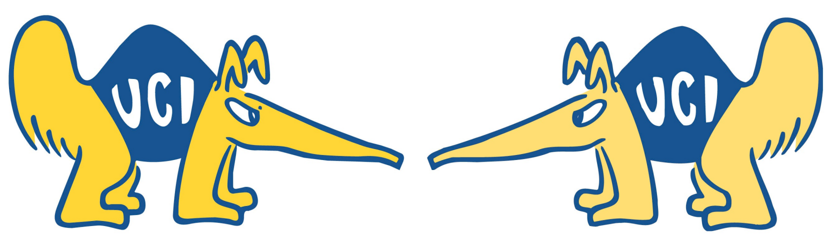
Two Tales about Modular Flavor Symmetries

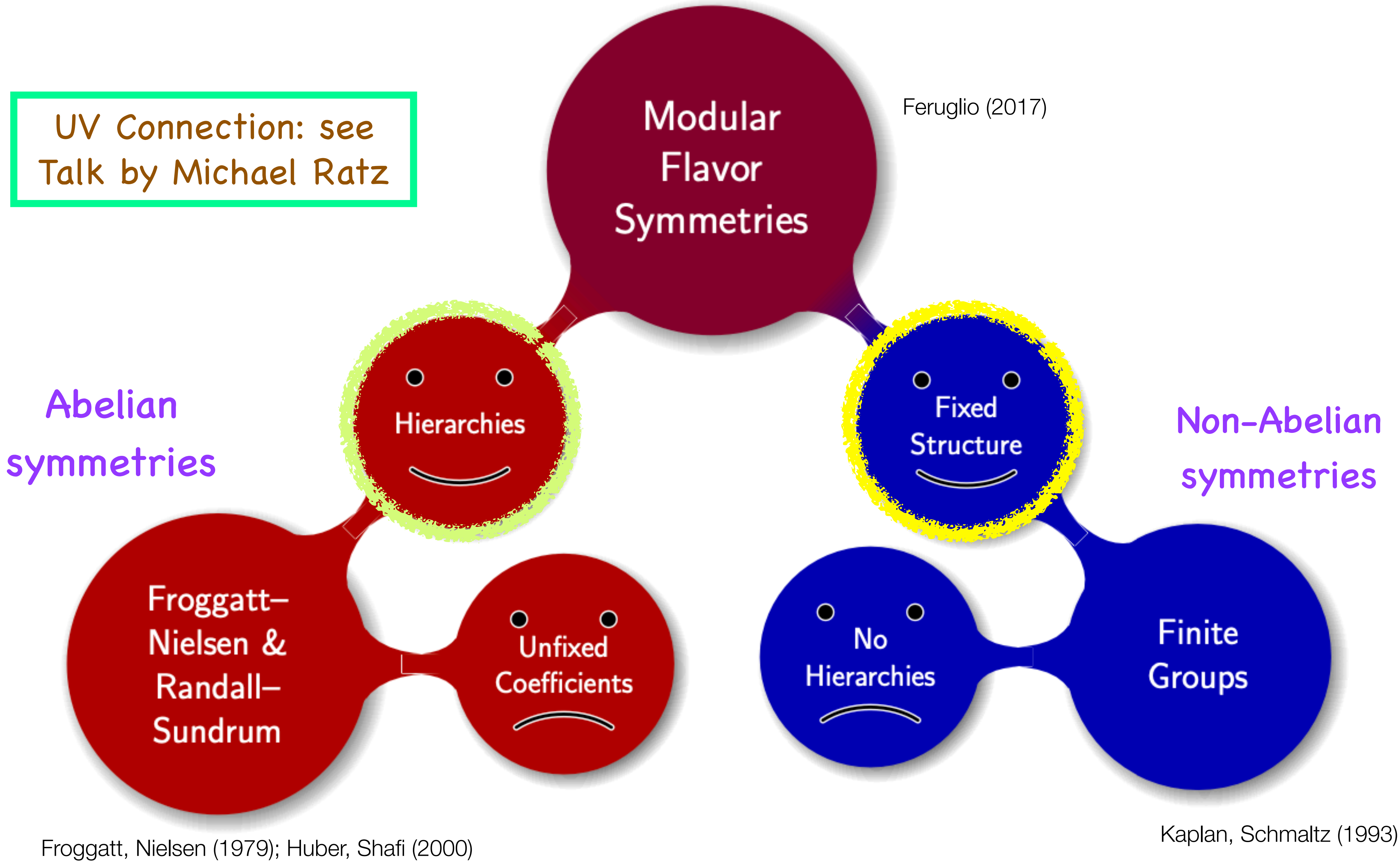
Mu-Chun Chen, University of California at Irvine



FLASY 2025, Rome, Italy, July 1, 2025

Theories of Flavor

UV Connection: see Talk by Michael Ratz



Modular Flavor Symmetries

Feruglio (2017)

- Imposing modular symmetry Γ on the Lagrangian:

$$\mathscr{L} \supset \sum Y_{i_1,\, i_2,\, \ldots\ldots\, i_n} \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}$$

$$\tau \xrightarrow{\gamma} \gamma\tau := \frac{a\tau + b}{c\tau + d},$$

$$\Phi_j \xrightarrow{\gamma} (c\tau + d)^{k_j} \rho_{r_j}(\gamma) \Phi_j\,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

k_i : integers

representation matrix of Γ_N

- Yukawa Couplings = Modular Forms at level "N" w/ weight "k"

$$f_i(\gamma\tau) = (c\tau + d)^{-k} [\rho_N(\gamma)]_{ij} f_j(\tau)$$

$k = k_{i_1} + k_{i_2} + \ldots + k_{i_n}$

representation matrix of Γ_N

Predictive Power of Modular Symmetries

- Modular Invariance: modular weights + representation assignments $\Rightarrow$ extremely constrained framework

- Much fewer number input parameters $\Rightarrow$ predictions consistent with data

- This Talk: Two generic predictions from properties of modular forms

  - Invariants $\Rightarrow$ robust sum rules among mixing parameters: modular invariance (even τ-independendt), RG invariant

    MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz, PLB852 (2024) 138600

  - Near-critical behavior of mass matrices $\Rightarrow$ Mass hierarchy at near critical points, determined by modular weights

    MCC, X.-Q. Li, X.-G. Liu, M. Ratz, 2506.23343

Predictive Power of Modular Symmetries

- Ingredients
  - Modular invariance
  - Holomorphy
  - Finiteness
- However, typical observables are not holomorphic, e.g.

$$\mathscr{W} = \frac{\mathscr{M}(\tau)}{2}\Phi^2$$

$$\mathscr{K} = \frac{1}{(-i\tau + i\bar{\tau})^{k_\Phi}}\overline{\Phi}\Phi$$

$$\Rightarrow$$

$$m_{physical} = m_{physical}(\bar{\tau}, \tau) = |\mathscr{M}(\tau)|(-i\tau + i\bar{\tau})^{k_\Phi}$$

Predictive Power of Modular Symmetries

- Ingredients
  - Modular invariance
  - Holomorphy
  - Finiteness
- However, typical observables are not holomorphic, e.g.

$$\mathscr{W} = \frac{\mathscr{M}(\tau)}{2}\Phi^2$$

$$\mathscr{K} = \frac{1}{(-i\tau + i\bar{\tau})^{k_\Phi}}\bar{\Phi}\Phi$$

$$m_{physical} = m_{physical}(\bar{\tau}, \tau) = |\mathscr{M}(\tau)|(-i\tau + i\bar{\tau})^{k_\Phi}$$

Are there observables fulfilling the three properties?

Holomorphic Observables

- Typical model

$$\mathscr{W}|_{lepton} = Y_e^{ij} L_i H_d E_j + \frac{1}{2}\kappa_{ij}(\tau) L_i H_u L_j H_u$$

In diagonal Y_e basis:

- Modular invariant holomorphic observables

$$I_{ij}(\tau) = \frac{\mathscr{M}_{ii}(\tau)\mathscr{M}_{jj}(\tau)}{(\mathscr{M}_{ij}(\tau))^2} = \frac{\kappa_{ii}\,\kappa_{jj}}{\kappa_{ij}^2} = \frac{m_{ii}(\tau,\bar{\tau})m_{jj}(\tau,\bar{\tau})}{(m_{ij}(\tau,\bar{\tau}))^2}$$

I_{ij} invariant under renormalization group

Chang, Kuo (2002)

Renormalization Group Invariants

Chang, Kuo (2002)

- In P-diagonal basis: $P = C_e Y_e^\dagger Y_e$

$$\frac{d}{dt}\kappa = \tilde{P}\kappa\tilde{Q}^T + \tilde{Q}\kappa\tilde{P}^T + \tilde{\alpha}\kappa,$$

$$\tilde{P} = \mathrm{diag}\left(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3\right), \quad \tilde{Q} = \mathrm{diag}\left(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3\right)$$

- At 1-loop: $\tilde{P} = \frac{1}{16\pi^2}P, \quad \tilde{Q} = I, \quad \tilde{\alpha} = \frac{1}{16\pi^2}\alpha$

$$\dot{\kappa}_{ij} = \kappa_{ij}\left(\tilde{P}_i\tilde{Q}_j + \tilde{P}_j\tilde{Q}_i + \tilde{\alpha}\right), \quad \frac{d}{dt}I_{ij} = 2\left(\tilde{P}_i - \tilde{P}_j\right)\left(\tilde{Q}_i - \tilde{Q}_j\right)I_{ij}$$

$\Rightarrow$ I_{ij} is RG invariant

A Toy Modular A_4 Model

Feruglio (2017)

- Lepton sector of the (supersymmetric) standard model

	(E_1^c, E_2^c, E_3^c)	L	H_d	H_u	φ
$\mathrm{SU(2)_L \times U(1)}_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$
Γ_3	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_d	k_u	k_φ

- Charged fermion masses obtained (with couplings to flavon) by adjusting 3 parameters
- Weinberg Operator:

$$\mathscr{W}_\nu = \frac{1}{\Lambda}\left[(H_u \cdot L)\, Y\, (H_u \cdot L)\right]_{\mathbf{1}}$$

- Kähler potential of leptons:

$$K_L = (-\mathrm{i}\tau + \mathrm{i}\overline{\tau})^{-1}\, (\overline{L}\, L)_{\mathbf{1}}$$

Invariants in Toy Modular A_4 Model

MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz (2024)

- Mass matrix in canonical basis:

$$m_\nu(\tau,\bar\tau) = (-\mathrm{i}\,\tau + \mathrm{i}\,\bar\tau)\frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_2(\tau) & -Y_3(\tau) \\ -Y_2(\tau) & 2Y_3(\tau) & -Y_1(\tau) \\ -Y_3(\tau) & -Y_1(\tau) & 2Y_2(\tau) \end{pmatrix} =: (-\mathrm{i}\,\tau + \mathrm{i}\,\bar\tau)v_u^2 \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} \\ \kappa_{12} & \kappa_{22} & \kappa_{23} \\ \kappa_{13} & \kappa_{23} & \kappa_{33} \end{pmatrix}$$

- Invariants

$$I_{12}(\tau) = 4\frac{Y_1(\tau)\,Y_3(\tau)}{\big(Y_2(\tau)\big)^2}, \qquad I_{13}(\tau) = 4\frac{Y_1(\tau)\,Y_2(\tau)}{\big(Y_3(\tau)\big)^2}, \qquad I_{23}(\tau) = 4\frac{Y_2(\tau)\,Y_3(\tau)}{\big(Y_1(\tau)\big)^2}$$

- Algebraic constraint $$Y_2^2 + 2Y_1Y_3 = 0$$

- Thus

$$I_{12}(\tau) = -2, \qquad I_{13}(\tau) = -2\Big(1 + \frac{1}{3}j_3(\tau)\Big)^3, \qquad I_{23}(\tau) = -\frac{32}{I_{13}(\tau)}$$

Invariants in Toy Modular A_4 Model

- Two interesting relations: **RG invariant, independent of τ**

$$I_{12}(\tau) = -2, \qquad I_{13}(\tau) I_{23}(\tau) = -32$$

- Invariants I_{ij} : functions of physical observables

$$\left(m_1, m_2, m_3, \theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{12}, \alpha_{23}\right)$$

⇒ **sum rules among physical observables:**

RG invariant, τ independent

MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz (2024)

Invariants in Toy Modular A_4 Model

- Invariants I_{ij} : functions of physical observables

$$\left(m_1,\, m_2,\, m_3,\, \theta_{12},\, \theta_{23},\, \theta_{13},\, \delta,\, \alpha_{12},\, \alpha_{23}\right)$$

$$I_{12} = -2$$

$$I_{12} = \frac{a_0\left[\widetilde{m}_1\left(\mathrm{e}^{\mathrm{i}\,\delta}c_{23}s_{12} + c_{12}s_{13}s_{23}\right)^2 + \widetilde{m}_2\left(\mathrm{e}^{\mathrm{i}\,\delta}c_{12}c_{23} - s_{12}s_{13}s_{23}\right)^2 + \mathrm{e}^{2\,\mathrm{i}\,\delta}m_3c_{13}^2s_{23}^2\right]}{c_{13}^2\left[\widetilde{m}_1c_{12}\left(\mathrm{e}^{\mathrm{i}\,\delta}c_{23}s_{12} + c_{12}s_{13}s_{23}\right) + \widetilde{m}_2s_{12}\left(s_{12}s_{13}s_{23} - \mathrm{e}^{\mathrm{i}\,\delta}c_{12}c_{23}\right) - \mathrm{e}^{2\,\mathrm{i}\,\delta}m_3s_{13}s_{23}\right]^2}$$

$$\widetilde{m}_1 := m_1\,\mathrm{e}^{\mathrm{i}\,\varphi_1}$$

$$\widetilde{m}_2 := m_2\,\mathrm{e}^{\mathrm{i}\,\varphi_2}.$$

$$a_0 := \left(\widetilde{m}_1c_{12}^2 + \widetilde{m}_2s_{12}^2\right)c_{13}^2 + \mathrm{e}^{2\,\mathrm{i}\,\delta}m_3s_{13}^2\,,$$

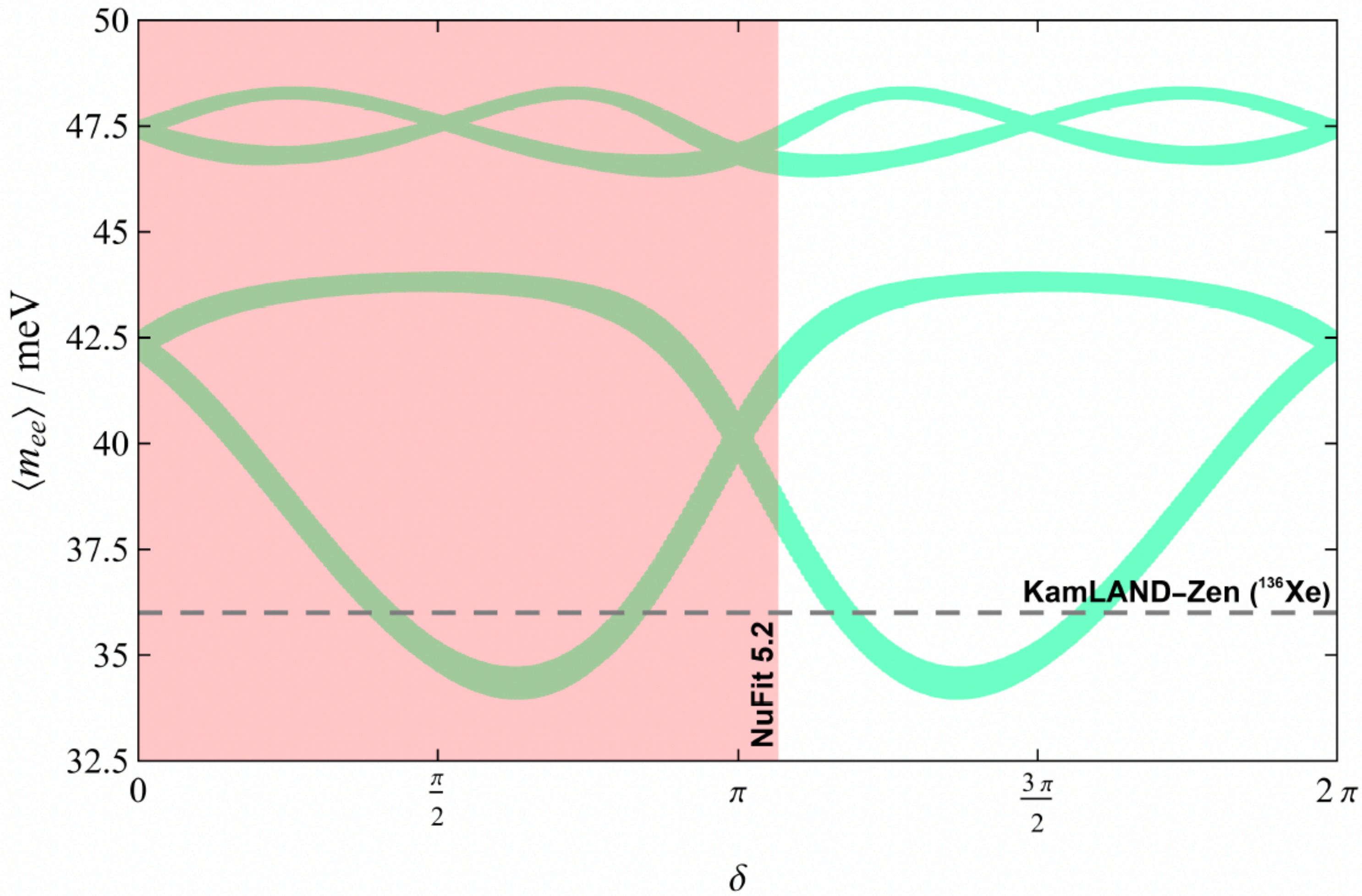
$$a_1 := \left[\left(\mathrm{e}^{2\,\mathrm{i}\,\delta}s_{12}^2 - c_{12}^2s_{13}^2\right)\sin(2\theta_{23}) - \mathrm{e}^{\mathrm{i}\,\delta}\cos(2\theta_{23})\sin(2\theta_{12})s_{13}\right],$$

$$a_2 := \left[\mathrm{e}^{\mathrm{i}\,\delta}\cos(2\theta_{23})\sin(2\theta_{12})s_{13} + \left(\mathrm{e}^{2\,\mathrm{i}\,\delta}c_{12}^2 - s_{12}^2s_{13}^2\right)\sin(2\theta_{23})\right].$$

MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz (2024)

Invariants in Toy Modular A_4 Model

- Predictions from $I_{12} = -2$ invariant, Inverted Ordering



MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz (2024)

Reality of I_{12} $\Rightarrow$

Constraints on CP phases:

Given δ $\Rightarrow$

Majorana phases α_{12}, α_{13}

Invariants in Toy Modular A_4 Model

$$I_{12} = \frac{a_0\left[\widetilde{m}_1\left(e^{i\delta}c_{23}s_{12}+c_{12}s_{13}s_{23}\right)^2+\widetilde{m}_2\left(e^{i\delta}c_{12}c_{23}-s_{12}s_{13}s_{23}\right)^2+e^{2i\delta}m_3c_{13}^2s_{23}^2\right]}{c_{13}^2\left[\widetilde{m}_1c_{12}\left(e^{i\delta}c_{23}s_{12}+c_{12}s_{13}s_{23}\right)+\widetilde{m}_2s_{12}\left(s_{12}s_{13}s_{23}-e^{i\delta}c_{12}c_{23}\right)-e^{2i\delta}m_3s_{13}s_{23}\right]^2}\,,$$

$$I_{13} = \frac{a_0\left[\widetilde{m}_1\left(c_{12}c_{23}s_{13}-e^{i\delta}s_{12}s_{23}\right)^2+\widetilde{m}_2\left(c_{23}s_{12}s_{13}+e^{i\delta}c_{12}s_{23}\right)^2+e^{2i\delta}m_3c_{13}^2c_{23}^2\right]}{c_{13}^2\left[\widetilde{m}_1c_{12}\left(c_{12}c_{23}s_{13}-e^{i\delta}s_{12}s_{23}\right)+\widetilde{m}_2s_{12}\left(c_{23}s_{12}s_{13}+e^{i\delta}c_{12}s_{23}\right)-e^{2i\delta}m_3c_{23}s_{13}\right]^2}\,,$$

$$I_{23} = \left[e^{2i\delta}m_3c_{13}^2s_{23}^2+\widetilde{m}_1\left(e^{i\delta}c_{23}s_{12}+c_{12}s_{13}s_{23}\right)^2+\widetilde{m}_2\left(e^{i\delta}c_{12}c_{23}-s_{12}s_{13}s_{23}\right)^2\right]$$
$$\times\frac{4\left[e^{2i\delta}m_3c_{13}^2c_{23}^2+\widetilde{m}_2\left(c_{23}s_{12}s_{13}+e^{i\delta}c_{12}s_{23}\right)^2+\widetilde{m}_1\left(c_{12}c_{23}s_{13}-e^{i\delta}s_{12}s_{23}\right)^2\right]}{\left[\widetilde{m}_1a_1+\widetilde{m}_2a_2-e^{2i\delta}m_3\sin(2\theta_{23})c_{13}^2\right]^2}\,,$$

$$\widetilde{m}_1 := m_1\,e^{i\varphi_1}$$

$$\widetilde{m}_2 := m_2\,e^{i\varphi_2}.$$

$$a_0 := \left(\widetilde{m}_1c_{12}^2+\widetilde{m}_2s_{12}^2\right)c_{13}^2+e^{2i\delta}m_3s_{13}^2\,,$$

$$a_1 := \left[\left(e^{2i\delta}s_{12}^2-c_{12}^2s_{13}^2\right)\sin(2\theta_{23})-e^{i\delta}\cos(2\theta_{23})\sin(2\theta_{12})s_{13}\right]\,,$$

$$a_2 := \left[e^{i\delta}\cos(2\theta_{23})\sin(2\theta_{12})s_{13}+\left(e^{2i\delta}c_{12}^2-s_{12}^2s_{13}^2\right)\sin(2\theta_{23})\right]\,.$$

Invariants in Toy Modular A_4 Model

- No simultaneous solution for I_{ij} that is consistent with data
  - Agree with previous analysis by scanning parameter space (i.e. toy modular A4 model does not fit all data)
  - Here, arrived at conclusion without the need to scan

MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz (2024)

Invariants in Toy Modular A_5 Model

MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz (2024)

- In a model based on modular A5:

$$I_{12} = \frac{2\sqrt{6}}{3}\frac{Y_1(\tau)Y_4(\tau)}{Y_5^2(\tau)}\,, \qquad I_{13} = \frac{2\sqrt{6}}{3}\frac{Y_1(\tau)Y_3(\tau)}{Y_2^2(\tau)}\,, \qquad I_{23} = 6\frac{Y_3(\tau)Y_4(\tau)}{Y_1^2(\tau)}$$

- Algebraic relations among the invariants

$$\begin{aligned} 0 &= 4 + 18I_{12} + 18I_{13} + 9I_{12}I_{13} + I_{12}I_{13}I_{23}\,, \\ 0 &= 8 + 12I_{12} - 108I_{12}^2 + 12I_{13} + 414I_{12}I_{13} + 108I_{12}^2I_{13} - 108I_{13}^2 + 108I_{12}I_{13}^2 + 81I_{12}^2I_{13}^2 \\ &\quad - I_{12}^2I_{23} - I_{13}^2I_{23}\,. \end{aligned}$$

- Exchange symmetry: $I_{12} \leftrightarrow I_{13} \;\Rightarrow\; \mu - \tau$ symmetry built in

Determinants & VVMFs

- Determinants of mass matrices are 1-dim VVMFs

$$\det M(\tau) \xrightarrow{\gamma} (c\,\tau + d)^{\sum_i k_{\varphi_i^{\mathcal{C}}} + k_{\varphi_i}} \left(\det \rho^{\mathcal{C}} \rho\right)^* \det M(\tau)$$

X.-G. Liu, G.-J. Ding (2022)

- 1-dim VVMFs form a finite-dimensional linear space generated by η^2, E_4, E_6

$$\mathcal{M}_k^{(1\mathrm{D})}(\mathrm{SL}(2,\mathbb{Z})) = \sum_{\substack{a+b+c=k \\ a,b,c\geq 0}} \alpha_{abc}\, \eta^{2c}\, {E_4}^a\, {E_6}^b$$

η : Dedekind η function

$E_{4,\,6}$: Eisenstein series

- We know a lot about the determinants from theory of modular forms

Location of Zeros

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

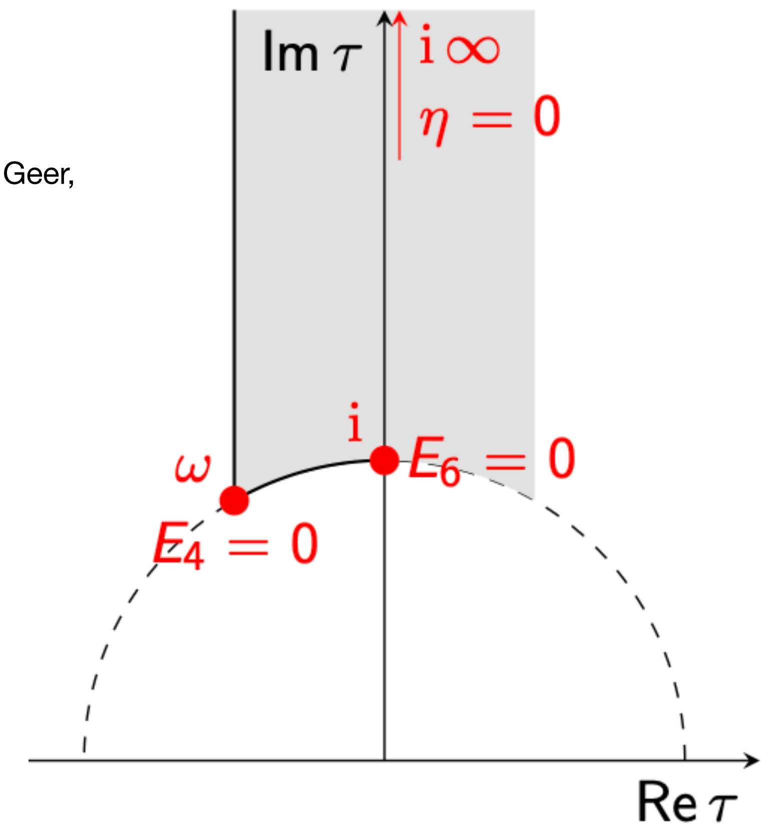
c.f. Feruglio, Gherardi, Romanino, Titov (2021); Novichkov, Penedo, Petcov (2021); Feruglio (2023)

- Close to zeros of det M $\Rightarrow$ hierarchical masses
- Orders of zeros constrained by Valence Formula

Prop 2 in Bruinier, van der Geer, Harder, Zagier (2008)

$$\frac{1}{12}n_{\mathrm{i}\infty} + \frac{1}{2}n_{\mathrm{i}} + \frac{1}{3}n_{\omega} + \sum_{\substack{\tau_0 \neq \\ \mathrm{i},\omega,\mathrm{i}\infty}} n_{\tau_0} = \frac{k}{12}$$

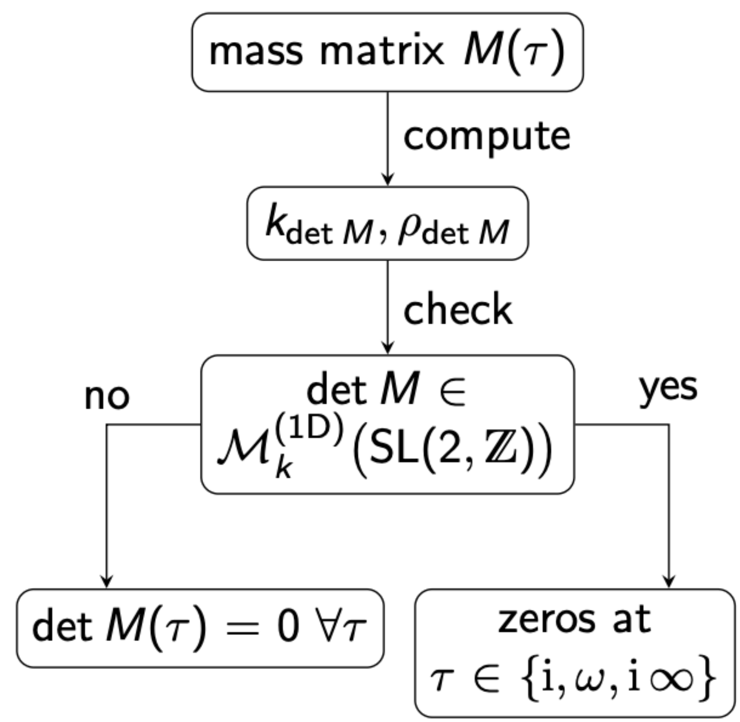
- Absence of cancellations of different terms for $k_{\det M} < 12$, the zeros can only be at $\tau = \omega,\, i,\, i\infty$



Possible Representations of Det M

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

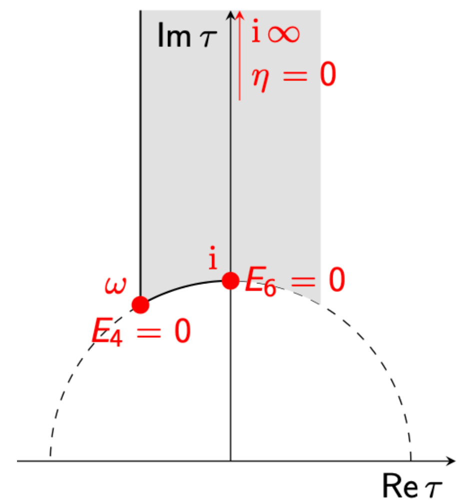
- det M can be labeled by a $\mathbb{Z}_{12}$ phase: 1_p w/ $p \in \{0, \ldots, 11\}$
  - Locations and orders of zeros are determined
- A given determinant:
  - May or may not fit into $$\mathcal{M}_k^{(1D)}(\mathrm{SL}(2,\mathbb{Z}))$$
  - If not, determinant = 0



Determinants of Weights up to 6

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

$k_{\det M}$	Modular Form	Zeros	$(n_{\mathrm{i}\infty}, n_{\mathrm{i}}, n_\omega)$	irrep
1	η^2	$\mathrm{i}\infty$	$(1,0,0)$	$\mathbf{1}_1$
2	η^4	$\mathrm{i}\infty$	$(2,0,0)$	$\mathbf{1}_2$
3	η^6	$\mathrm{i}\infty$	$(3,0,0)$	$\mathbf{1}_3$
4	E_4	ω	$(0,0,1)$	$\mathbf{1}_0$
	η^8	$\mathrm{i}\infty$	$(4,0,0)$	$\mathbf{1}_4$
5	$\eta^2 E_4$	ω and $\mathrm{i}\infty$	$(1,0,1)$	$\mathbf{1}_1$
	η^{10}	$\mathrm{i}\infty$	$(5,0,0)$	$\mathbf{1}_5$
6	E_6	i	$(0,1,0)$	$\mathbf{1}_0$
	$\eta^4 E_4$	ω and $\mathrm{i}\infty$	$(2,0,1)$	$\mathbf{1}_2$
	η^{12}	$\mathrm{i}\infty$	$(6,0,0)$	$\mathbf{1}_6$



$$\det M(\tau) \xmapsto{\gamma}$$

$$(c\tau + d)^{\sum_i k_{\varphi_i^{\mathcal{C}}} + k_{\varphi_i}} \left(\det \rho^{\mathcal{C}} \rho\right)^* \det M(\tau)$$

$$\frac{1}{12} n_{\mathrm{i}\infty} + \frac{1}{2} n_{\mathrm{i}} + \frac{1}{3} n_\omega + \sum_{\substack{\tau_0 \neq \\ \mathrm{i},\omega,\mathrm{i}\infty}} n_{\tau_0} = \frac{k}{12}$$

Determinants of Weights 7–9

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

$k_{\det M}$	Modular Form	Zeros	$(n_{\mathrm{i}\infty}, n_{\mathrm{i}}, n_{\omega})$	irrep
7	$\eta^6 E_4$	ω and $\mathrm{i}\infty$	$(3,0,1)$	$\mathbf{1}_3$
	$\eta^2 E_6$	i and $\mathrm{i}\infty$	$(1,1,0)$	$\mathbf{1}_1$
	η^{14}	$\mathrm{i}\infty$	$(7,0,0)$	$\mathbf{1}_7$
8	E_4^2	ω	$(0,0,2)$	$\mathbf{1}_0$
	$\eta^4 E_6$	i and $\mathrm{i}\infty$	$(2,1,0)$	$\mathbf{1}_2$
	$\eta^8 E_4$	ω and $\mathrm{i}\infty$	$(4,0,1)$	$\mathbf{1}_4$
	η^{16}	$\mathrm{i}\infty$	$(8,0,0)$	$\mathbf{1}_8$
9	$\eta^{10} E_4$	ω and $\mathrm{i}\infty$	$(5,0,1)$	$\mathbf{1}_5$
	$\eta^6 E_6$	i and $\mathrm{i}\infty$	$(3,1,0)$	$\mathbf{1}_3$
	$\eta^2 E_4^2$	ω and $\mathrm{i}\infty$	$(1,0,2)$	$\mathbf{1}_1$
	η^{18}	$\mathrm{i}\infty$	$(9,0,0)$	$\mathbf{1}_9$

Determinants of Weights 10, 11

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

$k_{\det M}$	Modular Form	Zeros	$(n_{\mathrm{i}\infty}, n_{\mathrm{i}}, n_\omega)$	irrep
10	$E_4 E_6$	ω and i	$(0,1,1)$	$\mathbf{1}_0$
	$\eta^4 E_4^2$	ω and $\mathrm{i}\infty$	$(2,0,2)$	$\mathbf{1}_2$
	$\eta^8 E_6$	i and $\mathrm{i}\infty$	$(4,1,0)$	$\mathbf{1}_4$
	$\eta^{12} E_4$	ω and $\mathrm{i}\infty$	$(6,0,1)$	$\mathbf{1}_6$
	η^{20}	$\mathrm{i}\infty$	$(10,0,0)$	$\mathbf{1}_{10}$
11	$\eta^2 E_4 E_6$	ω, i and $\mathrm{i}\infty$	$(1,1,1)$	$\mathbf{1}_1$
	$\eta^6 E_4^2$	ω and $\mathrm{i}\infty$	$(3,0,2)$	$\mathbf{1}_3$
	$\eta^{10} E_6$	i and $\mathrm{i}\infty$	$(5,1,0)$	$\mathbf{1}_5$
	$\eta^{14} E_4$	ω and $\mathrm{i}\infty$	$(7,0,1)$	$\mathbf{1}_7$
	η^{22}	$\mathrm{i}\infty$	$(11,0,0)$	$\mathbf{1}_{11}$

Orders of Zeros and Mass Hierarchy

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

- Convincing explanation of hierarchical masses by modular flavor symmetries ⇒ requires zeros of high orders
- At $\tau = i$ or ω : zeros at most orders 1 or 2
- At $\tau = i\infty$: only occur for nontrivial 1-plets: $1_{p>0}$
- Certain finite modular groups do not have appropriate $1_{p>0}$ -plets
- Perfect groups have no nontrivial 1-plets, e.g.
    - A_5 : hierarchy can only be obtained by balancing coefficients

MCC, Fallbacher, Ratz, Trautner, Vaudrevange (2015)

Orders of Zeros and Mass Hierarchy

MCC, X.-Q. Li, X.-G. Liu, M. Ratz (2025)

- Convincing explanation of hierarchical masses by modular flavor symmetries ⇒ requires zeros of high orders

- At $\tau = i$ or ω : zeros at most orders 1 or 2

- At $\tau = i\infty$: only occur for nontrivial 1-plets: $1_{p>0}$

- Certain finite modular groups do not have appropriate $1_{p>0}$-plets

- Perfect groups have no nontrivial 1-plets, e.g.

  - A_5 : hierarchy can only be obtained by balancing coefficients

- Bottom line: given finite modular group + modular weights of matter fields ⇒ can immediately tell whether mass hierarchy can emerge as properties of modular forms

Acknowledgements

Xueqi Li
(UCI Grad)

Xiang-Gan Liu
(UCI PD)

Omar Medina
(IFIC Valencia Grad; former UCI visiting student)

Michael Ratz
(UCI)

MCC, X.-Q. Li, X.-G. Liu, O. Medina, M. Ratz, PLB852 (2024) 138600

MCC, X.-Q. Li, X.-G. Liu, M. Ratz, 2506.23343

Conclusions

- Fundamental origin of fermion mass & mixing patterns still unknown
- Uniqueness of Neutrino masses exciting opportunities to explore BSM Physics
- Modular Flavor Symmetries:
  - Significant reduction of the number of parameters
  - Two generic predictions:
    - Modular Invariant (and even τ-independent), RG Invariants:
      - Robust sum rules among physical observables
      - Independent of renormalization scale, model parameters
    - Near-critical behavior of mass matrices:
      - Mass hierarchy can arise due to properties of modular forms
      - Expansion parameter: deviation from fixed point
      - Classification of symmetries based on modular weights
      - Preference for $\tau = i\infty$

NEUTRINO 2026

June 22-26, 2026, University of California, Irvine, U.S.A.

About Irvine, California

a metropolitan city located at about 40 miles (64 km) south of Los Angeles, 70 miles (112 km) north of San Diego, on the beautiful coast of the Pacific Ocean with 11,000 ft (3500 m) towering San Bernadino Mountains in its backdrop.

70th Anniversary of Neutrino Discovery

by George Cowan and Fred Reines. Fred Reines (1995 Nobel Laureate) was the founding Dean of School of Physical Sciences at UC Irvine.

Contact Us

co-Chairs: Mu-Chun Chen, Michael Smy | E-Mail: neutrino2026uci@gmail.com