explain leptonic mixing



non-Abelian symmetry broken spontaneously $\longrightarrow$ residual Abelian symmetries

Explicit symmetry introduced to induce non-zero θ_{13}

Sum rules are predicted and can be used to distinguish different flavour symmetries

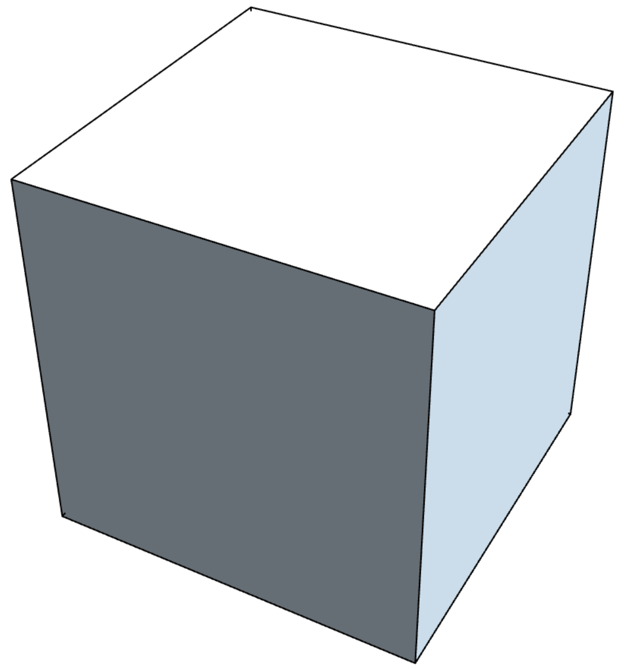
S4 Leptonic Flavour Symmetry

Hagedorn, King & Luhn, 1205.3114

Ding & Zhou, 1304.2645

Luhn, 1306.2358

King & Luhn, 1607.05276



Consider S_4 as high scale symmetry

$$S^2 = T^3 = (ST)^3 = U^2 = (SU)^2 = (TU)^2 = (STU)^4 = 1$$

$$T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Ma & Rajasekaran, 0106291

S_4 broken spontaneously by $\phi = \left(\phi_1, \phi_2, \phi_3\right)^T$, $3'$ of S_4

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

$$I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

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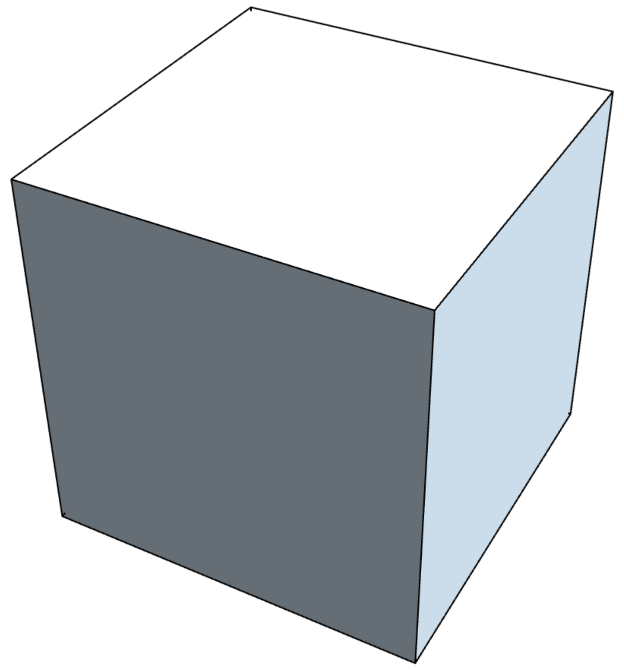

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$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2$$

$$\beta = \frac{g_2}{g_1}$$

$$I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2$$

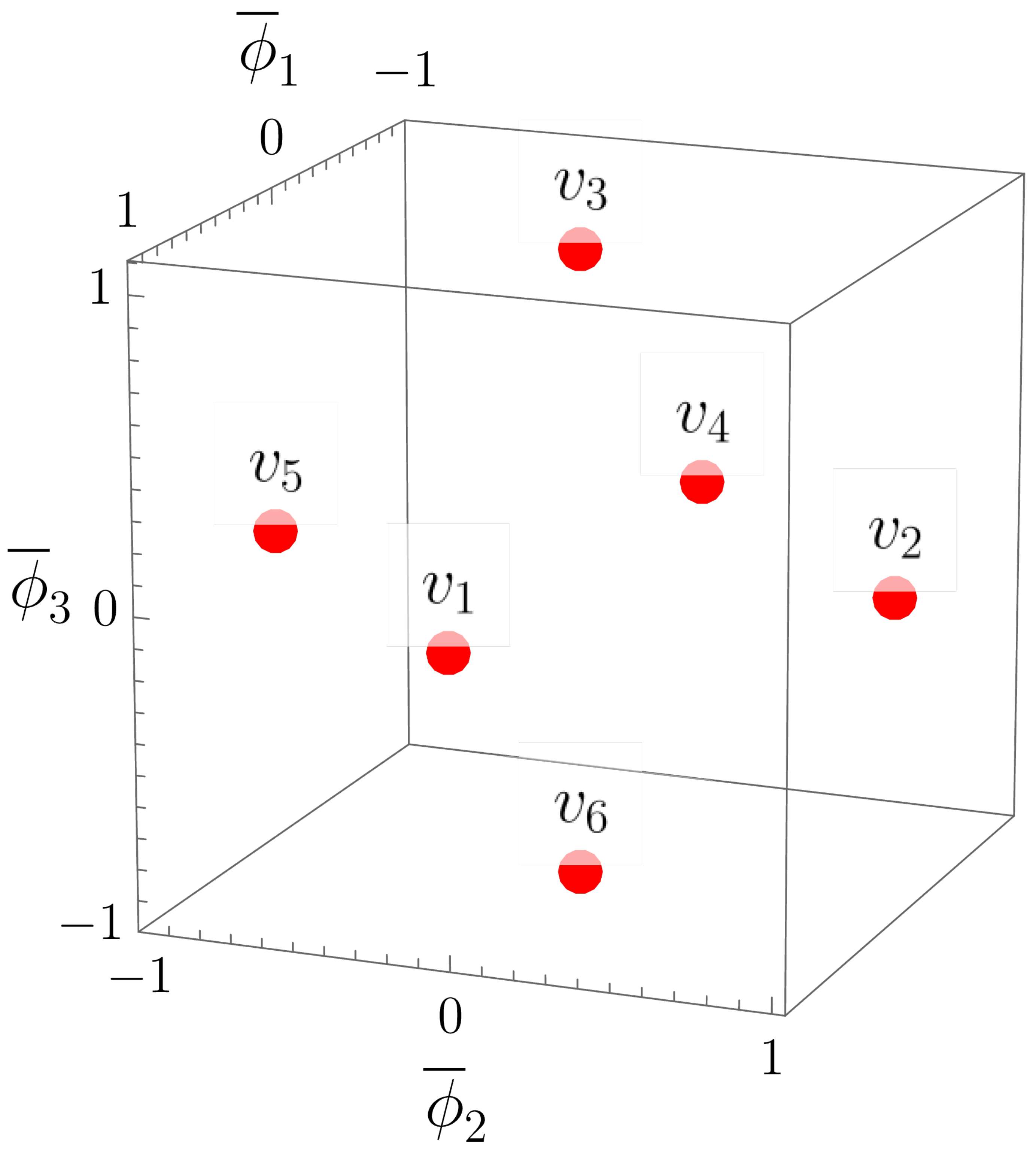
$$I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$

S4 Leptonic Flavour Symmetry

VEVs of scalars can be found, $\mathbb{Z}_2$-invariant alignments:

$$v_m \in \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \right\} v \qquad v = \frac{\mu}{\sqrt{g_1}}$$

2409.16359 Fu, King, Marsili, Pascoli, JT, Zhou

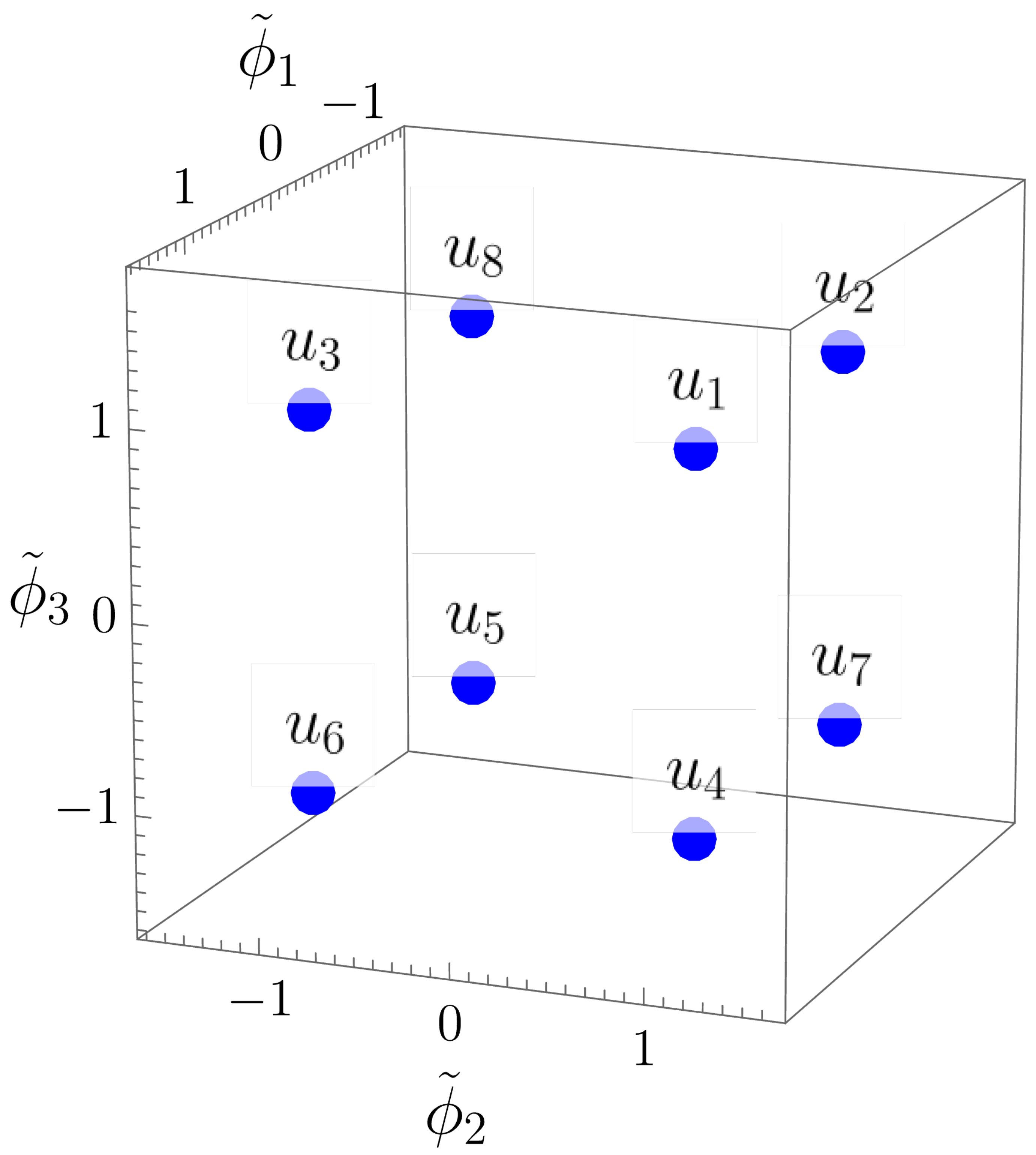


S4 Leptonic Flavour Symmetry

VEVs of scalars can be found, $\mathbb{Z}_3$-invariant alignments:

$$u_n = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\} u \qquad u = \frac{\mu}{\sqrt{3g_1 + 2g_2}}$$

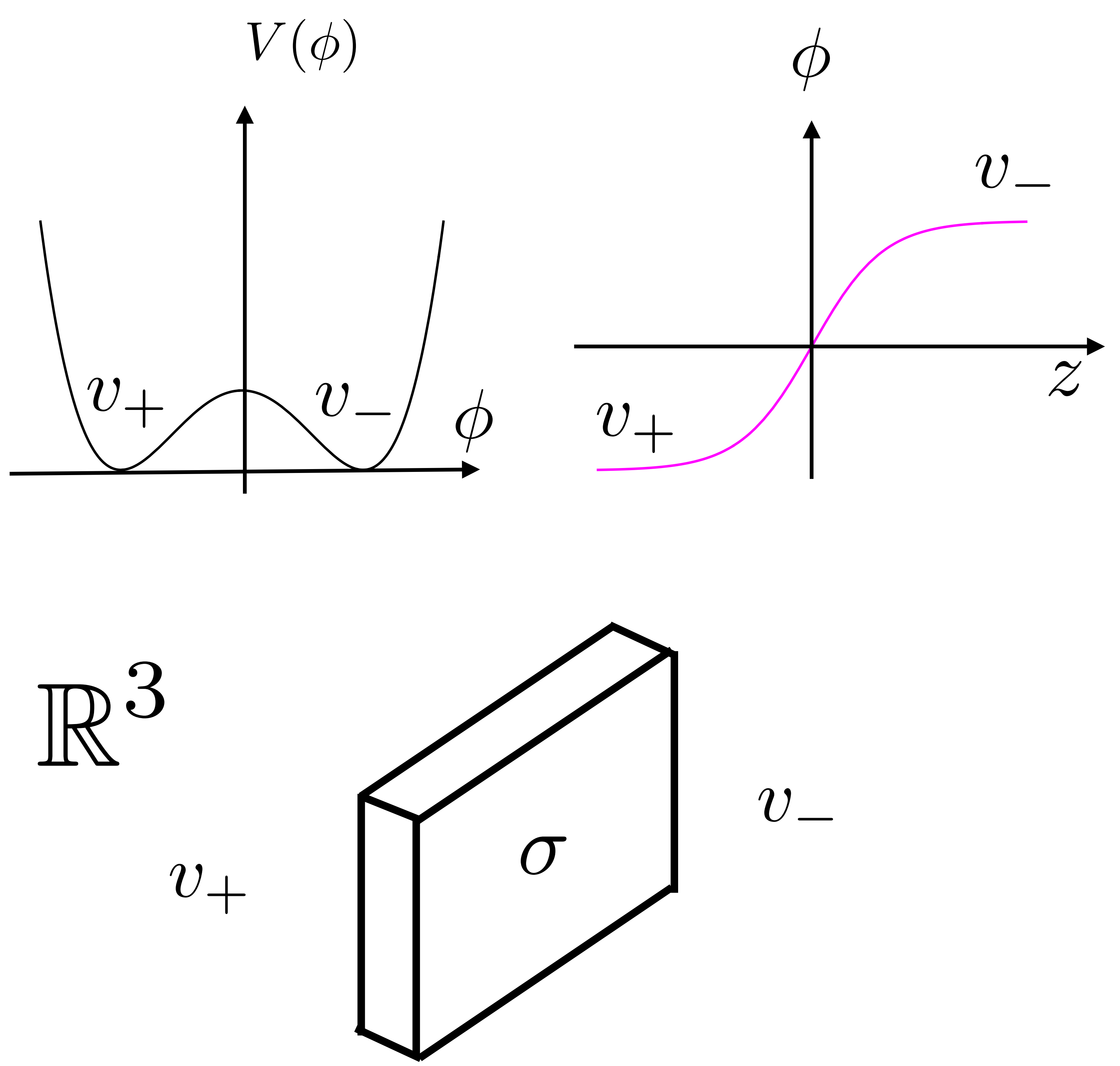
2409.16359 Fu, King, Marsili, Pascoli, JT, Zhou



topological defects

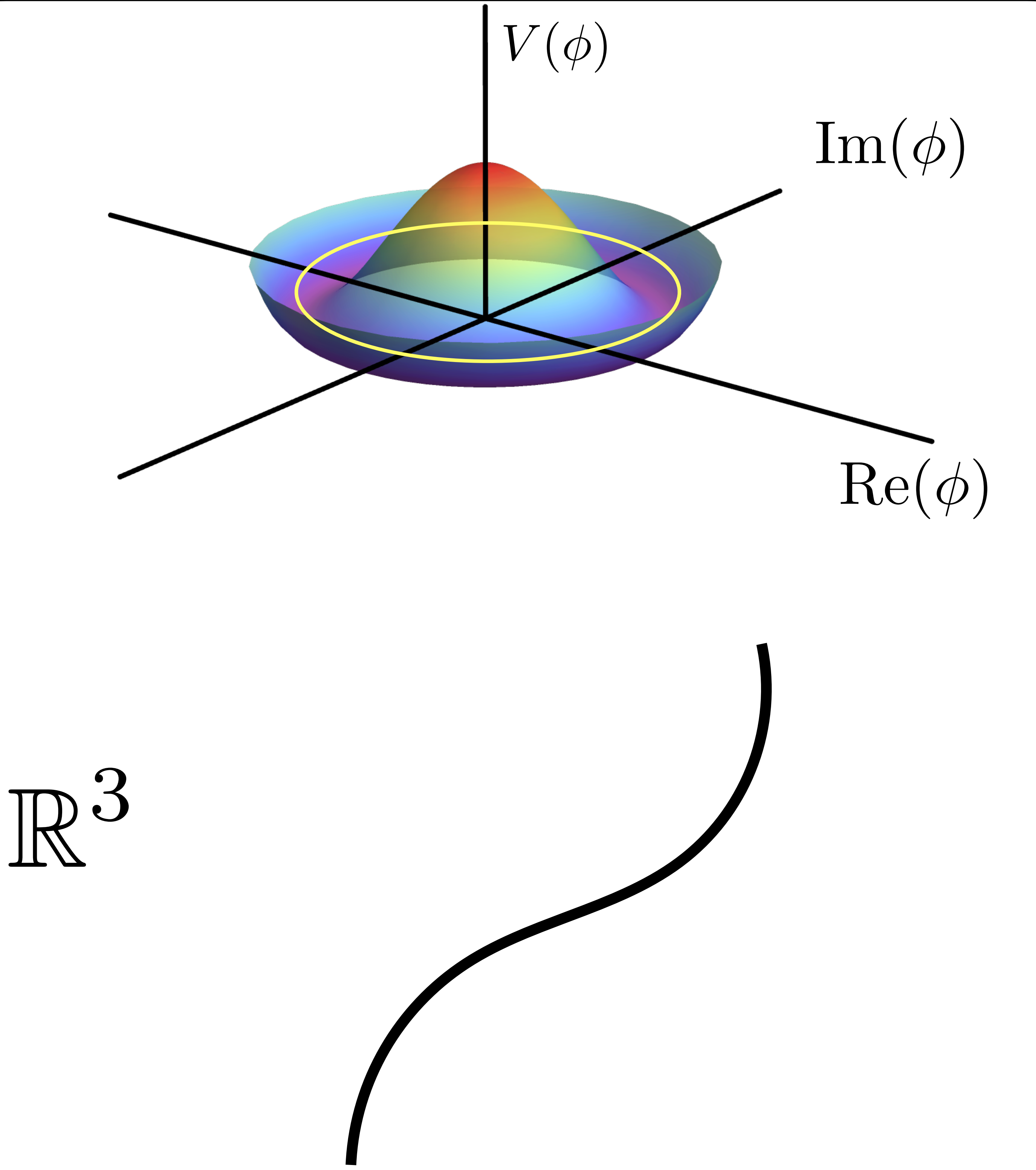
During SSB from $G_{GUT} \rightarrow H \rightarrow \cdots \rightarrow G_{SM}$ topological defects may form.

Domain wall



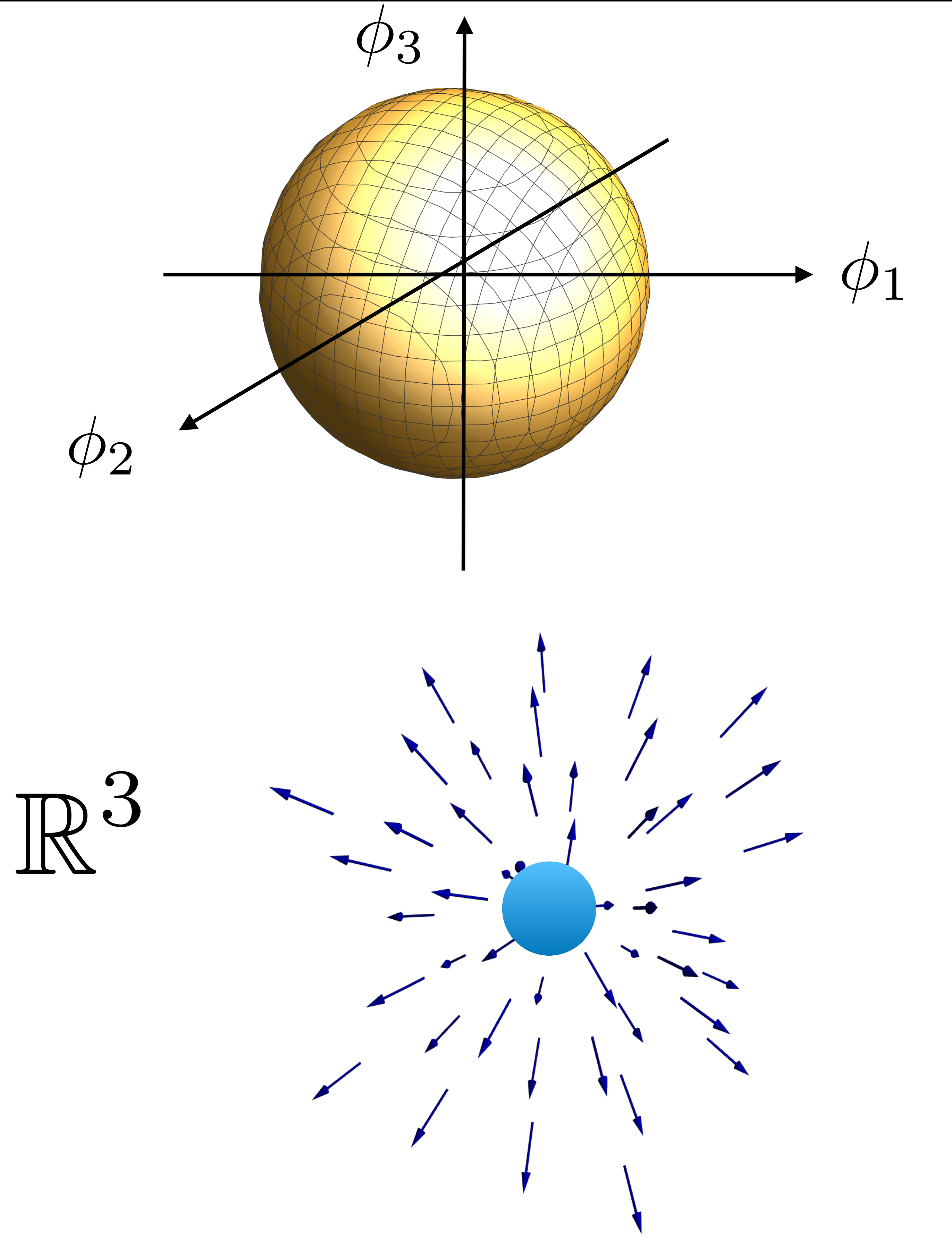
$\pi_0(\mathrm{G/H}) \neq 0$

Cosmic string



$\pi_1(\mathrm{G/H}) \neq 0$

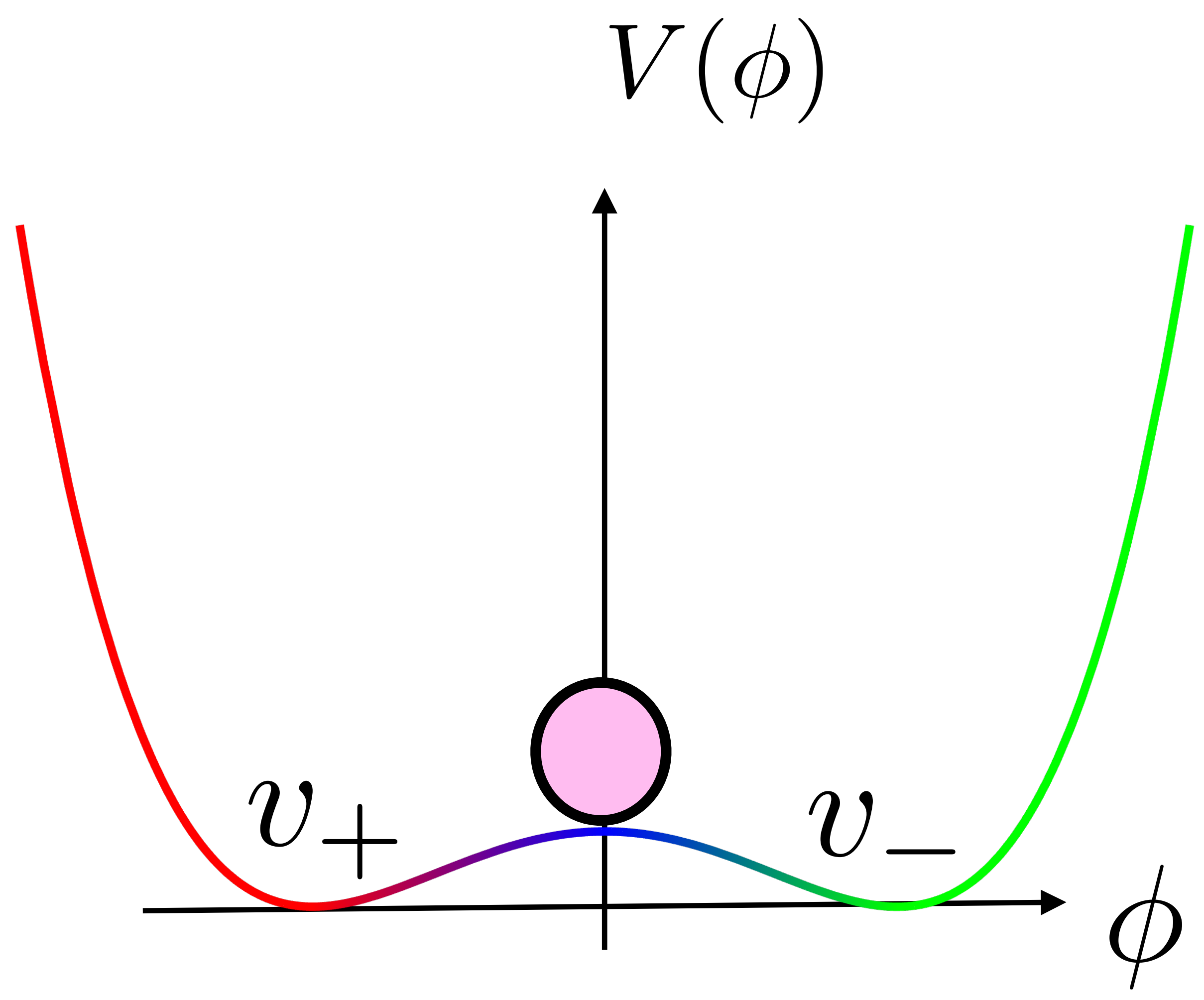
Monopole



$\pi_2(\mathrm{G/H}) \neq 0$

Domain Walls

Consider a real scalar field with $\mathbb{Z}_2$-symmetric potential



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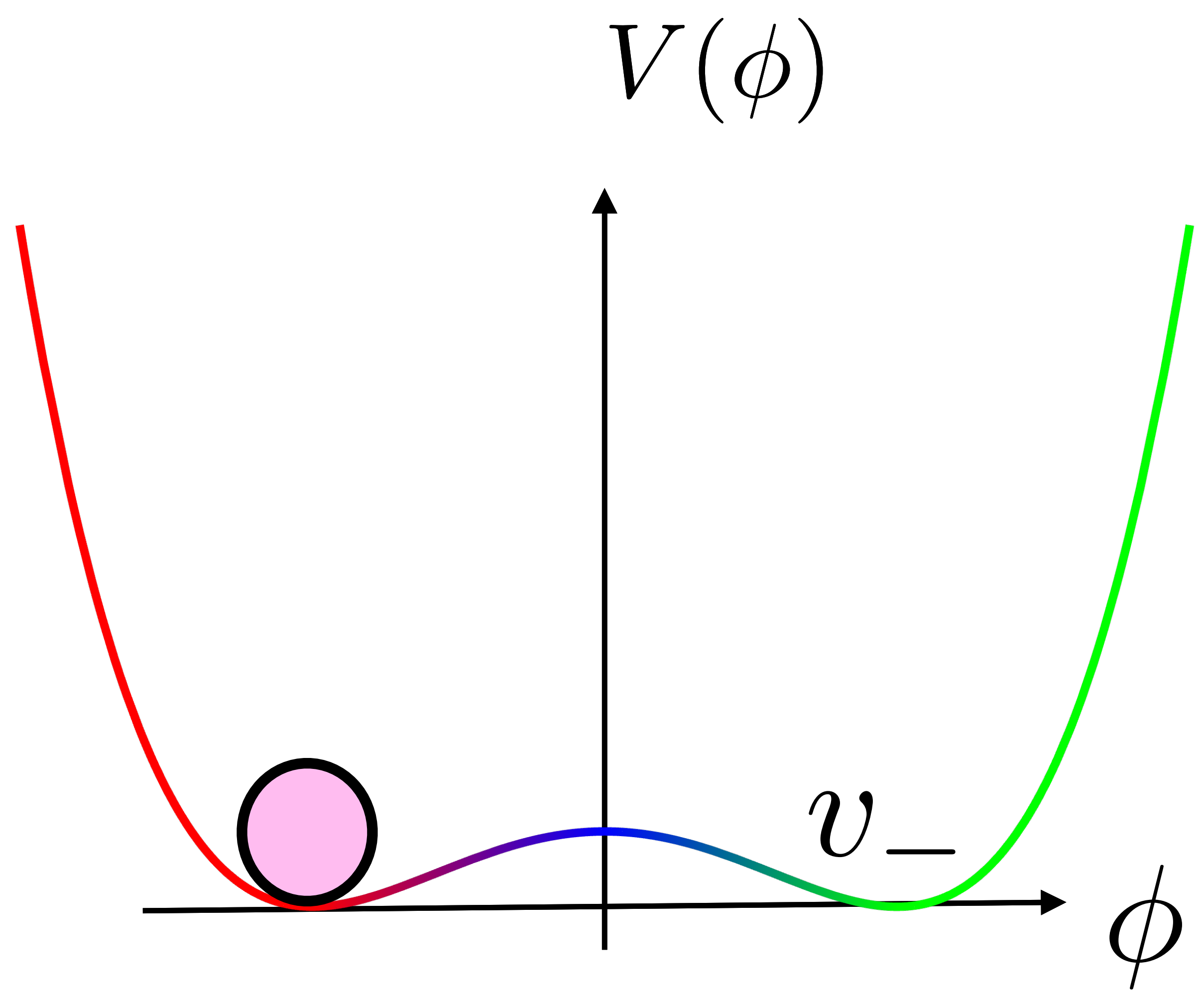

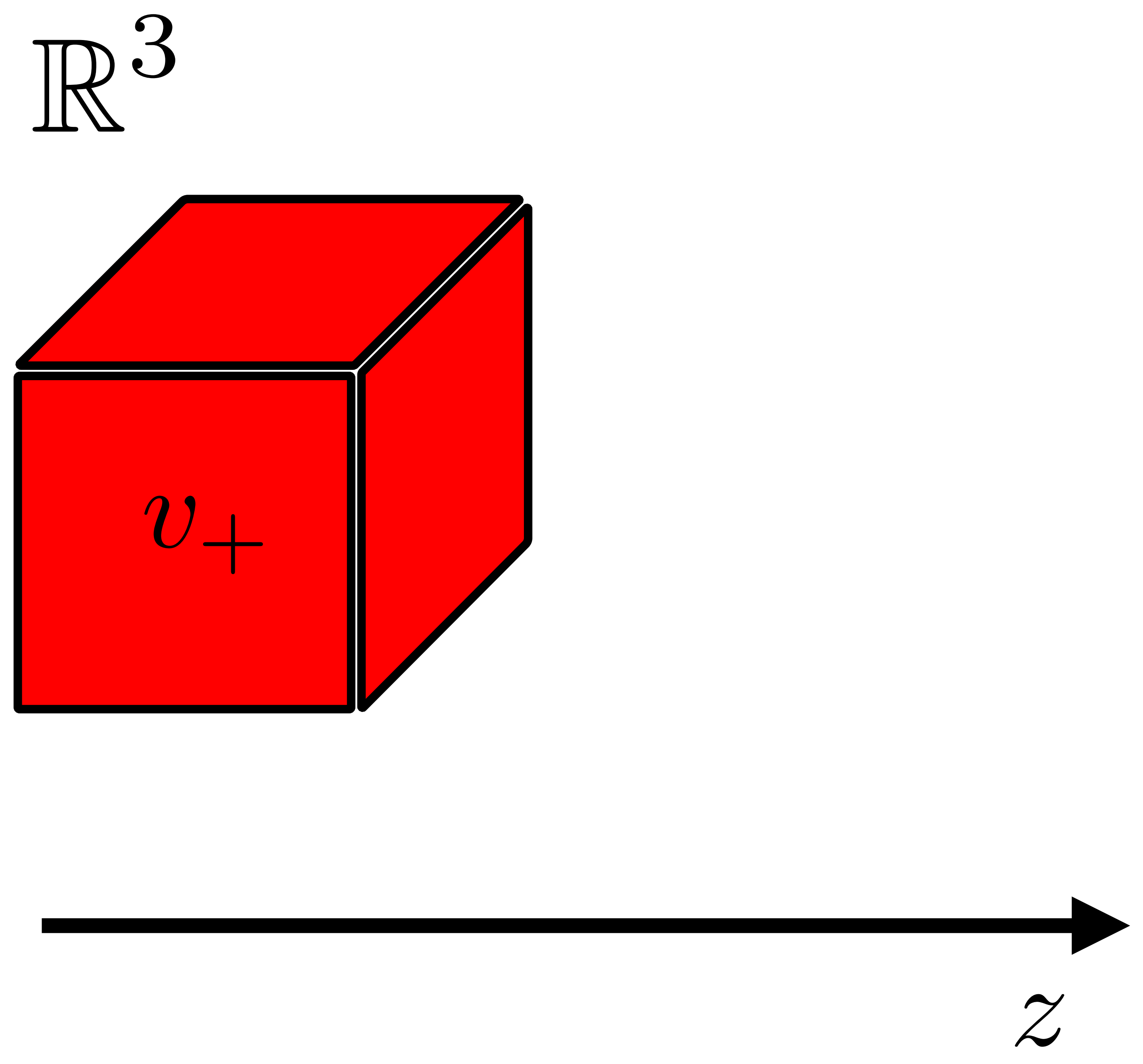

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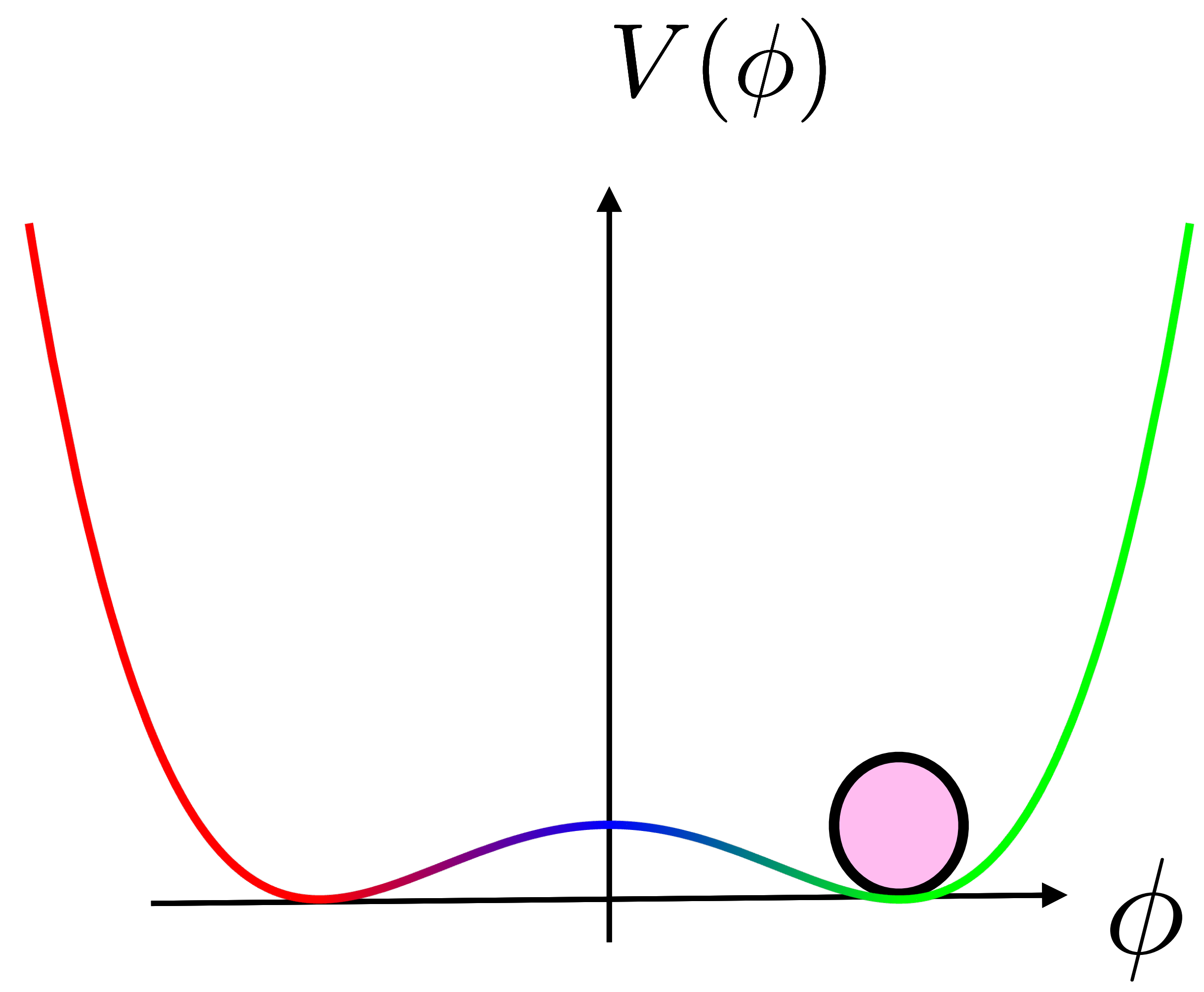

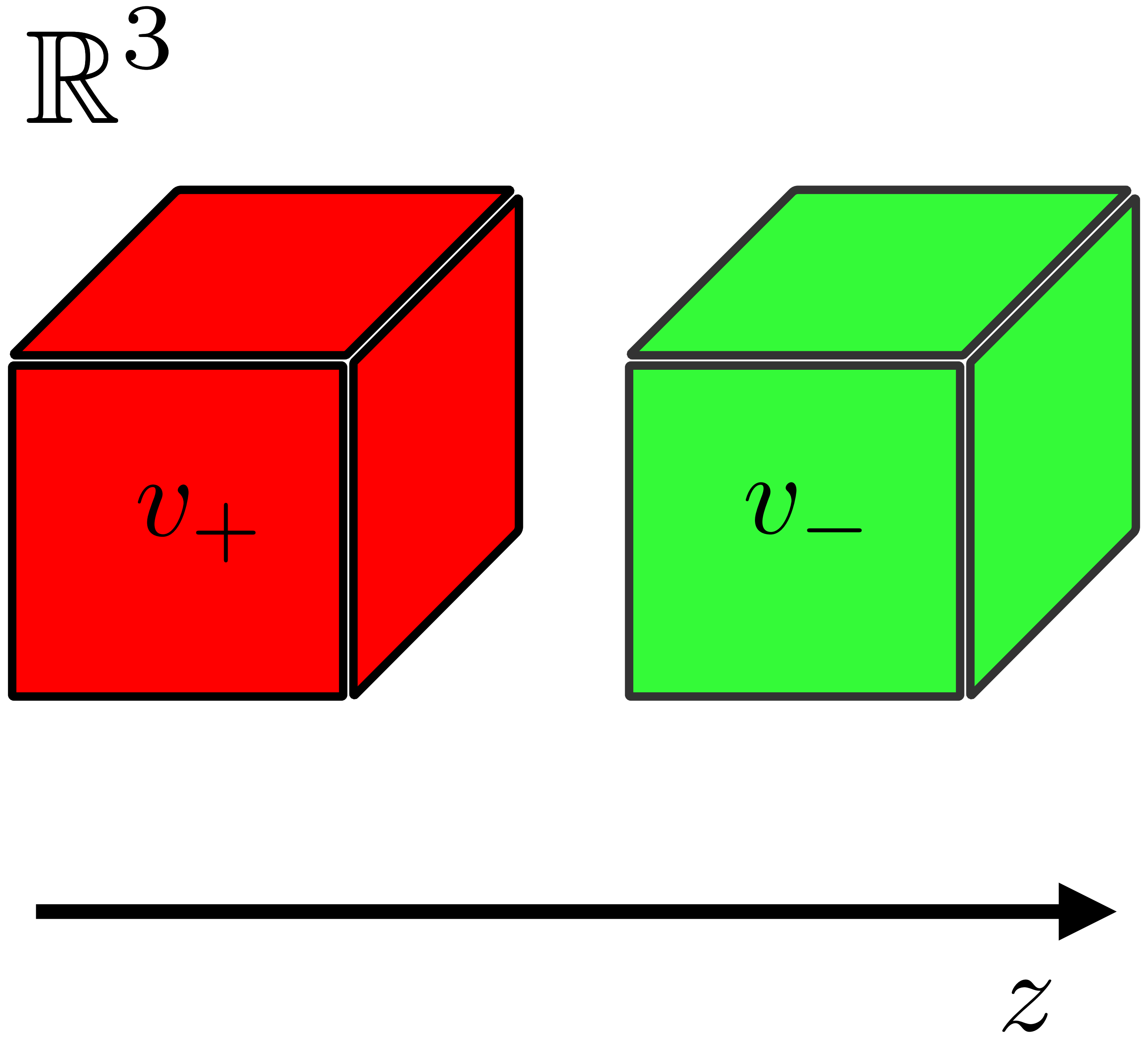

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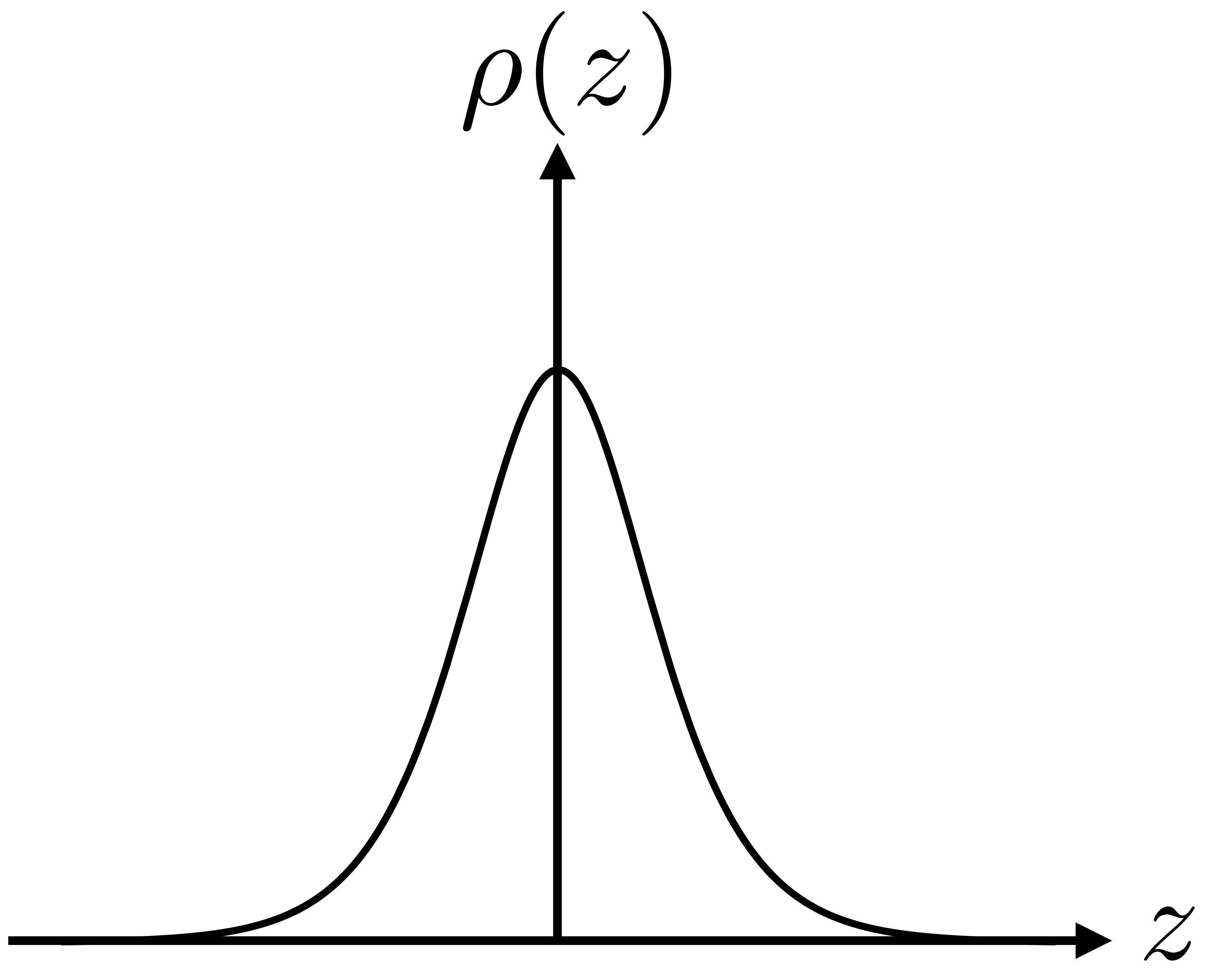
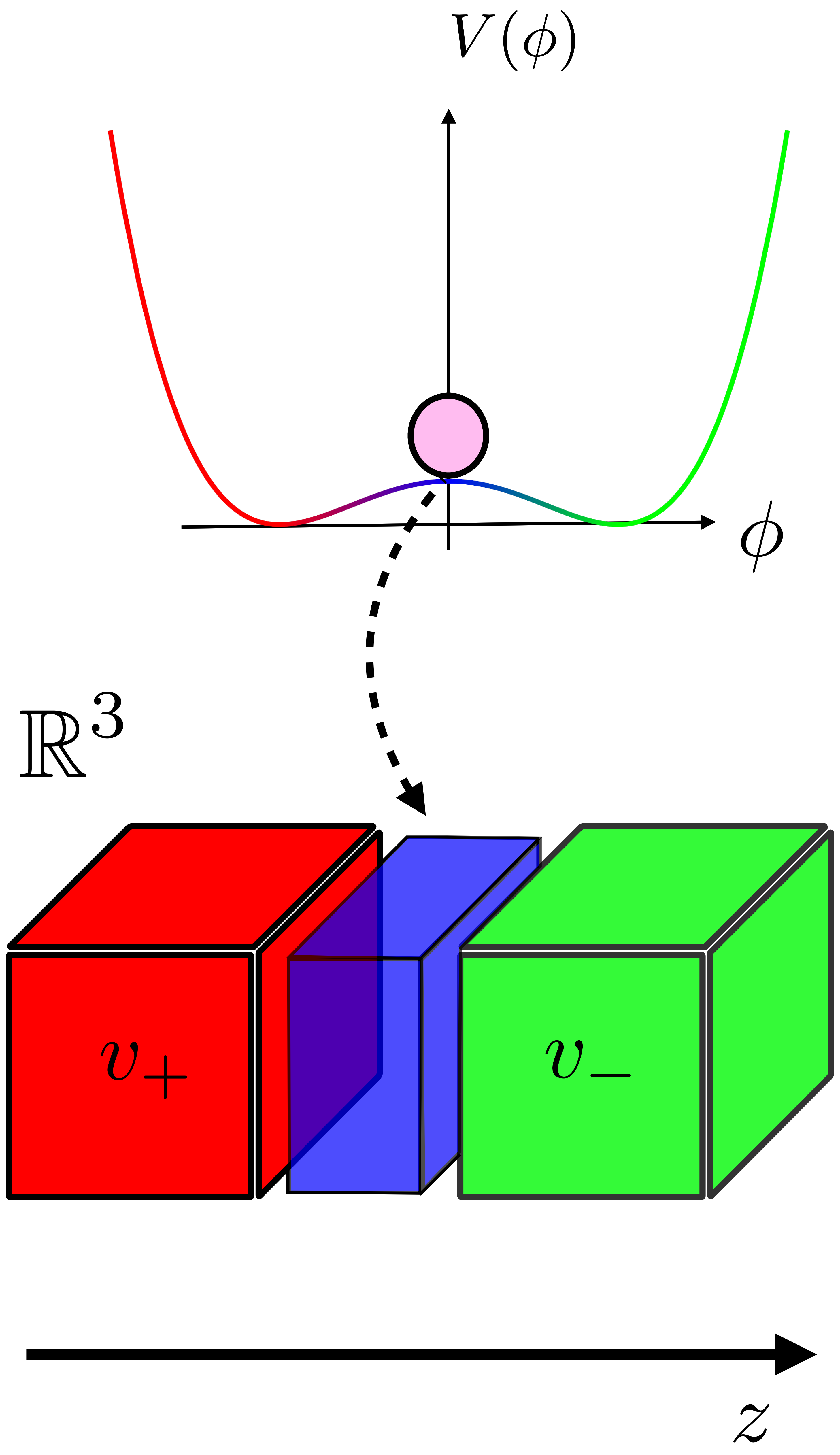




Domain Walls

Static EoM for scalar field:

$$\frac{d^2\phi_i(z)}{dz^2} = \frac{\partial V(\phi)}{\partial \phi_i}$$

Energy density of domain wall:

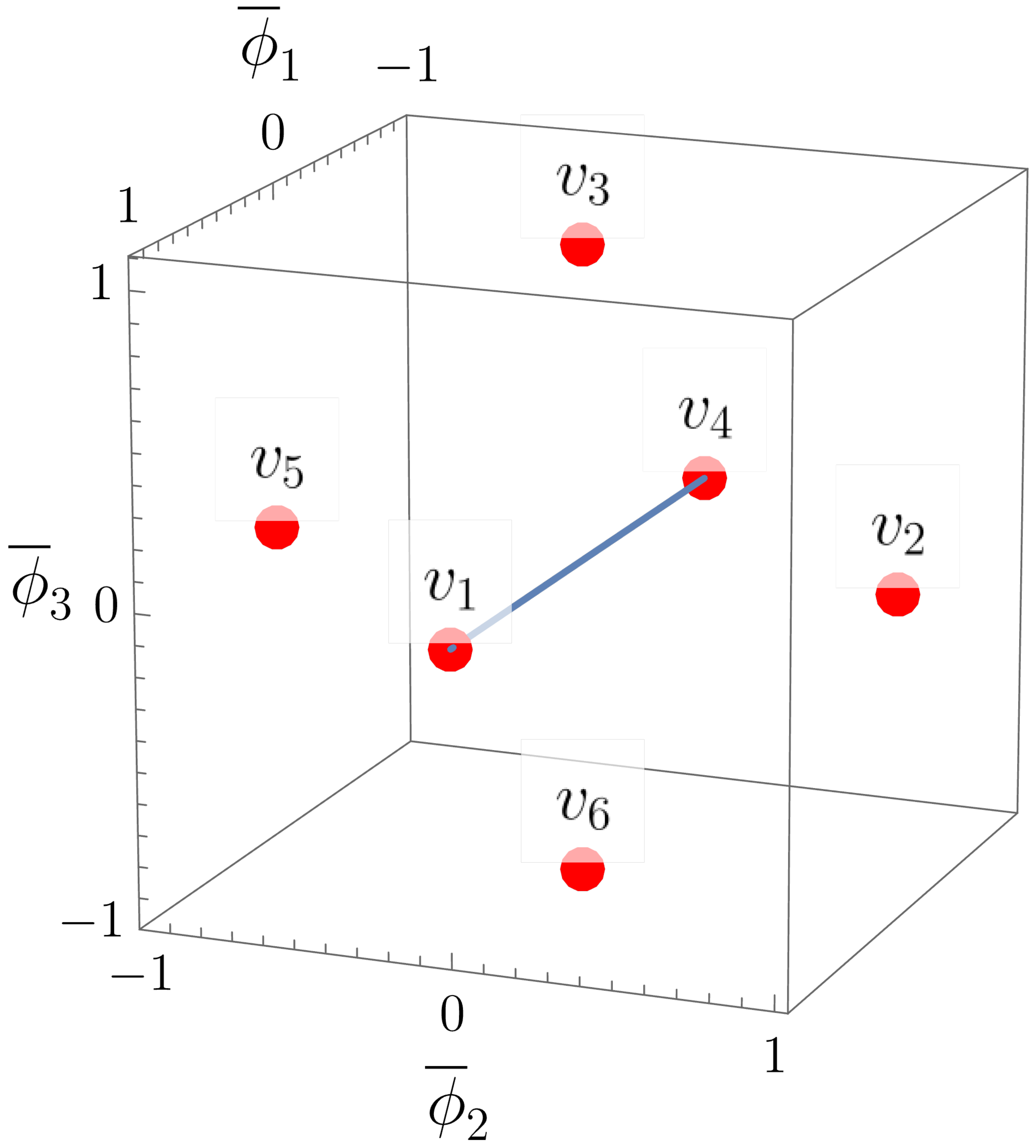
$$\rho(z) = \sum_i \underbrace{\frac{1}{2}\left[\phi_i'(z)\right]^2}_{\text{Gradient}} + \underbrace{\Delta V(\phi(z))}_{\text{Potential}}$$

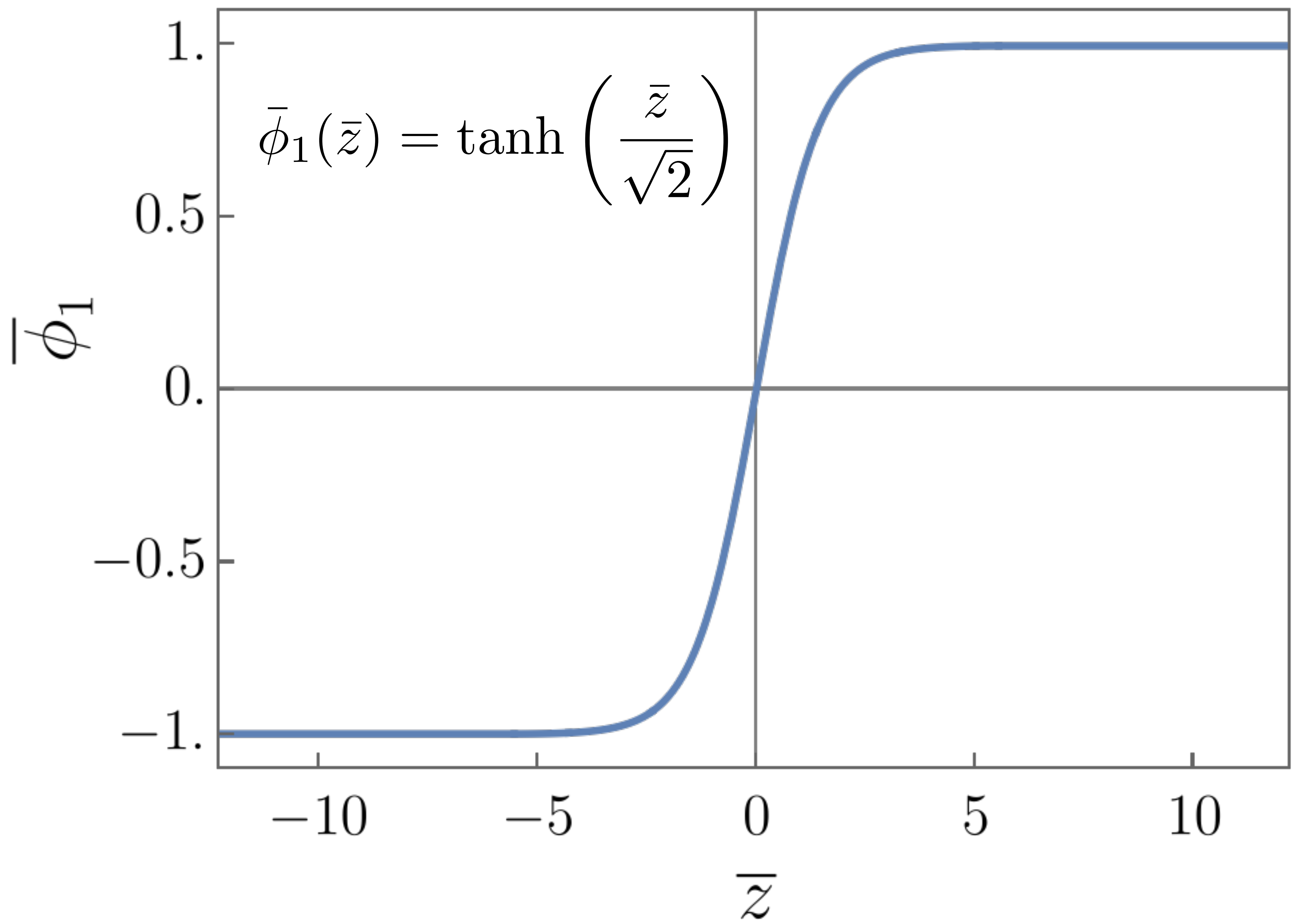
Wall tension (m/L^2):

$$\sigma = \int_{-\infty}^{+\infty} dz \rho(z)$$

$\mathbb{Z}_2$ Domain Walls: SI type

$$\bar{\phi}_1''(\bar{z}) = \bar{\phi}_1 \left[-1 + \bar{\phi}_1^2\right],$$
$$\bar{\phi}_1\big|_{\bar{z}\to+\infty} = +1, \quad \bar{\phi}_1\big|_{\bar{z}\to-\infty} = -1.$$





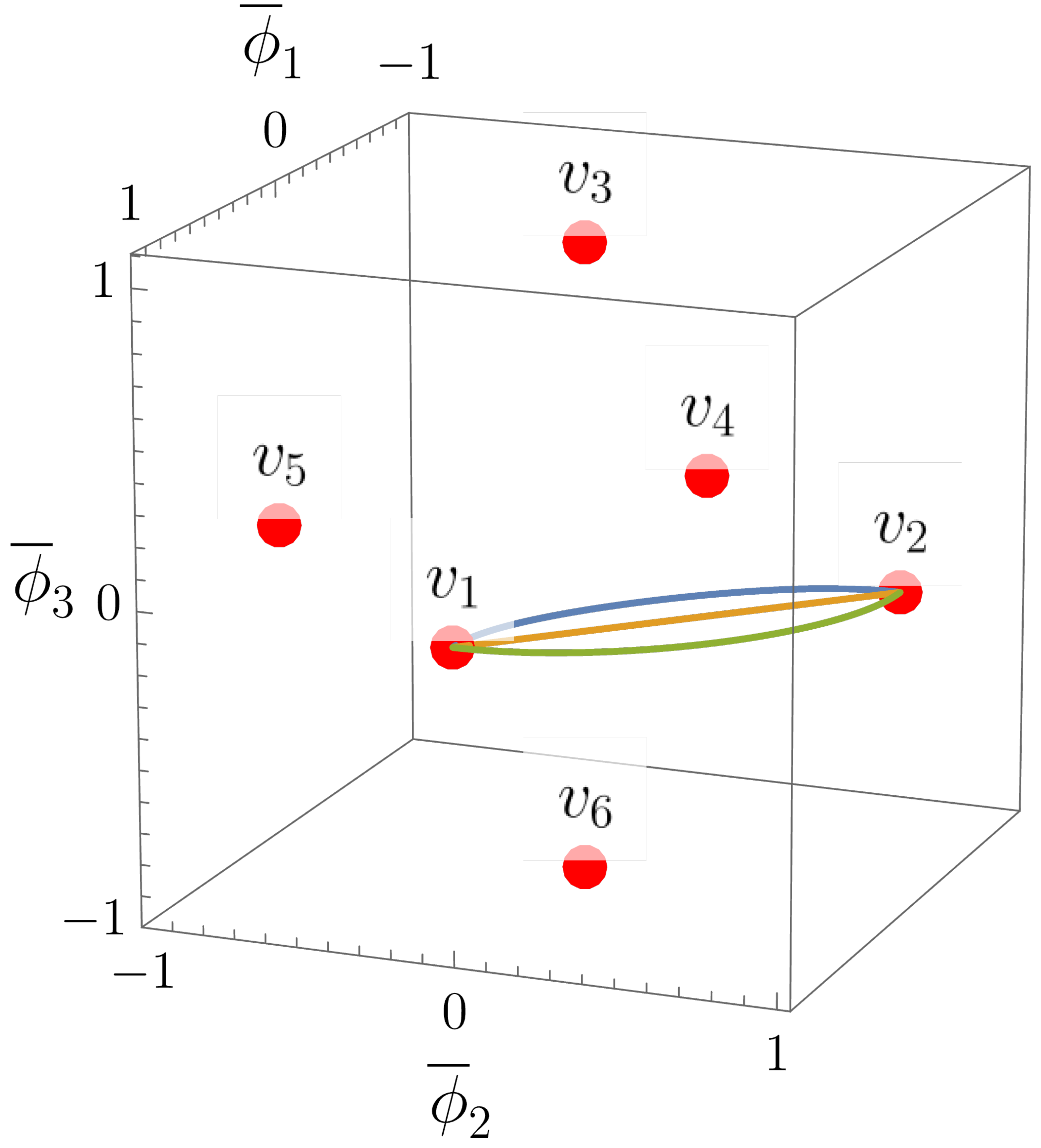
$$\bar{\sigma}_{\rm SI} = \int_{-\infty}^{+\infty} d\bar{z} \left\{ \frac{1}{2}\bar{\phi}_1'^2(\bar{z}) + \frac{1}{4}\left[\bar{\phi}_1^2(\bar{z}) - 1\right]^2 \right\} = \frac{2\sqrt{2}}{3}$$

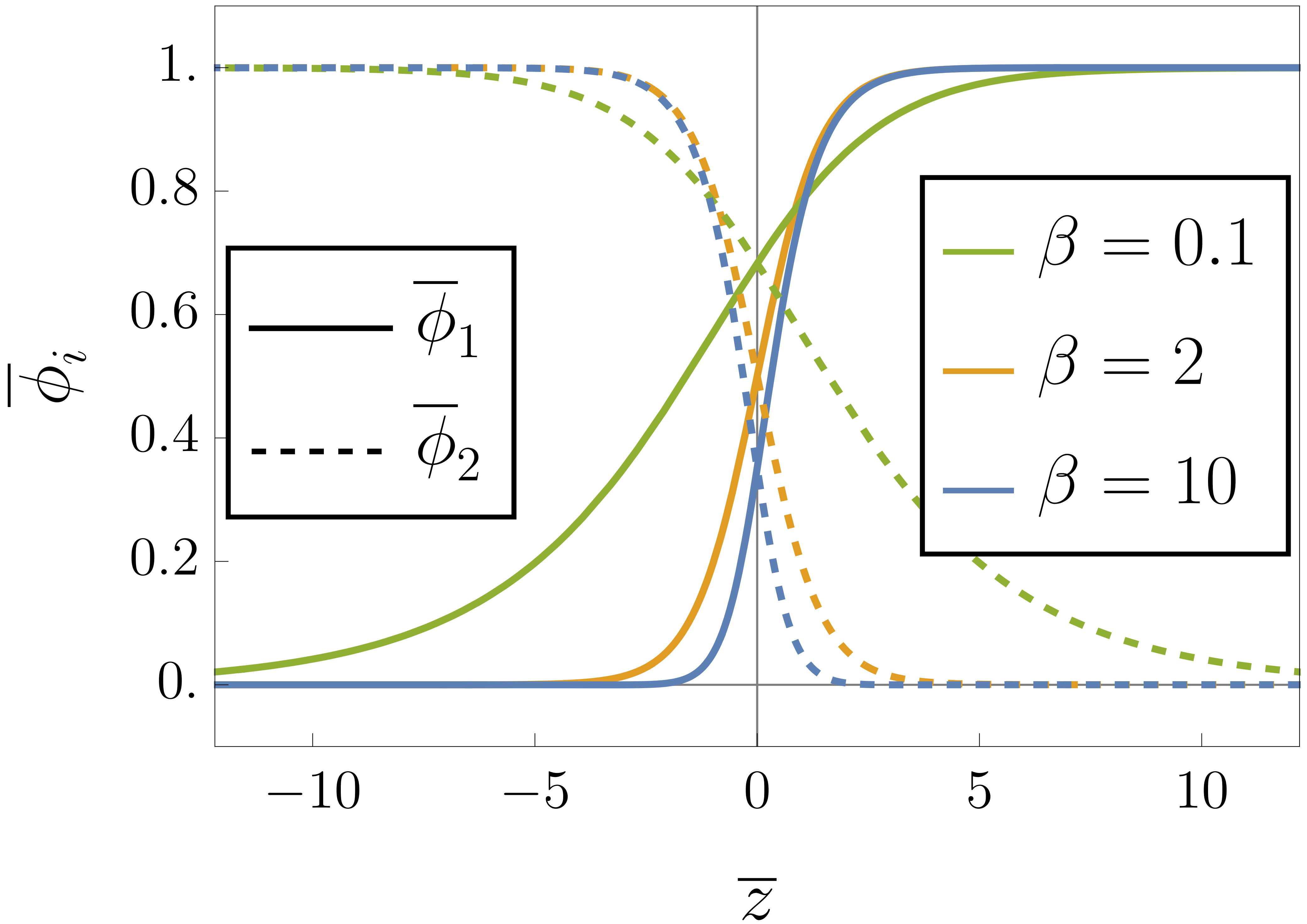
Tension dependent of $\beta = \dfrac{g_2}{g_1}$

$\mathbb{Z}_2$ Domain Walls: SII-type

$$\bar{\phi}_1''(\bar{z}) = \bar{\phi}_1\left[-1+\bar{\phi}_1^2+\bar{\phi}_2^2+\beta\bar{\phi}_2^2\right]$$
$$\bar{\phi}_2''(\bar{z}) = \bar{\phi}_2\left[-1+\bar{\phi}_1^2+\bar{\phi}_2^2+\beta\bar{\phi}_1^2\right]$$

$$\begin{pmatrix}\bar{\phi}_1\\ \bar{\phi}_2\end{pmatrix}\Bigg|_{\bar{z}\to+\infty} = \begin{pmatrix}1\\0\end{pmatrix},\quad \begin{pmatrix}\bar{\phi}_1\\ \bar{\phi}_2\end{pmatrix}\Bigg|_{\bar{z}\to-\infty} = \begin{pmatrix}0\\1\end{pmatrix}$$

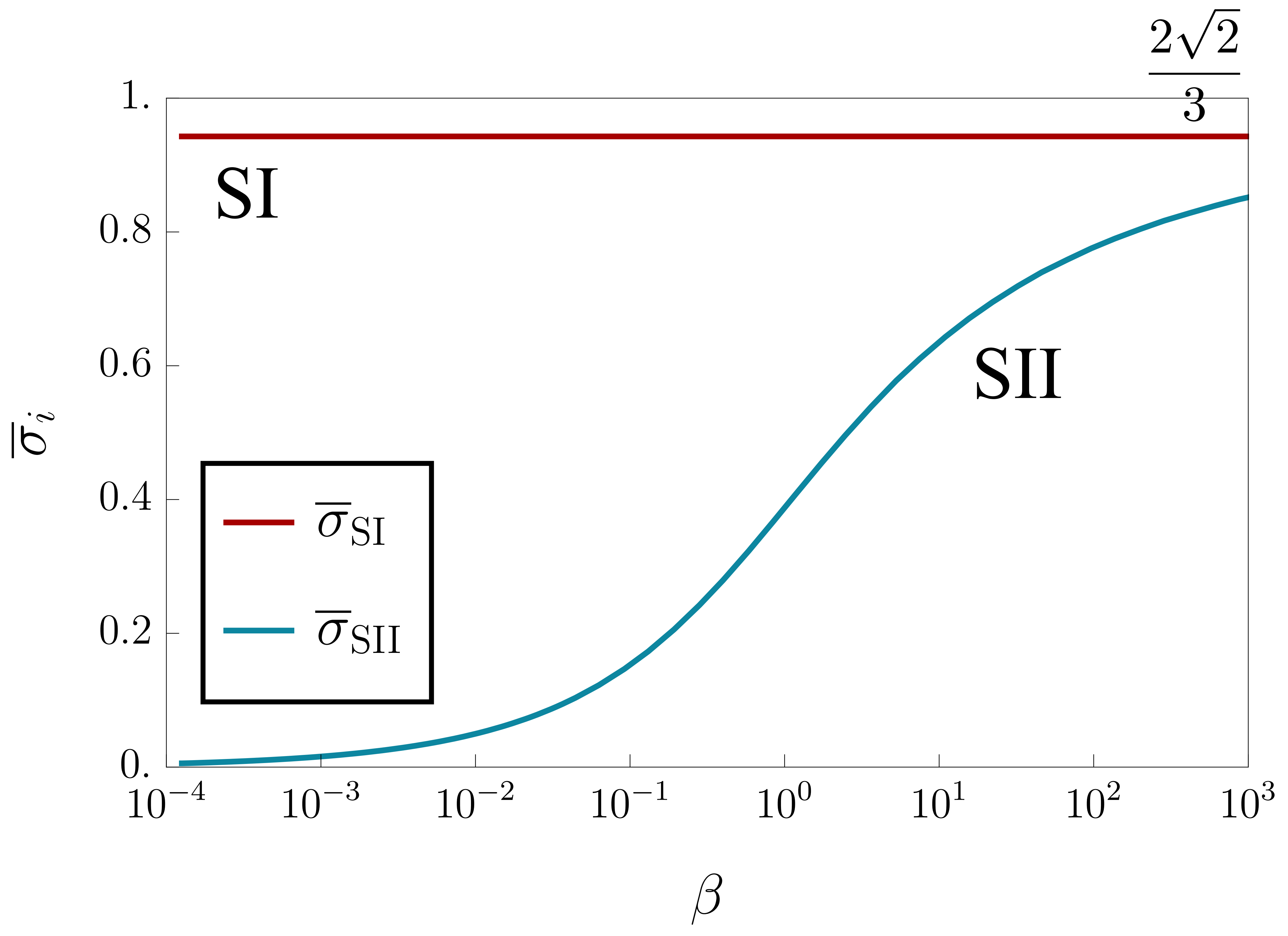


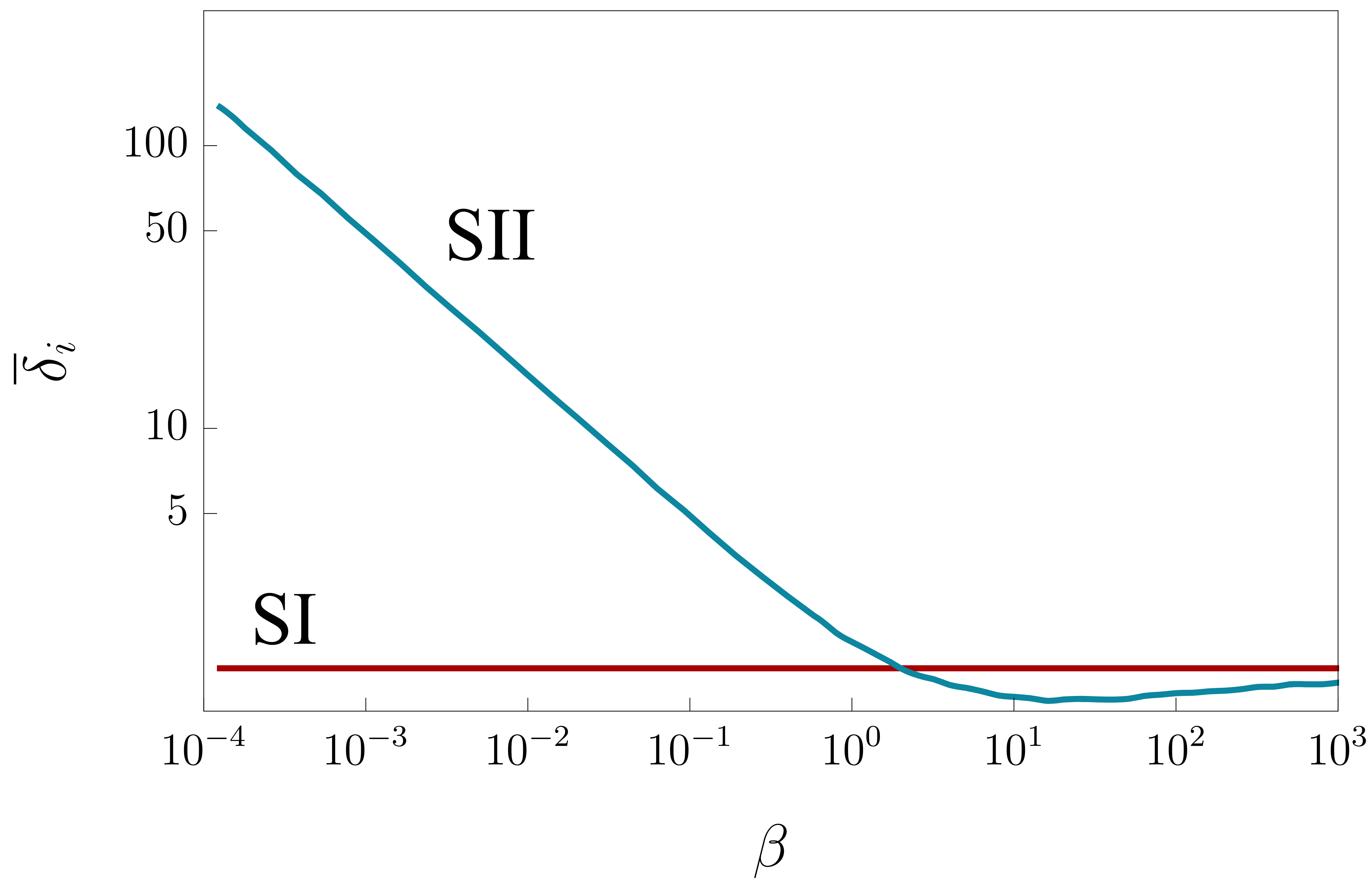


$$\bar{\sigma}_{\rm SII} = \int_{-\infty}^{+\infty} d\bar{z}\left[\frac{1}{2}\sum_{i=1}^{2}\bar{\phi}_i'^2 + \frac{1}{4}\left(\bar{\phi}_1^2+\bar{\phi}_2^2-1\right)^2 + \frac{\beta}{2}\bar{\phi}_1^2\bar{\phi}_2^2\right]$$

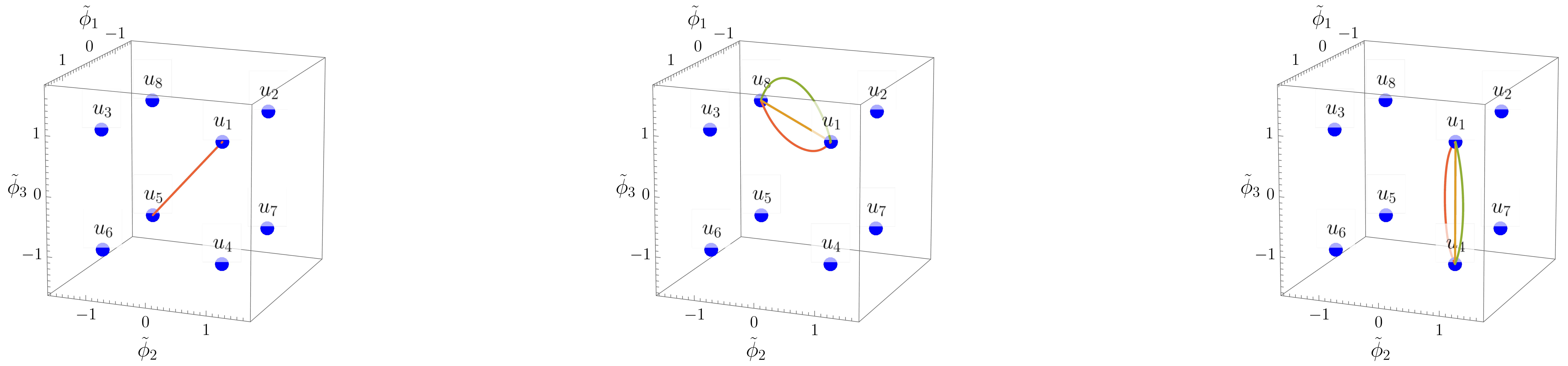
Tension **dependent** of $\beta = \dfrac{g_2}{g_1}$

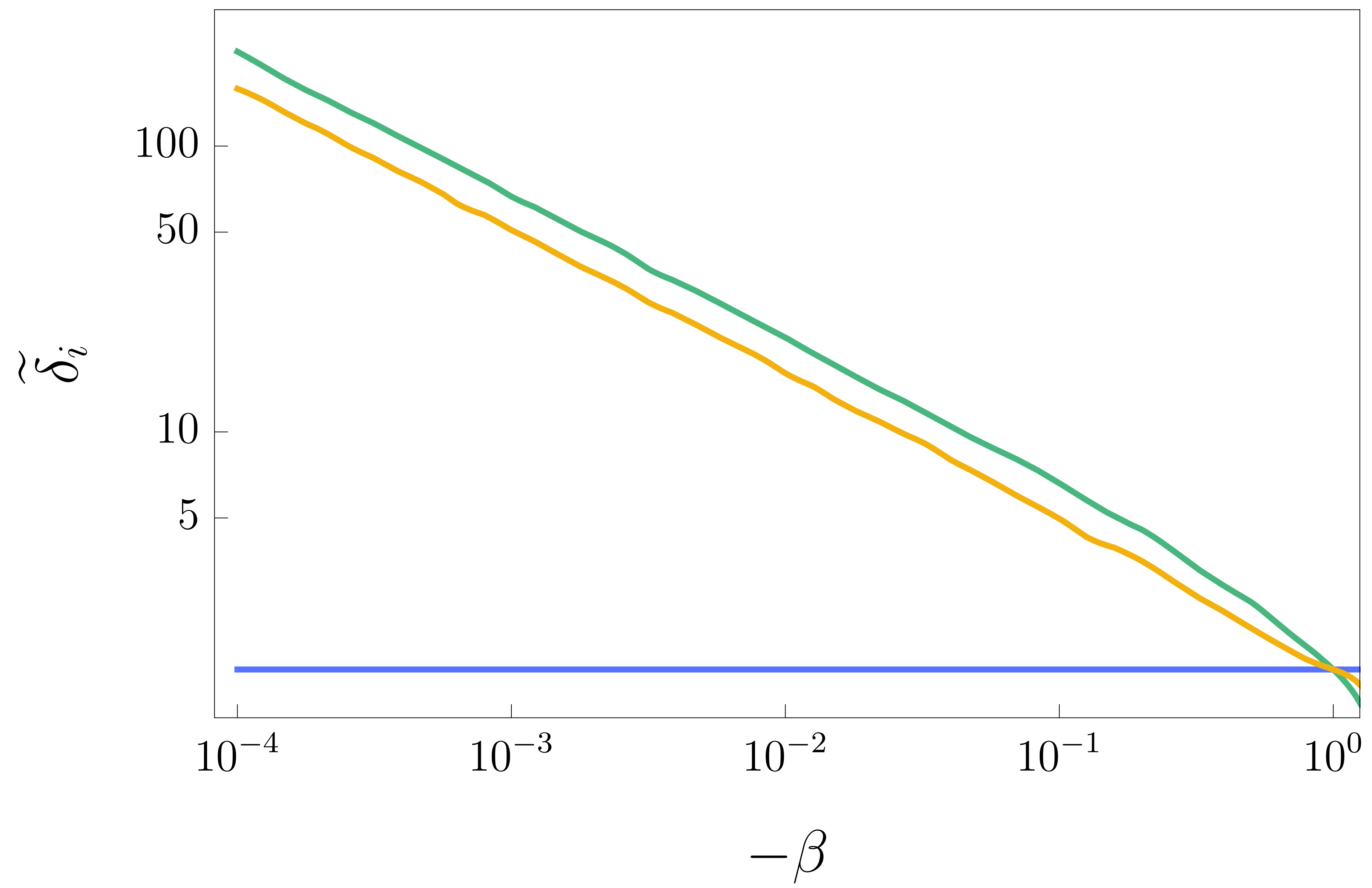
$\mathbb{Z}_2$ Domain Wall Tensions

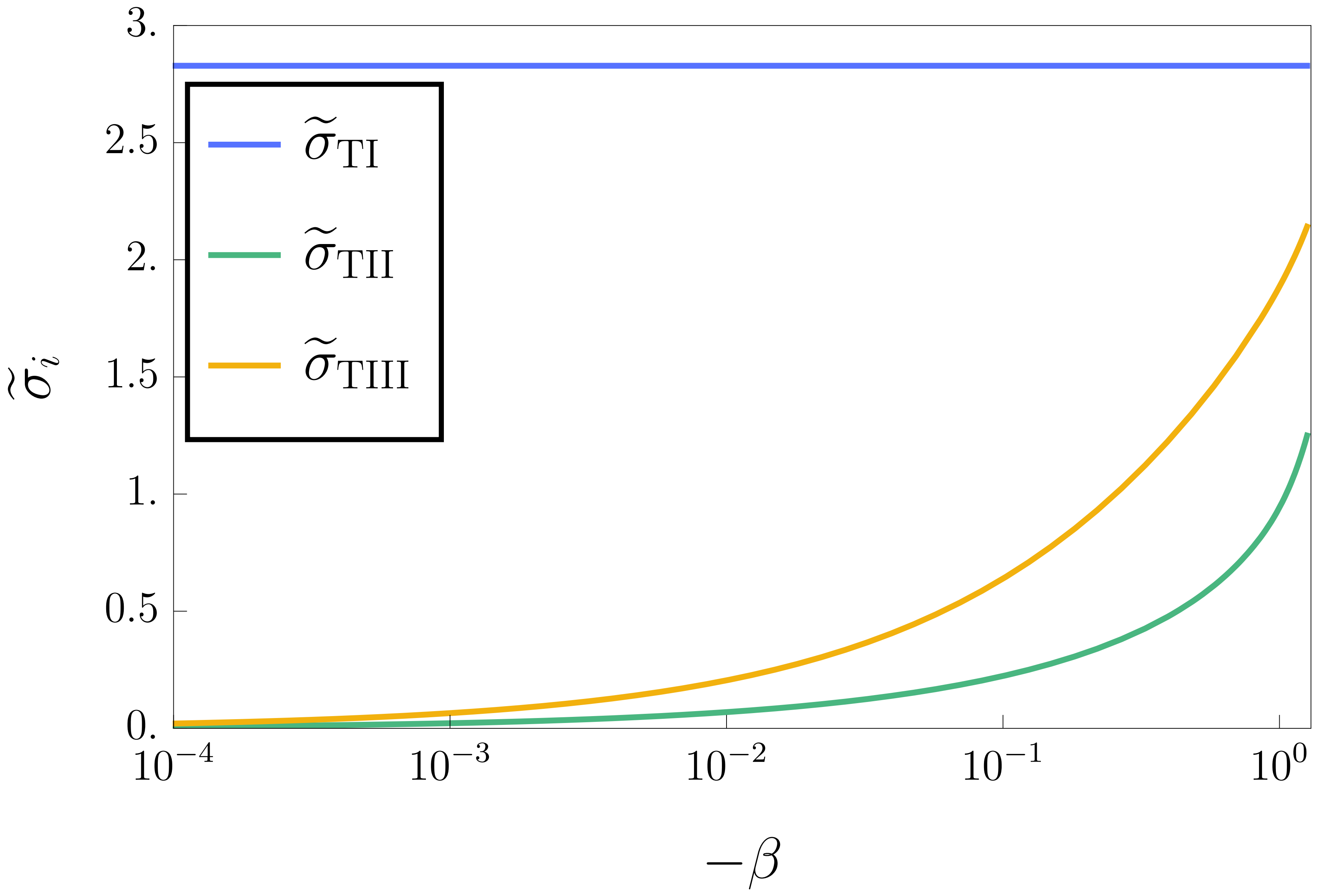




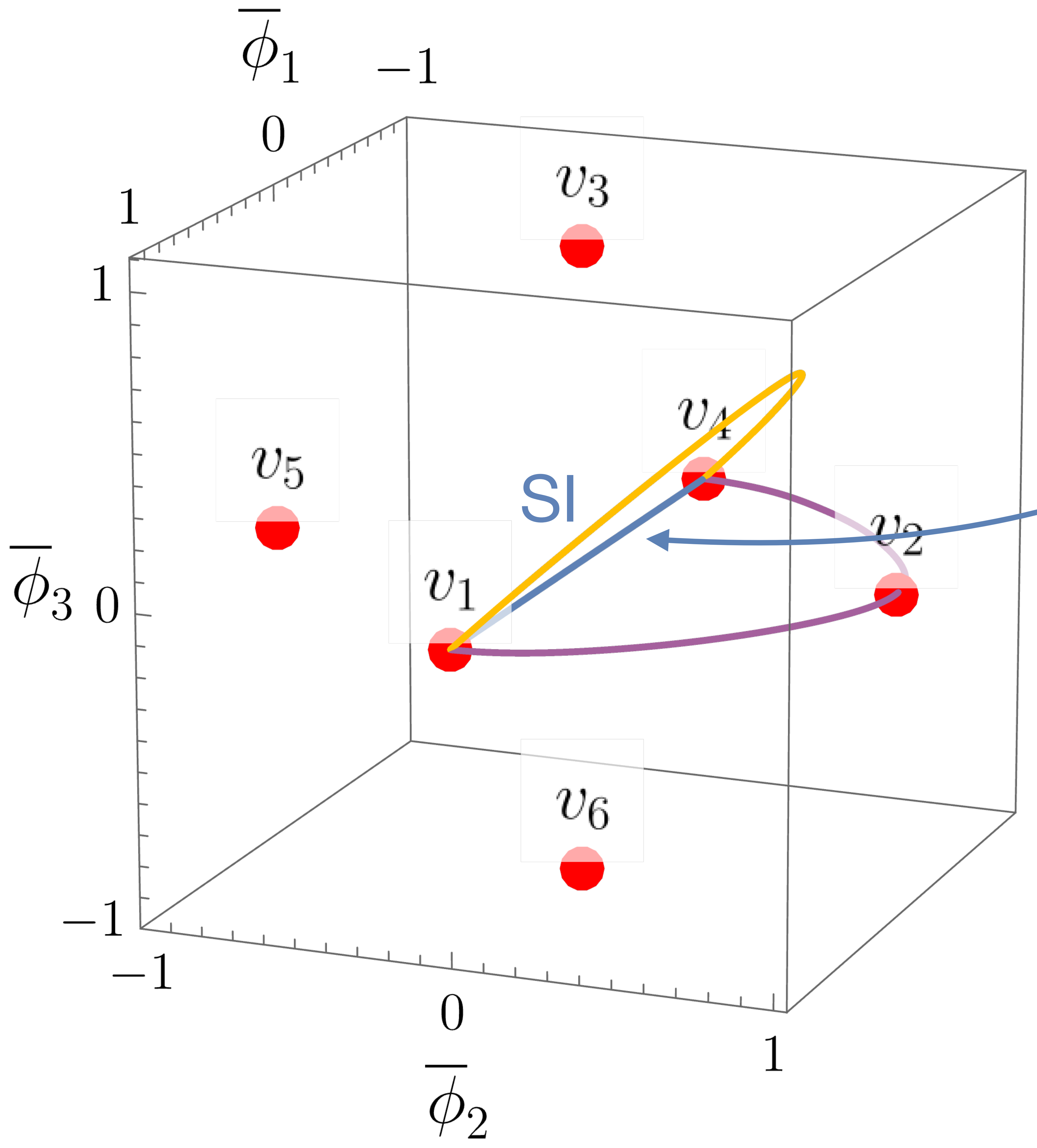
$\mathbb{Z}_3$ Domain Walls: T-Type







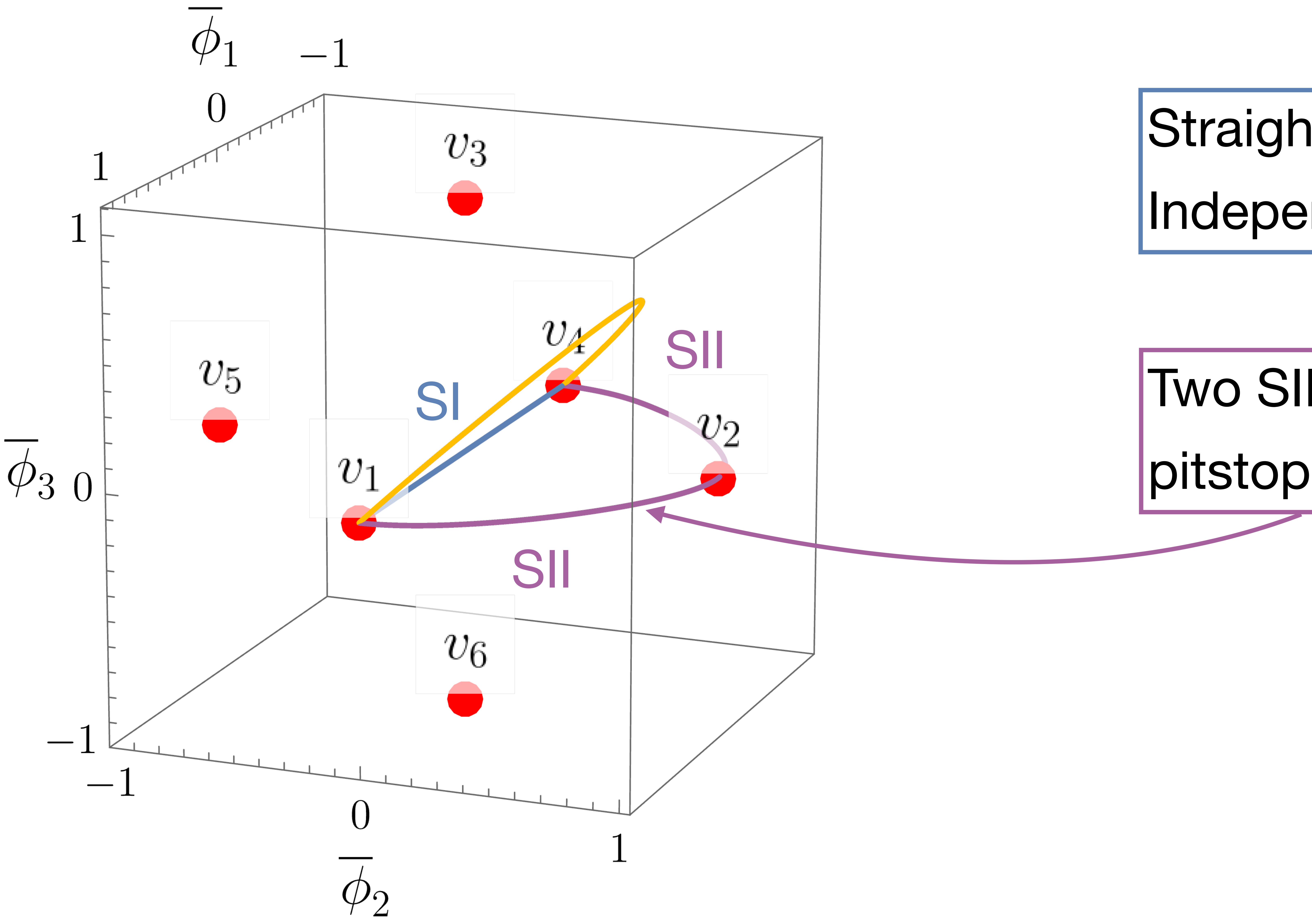
Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case



Straight line SI solution

Independent of $\beta = g_2/g_1$

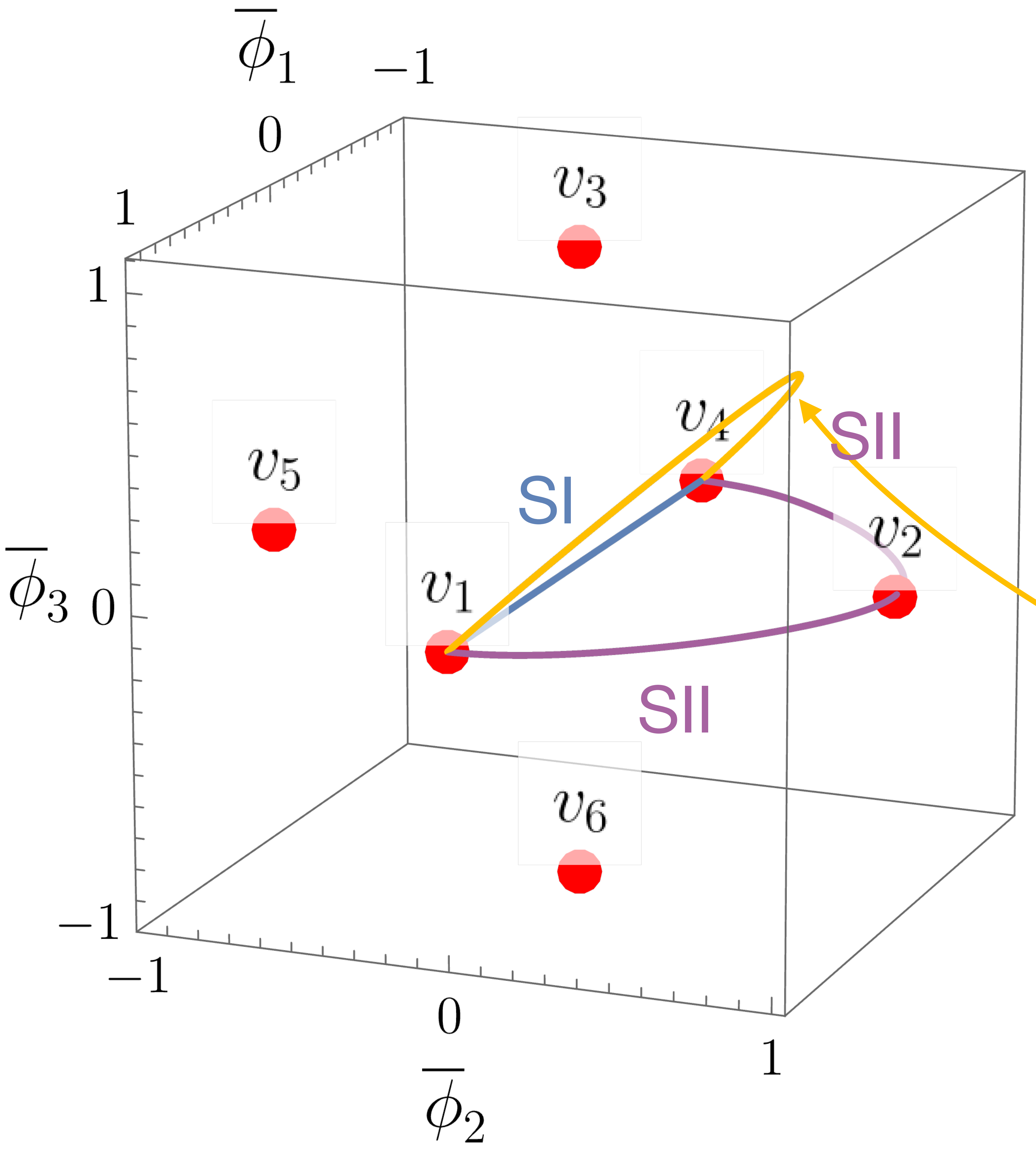
Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case



Straight line SI solution
Independent of $\beta = g_2/g_1$

Two SIII solutions with
pitstop at v_2

Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case

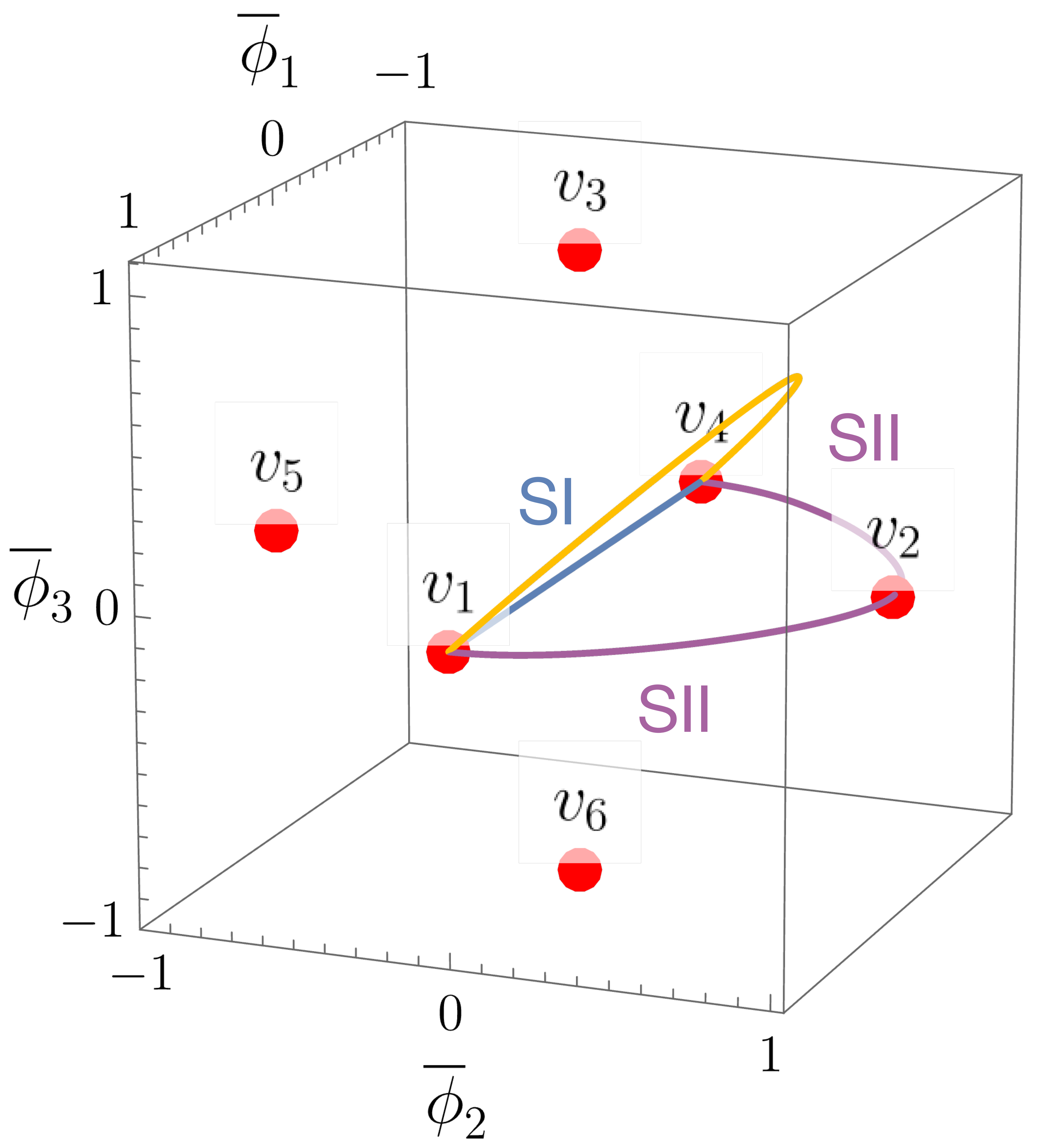


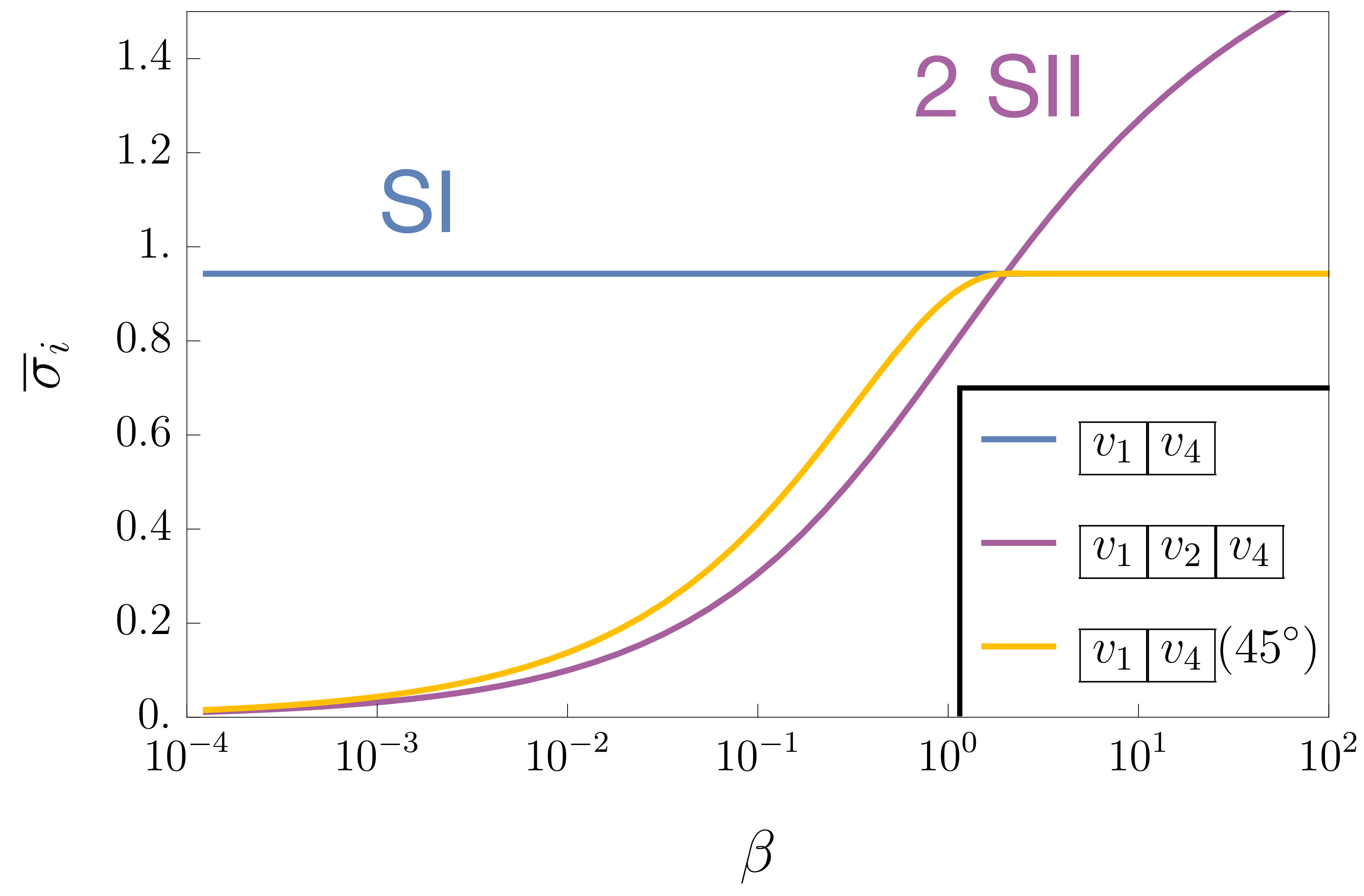
Straight line SI solution
Independent of $\beta = g_2/g_1$

Two SIII solutions with pitstop at v_2

Intermediate solution (still satisfies EoM)

Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case

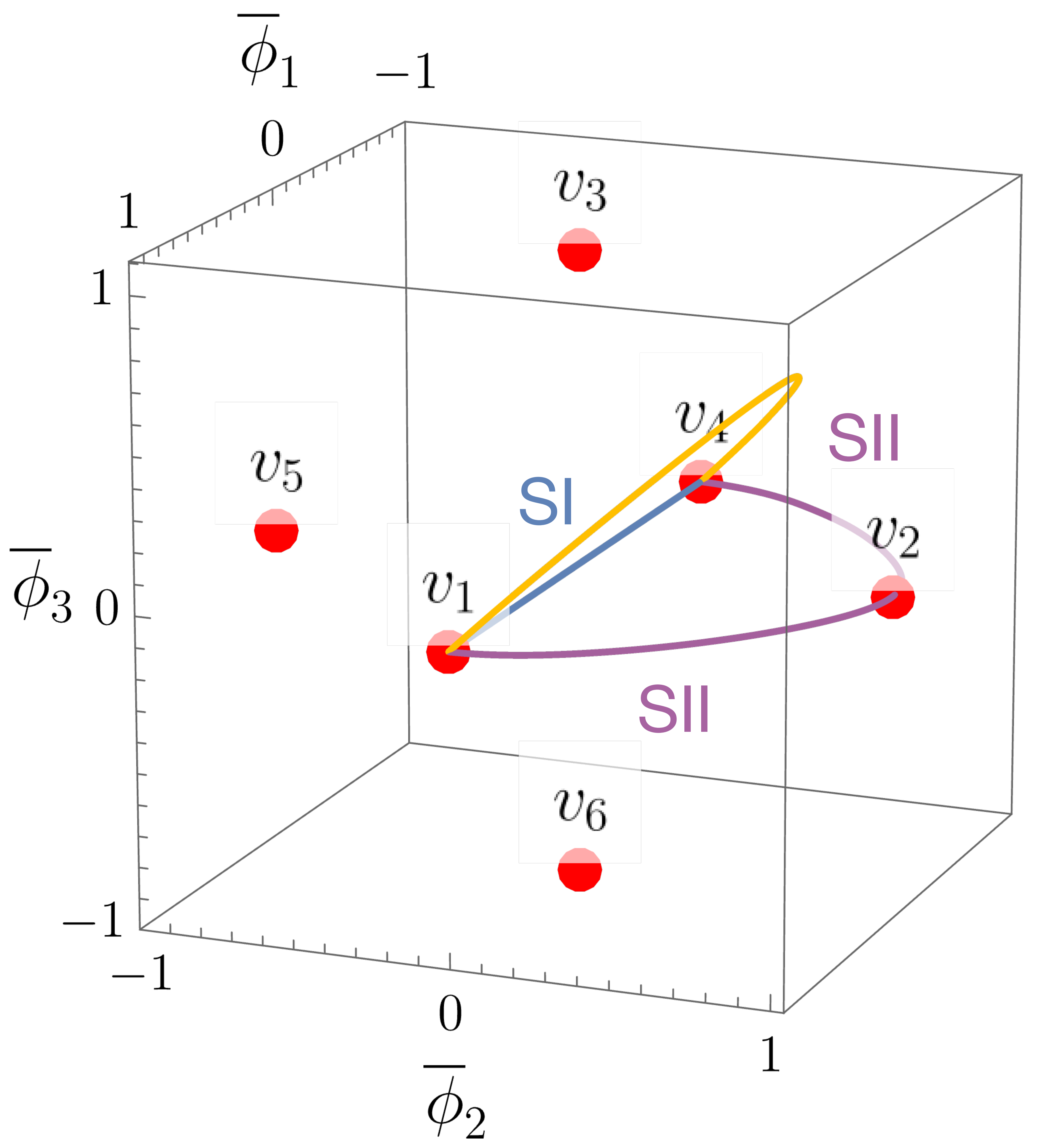


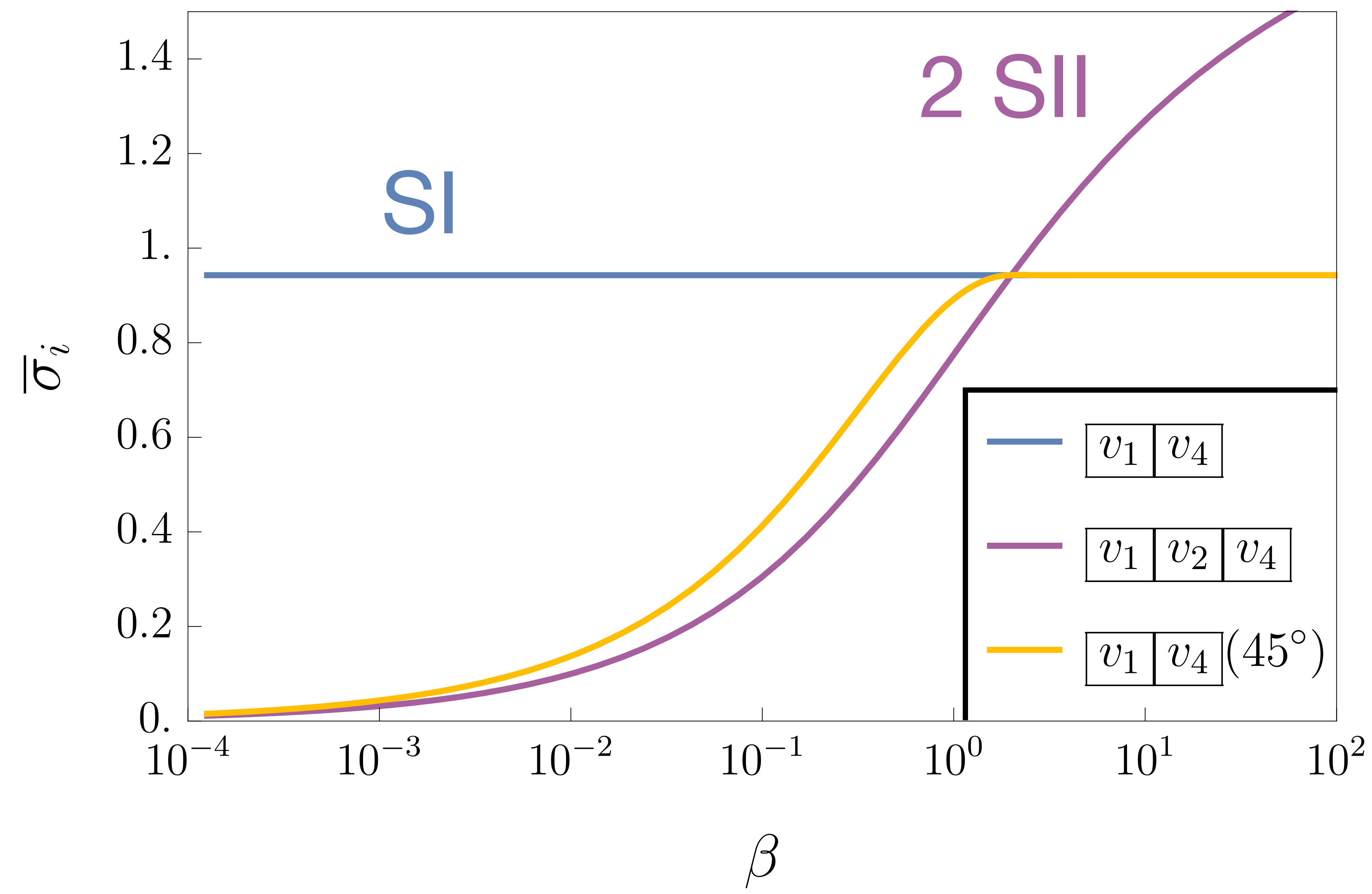


$$\beta \ll 2 : \sigma(\mathrm{SI}) > 2\sigma(\mathrm{SII})$$

SI DW **unstable** & would decay to SII

Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case

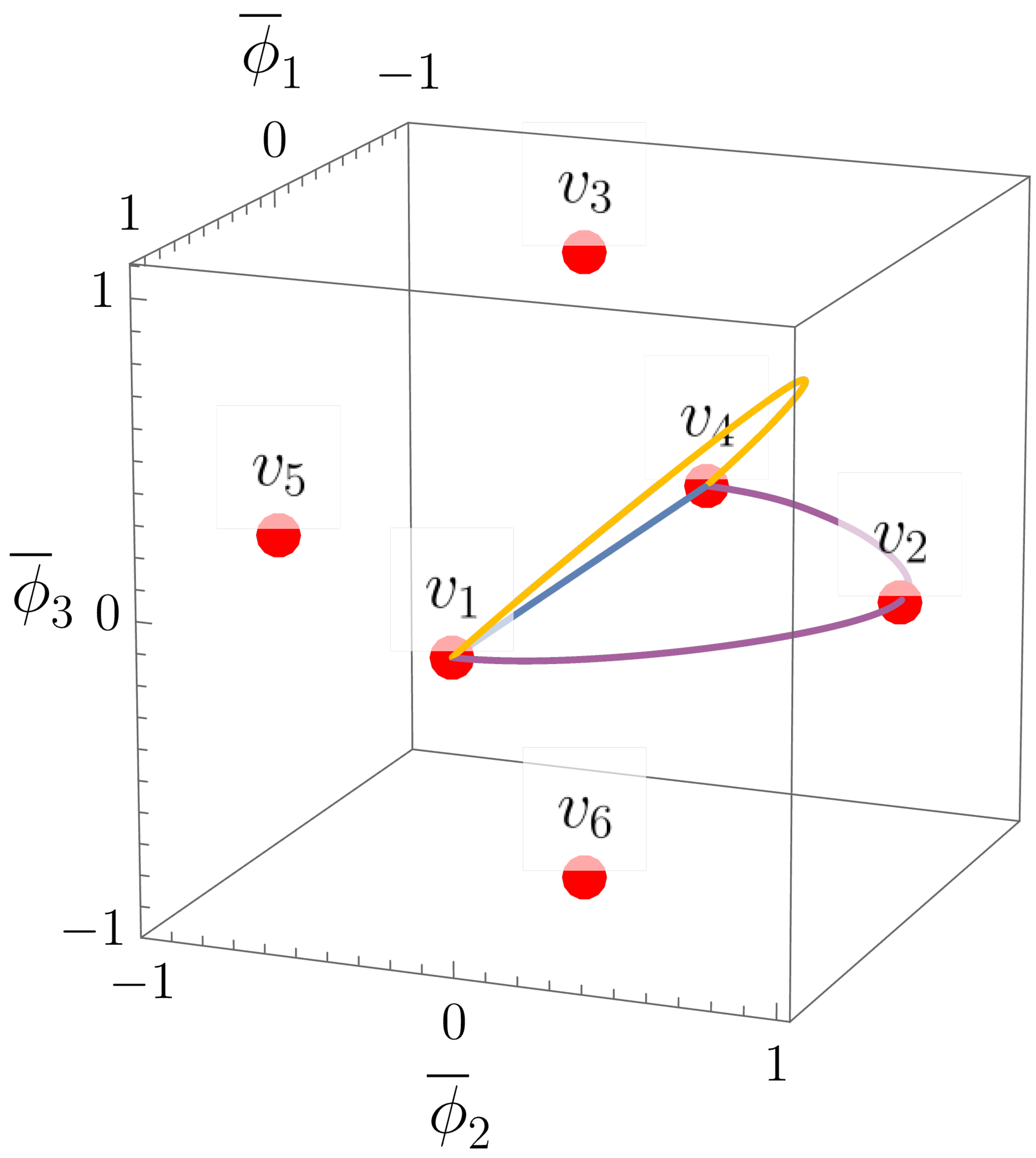


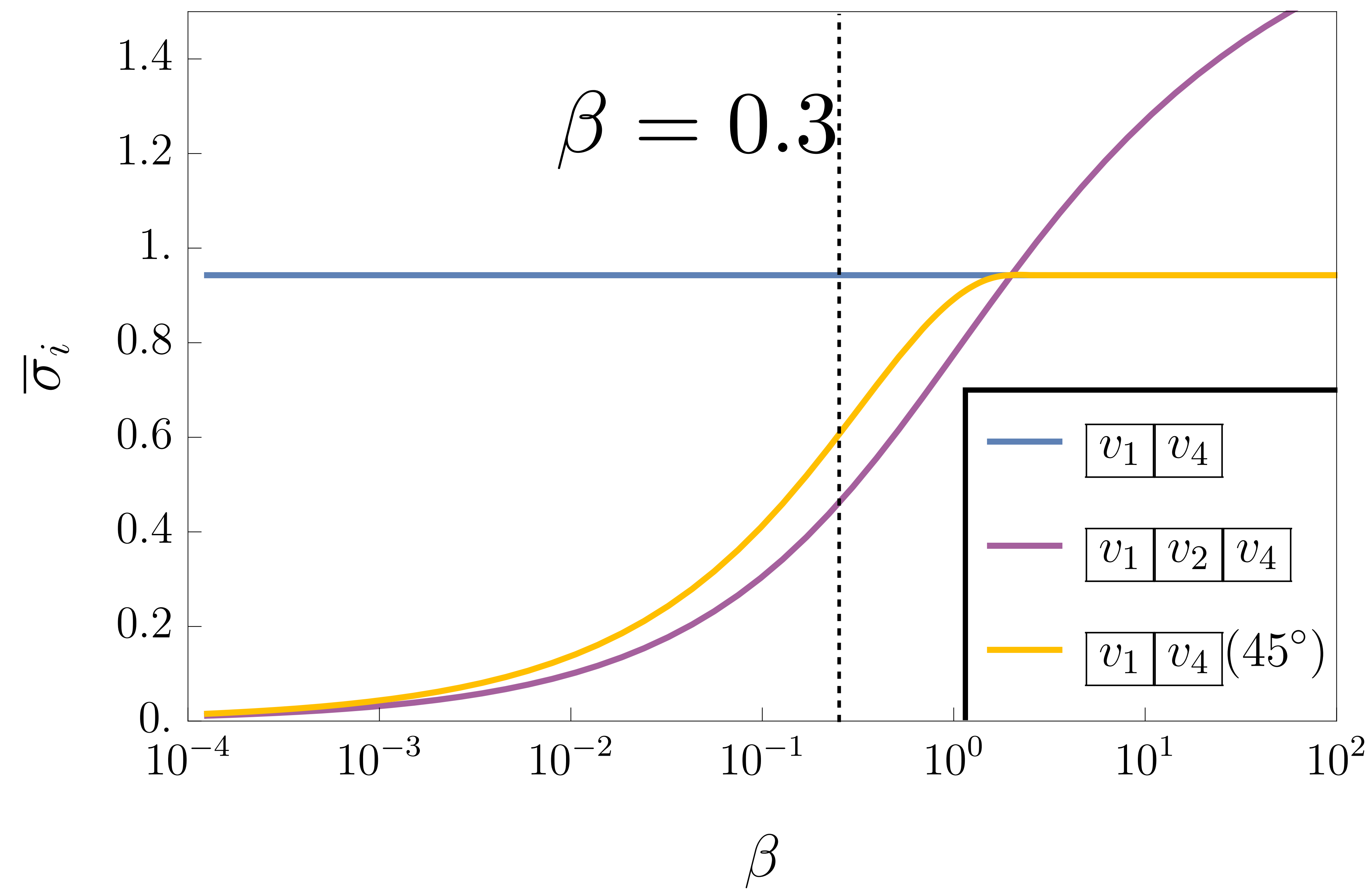


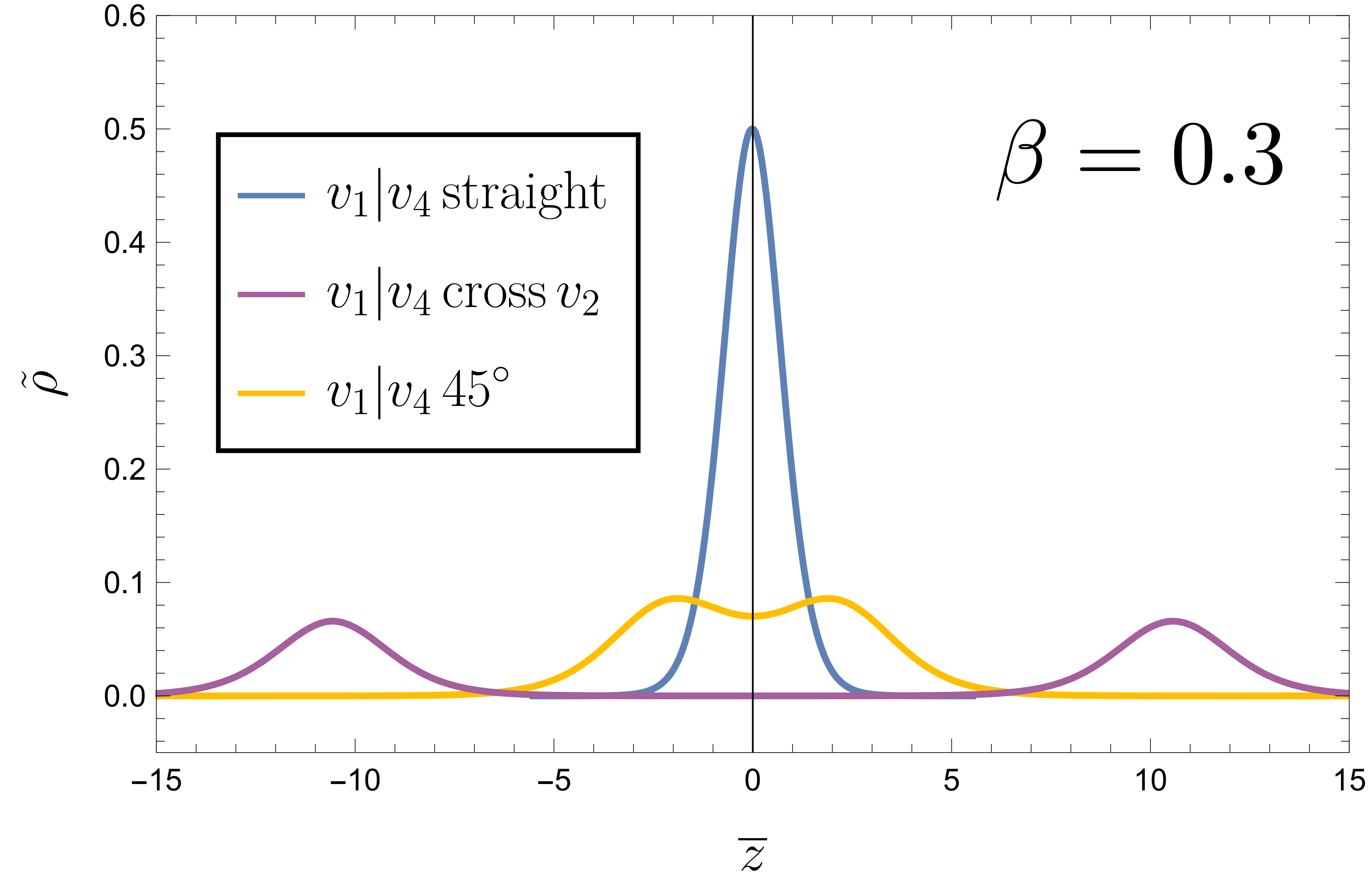
$$\beta \gg 2 : \sigma(\mathrm{SI}) < 2\sigma(\mathrm{SII})$$

2 SII DW **unstable** & would decay to SI

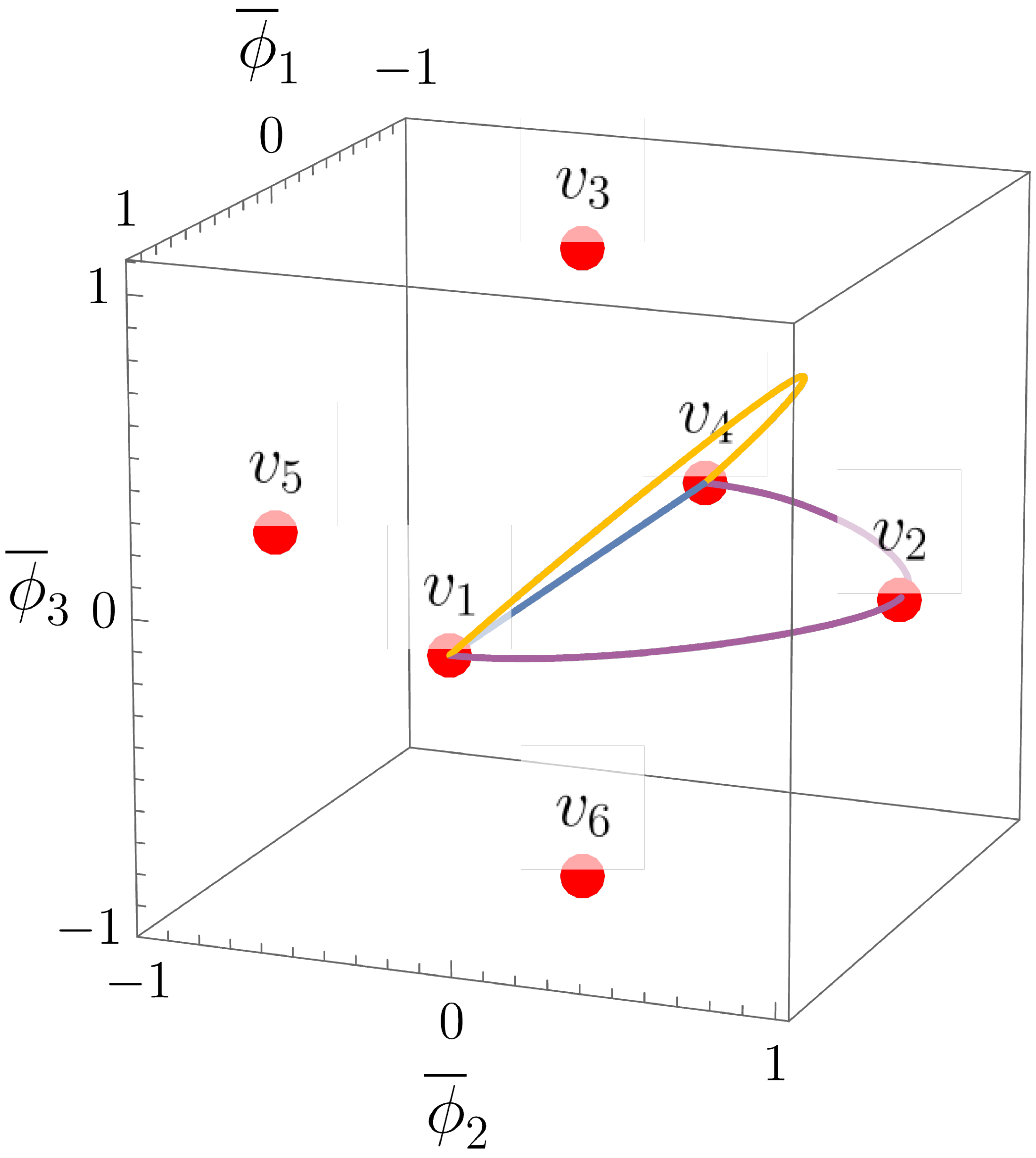
Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case



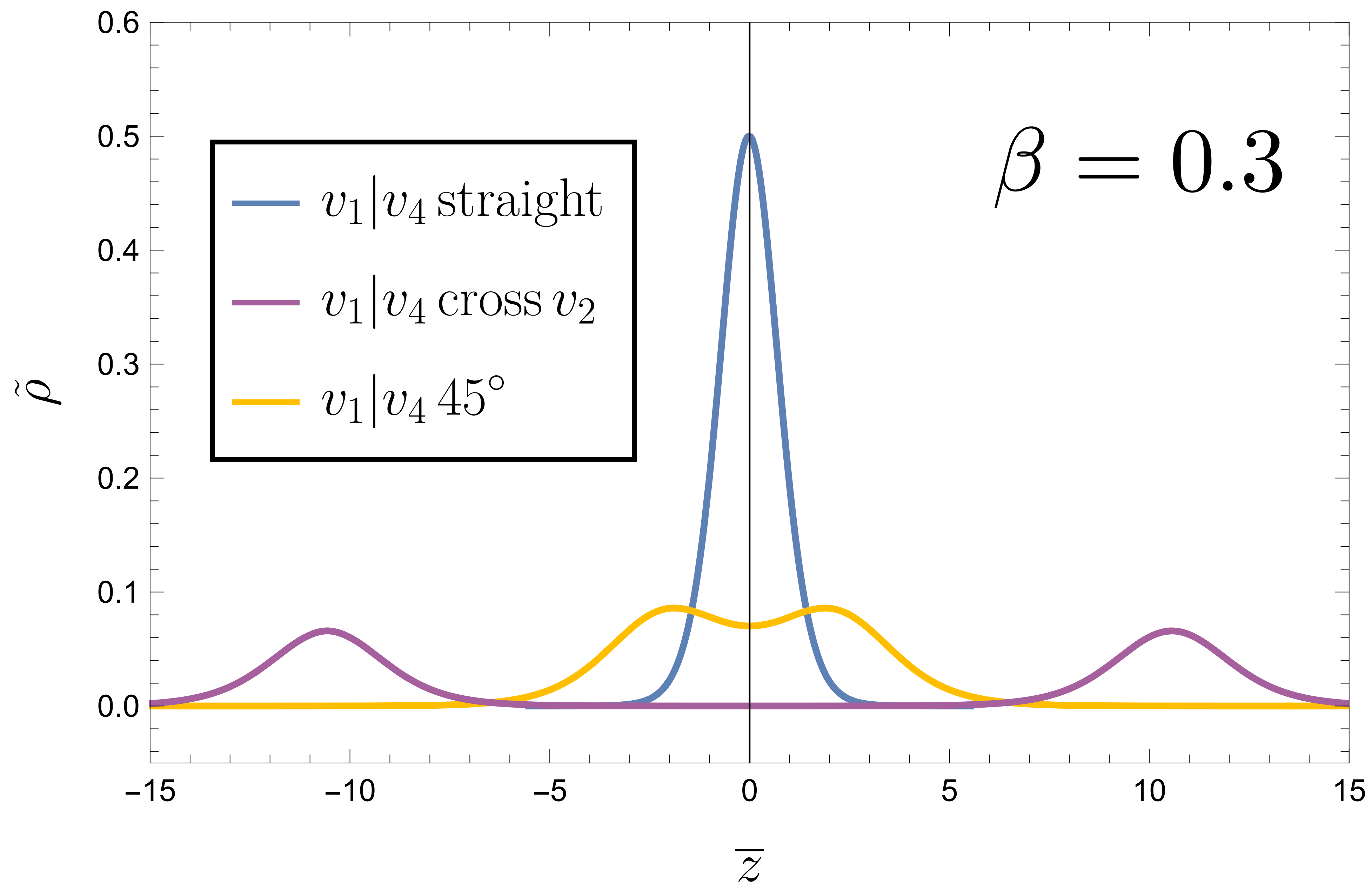




Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case



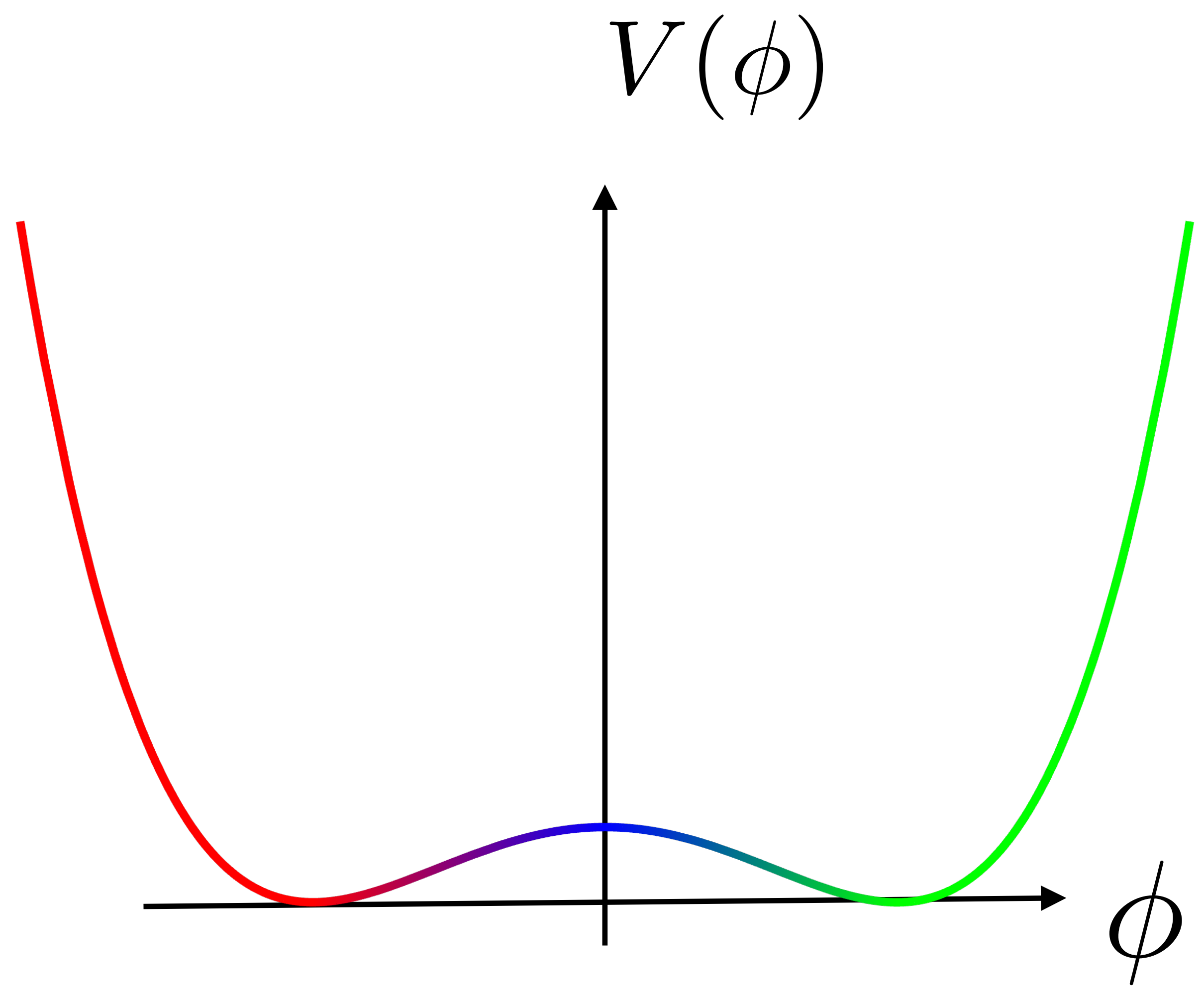
For $\beta = 0.3$ since SI-type DW form will decay to two SII type DWs



Stability of non-Abelian Domain Walls: $\mathbb{Z}_2$ case

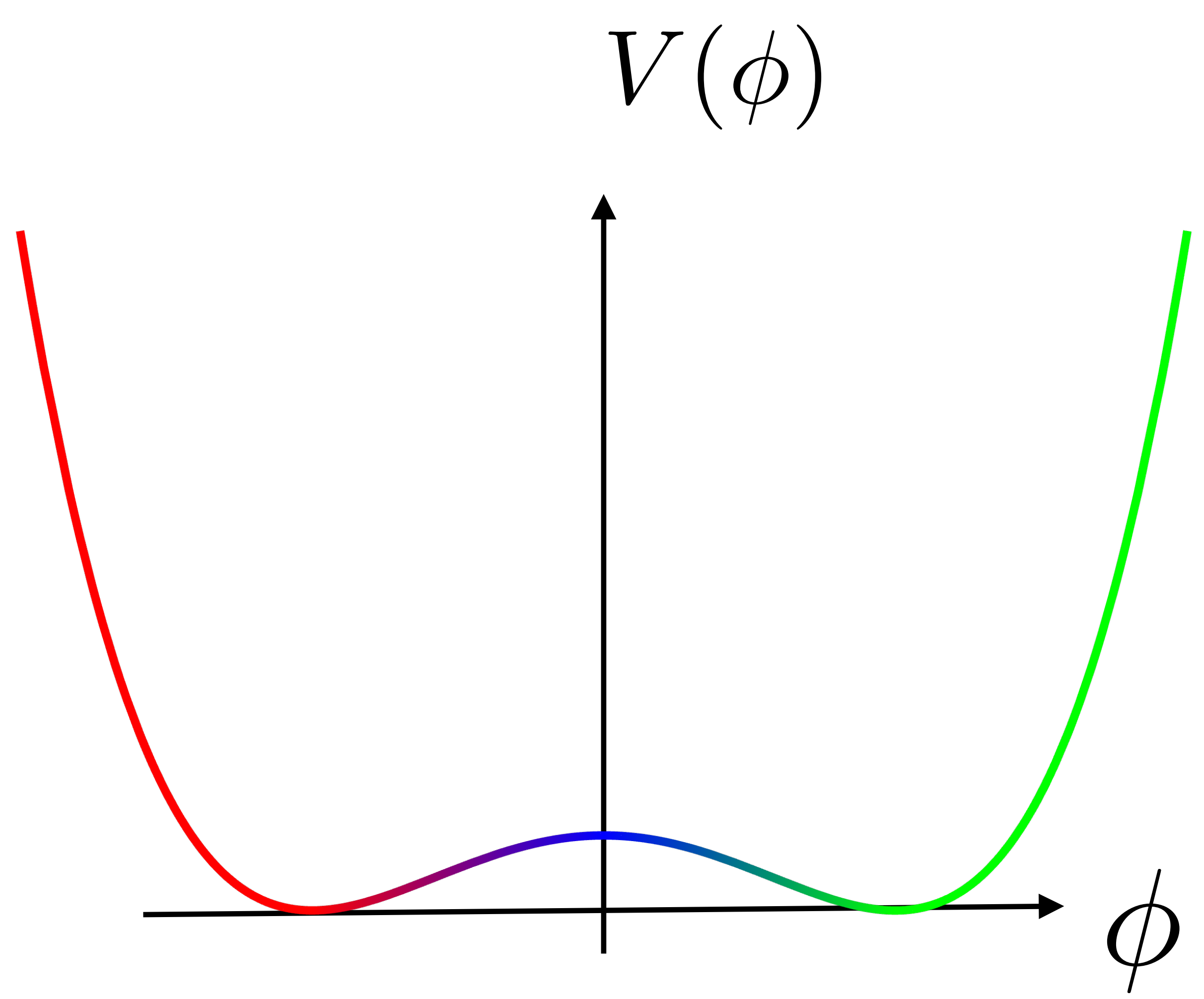
Analogous discussion for $\mathbb{Z}_3$ type domain walls, see paper

Annihilation of Domain Walls



Static Domain Walls

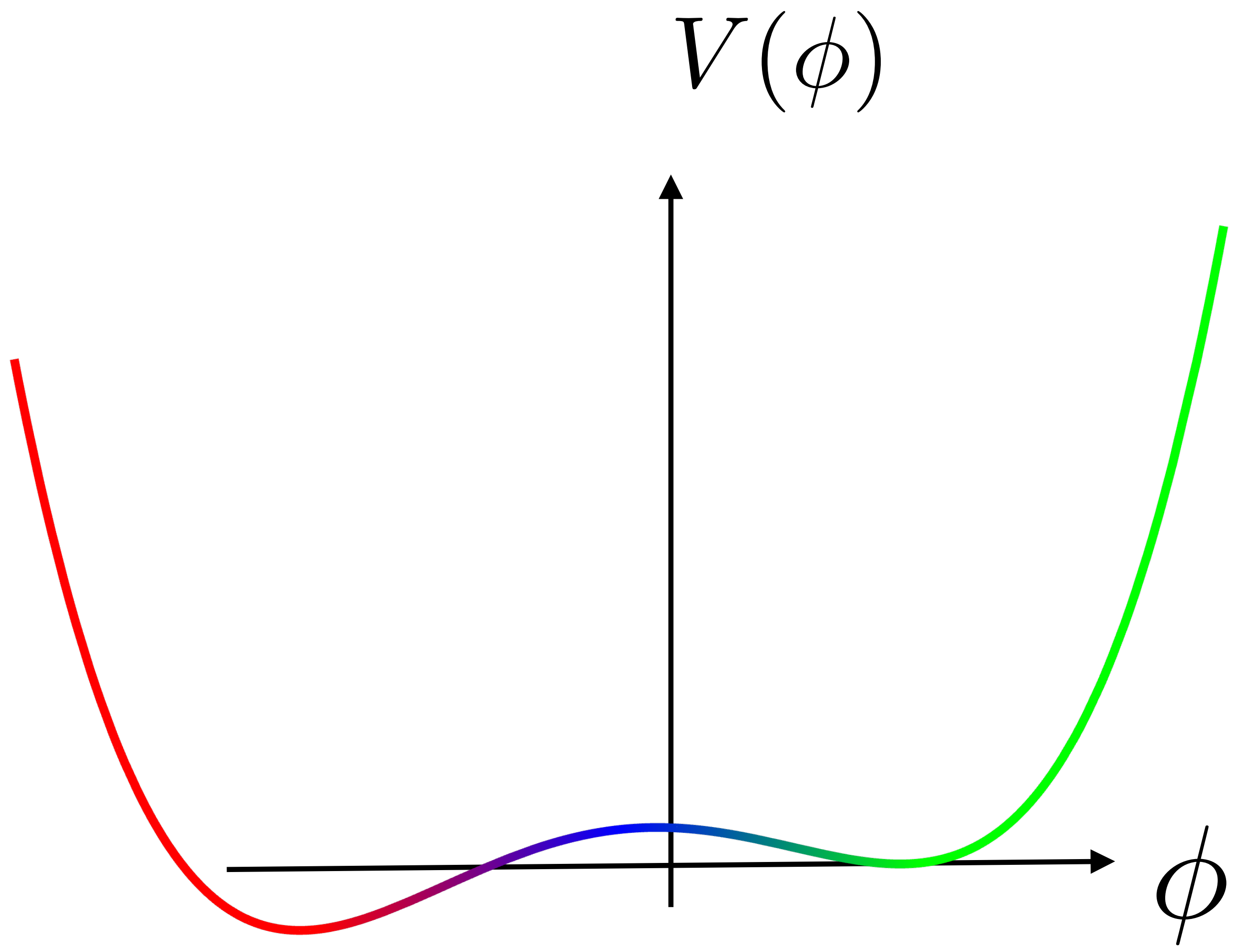
Annihilation of Domain Walls



Static Domain Walls

Bias $\Longrightarrow$ may correct LO mixing pattern

$$V_{\rm bias}^{ij} = \epsilon^{ij} v^4$$



Bias $\Longrightarrow$ accelerating DWs
$\Longrightarrow$ Collisions & GW Emission

Gravitational Waves

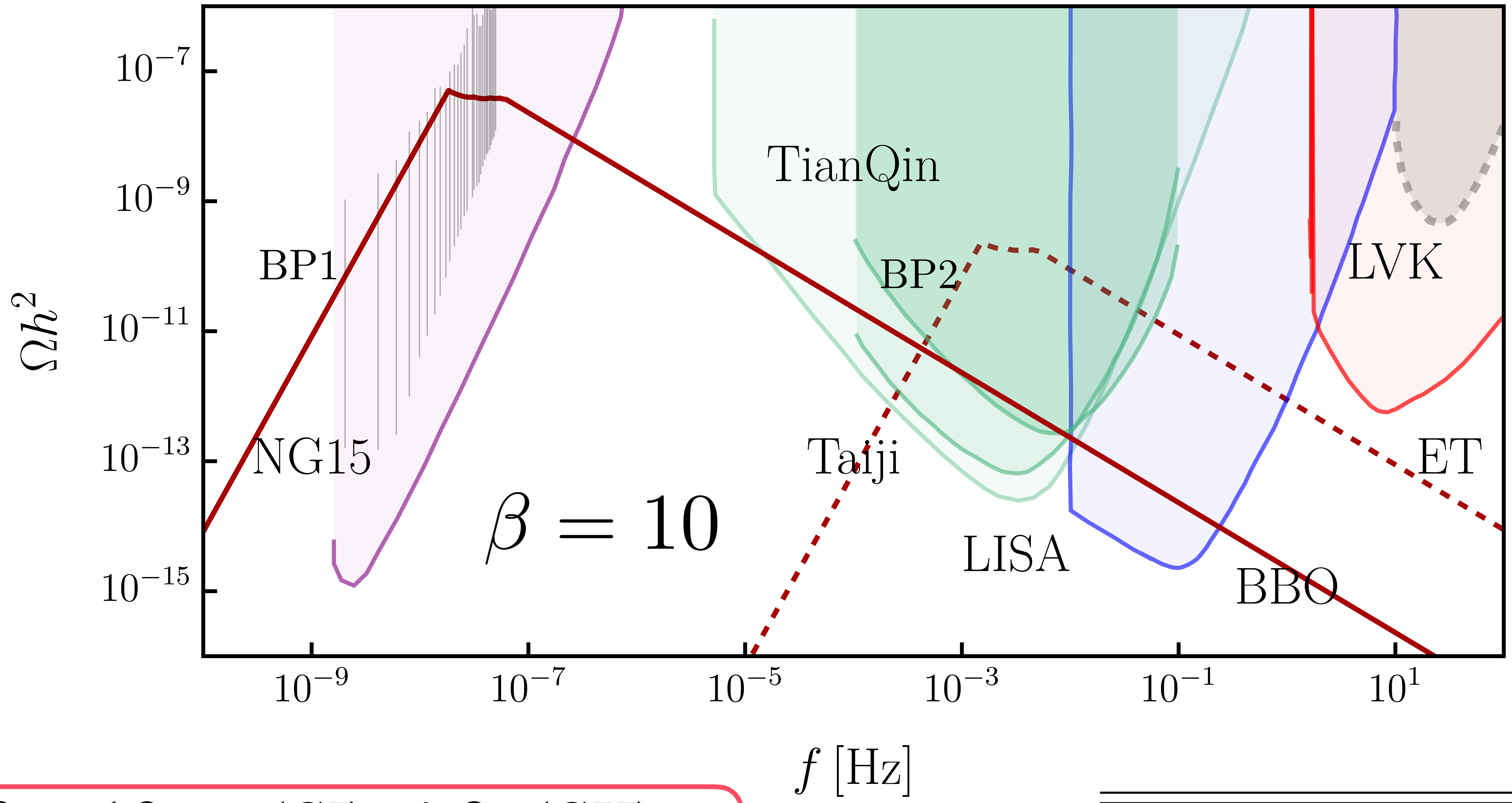
Decay of higher energy density domain walls to lower energy ones can change domain wall network and influence gravitational wave emission. Relegate for future work

Consider bias term to annihilate away DWs & bias could be connected to Abelian residual Abelian symmetry breaking

$$f_{\rm peak} \simeq 3 \times 10^3\,{\rm Hz} \left(\frac{10}{{\rm g}_*({\rm T_{ann}})}\right)^{1/2} \left(\frac{\rm V_{bias}}{\sigma\,{\rm TeV}}\right)^{1/2}$$

$$\Omega_{\rm gw} h^2 \simeq 0.9 \times 10^{-67} \left(\frac{10}{g_*\,(T_{\rm ann})}\right)^{1/3} \left(\frac{\sigma}{\rm TeV^3}\right)^4 \left(\frac{\rm TeV^4}{V_{\rm bias}}\right)^2$$

Gravitational Waves: S-type, $\mathbb{Z}_2$



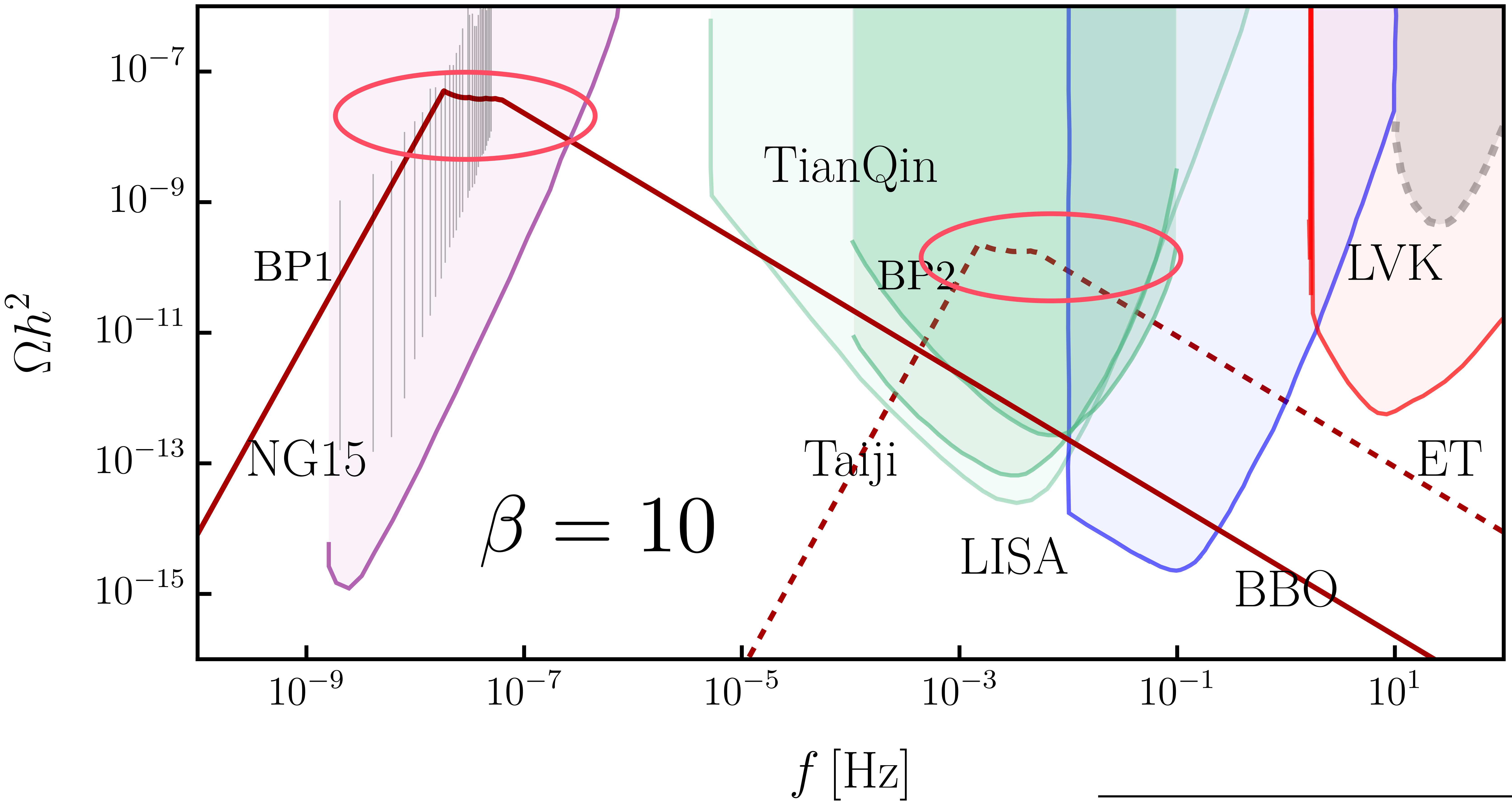
$\beta = 10, \quad \sigma(SI) \ngtr 2\sigma(SII)$

So SI & SII type walls stable & both contribute to GW signal

	$\sigma^{1/3}$/TeV	$V_{\text{bias}}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Gravitational Waves: S-type, $\mathbb{Z}_2$

Multipeak structure from different Domain walls i.e. SI, SII



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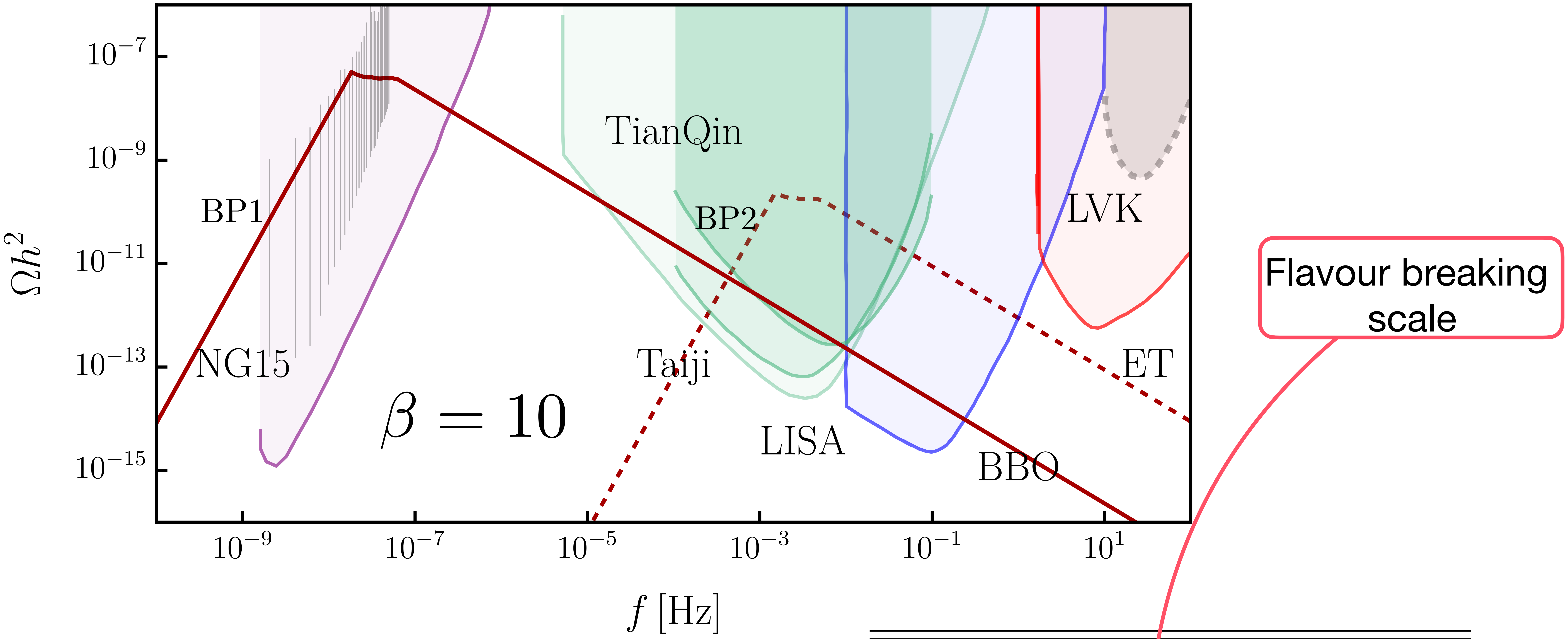

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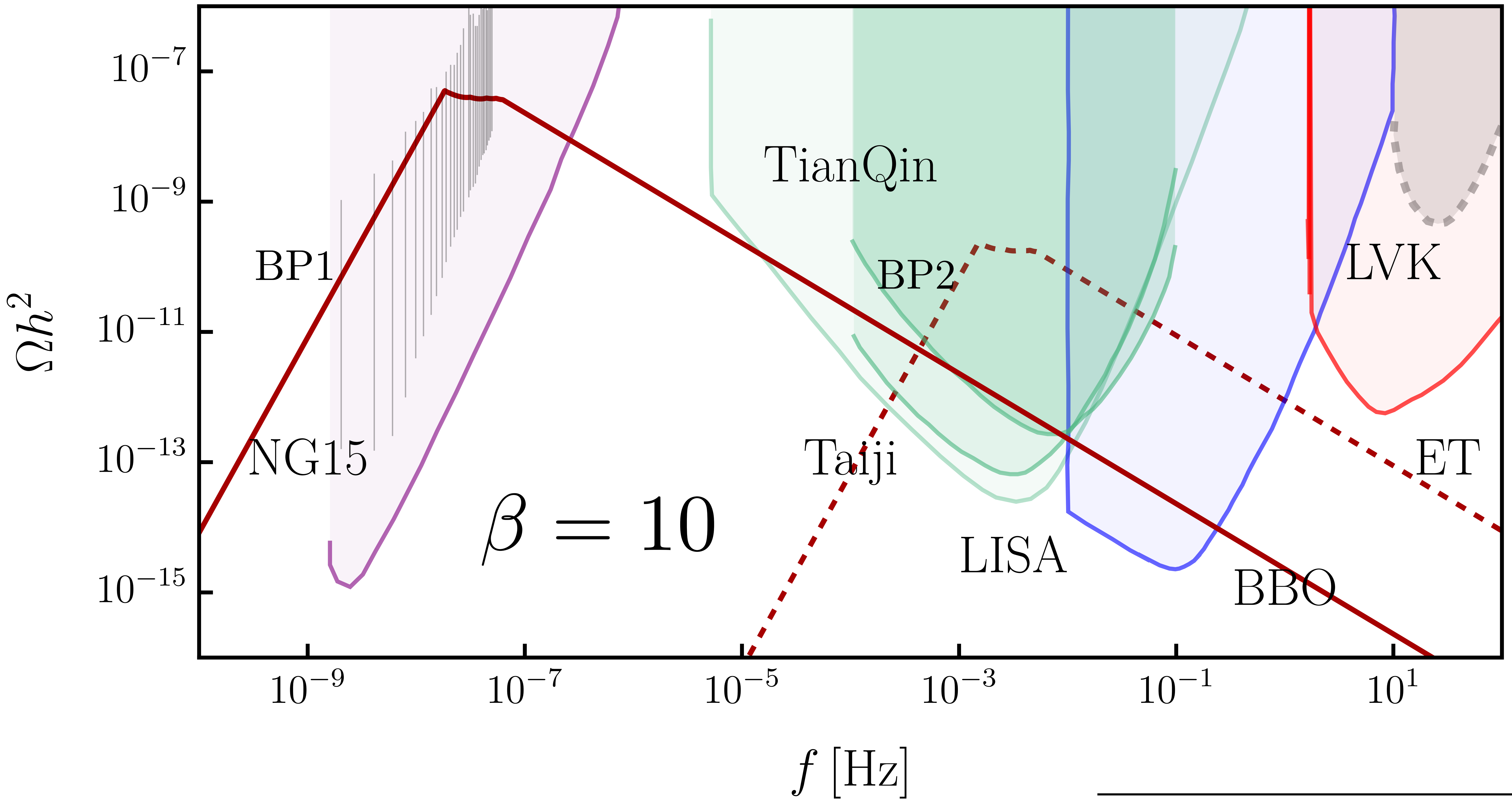
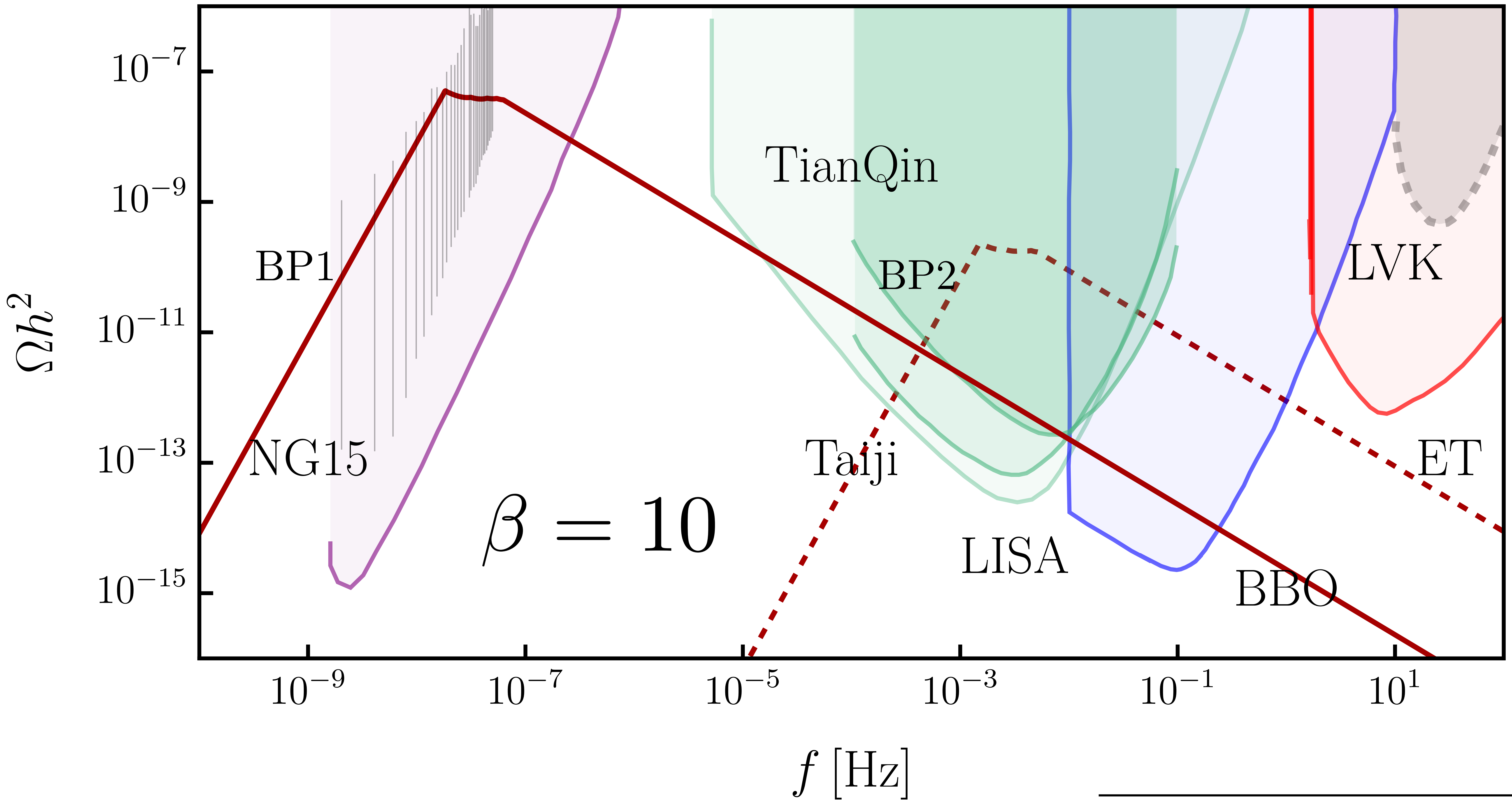


$\epsilon \sim 10^{-25}$

	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Gravitational Waves: S-type, $\mathbb{Z}_2$



$\epsilon \sim 10^{-18}$

	$\sigma^{1/3}$/TeV	$V_{\text{bias}}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Summary

Non-Abelian symmetries can solve flavour problem & give rise to rich topological including domain walls

S_4 has 5 types of walls: $\mathbb{Z}_2$ (SI, SII) & $\mathbb{Z}_3$ (TI,TII,TIII)

For certain parameter space, DWs are stable and give rise to a unique multipeak GW signal. Different experiments are sensitive to different flavour breaking scales

Much to explore: linking bias term with explicit breaking of residual symmetries, unstable domain walls exist in much of parameter space and may give rise to unique GW signatures, requires lattice simulation

Backup slides

S4 irreps

$$1^2 + 1'^2 + 2^2 + 3^2 + 3'^2$$

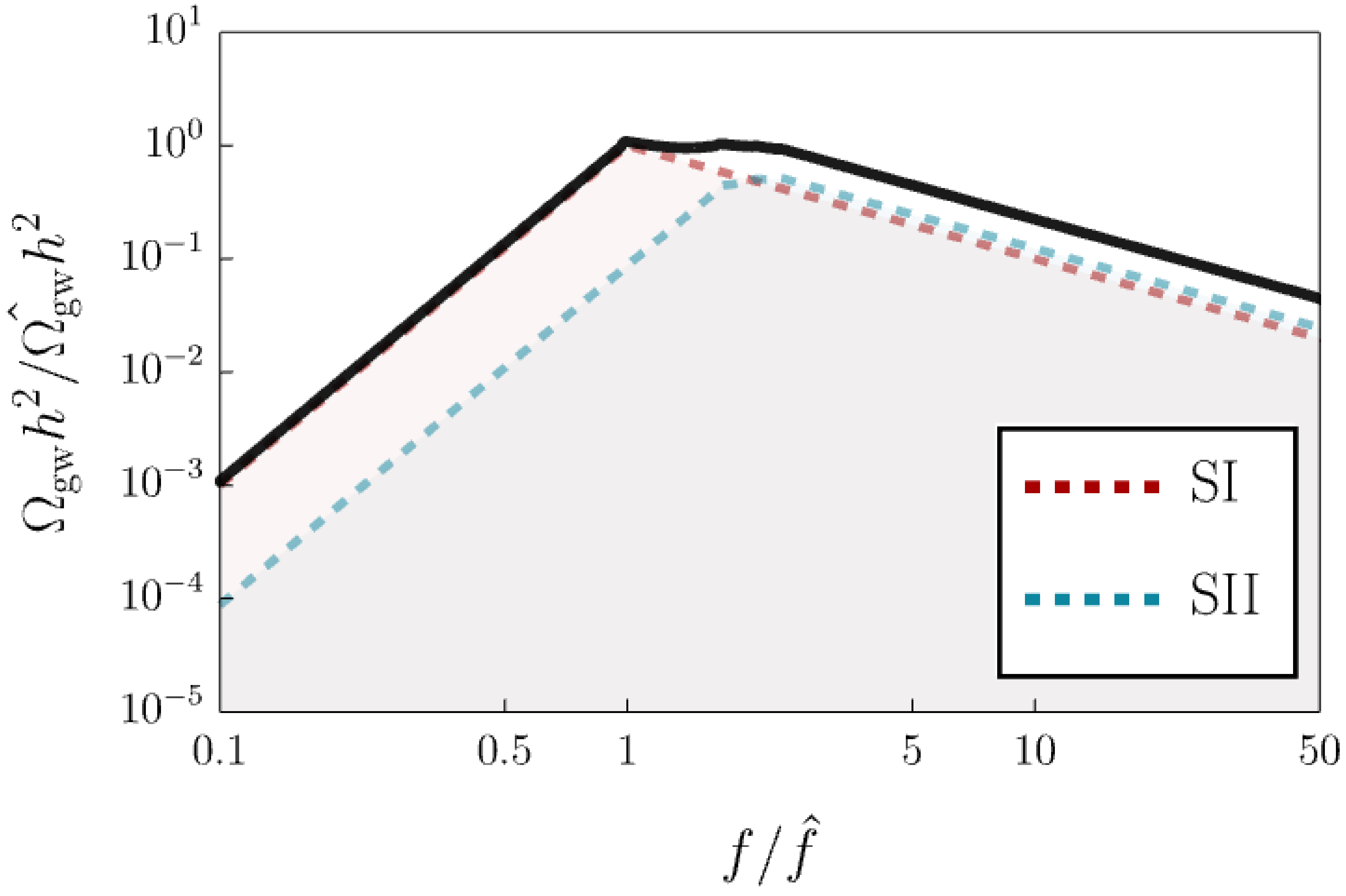
$$\left(M_\phi^2\right)_{ij} = \left.\frac{\partial^2 V(\phi)}{\partial\phi_i \partial\phi_j}\right|_{\langle\phi\rangle}$$

$$m_1^2 = 2g_1 v^2, \quad m_2^2 = m_3^2 = g_2 v^2;$$
$$m_1^2 = 2\left(3g_1 + 2g_2\right) u^2, \quad m_2^2 = m_3^2 = -2g_2 u^2;$$

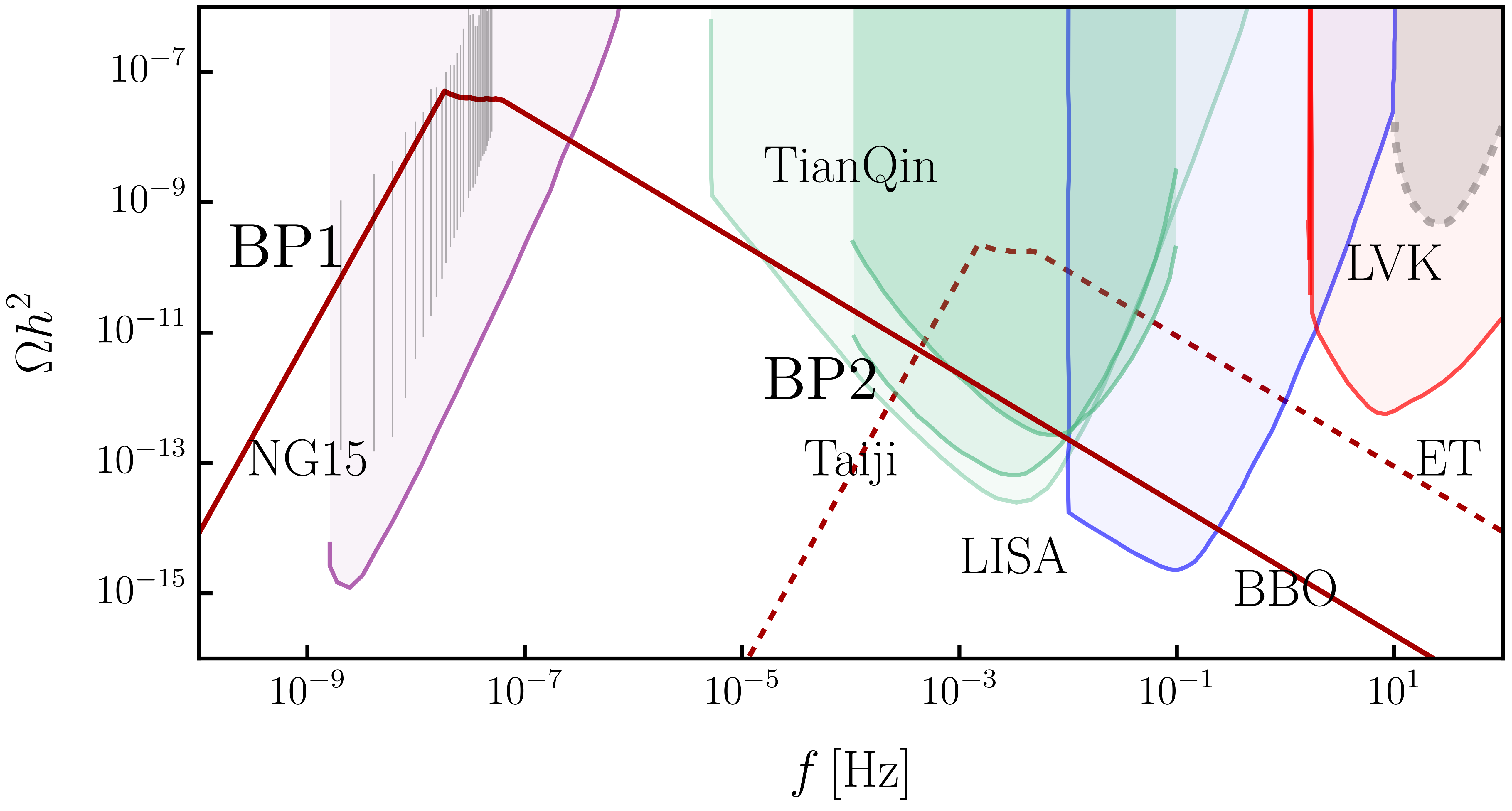
$g_2 < 0 \implies$ flavon breaks $\mathbb{Z}_2$, $g_2 > 0 \implies$ flavon breaks $\mathbb{Z}_3$

Gravitational Waves: S-type, $\mathbb{Z}_2$

$\beta = 10$



$\beta = 10$

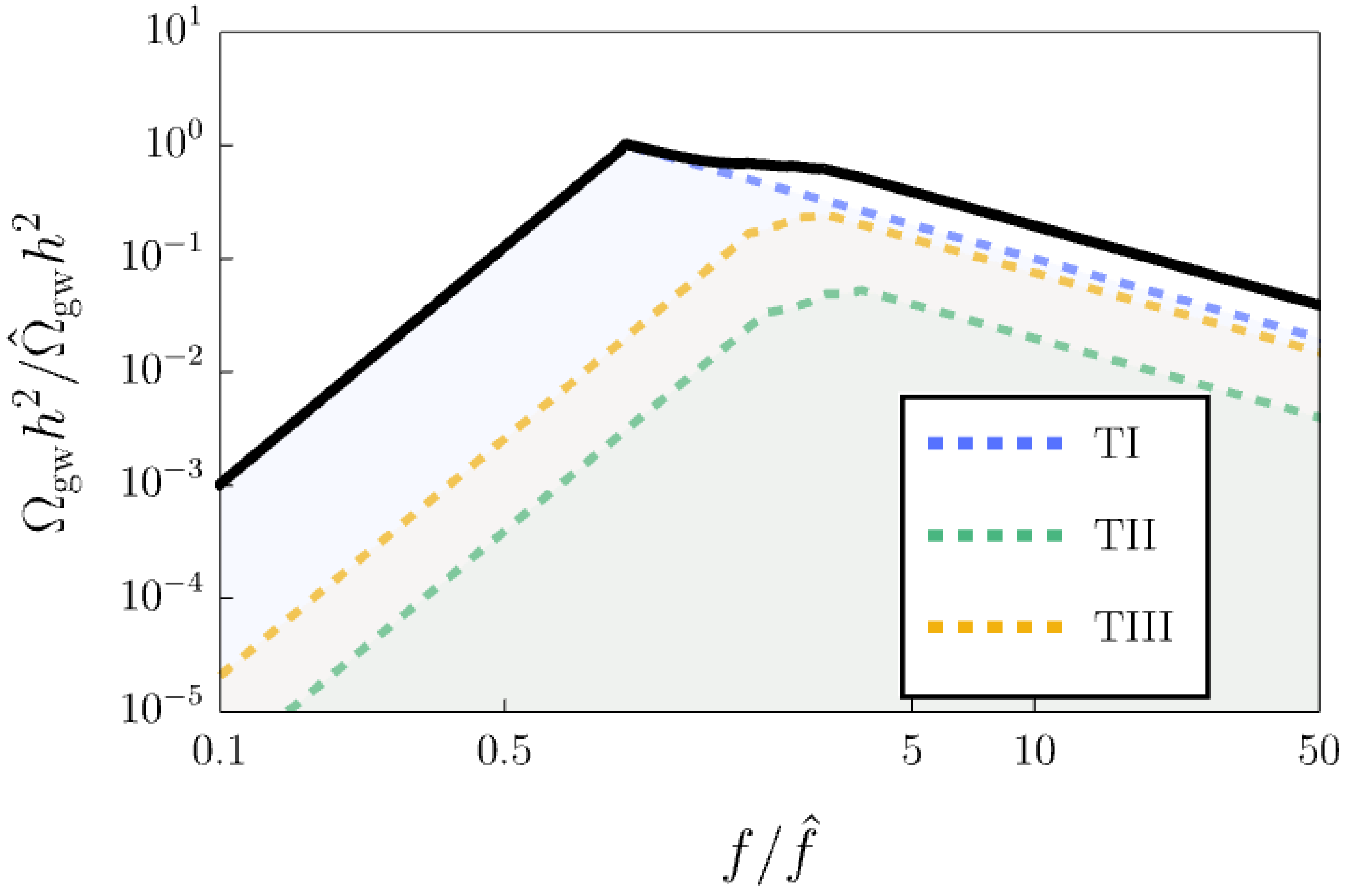


$$n_{\rm SI} : n_{\rm SII} = 3 : 12$$

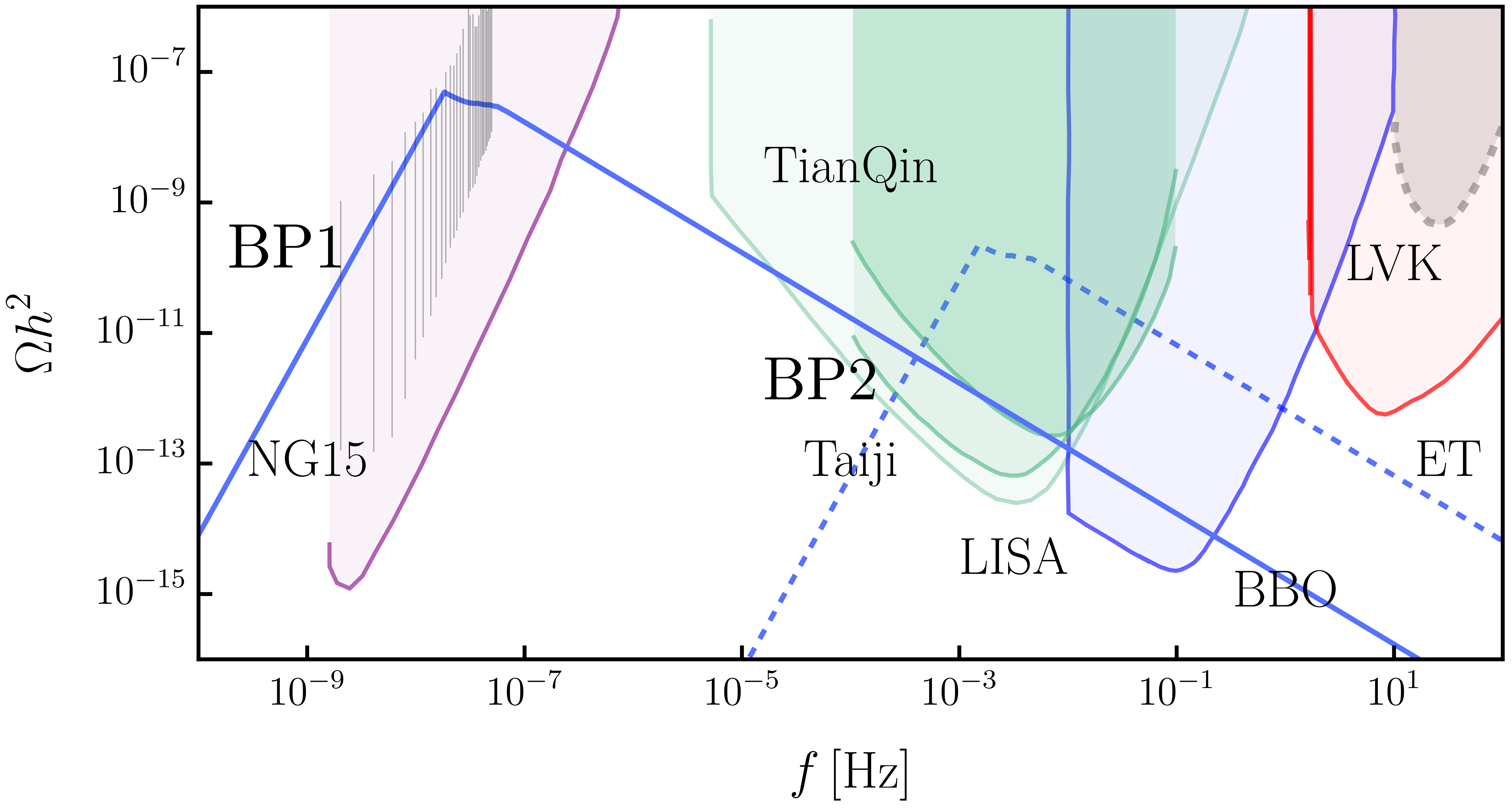
	$\sigma^{1/3}/\text{TeV}$	$V_{\rm bias}^{1/4}/\text{TeV}$
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Gravitational Waves: T-type, $\mathbb{Z}_3$

$\beta = -1.2$



$\beta = -1.2$



$$n_{\rm TI} : n_{\rm TII} : n_{\rm TIII} = 4 : 12 : 12$$

	$\sigma^{1/3}$/TeV	$V_{\rm bias}^{1/4}$/TeV
BP1	3×10^2	$10^{-3.75}$
BP2	3×10^5	10^1

Domain wall characteristics

Domain walls annihilate when pressure from the wall tension equals pressure from bias

$$p_T \sim \frac{\sigma}{t} \approx V_{\text{bias}} \implies t_{\text{ann}} = \frac{\sigma}{V_{\text{bias}}}$$

We took the curvature to be equal to time, $R \sim t$

$$t_{\text{ann}} = \frac{f_\sigma v^3}{\epsilon v^4} = \frac{f_\sigma}{\epsilon v}$$

Assuming radiation domination, $H \propto 1/(2t)$

$$H \simeq 1.66\sqrt{g_*}\frac{T^2}{M_{\text{Pl}}} \implies t = \frac{1}{2H} = \frac{1}{2 \cdot 1.66\sqrt{g_*}} \cdot \frac{M_{\text{Pl}}}{T^2} = \frac{0.301}{\sqrt{g_*}} \cdot \frac{M_{\text{Pl}}}{T^2}$$

Domain wall characteristics

Invert to find annihilation temperature:

$$T = \left(\frac{0.301 M_{\rm Pl}}{\sqrt{g_*} t}\right)^{1/2}$$

Plug in constants and take $g_* \sim 10$

$$T_{\rm ann} \simeq 3 \times 10^7 \,{\rm TeV} \left(\frac{\epsilon_b}{f_\sigma}\right)^{1/2} \left(\frac{v}{\rm TeV}\right)^{1/2}$$

For BP1: $\epsilon \sim 10^{-25}$ $\quad T_{\rm ann} \sim 0.18$ GeV

For BP2: $\epsilon \sim 10^{-18}$ $\quad T_{\rm ann} \sim 18$ TeV

Domain wall characteristics

The observed peak frequency is the redshifted Hubble scale at annihilation:

$$f_{\text{peak}} = \frac{a(T_{\text{ann}})}{a_0} \cdot H(T_{\text{ann}}) \qquad\qquad H \approx 1.66\sqrt{g_*}\frac{T_{\text{ann}}^2}{M_{\text{Pl}}}$$

Redshift factor used $g_*(T_{\text{ann}}) \sim 10\,, T_0 \sim 10^{-13}$ GeV, $g_*(T_0) = 3.91$

$$\frac{a(T_{\text{ann}})}{a_0} = \left(\frac{g_{*s}(T_0)}{g_*(T_{\text{ann}})}\right)^{1/3} \cdot \frac{T_0}{T_{\text{ann}}}$$

$$f_{\text{peak}} \approx \frac{T_0}{M_{\text{Pl}}}\left(\frac{g_{*s}(T_0)}{g_*(T_{\text{ann}})}\right)^{1/3} \cdot T_{\text{ann}} \quad \Rightarrow \quad f_{\text{peak}} \approx 1.1 \times 10^{-7}\ \text{Hz}/\text{GeV} \cdot T_{\text{ann}}$$

Using T_{ann} expression from before: $f_{\text{peak}} \simeq 3 \times 10^3\ \text{Hz} \cdot \left(\frac{\epsilon_b v}{f_\sigma \text{TeV}}\right)^{1/2}$

Domain wall characteristics

Energy Density in GW from domain wall collapse at the time of annihilation:

$$\rho_{\mathrm{GW}}\left(T_{\mathrm{ann}}\right) \sim G\sigma^2 H^2\left(T_{\mathrm{ann}}\right)$$

$$\rho_{\mathrm{GW}} \sim \frac{1}{M_{\mathrm{Pl}}^2} \cdot \left(f_\sigma^2 v^6\right) \cdot \left(\frac{T_{\mathrm{ann}}^4}{M_{\mathrm{Pl}}^2}\right) = \frac{f_\sigma^2 v^6 T_{\mathrm{ann}}^4}{M_{\mathrm{Pl}}^4}$$

$$T_{\mathrm{ann}} \sim \left(\frac{V_{\mathrm{bias}}}{\sigma}\right)^{1/2} M_{\mathrm{Pl}}^{1/2} \sim \left(\frac{\epsilon_b v}{f_\sigma}\right)^{1/2} M_{\mathrm{Pl}}^{1/2}$$

$$\rho_{\mathrm{GW}} \sim \frac{\epsilon_b^2 v^8}{M_{\mathrm{Pl}}^2}$$

Domain wall characteristics

Need to redshift this to today:

$$\rho_{\mathrm{GW},0} = \rho_{\mathrm{GW}}\left(T_{\mathrm{ann}}\right) \cdot \left(\frac{T_0}{T_{\mathrm{ann}}}\right)^4 \cdot \left(\frac{g_{*s}\left(T_0\right)}{g_{*s}\left(T_{\mathrm{ann}}\right)}\right)^{4/3}$$

Also convert to fractional energy density per logarithmic frequency:

$$\Omega_{\mathrm{GW}} h^2 = \frac{h^2}{\rho_c} \cdot \rho_{\mathrm{GW},0} \sim \frac{h^2}{\rho_c} \cdot \frac{\epsilon_b^2 v^8}{M_{\mathrm{Pl}}^2} \cdot \left(\frac{T_0}{T_{\mathrm{ann}}}\right)^4$$

Plug in appropriate numerical factors:

$$\Omega_{\mathrm{gw}} h^2 \simeq 0.9 \times 10^{-67} \left(\frac{10}{g_*\left(T_{\mathrm{ann}}\right)}\right)^{1/3} \left(\frac{\sigma}{\mathrm{TeV}^3}\right)^4 \left(\frac{\mathrm{TeV}^4}{V_{\mathrm{bias}}}\right)^2$$