Mapping the theory space of the flavor puzzle: the Froggatt-Nielsen case

Claudia Cornella || CERN

Based on: 2306.08026 with D. Curtin, E. Neil, J. Thompson
2501.00629, with D. Curtin, G. Krnjaic, M. Mellors

Photo: Santa Maria in Trastevere

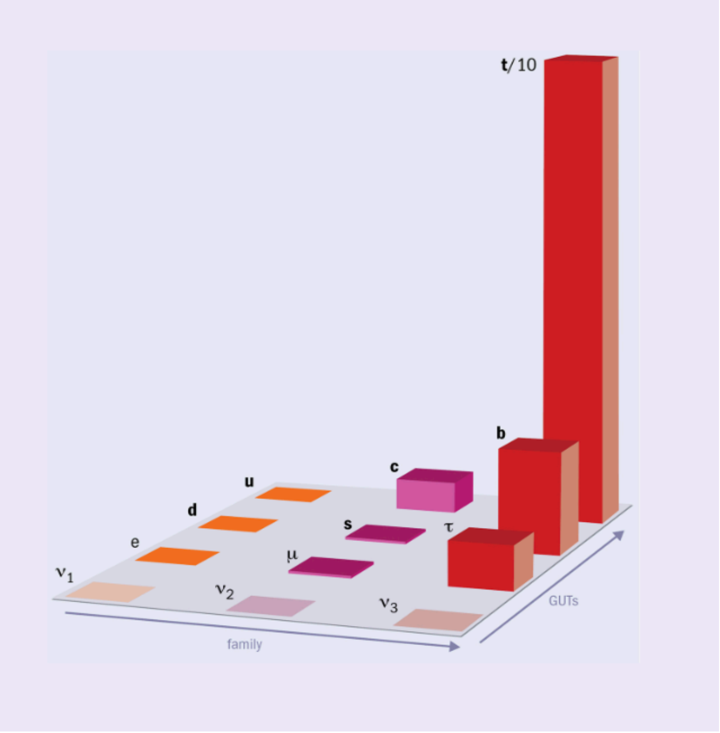
Symmetry-based solutions to the Flavor Puzzle

Flavor puzzle = a series of puzzling observations:

3 copies of each species, identical from the point of view of gauge interactions, yet:

- 12 orders of magnitude from neutrinos to the top mass
- mixing looks very different in lepton vs quark sector

All *technically natural*, still suggestive of an organising principle beyond the SM.



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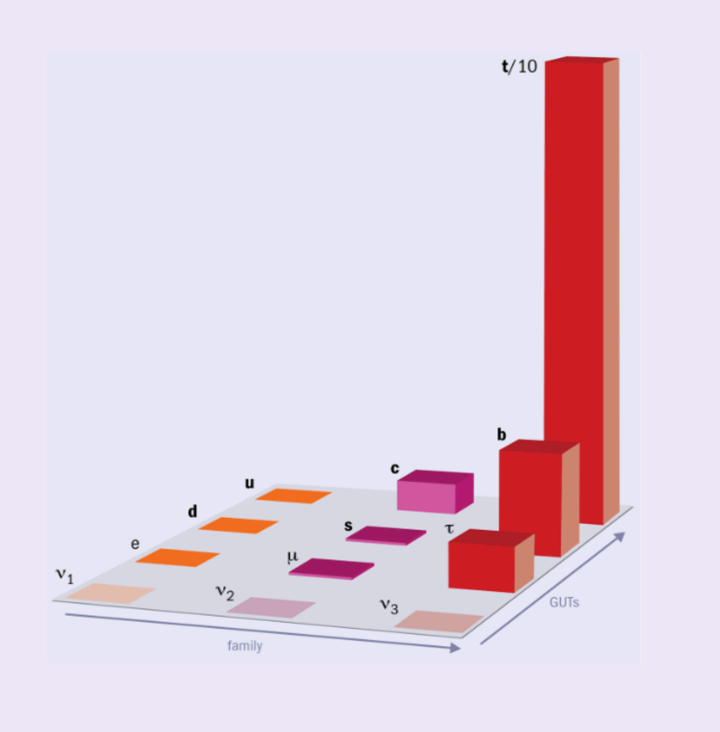

Common approach: **describe in terms of a symmetry and its breaking**

- "Deeper" origin can vary:
  - flavor-dependent gauge interactions
  - geometry, e.g. localisation of fermions in extra dimension
- Many examples: discrete, $U(2)^n$ from flavor deconstruction, Froggatt-Nielsen

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Ingredients:

- a symmetry

$$Y = \begin{bmatrix} & & \blacksquare \\ & & \blacksquare \\ & & \blacksquare \end{bmatrix}$$

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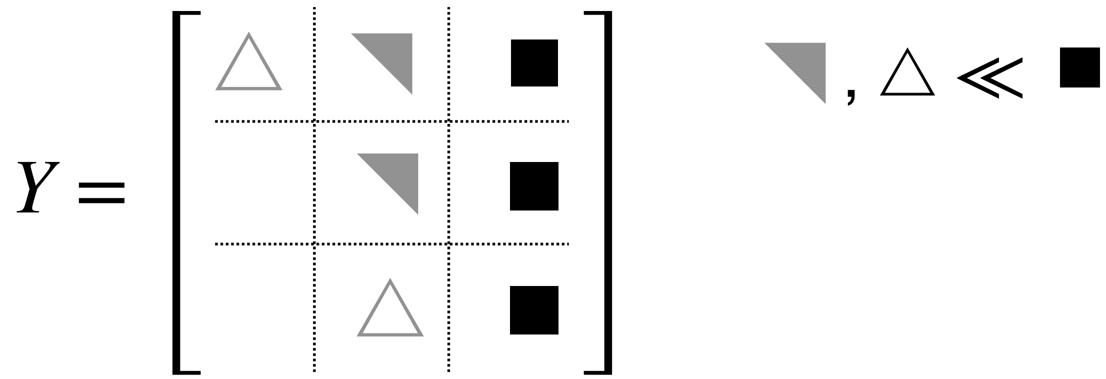

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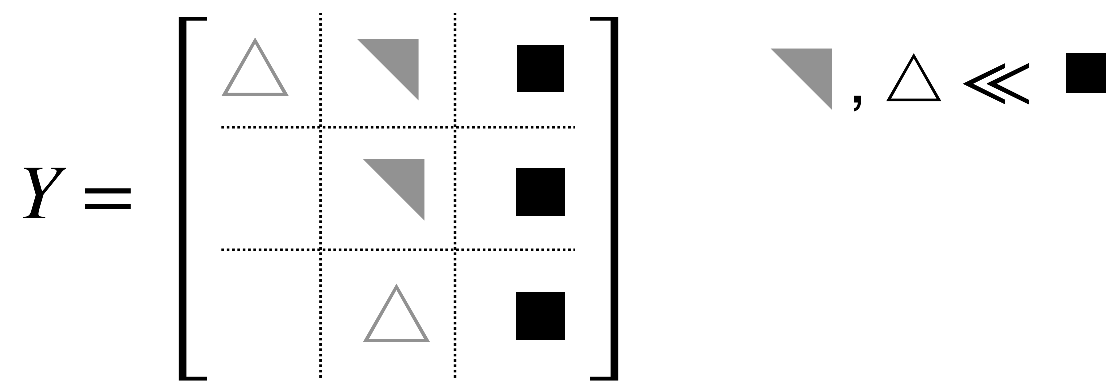
- some coefficients c_{ij}

  $\sim \mathcal{O}(1)$ if we want the pattern to come only from the symmetry ansatz



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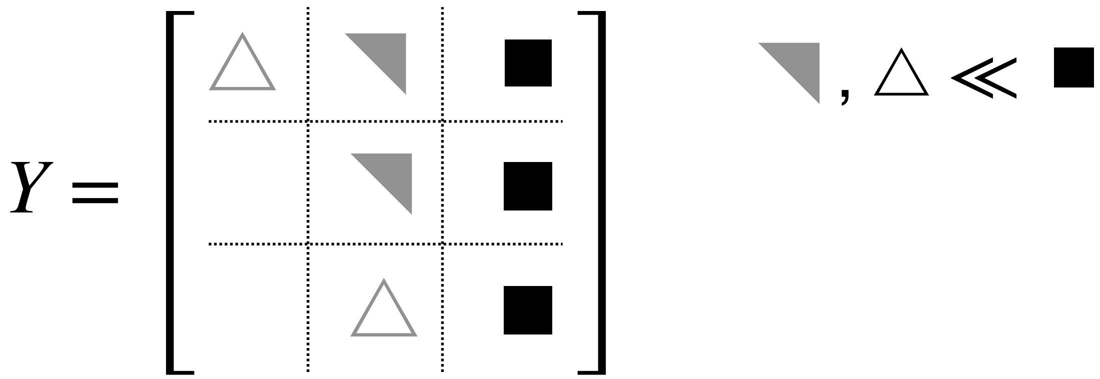

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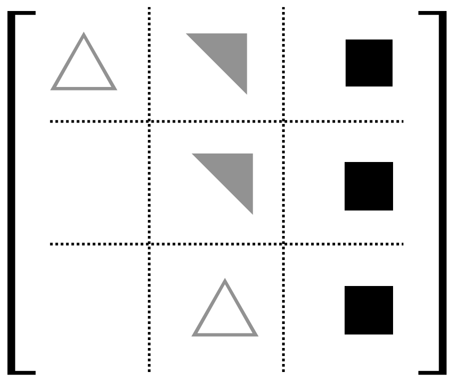
$Y =$ 

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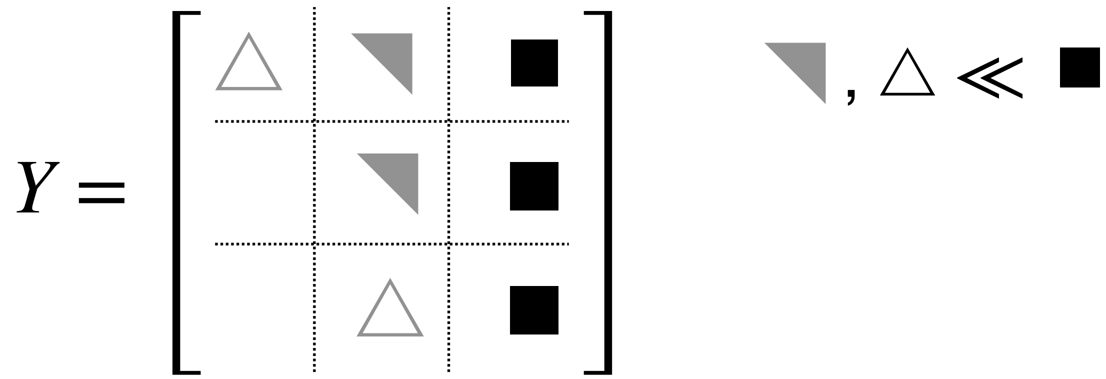

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We'll use the global definition to explore which FN ansätze are good solutions to the quark & lepton flavor puzzle(s)

A quick review of the Froggatt-Nielsen mechanism

(in its simplest version)

- The SM is extended by a $U(1)_X$ symmetry, spontaneously broken by a scalar ϕ , the *flavon*, with $\epsilon = \frac{\langle \phi \rangle}{\Lambda_F} \ll 1$

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- The same selection rules hold for other higher-dimensional operators

$$\frac{c_{ijkl}}{\Lambda^2} \left(\bar{\psi}_i \psi_j\right) \left(\bar{\psi}_k \psi_l\right) \epsilon^{n_{ijkl}}$$

$$n_{ijkl} \equiv |X_{\psi_i} - X_{\psi_j}| + X_{\psi_k} - X_{\psi_l}|$$

⇒ predictions for flavor-violating processes bear the fingerprint of textures!

What we want to do

- Define a notion of global goodness for a given texture
  [we know one can "always" fit almost any FN model to the SM, but that's not the point;The model should 'want' to look like the SM with 'O(1)' parameters]
- Use it to rank textures for quarks & leptons [*separately*]
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Previous works have tackled FN textures from various angles, e.g:

- Bayesian analysis of specific textures: Altarelli, Feruglio, Masina [0210342, 1207.0587]
- Finding "locally" good textures with small charges: Fedele, Mastrodii, Valli [2009.05587]
- Leptonic FN + CPV in MSSM Aloni et al. [2104.02679]
- coincidentally with our lepton paper, Ibe, Shirai, Watanabe [2412.19484] performed a Bayesian scan very similar in spirit to our work

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For each choice of coefficients:

- compute masses and mixings
- quantify how well they reproduce data via $\delta_{\max} \equiv \max_{\mathcal{O}} \left[\frac{\mathcal{O}_{\rm FN}}{\mathcal{O}_{\rm exp}}, \frac{\mathcal{O}_{\rm exp}}{\mathcal{O}_{\rm FN}}\right]$

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- Define: $F_a \equiv$ fraction of models | $\delta_{\max} < a$

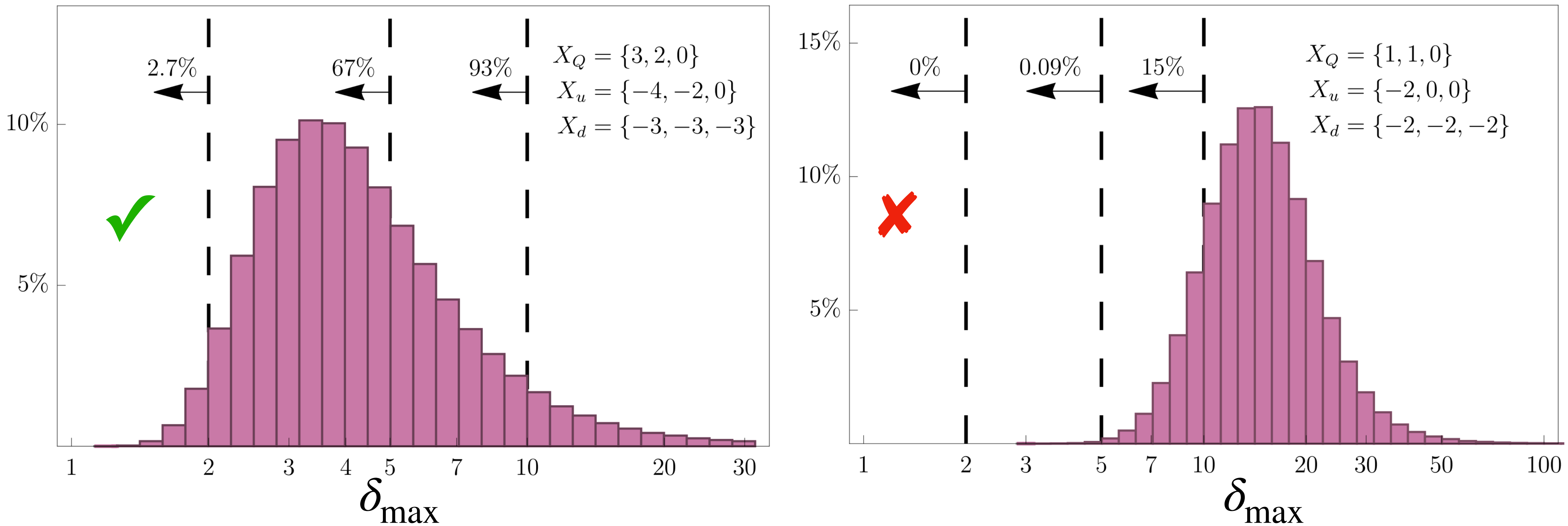
F_a *estimates "how much" of the parameter space is data-like for a given texture*

$F_2 = 0.5 \rightarrow$ half of models are within a factor 2 from data

Rank textures by F_2 [or smaller deviations, if statistics permits]

Methodology

Example: distribution of $\delta_{\max}$ for a "good" and a "bad" texture



[Obviously $F_a \to 0$ as $a \to 1$, but top charge assignments tend to give $F_5 \sim 50\,\%$, $F_2 \sim$ few %]

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in which sense?

What is a “natural fit”?

1. **It should involve only O(1) coefficients** —i.e. reproduce masses and mixings due to FN charges & ϵ, without relying on accidental hierarchies in the c_i

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2. **It should be *collectively* untuned**, i.e. stable under collective deformations of the c_k

   A standard approach is to use the Barbieri-Giudice tuning measure:

$$\Delta_{\rm BG}^K = \max_k \left| \frac{\partial \log \mathcal{O}_K}{\partial \log c_k} \right| \;\Rightarrow\; \Delta_{\rm BG} = \sum_K \Delta_{\rm BG}^K$$

   …but not suitable for models that derive from a UV competition where jiggling one UV parameter will jiggle all the IR coefficients.

   Instead:

$$\Delta_{\rm tot}^K = \sqrt{\sum_s (\lambda_s^K)^2} \;,\; \lambda_s^K \in \text{Eig}\left(\frac{\partial^2 \log \mathcal{O}_K}{\partial \log c_k \, \partial \log c_l} \right) \;\Rightarrow\; \Delta_{\rm tot} = \sum_K \Delta_{\rm tot}^K$$

Methodology

We now have, for a given type of FN model, a way of ranking charge assignments by how data-like they want to be.

Next: interrogate the “top“ textures for their predictions (e.g. for flavor-violating processes)

- find an ensemble of “natural” fits starting from the coefficient choices with small δ_{max}
- for each fit, generate random O(1) coefficients for SMEFT operators
- get distributions of predictions

The quark sector

Setup $$L_Y \supset -\, c^u_{ij} \left(\frac{\langle\phi\rangle}{\Lambda_F}\right)^{|X_{Q_i}-X_{u_j}|} \bar{Q}_i\, \tilde{H}\, u_j \;-\; c^d_{ij} \left(\frac{\langle\phi\rangle}{\Lambda_F}\right)^{|X_{Q_i}-X_{d_j}|} \bar{Q}_i\, H\, d_j$$

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Phenomenologically viable textures for $|X|_{\max} = 4$

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1	3	2	-4	-2	-3	-3	-3	2.7	67	0.17
2	3	2	-4	-2	-4	-3	-3	2.5	66	0.18
3	3	2	-3	-1	-3	-2	-2	1.9	56	0.12
4	3	2	-4	-1	-3	-3	-3	1.5	65	0.16
5	4	3	-4	-2	-4	-3	-3	1.2	52	0.23
6	3	2	-4	-1	-3	-3	-2	1.1	63	0.15
7	4	2	-4	-2	-4	-3	-3	1.1	47	0.21
8	3	2	-3	-1	-2	-2	-2	0.9	41	0.11
9	3	2	-3	-1	-3	-3	-2	0.9	55	0.14
10	3	2	-4	-2	-3	-3	-2	0.9	59	0.16
11	2	1	-3	-1	-2	-2	-2	0.8	52	0.06
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Row 3 ← [Leurer, Nir, Seiberg]

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- textures with minimal charges (0,1) are at most locally good, never globally
- good textures have near degenerate charges in the down sector

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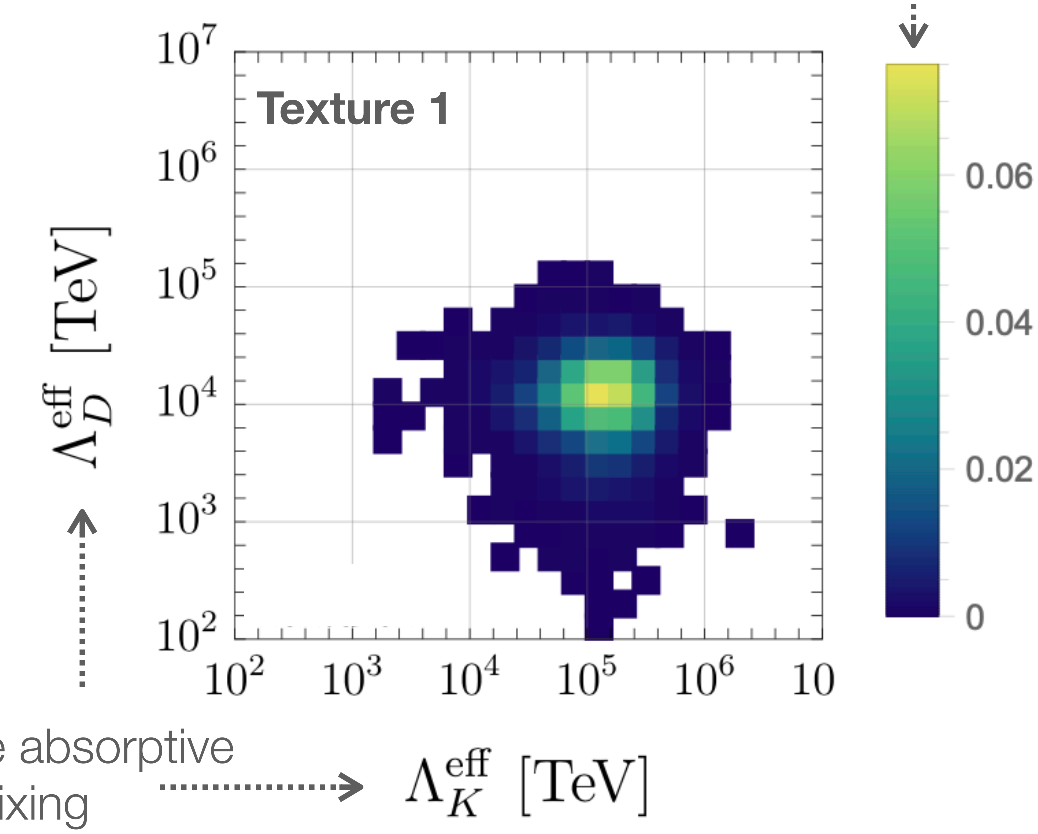
large right-handed rotations are needed to diagonalize down-type Yukawas

→ large FCNCs (of BSM origin)

→ K mixing bounds push $\Lambda_F \gtrsim 10^5\,\mathrm{TeV}$, close to the flavor anarchic case!

fraction of models with a certain $\Lambda_{K,D}$



Bounds on Λ_F set by the absorptive part of D and K mixing

The quark sector

Near degeneracy has important consequences.

$\Rightarrow$ hierarchies are explained, but no visible “collider” signal

$\Rightarrow$ even with a super-power collider, it would be difficult to distinguish among different good textures because the O(1) contribution from RH currents tends to dominate, “flattening” predictions

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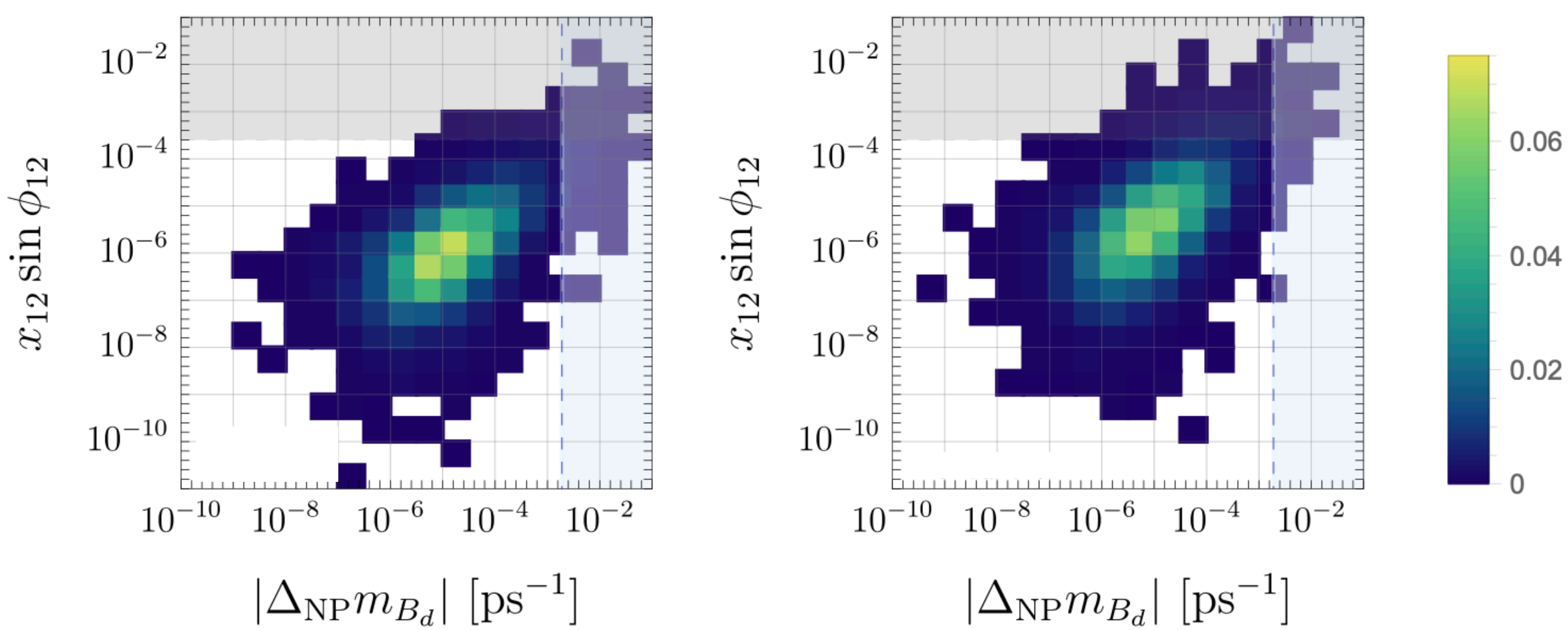
Example: predictions for D and Bd mixing for two different textures, fixing $\Lambda_F = \Lambda_K$



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Caveat: this does <u>not</u> mean that a ”good” low-scale explanation of quark flavor masses and mixings is not possible within FN.

Need to go beyond this simple setup, e.g.

- going much higher up in charges ($X_{\rm max} \sim 10$)
- add additional discrete symmetries, e.g. some $\mathbb{Z}_N$ that makes Yukawa matrices upper triangular to begin with [see e.g. Greljio, Smolkovič, Valenti,2407.02998]
- have multiple flavons

The lepton sector

Leptons show *large mixings* and *mild mass hierarchies* [for νs]
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…but also **opportunities**:

- cLFV — especially in the $\mu - e$ sector — can be tested with extreme precision ⇒ potential access even to high-scale flavor models
- Cosmology measurements and $0\nu\beta\beta$ searches are set for big advancements

The lepton sector

Setup

For charged leptons, implement FN analogously to quarks.

For neutrinos, two options:

Dirac

$$\mathscr{L}_D \supset c_{ij}^{\nu} \epsilon^{n_{ij}^{\nu}} H L_i N_j \,, \quad n_{ij}^{\nu} \equiv |X_{L_i} + X_{N_j}|$$

smallness of m_ν comes entirely from FN

Majorana

$$\mathscr{L}_W \supset -\frac{c_{ij}^{W} \, \epsilon^{n_{ij}^{W}}}{\Lambda_W} (L_i H)(L_j H), \quad n_{ij}^{W} \equiv |X_{L_i} + X_{L_j}|$$

may or may not be related to Λ_F

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Λ_W may or may not be related to Λ_F

....Λ_W helps!

Scan

Scan all integer lepton flavour charges up to $|X_\ell|_{\max} = 7(9)$ for Dirac (Majorana)

Compare to $\mathcal{O} = \left\{ m_\ell\ ,\ \Delta m^2_{ij}\ ,\ |V_{ij}|\ ,\ \sum m_\nu \right\}$

Note: cosmological or lab input has no effect on results

Best textures for Dirac and Majorana neutrinos

Top Dirac textures

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	N_1	N_2	N_3	ϵ	NO
6	5	5	-3	-2	0	9	8	8	0.10	96
3	3	3	2	-1	-6	9	9	8	0.07	99
3	3	3	2	-5	-6	9	9	8	0.07	99
7	7	6	-4	-2	0	9	9	9	0.14	99
7	7	6	-4	-3	-1	9	7	7	0.11	99
3	3	3	2	0	-5	9	9	8	0.07	99
3	3	3	2	0	-1	9	9	8	0.07	99
6	5	5	-3	-2	0	9	7	7	0.08	97
7	3	3	2	0	-5	9	9	9	0.08	93
6	6	6	-4	-3	-1	9	6	5	0.07	99

Top Majorana textures

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	ϵ	$\log \Lambda_W$	NO
2	0	-1	7	6	4	0.24	15	91
5	5	-2	7	-2	-3	0.08	12	3
4	4	3	5	2	0	0.23	11	96
7	6	5	7	3	0	0.39	11	97
6	6	5	5	1	-1	0.30	10	96
7	7	6	2	-1	-3	0.23	7.6	96
5	5	4	6	2	0	0.30	11	96
7	7	6	4	0	-2	0.30	9	96
5	5	-2	7	-2	-7	0.08	12	3
1	1	-1	-7	-5	-4	0.18	15	2

Λ_W expressed in GeV

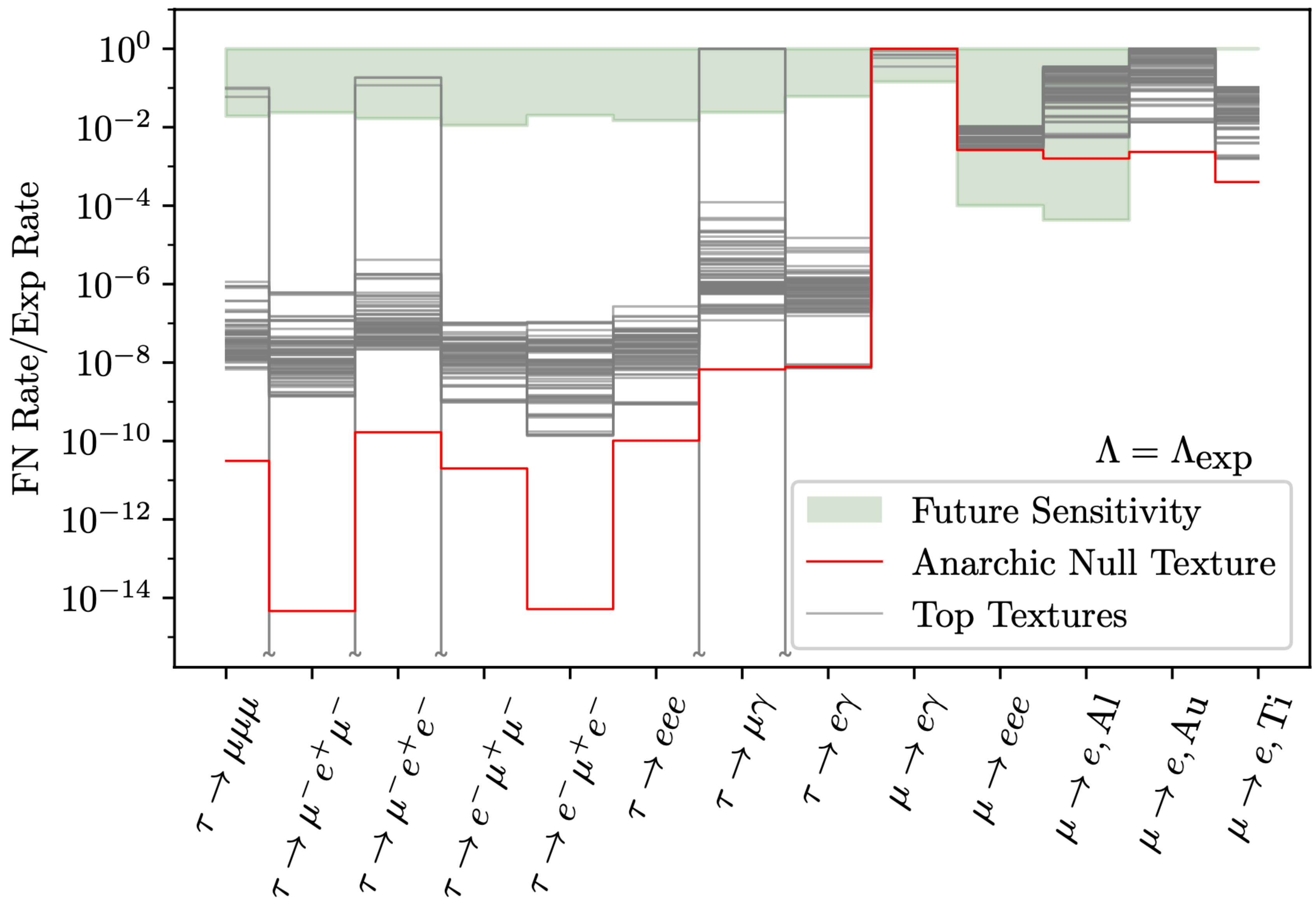
% of “models” predicting normal ordering

Textures reproducing masses and mixings within a factor $\delta_{max} < 5,\ 2,\ 1.35$
for ~50%, 2-5% and 0.03% of O(1) coefficient choices.

- Dirac requires large FN charges for RH neutrinos.
- FN favours normal ordering. Majorana neutrinos can also have inverted ordering.
-

CLFV predictions from Dirac FN models

Average predicted cLFV decay rates (relative to current constraint)
for the 100 top Dirac FN textures



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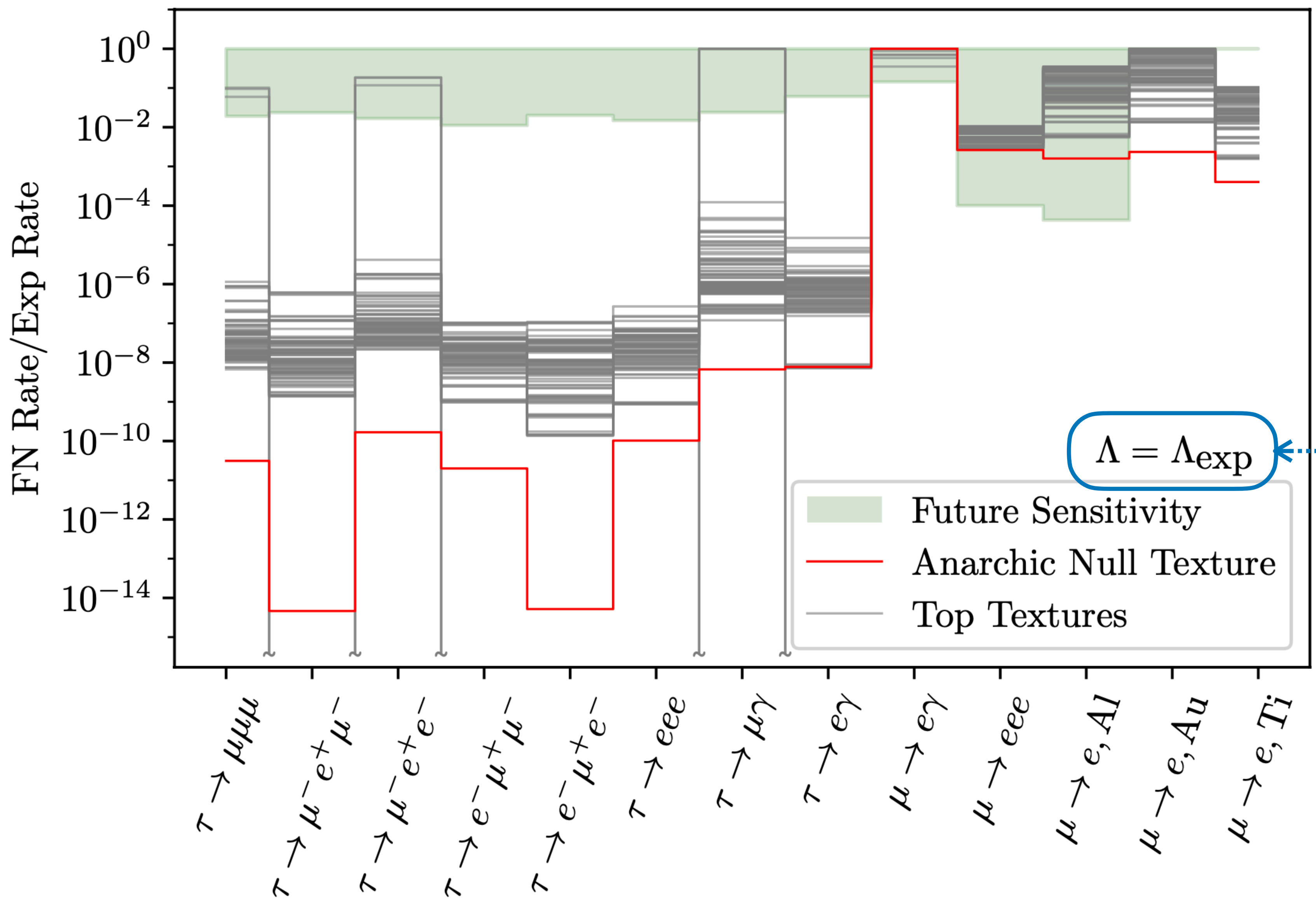

Λ chosen to saturate the most stringent bound (usually $\mu \to e\gamma$)

$\Rightarrow$ **best-case scenario**:
gives the **highest possible rates** for future CLFV signals

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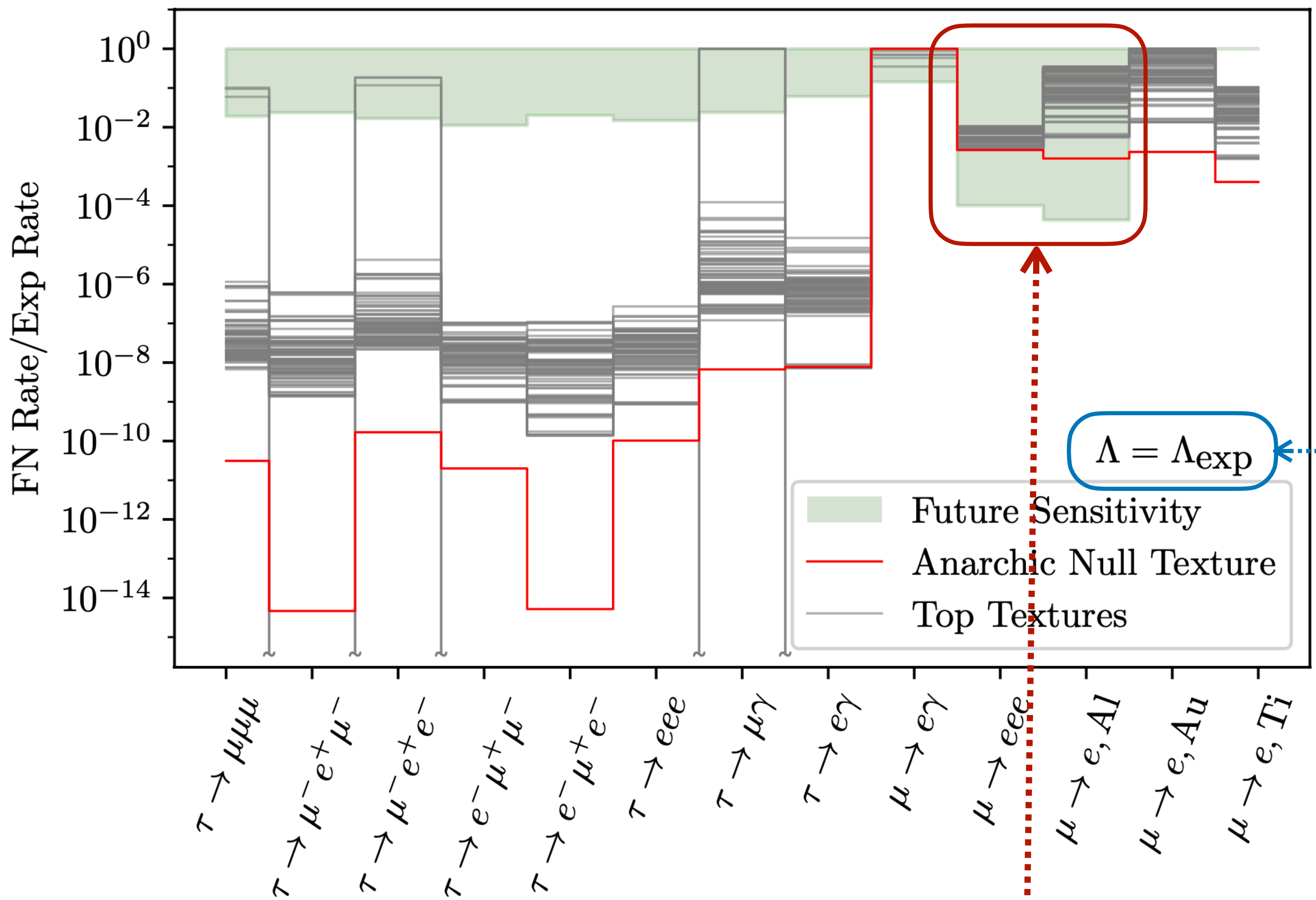

Λ chosen to saturate the most stringent bound (usually $\mu \to e\gamma$)

$\Rightarrow$ **best-case scenario**:
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For this best-case scenario, Dirac FN textures yield **measurable $\mu \to 3e$ and μ-e conversion** in nuclei,
the latter with **significant spread between different textures**

CLFV predictions from Majorana FN models

Average predicted cLFV decay rates (relative to current constraint)
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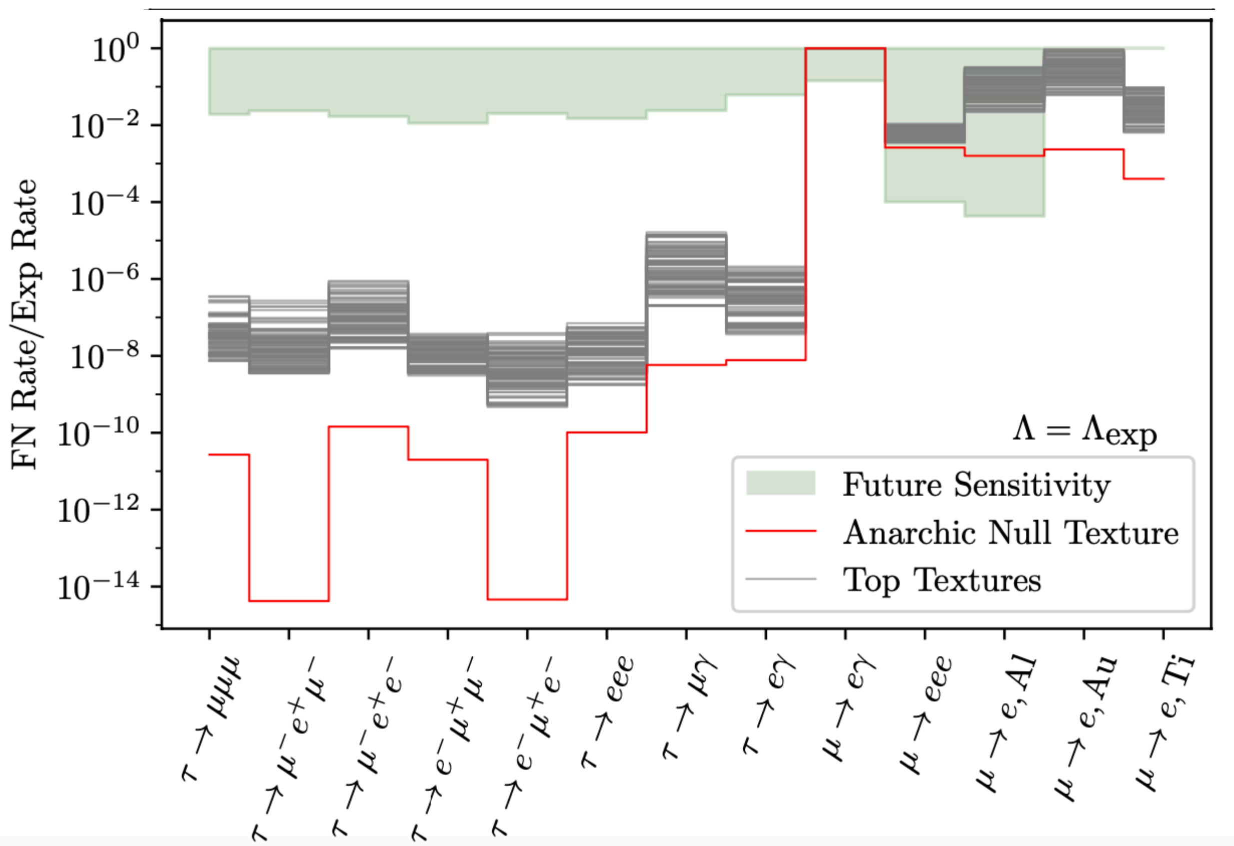
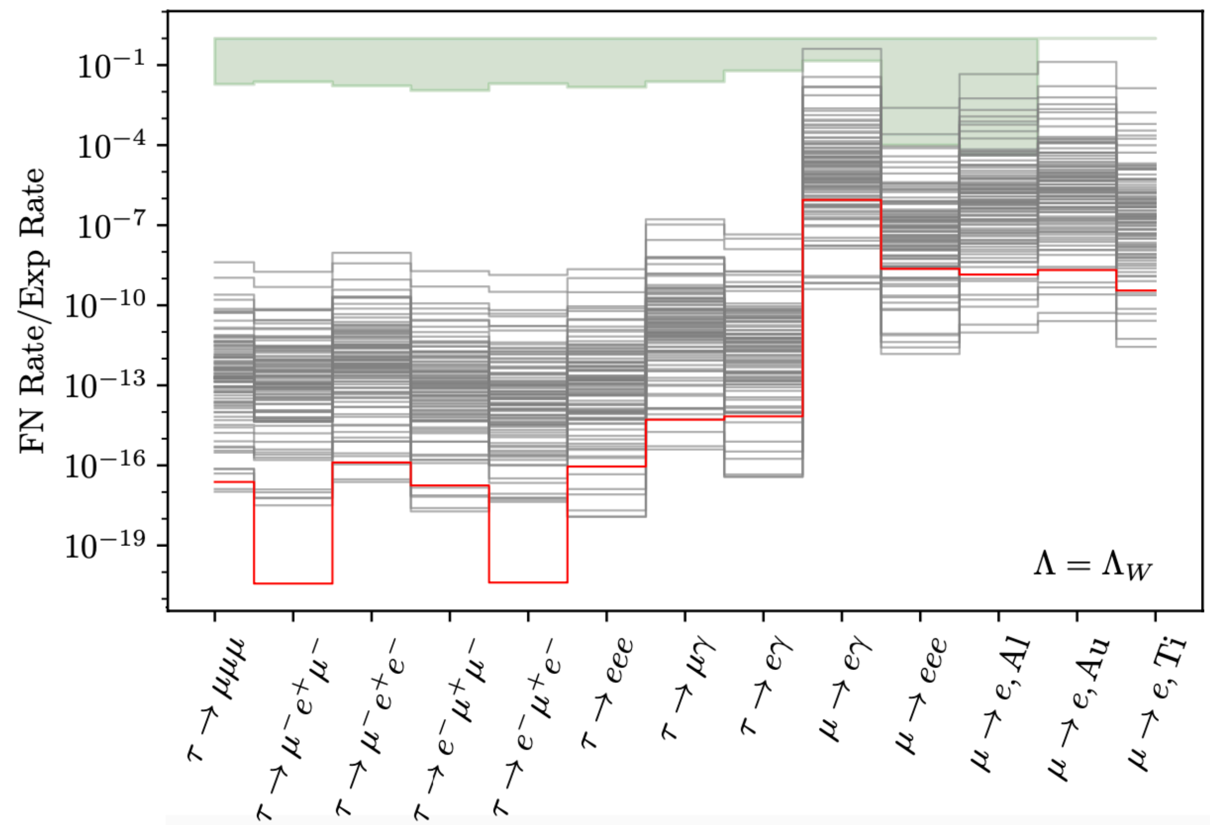
Λ fixed to most stringent constraint
(generally lower than Λ_W)



$\Lambda = \Lambda_W$
(fixed by $\sum_\nu m_\nu$)



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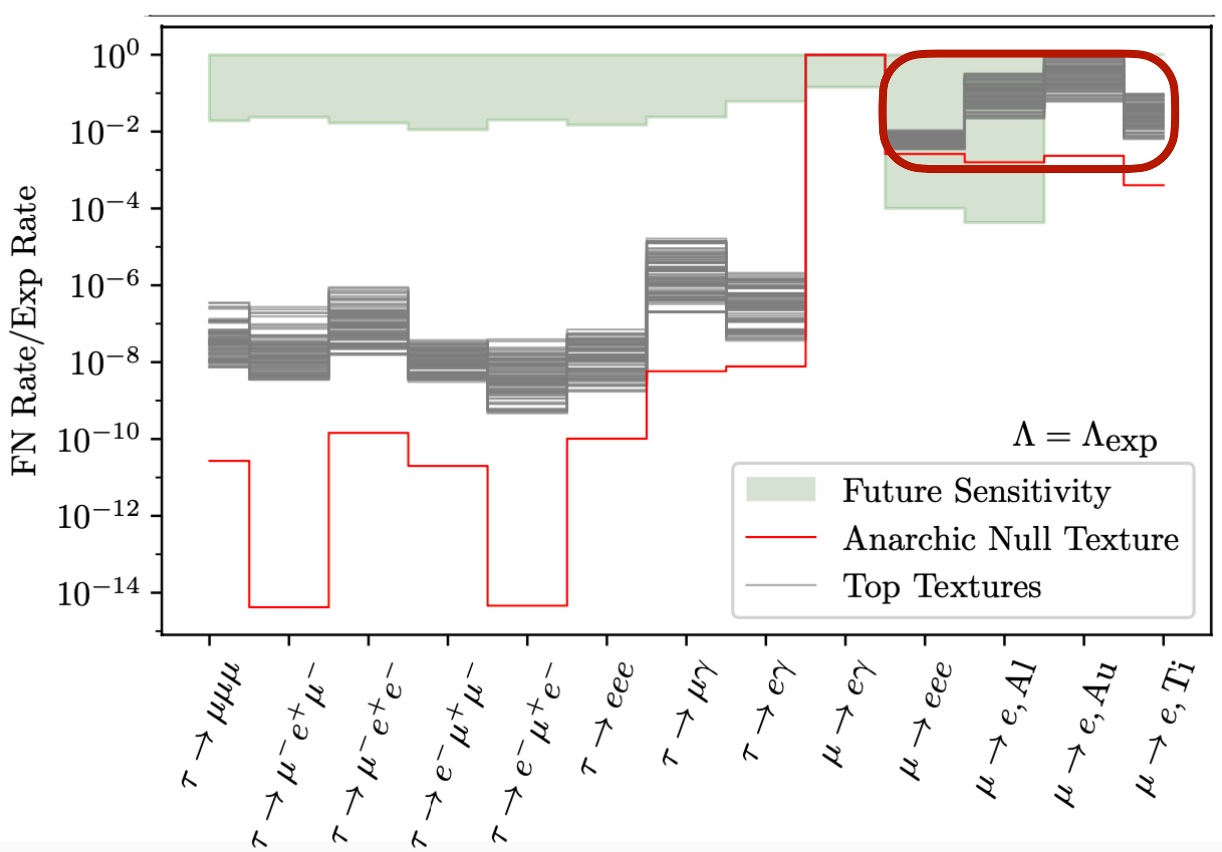
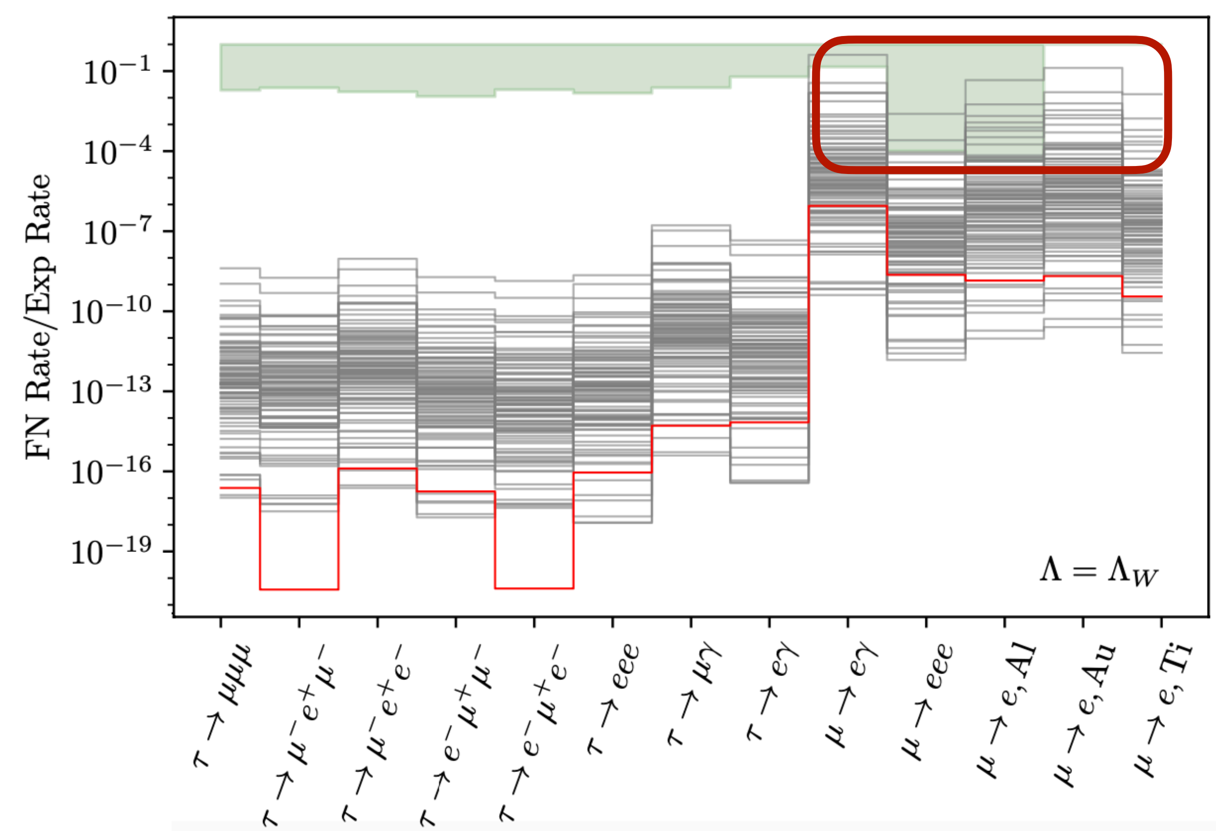


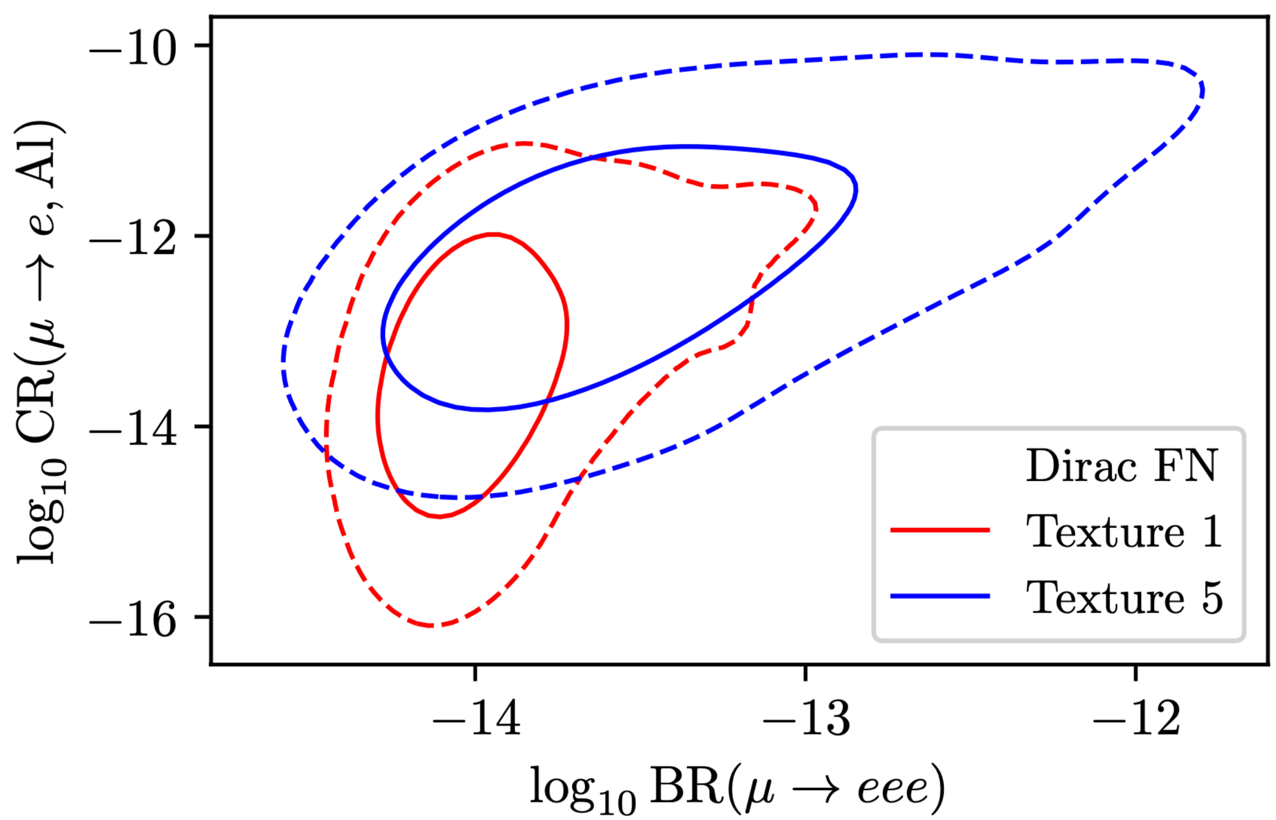


$\mu \to e\gamma, \mu \to 3e$ and $\mu - e$ conversion are the most promising observables

with $\Lambda = \Lambda_W$, only some textures give signals measurable in the near future

Discriminating FN textures via cLFV correlation

Correlations between μ → 3e and μ–e conversion for two good Dirac textures

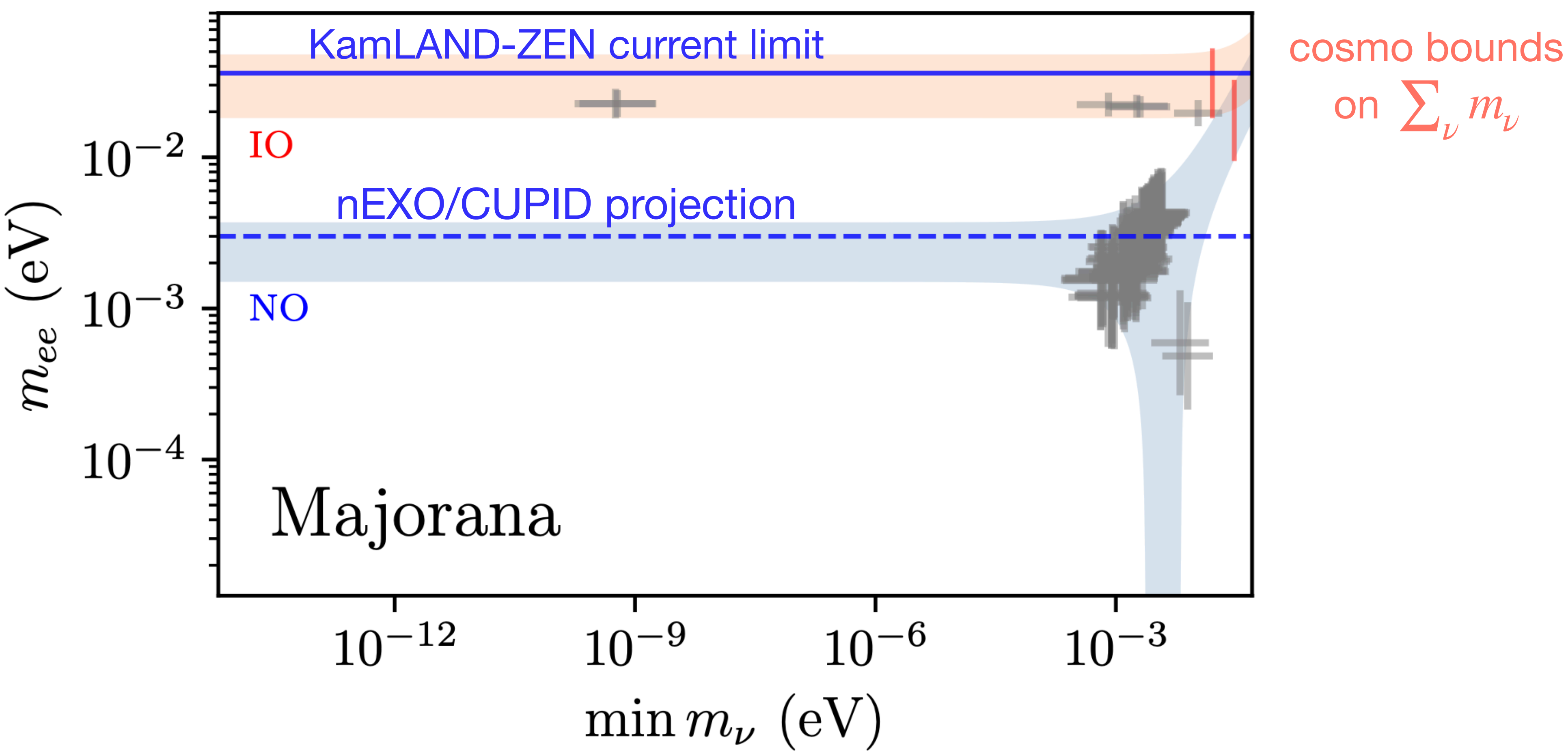


Different textures predict not just different signal strengths, but also different **correlations** between observables.

⇒ Detecting **multiple** cLFV signals could help **favour or exclude** certain textures.

Neutrinoless double β decay vs lightest ν mass

(for the 100 best Majorana FN textures)



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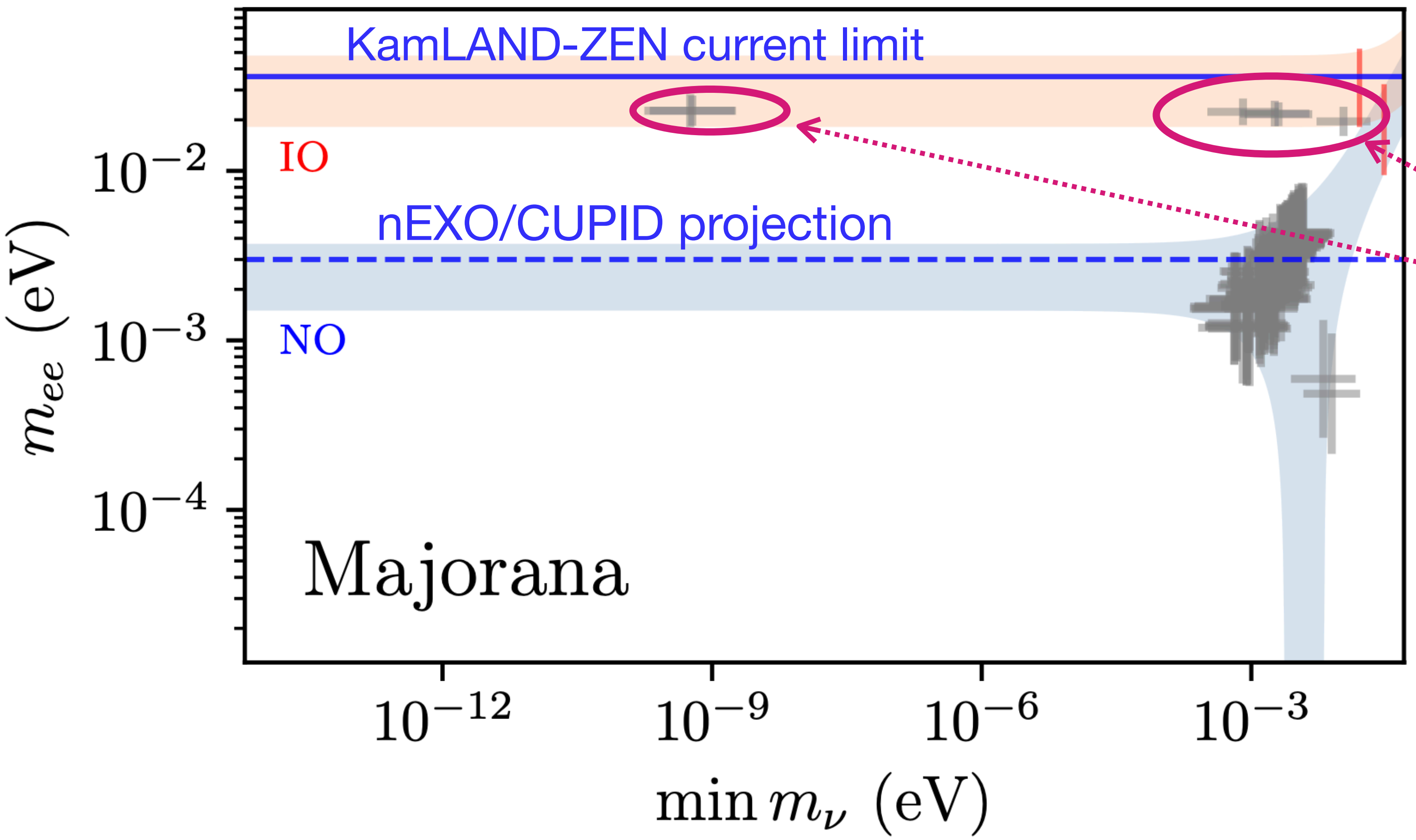

The (few) **IO textures** cluster **at the edge of the KamLAND-ZEN limit**
$\Rightarrow$ a *genuine prediction*, not imposed by the scan!

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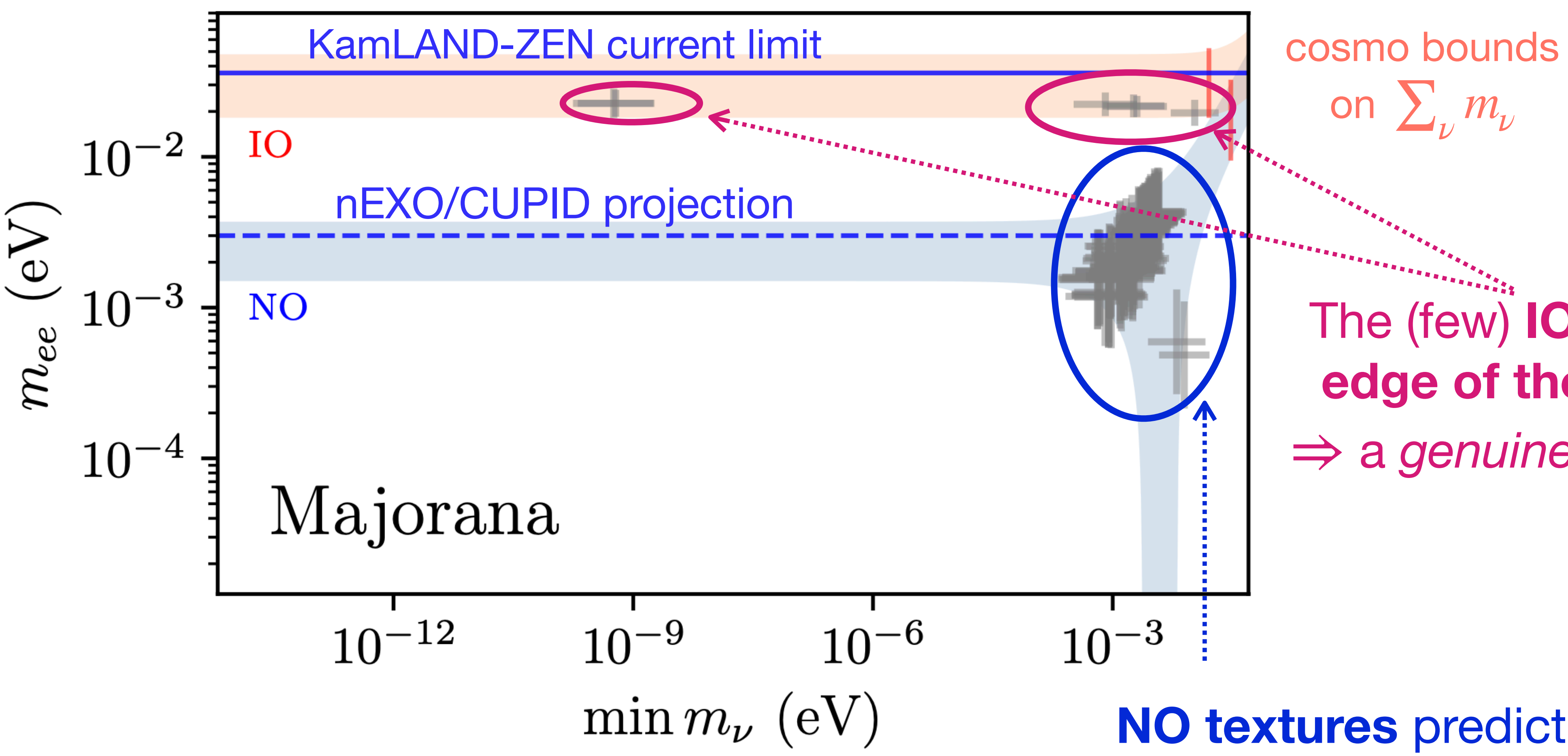

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NO textures predict m_{ee} **well within** the reach of **nEXO/CUPID or just below**

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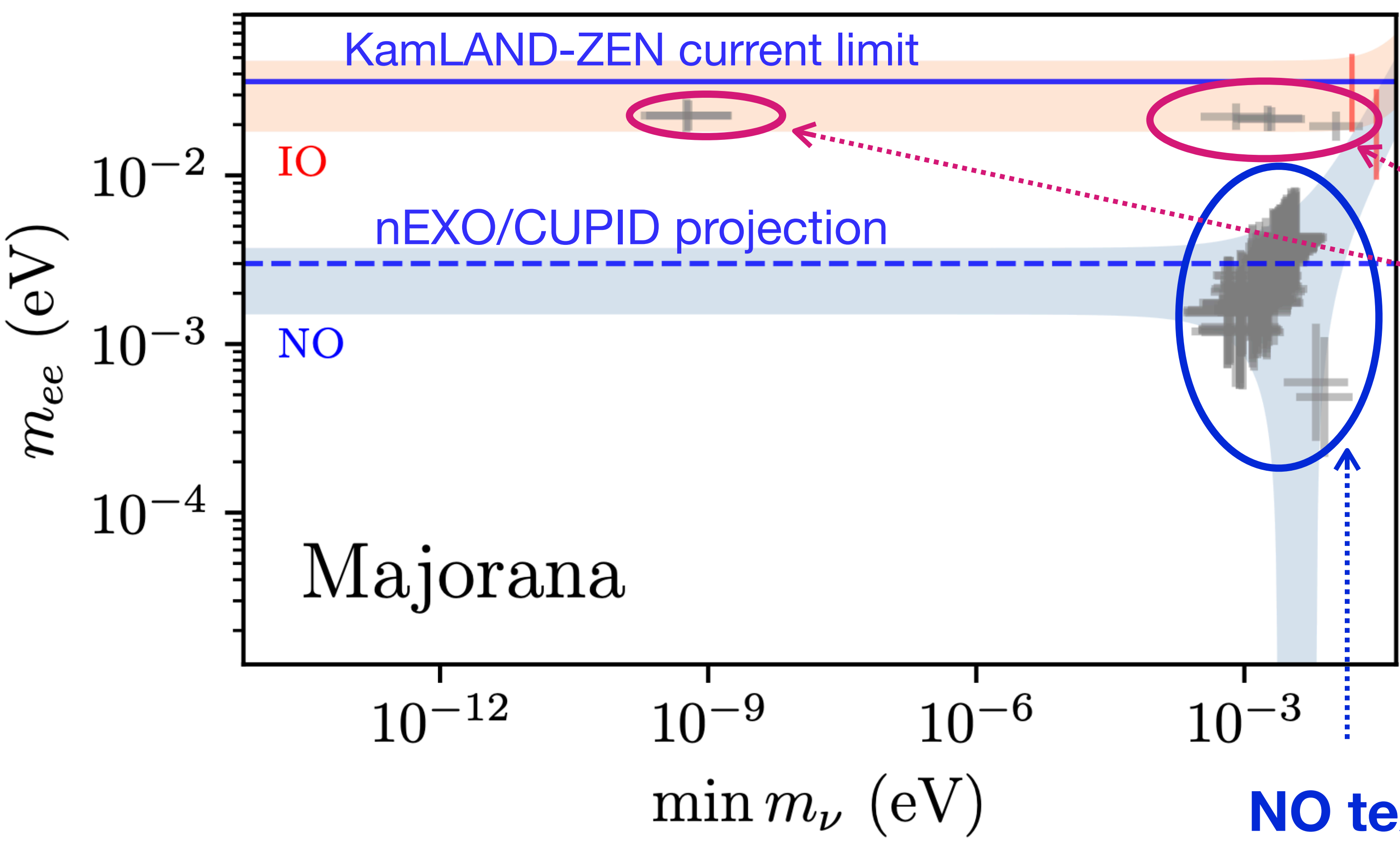

cosmo bounds on $\sum_\nu m_\nu$

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NO textures predict m_{ee} **well within** the reach of **nEXO/CUPID or just below**

All these textures are either detectable in next-gen $0\nu\beta\beta$ or within a 10x improvement

⇒opens the door to a **complete "near"-future test** of the FN Majorana scenario

Conclusions

- We aimed at giving a **bird's eye view of FN** models in the quark & lepton sectors, ranking textures by how "generically" they reproduce data with O(1) coefficients and studying their phenomenology.

- In the quark sector:
  - Top textures involve quasi-degenerate RH down-quark charges $\Rightarrow$ large RH rotations, hence large FCNCS, hence $\Lambda_F \gtrsim 10^5\,\mathrm{TeV}$
  - Direct collider phenomenology unlikely in the minimal setup without additional hypothesis (higher charges, discrete symmetries…)

- In the lepton sector:
  - FN favors normal ordering and predicts interesting cLFV patterns
  - Most top Majorana FN textures predict measurable rates for $0\nu\beta\beta$

- These ideas can be **extended to other symmetry-based solutions** to the flavor puzzle