S3 AND Q6 AS FLAVOR SYMMETRIES:
INTERPLAY BETWEEN QUARK AND HIGGS SECTORS

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HOW TO GO BSM?

- Many ways to go BSM
- Usually: add symmetries,
  add particles,
  add interactions
- All of the above
- Messy…
- I will concentrate on
  masses
  and mixings
- And the possibility of
  dark matter

SOME ASPECTS OF THE FLAVOUR PROBLEM

- Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- Neutrino masses unknown, only difference of squared masses.
- Type of hierarchy (normal or inverted) also unknown
- Higgs sector under study

- Quark mixing angles

$$\theta_{12} \approx 13.0^o$$
$$\theta_{23} \approx 2.4^o$$
$$\theta_{13} \approx 0.2^o$$

- Neutrino mixing angles

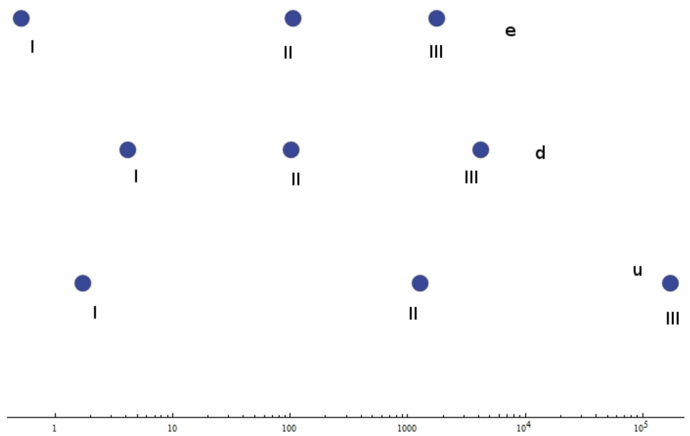
$$\Theta_{12} \approx 33.8^\circ$$
$$\Theta_{23} \approx 48.6^\circ$$
$$\Theta_{13} \approx 8.6^\circ$$

- Small mixing in quarks, large mixing in neutrinos. Very different
- Is there an underlying symmetry?

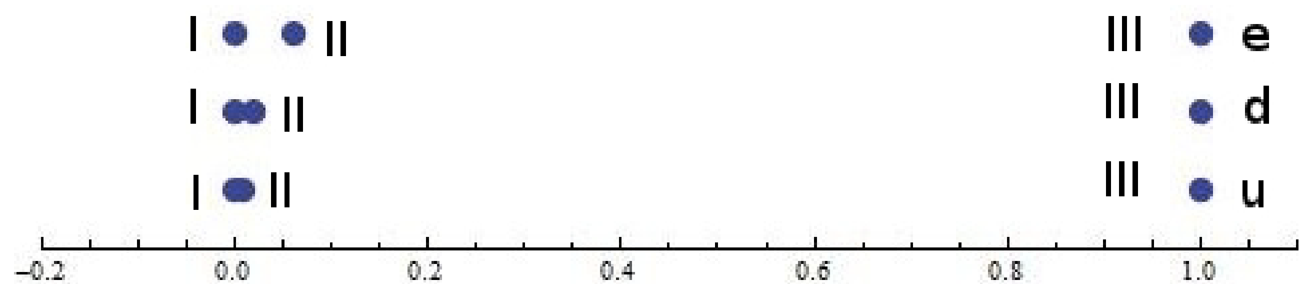
?

HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs, L-R models, etc
- Look at low energy phenomenology
- At some point they should intersect…
- In here, look at low energy phenomenology:
  - Try a flavor symmetry with 2+1 structure
  - Explore how generally it can be applied
  - Lots of scalars…quarks ok, what about neutrinos?
  - Compare it with the data — prospects for dark matter
- See how predictive it turns out



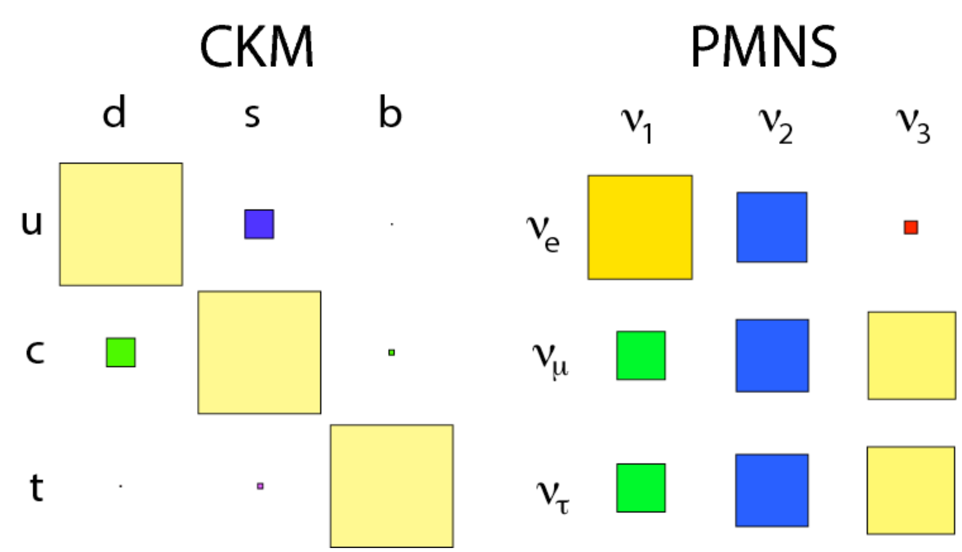
Plot of mass ratios



Logarithmic plot of quark masses

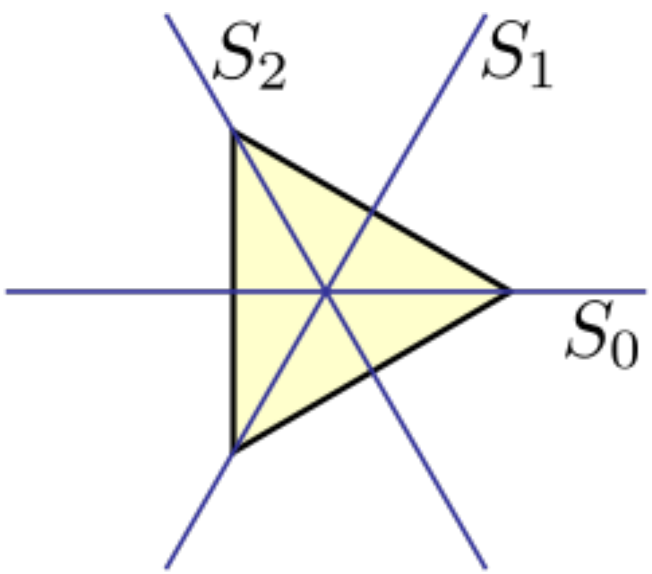
$$
\begin{bmatrix} |V_{\mathrm{ud}}| & |V_{\mathrm{us}}| & |V_{\mathrm{ub}}| \\ |V_{\mathrm{cd}}| & |V_{\mathrm{cs}}| & |V_{\mathrm{cb}}| \\ |V_{\mathrm{td}}| & |V_{\mathrm{ts}}| & |V_{\mathrm{tb}}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},
$$

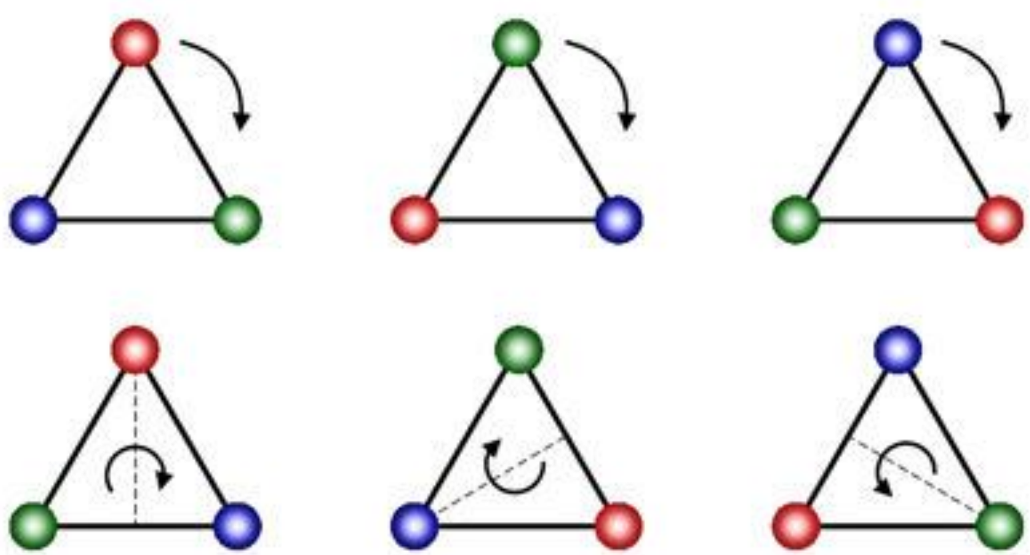
Suggests a 2⊕1 structure for quarks… for leptons?



WE WILL EXPLORE S3 AND Q6

- S3 is the smallest non-abelian group
- Has irreducible representations $\mathbf{2}, \mathbf{1}_S, \mathbf{1}_A$
- Permutations of three objects or rotations and reflections that leave invariant an equilateral triangle





- Q6 is double covering of S3, has irreps

$$\mathbf{2}_1, \mathbf{2}_2, \mathbf{1}_{++}, \mathbf{1}_{--}, \mathbf{1}_{+-}, \mathbf{1}_{-+}$$

A sample of S3 models

S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)

E. Derman, Phys. Rev. D19, 317 (1979)

D. Wyler, Phys. Rev. D19, 330 (1979)

R. Yahalom, Phys. Rev. D29, 536 (1984)

Y. Koide, Phys. Rev. D60, 077301 (1999)

A. Mondragon et al, Phys. Rev. D59, 093009, (1999)

J. Kubo, A. Mondragon, et al, Prog. Theor. Phys. 109, 795 (2003)

J. Kubo et al, Phys. Rev. D70, 036007 (2004)

S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)

A. Mondragon et al, Phys. Rev. D76, 076003, (2007)

S. Kaneko et al, hep-ph/0703250, (2007)

S. Chen et al, Phys. Rev. D70, 073008 (2004)
T. Teshima et al, Phys.Rev. D84 (2011) 016003 Phys.Rev. D85 105013 (2012)

F. Gonzalez Canales, A&M. Mondragon Fort. der Physik 61, Issue 4-5 (2013)

H.B. Benaoum, Phys. RevD.87.073010 (2013)

E. Ma and B. Melic, arXiv:1303.6928

F. Gonzalez Canales, A. &M Mondragon, U. Saldaña, L. Velasco, arXiv:1304.6644

R. Jora et al, Int.J.Mod.Phys. A28 (2013),1350028

A. E. Cárcamo Hernández, E. Cataño Mur, R. Martinez, Phys.Rev. D90 (2014) no.7, 073001

A.E. Cárcamo, I. de Medeiros E. Schumacheet, Phys.Rev. D93 (2016) no.1, 016003

A.E. Cárcamo, R. Martinez, F. Ochoa, Eur.Phys.J. C76 (2016)

D Das, P Pal, Pays Rev D98 (2018)

C Espinoza, E Garcés, MM, H. Reyes, PLB 788 (2019)

JC Gómez Izquierdo, MM, EPJC 79 (2019)

O. Felix-Beltran, M.M., et al, J.Phys.Conf.Ser. 171, 012028 (2009)

A. Dicus, S Ge, W Repko, Phys. Rev D82 (2010)

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

G. Bhattacharyya et al, Phys. Rev. D83, 011701 (2011)

D. Meloni, JHEP 1205 (2012) 124

S. Dev et al, Phys.Lett. B708 (2012) 284-289

S. Zhou, Phys.Lett. B704 (2011) 291-295

D. Meloni et al, Nucl. Part. Phys. 38 015003, (2011)

E. Ma and B. Melic, Phys.Lett. B725 (2013)

E. Barradas et al, 2014

P. Das et al, PhyrRev D89 (2014,) 2016

ZZ Zhing, D Zhang JHEP 03 2019)

S Pramanick, Phys Rev D100 (2019)

Emmanuel-Costa et al, JHEP (2016)

Kuncinas et al, PRD (2020)

M. Gómez-Bock, A. Pérez, MM, EPJC81 (2021)

T. Kobayashi, K. Tanaka, T.H. Tatsuishi, PRD98 (2018)

D. Meloni, M. Parriciatu, JHEP (2023)

H. Okada, Y. Orikasa, arXiv:2501.15748

Just a sample, there are many more… I apologize for those not included

2+1 STRUCTURE

- 2+1 structure works well for quarks:
  S3, S4, Q6
- Neutrinos?
  Mixing angles known
  Masses? Only difference of squared masses
- Type I seesaw works well, other ways?
- Scalar sector? More Higgses?
  Residual symmetries possible once full minimization of scalar potential is done
- Dark Matter?

- Explore non-minimal models in scalars
- Neutrino masses: ISSM or radiatively
- Quarks:
  2+1 works well
- Scalars, allow for complex vev's
- Phenomenology

S3 AND Q6 MODELS (UNDER CONSTRUCTION)

- S3 x Z2 $U(1)_{B\text{-}L}$ flavored model:
  - 2+1 in quarks
  - 1+2 in leptons $\Longrightarrow$ mu-tau symmetry
  - ISSM mechanism for neutrino masses
  - Cobimaximal mixing in neutrinos, with deviations from leptonic sector $\Longrightarrow$ we can fit observables
  - Many scalars, some with complex vev's (new in quarks)
  - DM sector not yet worked out
- Q6 x Z4 xZ2
  - 2+1 in quarks, leptons and Higgs
  - Many scalars, some with complex vev's
  - Neutrino masses radiatively generated at two-loops
  - Cobimaximal mixing, with deviations from leptonic sector
  - Higgs sector and DM worked out,
    consistent with known phenomenological constraints

S3 X Z2 FLAVORED U(1)B-L MODEL

- Usual U(1)B-L has 3NR and 3S and 3s fermionic singlets and one Higgs
  Khalil PRD 2010

- In our version we have 3NR, 3S and 3s neutrinos, plus 3 Higgs doublets and 3 singlets (lots of exotics!)

  Work with J.C. Gómez-Izquierdo, C. Espinoza, L. Gutiérrez-Luna, M.M arXiv:2411.03392

- Additional Z2 to forbid some Yukawa couplings

Matter	$Q_I, d_{IR}, u_{I,R}, H_I, L_J, e_{JR}, N_{J,R}, S_{JL}, s_{JL}$	$L_1, e_{1R}, N_{1R}, S_{1L}, s_{1L}$	$Q_3, d_{3R}, u_{3R}, H_3, \phi_3$	ϕ_I
$\mathbf{S}_3$	2	1_S	1_S	2
$\mathbf{Z}_2$	1	-1	1	-1

TABLE II. Flavored $B-L$ model. Here, $I=1,2$ and $J=2,3$.

Φ_i, S_i, s_i at high energies (TeV), H_i at low energies (eW)

LAGRANGIAN

- Lepton sector Lagrangian

$$
\begin{aligned}
-\mathcal{L}_\ell = {} & y_1^e \bar{L}_1 H_3 e_{1R} + y_2^e \left[(\bar{L}_2 H_2 + \bar{L}_3 H_1) e_{2R} + (\bar{L}_2 H_1 - \bar{L}_3 H_2) e_{3R}\right] + y_3^e \left[\bar{L}_2 H_3 e_{2R} + \bar{L}_3 H_3 e_{3R}\right] + y_1^D \bar{L}_1 \tilde{H}_3 N_{1R} \\
& + y_2^D \left[(\bar{L}_2 \tilde{H}_2 + \bar{L}_3 \tilde{H}_1) N_{2R} + (\bar{L}_2 \tilde{H}_1 - \bar{L}_3 \tilde{H}_2) N_{3R}\right] + y_3^D \left[\bar{L}_2 \tilde{H}_3 N_{2R} + \bar{L}_3 \tilde{H}_3 N_{3R}\right] + y_1^R \overline{(N_{1R})^c} \phi_3 S_1 \\
& + y_2^R \overline{(N_{1R})^c} [\phi_1 S_2 + \phi_2 S_3] + y_3^R [\overline{(N_{2R})^c} \phi_1 + \overline{(N_{3R})^c} \phi_2] S_1 + y_4^R \left[\overline{(N_{2R})^c} \phi_3 S_2 + \overline{(N_{3R})^c} \phi_3 S_3\right] + M_1 \overline{S_1^c} S_1 \\
& + M_2 \left[\overline{S_2^c} S_2 + \overline{S_3^c} S_3\right] + h.c. \qquad (10)
\end{aligned}
$$

s not shown for simplicity, not relevant here

- Quark sector Lagrangian

$$
\begin{aligned}
-\mathcal{L}_q = {} & y_1^d \left[\bar{Q}_{1L} (H_1 d_{2R} + H_2 d_{1R}) + \bar{Q}_{2L} (H_1 d_{1R} - H_2 d_{2R})\right] + y_2^d \left[\bar{Q}_{1L} H_3 d_{1R} + \bar{Q}_{2L} H_3 d_{2R}\right] + y_3^d \left[\bar{Q}_{1L} H_1 + \bar{Q}_{2L} H_2\right] d_{3R} \\
& + y_4^d \bar{Q}_{3L} \left[H_1 d_{1R} + H_2 d_{2R}\right] + y_5^d \bar{Q}_{3L} H_3 d_{3R} + y_1^u \left[\bar{Q}_{1L} \left(\tilde{H}_1 u_{2R} + \tilde{H}_2 u_{1R}\right) + \bar{Q}_{2L} \left(\tilde{H}_1 u_{1R} - \tilde{H}_2 u_{2R}\right)\right] \\
& + y_2^u \left[\bar{Q}_{1L} \tilde{H}_3 u_{1R} + \bar{Q}_{2L} \tilde{H}_3 u_{2R}\right] + y_3^u \left[\bar{Q}_{1L} \tilde{H}_1 + \bar{Q}_{2L} \tilde{H}_2\right] u_{3R} + y_4^u \bar{Q}_{3L} \left[\tilde{H}_1 u_{1R} + \tilde{H}_2 u_{2R}\right] \\
& + y_5^u \bar{Q}_{3L} \tilde{H}_3 u_{3R} + h.c. \qquad (11)
\end{aligned}
$$

SCALAR POTENTIAL

- Scalar potential is of the form:

$$V = V(H) + V(\phi) + V(H,\phi),$$

- V(H) usual S3-3H potential, V(*Φ*) of B-L sector, V(H,*Φ*) mixing term

$$V(H) = M^2(H^\dagger H) + \frac{a}{2}(H^\dagger H)^2.$$

$$V(\phi) = \mu_{BL}^2(\phi^\dagger\phi) + \frac{\lambda}{2}(\phi^\dagger\phi)^2$$

$$V(H,\phi) = -L\left(H^\dagger H\right)\left(\phi^\dagger\phi\right) \ ,$$

Vev's for eW sector complex:

$$\langle H_1\rangle = v_1,\ \langle H_2\rangle = iv_2 \text{ and } \langle H_3\rangle = v_3$$

no residual symmetry

Vev's for B-L sector real, no mixing:

$$w_1 = w_2 \quad \text{or} \quad \langle\phi_2\rangle = \langle\phi_1\rangle.$$

QUARKS

- Quark mass matrices after minimizing potential

$$\mathbf{M}_d = \frac{1}{\sqrt{2}} \begin{pmatrix} y_2^d v_3 + i y_1^d v_2 & y_1^d v_1 & y_3^d v_1 \\ y_1^d v_1 & y_2^d v_3 - i y_1^d v_2 & i y_3^d v_2 \\ y_4^d v_1 & i y_4^d v_2 & y_5^d v_3 \end{pmatrix}, \mathbf{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_2^u v_3 - i y_1^u v_2 & y_1^u v_1 & y_3^u v_1 \\ y_1^u v_1 & y_2^u v_3 + i y_1^u v_2 & -i y_3^u v_2 \\ y_4^u v_1 & -i y_4^u v_2 & y_5^u v_3 \end{pmatrix}.$$

- M23 = M32 via appropriate rotation and assume hierarchical structure in complex Yukawas and vevs, in polar form

Fritzsch and Xing (1999)

$$\bar{\mathbf{M}}_q = \begin{pmatrix} |\tilde{A}_q| & |\tilde{b}_q| & 0 \\ |\tilde{b}_q| & |\tilde{B}_q| & |\tilde{C}_q| \\ 0 & |\tilde{C}_q| & |\tilde{h}_q| \end{pmatrix}.$$

$|\langle H_1 \rangle| < |\langle H_2 \rangle| < |\langle H_3 \rangle|$ and $y_3^q, y_4^q \ll y_1^q$ $\qquad$ $\tilde{m}_{q_i} = m_{q_i}/m_{q_3}$, $\qquad$ $1 > |\tilde{h}_q| > \tilde{m}_{q_2} > |\tilde{m}_{q_1}| > |\tilde{A}_q|$

- 10 parameters + 2 phases = too many, but we reparametrize 3 in terms of the mass ratios 6 parameters in total with these assumptions:

$|h_q|$, $|A_q|$ $(q = u, d)$ $\qquad$ $\bar{\beta} = \beta_q - \alpha_q$ and $\bar{\gamma} = \gamma_q - \alpha_q$

Still too many parameters

GATTO-SARTORI-TONIN LIMIT

- We take a particular benchmark $|\tilde{h}_q| \approx 1 - \tilde{m}_{q_2}$ and $|\tilde{A}_q| \approx 0$
- We obtain Gatto-Sartori-Tonin relations as limiting case, 4 free parameters, fit everything correctly
- If Yukawa couplings considered real not possible to fit Jarlskog invariant
- We fit all 6, BFP goes to GST (as expected):

$$\mathcal{L}_{ij} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{1}{2} \frac{(v_{ij} - v_{ij}^{\exp})^2}{\sigma^2} \right\}$$

$$\Delta\chi^2 \equiv \chi^2 - \chi^2_{\min}$$

$$\Delta\chi^2 = -2 \log (\mathcal{L}/\mathcal{L}_{\max})$$

$$|\tilde{h}_u| = 0.8919159, \qquad |\tilde{A}_u| = 6.85834 \times 10^{-6}, \qquad \bar{\beta} = 1.005637;$$

$$|\tilde{h}_d| = 0.8693808, \qquad |\tilde{A}_d| = 4.85842 \times 10^{-4}, \qquad \bar{\gamma} = 0.956388.$$

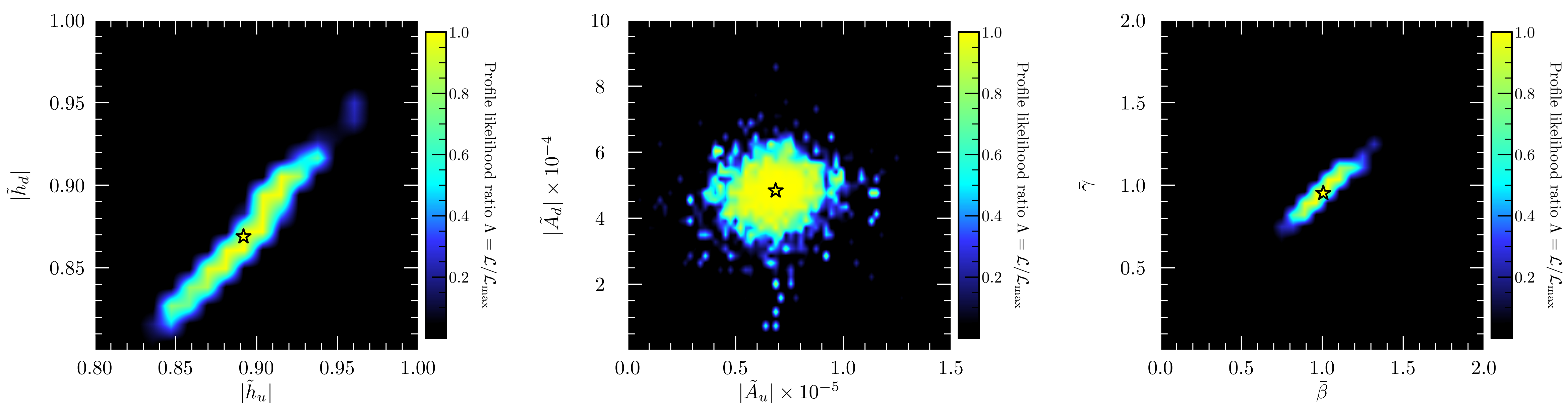


FIG. 1. Regions of the free parameter space where the model can fit accurately the experimental observations regarding the CKM matrix. Dark regions are not compatible with observations at all, while the best fit point (BFP) is depicted with a star.

CHARGED LEPTONS

- After minimization of the scalar potential:

$$\mathbf{M}_e = \begin{pmatrix} a_e & 0 & 0 \\ 0 & c_e + i b_e & d_e \\ 0 & d_e & c_e - i b_e \end{pmatrix}, \quad \mathbf{M}_D = \begin{pmatrix} a_D & 0 & 0 \\ 0 & c_D - i b_D & d_D \\ 0 & d_D & c_D + i b_D \end{pmatrix}, \quad \mathbf{M}_R = \begin{pmatrix} a_R & b_R & b_R \\ b'_R & c_R & 0 \\ b'_R & 0 & c_R \end{pmatrix}$$

$$\mathbf{M}_2 = \mathrm{Diag.}(M_1, M_2, M_2)$$

$$a_e = y_1^e v_3, \quad c_e = y_3^e v_3, \quad b_e = y_2^e v_2, \quad d_e = y_2^e v_1,$$
$$a_D = y_1^D v_3, \quad c_D = y_3^D v_3, \quad b_D = y_2^D v_2, \quad d_D = y_2^D v_1,$$
$$a_R = y_1^R \langle \phi_3 \rangle, \quad b_R = y_2^R \langle \phi_1 \rangle, \quad b'_R = y_3^R \langle \phi_1 \rangle, \quad c_R = y_4^R \langle \phi_3 \rangle$$

- For charged leptons one phase that contributes to PMNS matrix, Yukawas are complex

$$\mathbf{M}_e = \begin{pmatrix} a_e & 0 & 0 \\ 0 & A_e & d_e \\ 0 & d_e & B_e \end{pmatrix} \quad \mathbf{O}_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_e & \sin\theta_e \\ 0 & -\sin\theta_e & \cos\theta_e \end{pmatrix} \quad \tan\theta_e = \sqrt{\frac{|A_e| - m_\mu}{m_\tau - |A_e|}},$$

$$m_\tau > |A_e| > m_\mu.$$

- Neutrino masses from ISSM: $\mathbf{M}_\nu = \mathbf{M}_D (\mathbf{M}^T)_R^{-1} \mathbf{M}_2 \mathbf{M}_R^{-1} \mathbf{M}_D^T$

NEUTRINOS – ISSM AND CONTRIBUTION FROM LEPTONS

- Light neutrinos acquire mass through inverted seesaw

$$-\mathcal{L} = \frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \overline{(N_R)^c} & \overline{(S)^c}\end{pmatrix}\overbrace{\begin{pmatrix} 0 & \mathbf{M}_D & 0 \\ \mathbf{M}_D^T & 0 & \mathbf{M}_R \\ 0 & \mathbf{M}_R^T & \mathbf{M}_2\end{pmatrix}}^{\mathcal{M}_\nu}\begin{pmatrix}(\nu_L)^c \\ N_R \\ S\end{pmatrix} + h.c.$$

$$\mathcal{U}_\nu \approx \begin{pmatrix}\left(\mathbf{1}_{3\times3} - \frac{1}{2}\mathcal{A}\mathcal{A}^\dagger\right) & \mathcal{A} \\ -\mathcal{A}^\dagger & \left(\mathbf{1}_{6\times6} - \frac{1}{2}\mathcal{A}^\dagger\mathcal{A}\right)\end{pmatrix}\begin{pmatrix}\mathbf{U}_\nu & 0 \\ 0 & \mathbf{V}_R\end{pmatrix}$$

$$\mathcal{U}_\nu^\dagger \mathcal{M}_\nu \mathcal{U}_\nu^* = \hat{\mathcal{M}}_\nu \text{ with } \hat{\mathcal{M}}_\nu = \mathrm{Diag.}\left(\hat{\mathbf{M}}_\nu, \hat{\mathbf{M}}_R\right)$$

$$\mathbf{U}_\nu^\dagger \mathbf{M}_\nu \mathbf{U}_\nu^* = \hat{\mathbf{M}}_\nu \qquad \mathbf{V}_R^\dagger \mathcal{M}_R \mathbf{V}_R^* = \hat{\mathbf{M}}_R.$$

$$\mathcal{A} = \left((\mathbf{M}_D(\mathbf{M}_R^T)^{-1}\mathbf{M}_2\mathbf{M}_R^{-1})_{3\times3} \quad (\mathbf{M}_D(\mathbf{M}_R^T)^{-1})_{3\times3}\right) \approx \left((0)_{3\times3} \quad (\mathbf{A})_{3\times3}\right)$$

$$\mathbf{M}_\nu = \mathbf{M}_D(\mathbf{M}^T)_R^{-1}\mathbf{M}_2\mathbf{M}_R^{-1}\mathbf{M}_D^T$$

- Then in the PMNS, η is mixing between light and heavy scalars:

$$\mathbf{U} = \mathbf{U}_l^\dagger\,(1-\eta)\,\mathbf{U}_\nu \qquad \eta = \mathcal{A}\mathcal{A}^\dagger/2$$

- Neutrinoless double beta decay and charged LFV get extra contributions from heavy scalars

NEUTRINOS

- In our model, assume neutrino Yukawas are real, then:

$\mathbf{M}_\nu = \mathbf{M}_D (\mathbf{M}^T)^{-1}_R \mathbf{M}_2 \mathbf{M}_R^{-1} \mathbf{M}_D^T$

$$\mathbf{A} \equiv \begin{pmatrix} a_1 & -a_2 & -a_2 \\ a_3 & a_4^* & a_5^* \\ a_3^* & a_5 & a_4 \end{pmatrix}, \qquad \eta = \begin{pmatrix} a_\nu & b_\nu & b_\nu^* \\ b_\nu^* & c_\nu & d_\nu \\ b_\nu & d_\nu^* & c_\nu \end{pmatrix}, \qquad \mathbf{M}_\nu = \begin{pmatrix} A_\nu & B_\nu & B_\nu^* \\ B_\nu & C_\nu^* & D_\nu \\ B_\nu^* & D_\nu & C_\nu \end{pmatrix}$$

- M_ν has cobimaximal texture, 4 parameters, $\theta_{23} = \pi/4$ and $\delta_{CP} = -\pi/2$ are fixed
- Parameters can be expressed in terms of masses and mixing angles
- The contribution from the leptonic sector breaks this texture $\Longrightarrow$ deviations from cobimaximal with 2 lepton parameters

  $\Longrightarrow$ possible to fit correctly all neutrino parameters

$$\sin^2\theta_{13} = |(\mathbf{U})_{13}|^2 = \sin^2\rho_{13};$$
$$\sin^2\theta_{23} = \frac{|(\mathbf{U})_{23}|^2}{1 - |\mathbf{U}_{13}|^2} = \frac{1}{2}\left[1 + \sin 2\theta_e \cos\eta_\ell\right];$$
$$\sin^2\theta_{12} = \frac{|(\mathbf{U})_{12}|^2}{1 - |\mathbf{U}_{13}|^2} = \sin^2\rho_{12}.$$

$$\sin\delta_{CP} = -\frac{\cos 2\theta_e}{\sqrt{1 - \sin^2 2\theta_e \cos^2\eta_\ell}}.$$

LEPTON FLAVOR VIOLATION

- BR charged lepton flavor violating decays, depends on η

$$BR(\ell_\alpha \to \ell_\beta \gamma) \approx \frac{\alpha_W^3 \sin^2\theta_W}{256\pi^2} \frac{m_{\ell_\alpha}^4}{m_W^4} \frac{m_{\ell_\alpha}}{\Gamma_{\ell_\alpha}} \left| \sum_{i=1}^{3} \mathbf{U}_{\alpha i} \mathbf{U}_{\beta i}^* G_\gamma \left(\frac{m_i^2}{m_W^2}\right) + \sum_{j=1}^{6} \mathcal{K}_{\alpha j} \mathcal{K}_{\beta j}^* G_\gamma \left(\frac{m_{jR}^2}{m_W^2}\right) \right|^2$$

- We define the likelihood function like

$$\log \mathcal{L} = \log \mathcal{L}_{\delta_{CP}} + \log \mathcal{L}_{\theta_{23}} + \log \mathcal{L}_{\Delta m_{21}^2} + \log \mathcal{L}_{\Delta m_{31}^2} + \log \mathcal{L}_{m_{\mathrm{tot}}} + \log \mathcal{L}_{\mu \to e\gamma} + \log \mathcal{L}_{\tau \to e\gamma} + \log \mathcal{L}_{\tau \to \mu\gamma}$$

- We check that m_{ee} is within GERDA limits

$$\begin{aligned} |m_{ee}| &\approx \left| \sum_{i=1}^{3} (\mathbf{U})_{ei}^2 m_i + \sum_{i=1}^{6} (\mathcal{K})_{ei}^2 p^2 \frac{m_{iR}}{p^2 - m_{iR}^2} \right|; \\ &\approx \left| \sum_{i=1}^{3} (\mathbf{U})_{ei}^2 m_i - p^2 \sum_{i=1}^{6} \frac{(\mathcal{K})_{ei}^2}{m_{iR}} \right|, \end{aligned}$$

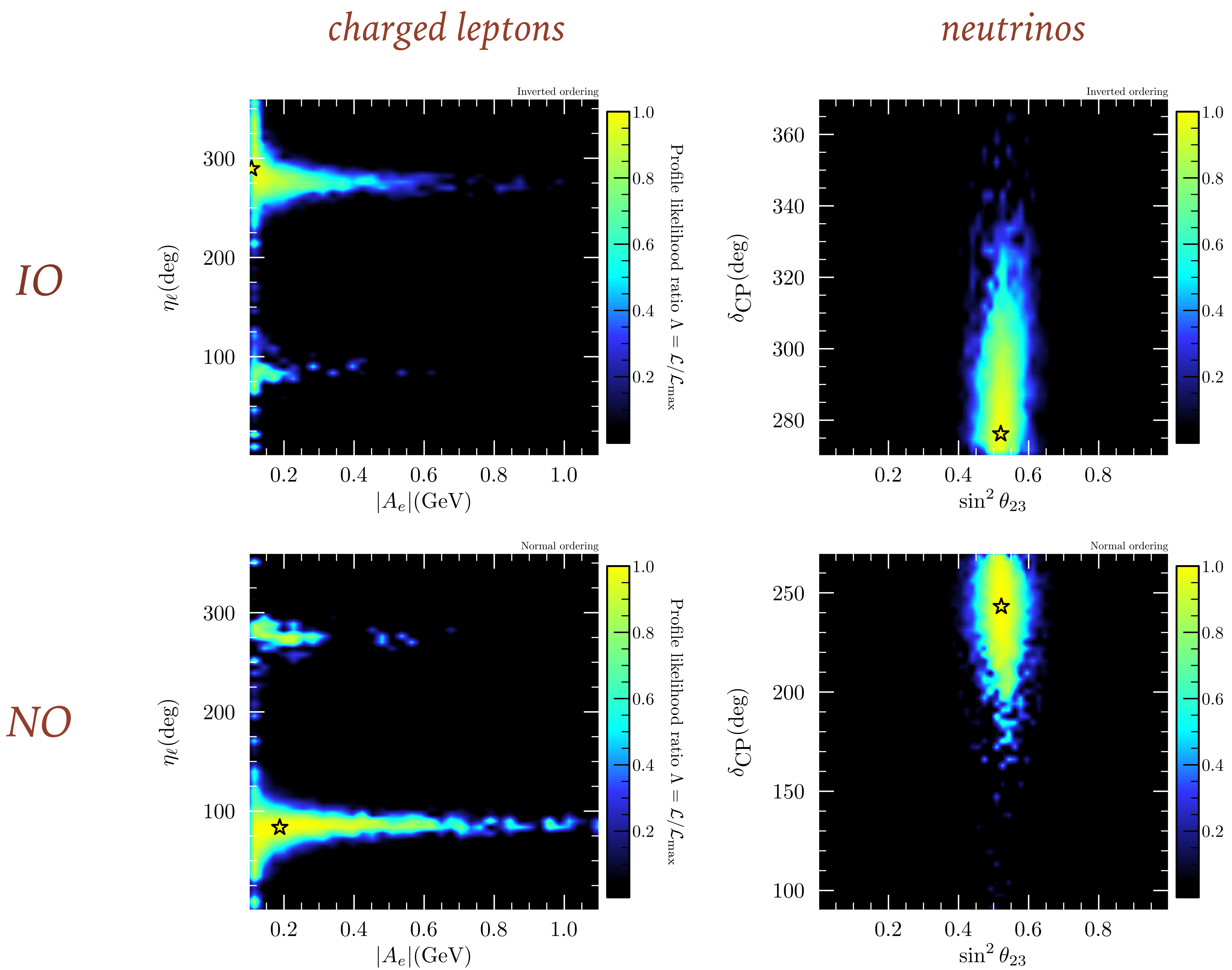


FIG. 2. Regions of the free parameters where the model can fit accurately the experimental observation regarding the observables in the lepton sector (left panels), the right panels show the values of $\sin^2\theta_{23}$ and δ_{CP} predicted by the model that are most compatible with current observations. Top (bottom) panel is for Inverted (Normal) Hierarchy. Dark regions are not compatible with observations at all, while the best fit point (BFP) is depicted with a star.

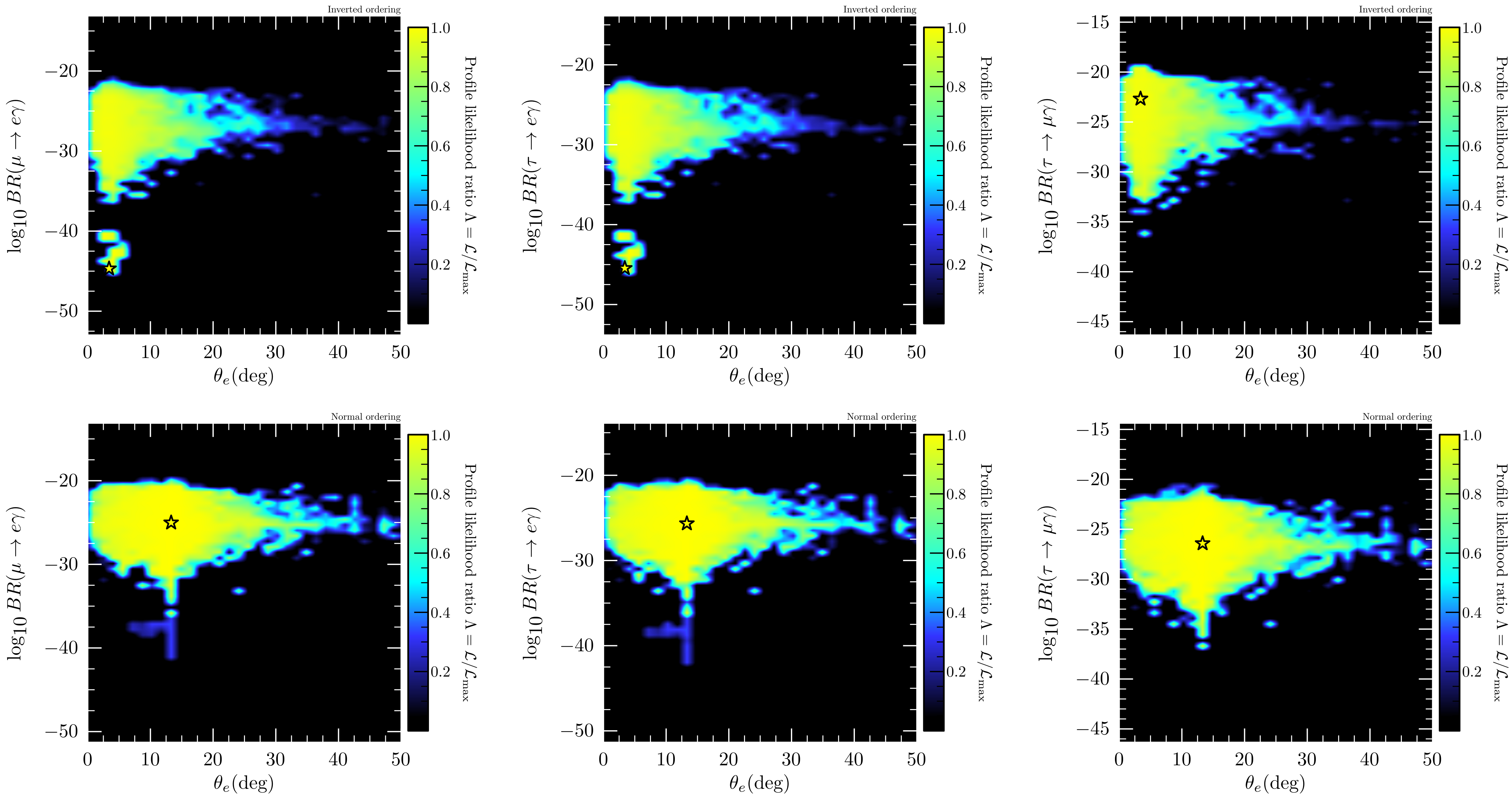


FIG. 3. Predicted branching fractions as functions of θ_e. Dark regions are not compatible with observations at all, while the best fit point (BFP) is depicted with a star.

RECAP S3 B-L

- 2+1 + complex vev's still works for quarks (new)
  - Gatto-Sartori-Toni relations as a limit
- Leptons: complex vev's and 2+1 assignment $\Longrightarrow$ we recover cobimaximal plus contribution from leptons
- Despite the large amount of extra scalars, effective number of parameters is reduced greatly by symmetry (6)
- We can get values for LFV and m_{ee}, consistent with data and also BFPs values

  $\Longrightarrow$ testable or falsifiable
- To do, Higgs and DM sectors, leptogenesis also possible

Q6, LOTS OF SCALARS, COBIMAXIMAL MIXING AND DM

$$\mathcal{G} = SU(3)_C \times SU(2)_L \times U(1)_Y \times Q_6 \times Z_4 \times Z_2 \xrightarrow{v_\sigma, v_\rho, v_\xi} SU(3)_C \times SU(2)_L \times U(1)_Y \times \widetilde{Z}_2 \times Z_2 \xrightarrow{v_1, v_2, v_3} SU(3)_C \times U(1)_Q \times \widetilde{Z}_2 \times Z_2,$$

Z2,Z2 will allow for 3 DM candidates

3 H_i doublets + 1H_4 inert doublet + 6 scalar neutral singlets + 3N_R

High energies

	q_L	q_{3L}	u_R	u_{3R}	d_R	d_{3R}	l_{1L}	l_L	e_{1R}	e_R	N_{1R}	N_R
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	-1	0	0
Q_6	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$	$\mathbf{1}_{-+}$	$\mathbf{2}_2$
Z_2	0	0	0	0	0	0	0	0	0	0	1	1
Z_4	0	0	0	0	0	0	0	0	0	0	1	1

Table I: Fermion content with the $SU(3)_C \times SU(2)_L \times U(1)_Y \times Q_6 \times Z_2 \times Z_4$ assignments.

High energies

	H	H_3	H_4	φ_1	φ_2	σ	ξ	ρ
$SU(3)_C$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_Y$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	0
Q_6	$\mathbf{2}_1$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{1}_{++}$	$\mathbf{2}_1$	$\mathbf{1}_{--}$
Z_2	0	0	1	0	1	0	0	0
Z_4	0	0	1	1	2	2	2	2

Table II: Scalar content with the $SU(3)_C \times SU(2)_L \times U(1)_Y \times Q_6 \times Z_2 \times Z_4$ assignments.

QUARKS

- General form of mass matrices

$$\mathbf{M}_d = \begin{pmatrix} 0 & y_1^d\langle H_3^0\rangle & y_2^d\langle H_1^0\rangle \\ -y_1^d\langle H_3^0\rangle & 0 & y_2^d\langle H_2^0\rangle \\ y_3^d\langle H_1^0\rangle & y_3^d\langle H_2^0\rangle & y_4^d\langle H_3^0\rangle \end{pmatrix}, \qquad \mathbf{M}_u = \begin{pmatrix} 0 & y_1^u\langle \tilde{H}_3^0\rangle & y_2^u\langle \tilde{H}_1^0\rangle \\ -y_1^u\langle \tilde{H}_3^0\rangle & 0 & y_2^u\langle \tilde{H}_2^0\rangle \\ y_3^u\langle \tilde{H}_1^0\rangle & y_3^u\langle \tilde{H}_2^0\rangle & y_4^u\langle \tilde{H}_3^0\rangle \end{pmatrix}.$$

- We minimize the scalar potential, with one inert vev:

$$\left(\langle H_1^0\rangle, \langle H_2^0\rangle\right) = \left(0, v_2/\sqrt{2}\right) \text{ and } \langle H_3^0\rangle = v_3/\sqrt{2}$$

$$\mathbf{M}_q = \begin{pmatrix} 0 & A_q & 0 \\ -A_a & 0 & b_q \\ 0 & c_q & F_q \end{pmatrix}$$

NNI

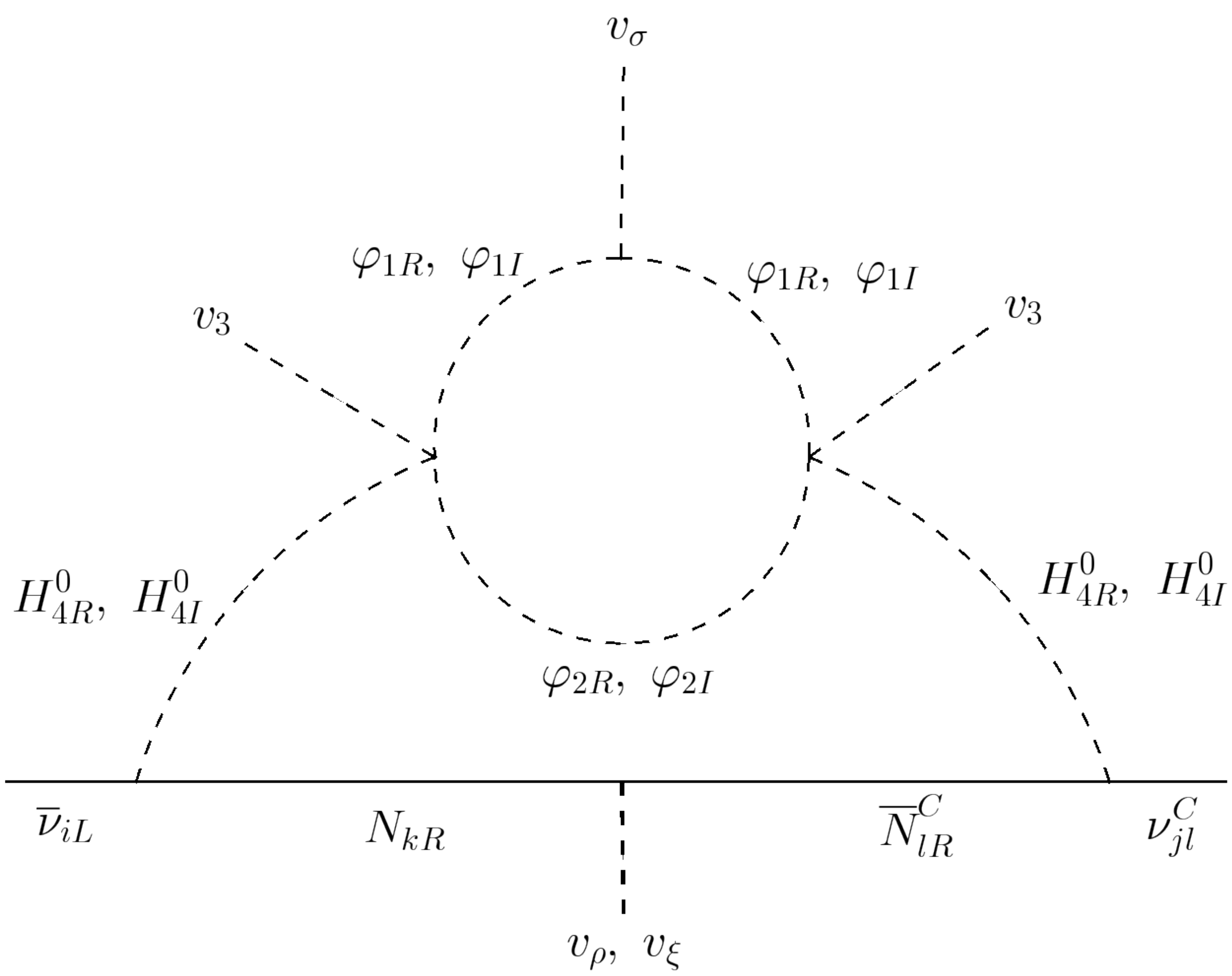
- Reparameterizing in terms of mass ratios leaves

$$y_q \equiv |F_q|/m_{q_3} \qquad\qquad 1 > y_q > \tilde{m}_{q_2} > \tilde{m}_{q_1}$$

- 4 free parameters: yu, yd, phi_u, phi_d
  VCKM can be fitted exactly (work in progress)

NEUTRINOS

- Active neutrino masses generated at two-loops



NEUTRINO MASS MATRICES

- After minimization of potential

$$M_\nu = \frac{1}{16\pi^2}\widetilde{Y}_{\nu D}\begin{pmatrix} m_{N_1}f_1 & 0 & 0 \\ 0 & m_{N_2}f_2 & 0 \\ 0 & 0 & m_{N_3}f_3 \end{pmatrix}\widetilde{Y}^T_{\nu D}, \qquad \widetilde{Y}_{\nu D} = Y_{\nu D}R_N,$$

$$Y_{\nu D} = \begin{pmatrix} y_1^{(\nu)} & 0 & 0 \\ 0 & 0 & y_2^{(\nu)} \\ 0 & -y_2^{(\nu)} & 0 \end{pmatrix}, \qquad (M_N)_{diag} = \begin{pmatrix} m_{N_1} & 0 & 0 \\ 0 & m_{N_2} & 0 \\ 0 & 0 & m_{N_3} \end{pmatrix} = R_N^T M_N R_N$$

$$f_k = \frac{m_R^2}{m_R^2 - m_{N_k}^2}\ln\left(\frac{m_R^2}{m_{N_k}^2}\right) - \frac{m_I^2}{m_I^2 - m_{N_k}^2}\ln\left(\frac{m_I^2}{m_{N_k}^2}\right), \qquad k = 1,2,3$$

$$m_R = m_{\mathrm{Re}\,H_4^0},\ m_I = m_{\mathrm{Im}\,H_4^0}$$

$$M_N = \begin{pmatrix} y_{1N}\frac{v_\sigma}{\sqrt{2}} & y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} \\ y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{2N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & y_{3N}\frac{v_\rho}{\sqrt{2}} \\ y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & y_{3N}\frac{v_\rho}{\sqrt{2}} & y_{2N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} \end{pmatrix}$$

$$\Downarrow$$

$$M_\nu \simeq \frac{m_R^2 - m_I^2}{8\pi^2\left(m_R^2 + m_I^2\right)}\begin{pmatrix} y_{1\nu}^2 y_{1N}\frac{v_\sigma}{\sqrt{2}} & y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} \\ y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} & y_{2\nu}^2 y_{2N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & -y_{2\nu}^2 y_{3N}\frac{v_\rho}{\sqrt{2}} \\ y_{1\nu}y_{2\nu}y_{4N}\frac{v_\xi}{\sqrt{2}}e^{-i\theta} & -y_{2\nu}^2 y_{3N}\frac{v_\rho}{\sqrt{2}} & y_{2\nu}^2 y_{2N}\frac{v_\xi}{\sqrt{2}}e^{i\theta} \end{pmatrix},$$

$$\boxed{\mathbf{M}_\nu = \begin{pmatrix} A_\nu & \tilde{B}_\nu & \tilde{B}_\nu^* \\ \tilde{B}_\nu & \tilde{C}_\nu^* & D_\nu \\ \tilde{B}_\nu^* & D_\nu & \tilde{C}_\nu \end{pmatrix}}$$

CHARGED LEPTONS

- Form of mass matrix after minimization of potential

$$M_l = \begin{pmatrix} a_l & 0 & b_l \\ 0 & 0 & d_l \\ c_l & -d_l & 0 \end{pmatrix}$$

- After reparameterization only 1 free parameter

$$|m_\tau| > |a_l| \approx (|m_\tau|/|m_\mu|)|m_e$$

- Diagonalization matrix U_{1L} almost identity
- Again charged lepton will provide deviation from cobimaximal in the PMNS matrix $\implies$ 5 free parameters

$$\gamma_{12},\ \gamma_{13} \text{ and } |a_l|$$

$$\eta_\mu \text{ and } \eta_\tau$$

The limit gives t

$$|a_l| = (|m_\tau|/|m_\mu|)|m_e|$$

$$|\bar{m}_e| = |m_e|/|m_\mu| \sim \mathcal{O}\left(10^{-3}\right)$$

$$\boxed{\begin{aligned} \sin\theta_{13} &\approx \sin\gamma_{13}\left[1 - \frac{|\bar{m}_e|}{\sqrt{2}}\cot\gamma_{13}\sin\eta_\tau\right]; \\ \sin\theta_{12} &\approx \sin\gamma_{12}\left[1 + \frac{|\bar{m}_e|}{\sqrt{2}}\tan\gamma_{13}\sin\eta_\tau\right]; \\ \sin\theta_{23} &\approx \frac{1}{\sqrt{2}}\left[1 + \frac{|\bar{m}_e|}{\sqrt{2}}\tan\gamma_{13}\sin\eta_\tau\right]; \\ \sin\delta_{CP} &\approx -1 + \frac{|\bar{m}_e|}{\sqrt{2}}\tan\gamma_{13}\sin\eta_\tau. \end{aligned}}$$

3 free parameters for PMNS

η_μ negligible

FIT FOR LEPTONS

- We fit with the 5 effective parameters, $\chi^2 = 0.497$:

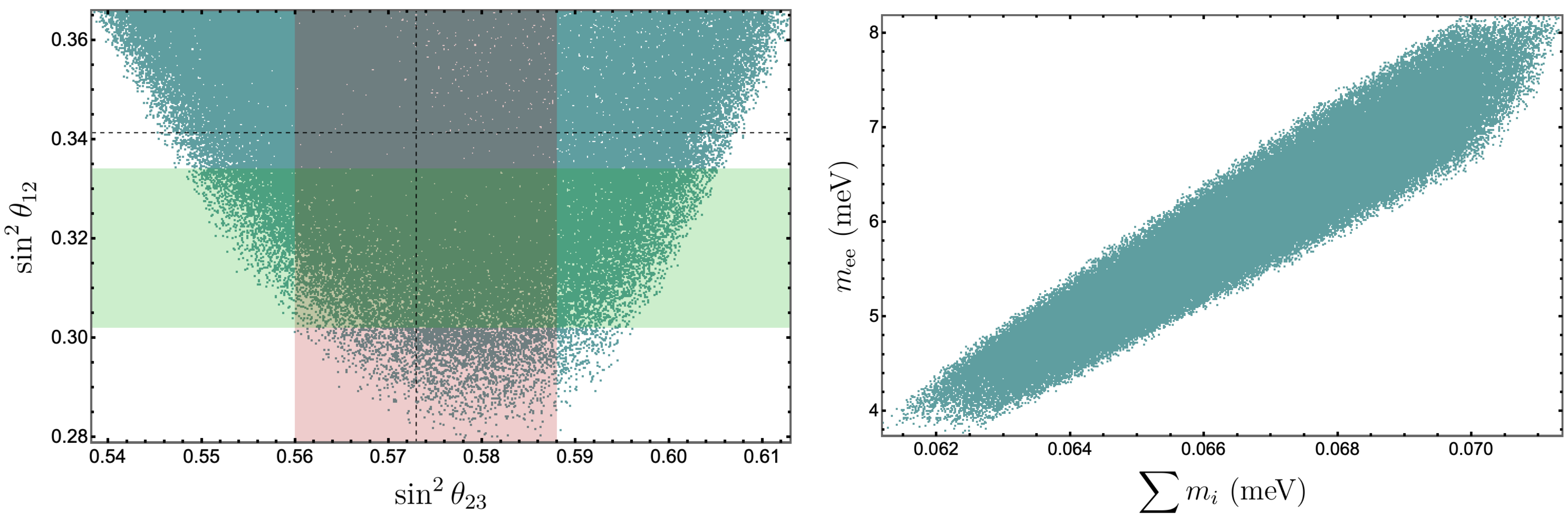
$$\chi^2 = \frac{\left(\Delta m_{21}^{2\;\mathrm{exp}} - \Delta m_{21}^{2\;\mathrm{th}}\right)^2}{\sigma^2_{\Delta m^2_{21}}} + \frac{\left(\Delta m_{31}^{2\;\mathrm{exp}} - \Delta m_{31}^{2\;\mathrm{th}}\right)^2}{\sigma^2_{\Delta m^2_{31}}} + \frac{\left(\sin^2\theta_{12}^{(l)\,\mathrm{exp}} - \sin^2\theta_{12}^{(l)\mathrm{th}}\right)^2}{\sigma^2_{\sin^2\theta_{12}^{(l)}}}$$
$$+ \frac{\left(\sin^2\theta_{23}^{(l)\,\mathrm{exp}} - \sin^2\theta_{23}^{(l)\mathrm{th}}\right)^2}{\sigma^2_{\sin^2\theta_{23}^{(l)}}} + \frac{\left(\sin^2\theta_{13}^{(l)\,\mathrm{exp}} - \sin^2\theta_{13}^{(l)\mathrm{th}}\right)^2}{\sigma^2_{\sin^2\theta_{13}^{(l)}}} + \frac{\left(\delta_{\mathrm{CP}}^{\mathrm{exp}} - \delta_{\mathrm{CP}}^{\mathrm{th}}\right)^2}{\sigma^2_{\delta_{\mathrm{CP}}}},$$

- And obtain m_i and m_{ee}:

$$\sum m_i \qquad\qquad m_{ee} = \left|\sum_i \mathbf{U}^2_{ei} m_{\nu i}\right|$$

Observable	range	Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$\sin^2\theta^{(l)}_{12}/10^{-1}$	$\sin^2\theta^{(l)}_{13}/10^{-2}$	$\sin^2\theta^{(l)}_{23}/10^{-1}$	$\delta^{(l)}_{\mathrm{CP}}(^\circ)$
Experimental	1σ	$7.50^{+0.22}_{-0.20}$	$2.55^{+0.02}_{-0.03}$	3.18 ± 0.16	$2.200^{+0.069}_{-0.062}$	5.74 ± 0.14	194^{+24}_{-22}
Value [97]	3σ	$6.94 - 8.14$	$2.47 - 2.63$	$2.71 - 3.69$	$2.000 - 2.405$	$4.34 - 6.10$	$128 - 359$
Experimental	1σ	7.49 ± 0.19	$2.513^{+0.021}_{-0.019}$	$3.08^{+0.12}_{-0.11}$	$2.215^{+0.058}_{-0.056}$	$4.7^{+0.17}_{-0.13}$	212^{+26}_{-41}
Value [98]	3σ	$6.92 - 8.05$	$2.451 - 2.578$	$2.75 - 3.45$	$2.03 - 2.388$	$4.35 - 5.85$	$124 - 364$
Fit	$1\sigma - 3\sigma$	7.69	2.54	3.41	2.24	5.73	219.7

Table III: Model predictions for the scenario of normal order (NO) neutrino mass.



(a) (b)

Figure 2: Correlation plot between mixing angle, effective Majorana mass, and sum lightest mass neutrino.

NO shown

IO disfavored

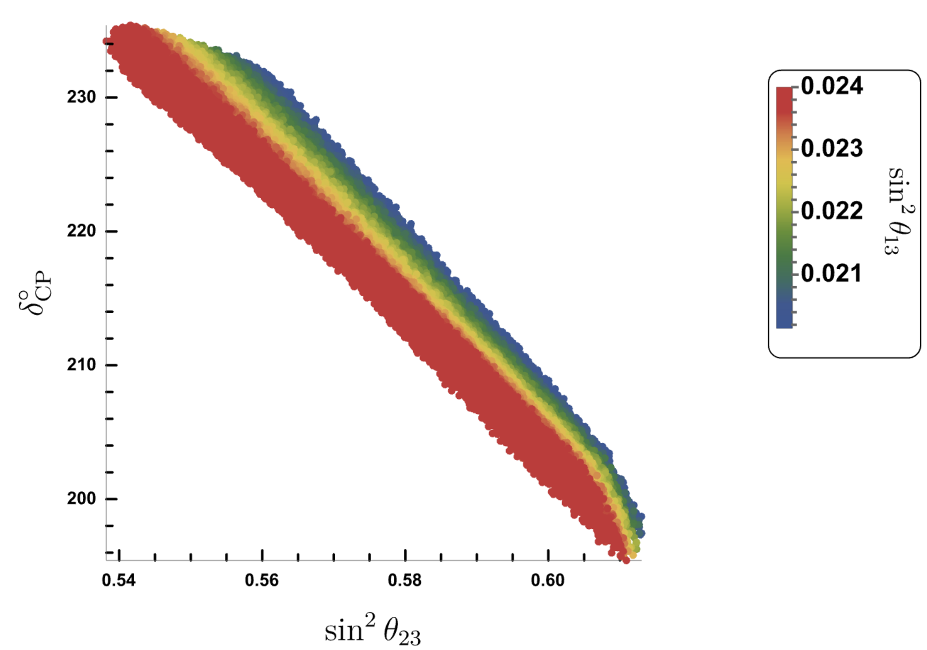


Figure 3: Correlation plot between mixing angles and CP violation phase, for different values of $\sin^2\theta_{13}$.

SCALAR SECTOR + DM

- Scalar potential V+ V_{DM}

$$
\begin{aligned}
V \;=\; & -\mu_2^2\left(H_2^\dagger H_2\right)-\mu_3^2\left(H_3^\dagger H_3\right)-\mu_{13}^2\left(H_3^\dagger H_1+H_1^\dagger H_3\right)-\mu_{23}^2\left(H_2^\dagger H_3+H_3^\dagger H_2\right)-\mu_{12}^2\left(H_1^\dagger H_2+H_2^\dagger H_1\right)-\mu_4^2\left(\sigma^*\sigma\right)\\
& -\mu_5^2\left(\xi_1^*\xi_1\right)-\mu_6^2\left(\xi_2^*\xi_2\right)-\mu_7^2\left(\rho^*\rho\right)+\lambda_1\left(H^\dagger H\right)^2_{\mathbf{1}_{++}}+\lambda_2\left(H_3^*H_3\right)^2+\lambda_3\left(\sigma^*\sigma\right)^2+\lambda_4\left(\xi^\dagger\xi\right)^2_{\mathbf{1}_{++}}+\lambda_5\left(\rho^*\rho\right)^2\\
& +\lambda_6\left(H^\dagger H\right)_{\mathbf{1}_{++}}\left(H_3^*H_3\right)+\lambda_7\left(H^\dagger H\right)_{\mathbf{1}_{++}}\left(\sigma^*\sigma\right)+\lambda_8\left(H^\dagger H\right)_{\mathbf{1}_{++}}\left(\rho^*\rho\right)+\lambda_9\left(\xi^\dagger\xi\right)_{\mathbf{1}_{++}}\left(H_3^*H_3\right)+\lambda_{10}\left(\xi^\dagger\xi\right)_{\mathbf{1}_{++}}\left(\sigma^*\sigma\right)\\
& +\lambda_{11}\left(\xi^\dagger\xi\right)_{\mathbf{1}_{++}}\left(\rho^*\rho\right)+\lambda_{12}\left(H^\dagger H\right)_{\mathbf{1}_{+}}\left(\xi^\dagger\xi\right)_{\mathbf{1}_{++}}+\lambda_{13}\left(H^\dagger H\right)_{\mathbf{1}_{--}}\left(\xi^\dagger\xi\right)_{\mathbf{1}_{--}}+\lambda_{14}\left(H^\dagger H\right)_{\mathbf{1}_{--}}\left(\sigma^*\rho\right)\\
& +\lambda_{15}\left(\xi^\dagger\xi\right)_{\mathbf{1}_{--}}\left(\sigma^*\rho\right)+\lambda_{16}\left(H_3^*H_3\right)\left(\sigma^*\sigma\right)+\lambda_{17}\left(H_3^*H_3\right)\left(\rho^*\rho\right)+\lambda_{18}\left(\sigma^*\sigma\right)\left(\rho^*\rho\right)+h.c. \qquad (31)
\end{aligned}
$$

- One vev=0, so we can rotate to the 2HDM basis and work in alignment limit
- DM potential

$$
\begin{aligned}
V \supset\; & \mu_8^2 H_4^\dagger H_4+\mu_9^2\varphi_1^*\varphi_1+\mu_{10}^2\varphi_2^*\varphi_2+\mu_{\rm SB}^2(\varphi_1^2+{\rm h.c.})+\kappa_1(\varphi_1^*\varphi_1)^2+\kappa_2(\varphi_2^*\varphi_2)^2+\kappa_3(\varphi_1^*\varphi_1)(\varphi_2^*\varphi_2)\\
& +\left[\kappa_4(\varphi_1^2\varphi_2^2)+\kappa_5(H_4^\dagger H_3)(\varphi_1^*\varphi_2)+\kappa_6(H_3^\dagger H_4)(\varphi_1\varphi_2)+{\rm h.c.}\right]\\
& +\sum_{i=1}^{2}(\varphi_i^*\varphi_i)\left[\kappa_{6+i}(H_4^\dagger H_4)+\kappa_{8+i}(H_3^\dagger H_3)+\kappa_{10+i}(H^\dagger H)_{1++}\right]\\
& +\kappa_{13}(H_4^\dagger H_4)(H^\dagger H)_{1++}+\kappa_{14}(H_4^\dagger H_4)(H_3^\dagger H_3),
\end{aligned}
$$

SCALAR SECTOR AND DARK MATTER ANALYSIS

- Implement model in SARAH, neglect off-diagonal terms and masses of first and second generation
- Then implement SARAH-SPheno
- Scalar potential analyzed with EVADE
- Exclusion limits with Higgs/Tools, Higgs/Predictions, Higgs/Bounds
- To generate input for Higgs/Toos we use CalcHEP and Micromegas
- Several dark matter candidates: lightest right-handed neutrino N_R one component of φ_1 and φ_2 , we denote them ϕ_1 and ϕ_2
- We calculate relic density

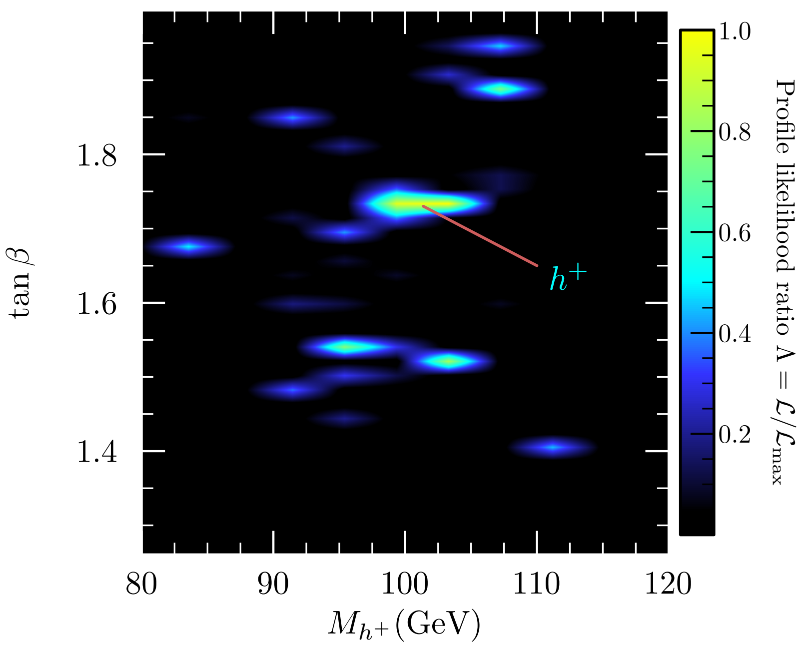
- Hard cuts, anything that doesn't pass this
- Then couplings and decays using Higgs/Tools, Higgs/Predictions
- Likelihood function Higgs

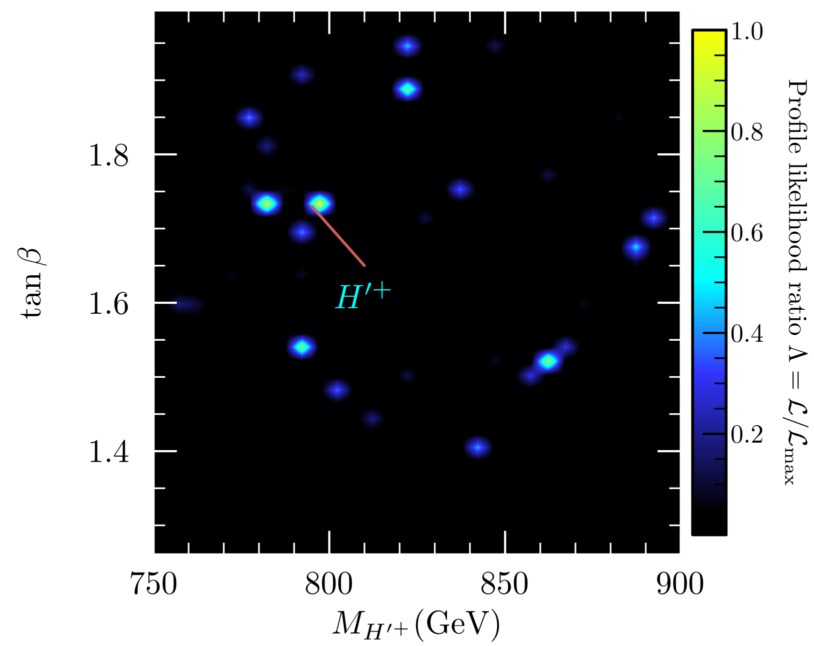
$$\log \mathcal{L}_{\text{scalar}} = \log \mathcal{L}_{\text{Higgs}} + \log \mathcal{L}_{H_0 \to \gamma\gamma}$$

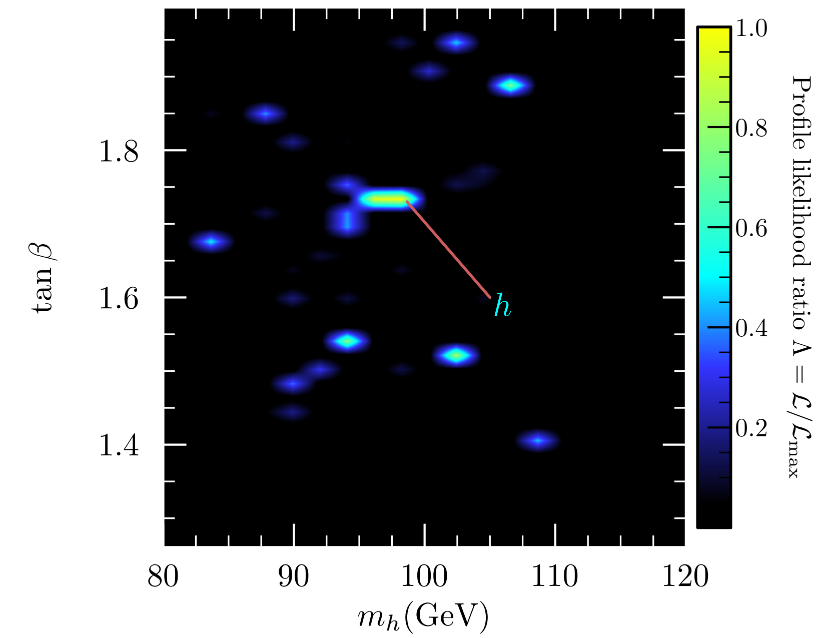
- Likelihood function DM

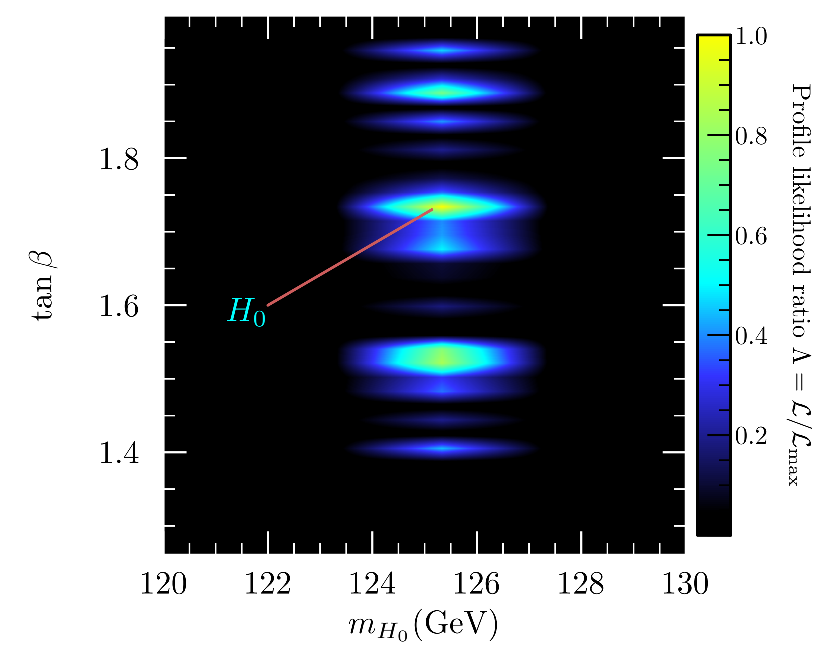
$$\log \mathcal{L} = \log \mathcal{L}_{\text{scalar}} + \log \mathcal{L}_{\text{DD}} + \log \mathcal{L}_{\Omega h^2}$$

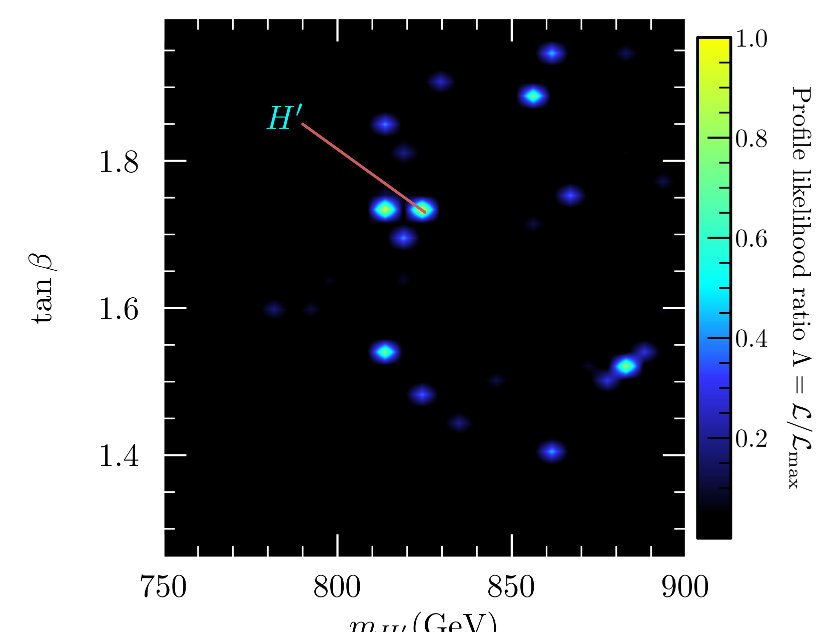
HIGGS SECTOR RESULTS











Interesting BFP:

$$(m_h, m_{H_0}, m_{H'}) = (98.7, 125.1, 825)\ \text{GeV}$$

$$(m_A, m_{A'}, m_{h^+}, m_{H'^+}) = (348.3, 795.6, 101.4, 795)\ \text{GeV and } \tan\beta = 1.73.$$

BFP signals out a neutral scalar at $\sim$ 98 GeV
and a pseudoscalars at $\sim$ 348 and 795 GeV

CMS new scalar at $\sim$ 95 GeV ?
pseudoscalar $\sim$ 365 GeV ?   pbbly toponium

DM RESULTS

Direct detection

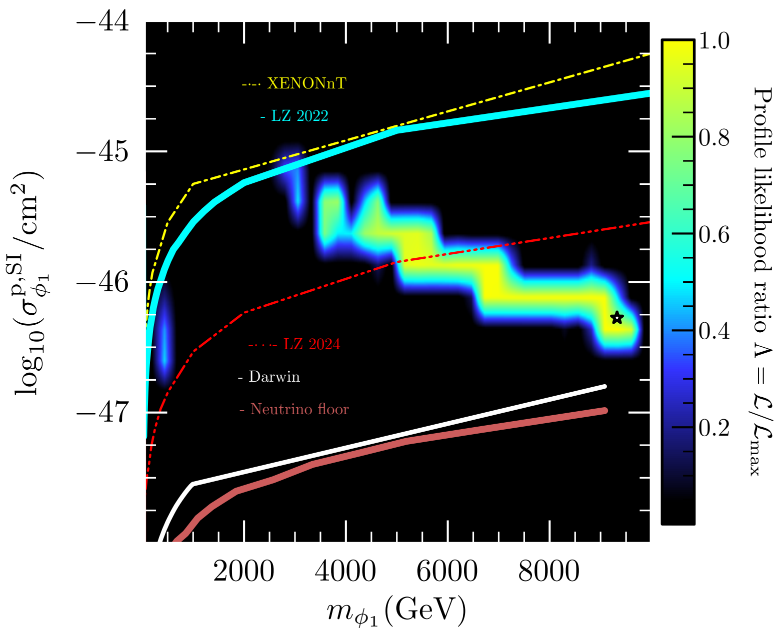
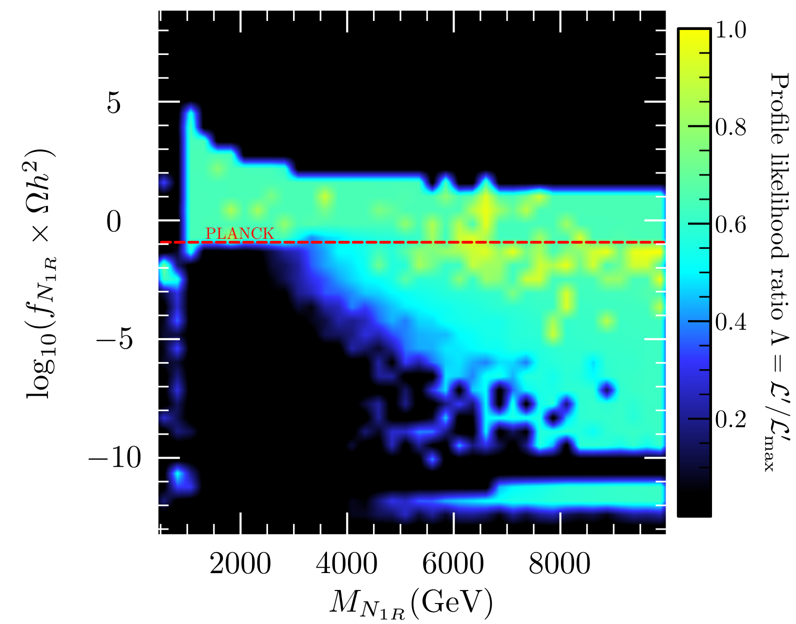
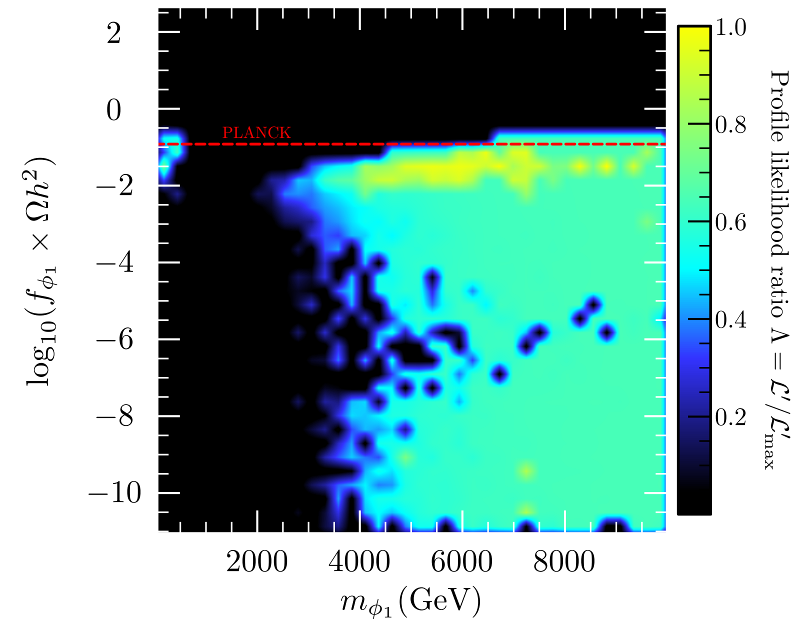
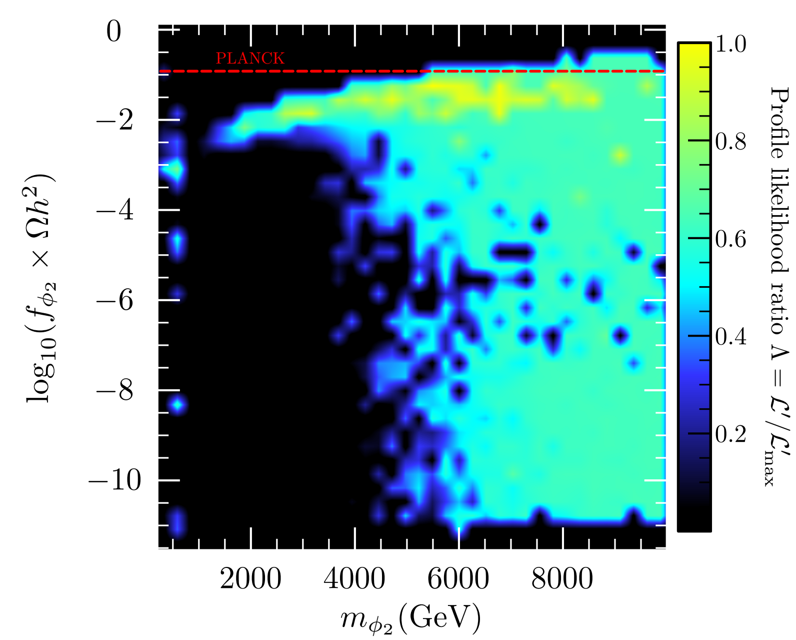


Figure 8: DM-proton spin-independent elastic scattering cross section as a function of the masses of the scalar DM candidates. Brightest areas are most consistent with all imposed constraints, dark areas are excluded by the analysis. The best fit point (BFP) is marked with a small star. For comparison, exclusion limits of the XENONnT, LZ 2022 and LZ 2024 experiments are shown, alongside with the projection of the DARWIN experiment and the neutrino floor.







DM relic abundance weighed by respective DM fractions of N1R, ϕ_1 and ϕ_2

$$\log \mathcal{L}' = \log \mathcal{L} - \log \mathcal{L}_{\Omega h^2}$$

TOTAL DM

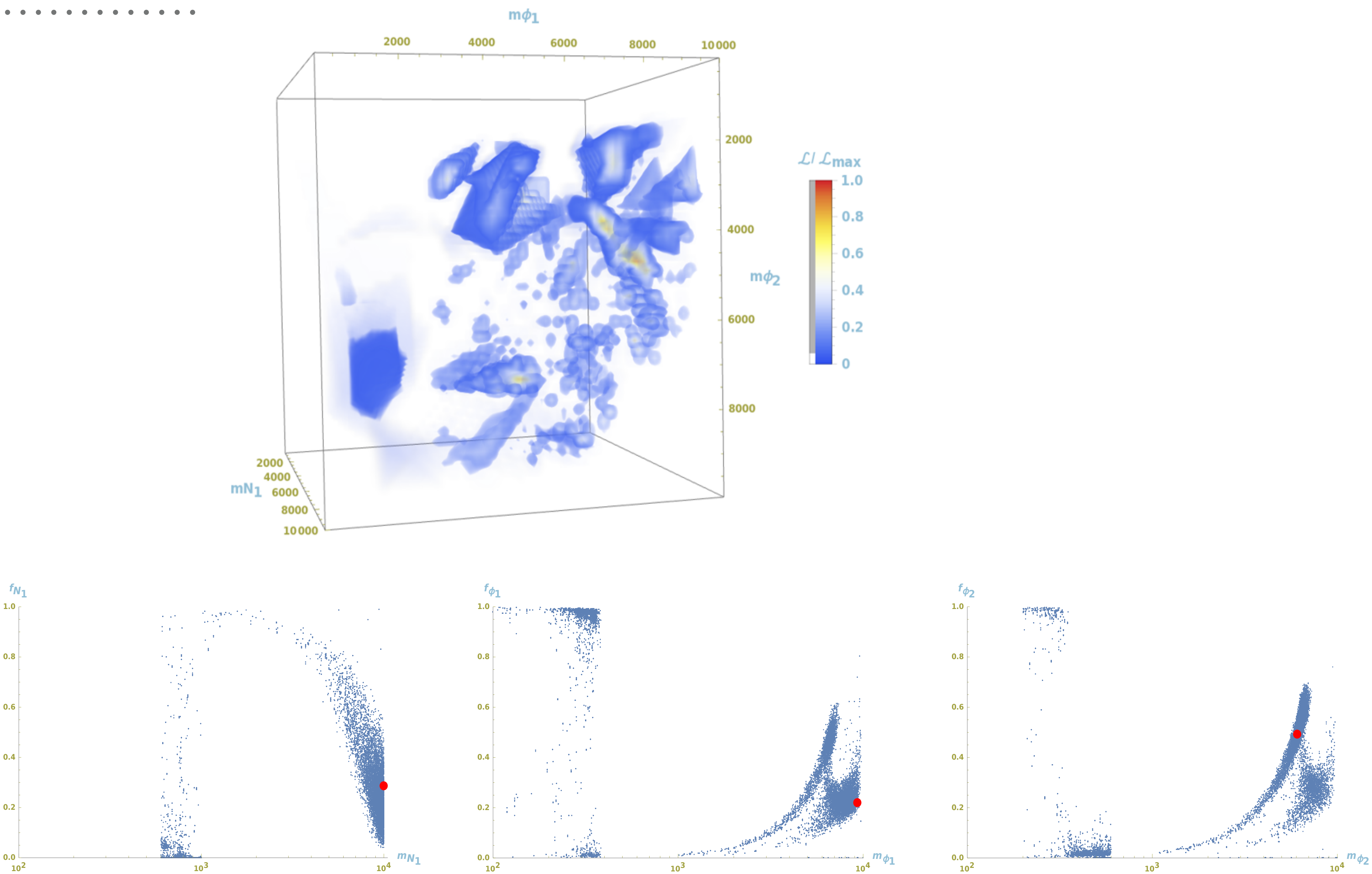


Figure 6: Top panel: Scattered plot of points in parameter space that lie inside the experimental Planck interval for the DM abundance, bright/red points are most consistent with the global constraints (all masses are in GeV). Bottom panel: The fraction of the relic density contributed by each DM candidate. The corresponding masses and fractions for the best fit point (BFP) are marked in red and have values $(m_{N_1}, m_{\phi_1}, m_{\phi_2}) = (9991, 9323, 6045)$ GeV and $(f_{N_1}, f_{\phi_1}, f_{\phi_2}) = (0.2867077, 0.2200167, 0.4932755)$.

RECAP Q6

- 2+1 in quarks and leptons works well
- 2-loop generated neutrino masses
    - again cobimaximal pattern "corrected" by lepton sector
- Number of effective parameters 4 in quark sector and 5 in neutrino sector
- Higgs sector with interesting new possibilities
  and passes all known constraints
- Viable DM sector with three candidates
- To do: more phenomenology…

SUMMARY AND CONCLUSIONS

- 2+1 structure as S3 and Q6 works well for quarks and leptons in different settings
    - S3 complex vev's, 1+2 quarks, 2+1 leptons, ISSM
    - Q6 1+2 quarks + leptons, 2-loop for neutrinos
- Both models have cobimaximal mixing "corrected/broken" by leptonic sector
- Symmetries drives the form of the mass matrices and predictions
- Allows for defining few(er) effective parameters despite large number of scalars
- Scalar potential plays crucial role in the shaping of the mass matrices
- DM candidates: S3 not yet studied
  Q6, 2 scalars + 1 N_R with possibilities of future detection
- Small symmetries can go a long way

Thank you for your attendance and attention!