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Light New Physics, Non-Conserved currents and MFV

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Light New Physics and Flavour

1) To evade direct searches, New Physics (NP) must be either light and very weakly coupled to the SM or heavy. In recent years, interest in light NP has been growing significantly.

2) Considering a light vector coupled to an anomalous current, flavour universal WZ terms are generically expected. Flavour Violation is generated by SM interactions leading to a MFV scenario

3) Light NP is described by an EFT. Providing UV explicit models that realizes such an EFT is not merely a theoretical exercise but it can also reveal important phenomenological implications.

4) I will discuss an explicit example, try do address the recent "anomalous" result in $B \to K^{(*)} + E_{miss}$

based on
- L. Di Luzio, MN, C. Toni, arXiv: 2505.11499 and 2204.05945

New Physics

Experimental evidences:

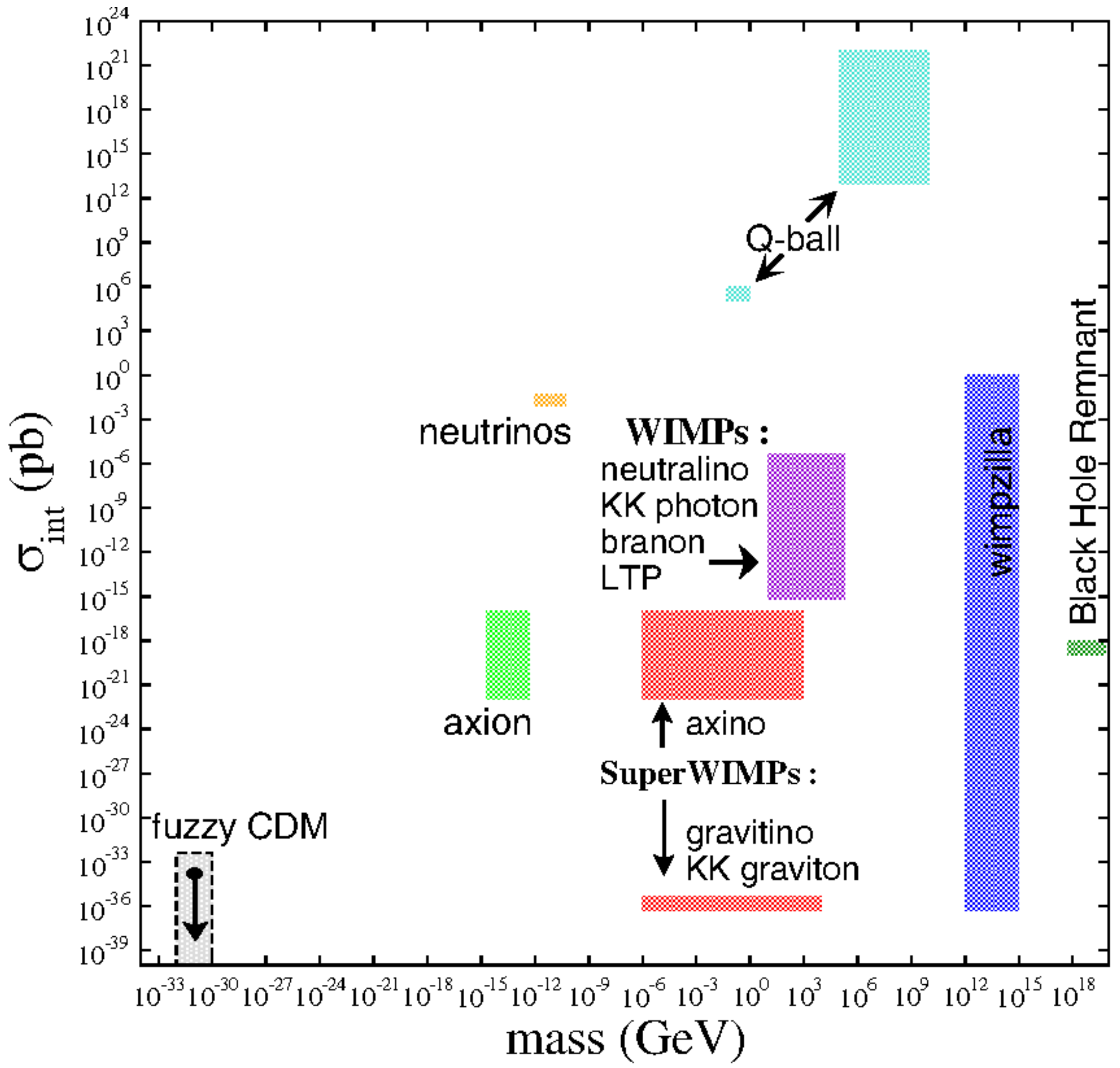
- Neutrino masses
- Dark Matter
- Baryon Asymmetry of the Universe
- (Gravity)

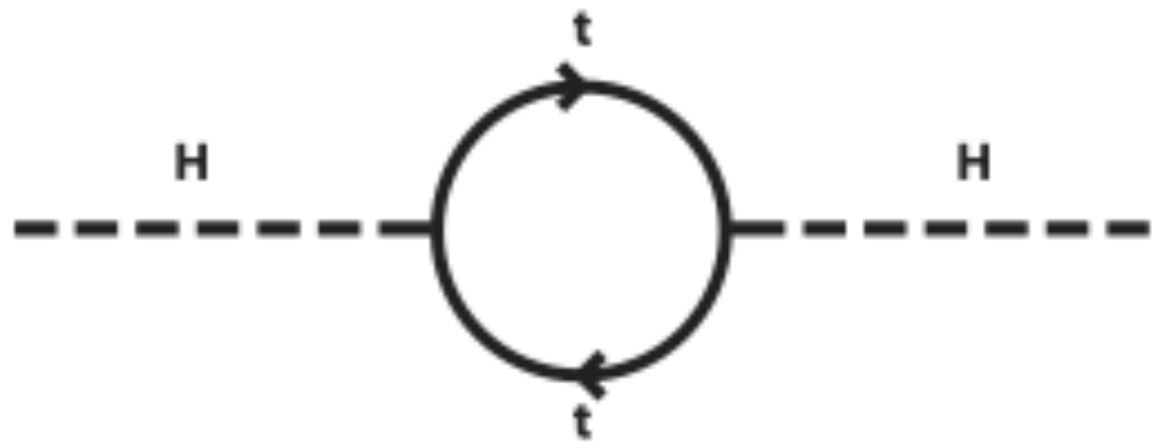
Theoretical problems/puzzles/hints:

- Hierarchy or Naturalness problem
- Flavour puzzle
- Strong CP problem
- Family replication
- GUT
- …..

What is the energy scale of New Physics at its coupling to the Standard Model?

Can we study it on shell?

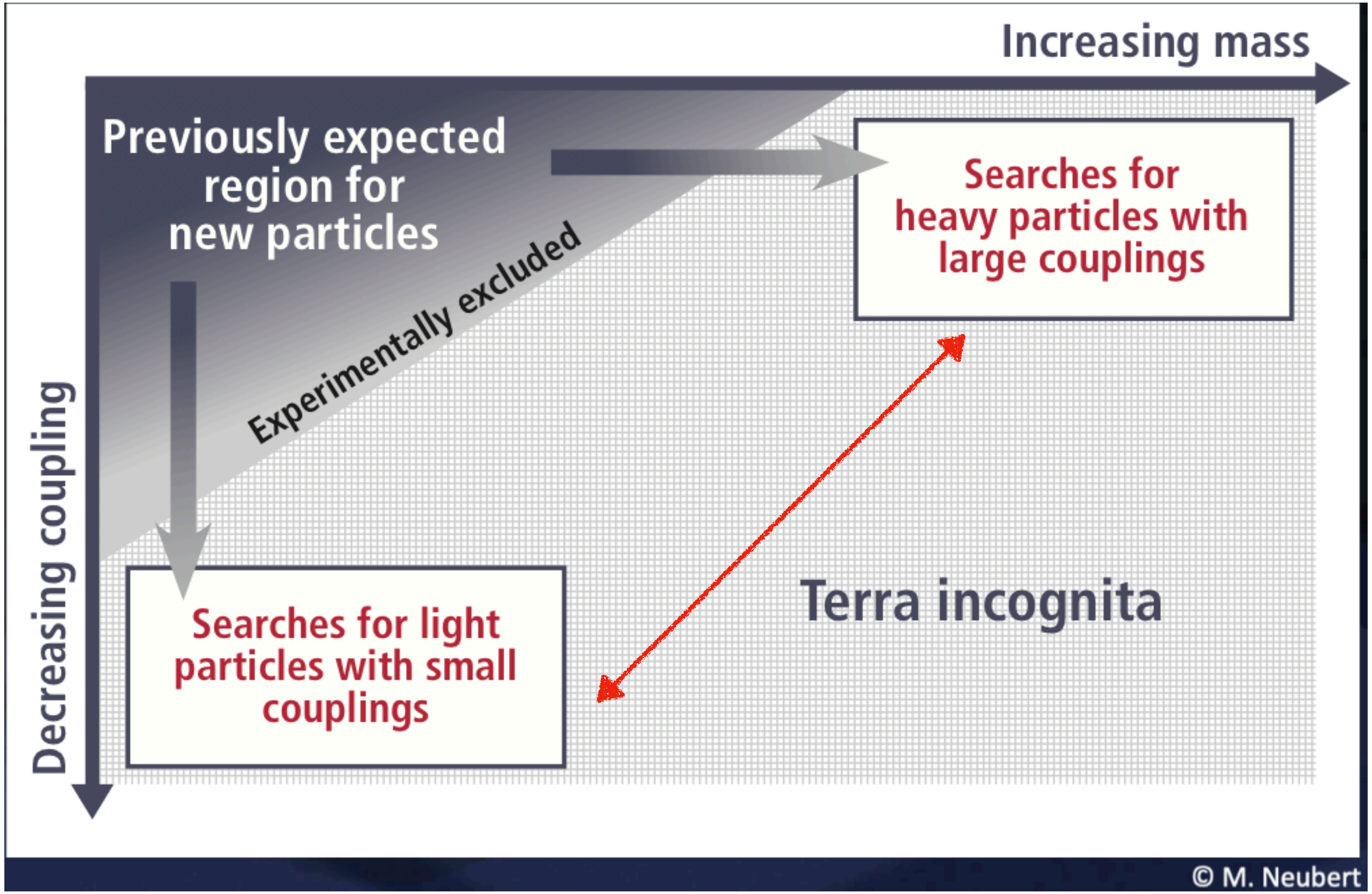




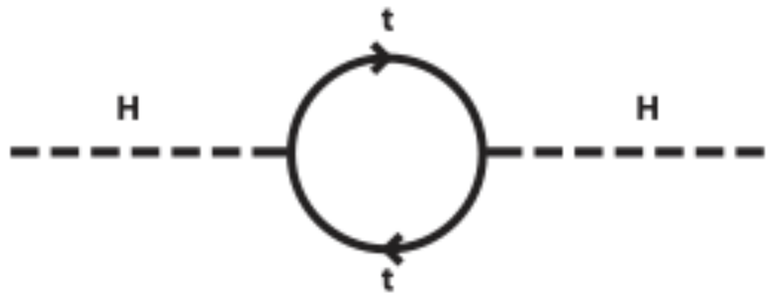
$$m_H^2 = m_{\text{tree}}^2 + \delta m_H^2$$

$$\delta m_H^2 = \frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 \approx (0.3\,\Lambda)^2$$

Coupling vs Mass Range

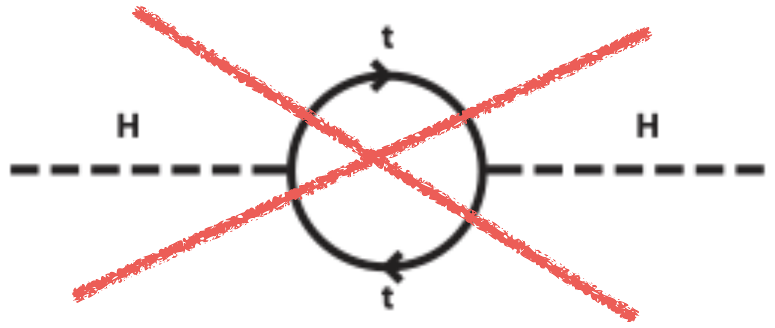


Why?



- SUSY
- Composite Higgs

Why? Why not

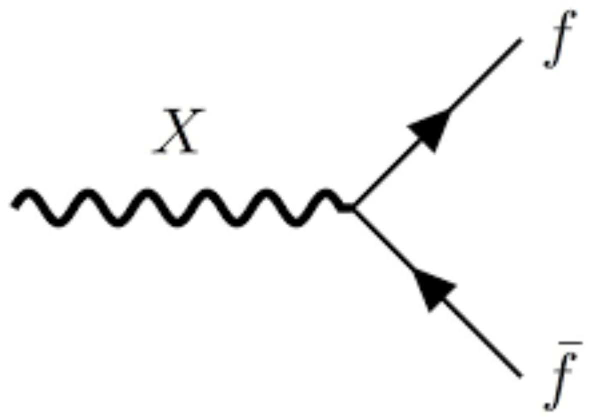


- Didn't find NP at high energy
- Some problems can be addressed in this regime: axions, portal to DM sector, etc.
- Ideas for new experiments on smaller scale that LHC
- No "No-Lose Theorem"

UV completing the light NP physics could be phenomenologically important!

Light New Vector and (Anomalous) Currents

Consider, for example, a light gauged vector coupled to the baryon number current:



$$\mathcal{L} \supset g_X \frac{1}{3} Z'_{B\mu} \left(\bar{q}\gamma^\mu q\right)$$

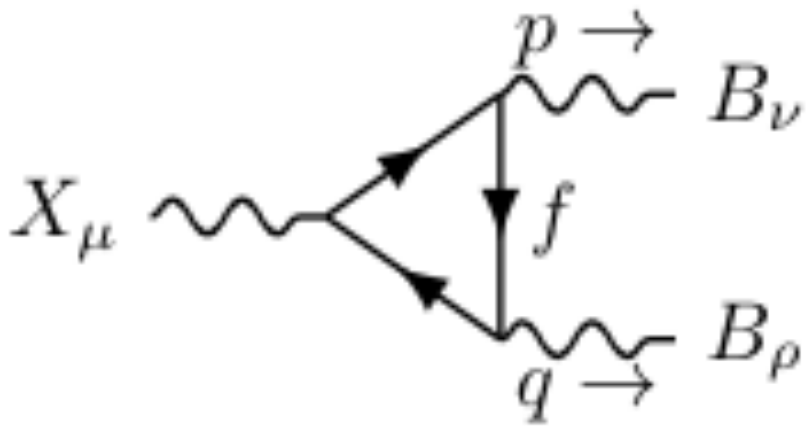
(Notation: X=Z' interchangeably
in what follows)

Background:
- D'Hoker, Farhi, 1984
- Preskill 1991
- Feruglio, Masiero, Maiani 1992

New constraints for light vectors:
- Dror, Lasenby, Pospelov 1707.01503
- Dror, Lasenby, Pospelov 1705.06726

Naively, NP physics at low energy implies two parameters: the coupling and the mass

SM + X EFT is non-renormalizable and the current is anomalous at quantum level:



$$\partial^\mu J_\mu^{\text{baryon}} = \frac{\mathcal{A}}{16\pi^2}\left(g^2 W^a_{\mu\nu}(\tilde{W}^a)^{\mu\nu} - g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right)$$

A=3/2

EFT must be completed at a scale $\lesssim \dfrac{4\pi m_X}{g_X} \Big/ \left(\dfrac{3g^2}{16\pi^2}\right)$ [Preskill 1991]

Need to include also
Wess-Zumino terms in the EFT:

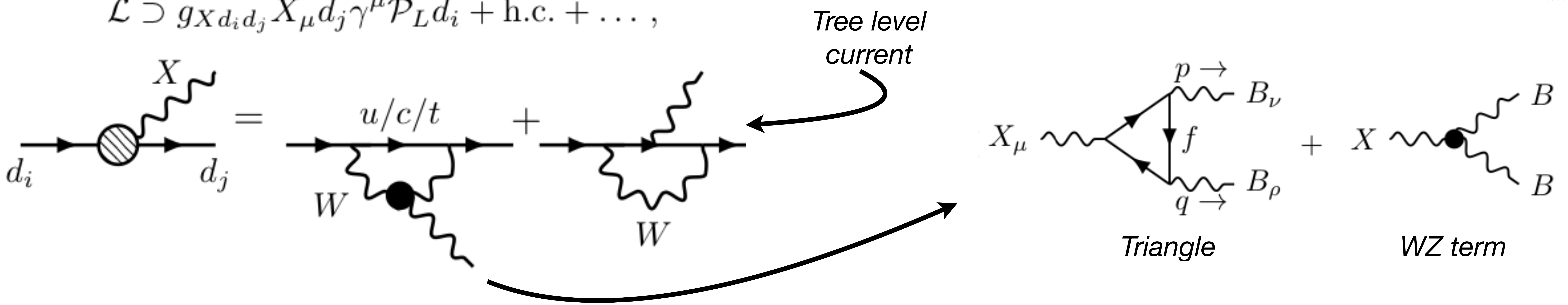
$$\mathcal{L} \supset C_B g_X g'^2 \epsilon^{\mu\nu\rho\sigma} X_\mu B_\nu \partial_\rho B_\sigma$$
$$+ C_W g_X g^2 \epsilon^{\mu\nu\rho\sigma} X_\mu \left(W^a_\nu \partial_\rho W^a_\sigma + \frac{1}{3} g \epsilon^{abc} W^a_\nu W^b_\rho W^c_\sigma\right)$$

Flavour and E/m enhancements

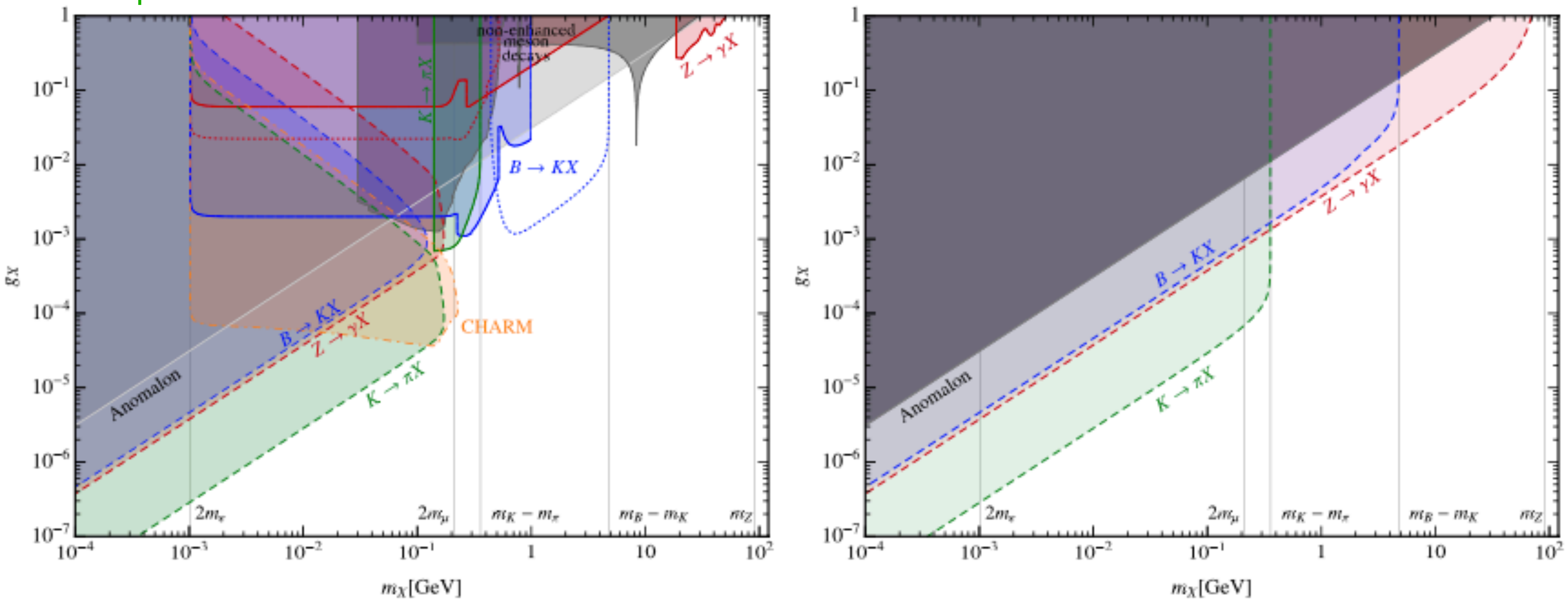
Generically, there are E/mx enhancements of the longitudinal polarisation of X leading to strong bounds.

$$\Gamma(A \to BX) \propto \frac{g_X^2}{m_X^2}$$

$$\mathcal{L} \supset g_{X d_i d_j} X_\mu \bar{d}_j \gamma^\mu \mathcal{P}_L d_i + \text{h.c.} + \dots ,$$



example from 1707.01503



Quark Current is flavour universal in this case and extra Flavour Violation is linked to the SM

Minimal Flavour Violation
(with an extra state X)

However phenomenology is non-trivial:

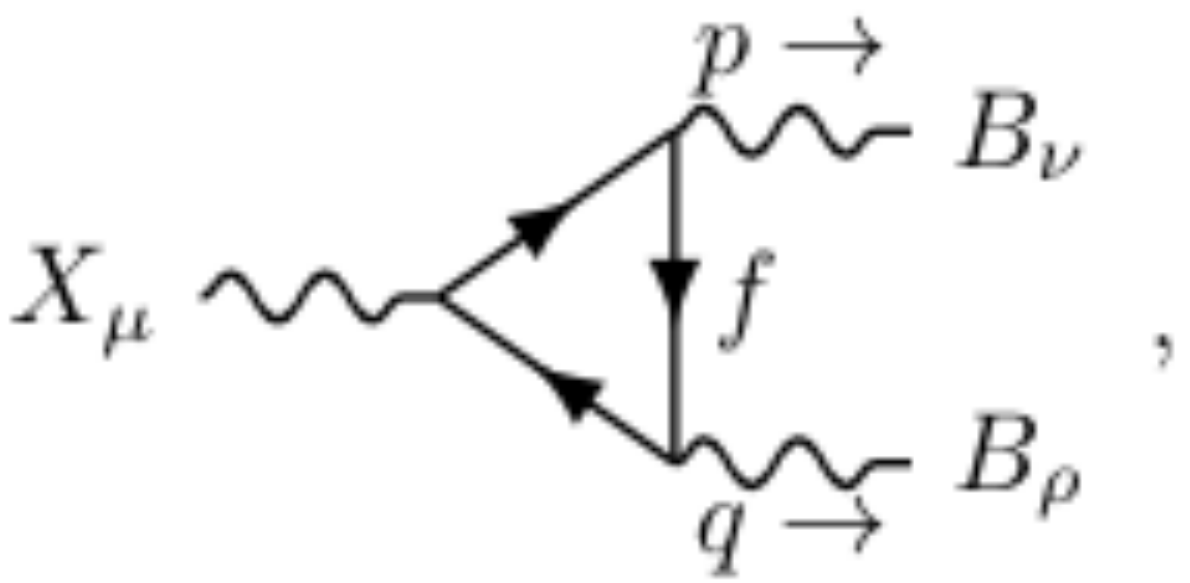
$$K \to \pi X$$
$$Z \to \gamma X$$
$$B \to K X$$

What is the UV origin of this EFT?

Anomalons in the UV

Let us assume X is a gauge boson. In the UV we need new states have to cancel the mass independent part of the triangular diagram:

$$\mathcal{M}^{\mu\nu\rho} \equiv \sum_{f, f_{SM}}$$



Extra states are called anomalons in what follows. NP carries quantum number under both the SM gauge symmetry and the new symmetry

Anomalons are chiral with respect to the full group SM x U(1). Chiral fermions cannot get an explicit mass term.

The E/m enhancement is due to longitudinal d.o.f. The effect can be understood with the equivalence theorem: $X \rightarrow \xi$

Do the Anomalons couple to the Goldstone boson of the new symmetry?

This depends on the physics that generate the mass term: $H\bar{f}_L f_R$ and/or $S\bar{f}_L f_R$ $\quad S \propto e^{i\xi}$

To escape direct searches, anomalons take mass (mostly) from the breaking of the new gauge symmetry

E/m enhancement remains in low energy observables

Depending of the details of the model the new states (anomalons) might have important phenomenology

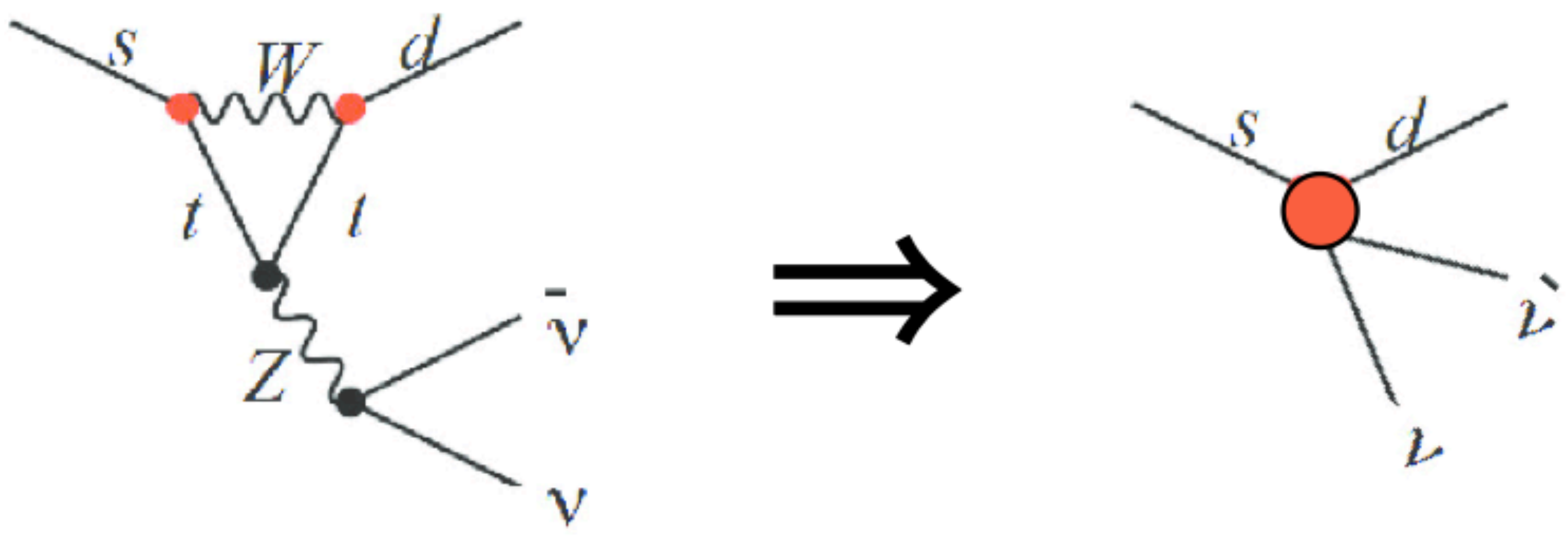
An explicit physics case

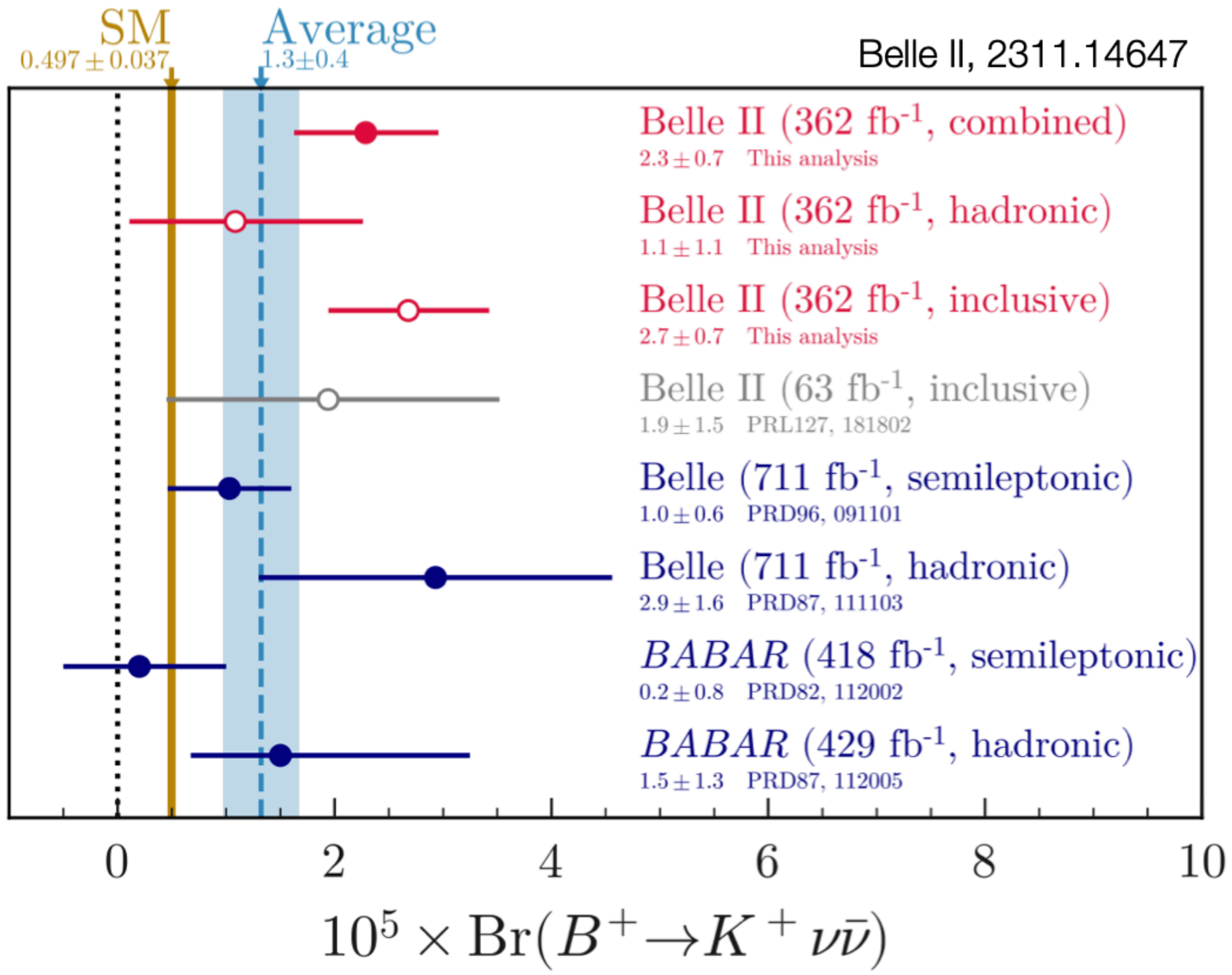
$B \to K + \text{invisible}$

- In the SM is FCNC, generated at 1-loop
- Recent Belle II results is showing a tensions of about 3σ
- Best fit about 5 times the SM, goes back to 3 times when averaged with other measurements
- Short distance New Physics can provvide good fit (see for example Allwicher et al. 2309.02246)
- A Light New Physics particle might appear in the final state mimicking the effect the missing energy of the SM neutrinos (see for example 2503.19025)

$$B \to K + \text{invisible} \;\Rightarrow\; B \to K + X$$

- A spin 1 particle with a mass of about 2 GeV is a viable candidate





EFT framework of the anomalous X

- At low energy we get an (anomalous) current and a WZ term

$$\mathcal{L}_{eff} = \mathcal{L}_{eff} + \mathcal{L}_{WZ} = -g_X X_\mu J^\mu_X + C_{BB}\frac{g_X g'^2}{24\pi^2} X B\partial B + C_{WW}\frac{g_X g^2}{24\pi^2} X\left(W\partial W + \frac{g}{3}WWW\right)$$

- Where the current is given by $J^\mu_X = \sum_{f\in SM} \alpha_f \overline{f}\gamma^\mu f$ $\qquad f = \{q^i_L, u^i_R, d^i_R, \ell^i_L, e^i_R\}$

- The current is not conserved in the sense $\partial_\mu J^\mu_X = \mathcal{A}_{XYY}\frac{g'^2}{16\pi^2}B\tilde{B} + \mathcal{A}_{XWW}\frac{g^2}{16\pi^2}W\tilde{W} + \dots$

(Needs proper regularisation of the loop integrals)

- But to restore the symmetry WZ terms know this fact: $C_{BB} = -\mathcal{A}_{XYY}$ $\qquad C_{WW} = -\mathcal{A}_{XWW}$

- Assuming a flavour conserving current, phenomenology is effectively dictated by 3 parameters: the two coefficient of the anomaly and the mass of the vector: $g_X\mathcal{A}_{XYY}, g_X\mathcal{A}_{XWW}$ and M_X

Fit to the data

- The vector mass is now fixed at 2.1 GeV and we are left with two observables

$$\Gamma(Z \to \gamma X) \approx \frac{g_X^2 g^2 g'^2}{1536\pi^5}(\mathcal{A}_{XWW}^{\rm SM} - \mathcal{A}_{XYY}^{\rm SM})^2 \frac{m_Z^3}{m_X^2}$$

The strongest constraint on this decays comes from the L3 experiment at LEP and reads [64]

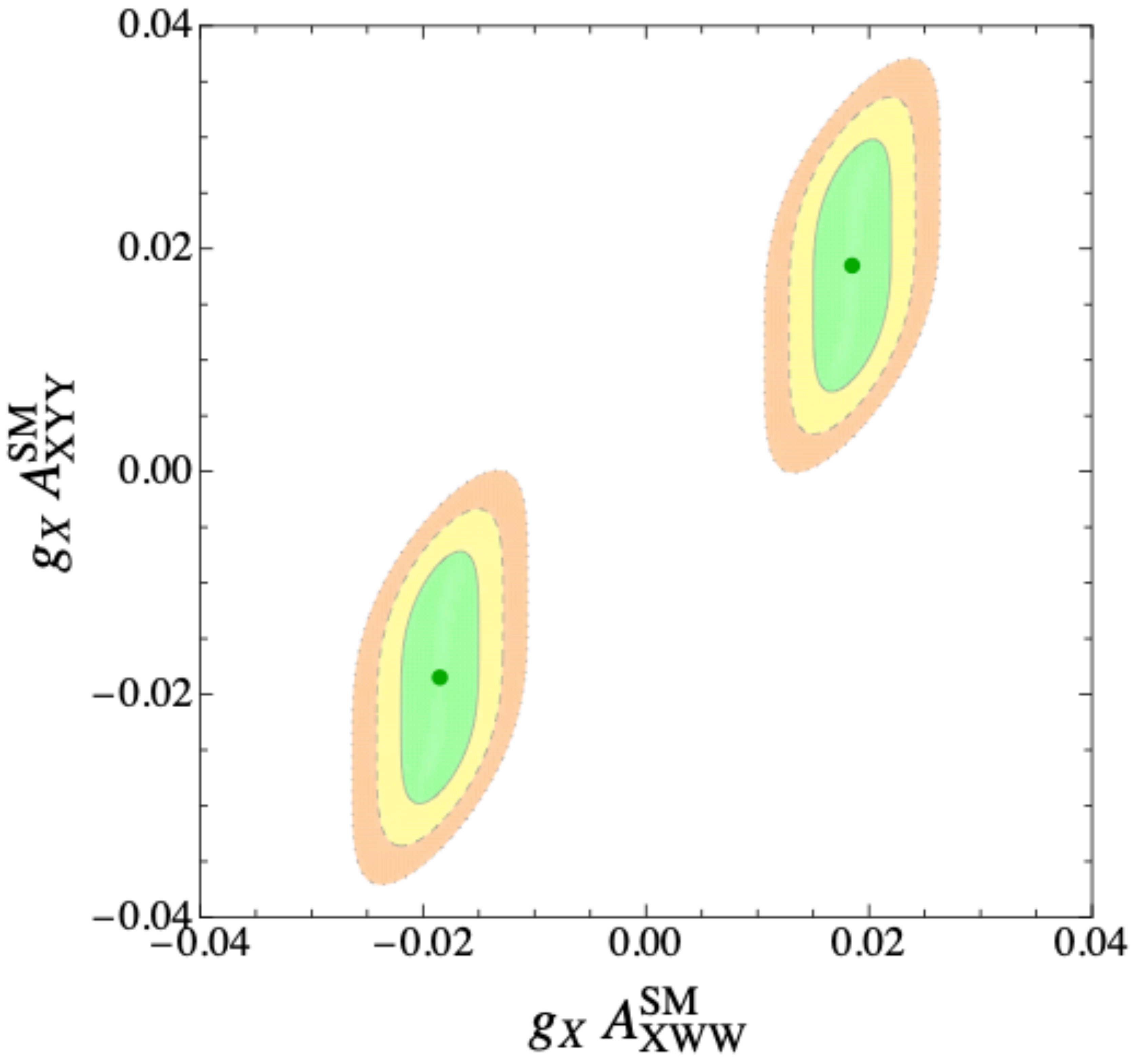
$$\mathcal{B}(Z \to \gamma X) \lesssim 10^{-6} \text{ at 95\% C.L.} \,. \qquad (20)$$

$$\Gamma(B \to KX) \simeq \frac{m_B^3}{64\pi m_X^2}|g_{bsX}|^2 \left(1 - \frac{m_K^2}{m_B^2}\right)^2 |f_K(m_X^2)|^2 \frac{2Q}{m_B}\,,$$

$$g_{\xi d_i d_j} = -\frac{3 g_X g^4}{(4\pi)^4}\mathcal{A}_{XWW}^{\rm SM} \sum_{\alpha=u,c,t} V_{\alpha i} V_{\alpha j}^* F(m_\alpha^2/m_W^2)\,, \qquad (13)$$

with V denoting the CKM matrix and the loop function

$$F(x) = \frac{x(1 + x(\ln x - 1))}{(1-x)^2}\,. \qquad (14)$$



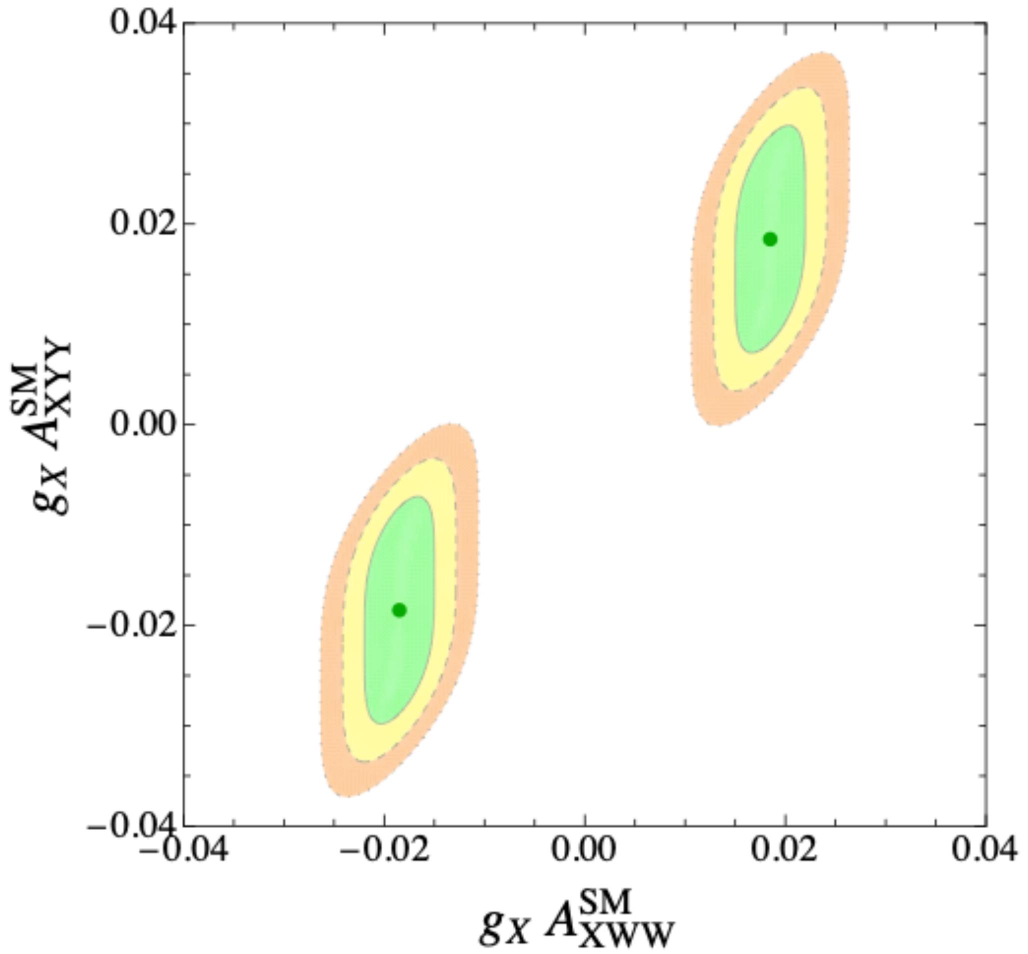
Towards explicit models

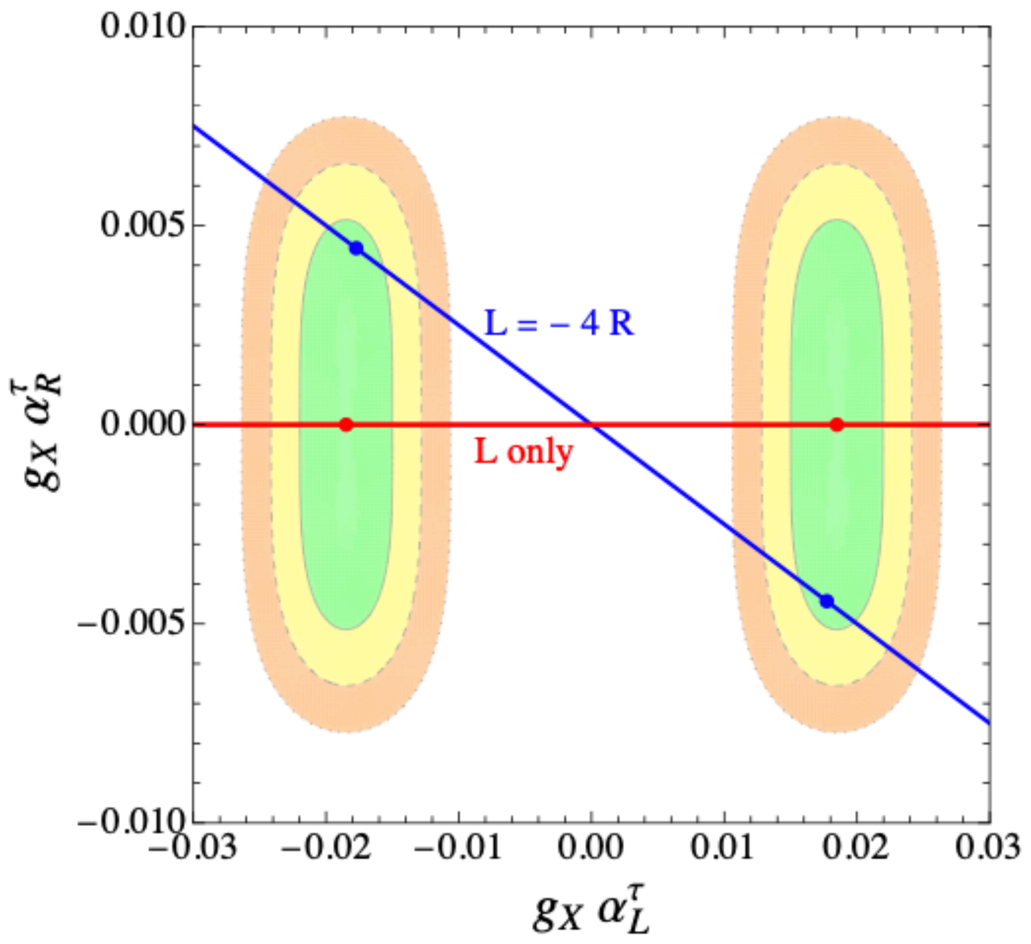
- The EFT analysis fixes only the anomaly coefficients, but leaves freedom in the choice of the current.

- The current must be flavour conserving both in the quark and lepton sector. Cannot couple to electrons or muons otherwise the X will decay in charged particle visible at LHCb. The natural solution is to couple the vector to a (chiral) tau current

$$J_X^\mu = \alpha_L^\tau \, \overline{\ell}_L^\tau \gamma^\mu \ell_L^\tau + \alpha_R^\tau \overline{\tau}_R \gamma^\mu \tau_R$$

- Now the problem is mapped in a different space (the direction of the gauging of the tau current):





- With very reasonable and general assumptions we identify only 2 viable renormalizable models: L and L=-4R

Models and predictions

B. $L = -4R$ model

Field	Lorentz	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$
q_L^i	$(\frac{1}{2},0)$	3	2	1/6	0
u_R^i	$(0,\frac{1}{2})$	3	1	2/3	0
d_R^i	$(0,\frac{1}{2})$	3	1	$-1/3$	0
$\ell_L^{1,2}$	$(\frac{1}{2},0)$	1	2	$-1/2$	0
$e_R^{1,2}$	$(0,\frac{1}{2})$	1	1	-1	0
ℓ_L^3	$(\frac{1}{2},0)$	1	2	$-1/2$	4
e_R^3	$(0,\frac{1}{2})$	1	1	-1	-1
H	$(0,0)$	1	2	1/2	0
$\mathcal{L}_L$	$(\frac{1}{2},0)$	1	3	1	-3
$\mathcal{L}_R$	$(0,\frac{1}{2})$	1	3	1	-2
$\mathcal{N}_R^1$	$(0,\frac{1}{2})$	1	1	0	2
$\mathcal{N}_R^2$	$(0,\frac{1}{2})$	1	1	0	4
$\mathcal{E}_L$	$(\frac{1}{2},0)$	1	1	-1	4
$\mathcal{E}_R$	$(0,\frac{1}{2})$	1	1	-1	4
$\mathcal{S}_1$	$(0,0)$	1	1	0	1
$\mathcal{S}_5$	$(0,0)$	1	1	0	5

SM fields, tau is charged under the new U(1)

Anomalons, needed in the UV to cancel gauge anomalies

Yukawa for the tau not is allowed, needs extra states

Scalar sector to ensure a suitable U(1) breaking

- Flavour safe models because of the MFV structure
- Anomalons are EW states and cannot be decoupled (clear target for both models)
- $Z \to X\gamma$ is a clear target for future collider (model B)
- Accidental symmetries of the Standard Model automatically preserved (model B)

Conclusions

1) To evade direct searches, New Physics (NP) must be either light and very weakly coupled to the SM or heavy. In recent years, interest in light NP has been growing significantly.

2) Considering a light vector couple to an anomalous current, flavour universal terms are generically expected. However, Flavour Violation is generated by SM interactions leading to a MFV scenario

3) Light NP is described by an EFT. Providing UV explicit models that realizes such an EFT is not merely a theoretical exercise but it can also reveal important phenomenological implications.

4) I discussed an explicit example, try do address the recent "anomalous" result in $B \to K^{(*)} + E_{miss}$

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Backup

m	n	$\pm\mathcal{Y}_{\mathcal{L}}$	$\pm(\mathcal{X}_{\mathcal{L}_L}+\mathcal{X}_{\mathcal{L}_R})$	$\mathcal{X}_S/\alpha_L^\tau$
2	5	-2	3/2	1/20
4	3	-1	5/4	1/4
∞	2	$-1/2$	1	1

TABLE II. Solutions of Eqs. (31)–(33) featuring rational m and a LP in the n-plet that is neutral. Highlighted in color the cases that can explain the $B \to K^{(*)} E_{\rm miss}$ excess, while remaining consistent with $Z \to \gamma X$ at 1σ.

IV. UV COMPLETIONS FOR τ MASS GENERATION

In this section, we present a UV completion for the two viable scenarios identified within the class of simplified models discussed in the previous section. A general feature of these constructions is the presence of vector-like fermions, which are required to generate a mass for the τ lepton via a FN mechanism [17–19].

A. L-only model

Field	Lorentz	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$	Z_2^A	Z_2^τ
q_L^i	$(\frac{1}{2},0)$	3	2	1/6	0	+	+
u_R^i	$(0,\frac{1}{2})$	3	1	2/3	0	+	+
d_R^i	$(0,\frac{1}{2})$	3	1	$-1/3$	0	+	+
$\ell_L^{1,2}$	$(\frac{1}{2},0)$	1	2	$-1/2$	0	+	+
$e_R^{1,2}$	$(0,\frac{1}{2})$	1	1	-1	0	+	+
e_R^3	$(0,\frac{1}{2})$	1	1	-1	0	+	$-$
ℓ_L^3	$(\frac{1}{2},0)$	1	2	$-1/2$	1	+	$-$
H	$(0,0)$	1	2	1/2	0	+	+
$\mathcal{L}_L$	$(\frac{1}{2},0)$	1	2	$-1/2$	0	$-$	+
$\mathcal{L}_R$	$(0,\frac{1}{2})$	1	2	$-1/2$	1	$-$	+
$\mathcal{E}_L$	$(\frac{1}{2},0)$	1	2	$-1/2$	0	+	$-$
$\mathcal{E}_R$	$(0,\frac{1}{2})$	1	2	$-1/2$	0	+	$-$
$\mathcal{S}$	$(0,0)$	1	1	0	1	+	+

TABLE III. Field content of the L-only model.

The field content of the model is summarized in Table IV. In this construction, gauge anomaly cancellation is achieved by introducing an $\mathrm{SU}(2)_L$ doublet anomalon ($n=2$), without the need for any SM-singlet fermions. This is possible because ℓ_L^3 and $\mathcal{L}_R$, the only fermions charged under $\mathrm{U}(1)_X$, form a vector-like pair under the full gauge group.

To forbid an explicit mass term between ℓ_L^3 and $\mathcal{L}_R$, we introduce a discrete Z_2^A symmetry acting non-trivially only on the anomalon fields $\mathcal{L}_{L,R}$. This allows for a Yukawa interaction of the form given in Eq. (29). We further assume a separate discrete symmetry Z_2^τ, interpreted as a τ-flavor symmetry, which distinguishes e_R^3 from $e_R^{1,2}$. To generate the τ lepton mass via a FN mechanism, we introduce a vector-like fermion pair $\mathcal{E}_{L,R}$, charged under Z_2^τ and vector-like under the full gauge group. The Yukawa Lagrangian relevant to the τ sector is then:

$$-\mathcal{L}_Y^\tau = y_L\,\bar{\ell}_L^3\mathcal{E}_R\mathcal{S} + M\,\bar{\mathcal{E}}_L\mathcal{E}_R + y_R\,\bar{\mathcal{E}}_L H\, e_R^3 + \text{h.c.}\,. \tag{40}$$

Assuming M is sufficiently large, the heavy FN mediator $\mathcal{E}$ can be integrated out, yielding the effective operator:

$$-\mathcal{L}_Y^\tau = -\frac{y_L y_R}{M}\,\bar{\ell}_L^3 H\, e_R^3\,\mathcal{S} + \text{h.c.}\,, \tag{41}$$

which generates the τ lepton mass (upon removing an overall sign via a chiral transformation):

$$m_\tau = \frac{y_L y_R}{2}\frac{v v_X}{M} = 1.78\,\text{GeV} \times y_L y_R \left(\frac{v_X/M}{0.014}\right). \tag{42}$$

In this limit, the effective IR X-current charges are simply $\alpha_L^\tau = 1$ and $\alpha_R^\tau = 0$. The prediction of this model is shown in red in the right panel of Fig. 1.

Specifying the anomalon mass in Eq. (36) to the case $n=2$ and $\alpha_L^\tau = 1$ we obtain

$$m_{\mathcal{L}} = \frac{y_{\mathcal{L}}}{\sqrt{2}} 120\,\text{GeV}\left(\frac{m_X}{2.1\,\text{GeV}}\right)\left(\frac{0.018}{g_X}\right), \tag{43}$$

normalized to the best fit point $(m_X, g_X) = (2.1\,\text{GeV}, 0.018)$. The requirement of perturbative unitarity allows one to infer an upper bound on the Yukawa coupling, $y_{\mathcal{L}} \le \sqrt{4\pi}$ [76], thus $m_{\mathcal{L}} \le 290$ GeV.

B. $L = -4R$ model

Field	Lorentz	$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_X$
q_L^i	$(\frac{1}{2},0)$	3	2	1/6	0
u_R^i	$(0,\frac{1}{2})$	3	1	2/3	0
d_R^i	$(0,\frac{1}{2})$	3	1	$-1/3$	0
$\ell_L^{1,2}$	$(\frac{1}{2},0)$	1	2	$-1/2$	0
$e_R^{1,2}$	$(0,\frac{1}{2})$	1	1	-1	0
ℓ_L^3	$(\frac{1}{2},0)$	1	2	$-1/2$	4
e_R^3	$(0,\frac{1}{2})$	1	1	-1	-1
H	$(0,0)$	1	2	1/2	0
$\mathcal{L}_L$	$(\frac{1}{2},0)$	1	3	1	-3
$\mathcal{L}_R$	$(0,\frac{1}{2})$	1	3	1	-2
$\mathcal{N}_R^1$	$(0,\frac{1}{2})$	1	1	0	2
$\mathcal{N}_R^2$	$(0,\frac{1}{2})$	1	1	0	4
$\mathcal{E}_L$	$(\frac{1}{2},0)$	1	1	-1	4
$\mathcal{E}_R$	$(0,\frac{1}{2})$	1	1	-1	4
$\mathcal{S}_1$	$(0,0)$	1	1	0	1
$\mathcal{S}_5$	$(0,0)$	1	1	0	5

TABLE IV. Field content of the $L=-4R$ model.

In this model gauge anomalies are canceled by an EW triplet ($n=3$) and two SM-singlet fields. We also introduce a fermion field, $\mathcal{E}$, vector-like under the full gauge group, and a new scalar field $\mathcal{S}_5$ to give mass to the τ lepton through a FN mechanism. We show the field content of the model in Table IV.

By adding a proper term in the scalar potential, $\Delta V(H,\mathcal{S}_1,\mathcal{S}_5)$,[6] the following VEV configurations are generated

$$\langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}, \quad \langle\mathcal{S}_1\rangle = \frac{V_1}{\sqrt{2}}, \quad \langle\mathcal{S}_5\rangle = \frac{V_5}{\sqrt{2}}, \tag{44}$$

with $v \approx 246$ GeV and $V_{1,5}$ being the order parameters of $\mathrm{U}(1)_X$ breaking. The latter are responsible for the mass of the $\mathrm{U}(1)_X$ gauge boson, that is

$$m_X = g_X\sqrt{V_1^2 + 25V_5^2} \equiv g_X v_X\,, \tag{45}$$

where g_X is the $\mathrm{U}(1)_X$ gauge coupling entering the covariant derivative, *i.e.* $D^\mu\mathcal{S}_{1,5} \equiv (\partial^\mu + i g_X \mathcal{X}_{\mathcal{S}_{1,5}} X^\mu)\mathcal{S}_{1,5}$.

The renormalizable Yukawa Lagrangian involving uncolored fields reads $(a,b=1,2)$

$$\begin{aligned}-\mathcal{L}_Y = {} & y_{\mathcal{L}}\bar{\mathcal{L}}_L\mathcal{L}_R\mathcal{S}_1^* + M\bar{E}_L E_R + y_{ab}\bar{\ell}_L^a e_R^b H \\ & + y_L\bar{\ell}_L^3 E_R H + y_R S_5 \bar{E}_L e_R^3 + \text{h.c.}\,.\end{aligned} \tag{46}$$

The accidental global symmetry left unbroken by $\mathcal{L}_Y$ is $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2 \times \mathrm{U}(1)_{3+E} \times \mathrm{U}(1)_{\mathcal{L}}$, where $U(1)_{3+E}$ can be identified as the generalized τ number and $U(1)_{\mathcal{L}}$ is the anomalon number.

To identify the τ lepton, we introduce the mass matrix $\mathcal{M}_E$, defined via

$$\begin{pmatrix}\bar{e}_L^3 & \bar{\mathcal{E}}_L\end{pmatrix}\begin{pmatrix}0 & \frac{1}{\sqrt{2}}y_L v\\ \frac{1}{\sqrt{2}}y_R V_5 & M\end{pmatrix}\begin{pmatrix}e_R^3\\ \mathcal{E}_R\end{pmatrix}, \tag{47}$$

where e_L^3 denotes the charged component of the doublet ℓ_L^3. In particular, from

$$\mathcal{M}_E\mathcal{M}_E^T = \begin{pmatrix}\frac{1}{2}y_L^2 v^2 & \frac{1}{\sqrt{2}}M y_L v\\ \frac{1}{\sqrt{2}}M y_L v & M^2 + \frac{1}{2}y_R^2 V_5^2\end{pmatrix}, \tag{48}$$

and

$$\mathcal{M}_E^T\mathcal{M}_E = \begin{pmatrix}\frac{1}{2}y_R^2 V_5^2 & \frac{1}{\sqrt{2}}M y_R V_5\\ \frac{1}{\sqrt{2}}M y_R V_5 & M^2 + \frac{1}{2}y_L^2 v^2\end{pmatrix}, \tag{49}$$

we obtain the mixing angles (taking for simplicity real parameters)

$$\begin{aligned}\tan 2\theta_L &= \frac{\sqrt{2}M y_L v}{\frac{1}{2}y_L^2 v^2 - M^2 - \frac{1}{2}y_R^2 V_5^2}\\ &\approx -\sqrt{2}y_L v/M\,, \end{aligned}\tag{50}$$

$$\begin{aligned}\tan 2\theta_R &= \frac{\sqrt{2}M y_R V_5}{\frac{1}{2}y_R^2 V_5^2 - M^2 - \frac{1}{2}y_L^2 v^2}\\ &\approx -\sqrt{2}y_R V_5/M\,, \end{aligned}\tag{51}$$

where the approximation holds in the $M \gg V_5, v$ limit. We hence obtain the τ chiral components in terms of gauge eigenstates:

$$\tau_{L,R} = \cos\theta_{L,R}e_{L,R}^3 + \sin\theta_{L,R}E_{L,R}\,, \tag{52}$$

$$\mathcal{T}_{L,R} = -\sin\theta_{L,R}e_{L,R}^3 + \sin\theta_{L,R}E_{L,R}\,. \tag{53}$$

Projecting Eq. (46) on the mass eigenstates, we obtain

$$\begin{aligned} m_\tau &\approx \frac{y_L y_R}{2} v \frac{V_5}{M}\\ &= 1.78\,\text{GeV}\, y_L y_R\left(\frac{V_5/M}{0.014}\right), \end{aligned}\tag{54}$$

$$m_{\mathcal{T}} \approx M\,, \tag{55}$$

implying that θ_R must be small to account for the τ-lepton mass.

Note that this setup induces a deviation from unitarity in the PMNS matrix, due to the modified structure of the third-generation lepton doublet:

$$\ell_L^3 = \begin{pmatrix}\cos\theta_L\tau_L\\ \nu_\tau\end{pmatrix}, \tag{56}$$

which implies that θ_L is constrained to be close to zero at the percent level, consistent with neutrino oscillation data (see *e.g.* [77]).

Hence, in the limit of negligible $\theta_{L,R}$, the effective couplings of the IR X-current are given by

$$\alpha_L^\tau = 4\,, \quad \alpha_R^\tau = -1 + 5\sin^2\theta_R \approx -1\,. \tag{57}$$

In the right panel of Fig. 1, we display the prediction of this model in blue. Given the best fit point $(m_X, g_X) = (2.1\,\text{GeV}, 0.0042)$, this yields $v_X \approx 467$ GeV, along with a predicted rate for the $Z \to \gamma X$ decay

$$\mathcal{B}(Z\to\gamma X)_{\text{best-fit}} = 5.6\times 10^{-7}\,. \tag{58}$$

This is sizeable, but still compatible with the LEP bound [64] in Eq. (20). Hence, a future collider such as FCC-ee [78] operating at the Z-pole could provide a test of this hypothesis.

The anomalons pick up the mass term

$$\begin{aligned} m_{\mathcal{L}} &= \frac{y_{\mathcal{L}}}{\sqrt{2}}V_1 \approx \frac{y_{\mathcal{L}}}{\sqrt{2}}\frac{m_X}{g_X}\\ &= \frac{y_{\mathcal{L}}}{\sqrt{2}}500\,\text{GeV}\left(\frac{m_X}{2.1\,\text{GeV}}\right)\left(\frac{0.0042}{g_X}\right), \end{aligned}\tag{59}$$

where we assumed $v_X \approx V_1 \gg V_5$. The perturbative unitarity bound on the Yukawa coupling reads in this case $y_{\mathcal{L}} \le \sqrt{8\pi/3}$ [76], thus $m_{\mathcal{L}} \lesssim 1.0$ TeV.

The SM-singlet states $\mathcal{N}_R^{1,2}$ pick-up a mass term from the $d=6$ operators

$$\mathcal{L}_{\mathcal{N}} = \frac{\lambda_1}{\Lambda^2}\mathcal{S}\mathcal{S}_5^*\mathcal{N}_R^1\mathcal{N}_R^1 + \frac{\lambda_2}{\Lambda^2}\mathcal{S}^*\mathcal{S}_5^*\mathcal{N}_R^1\mathcal{N}_R^2 + \text{h.c.}\,. \tag{60}$$

While their masses are controlled by the cutoff scale Λ, these SM-singlet states do not mix with SM fermions and therefore play only a marginal role in the low-energy phenomenology.

A final comment about neutrino masses is in order. We work in the limit of massless neutrinos. However, some extra dynamics is required in order to generate neutrino masses and a non-trivial PMNS. This dynamics can be pushed up to scales $\gg$ TeV, and therefore it decouples from the relevant phenomenology we focus on.

[6] It is worth noting that, in the presence of only renormalizable operators, a massless Goldstone boson arises at the tree level. Its mass can be lifted either through loop corrections or by introducing a non-hermitian effective operator, such as $S_5^\dagger S_1^5$, in the scalar potential.