FLASY 2025, the 11th Workshop on Flavour Symmetries and Consequences in Accelerators and Cosmology, 30 June 2025 to 4 July 2025 Rome

The Modular Blueprint of Flavor: Quarks and Leptons Entangled

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Finanziato dall'Unione europea
NextGenerationEU
Ministero dell'Università e della Ricerca
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The modular group acts on the modulus as follows $\tau \to \gamma\tau = \dfrac{a\tau + b}{c\tau + d}$, $\mathrm{Im}(\tau) > 0$ where $\gamma \in SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, a, b, c, d \in \mathbb{Z}, ac - bd = 1 \right\}$

Consider the “Principal congruence group of level N”

$$\Gamma(N) = \left\{ \gamma \in SL(2,\mathbb{Z}) \,|\, \gamma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

→

The Quotient group gives the “Finite Modular Group”

$\Gamma_N \equiv SL(2,\mathbb{Z})/\pm\Gamma(N)$ or $\Gamma'_N \equiv SL(2,\mathbb{Z})/\Gamma(N)$

N	2	3	4	5
Γ_N	S_3	A_4	S_4	A_5
Γ'_N	S_3	$A'_4 \equiv T'$	S'_4	A'_5

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Fields transform as $\phi(\tau) \to (c\tau + d)^{-k_\phi}\, \rho_\phi(\gamma)\, \phi(\tau)$

Weights

Irreps of Γ_N (Γ'_N)

Yukawa couplings are in multiplets transforming as $Y(\tau) \to (c\tau + d)^{k_Y}\, \rho_Y(\gamma)\, Y(\tau)$

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Construct the Yukawa sector of the Lagrangian by introducing a set of real or complex model parameters

Derive the model's predictions for the observables and compare them with experimental data to constrain those parameters

Experimental inputs

For instance in the following examples

Quark sector

Observable	Central value	$\pm 1\sigma$
m_u/m_c (10^{-3})	1.93	0.60
m_c/m_t (10^{-3})	2.82	0.12
m_d/m_s (10^{-2})	5.05	0.62
m_s/m_b (10^{-2})	1.82	0.10
m_t [GeV]	87.5	2.1
m_b [GeV]	0.97	0.01

Observable	Central value	$\pm 1\sigma$
$\sin^2\theta^q_{12}$ (10^{-2})	5.08	0.03
$\sin^2\theta^q_{13}$ (10^{-5})	1.22	0.09
$\sin^2\theta^q_{23}$ (10^{-3})	1.61	0.05
$\delta_{\mathrm{CKM}}/\pi$	0.385	0.017

Values of quark masses and mixings renormalized around 2×10^{16} GeV, assuming the supersymmetry breaking scale M_{SUSY} = 10 TeV and tan β = 10

Lepton sector

Observable	Central value	$\pm 1\sigma$
m_e/m_μ (10^{-3})	4.74	0.04
m_μ/m_τ (10^{-2})	5.88	0.05
$\delta m^2/\lvert\Delta m^2\rvert$ (10^{-2}) (NO)	2.95	0.06
$\delta m^2/\lvert\Delta m^2\rvert$ (10^{-2}) (IO)	2.99	0.07
m_τ [GeV]	1.293	0.007
δm^2 $(10^{-5}\,\mathrm{eV}^2)$	7.37	0.17
$\lvert\Delta m^2\rvert$ $(10^{-3}\,\mathrm{eV}^2)$ (NO)	2.495	0.020
$\lvert\Delta m^2\rvert$ $(10^{-3}\,\mathrm{eV}^2)$ (IO)	2.465	0.020

We choose NO, currently preferred by global analysis

Observable	Central value	$\pm 1\sigma$
$\sin^2\theta^\ell_{12}$ (10^{-1})	3.03	0.14
$\sin^2\theta^\ell_{13}$ (10^{-2})	2.23	0.05
$\sin^2\theta^\ell_{23}$ (10^{-1}) (NO)	4.73	0.24
$\sin^2\theta^\ell_{23}$ (10^{-1}) (IO)	5.45	0.23
$\delta_{\mathrm{PMNS}}/\pi$ (NO)	1.20	0.22
$\delta_{\mathrm{PMNS}}/\pi$ (IO)	1.48	0.12

We prefer to use $\sin^2\theta^\ell_{23} = 0.5$ with 1σ error given by 1/6 of the 3σ allowed region because of the octant ambiguity

We prefer not to use δ_{CP} because of the large uncertainty

effects on the neutrino mass ratios and mixing angles known to be small to a good approximation

S. Antusch et al., J. High Energy Phys. 03 (2005) 024

S. Antusch and V. Maurer, J. High Energy Phys. 11 (2013) 115.

Parameters and Observables

Parameters: N_{par}

The modulus τ

$\mathbf{P}_q^{up}$ $\mathbf{P}_q^{down}$

To construct the Yukawa matrices (Y^u, Y^d)

$\mathbf{P}^{charged}_{leptons}$ $\mathbf{P}_\nu$

To construct the Yukawa matrices (Y^ℓ, C^ν)

Observables: N_{obs}

$(\sin^2\theta_{12}^q, \sin^2\theta_{13}^q, \sin^2\theta_{23}^q, \delta_{CP}^q)$

$(m_u, m_d, m_c, m_s, m_t, m_b)$

$(\sin^2\theta_{12}^\ell, \sin^2\theta_{13}^\ell, \sin^2\theta_{23}^\ell, \Delta m^2, \delta m^2)$

(m_e, m_μ, m_τ)

Unknowns ⟶ $(\delta_{CP}^\ell, m_1, m_{\beta\beta}, m_\beta)$

To test the model

$$\chi^2_{\text{leptons}}(\mathbf{P}_{\text{leptons}}) = \sum_{i=1}^{8}\left(\frac{\mathrm{O}_i^{\text{theo}}(\mathbf{P}_{\text{leptons}}) - \mathrm{O}_i^{\text{exp}}}{\sigma_i^{\text{exp}}}\right)^2$$

$$\chi^2_{\text{quarks}}(\mathbf{P}_{\text{quarks}}) = \sum_{i=9}^{18}\left(\frac{\mathrm{O}_i^{\text{theo}}(\mathbf{P}_{\text{quarks}}) - \mathrm{O}_i^{\text{exp}}}{\sigma_i^{\text{exp}}}\right)^2$$

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Statistical analysis well defined, the model is over-determined and the χ^2statistic, its associated p-value, and all subsequent confidence intervals can be meaningfully computed.

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The χ^2 distribution undefined, no meaningful p-value

Model **over-parameterized**: it can always pass through all the data points but likely does not capture the underlying physics, similar to fitting N points with an N–1 degree polynomial: it can always fit perfectly, but the model has no predictive power

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It is not hard to find in the literature sentences such as “the model is fine because all the observables are within their 3σ experimentally allowed regions” or where only the $\Delta\chi^2$ is mentioned without also looking at the minimum χ^2_{min} or where the number of d.o.f. is completely wrong

For instance

$$\left[\begin{array}{l} N_{\mathrm{obs}} = 18 \\ N_{\mathrm{par}} = 17 \end{array}\right. \longrightarrow N_{\mathrm{dof}} = 1$$

p-value can be calculated depending on the χ^2_{min}

- $\Delta\chi^2$ = 1 corresponds to the **68.27% confidence region (1σ)**
- $\Delta\chi^2$ = 4 corresponds to the **95.45% confidence region (2σ)**
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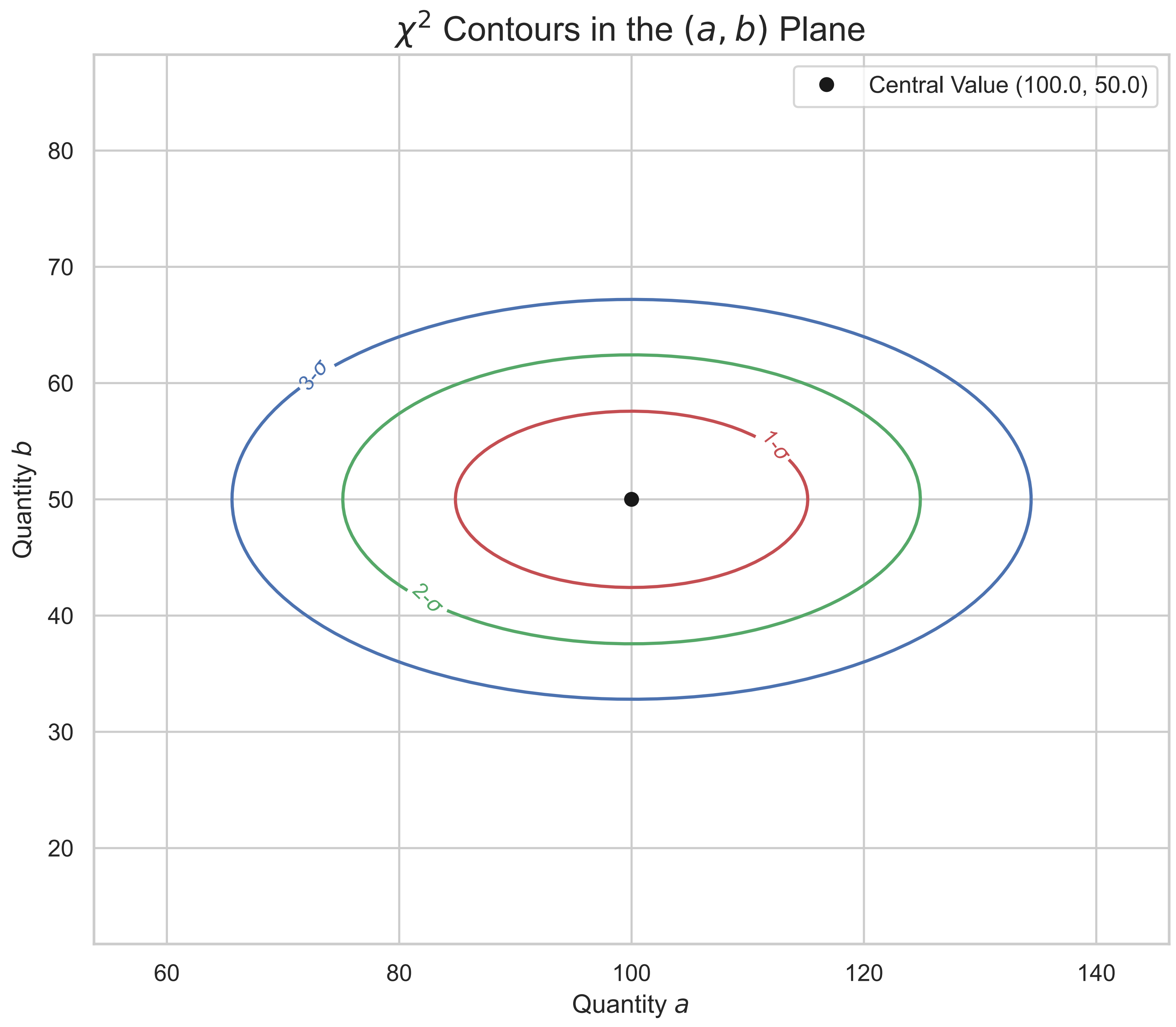
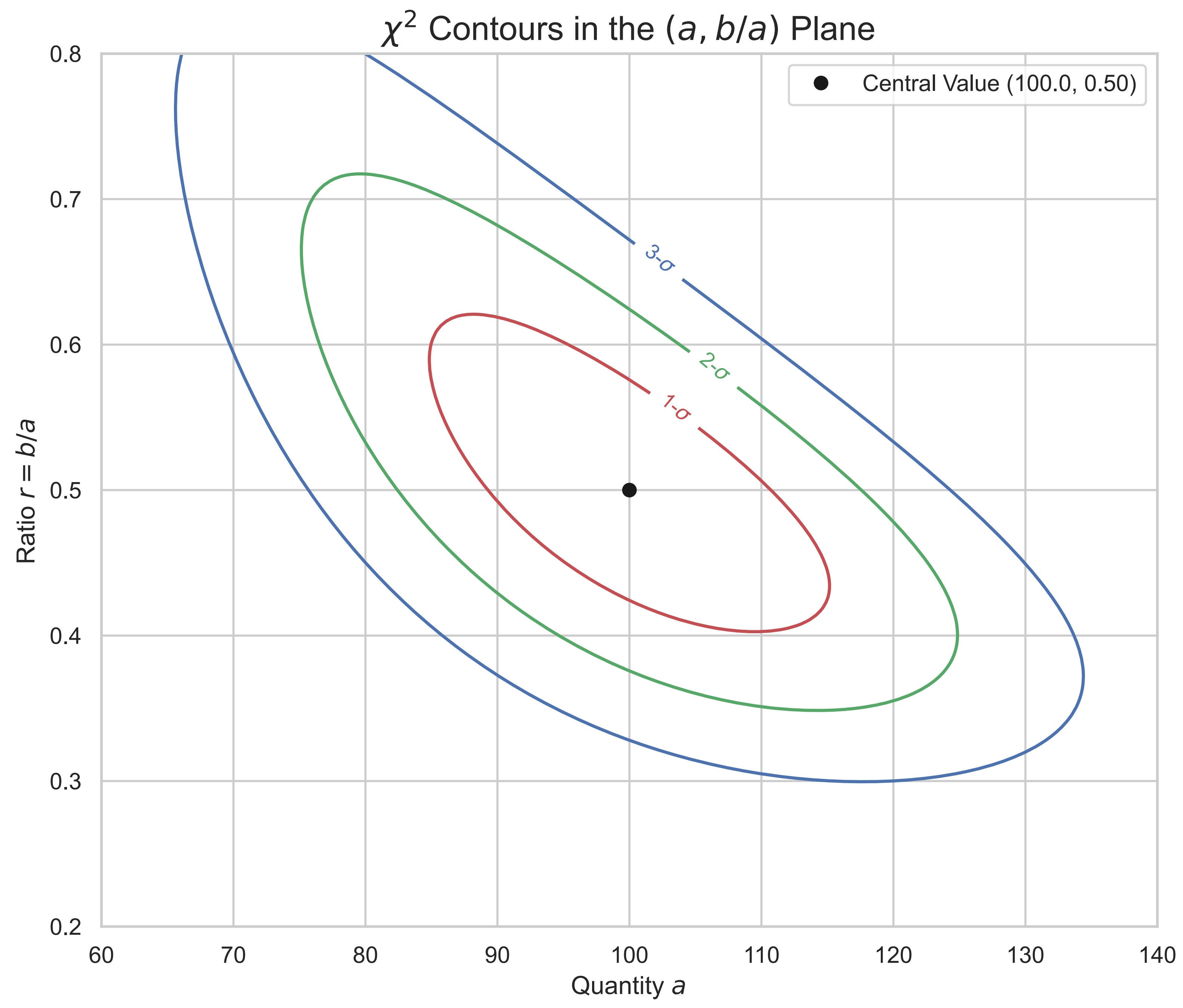

χ^2 Contours in the (a, b) Plane


χ^2 Contours in the $(a, b/a)$ Plane

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Quark-lepton correlations (in a non-GUT framework) are a distinctive feature of models with modular symmetries and can help help clarify how the model works

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2O - the binary octahedral group	14	$N_{\mathrm{dof}} > 0$

S_3 modular invariant model in a N = 1 rigid SUSY

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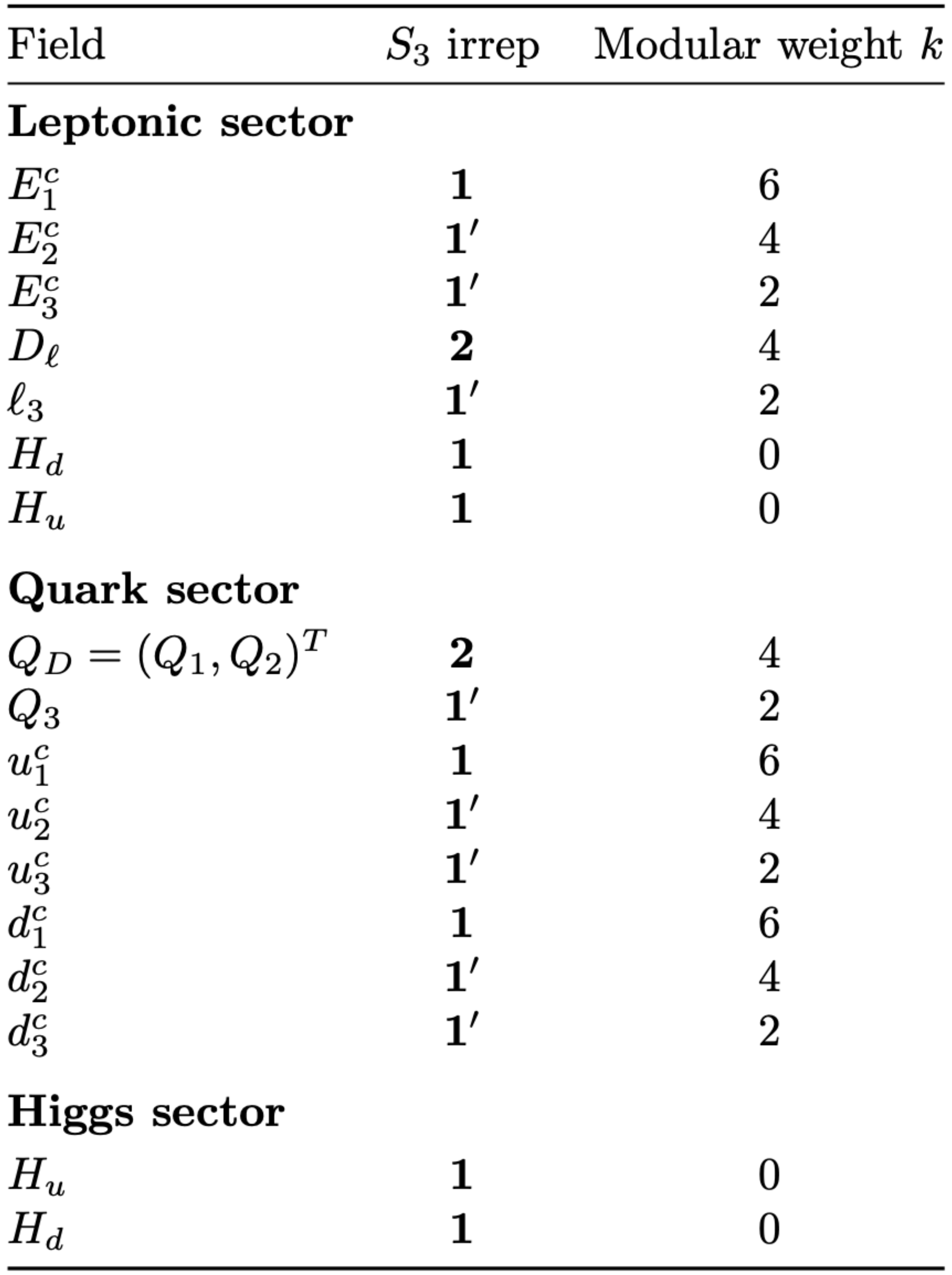

Field	S_3 irrep	Modular weight k
Leptonic sector		
E_1^c	$\mathbf{1}$	6
E_2^c	$\mathbf{1}'$	4
E_3^c	$\mathbf{1}'$	2
D_ℓ	$\mathbf{2}$	4
ℓ_3	$\mathbf{1}'$	2
H_d	$\mathbf{1}$	0
H_u	$\mathbf{1}$	0
Quark sector		
$Q_D = (Q_1, Q_2)^T$	$\mathbf{2}$	4
Q_3	$\mathbf{1}'$	2
u_1^c	$\mathbf{1}$	6
u_2^c	$\mathbf{1}'$	4
u_3^c	$\mathbf{1}'$	2
d_1^c	$\mathbf{1}$	6
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d_3^c	$\mathbf{1}'$	2
Higgs sector		
H_u	$\mathbf{1}$	0
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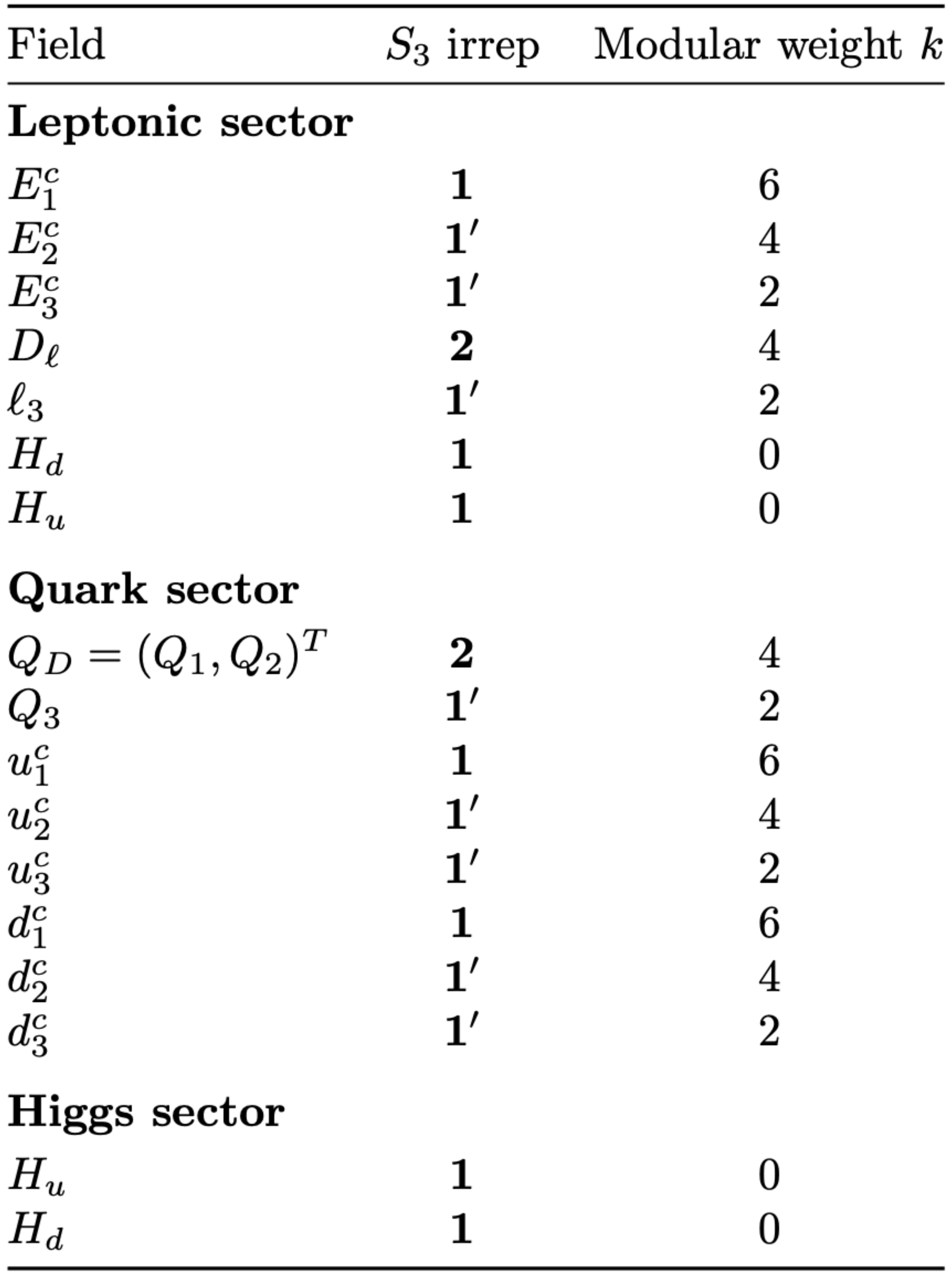
S_3 modular invariant model in a N = 1 rigid SUSY



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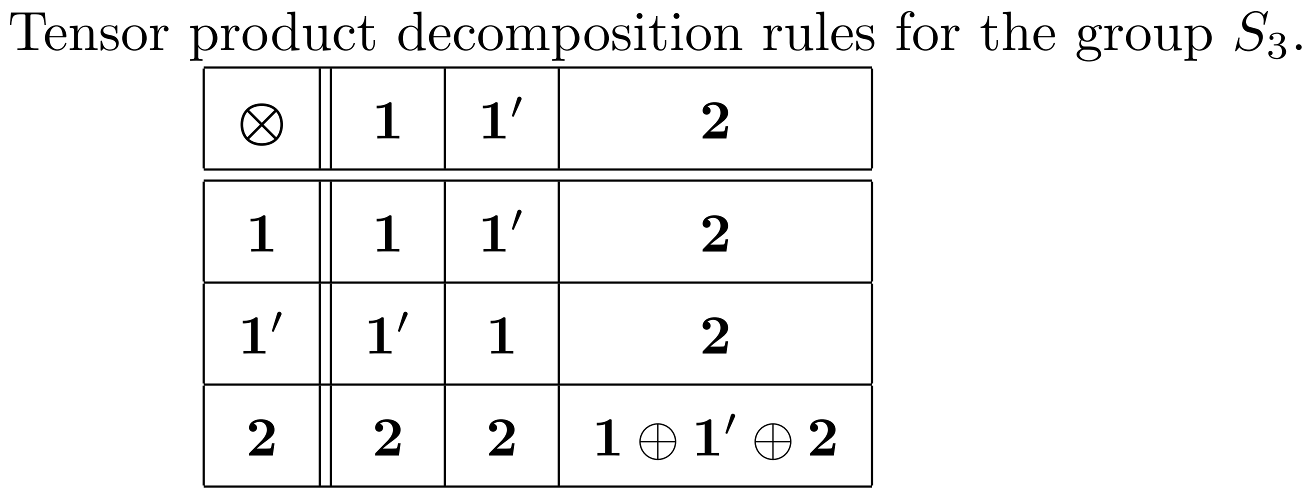

Tensor product decomposition rules for the group S_3.

$\otimes$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$
$\mathbf{1'}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{2}$
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Parameters

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$(\alpha_u, \tilde{\alpha}_u, \beta_u, \tilde{\beta}_u, \alpha_{uD}, \beta_{uD}, \gamma_{uD})$	7
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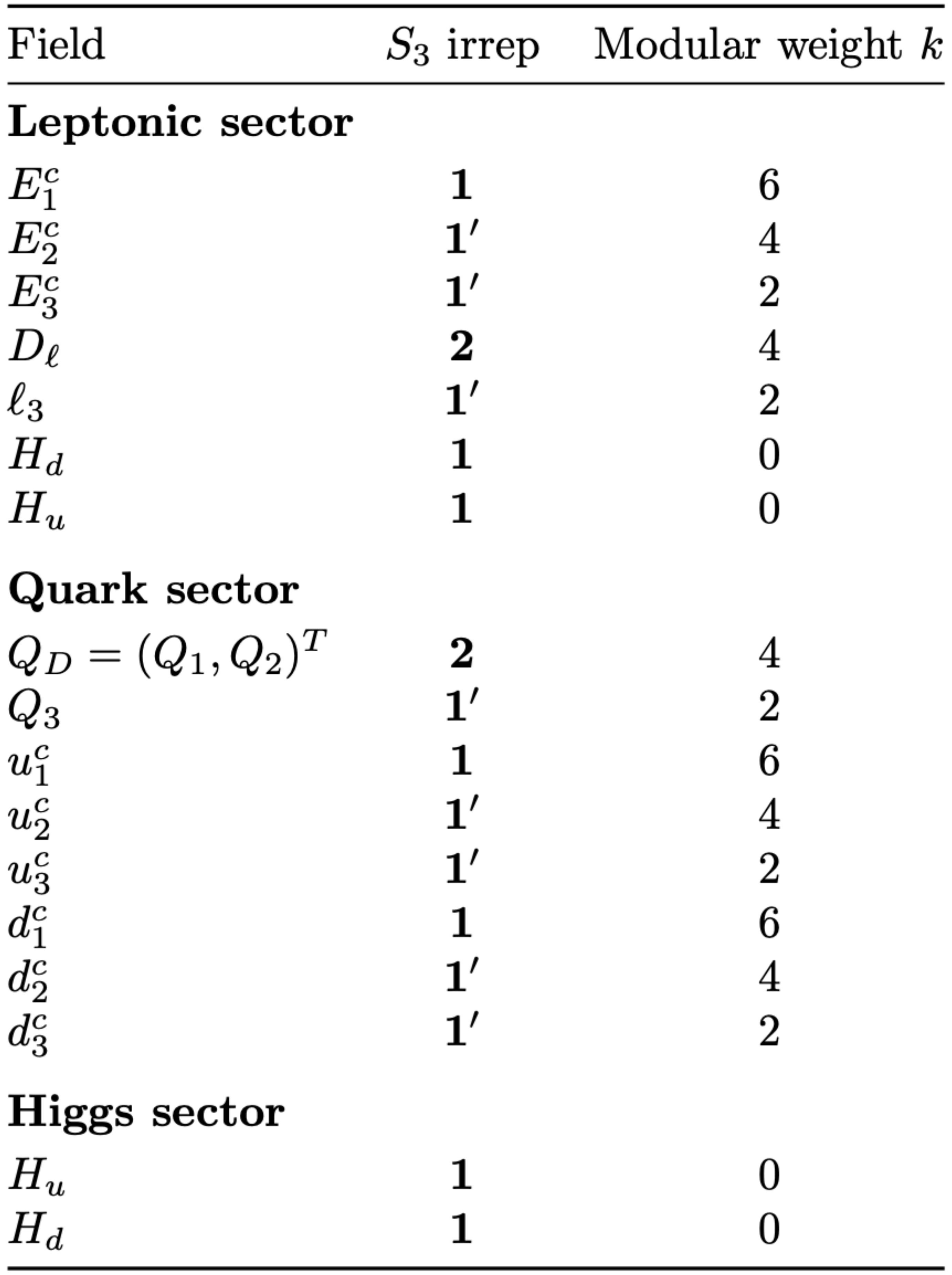

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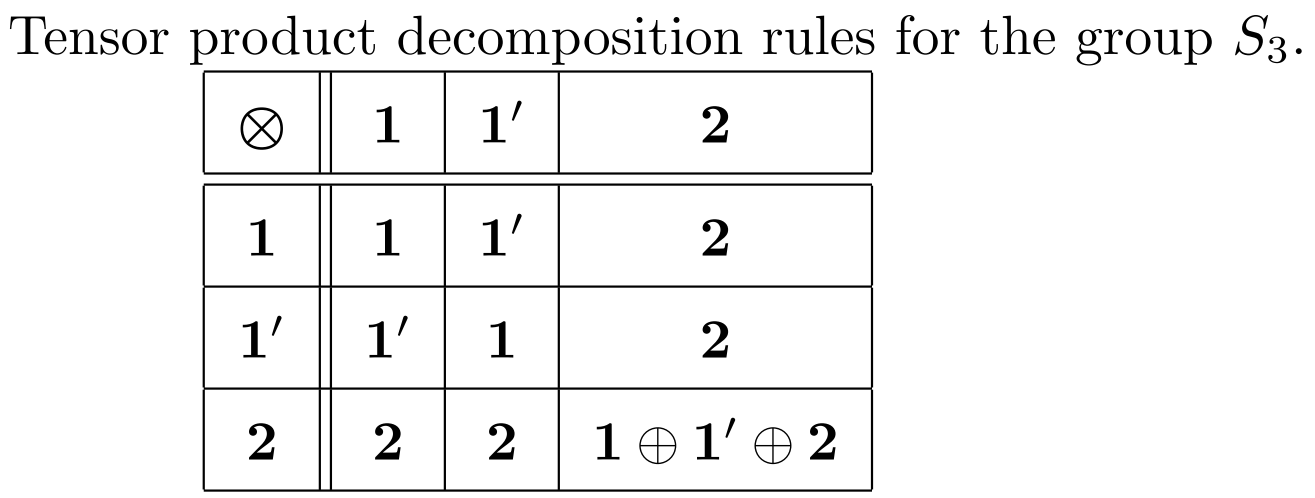

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$Q_D = (Q_1, Q_2)^T$	$\mathbf{2}$	4
Q_3	$\mathbf{1'}$	2
u_1^c	$\mathbf{1}$	6
u_2^c	$\mathbf{1'}$	4
u_3^c	$\mathbf{1'}$	2
d_1^c	$\mathbf{1}$	6
d_2^c	$\mathbf{1'}$	4
d_3^c	$\mathbf{1'}$	2
Higgs sector		
H_u	$\mathbf{1}$	0
H_d	$\mathbf{1}$	0

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Tensor product decomposition rules for the group S_3.

$\otimes$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$
$\mathbf{1'}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{2}$
$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{2}$

Parameters

$\tau = \mathrm{Re}(\tau) + \mathrm{Im}(\tau)$	2
$(\alpha_u, \tilde{\alpha}_u, \beta_u, \tilde{\beta}_u, \alpha_{uD}, \beta_{uD}, \gamma_{uD})$	7
$(\alpha_d, \tilde{\alpha}_d, \beta_d, \tilde{\beta}_d, \alpha_{dD}, \beta_{dD}, \gamma_{dD})$	7
$(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}, \alpha_D, \beta_D, \gamma_D)$	7
$(g_{8s}, g_8', \tilde{g}_8', g_6', g_w')$	5

$$N_{\mathrm{par}} = 26 + 2 = 28$$

$$N_{\mathrm{dof}} = N_{\mathrm{obs}} - N_{\mathrm{par}} = 18 - 28 = -10 \quad !!$$

S_3 modular invariant model in a N = 1 rigid SUSY

Field	S_3 irrep	Modular weight k
Leptonic sector		
E_1^c	$\mathbf{1}$	6
E_2^c	$\mathbf{1}'$	4
E_3^c	$\mathbf{1}'$	2
D_ℓ	$\mathbf{2}$	4
ℓ_3	$\mathbf{1}'$	2
H_d	$\mathbf{1}$	0
H_u	$\mathbf{1}$	0
Quark sector		
$Q_D = (Q_1, Q_2)^T$	$\mathbf{2}$	4
Q_3	$\mathbf{1}'$	2
u_1^c	$\mathbf{1}$	6
u_2^c	$\mathbf{1}'$	4
u_3^c	$\mathbf{1}'$	2
d_1^c	$\mathbf{1}$	6
d_2^c	$\mathbf{1}'$	4
d_3^c	$\mathbf{1}'$	2
Higgs sector		
H_u	$\mathbf{1}$	0
H_d	$\mathbf{1}$	0

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Tensor product decomposition rules for the group S_3.

$\otimes$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{2}$
$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$

Parameters

$\tau = \mathrm{Re}(\tau) + \mathrm{Im}(\tau)$ 2

$(\alpha_u, \tilde{\alpha}_u, \beta_u, \tilde{\beta}_u, \alpha_{uD}, \beta_{uD}, \gamma_{uD})$ 7

$(\alpha_d, \tilde{\alpha}_d, \beta_d, \tilde{\beta}_d, \alpha_{dD}, \beta_{dD}, \gamma_{dD})$ 7

$(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}, \alpha_D, \beta_D, \gamma_D)$ 7

$(g_{8s}, g_8', \tilde{g}_8', g_6', g_w')$ 5

$$N_{\text{par}} = 26 + 2 = 28$$

$$N_{\text{dof}} = N_{\text{obs}} - N_{\text{par}} = 18 - 28 = -10 \quad !!$$

$$W_\nu = \frac{1}{\Lambda}\Big[g_{8s}\,(D_\ell D_\ell)_1\,Y_1^{(8)} + g_8'\,(D_\ell D_\ell)_2 \cdot D_{2a}^{(8)} + \tilde{g}_8'\,(D_\ell D_\ell)_2 \cdot D_{2b}^{(8)} + g_{p6}\,(D_\ell \ell_3)_2 \cdot D_2^{(6)} + g_w'\,(\ell_3 \ell_3)\,Y_1^{(4)}\Big] H_u H_u \ .$$

S_3 modular invariant model in a N = 1 rigid SUSY

Field	S_3 irrep	Modular weight k
Leptonic sector		
E_1^c	$\mathbf{1}$	6
E_2^c	$\mathbf{1}'$	4
E_3^c	$\mathbf{1}'$	2
D_ℓ	$\mathbf{2}$	4
ℓ_3	$\mathbf{1}'$	2
H_d	$\mathbf{1}$	0
H_u	$\mathbf{1}$	0
Quark sector		
$Q_D = (Q_1, Q_2)^T$	$\mathbf{2}$	4
Q_3	$\mathbf{1}'$	2
u_1^c	$\mathbf{1}$	6
u_2^c	$\mathbf{1}'$	4
u_3^c	$\mathbf{1}'$	2
d_1^c	$\mathbf{1}$	6
d_2^c	$\mathbf{1}'$	4
d_3^c	$\mathbf{1}'$	2
Higgs sector		
H_u	$\mathbf{1}$	0
H_d	$\mathbf{1}$	0

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Tensor product decomposition rules for the group S_3.

$\otimes$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{2}$
$\mathbf{1}'$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{2}$
$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2}$

Parameters

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$(\alpha_d, \tilde{\alpha}_d, \beta_d, \tilde{\beta}_d, \alpha_{dD}, \beta_{dD}, \gamma_{dD})$	7
$(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}, \alpha_D, \beta_D, \gamma_D)$	7
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S₃-invariant scalar product between two doublets

S_3 modular invariant model in a N = 1 rigid SUSY

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Leptonic sector		
E_1^c	$\mathbf{1}$	6
E_2^c	$\mathbf{1'}$	4
E_3^c	$\mathbf{1'}$	2
D_ℓ	$\mathbf{2}$	4
ℓ_3	$\mathbf{1'}$	2
H_d	$\mathbf{1}$	0
H_u	$\mathbf{1}$	0
Quark sector		
$Q_D = (Q_1, Q_2)^T$	$\mathbf{2}$	4
Q_3	$\mathbf{1'}$	2
u_1^c	$\mathbf{1}$	6
u_2^c	$\mathbf{1'}$	4
u_3^c	$\mathbf{1'}$	2
d_1^c	$\mathbf{1}$	6
d_2^c	$\mathbf{1'}$	4
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H_u	$\mathbf{1}$	0
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Tensor product decomposition rules for the group S_3.

$\otimes$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1'}$	$\mathbf{2}$
$\mathbf{1'}$	$\mathbf{1'}$	$\mathbf{1}$	$\mathbf{2}$
$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1} \oplus \mathbf{1'} \oplus \mathbf{2}$

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$\tau = \mathrm{Re}(\tau) + \mathrm{Im}(\tau)$	2
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$(\alpha_d, \tilde{\alpha}_d, \beta_d, \tilde{\beta}_d, \alpha_{dD}, \beta_{dD}, \gamma_{dD})$	7
$(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}, \alpha_D, \beta_D, \gamma_D)$	7
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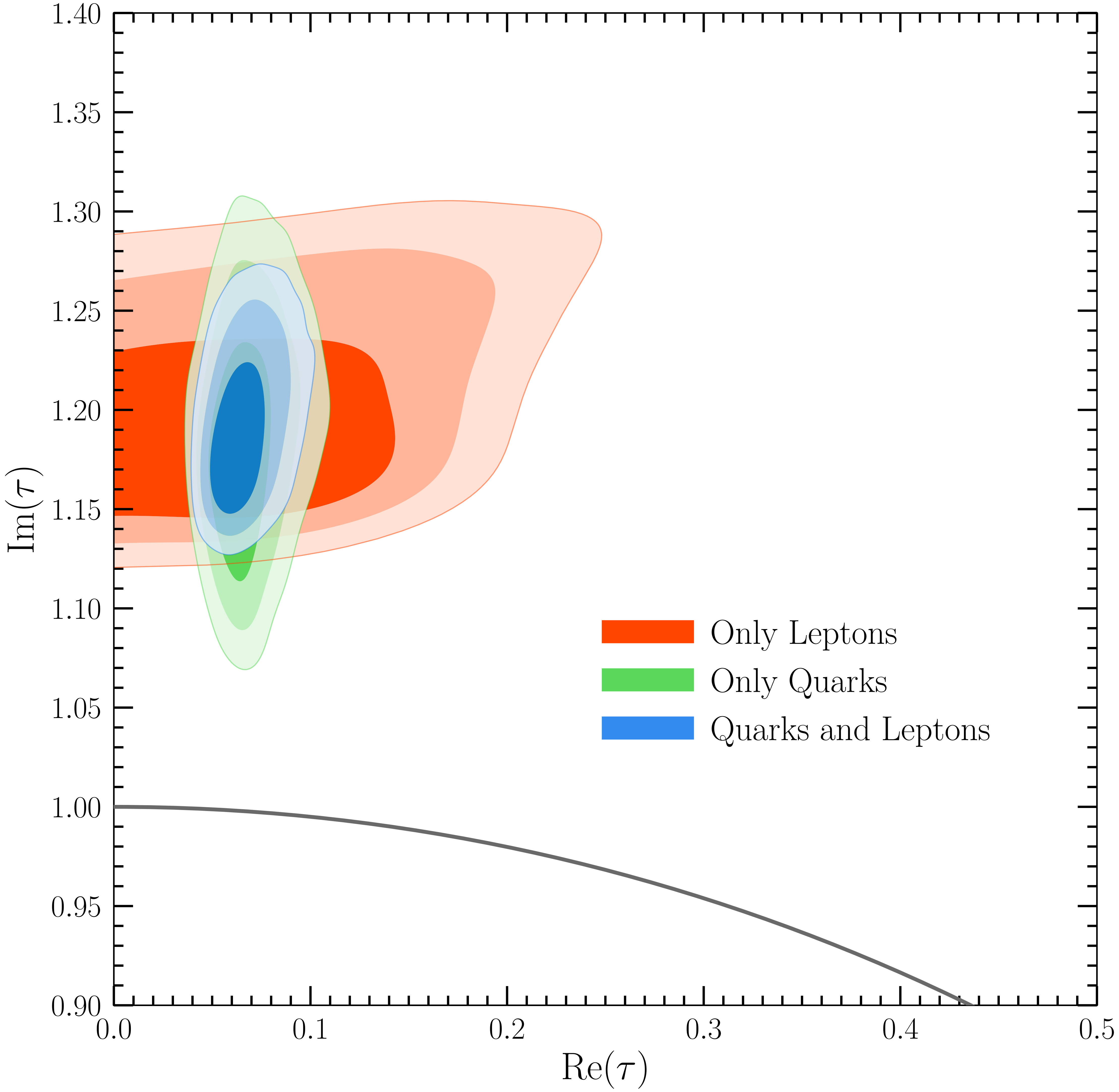
chosen as 10^{-16} eV

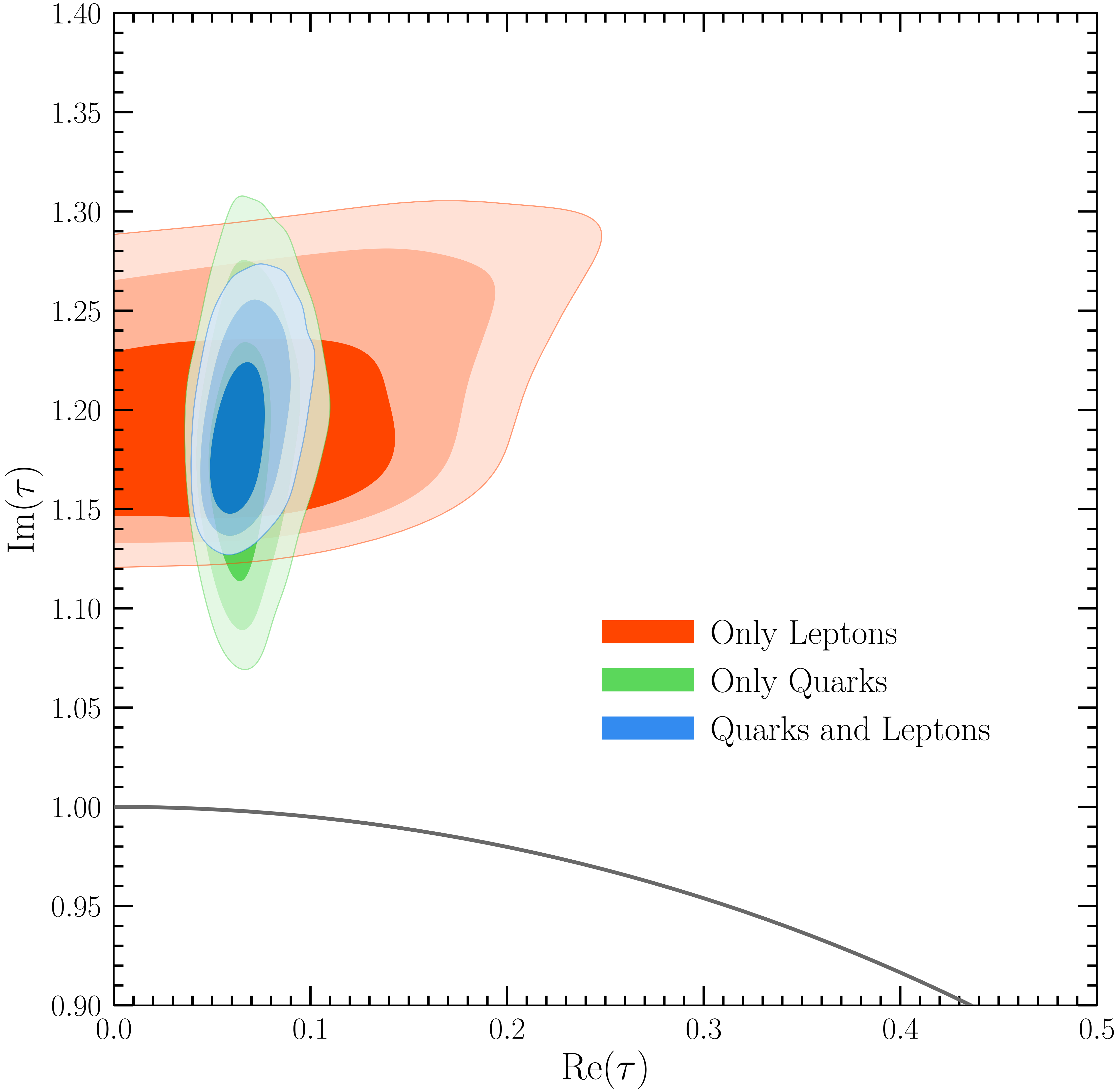
S₃-invariant scalar product between two doublets

S_3 unified quark-lepton model

S_3 unified quark-lepton model

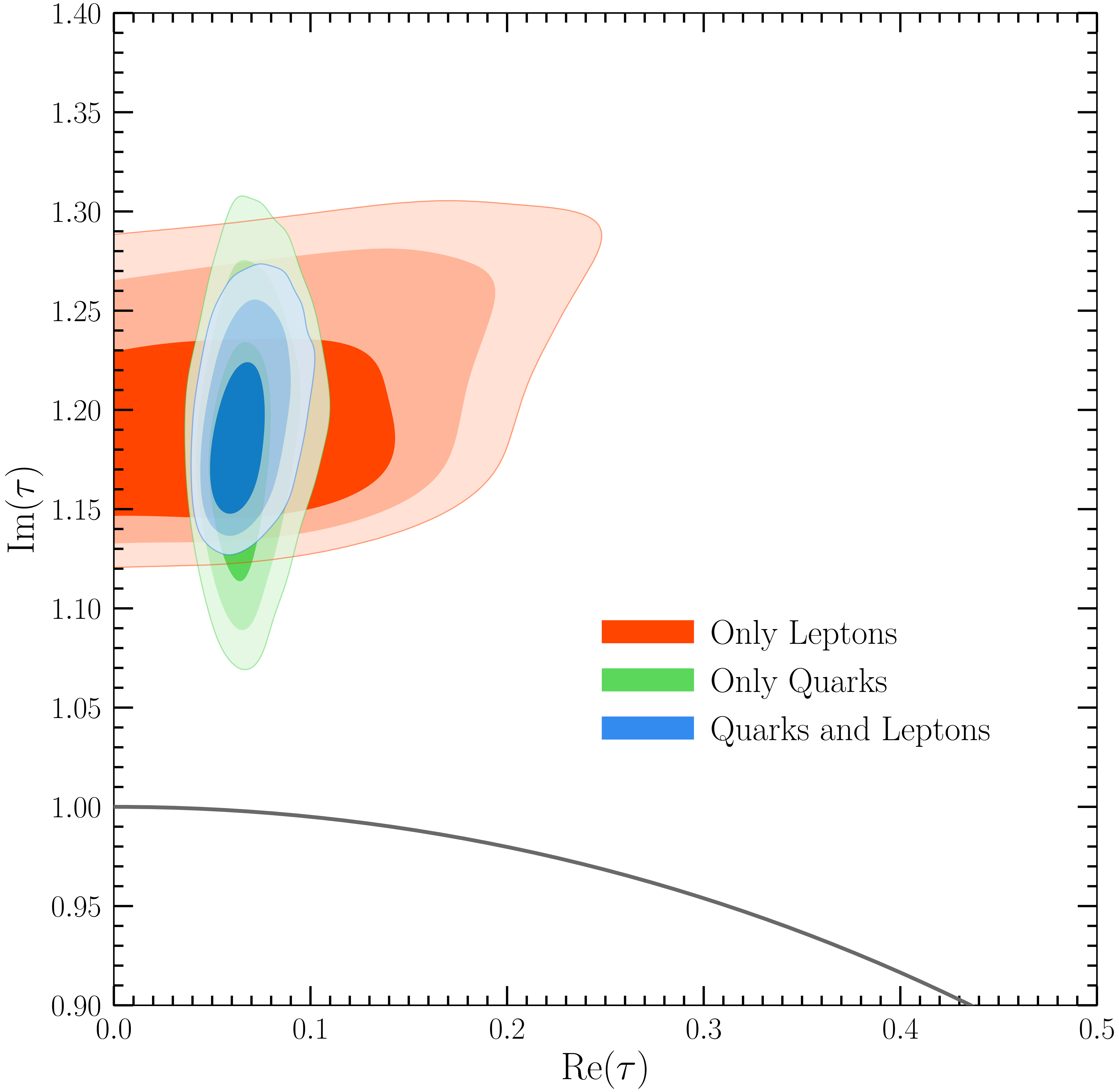


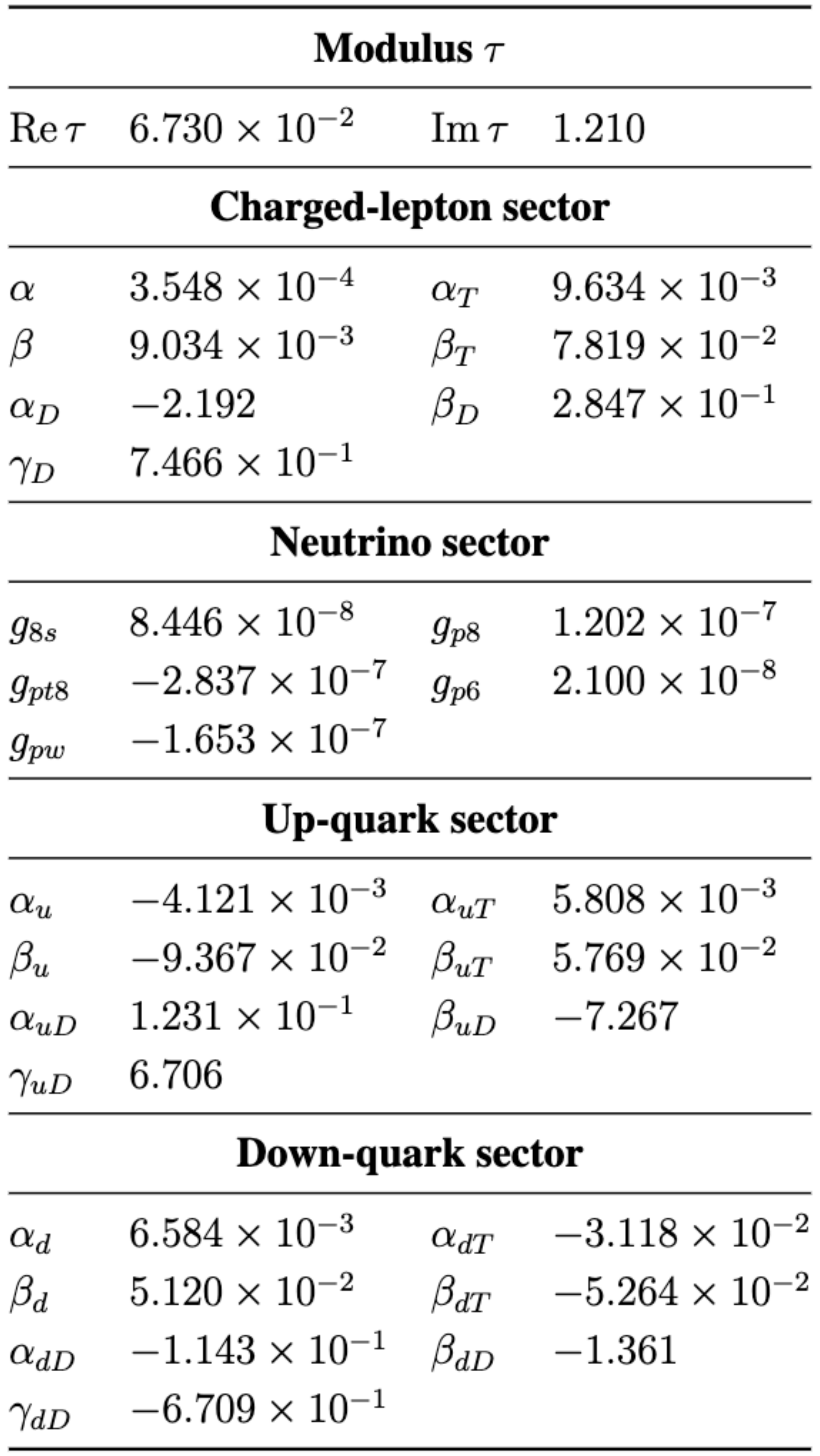
S_3 unified quark-lepton model



Modulus τ			
$\mathrm{Re}\,\tau$	6.730×10^{-2}	$\mathrm{Im}\,\tau$	1.210
Charged-lepton sector			
α	3.548×10^{-4}	α_T	9.634×10^{-3}
β	9.034×10^{-3}	β_T	7.819×10^{-2}
α_D	-2.192	β_D	2.847×10^{-1}
γ_D	7.466×10^{-1}		
Neutrino sector			
g_{8s}	8.446×10^{-8}	g_{p8}	1.202×10^{-7}
g_{pt8}	-2.837×10^{-7}	g_{p6}	2.100×10^{-8}
g_{pw}	-1.653×10^{-7}		
Up-quark sector			
α_u	-4.121×10^{-3}	α_{uT}	5.808×10^{-3}
β_u	-9.367×10^{-2}	β_{uT}	5.769×10^{-2}
α_{uD}	1.231×10^{-1}	β_{uD}	-7.267
γ_{uD}	6.706		
Down-quark sector			
α_d	6.584×10^{-3}	α_{dT}	-3.118×10^{-2}
β_d	5.120×10^{-2}	β_{dT}	-5.264×10^{-2}
α_{dD}	-1.143×10^{-1}	β_{dD}	-1.361
γ_{dD}	-6.709×10^{-1}		

S_3 unified quark-lepton model





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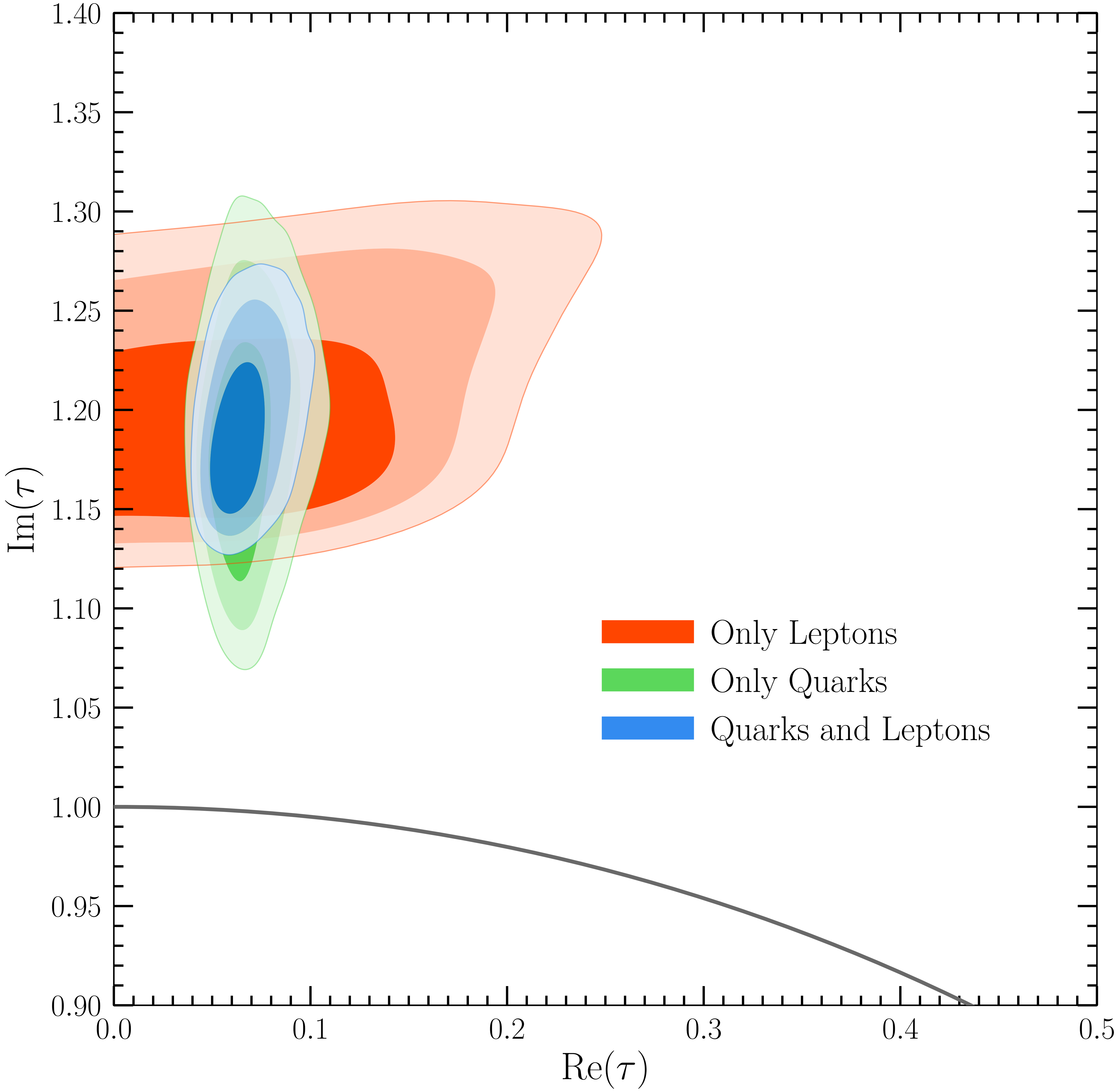
$$\chi^2_{\text{min}} \simeq 2.8 \times 10^{-8}$$

S_3 unified quark-lepton model



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Re τ	6.730×10^{-2}	Im τ	1.210
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$$\chi^2_{\text{min}} \simeq 2.8 \times 10^{-8}$$

The model fits the data but allowed regions are not well defined: they merely denotes $\Delta\chi^2 = 1{,}4{,}9$ and give an idea of where the modulus can be located and of the gradient of the χ^2 around the “only acceptable $\chi^2 = 0$” point

Interesting (the model being overfitted)

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1) As one might expect, the observables are almost completely uncorrelated

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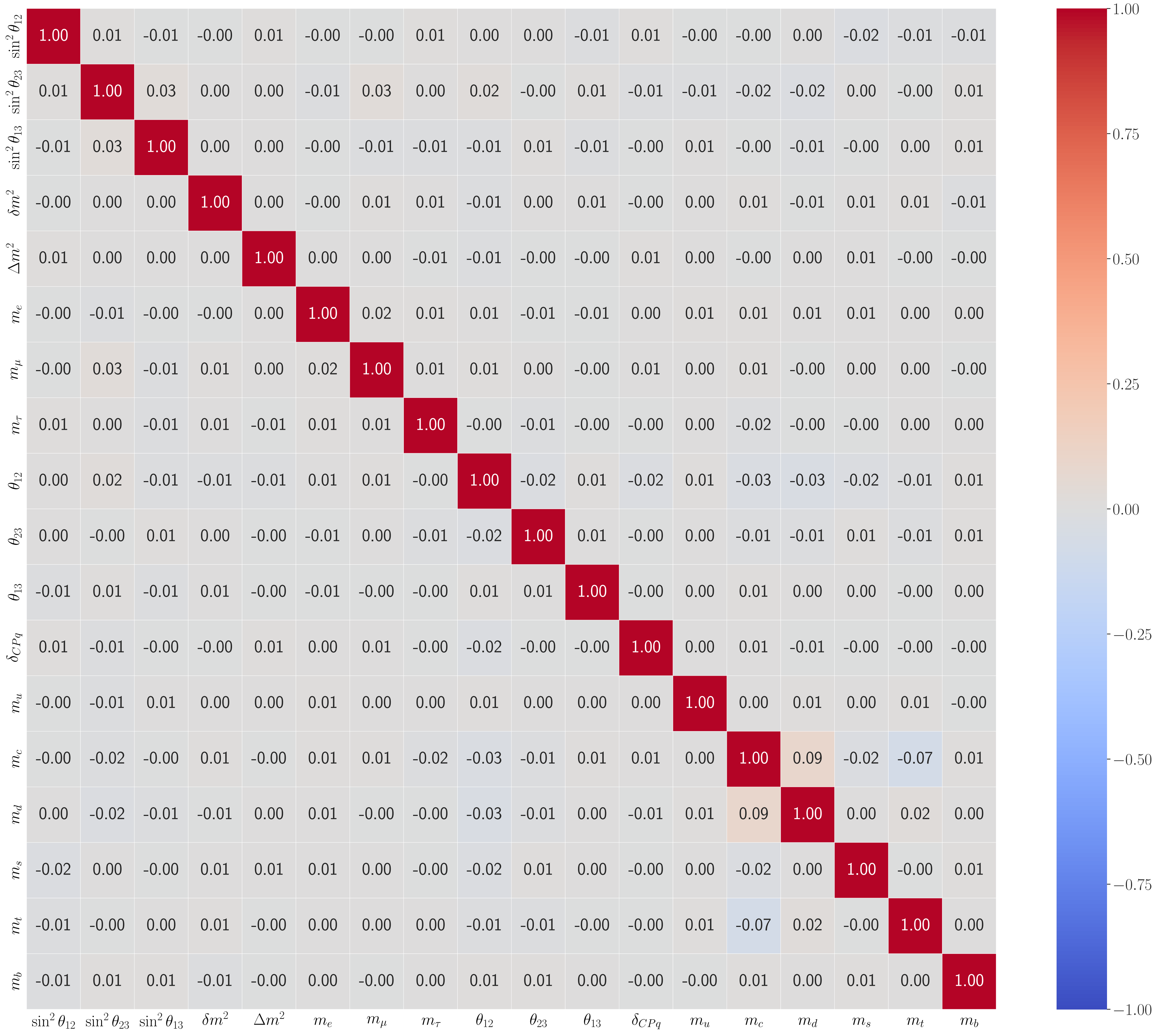
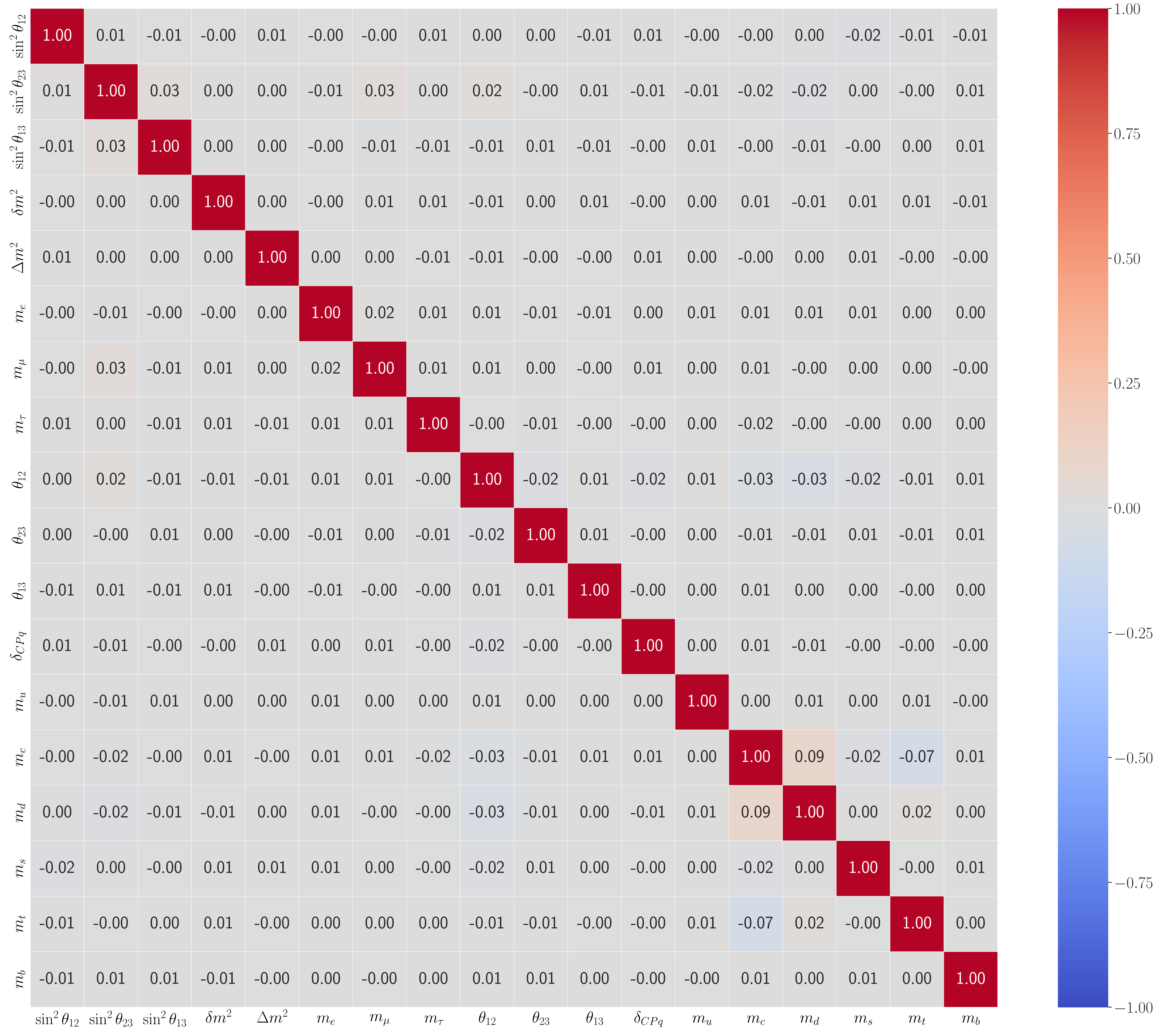


	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	m_e	m_μ	m_τ	θ_{12}	θ_{23}	θ_{13}	δ_{CPq}	m_u	m_c	m_d	m_s	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.01	-0.01	-0.00	0.01	-0.00	-0.00	0.01	0.00	0.00	-0.01	0.01	-0.00	-0.00	0.00	-0.02	-0.01	-0.01
$\sin^2\theta_{23}$	0.01	1.00	0.03	0.00	0.00	-0.01	0.03	0.00	0.02	-0.00	0.01	-0.01	-0.01	-0.02	-0.02	0.00	-0.00	0.01
$\sin^2\theta_{13}$	-0.01	0.03	1.00	0.00	0.00	-0.00	-0.01	-0.01	-0.01	0.01	-0.01	-0.00	0.01	-0.00	-0.01	-0.00	0.00	0.01
δm^2	-0.00	0.00	0.00	1.00	0.00	-0.00	0.01	0.01	-0.01	0.00	0.01	-0.00	0.00	0.01	-0.01	0.01	0.01	-0.01
Δm^2	0.01	0.00	0.00	0.00	1.00	0.00	0.00	-0.01	-0.01	-0.00	-0.00	0.01	0.00	-0.00	0.00	0.01	-0.00	-0.00
m_e	-0.00	-0.01	-0.00	-0.00	0.00	1.00	0.02	0.01	0.01	-0.01	-0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00
m_μ	-0.00	0.03	-0.01	0.01	0.00	0.02	1.00	0.01	0.01	0.00	-0.00	0.01	0.00	0.01	-0.00	0.00	0.00	-0.00
m_τ	0.01	0.00	-0.01	0.01	-0.01	0.01	0.01	1.00	-0.00	-0.01	-0.00	-0.00	0.00	-0.02	-0.00	-0.00	0.00	0.00
θ_{12}	0.00	0.02	-0.01	-0.01	-0.01	0.01	0.01	-0.00	1.00	-0.02	0.01	-0.02	0.01	-0.03	-0.03	-0.02	-0.01	0.01
θ_{23}	0.00	-0.00	0.01	0.00	-0.00	-0.01	0.00	-0.01	-0.02	1.00	0.01	-0.00	0.00	-0.01	-0.01	0.01	-0.01	0.01
θ_{13}	-0.01	0.01	-0.01	0.01	-0.00	-0.01	-0.00	-0.00	0.01	0.01	1.00	-0.00	0.00	0.01	0.00	0.00	-0.00	0.00
δ_{CPq}	0.01	-0.01	-0.00	-0.00	0.01	0.00	0.01	-0.00	-0.02	-0.00	-0.00	1.00	0.00	0.01	-0.01	-0.00	-0.00	-0.00
m_u	-0.00	-0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	1.00	0.00	0.01	0.00	0.01	-0.00
m_c	-0.00	-0.02	-0.00	0.01	-0.00	0.01	0.01	-0.02	-0.03	-0.01	0.01	0.01	0.00	1.00	0.09	-0.02	-0.07	0.01
m_d	0.00	-0.02	-0.01	-0.01	0.00	0.01	-0.00	-0.00	-0.03	-0.01	0.00	-0.01	0.01	0.09	1.00	0.00	0.02	0.00
m_s	-0.02	0.00	-0.00	0.01	0.01	0.01	0.00	-0.00	-0.02	0.01	0.00	-0.00	0.00	-0.02	0.00	1.00	-0.00	0.01
m_t	-0.01	-0.00	0.00	0.01	-0.00	0.00	0.00	0.00	-0.01	-0.01	-0.00	-0.00	0.01	-0.07	0.02	-0.00	1.00	0.00
m_b	-0.01	0.01	0.01	-0.01	-0.00	0.00	-0.00	0.00	0.01	0.01	0.00	-0.00	-0.00	0.01	0.00	0.01	0.00	1.00

Interesting (the model being overfitted)

1) As one might expect, the observables are almost completely uncorrelated

2) Model parameters, however, show significant correlations

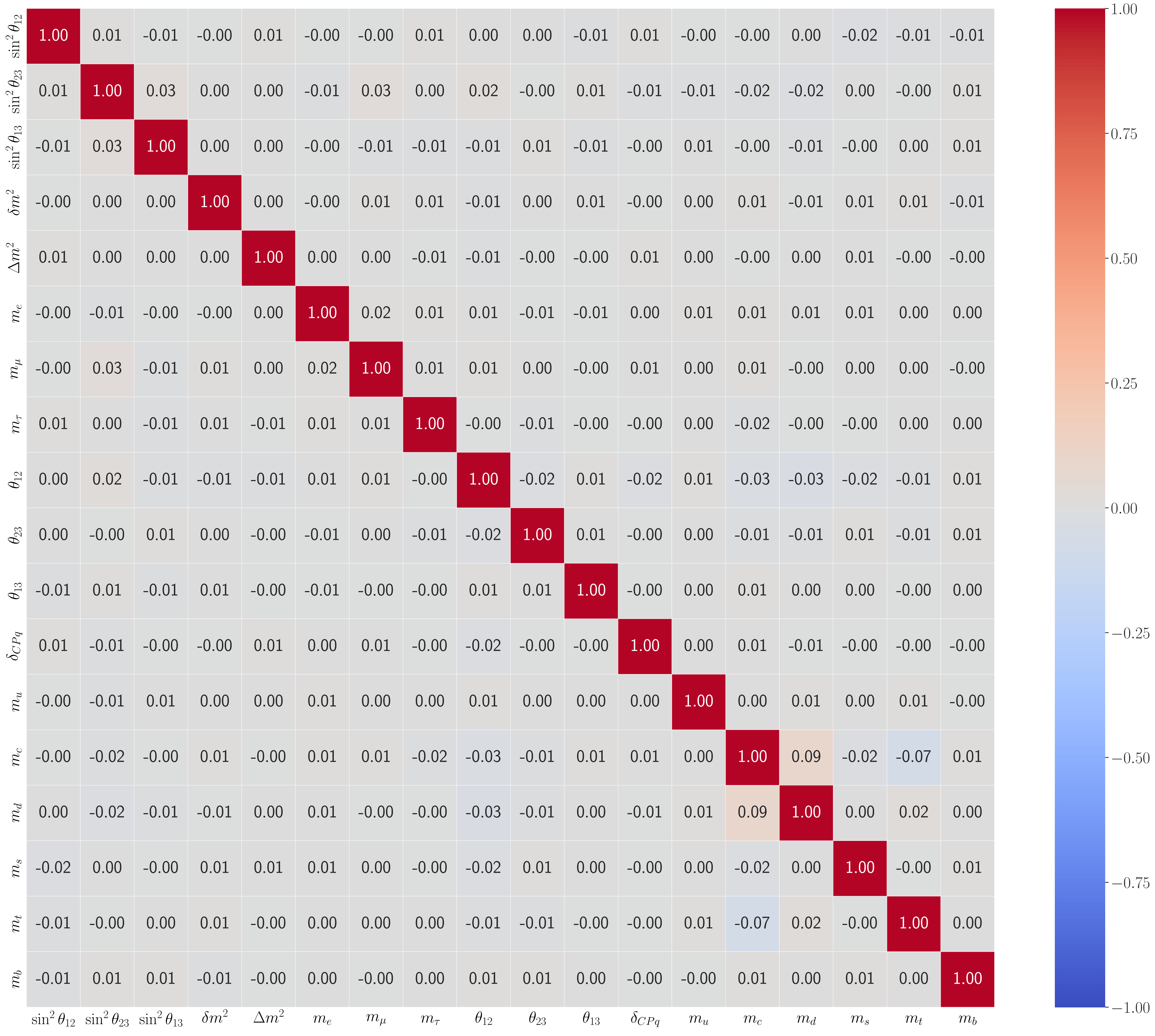


	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	m_e	m_μ	m_τ	θ_{12}	θ_{23}	θ_{13}	δ_{CPq}	m_u	m_c	m_d	m_s	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.01	-0.01	-0.00	0.01	-0.00	-0.00	0.01	0.00	0.00	-0.01	0.01	-0.00	-0.00	0.00	-0.02	-0.01	-0.01
$\sin^2\theta_{23}$	0.01	1.00	0.03	0.00	0.00	-0.01	0.03	0.00	0.02	-0.00	0.01	-0.01	-0.01	-0.02	-0.02	0.00	-0.00	0.01
$\sin^2\theta_{13}$	-0.01	0.03	1.00	0.00	0.00	-0.00	-0.01	-0.01	-0.01	0.01	-0.01	-0.00	0.01	-0.00	-0.01	-0.00	0.00	0.01
δm^2	-0.00	0.00	0.00	1.00	0.00	-0.00	0.01	0.01	-0.01	0.00	0.01	-0.00	0.00	0.01	-0.01	0.01	0.01	-0.01
Δm^2	0.01	0.00	0.00	0.00	1.00	0.00	0.00	-0.01	-0.01	-0.00	-0.00	0.01	0.00	-0.00	0.00	0.01	-0.00	-0.00
m_e	-0.00	-0.01	-0.00	-0.00	0.00	1.00	0.02	0.01	0.01	-0.01	-0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00
m_μ	-0.00	0.03	-0.01	0.01	0.00	0.02	1.00	0.01	0.01	0.00	-0.00	0.01	0.00	0.01	-0.00	0.00	0.00	-0.00
m_τ	0.01	0.00	-0.01	0.01	-0.01	0.01	0.01	1.00	-0.00	-0.01	-0.00	-0.00	0.00	-0.02	-0.00	-0.00	0.00	0.00
θ_{12}	0.00	0.02	-0.01	-0.01	-0.01	0.01	0.01	-0.00	1.00	-0.02	0.01	-0.02	0.01	-0.03	-0.03	-0.02	-0.01	0.01
θ_{23}	0.00	-0.00	0.01	0.00	-0.00	-0.01	0.00	-0.01	-0.02	1.00	0.01	-0.00	0.00	-0.01	-0.01	0.01	-0.01	0.01
θ_{13}	-0.01	0.01	-0.01	0.01	-0.00	-0.01	-0.00	-0.00	0.01	0.01	1.00	-0.00	0.00	0.01	0.00	0.00	-0.00	0.00
δ_{CPq}	0.01	-0.01	-0.00	-0.00	0.01	0.00	0.01	-0.00	-0.02	-0.00	-0.00	1.00	0.00	0.01	-0.01	-0.00	-0.00	-0.00
m_u	-0.00	-0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	1.00	0.00	0.01	0.00	0.01	-0.00
m_c	-0.00	-0.02	-0.00	0.01	-0.00	0.01	0.01	-0.02	-0.03	-0.01	0.01	0.01	0.00	1.00	0.09	-0.02	-0.07	0.01
m_d	0.00	-0.02	-0.01	-0.01	0.00	0.01	-0.00	-0.00	-0.03	-0.01	0.00	-0.01	0.01	0.09	1.00	0.00	0.02	0.00
m_s	-0.02	0.00	-0.00	0.01	0.01	0.01	0.00	-0.00	-0.02	0.01	0.00	-0.00	0.00	-0.02	0.00	1.00	-0.00	0.01
m_t	-0.01	-0.00	0.00	0.01	-0.00	0.00	0.00	0.00	-0.01	-0.01	-0.00	-0.00	0.01	-0.07	0.02	-0.00	1.00	0.00
m_b	-0.01	0.01	0.01	-0.01	-0.00	0.00	-0.00	0.00	0.01	0.01	0.00	-0.00	-0.00	0.01	0.00	0.01	0.00	1.00

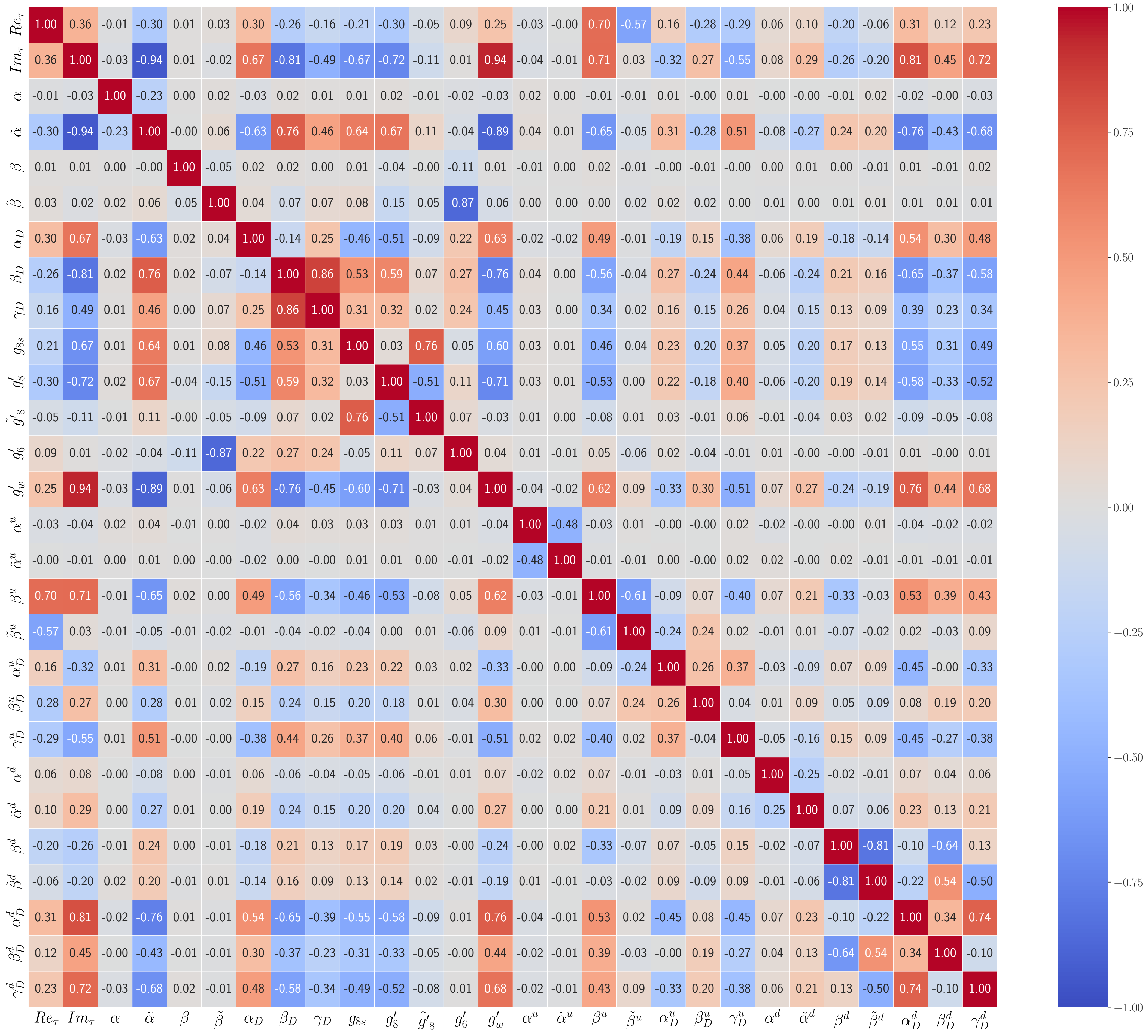
Interesting (the model being overfitted)

1) As one might expect, the observables are almost completely uncorrelated

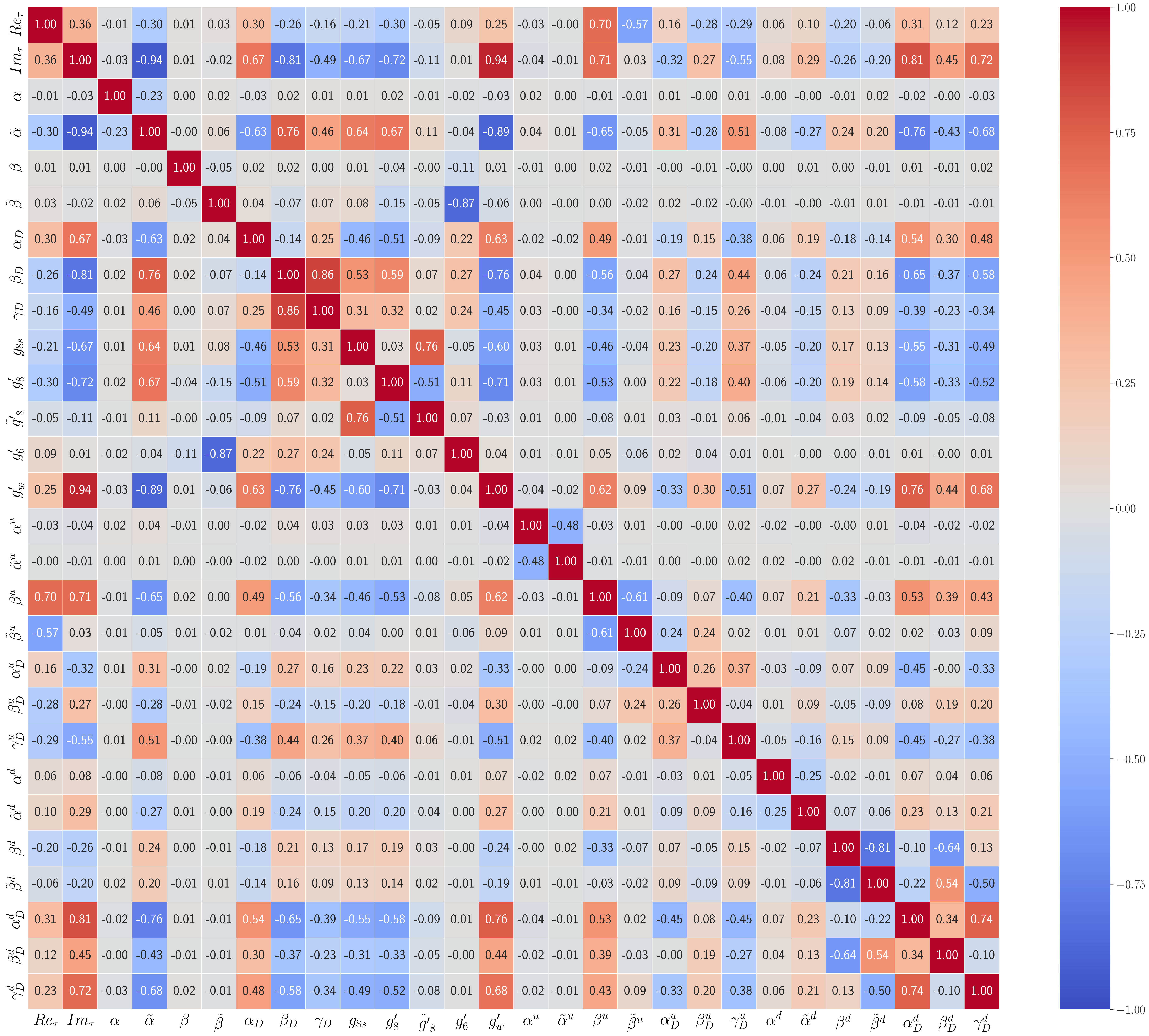
2) Model parameters, however, show significant correlations



	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	m_e	m_μ	m_τ	θ_{12}	θ_{23}	θ_{13}	δ_{CPq}	m_u	m_c	m_d	m_s	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.01	-0.01	-0.00	0.01	-0.00	-0.00	0.01	0.00	0.00	-0.01	0.01	-0.00	-0.00	0.00	-0.02	-0.01	-0.01
$\sin^2\theta_{23}$	0.01	1.00	0.03	0.00	0.00	-0.01	0.03	0.00	0.02	-0.00	0.01	-0.01	-0.01	-0.02	-0.02	0.00	-0.00	0.01
$\sin^2\theta_{13}$	-0.01	0.03	1.00	0.00	0.00	-0.00	-0.01	-0.01	-0.01	0.01	-0.01	-0.00	0.01	-0.00	-0.01	-0.00	0.00	0.01
δm^2	-0.00	0.00	0.00	1.00	0.00	-0.00	0.01	0.01	-0.01	0.00	0.01	-0.00	0.00	0.01	-0.01	0.01	0.01	-0.01
Δm^2	0.01	0.00	0.00	0.00	1.00	0.00	0.00	-0.01	-0.01	-0.00	-0.00	0.01	0.00	-0.00	0.00	0.01	-0.00	-0.00
m_e	-0.00	-0.01	-0.00	-0.00	0.00	1.00	0.02	0.01	0.01	-0.01	-0.01	0.00	0.01	0.01	0.01	0.01	0.00	0.00
m_μ	-0.00	0.03	-0.01	0.01	0.00	0.02	1.00	0.01	0.01	0.00	-0.00	0.01	0.00	0.01	-0.00	0.00	0.00	-0.00
m_τ	0.01	0.00	-0.01	0.01	-0.01	0.01	0.01	1.00	-0.00	-0.01	-0.00	-0.00	0.00	-0.02	-0.00	-0.00	0.00	0.00
θ_{12}	0.00	0.02	-0.01	-0.01	-0.01	0.01	0.01	-0.00	1.00	-0.02	0.01	-0.02	0.01	-0.03	-0.03	-0.02	-0.01	0.01
θ_{23}	0.00	-0.00	0.01	0.00	-0.00	-0.01	0.00	-0.01	-0.02	1.00	0.01	-0.00	0.00	-0.01	-0.01	0.01	-0.01	0.01
θ_{13}	-0.01	0.01	-0.01	0.01	-0.00	-0.01	-0.00	-0.00	0.01	0.01	1.00	-0.00	0.00	0.01	0.00	0.00	-0.00	0.00
δ_{CPq}	0.01	-0.01	-0.00	-0.00	0.01	0.00	0.01	-0.00	-0.02	-0.00	-0.00	1.00	0.00	0.01	-0.01	-0.00	-0.00	-0.00
m_u	-0.00	-0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	1.00	0.00	0.01	0.00	0.01	-0.00
m_c	-0.00	-0.02	-0.00	0.01	-0.00	0.01	0.01	-0.02	-0.03	-0.01	0.01	0.01	0.00	1.00	0.09	-0.02	-0.07	0.01
m_d	0.00	-0.02	-0.01	-0.01	0.00	0.01	-0.00	-0.00	-0.03	-0.01	0.00	-0.01	0.01	0.09	1.00	0.00	0.02	0.00
m_s	-0.02	0.00	-0.00	0.01	0.01	0.01	0.00	-0.00	-0.02	0.01	0.00	-0.00	0.00	-0.02	0.00	1.00	-0.00	0.01
m_t	-0.01	-0.00	0.00	0.01	-0.00	0.00	0.00	0.00	-0.01	-0.01	-0.00	-0.00	0.01	-0.07	0.02	-0.00	1.00	0.00
m_b	-0.01	0.01	0.01	-0.01	-0.00	0.00	-0.00	0.00	0.01	0.01	0.00	-0.00	-0.00	0.01	0.00	0.01	0.00	1.00

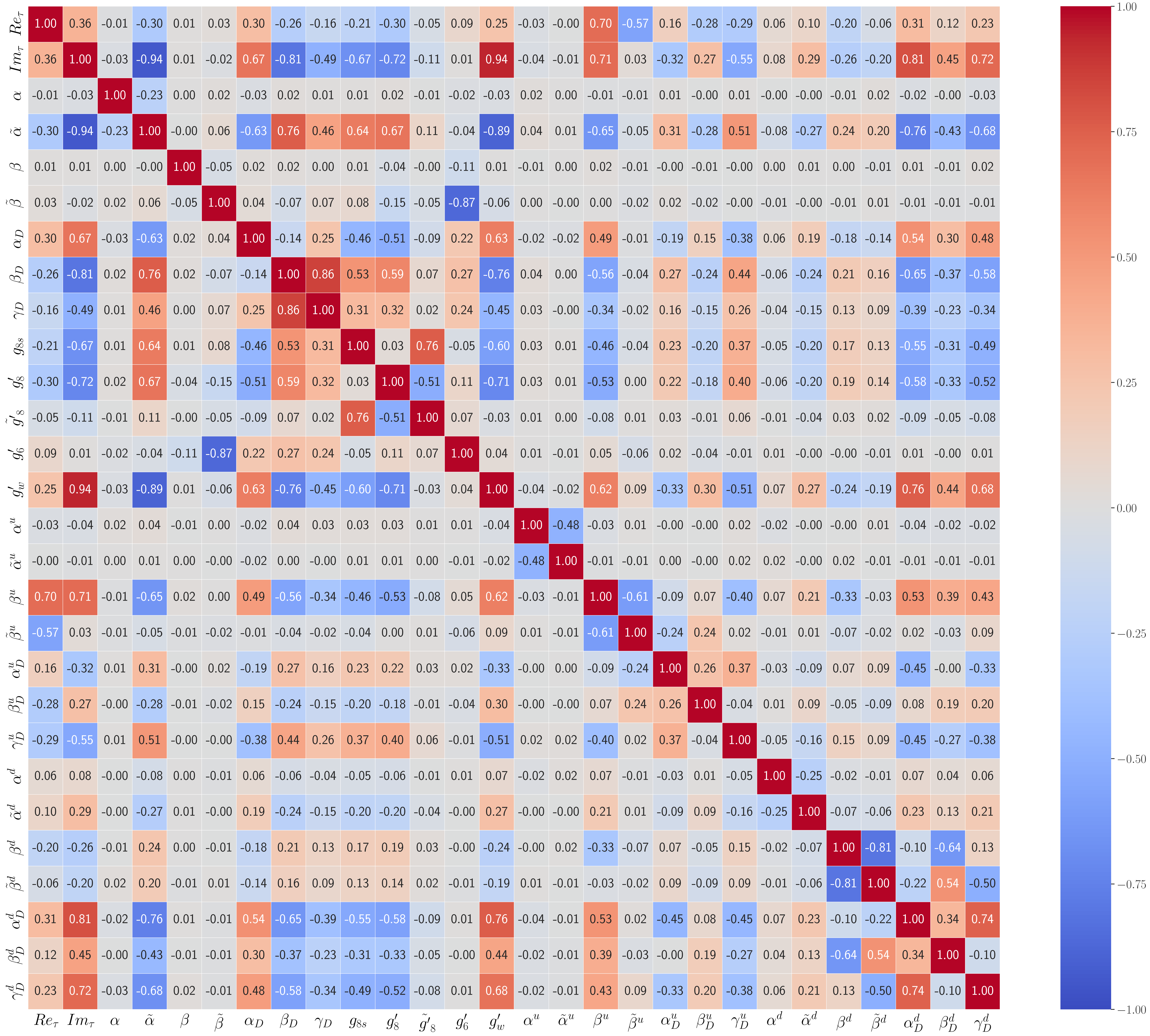


	Re_τ	Im_τ	α	$\tilde{\alpha}$	β	$\tilde{\beta}$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde{\alpha}^u$	β^u	$\tilde{\beta}^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde{\alpha}^d$	β^d	$\tilde{\beta}^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde{\alpha}$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde{\beta}$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde{\alpha}^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde{\beta}^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde{\alpha}^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde{\beta}^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00



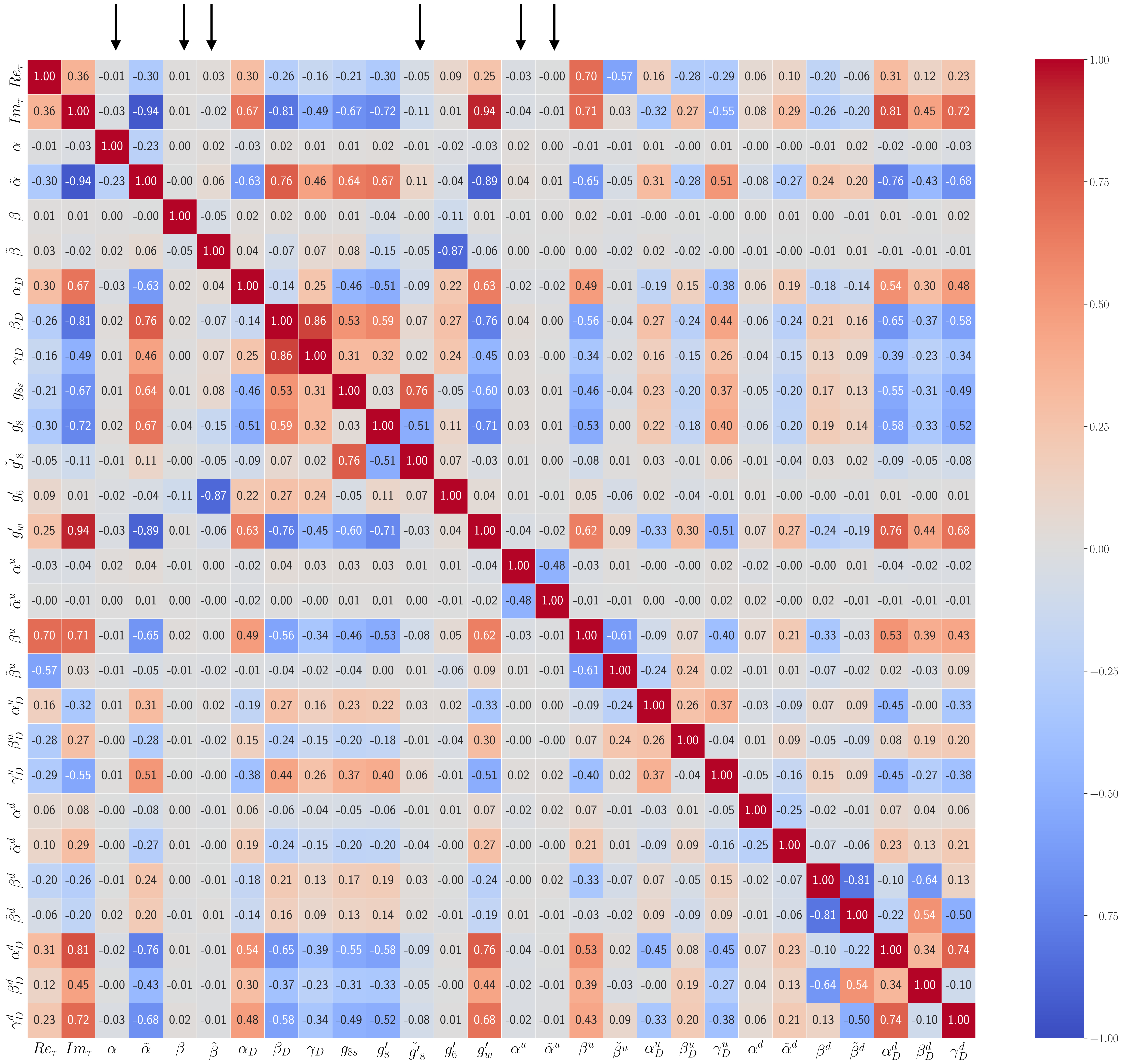
	Re_τ	Im_τ	α	$\tilde\alpha$	β	$\tilde\beta$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde\alpha^u$	β^u	$\tilde\beta^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde\alpha^d$	β^d	$\tilde\beta^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde\alpha$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde\beta$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde\alpha^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde\beta^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde\alpha^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde\beta^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00

Correlations tend to be governed by the modulus



	Re_τ	Im_τ	α	$\tilde{\alpha}$	β	$\tilde{\beta}$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde{\alpha}^u$	β^u	$\tilde{\beta}^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde{\alpha}^d$	β^d	$\tilde{\beta}^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde{\alpha}$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde{\beta}$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde{\alpha}^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde{\beta}^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde{\alpha}^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde{\beta}^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00

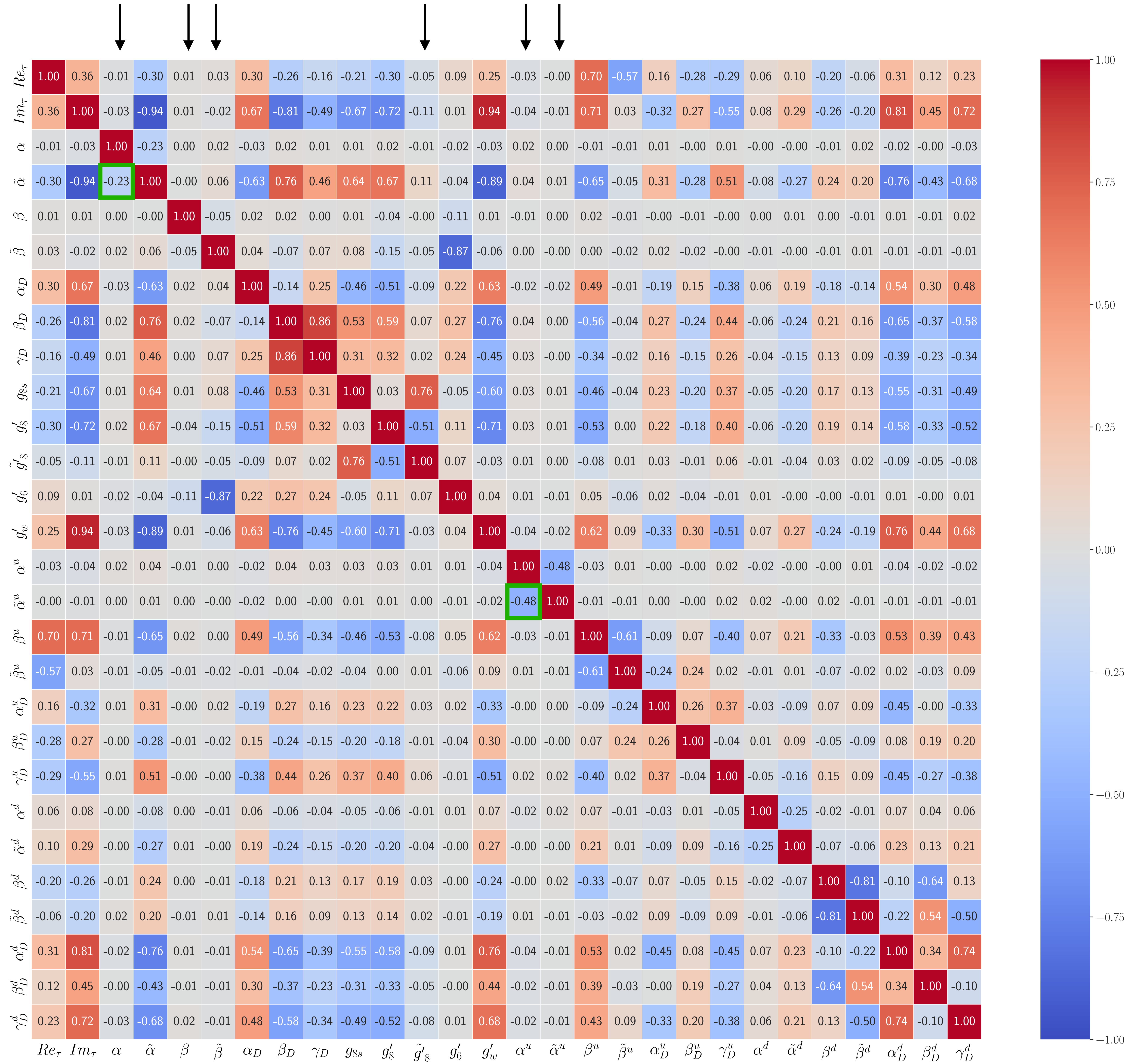
Correlations tend to be governed by the modulus



	Re_τ	Im_τ	α	$\tilde{\alpha}$	β	$\tilde{\beta}$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde{\alpha}^u$	β^u	$\tilde{\beta}^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde{\alpha}^d$	β^d	$\tilde{\beta}^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde{\alpha}$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde{\beta}$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde{\alpha}^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde{\beta}^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde{\alpha}^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde{\beta}^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00

Parameters that are only weakly correlated with τ also tend to be weakly correlated with all other parameters

Correlations tend to be governed by the modulus

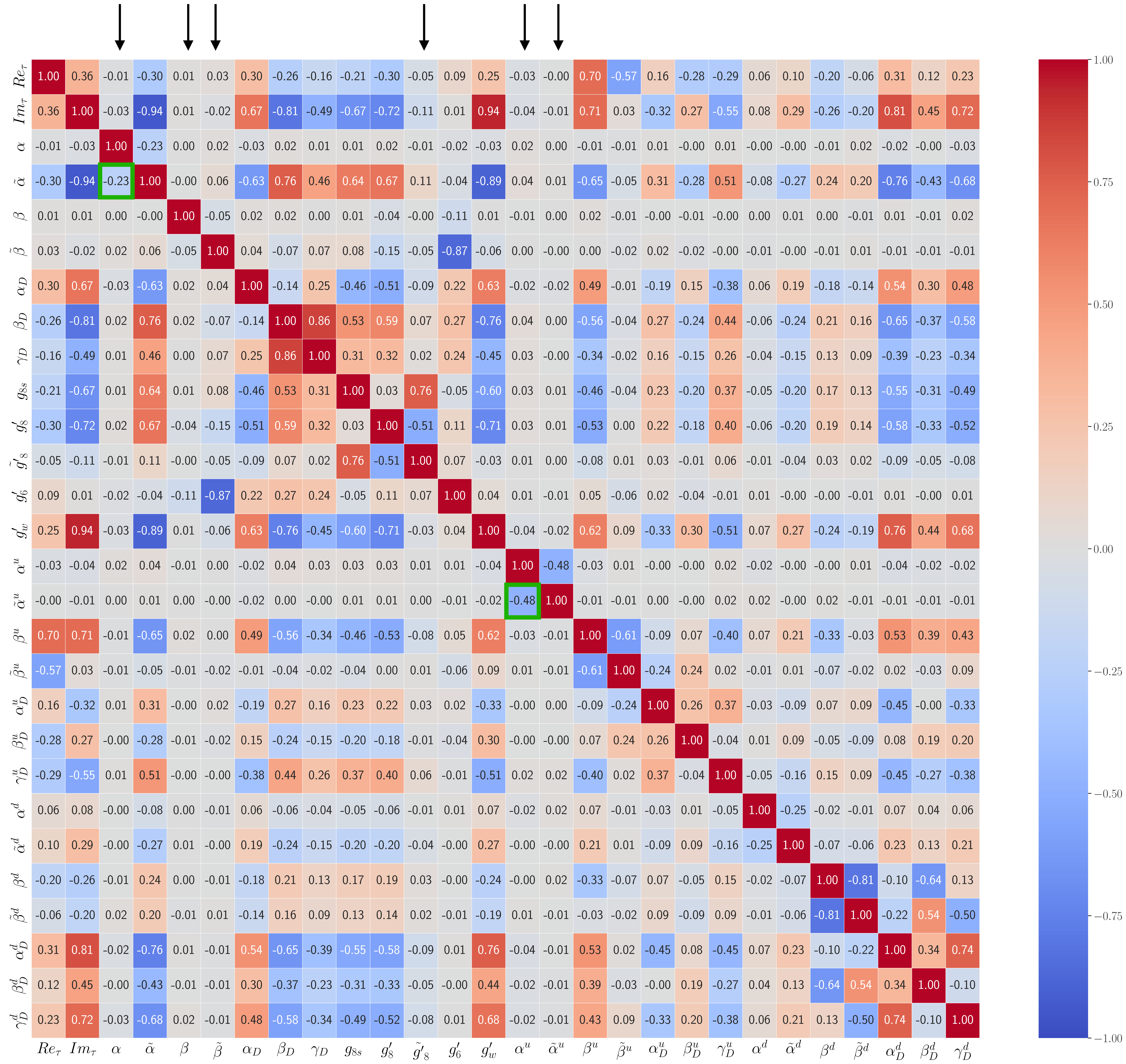


	Re_τ	Im_τ	α	$\tilde{\alpha}$	β	$\tilde{\beta}$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde{\alpha}^u$	β^u	$\tilde{\beta}^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde{\alpha}^d$	β^d	$\tilde{\beta}^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde{\alpha}$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde{\beta}$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde{\alpha}^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde{\beta}^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde{\alpha}^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde{\beta}^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00

Parameters that are only weakly correlated with τ also tend to be weakly correlated with all other parameters

$(\alpha, \tilde{\alpha})$ and $(\alpha_u, \tilde{\alpha}_u)$ are anti correlated because they appear in the same matrix element

Correlations tend to be governed by the modulus



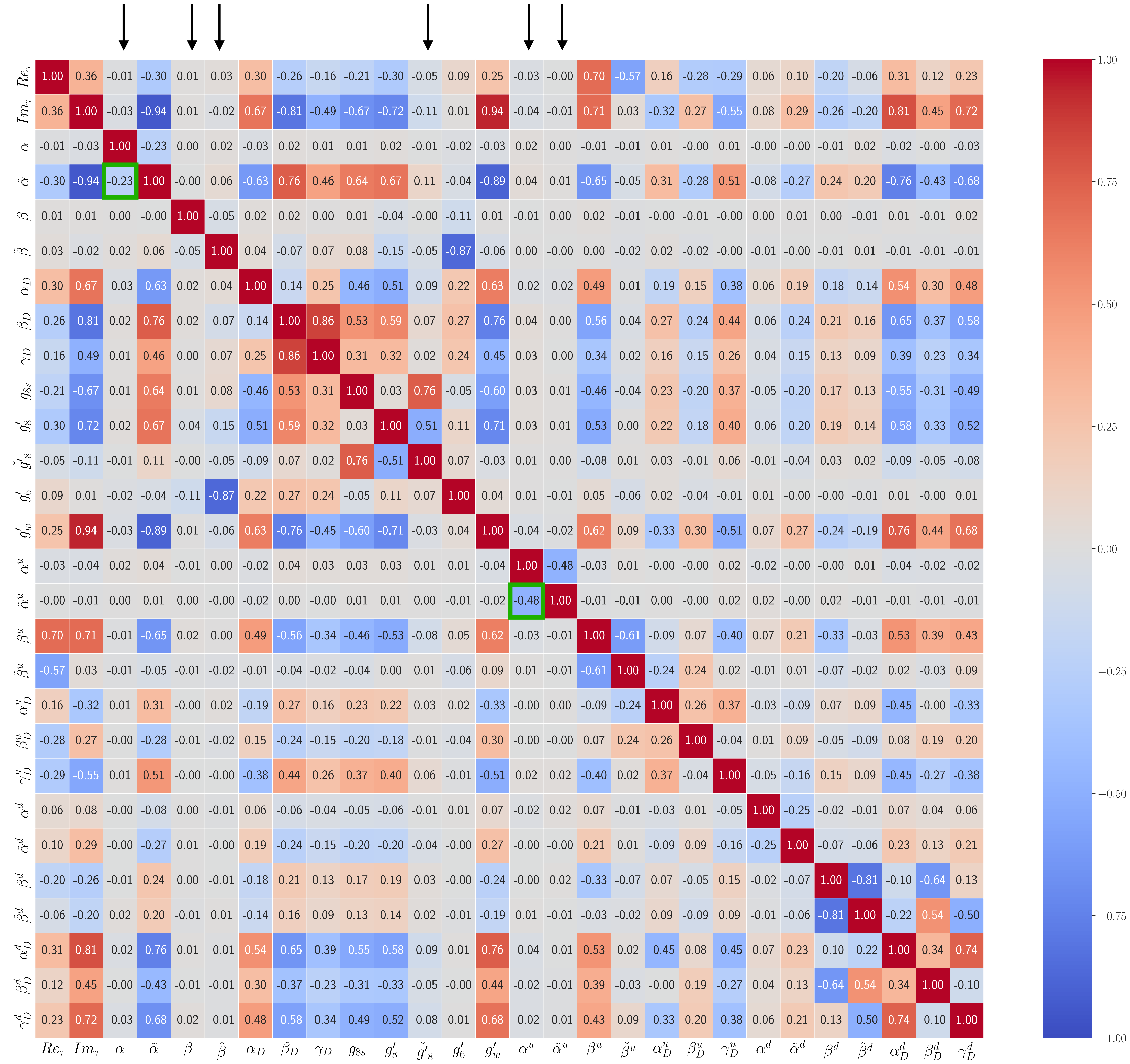
	Re_τ	Im_τ	α	$\tilde{\alpha}$	β	$\tilde{\beta}$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde{\alpha}^u$	β^u	$\tilde{\beta}^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde{\alpha}^d$	β^d	$\tilde{\beta}^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde{\alpha}$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde{\beta}$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde{\alpha}^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde{\beta}^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde{\alpha}^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde{\beta}^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00

Parameters that are only weakly correlated with τ also tend to be weakly correlated with all other parameters

$(\alpha, \tilde{\alpha})$ and $(\alpha_u, \tilde{\alpha}_u)$ are anti correlated because they appear in the same matrix element

In modular-invariant models, significant correlations among parameters naturally arise due to the rigid, symmetry-imposed structure of the theory, even when the number of parameters exceeds the number of observables

Correlations tend to be governed by the modulus



	Re_τ	Im_τ	α	$\tilde{\alpha}$	β	$\tilde{\beta}$	α_D	β_D	γ_D	g_{8s}	g'_8	$\tilde{g}'_8$	g'_6	g'_w	α^u	$\tilde{\alpha}^u$	β^u	$\tilde{\beta}^u$	α^u_D	β^u_D	γ^u_D	α^d	$\tilde{\alpha}^d$	β^d	$\tilde{\beta}^d$	α^d_D	β^d_D	γ^d_D
Re_τ	1.00	0.36	-0.01	-0.30	0.01	0.03	0.30	-0.26	-0.16	-0.21	-0.30	-0.05	0.09	0.25	-0.03	-0.00	0.70	-0.57	0.16	-0.28	-0.29	0.06	0.10	-0.20	-0.06	0.31	0.12	0.23
Im_τ	0.36	1.00	-0.03	-0.94	0.01	-0.02	0.67	-0.81	-0.49	-0.67	-0.72	-0.11	0.01	0.94	-0.04	-0.01	0.71	0.03	-0.32	0.27	-0.55	0.08	0.29	-0.26	-0.20	0.81	0.45	0.72
α	-0.01	-0.03	1.00	-0.23	0.00	0.02	-0.03	0.02	0.01	0.01	0.02	-0.01	-0.02	-0.03	0.02	0.00	-0.01	-0.01	0.01	-0.00	0.01	-0.00	-0.00	-0.01	0.02	-0.02	-0.00	-0.03
$\tilde{\alpha}$	-0.30	-0.94	-0.23	1.00	-0.00	0.06	-0.63	0.76	0.46	0.64	0.67	0.11	-0.04	-0.89	0.04	0.01	-0.65	-0.05	0.31	-0.28	0.51	-0.08	-0.27	0.24	0.20	-0.76	-0.43	-0.68
β	0.01	0.01	0.00	-0.00	1.00	-0.05	0.02	0.02	0.00	0.01	-0.04	-0.00	-0.11	0.01	-0.01	0.00	0.02	-0.01	-0.00	-0.01	-0.00	0.00	0.01	0.00	-0.01	0.01	-0.01	0.02
$\tilde{\beta}$	0.03	-0.02	0.02	0.06	-0.05	1.00	0.04	-0.07	0.07	0.08	-0.15	-0.05	-0.87	-0.06	0.00	-0.00	0.00	-0.02	0.02	-0.02	-0.00	-0.01	-0.00	-0.01	0.01	-0.01	-0.01	-0.01
α_D	0.30	0.67	-0.03	-0.63	0.02	0.04	1.00	-0.14	0.25	-0.46	-0.51	-0.09	0.22	0.63	-0.02	-0.02	0.49	-0.01	-0.19	0.15	-0.38	0.06	0.19	-0.18	-0.14	0.54	0.30	0.48
β_D	-0.26	-0.81	0.02	0.76	0.02	-0.07	-0.14	1.00	0.86	0.53	0.59	0.07	0.27	-0.76	0.04	0.00	-0.56	-0.04	0.27	-0.24	0.44	-0.06	-0.24	0.21	0.16	-0.65	-0.37	-0.58
γ_D	-0.16	-0.49	0.01	0.46	0.00	0.07	0.25	0.86	1.00	0.31	0.32	0.02	0.24	-0.45	0.03	-0.00	-0.34	-0.02	0.16	-0.15	0.26	-0.04	-0.15	0.13	0.09	-0.39	-0.23	-0.34
g_{8s}	-0.21	-0.67	0.01	0.64	0.01	0.08	-0.46	0.53	0.31	1.00	0.03	0.76	-0.05	-0.60	0.03	0.01	-0.46	-0.04	0.23	-0.20	0.37	-0.05	-0.20	0.17	0.13	-0.55	-0.31	-0.49
g'_8	-0.30	-0.72	0.02	0.67	-0.04	-0.15	-0.51	0.59	0.32	0.03	1.00	-0.51	0.11	-0.71	0.03	0.01	-0.53	0.00	0.22	-0.18	0.40	-0.06	-0.20	0.19	0.14	-0.58	-0.33	-0.52
$\tilde{g}'_8$	-0.05	-0.11	-0.01	0.11	-0.00	-0.05	-0.09	0.07	0.02	0.76	-0.51	1.00	0.07	-0.03	0.01	0.00	-0.08	0.01	0.03	-0.01	0.06	-0.01	-0.04	0.03	0.02	-0.09	-0.05	-0.08
g'_6	0.09	0.01	-0.02	-0.04	-0.11	-0.87	0.22	0.27	0.24	-0.05	0.11	0.07	1.00	0.04	0.01	-0.01	0.05	-0.06	0.02	-0.04	-0.01	0.01	-0.00	-0.00	-0.01	0.01	-0.00	0.01
g'_w	0.25	0.94	-0.03	-0.89	0.01	-0.06	0.63	-0.76	-0.45	-0.60	-0.71	-0.03	0.04	1.00	-0.04	-0.02	0.62	0.09	-0.33	0.30	-0.51	0.07	0.27	-0.24	-0.19	0.76	0.44	0.68
α^u	-0.03	-0.04	0.02	0.04	-0.01	0.00	-0.02	0.04	0.03	0.03	0.03	0.01	0.01	-0.04	1.00	-0.48	-0.03	0.01	-0.00	-0.00	0.02	-0.02	-0.00	-0.00	0.01	-0.04	-0.02	-0.02
$\tilde{\alpha}^u$	-0.00	-0.01	0.00	0.01	0.00	-0.00	-0.02	0.00	-0.00	0.01	0.01	0.00	-0.01	-0.02	-0.48	1.00	-0.01	-0.01	0.00	-0.00	0.02	0.02	-0.00	0.02	-0.01	-0.01	-0.01	-0.01
β^u	0.70	0.71	-0.01	-0.65	0.02	0.00	0.49	-0.56	-0.34	-0.46	-0.53	-0.08	0.05	0.62	-0.03	-0.01	1.00	-0.61	-0.09	0.07	-0.40	0.07	0.21	-0.33	-0.03	0.53	0.39	0.43
$\tilde{\beta}^u$	-0.57	0.03	-0.01	-0.05	-0.01	-0.02	-0.01	-0.04	-0.02	-0.04	0.00	0.01	-0.06	0.09	0.01	-0.01	-0.61	1.00	-0.24	0.24	0.02	-0.01	0.01	-0.07	-0.02	0.02	-0.03	0.09
α^u_D	0.16	-0.32	0.01	0.31	-0.00	0.02	-0.19	0.27	0.16	0.23	0.22	0.03	0.02	-0.33	-0.00	0.00	-0.09	-0.24	1.00	0.26	0.37	-0.03	-0.09	0.07	0.09	-0.45	-0.00	-0.33
β^u_D	-0.28	0.27	-0.00	-0.28	-0.01	-0.02	0.15	-0.24	-0.15	-0.20	-0.18	-0.01	-0.04	0.30	-0.00	-0.00	0.07	0.24	0.26	1.00	-0.04	0.01	0.09	-0.05	-0.09	0.08	0.19	0.20
γ^u_D	-0.29	-0.55	0.01	0.51	-0.00	-0.00	-0.38	0.44	0.26	0.37	0.40	0.06	-0.01	-0.51	0.02	0.02	-0.40	0.02	0.37	-0.04	1.00	-0.05	-0.16	0.15	0.09	-0.45	-0.27	-0.38
α^d	0.06	0.08	-0.00	-0.08	0.00	-0.01	0.06	-0.06	-0.04	-0.05	-0.06	-0.01	0.01	0.07	-0.02	0.02	0.07	-0.01	-0.03	0.01	-0.05	1.00	-0.25	-0.02	-0.01	0.07	0.04	0.06
$\tilde{\alpha}^d$	0.10	0.29	-0.00	-0.27	0.01	-0.00	0.19	-0.24	-0.15	-0.20	-0.20	-0.04	-0.00	0.27	-0.00	-0.00	0.21	0.01	-0.09	0.09	-0.16	-0.25	1.00	-0.07	-0.06	0.23	0.13	0.21
β^d	-0.20	-0.26	-0.01	0.24	0.00	-0.01	-0.18	0.21	0.13	0.17	0.19	0.03	-0.00	-0.24	-0.00	0.02	-0.33	-0.07	0.07	-0.05	0.15	-0.02	-0.07	1.00	-0.81	-0.10	-0.64	0.13
$\tilde{\beta}^d$	-0.06	-0.20	0.02	0.20	-0.01	0.01	-0.14	0.16	0.09	0.13	0.14	0.02	-0.01	-0.19	0.01	-0.01	-0.03	-0.02	0.09	-0.09	0.09	-0.01	-0.06	-0.81	1.00	-0.22	0.54	-0.50
α^d_D	0.31	0.81	-0.02	-0.76	0.01	-0.01	0.54	-0.65	-0.39	-0.55	-0.58	-0.09	0.01	0.76	-0.04	-0.01	0.53	0.02	-0.45	0.08	-0.45	0.07	0.23	-0.10	-0.22	1.00	0.34	0.74
β^d_D	0.12	0.45	-0.00	-0.43	-0.01	-0.01	0.30	-0.37	-0.23	-0.31	-0.33	-0.05	-0.00	0.44	-0.02	-0.01	0.39	-0.03	-0.00	0.19	-0.27	0.04	0.13	-0.64	0.54	0.34	1.00	-0.10
γ^d_D	0.23	0.72	-0.03	-0.68	0.02	-0.01	0.48	-0.58	-0.34	-0.49	-0.52	-0.08	0.01	0.68	-0.02	-0.01	0.43	0.09	-0.33	0.20	-0.38	0.06	0.21	0.13	-0.50	0.74	-0.10	1.00

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In modular-invariant models, significant correlations among parameters naturally arise due to the rigid, symmetry-imposed structure of the theory, even when the number of parameters exceeds the number of observables

These correlations reflect the non-linear modular constraints and the internal geometry of the parameter space, and are not a sign of fine-tuning

$SL(2,\mathbb{Z})$: model with modular-invariant Yukawa matrices. The determinant of the quark Yukawa couplings has a single zero at $\tau = i\infty$ to solve the strong CP problem.

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Parameters

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- 8 - for leptons
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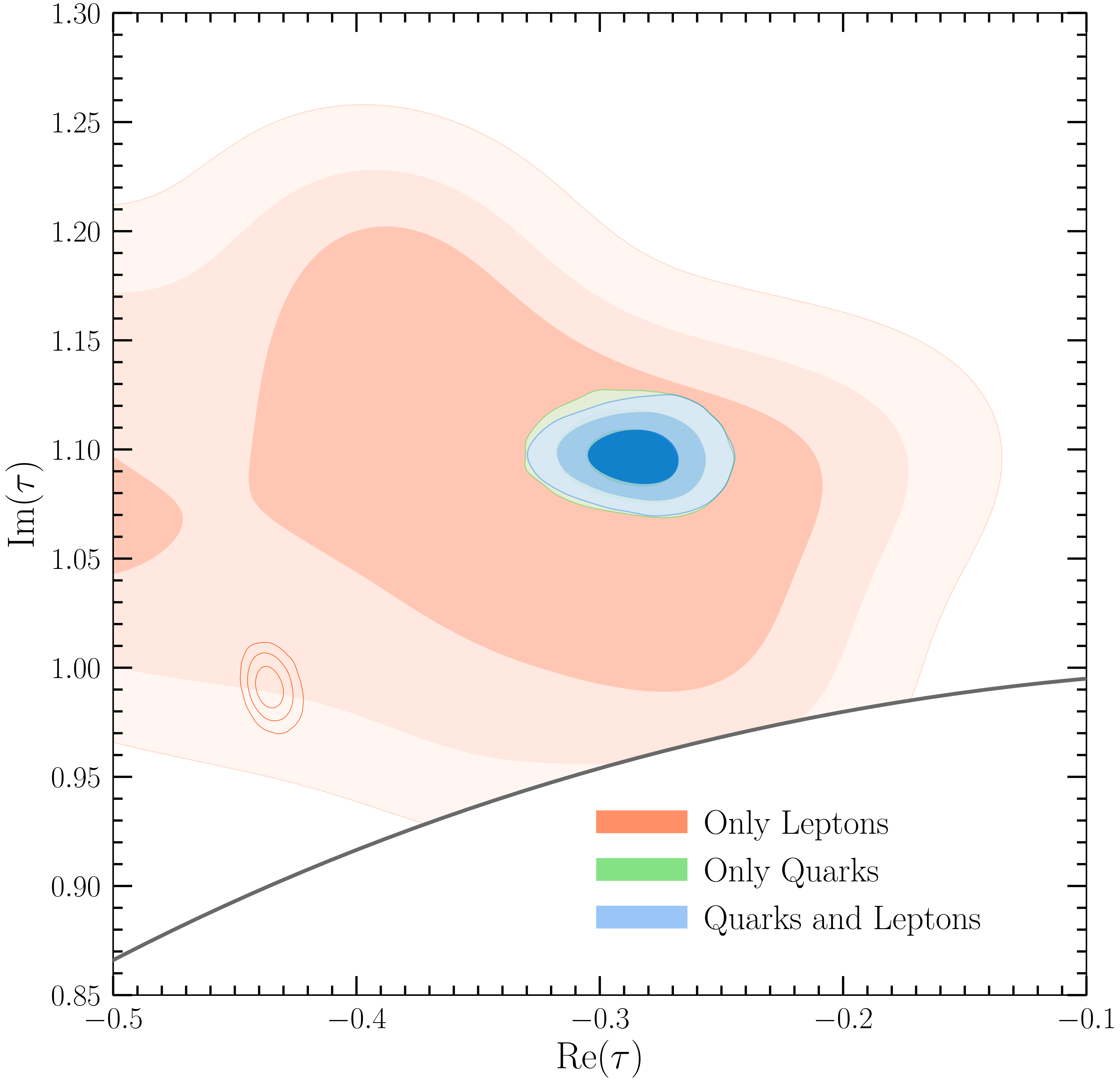
Also in the limiting case $N_{\text{dof}} = 0$ the χ^2 analysis is ill-defined and we can only draw qualitative conclusions from the analysis. The model can be "accepted" only if $\chi^2_{\text{min}} = 0$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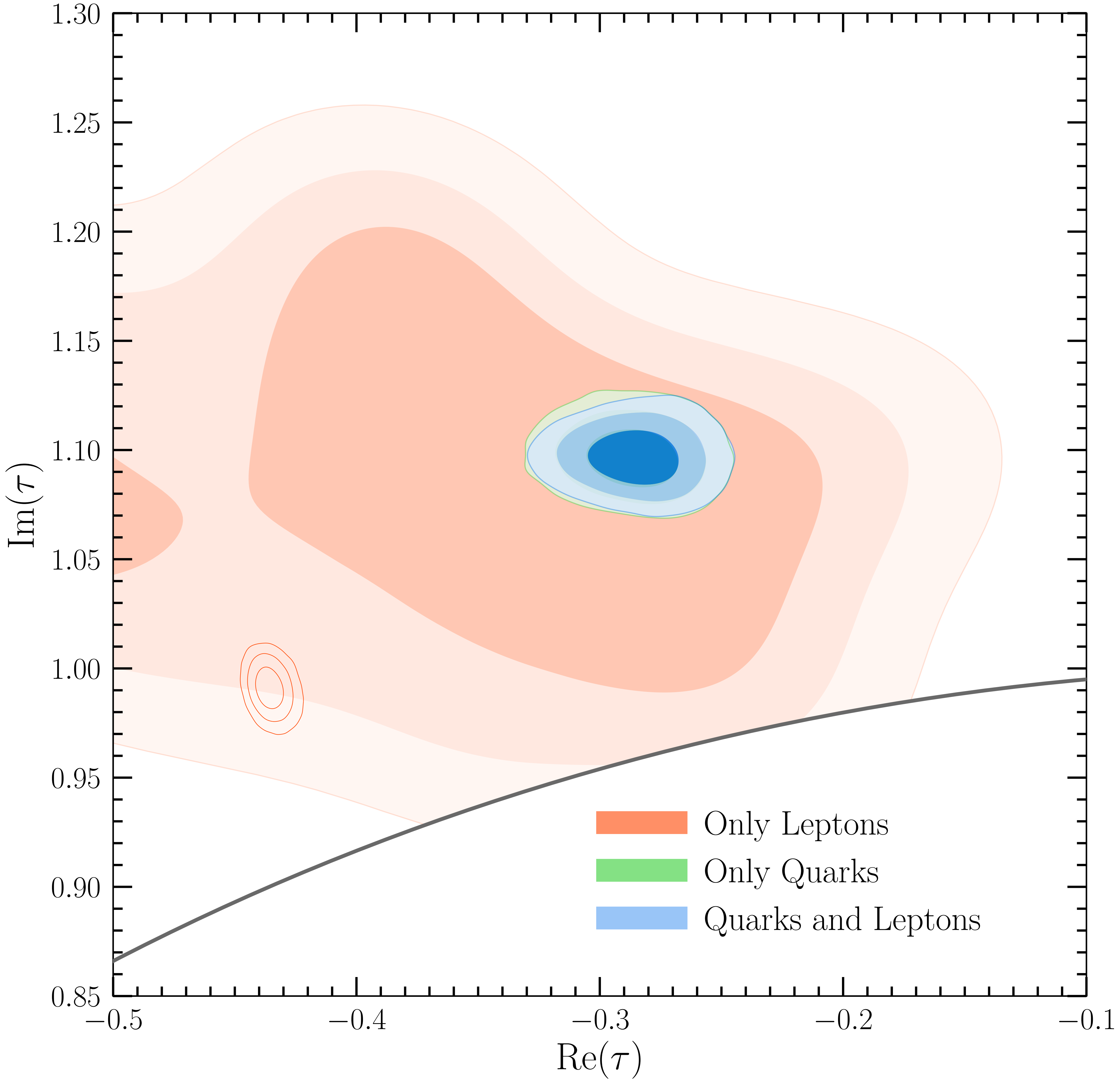
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
Im(τ)
−0.5
−0.4
−0.3
−0.2
−0.1
Re(τ)
Only Leptons
Only Quarks
Quarks and Leptons

Lepton allowed region for τ is much larger than the quark corresponding one, as expected and the combination in this plane essentially coincides with the quark allowed region



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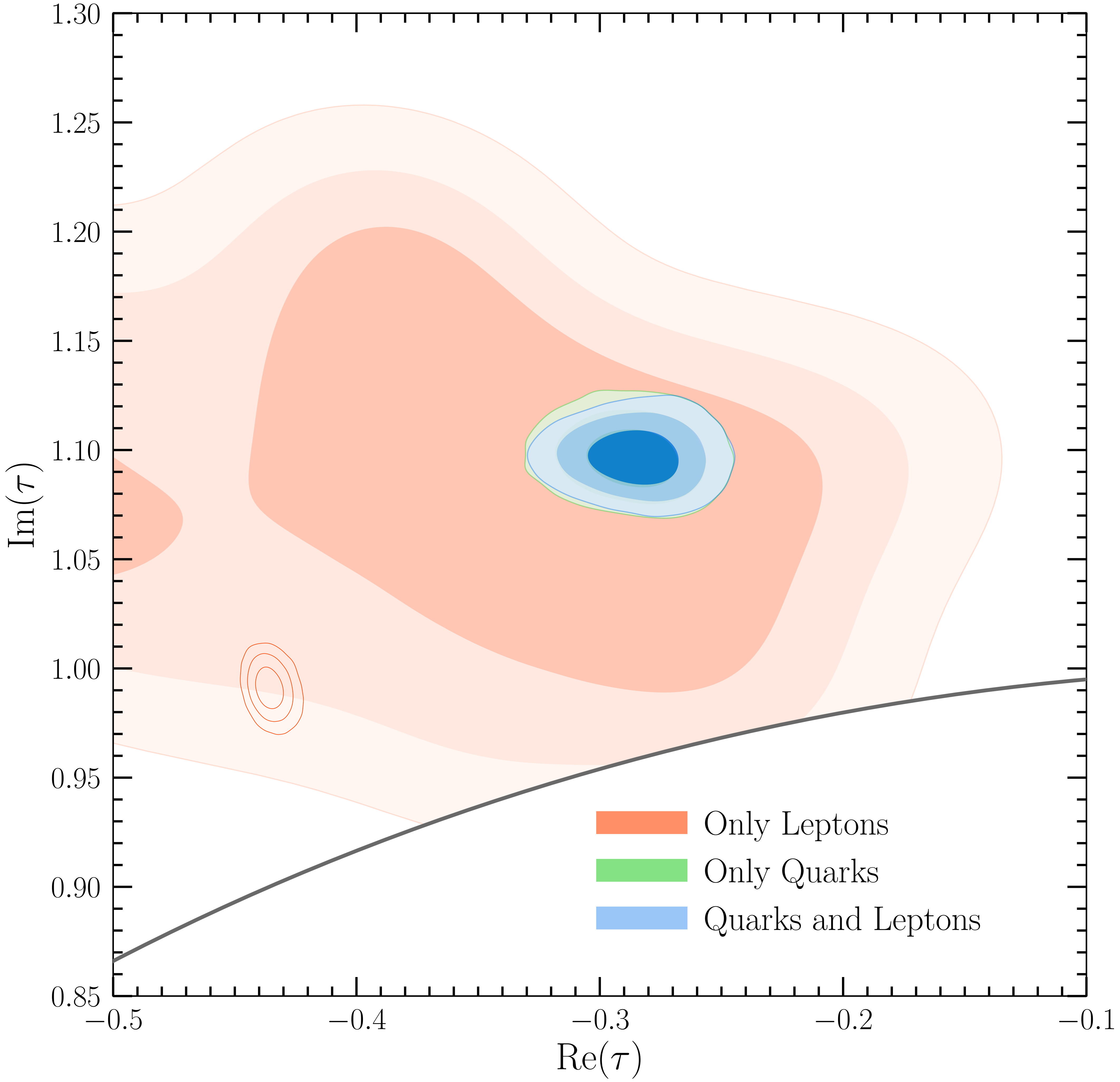
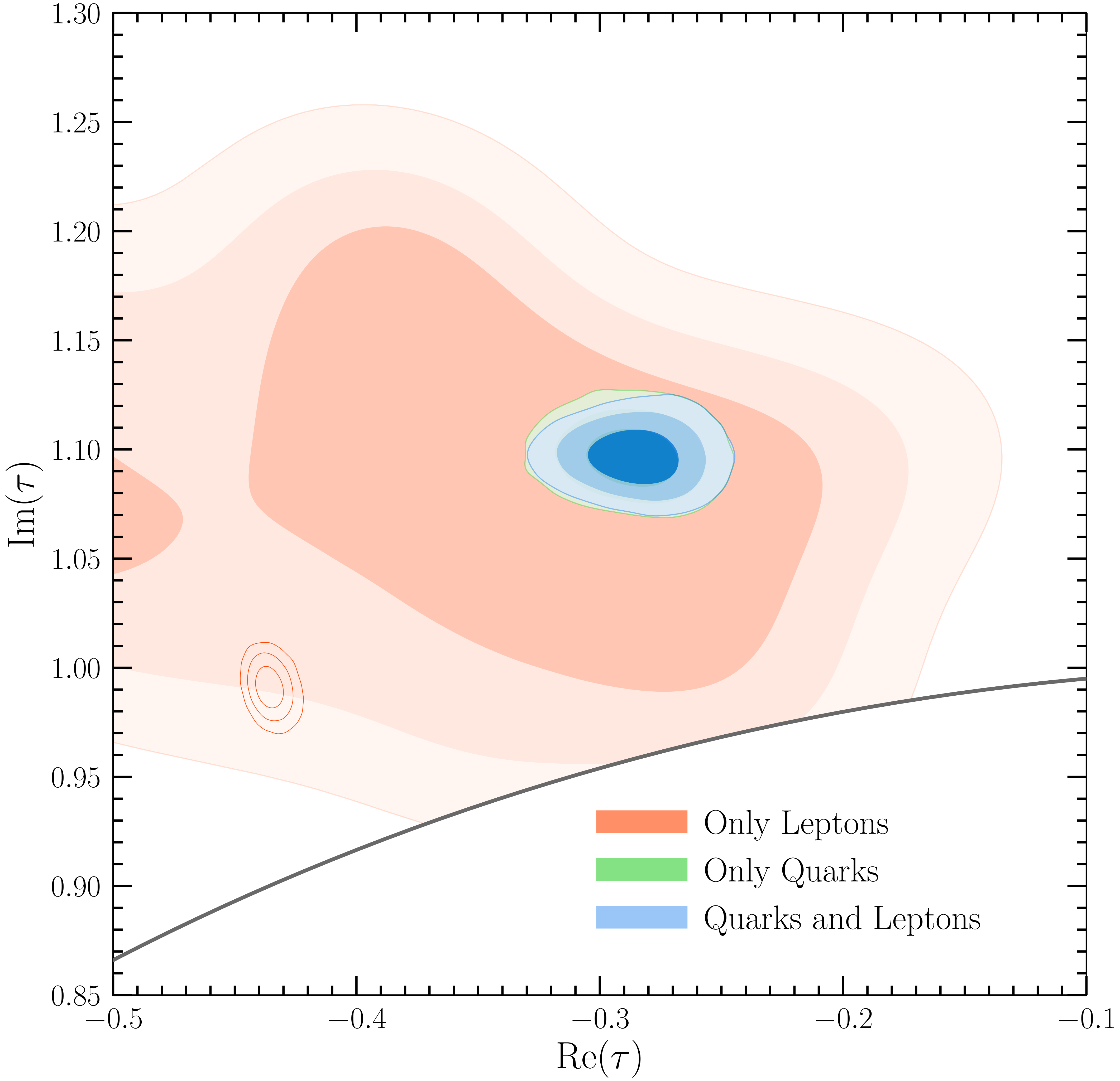


Parameter	Value
Lepton sector	
ℓ_{13}	2.51
ℓ_{23}	1.94
e_{13}	2.21
e_{23}	7.9×10^{-3}
c^e_{33}	5.61
c^ν_{33}	7.6×10^{-2}
$c_{E3}\, c_{L3}$	7.66×10^{-5}
$\frac{c^2_{L3}}{2\Lambda_L}$	$6.60 \times 10^{-2}/10^{16}$ GeV
Quark sector	
q_{13}	3.7×10^{-2}
q_{23}	7.5×10^{-2}
u_{13}	3.5×10^{-2}
u_{23}	19.98
d_{13}	3.44
d_{23}	2.03×10^{-1}
$c_{U3}\, c_{Q3}$	4.15×10^{-4}
$c_{D3}\, c_{Q3}$	4.76×10^{-4}
Modulus	
τ	$-0.286 + 1.096\, i$

Lepton allowed region for τ is much larger than the quark corresponding one, as expected and the combination in this plane essentially coincides with the quark allowed region



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Quark sector	
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$$\chi^2_{\text{min}} \simeq 3.7 \times 10^{-8}$$

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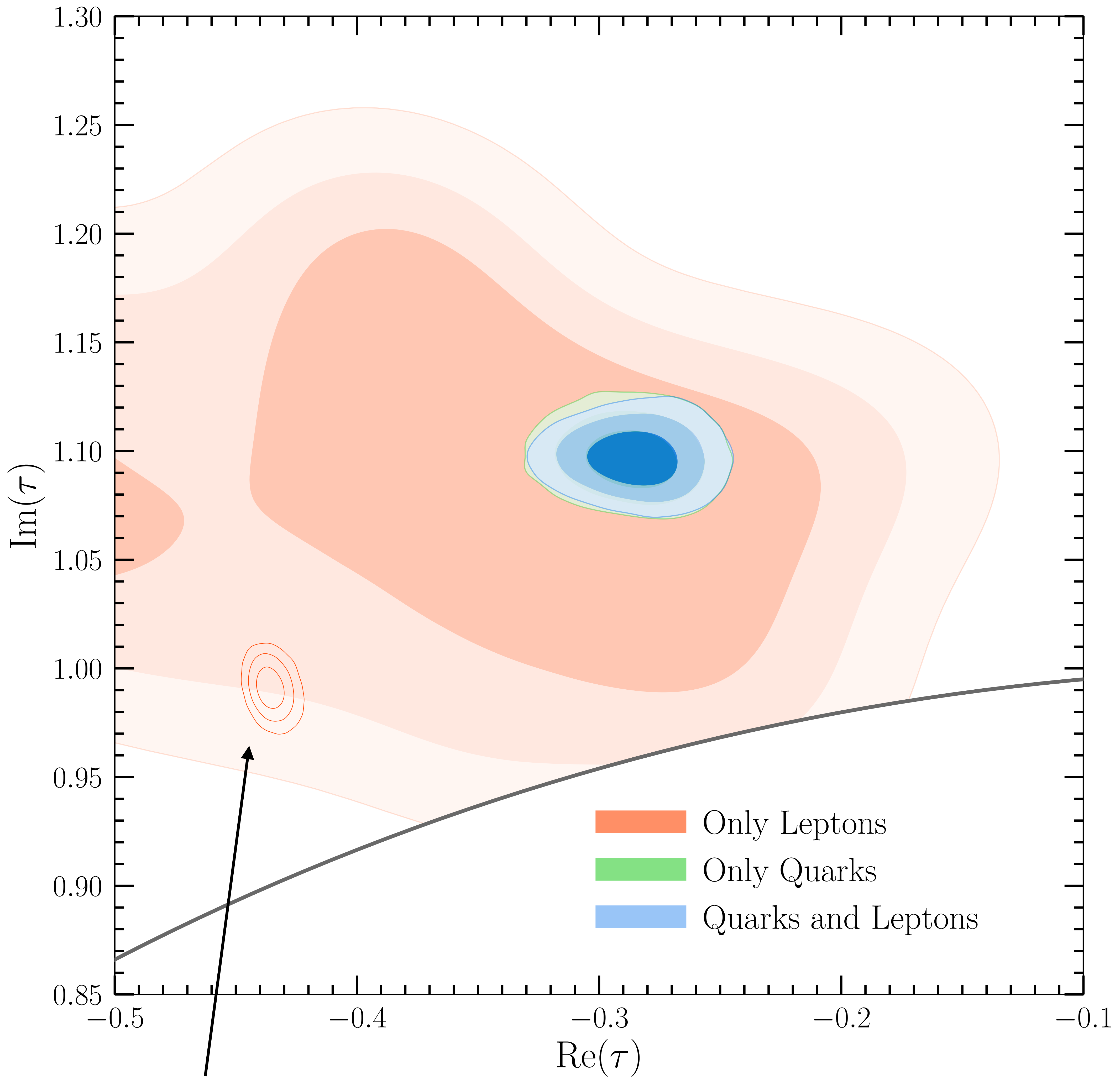

See Feruglio's talk for the explanation of the solid orange lines

Parameter	Value
Lepton sector	
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ℓ_{23}	1.94
e_{13}	2.21
e_{23}	7.9×10^{-3}
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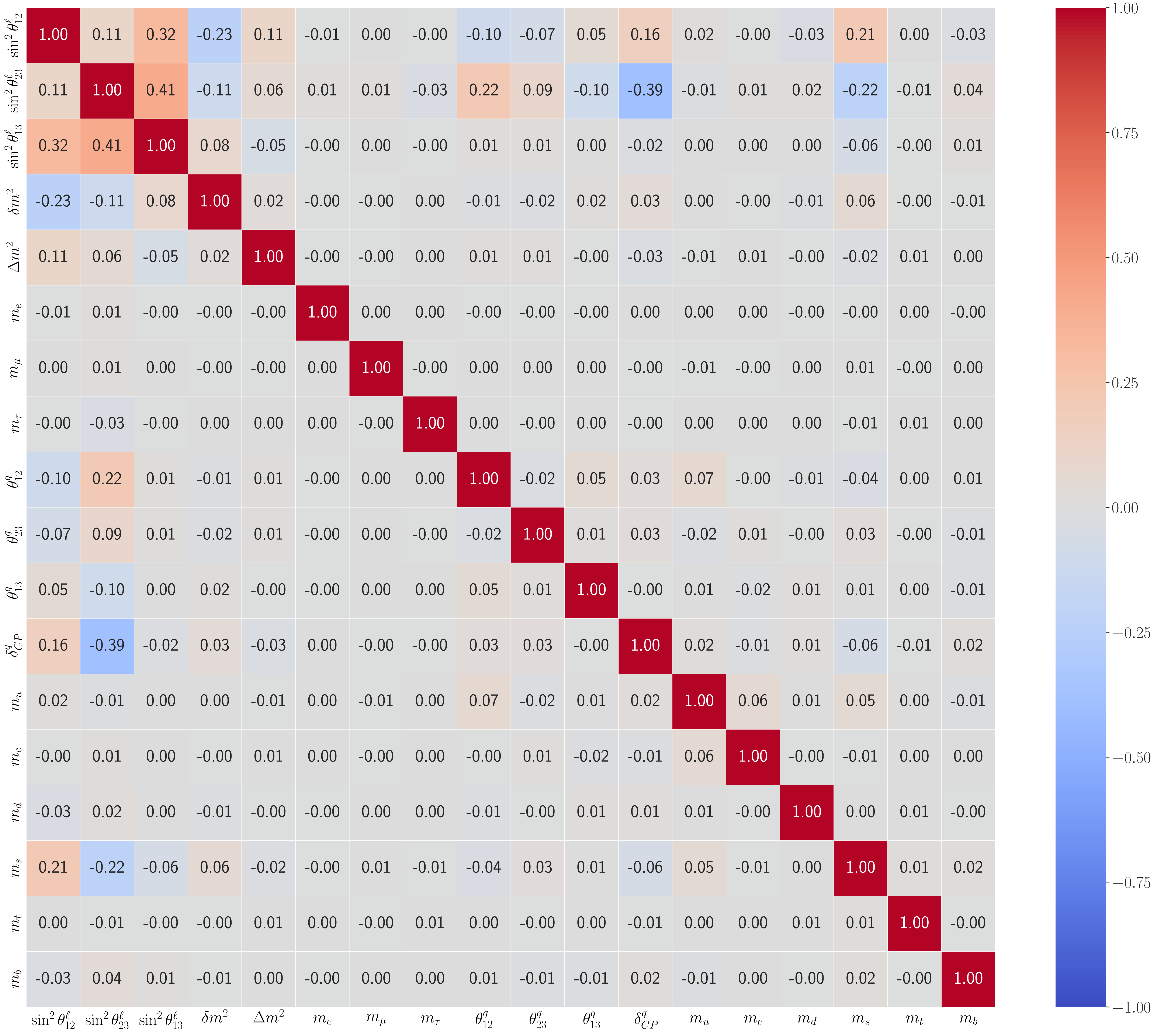
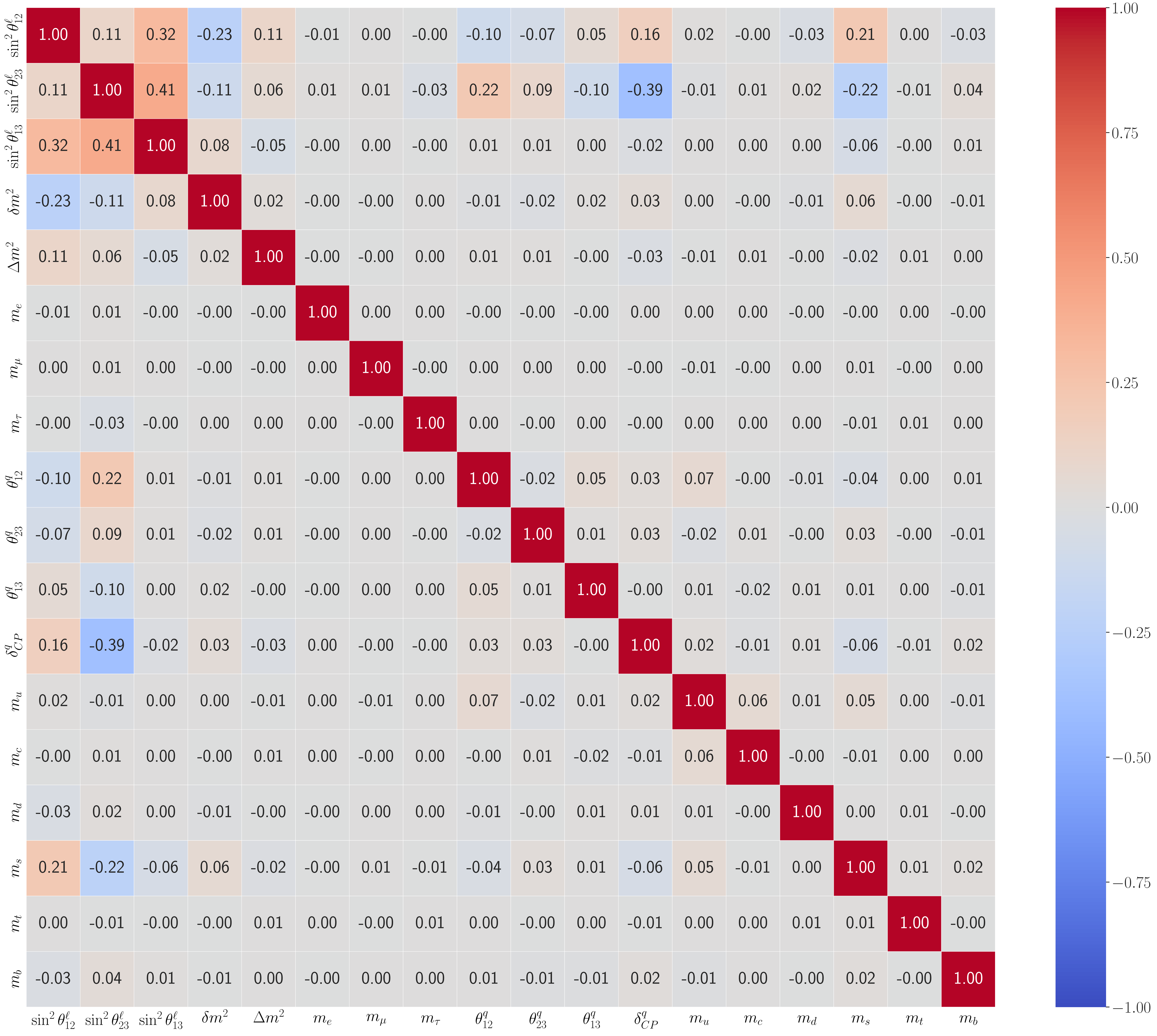
Correlations among Observables

Correlations among Observables



	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	Δm^2	m_e	m_μ	m_τ	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	m_u	m_c	m_d	m_s	m_t	m_b
$\sin^2\theta^\ell_{12}$	1.00	0.11	0.32	-0.23	0.11	-0.01	0.00	-0.00	-0.10	-0.07	0.05	0.16	0.02	-0.00	-0.03	0.21	0.00	-0.03
$\sin^2\theta^\ell_{23}$	0.11	1.00	0.41	-0.11	0.06	0.01	0.01	-0.03	0.22	0.09	-0.10	-0.39	-0.01	0.01	0.02	-0.22	-0.01	0.04
$\sin^2\theta^\ell_{13}$	0.32	0.41	1.00	0.08	-0.05	-0.00	0.00	-0.00	0.01	0.01	0.00	-0.02	0.00	0.00	0.00	-0.06	-0.00	0.01
δm^2	-0.23	-0.11	0.08	1.00	0.02	-0.00	-0.00	0.00	-0.01	-0.02	0.02	0.03	0.00	-0.00	-0.01	0.06	-0.00	-0.01
Δm^2	0.11	0.06	-0.05	0.02	1.00	-0.00	-0.00	0.00	0.01	0.01	-0.00	-0.03	-0.01	0.01	-0.00	-0.02	0.01	0.00
m_e	-0.01	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.00	0.00	-0.00
m_μ	0.00	0.01	0.00	-0.00	-0.00	0.00	1.00	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	0.00	0.01	-0.00	0.00
m_τ	-0.00	-0.03	-0.00	0.00	0.00	0.00	-0.00	1.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.01	0.01	0.00
θ^q_{12}	-0.10	0.22	0.01	-0.01	0.01	-0.00	0.00	0.00	1.00	-0.02	0.05	0.03	0.07	-0.00	-0.01	-0.04	0.00	0.01
θ^q_{23}	-0.07	0.09	0.01	-0.02	0.01	-0.00	0.00	-0.00	-0.02	1.00	0.01	0.03	-0.02	0.01	-0.00	0.03	-0.00	-0.01
θ^q_{13}	0.05	-0.10	0.00	0.02	-0.00	-0.00	0.00	0.00	0.05	0.01	1.00	-0.00	0.01	-0.02	0.01	0.01	0.00	-0.01
δ^q_{CP}	0.16	-0.39	-0.02	0.03	-0.03	0.00	-0.00	-0.00	0.03	0.03	-0.00	1.00	0.02	-0.01	0.01	-0.06	-0.01	0.02
m_u	0.02	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.07	-0.02	0.01	0.02	1.00	0.06	0.01	0.05	0.00	-0.01
m_c	-0.00	0.01	0.00	-0.00	0.01	0.00	-0.00	0.00	-0.00	0.01	-0.02	-0.01	0.06	1.00	-0.00	-0.01	0.00	0.00
m_d	-0.03	0.02	0.00	-0.01	-0.00	-0.00	0.00	0.00	-0.01	-0.00	0.01	0.01	0.01	-0.00	1.00	0.00	0.01	-0.00
m_s	0.21	-0.22	-0.06	0.06	-0.02	-0.00	0.01	-0.01	-0.04	0.03	0.01	-0.06	0.05	-0.01	0.00	1.00	0.01	0.02
m_t	0.00	-0.01	-0.00	-0.00	0.01	0.00	-0.00	0.01	0.00	-0.00	0.00	-0.01	0.00	0.00	0.01	0.01	1.00	-0.00
m_b	-0.03	0.04	0.01	-0.01	0.00	-0.00	0.00	0.00	0.01	-0.01	-0.01	0.02	-0.01	0.00	-0.00	0.02	-0.00	1.00

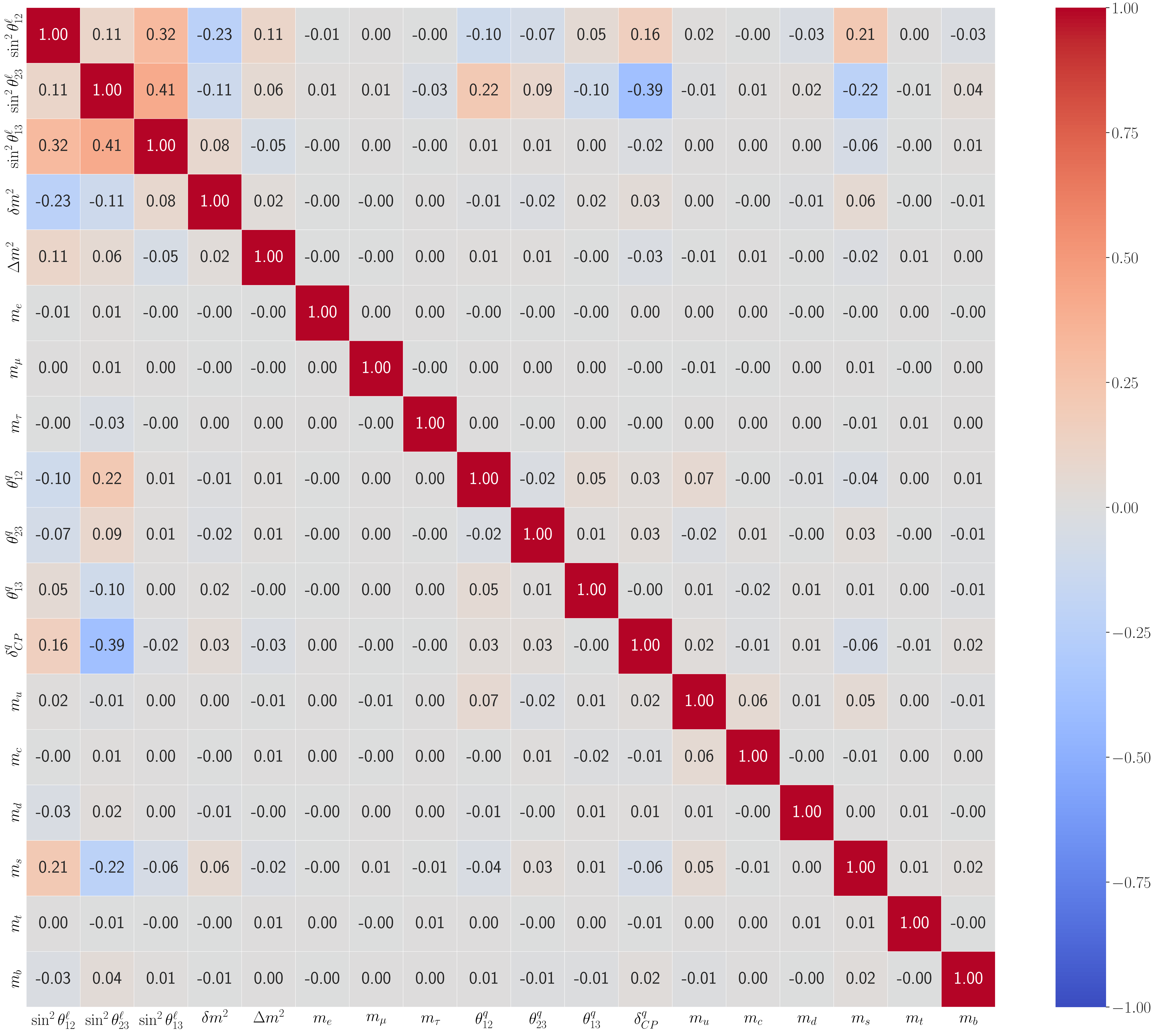
Correlations among Observables



	$\sin^2\theta_{12}^\ell$	$\sin^2\theta_{23}^\ell$	$\sin^2\theta_{13}^\ell$	δm^2	Δm^2	m_e	m_μ	m_τ	θ_{12}^q	θ_{23}^q	θ_{13}^q	δ_{CP}^q	m_u	m_c	m_d	m_s	m_t	m_b
$\sin^2\theta_{12}^\ell$	1.00	0.11	0.32	-0.23	0.11	-0.01	0.00	-0.00	-0.10	-0.07	0.05	0.16	0.02	-0.00	-0.03	0.21	0.00	-0.03
$\sin^2\theta_{23}^\ell$	0.11	1.00	0.41	-0.11	0.06	0.01	0.01	-0.03	0.22	0.09	-0.10	-0.39	-0.01	0.01	0.02	-0.22	-0.01	0.04
$\sin^2\theta_{13}^\ell$	0.32	0.41	1.00	0.08	-0.05	-0.00	0.00	-0.00	0.01	0.01	0.00	-0.02	0.00	0.00	0.00	-0.06	-0.00	0.01
δm^2	-0.23	-0.11	0.08	1.00	0.02	-0.00	-0.00	0.00	-0.01	-0.02	0.02	0.03	0.00	-0.00	-0.01	0.06	-0.00	-0.01
Δm^2	0.11	0.06	-0.05	0.02	1.00	-0.00	-0.00	0.00	0.01	0.01	-0.00	-0.03	-0.01	0.01	-0.00	-0.02	0.01	0.00
m_e	-0.01	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.00	0.00	-0.00
m_μ	0.00	0.01	0.00	-0.00	-0.00	0.00	1.00	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	0.00	0.01	-0.00	0.00
m_τ	-0.00	-0.03	-0.00	0.00	0.00	0.00	-0.00	1.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.01	0.01	0.00
θ_{12}^q	-0.10	0.22	0.01	-0.01	0.01	-0.00	0.00	0.00	1.00	-0.02	0.05	0.03	0.07	-0.00	-0.01	-0.04	0.00	0.01
θ_{23}^q	-0.07	0.09	0.01	-0.02	0.01	-0.00	0.00	-0.00	-0.02	1.00	0.01	0.03	-0.02	0.01	-0.00	0.03	-0.00	-0.01
θ_{13}^q	0.05	-0.10	0.00	0.02	-0.00	-0.00	0.00	0.00	0.05	0.01	1.00	-0.00	0.01	-0.02	0.01	0.01	0.00	-0.01
δ_{CP}^q	0.16	-0.39	-0.02	0.03	-0.03	0.00	-0.00	-0.00	0.03	0.03	-0.00	1.00	0.02	-0.01	0.01	-0.06	-0.01	0.02
m_u	0.02	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.07	-0.02	0.01	0.02	1.00	0.06	0.01	0.05	0.00	-0.01
m_c	-0.00	0.01	0.00	-0.00	0.01	0.00	-0.00	0.00	-0.00	0.01	-0.02	-0.01	0.06	1.00	-0.00	-0.01	0.00	0.00
m_d	-0.03	0.02	0.00	-0.01	-0.00	-0.00	0.00	0.00	-0.01	-0.00	0.01	0.01	0.01	-0.00	1.00	0.00	0.01	-0.00
m_s	0.21	-0.22	-0.06	0.06	-0.02	-0.00	0.01	-0.01	-0.04	0.03	0.01	-0.06	0.05	-0.01	0.00	1.00	0.01	0.02
m_t	0.00	-0.01	-0.00	-0.00	0.01	0.00	-0.00	0.01	0.00	-0.00	0.00	-0.01	0.00	0.00	0.01	0.01	1.00	-0.00
m_b	-0.03	0.04	0.01	-0.01	0.00	-0.00	0.00	0.00	0.01	-0.01	-0.01	0.02	-0.01	0.00	-0.00	0.02	-0.00	1.00

Notably, non-negligible correlations between observables begin to emerge, also for observables in different sectors

Correlations among Observables



	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	Δm^2	m_e	m_μ	m_τ	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	m_u	m_c	m_d	m_s	m_t	m_b
$\sin^2\theta^\ell_{12}$	1.00	0.11	0.32	-0.23	0.11	-0.01	0.00	-0.00	-0.10	-0.07	0.05	0.16	0.02	-0.00	-0.03	0.21	0.00	-0.03
$\sin^2\theta^\ell_{23}$	0.11	1.00	0.41	-0.11	0.06	0.01	0.01	-0.03	0.22	0.09	-0.10	-0.39	-0.01	0.01	0.02	-0.22	-0.01	0.04
$\sin^2\theta^\ell_{13}$	0.32	0.41	1.00	0.08	-0.05	-0.00	0.00	-0.00	0.01	0.01	0.00	-0.02	0.00	0.00	0.00	-0.06	-0.00	0.01
δm^2	-0.23	-0.11	0.08	1.00	0.02	-0.00	-0.00	0.00	-0.01	-0.02	0.02	0.03	0.00	-0.00	-0.01	0.06	-0.00	-0.01
Δm^2	0.11	0.06	-0.05	0.02	1.00	-0.00	-0.00	0.00	0.01	0.01	-0.00	-0.03	-0.01	0.01	-0.00	-0.02	0.01	0.00
m_e	-0.01	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.00	0.00	-0.00
m_μ	0.00	0.01	0.00	-0.00	-0.00	0.00	1.00	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	0.00	0.01	-0.00	0.00
m_τ	-0.00	-0.03	-0.00	0.00	0.00	0.00	-0.00	1.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.01	0.01	0.00
θ^q_{12}	-0.10	0.22	0.01	-0.01	0.01	-0.00	0.00	0.00	1.00	-0.02	0.05	0.03	0.07	-0.00	-0.01	-0.04	0.00	0.01
θ^q_{23}	-0.07	0.09	0.01	-0.02	0.01	-0.00	0.00	-0.00	-0.02	1.00	0.01	0.03	-0.02	0.01	-0.00	0.03	-0.00	-0.01
θ^q_{13}	0.05	-0.10	0.00	0.02	-0.00	-0.00	0.00	0.00	0.05	0.01	1.00	-0.00	0.01	-0.02	0.01	0.01	0.00	-0.01
δ^q_{CP}	0.16	-0.39	-0.02	0.03	-0.03	0.00	-0.00	-0.00	0.03	0.03	-0.00	1.00	0.02	-0.01	0.01	-0.06	-0.01	0.02
m_u	0.02	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.07	-0.02	0.01	0.02	1.00	0.06	0.01	0.05	0.00	-0.01
m_c	-0.00	0.01	0.00	-0.00	0.01	0.00	-0.00	0.00	-0.00	0.01	-0.02	-0.01	0.06	1.00	-0.00	-0.01	0.00	0.00
m_d	-0.03	0.02	0.00	-0.01	-0.00	-0.00	0.00	0.00	-0.01	-0.00	0.01	0.01	0.01	-0.00	1.00	0.00	0.01	-0.00
m_s	0.21	-0.22	-0.06	0.06	-0.02	-0.00	0.01	-0.01	-0.04	0.03	0.01	-0.06	0.05	-0.01	0.00	1.00	0.01	0.02
m_t	0.00	-0.01	-0.00	-0.00	0.01	0.00	-0.00	0.01	0.00	-0.00	0.00	-0.01	0.00	0.00	0.01	0.01	1.00	-0.00
m_b	-0.03	0.04	0.01	-0.01	0.00	-0.00	0.00	0.00	0.01	-0.01	-0.01	0.02	-0.01	0.00	-0.00	0.02	-0.00	1.00

Notably, non-negligible correlations between observables begin to emerge, also for observables in different sectors

Among the most correlated observables couples

$$(\sin^2\theta^\ell_{23}, \sin^2\theta^\ell_{13})$$

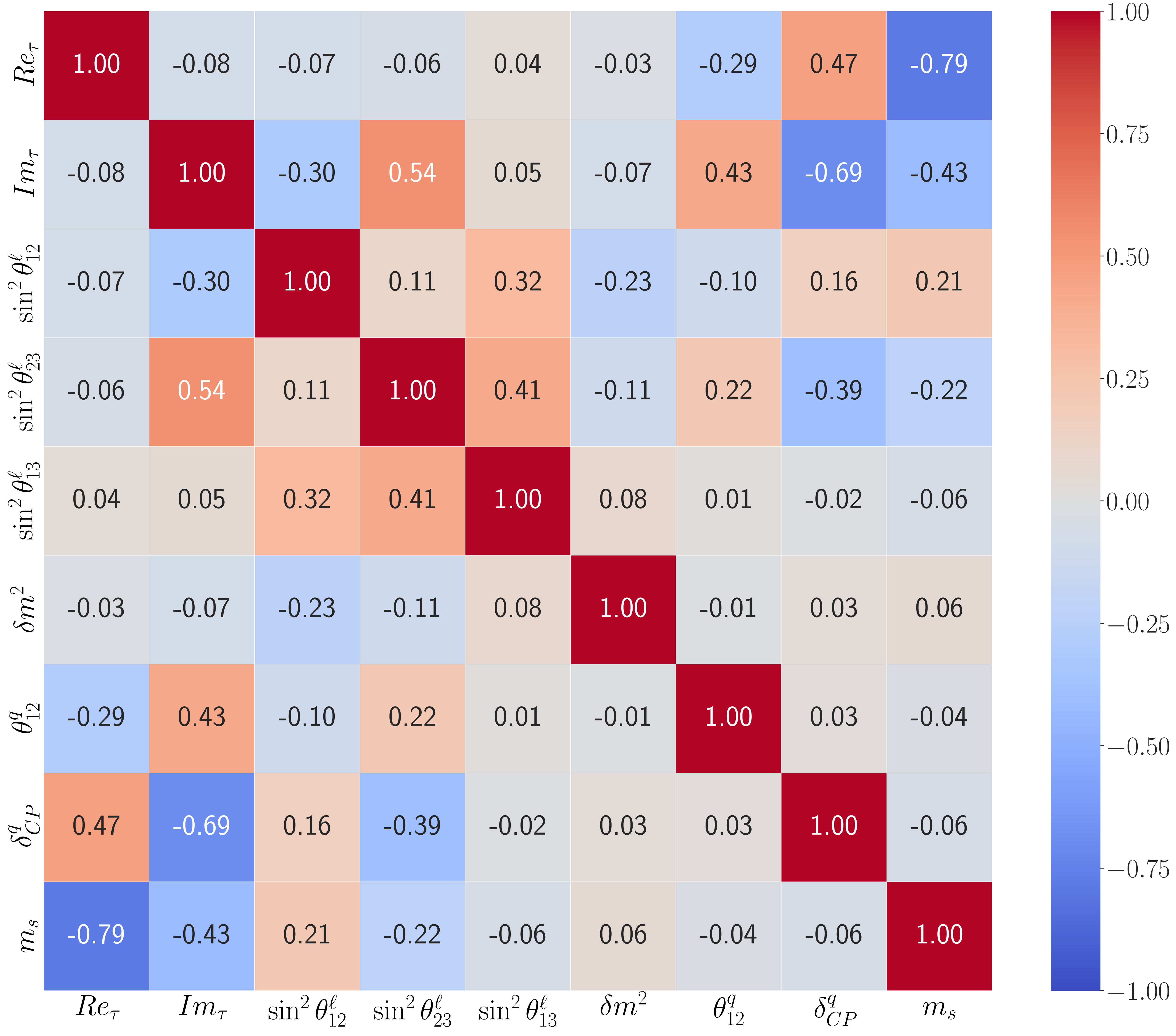
$$(\sin^2\theta^\ell_{23}, \delta^q_{CP})$$

$$(\sin^2\theta^\ell_{23}, m_s)$$

$$(\sin^2\theta^\ell_{12}, \delta m^2)$$

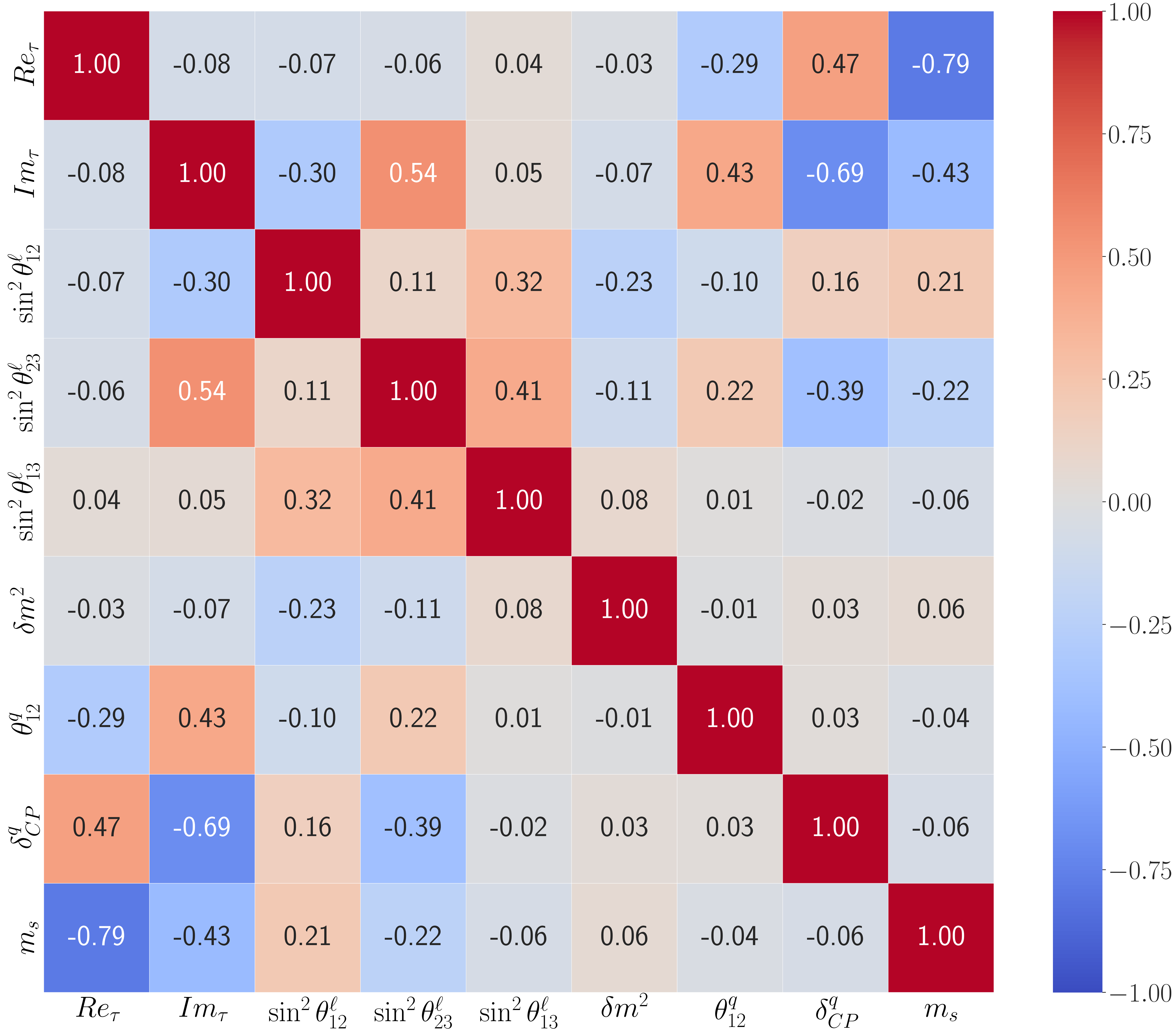
Most correlated Observables

Most correlated Observables



	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

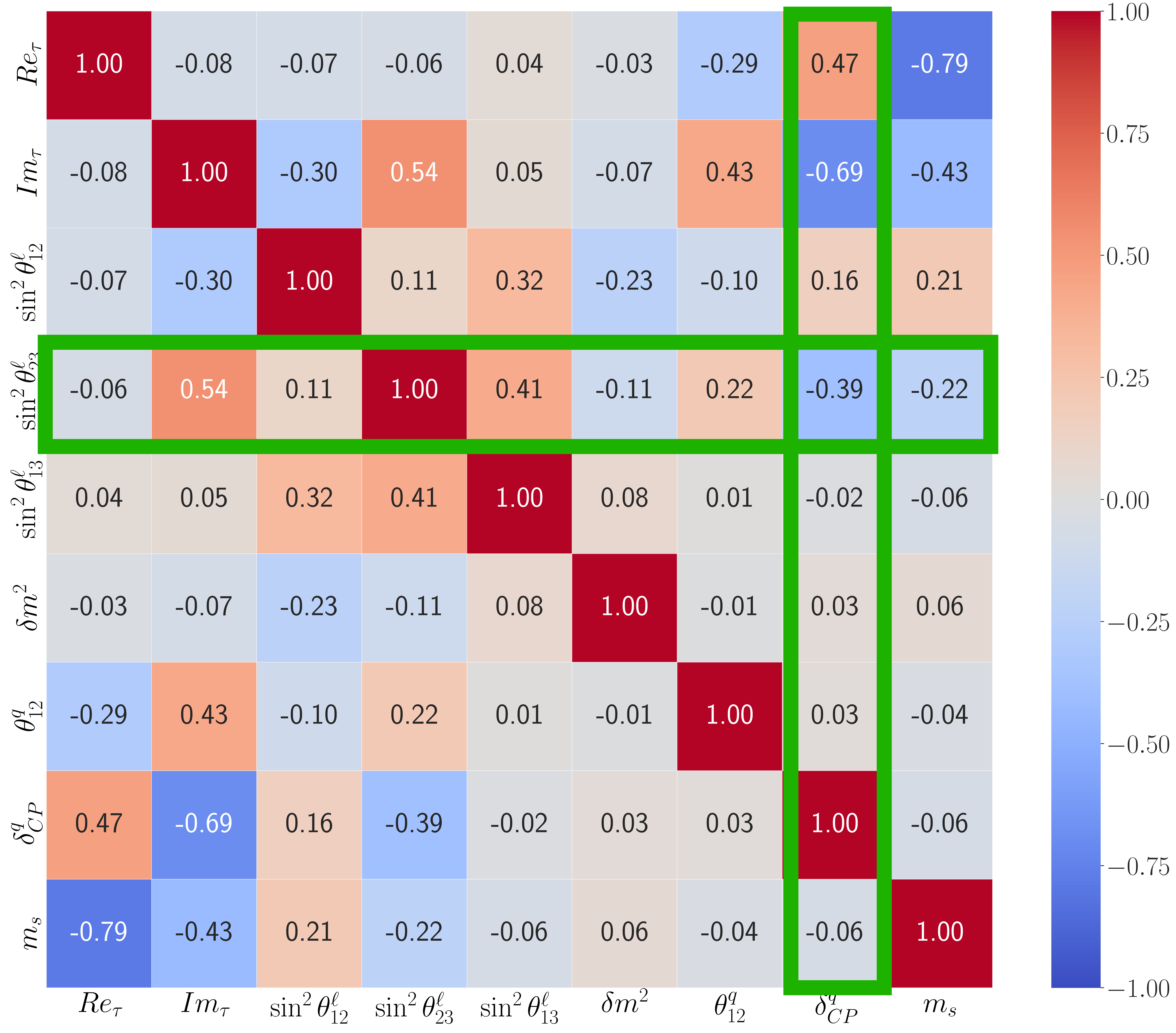
Most correlated Observables



	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

Most correlated Observables

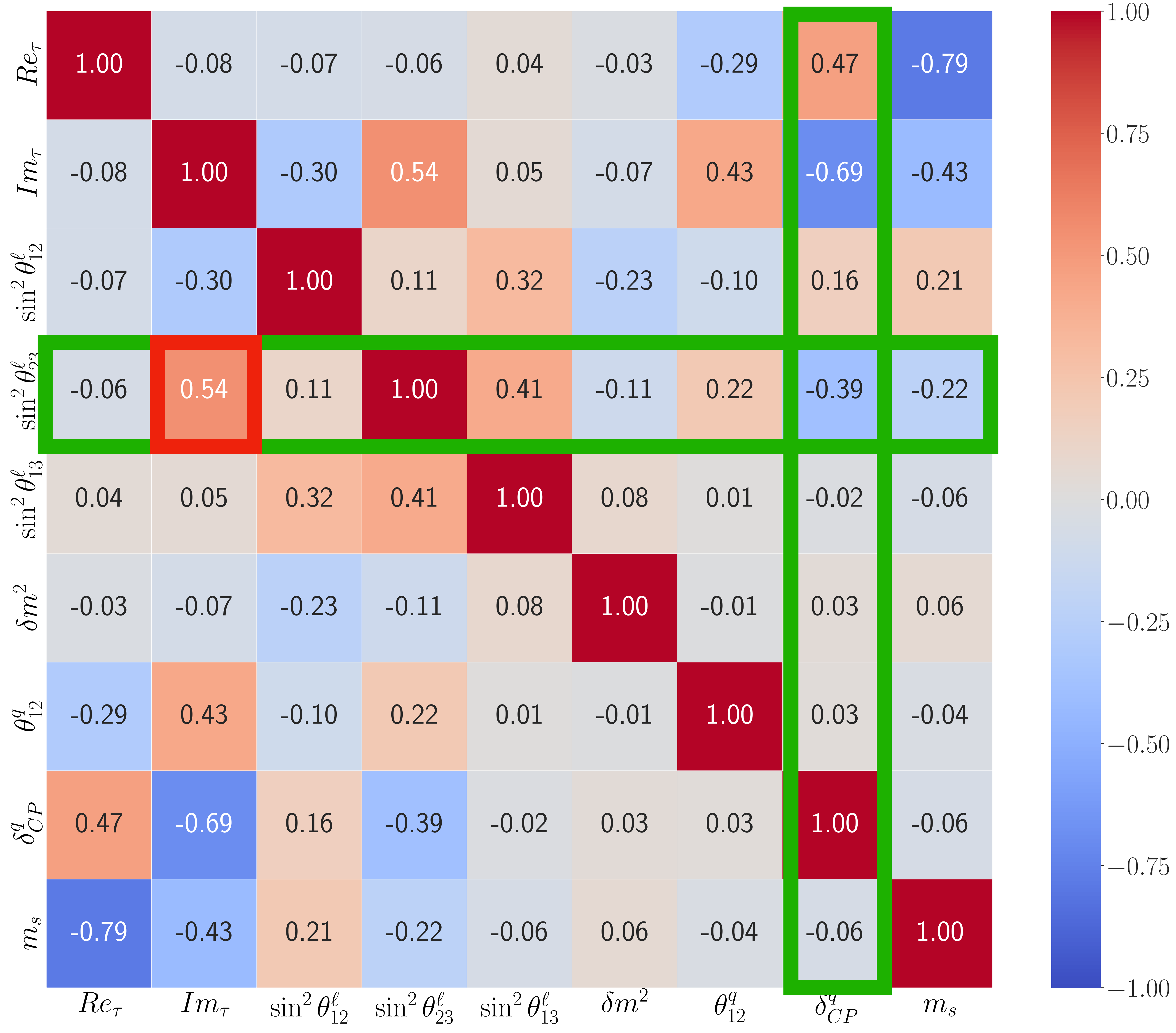


	Re_τ	Im_τ	$\sin^2\theta_{12}^\ell$	$\sin^2\theta_{23}^\ell$	$\sin^2\theta_{13}^\ell$	δm^2	θ_{12}^q	δ_{CP}^q	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta_{12}^\ell$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta_{23}^\ell$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta_{13}^\ell$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ_{12}^q	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ_{CP}^q	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

Consider for instance the pair $(\sin^2\theta_{23}^\ell, \delta_{\text{CP}}^q)$

Most correlated Observables



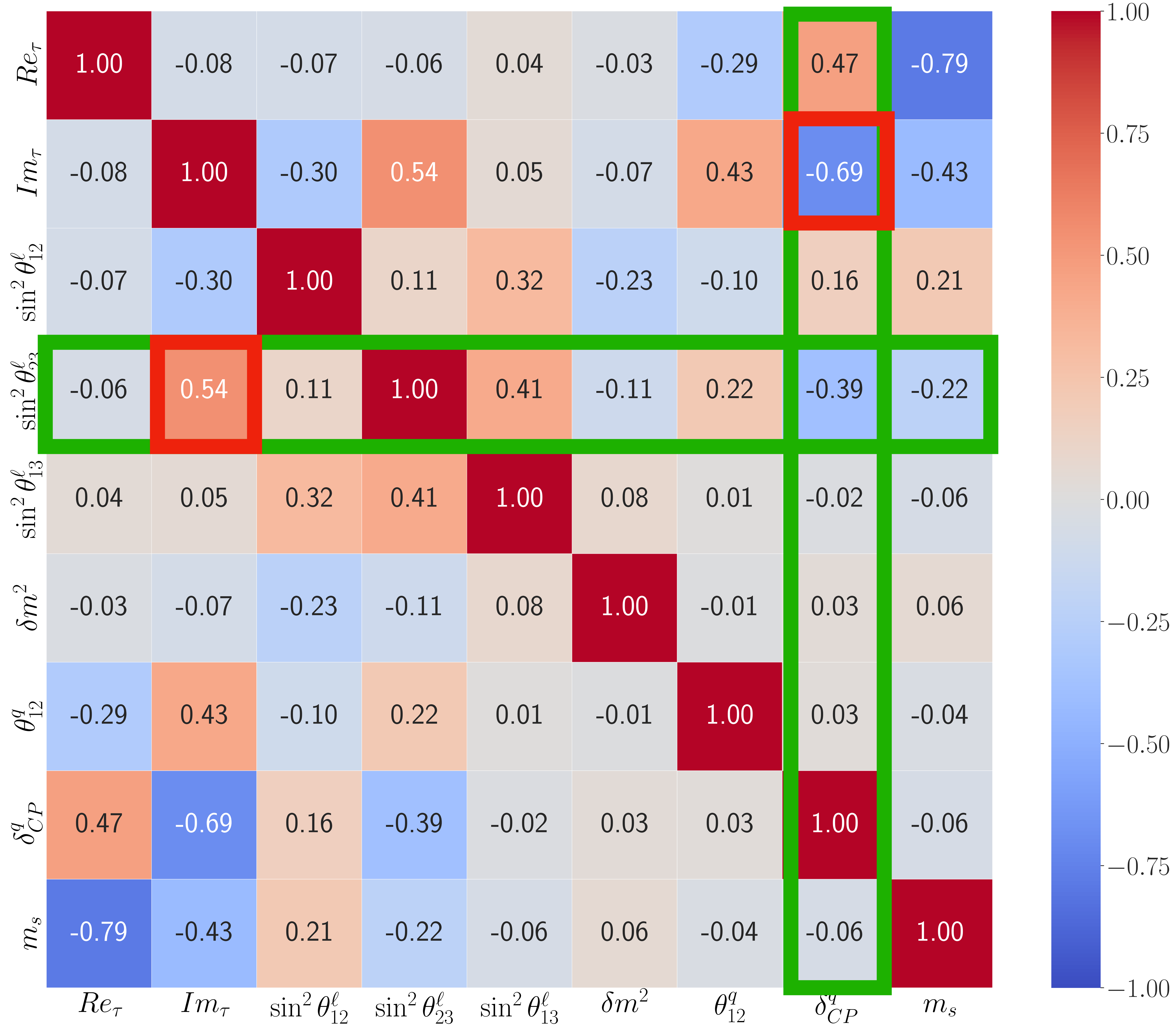
	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

Consider for instance the pair $(\sin^2\theta^\ell_{23}, \delta^q_{\mathrm{CP}})$

$\sin^2\theta^\ell_{23}$ positively correlated to Im(τ)

Most correlated Observables



	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

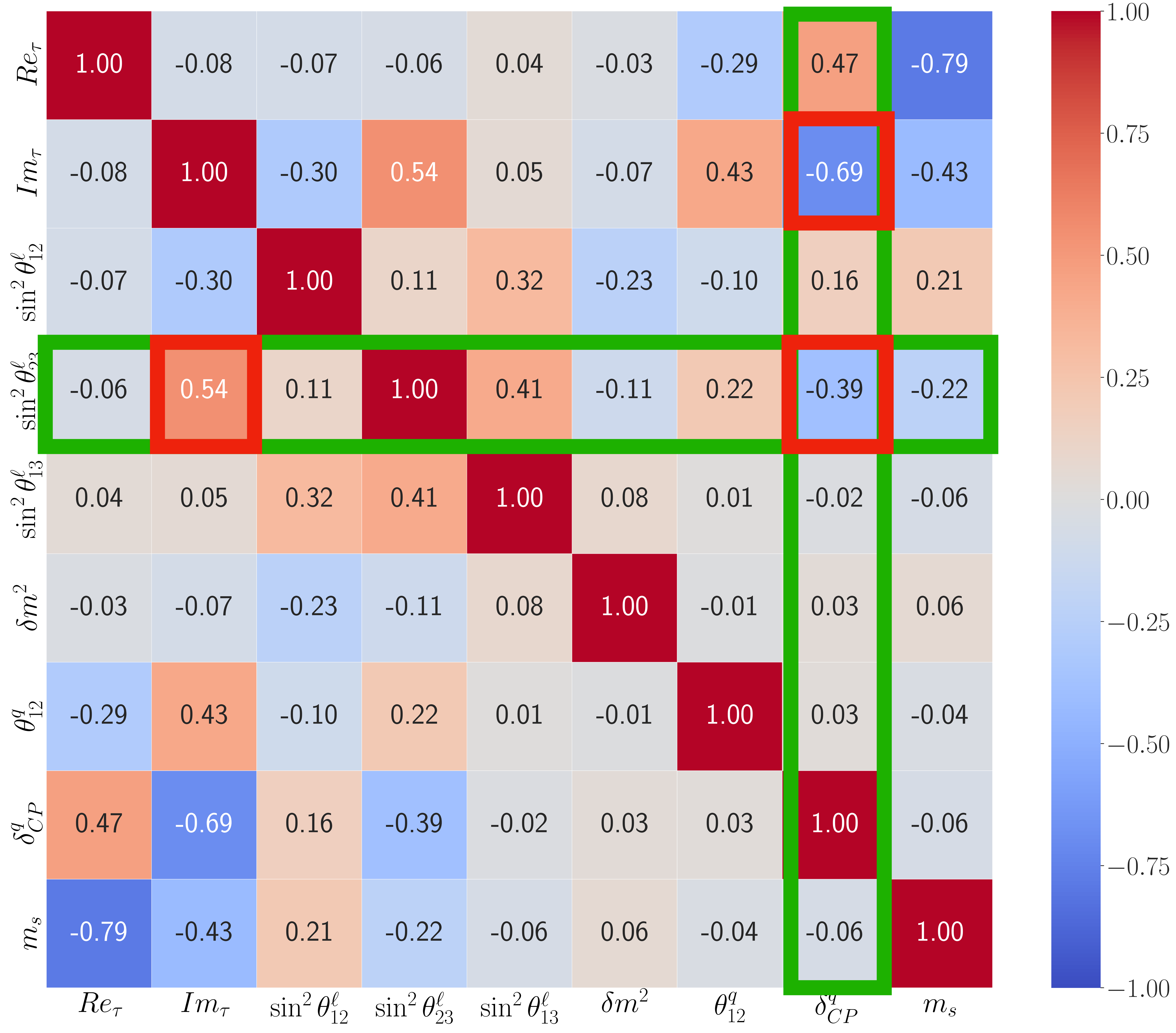
Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

Consider for instance the pair $(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$

$\sin^2\theta^\ell_{23}$ positively correlated to Im(τ)

δ^q_{CP} negatively correlated to Im(τ)

Most correlated Observables



	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

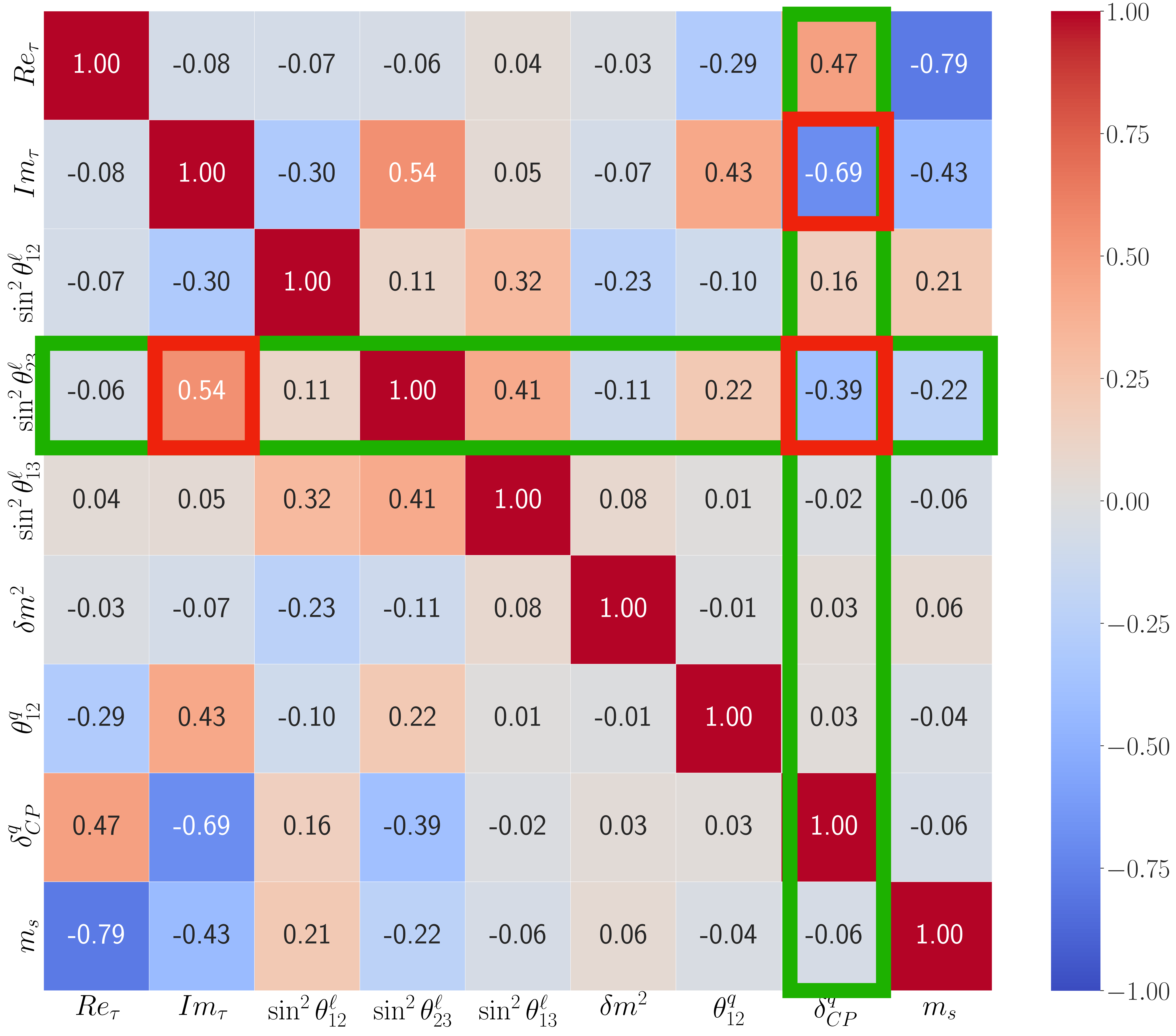
Consider for instance the pair $(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$

$\sin^2\theta^\ell_{23}$ positively correlated to Im(τ)

δ^q_{CP} negatively correlated to Im(τ)

$(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$ are negatively correlated

Most correlated Observables



	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

Consider for instance the pair $(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$

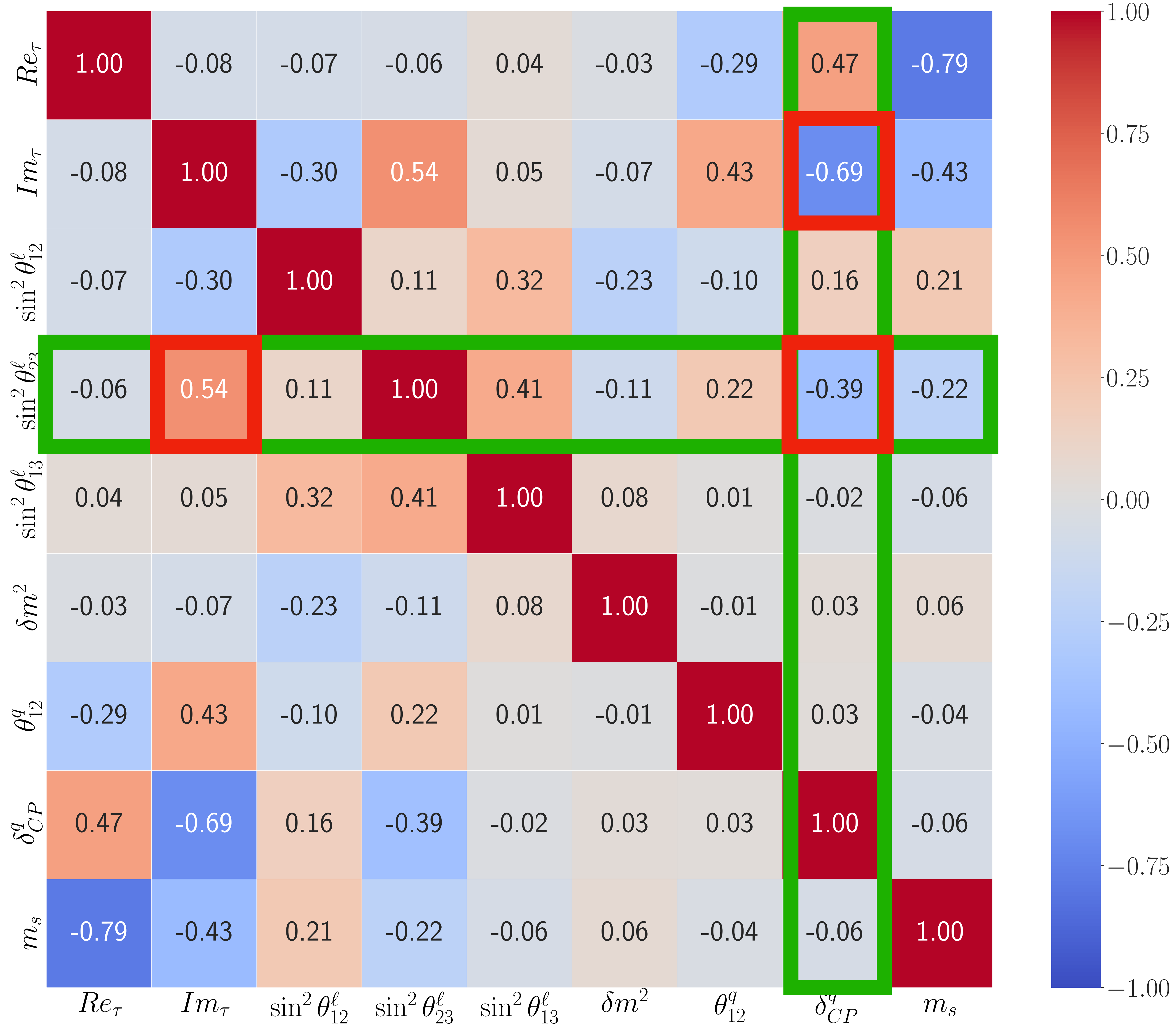
$\sin^2\theta^\ell_{23}$ positively correlated to Im(τ)

δ^q_{CP} negatively correlated to Im(τ)

$(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$ are negatively correlated

Same for m_s, θ^q_{12} and $\sin^2\theta^\ell_{23}$ and so on

Most correlated Observables



	Re_τ	Im_τ	$\sin^2\theta^\ell_{12}$	$\sin^2\theta^\ell_{23}$	$\sin^2\theta^\ell_{13}$	δm^2	θ^q_{12}	δ^q_{CP}	m_s
Re_τ	1.00	-0.08	-0.07	-0.06	0.04	-0.03	-0.29	0.47	-0.79
Im_τ	-0.08	1.00	-0.30	0.54	0.05	-0.07	0.43	-0.69	-0.43
$\sin^2\theta^\ell_{12}$	-0.07	-0.30	1.00	0.11	0.32	-0.23	-0.10	0.16	0.21
$\sin^2\theta^\ell_{23}$	-0.06	0.54	0.11	1.00	0.41	-0.11	0.22	-0.39	-0.22
$\sin^2\theta^\ell_{13}$	0.04	0.05	0.32	0.41	1.00	0.08	0.01	-0.02	-0.06
δm^2	-0.03	-0.07	-0.23	-0.11	0.08	1.00	-0.01	0.03	0.06
θ^q_{12}	-0.29	0.43	-0.10	0.22	0.01	-0.01	1.00	0.03	-0.04
δ^q_{CP}	0.47	-0.69	0.16	-0.39	-0.02	0.03	0.03	1.00	-0.06
m_s	-0.79	-0.43	0.21	-0.22	-0.06	0.06	-0.04	-0.06	1.00

Correlations must be governed by the modulus for lepton-quark correlated pairs of observables

Consider for instance the pair $(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$

$\sin^2\theta^\ell_{23}$ positively correlated to Im(τ)

δ^q_{CP} negatively correlated to Im(τ)

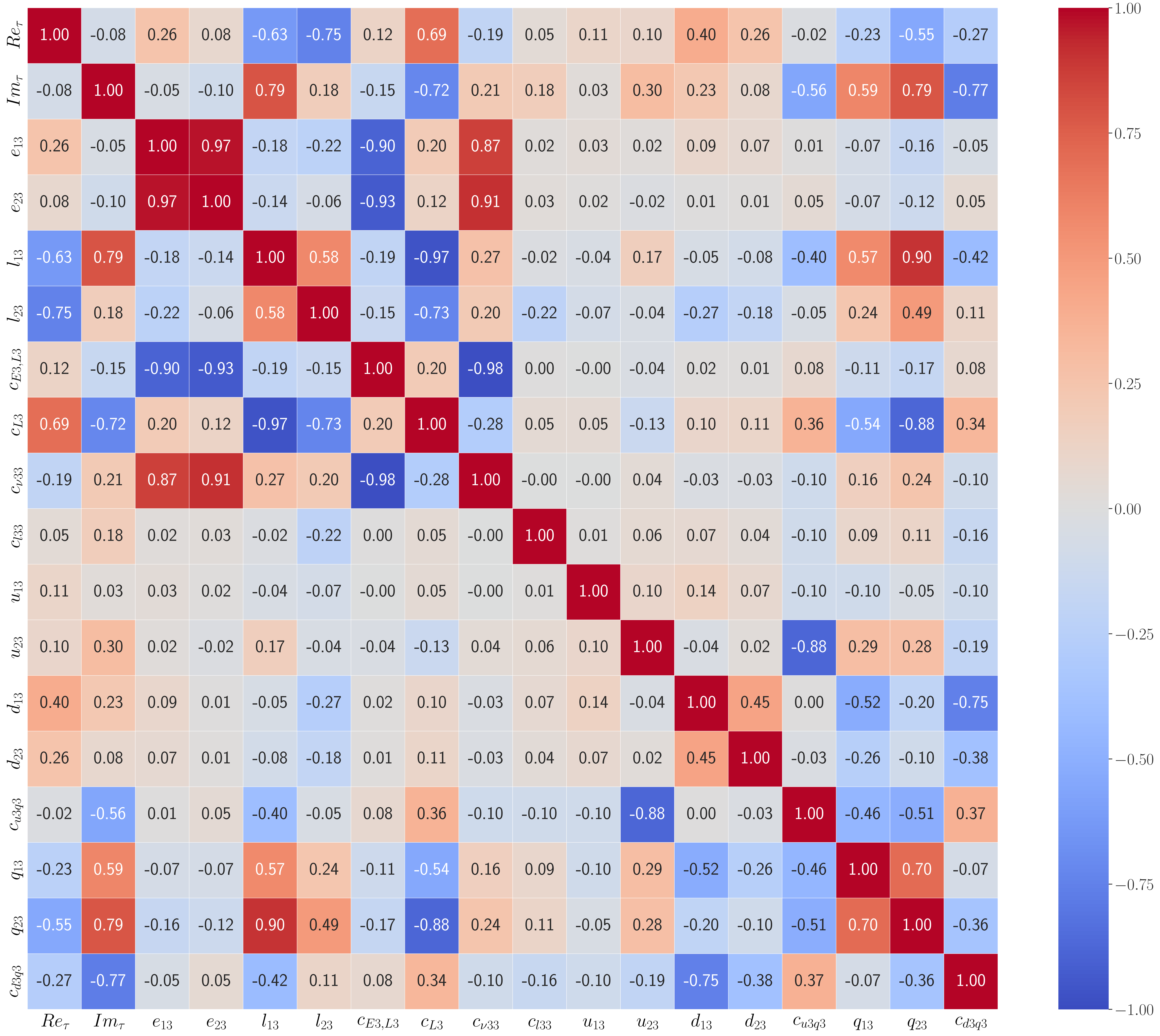
$(\sin^2\theta^\ell_{23}, \delta^q_{\text{CP}})$ are negatively correlated

Same for m_s, θ^q_{12} and $\sin^2\theta^\ell_{23}$ and so on

Things are different when observables are in the same sector, for instance $(\sin^2\theta^\ell_{23}, \sin^2\theta^\ell_{12})$ that are positive correlated but with opposite correlations to Im(τ). In this case one should look at the correlations with the other model parameters

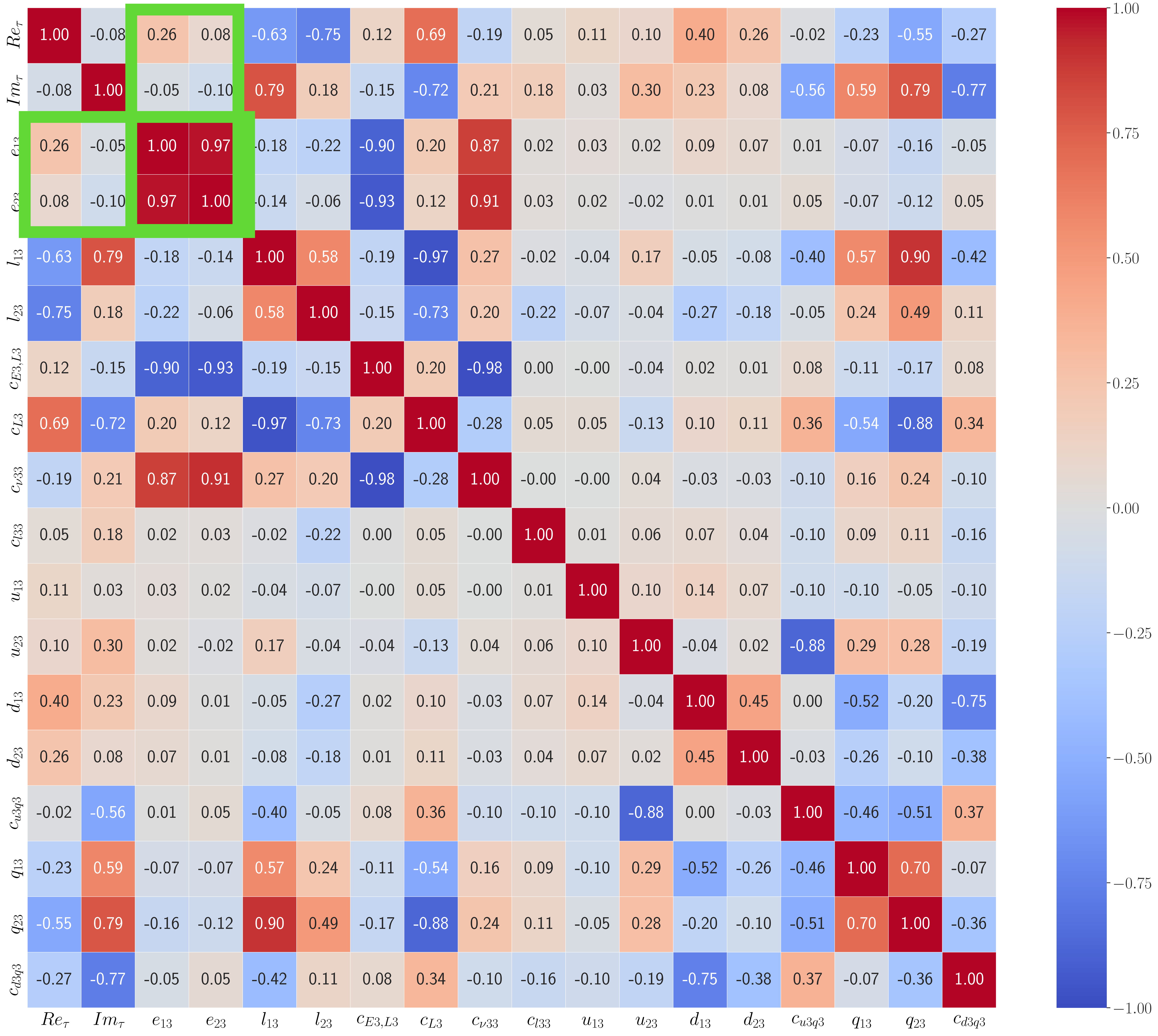
Correlations between Parameters

Correlations between Parameters



	Re_τ	Im_τ	e_{13}	e_{23}	l_{13}	l_{23}	$c_{E3,L3}$	c_{L3}	$c_{\nu 33}$	c_{l33}	u_{13}	u_{23}	d_{13}	d_{23}	c_{u3q3}	q_{13}	q_{23}	c_{d3q3}
Re_τ	1.00	-0.08	0.26	0.08	-0.63	-0.75	0.12	0.69	-0.19	0.05	0.11	0.10	0.40	0.26	-0.02	-0.23	-0.55	-0.27
Im_τ	-0.08	1.00	-0.05	-0.10	0.79	0.18	-0.15	-0.72	0.21	0.18	0.03	0.30	0.23	0.08	-0.56	0.59	0.79	-0.77
e_{13}	0.26	-0.05	1.00	0.97	-0.18	-0.22	-0.90	0.20	0.87	0.02	0.03	0.02	0.09	0.07	0.01	-0.07	-0.16	-0.05
e_{23}	0.08	-0.10	0.97	1.00	-0.14	-0.06	-0.93	0.12	0.91	0.03	0.02	-0.02	0.01	0.01	0.05	-0.07	-0.12	0.05
l_{13}	-0.63	0.79	-0.18	-0.14	1.00	0.58	-0.19	-0.97	0.27	-0.02	-0.04	0.17	-0.05	-0.08	-0.40	0.57	0.90	-0.42
l_{23}	-0.75	0.18	-0.22	-0.06	0.58	1.00	-0.15	-0.73	0.20	-0.22	-0.07	-0.04	-0.27	-0.18	-0.05	0.24	0.49	0.11
$c_{E3,L3}$	0.12	-0.15	-0.90	-0.93	-0.19	-0.15	1.00	0.20	-0.98	0.00	-0.00	-0.04	0.02	0.01	0.08	-0.11	-0.17	0.08
c_{L3}	0.69	-0.72	0.20	0.12	-0.97	-0.73	0.20	1.00	-0.28	0.05	0.05	-0.13	0.10	0.11	0.36	-0.54	-0.88	0.34
$c_{\nu 33}$	-0.19	0.21	0.87	0.91	0.27	0.20	-0.98	-0.28	1.00	-0.00	-0.00	0.04	-0.03	-0.03	-0.10	0.16	0.24	-0.10
c_{l33}	0.05	0.18	0.02	0.03	-0.02	-0.22	0.00	0.05	-0.00	1.00	0.01	0.06	0.07	0.04	-0.10	0.09	0.11	-0.16
u_{13}	0.11	0.03	0.03	0.02	-0.04	-0.07	-0.00	0.05	-0.00	0.01	1.00	0.10	0.14	0.07	-0.10	-0.10	-0.05	-0.10
u_{23}	0.10	0.30	0.02	-0.02	0.17	-0.04	-0.04	-0.13	0.04	0.06	0.10	1.00	-0.04	0.02	-0.88	0.29	0.28	-0.19
d_{13}	0.40	0.23	0.09	0.01	-0.05	-0.27	0.02	0.10	-0.03	0.07	0.14	-0.04	1.00	0.45	0.00	-0.52	-0.20	-0.75
d_{23}	0.26	0.08	0.07	0.01	-0.08	-0.18	0.01	0.11	-0.03	0.04	0.07	0.02	0.45	1.00	-0.03	-0.26	-0.10	-0.38
c_{u3q3}	-0.02	-0.56	0.01	0.05	-0.40	-0.05	0.08	0.36	-0.10	-0.10	-0.10	-0.88	0.00	-0.03	1.00	-0.46	-0.51	0.37
q_{13}	-0.23	0.59	-0.07	-0.07	0.57	0.24	-0.11	-0.54	0.16	0.09	-0.10	0.29	-0.52	-0.26	-0.46	1.00	0.70	-0.07
q_{23}	-0.55	0.79	-0.16	-0.12	0.90	0.49	-0.17	-0.88	0.24	0.11	-0.05	0.28	-0.20	-0.10	-0.51	0.70	1.00	-0.36
c_{d3q3}	-0.27	-0.77	-0.05	0.05	-0.42	0.11	0.08	0.34	-0.10	-0.16	-0.10	-0.19	-0.75	-0.38	0.37	-0.07	-0.36	1.00

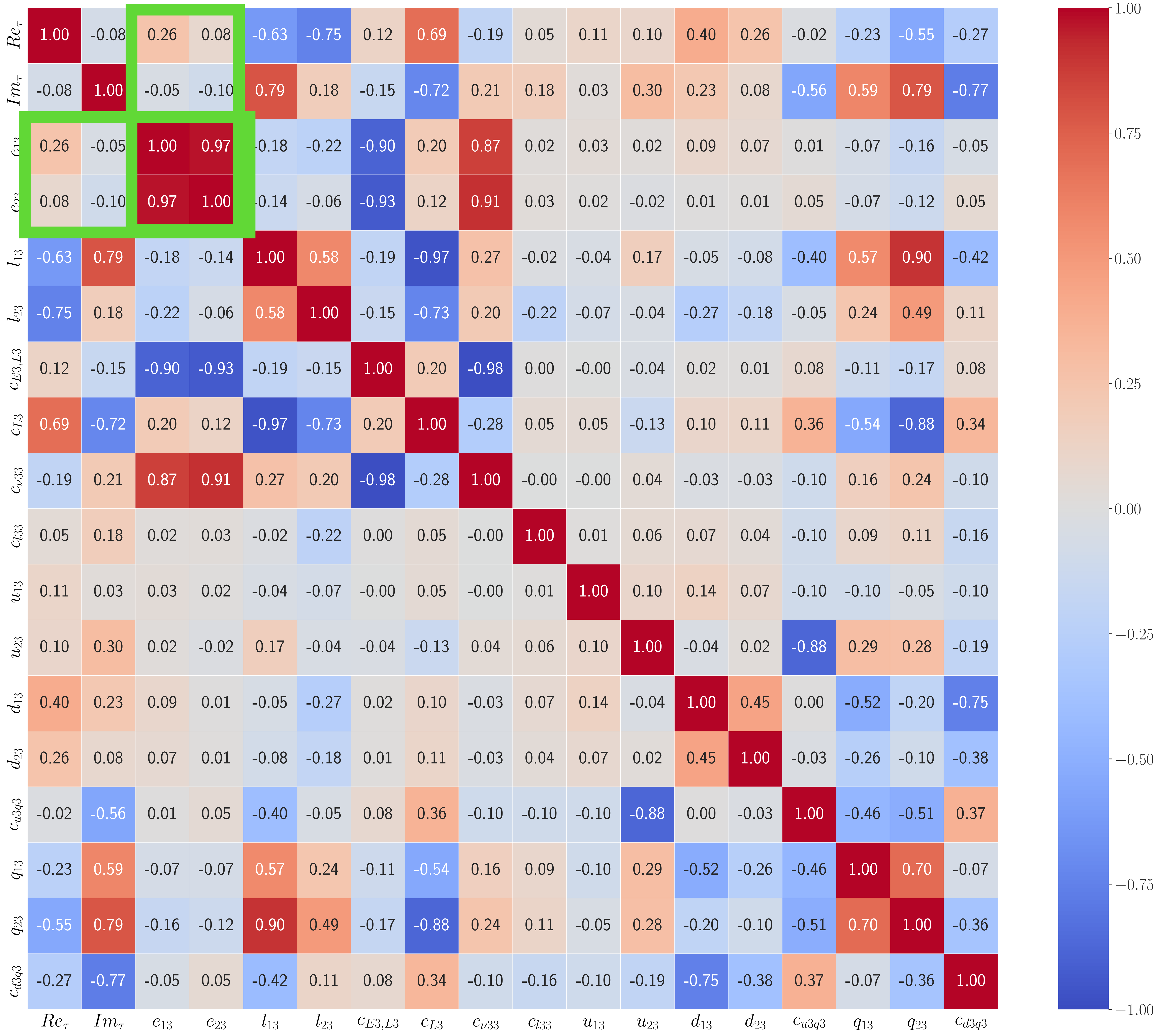
Correlations between Parameters



	Re_τ	Im_τ	e_{13}	e_{23}	l_{13}	l_{23}	$c_{E3,L3}$	c_{L3}	$c_{\nu 33}$	c_{l33}	u_{13}	u_{23}	d_{13}	d_{23}	c_{u3q3}	q_{13}	q_{23}	c_{d3q3}
Re_τ	1.00	-0.08	0.26	0.08	-0.63	-0.75	0.12	0.69	-0.19	0.05	0.11	0.10	0.40	0.26	-0.02	-0.23	-0.55	-0.27
Im_τ	-0.08	1.00	-0.05	-0.10	0.79	0.18	-0.15	-0.72	0.21	0.18	0.03	0.30	0.23	0.08	-0.56	0.59	0.79	-0.77
e_{13}	0.26	-0.05	1.00	0.97	-0.18	-0.22	-0.90	0.20	0.87	0.02	0.03	0.02	0.09	0.07	0.01	-0.07	-0.16	-0.05
e_{23}	0.08	-0.10	0.97	1.00	-0.14	-0.06	-0.93	0.12	0.91	0.03	0.02	-0.02	0.01	0.01	0.05	-0.07	-0.12	0.05
l_{13}	-0.63	0.79	-0.18	-0.14	1.00	0.58	-0.19	-0.97	0.27	-0.02	-0.04	0.17	-0.05	-0.08	-0.40	0.57	0.90	-0.42
l_{23}	-0.75	0.18	-0.22	-0.06	0.58	1.00	-0.15	-0.73	0.20	-0.22	-0.07	-0.04	-0.27	-0.18	-0.05	0.24	0.49	0.11
$c_{E3,L3}$	0.12	-0.15	-0.90	-0.93	-0.19	-0.15	1.00	0.20	-0.98	0.00	-0.00	-0.04	0.02	0.01	0.08	-0.11	-0.17	0.08
c_{L3}	0.69	-0.72	0.20	0.12	-0.97	-0.73	0.20	1.00	-0.28	0.05	0.05	-0.13	0.10	0.11	0.36	-0.54	-0.88	0.34
$c_{\nu 33}$	-0.19	0.21	0.87	0.91	0.27	0.20	-0.98	-0.28	1.00	-0.00	-0.00	0.04	-0.03	-0.03	-0.10	0.16	0.24	-0.10
c_{l33}	0.05	0.18	0.02	0.03	-0.02	-0.22	0.00	0.05	-0.00	1.00	0.01	0.06	0.07	0.04	-0.10	0.09	0.11	-0.16
u_{13}	0.11	0.03	0.03	0.02	-0.04	-0.07	-0.00	0.05	-0.00	0.01	1.00	0.10	0.14	0.07	-0.10	-0.10	-0.05	-0.10
u_{23}	0.10	0.30	0.02	-0.02	0.17	-0.04	-0.04	-0.13	0.04	0.06	0.10	1.00	-0.04	0.02	-0.88	0.29	0.28	-0.19
d_{13}	0.40	0.23	0.09	0.01	-0.05	-0.27	0.02	0.10	-0.03	0.07	0.14	-0.04	1.00	0.45	0.00	-0.52	-0.20	-0.75
d_{23}	0.26	0.08	0.07	0.01	-0.08	-0.18	0.01	0.11	-0.03	0.04	0.07	0.02	0.45	1.00	-0.03	-0.26	-0.10	-0.38
c_{u3q3}	-0.02	-0.56	0.01	0.05	-0.40	-0.05	0.08	0.36	-0.10	-0.10	-0.10	-0.88	0.00	-0.03	1.00	-0.46	-0.51	0.37
q_{13}	-0.23	0.59	-0.07	-0.07	0.57	0.24	-0.11	-0.54	0.16	0.09	-0.10	0.29	-0.52	-0.26	-0.46	1.00	0.70	-0.07
q_{23}	-0.55	0.79	-0.16	-0.12	0.90	0.49	-0.17	-0.88	0.24	0.11	-0.05	0.28	-0.20	-0.10	-0.51	0.70	1.00	-0.36
c_{d3q3}	-0.27	-0.77	-0.05	0.05	-0.42	0.11	0.08	0.34	-0.10	-0.16	-0.10	-0.19	-0.75	-0.38	0.37	-0.07	-0.36	1.00

Also, parameters that are weekly correlated with the modulus can be strongly correlated between them, as for instance (e_{13}, e_{23}) that appear in the elements (11) and (21) of the charged lepton mass matrix

Correlations between Parameters



	Re_τ	Im_τ	e_{13}	e_{23}	l_{13}	l_{23}	$c_{E3,L3}$	c_{L3}	$c_{\nu 33}$	c_{l33}	u_{13}	u_{23}	d_{13}	d_{23}	c_{u3q3}	q_{13}	q_{23}	c_{d3q3}
Re_τ	1.00	-0.08	0.26	0.08	-0.63	-0.75	0.12	0.69	-0.19	0.05	0.11	0.10	0.40	0.26	-0.02	-0.23	-0.55	-0.27
Im_τ	-0.08	1.00	-0.05	-0.10	0.79	0.18	-0.15	-0.72	0.21	0.18	0.03	0.30	0.23	0.08	-0.56	0.59	0.79	-0.77
e_{13}	0.26	-0.05	1.00	0.97	-0.18	-0.22	-0.90	0.20	0.87	0.02	0.03	0.02	0.09	0.07	0.01	-0.07	-0.16	-0.05
e_{23}	0.08	-0.10	0.97	1.00	-0.14	-0.06	-0.93	0.12	0.91	0.03	0.02	-0.02	0.01	0.01	0.05	-0.07	-0.12	0.05
l_{13}	-0.63	0.79	-0.18	-0.14	1.00	0.58	-0.19	-0.97	0.27	-0.02	-0.04	0.17	-0.05	-0.08	-0.40	0.57	0.90	-0.42
l_{23}	-0.75	0.18	-0.22	-0.06	0.58	1.00	-0.15	-0.73	0.20	-0.22	-0.07	-0.04	-0.27	-0.18	-0.05	0.24	0.49	0.11
$c_{E3,L3}$	0.12	-0.15	-0.90	-0.93	-0.19	-0.15	1.00	0.20	-0.98	0.00	-0.00	-0.04	0.02	0.01	0.08	-0.11	-0.17	0.08
c_{L3}	0.69	-0.72	0.20	0.12	-0.97	-0.73	0.20	1.00	-0.28	0.05	0.05	-0.13	0.10	0.11	0.36	-0.54	-0.88	0.34
$c_{\nu 33}$	-0.19	0.21	0.87	0.91	0.27	0.20	-0.98	-0.28	1.00	-0.00	-0.00	0.04	-0.03	-0.03	-0.10	0.16	0.24	-0.10
c_{l33}	0.05	0.18	0.02	0.03	-0.02	-0.22	0.00	0.05	-0.00	1.00	0.01	0.06	0.07	0.04	-0.10	0.09	0.11	-0.16
u_{13}	0.11	0.03	0.03	0.02	-0.04	-0.07	-0.00	0.05	-0.00	0.01	1.00	0.10	0.14	0.07	-0.10	-0.10	-0.05	-0.10
u_{23}	0.10	0.30	0.02	-0.02	0.17	-0.04	-0.04	-0.13	0.04	0.06	0.10	1.00	-0.04	0.02	-0.88	0.29	0.28	-0.19
d_{13}	0.40	0.23	0.09	0.01	-0.05	-0.27	0.02	0.10	-0.03	0.07	0.14	-0.04	1.00	0.45	0.00	-0.52	-0.20	-0.75
d_{23}	0.26	0.08	0.07	0.01	-0.08	-0.18	0.01	0.11	-0.03	0.04	0.07	0.02	0.45	1.00	-0.03	-0.26	-0.10	-0.38
c_{u3q3}	-0.02	-0.56	0.01	0.05	-0.40	-0.05	0.08	0.36	-0.10	-0.10	-0.10	-0.88	0.00	-0.03	1.00	-0.46	-0.51	0.37
q_{13}	-0.23	0.59	-0.07	-0.07	0.57	0.24	-0.11	-0.54	0.16	0.09	-0.10	0.29	-0.52	-0.26	-0.46	1.00	0.70	-0.07
q_{23}	-0.55	0.79	-0.16	-0.12	0.90	0.49	-0.17	-0.88	0.24	0.11	-0.05	0.28	-0.20	-0.10	-0.51	0.70	1.00	-0.36
c_{d3q3}	-0.27	-0.77	-0.05	0.05	-0.42	0.11	0.08	0.34	-0.10	-0.16	-0.10	-0.19	-0.75	-0.38	0.37	-0.07	-0.36	1.00

Also, parameters that are weekly correlated with the modulus can be strongly correlated between them, as for instance (e_{13}, e_{23}) that appear in the elements (11) and (21) of the charged lepton mass matrix

Once again, since we are in the case of an overfitted model, the presence of strong correlations between parameters is related to the non-linear modular constraints and the internal geometry of the parameter space, and is not a sign of fine-tuning

Model based on the 2O group

The group 2O, the binary octahedral group, is a group of order 48 closely related to the octahedral group O, the group of rotational symmetries of a cube (or an octahedron). Specifically, 2O is the double cover of the octahedral group and is isomorphic to S_4

Ding, Liu, Yao, *JHEP* 01 (2023); Ding, Liu, Lu, Weng, *JHEP* 11 (2023) 083, Ding, Lisi, AM, Petcov, *Phys.Rev.D* 111 (2025) 7, 075024

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2O has 8 irreducible representations $\begin{cases} \text{two one-dimensional irreps } \mathbf{1}, \mathbf{1}' \\ \text{three two-dimensional irreps } \mathbf{2},\ \hat{\mathbf{2}}, \hat{\mathbf{2}}' \end{cases}$ two three-dimensional irreps $\mathbf{3}, \mathbf{3}'$ one four-dimensional irrep $\mathbf{4}$

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We consider N= 1 global supersymmetry, generalized CP (gCP) symmetry → all coupling constants are real

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2O has 8 irreducible representations

- two one-dimensional irreps $\mathbf{1}, \mathbf{1}'$
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Particle content of the model

Three $SU(2)$ lepton doublets $L_i = \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}$ $i = e, \mu, \tau$

Three right-handed fermions $E^c = (e^c, \mu^c, \tau^c)$ that are $SU(2)$ singlets

Three right-handed neutrinos N_i^c $i = 1,2,3$ that are $SU(2)$ singlets

Six right-handed quarks $U^c = (u^c, c^c, t^c)$ and $D^c = (d^c, s^c, b^c)$ that are $SU(2)$ singlets

Three $SU(2)$ quark doublets $Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$ $u_i = u, c, t \quad d_i = d, s, b$

Two Higgs fields, H_u and H_d that are invariant singlets of 2O with zero modular weights.

Representation under the 2O modular group and weights

$$L \sim \mathbf{3}\,,\;\; E_D^c \equiv (e^c, \mu^c) \sim \hat{\mathbf{2}}'\,,\;\; \tau^c \sim \mathbf{1}'\,,\;\; N^c \sim \mathbf{3}$$

$$k_L = -1\,,\;\; k_{E_D^c} = 6\,,\;\; k_{\tau^c} = 5\,,\;\; k_{N^c} = 1$$

$$Q_D \equiv (Q_1, Q_2)^T \sim \mathbf{2}\,,\;\; Q_3 \sim \mathbf{1}'\,,\;\; U_D^c \equiv (u^c, c^c) \sim \hat{\mathbf{2}}'$$

$$t^c \sim \mathbf{1}'\,,\;\; D_D^c \equiv (d^c, s^c) \sim \mathbf{2}\,,\;\; b^c \sim \mathbf{1}'$$

$$k_{Q_D} = 3 - k_{U_D^c} = k_{Q_3} = 6 - k_{t^c} = 6 - k_{D_D^c} = -k_{b^c}$$

The Lagrangian of the model contains the most general superpotential with all possible singlets under 2O built with the modular forms organized in various multiplets of different weights

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$$\mathscr{W}_\nu = g H_u (N^c L)_{\mathbf{1}} + \Lambda \left(N^c N^c\right)_{\mathbf{2}} Y^{(2)}_{\mathbf{2}}$$ neutrinos

$$\mathscr{W}_E = g_1^E \left(E^c_D L\right)_{\hat{\mathbf{2}}'} Y^{(5)}_{\hat{\mathbf{2}}'} H_d + g_2^E \left(E^c_D L\right)_{\hat{\mathbf{4}}} Y^{(5)}_{\hat{\mathbf{4}}} H_d + g_3^E \left(\tau^c L\right)_{\mathbf{3}'} Y^{(4)}_{\mathbf{3}'} H_d$$ charged leptons

$$\mathscr{W}_u = g_1^u \left(U^c_D Q_D\right)_{\hat{\mathbf{4}}} Y^{(3)}_{\hat{\mathbf{4}}} H_u + g_2^u \left(t^c Q_D\right)_{\mathbf{2}} Y^{(6)}_{\mathbf{2}} H_u + g_3^u \left(t^c Q_3\right)_{\mathbf{1}} Y^{(6)}_{\mathbf{1}} H_u$$ up - quarks

$$\mathscr{W}_d = g_1^d \left(D^c_D Q_D\right)_{\mathbf{1}} Y^{(6)}_{\mathbf{1}} H_d + g_2^d \left(D^c_D Q_D\right)_{\mathbf{1}'} Y^{(6)}_{\mathbf{1}'} H_d + g_3^d \left(D^c_D Q_D\right)_{\mathbf{2}} Y^{(6)}_{\mathbf{2}} H_d + g_4^d \left(D^c_D Q_3\right)_{\mathbf{2}} Y^{(6)}_{\mathbf{2}} H_d + g_5^d \left(b^c Q_3\right)_{\mathbf{1}} H_d$$ down - quarks

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neutrinos

$$\mathscr{W}_{E} = g_1^E\left(E_D^cL\right)_{\hat{\mathbf{2}}'} Y_{\hat{\mathbf{2}}'}^{(5)} H_d + g_2^E\left(E_D^cL\right)_{\hat{\mathbf{4}}} Y_{\hat{\mathbf{4}}}^{(5)} H_d + g_3^E\left(\tau^cL\right)_{\mathbf{3}'} Y_{\mathbf{3}'}^{(4)} H_d$$

charged leptons

$$\mathscr{W}_{u} = g_1^u\left(U_D^cQ_D\right)_{\hat{\mathbf{4}}} Y_{\hat{\mathbf{4}}}^{(3)} H_u + g_2^u\left(t^cQ_D\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_u + g_3^u\left(t^cQ_3\right)_{\mathbf{1}} Y_{\mathbf{1}}^{(6)} H_u$$

up - quarks

$$\mathscr{W}_{d} = g_1^d\left(D_D^cQ_D\right)_{\mathbf{1}} Y_{\mathbf{1}}^{(6)} H_d + g_2^d\left(D_D^cQ_D\right)_{\mathbf{1}'} Y_{\mathbf{1}'}^{(6)} H_d + g_3^d\left(D_D^cQ_D\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_d + g_4^d\left(D_D^cQ_3\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_d + g_5^d\left(b^cQ_3\right)_{\mathbf{1}} H_d$$

down - quarks

Each term multiplied by an unknown real (gCP invariance) parameter

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$\mathscr{W}_d = g_1^d \left(D^c_D Q_D\right)_{\mathbf{1}} Y^{(6)}_{\mathbf{1}} H_d + g_2^d \left(D^c_D Q_D\right)_{\mathbf{1}'} Y^{(6)}_{\mathbf{1}'} H_d + g_3^d \left(D^c_D Q_D\right)_{\mathbf{2}} Y^{(6)}_{\mathbf{2}} H_d + g_4^d \left(D^c_D Q_3\right)_{\mathbf{2}} Y^{(6)}_{\mathbf{2}} H_d + g_5^d \left(b^c Q_3\right)_{\mathbf{1}} H_d$ down - quarks

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Light neutrino mass matrix is derived by the seesaw formula

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$$\mathscr{W}_{d} = g_1^d \left(D_D^cQ_D\right)_{\mathbf{1}} Y_{\mathbf{1}}^{(6)} H_d + g_2^d \left(D_D^cQ_D\right)_{\mathbf{1}'} Y_{\mathbf{1}'}^{(6)} H_d + g_3^d \left(D_D^cQ_D\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_d + g_4^d \left(D_D^cQ_3\right)_{\mathbf{2}} Y_{\mathbf{2}}^{(6)} H_d + g_5^d \left(b^cQ_3\right)_{\mathbf{1}} H_d$$ down - quarks

Each term multiplied by an unknown real (gCP invariance) parameter

Light neutrino mass matrix is derived by the seesaw formula

The model contains 14 unknowns

The modulus τ
2 real parameters

Lepton sector
4 real parameters

Quark sector
8 real parameters

Minimal model in the modular form literature (before arXiv:2506.19822)

Experimental inputs and model parameters

Experimental inputs and model parameters

Model should satisfy 18 experimental constraints (only Normal Ordering of neutrino masses allowed)

$$\mathrm{O}_i = \begin{cases} \sin^2\theta_{12}, \sin^2\theta_{23}, \sin^2\theta_{13} & i = 1,2,3 \quad \nu \text{ mixing}, \\ \delta m^2, \Delta m^2 & i = 4,5 \quad \nu \text{ masses}, \\ r_{e\mu}, r_{\mu\tau}, m_\tau & i = 6,7,8 \quad \text{charged lepton masses}, \\ \theta^q_{12}, \theta^q_{23}, \theta^q_{13}, \delta^q_{CP} & i = 9,10,11,12 \quad \text{quark mixing}, \\ r_{uc}, r_{ct}, r_{ds}, r_{sb}, m_t, m_b & i = 13,\ldots,18 \quad \text{quark masses}. \end{cases}$$

8 for leptons

10 for quarks

charged fermion masses and quark mixing
parameters extrapolated to the GUT scale

S. Antusch et al., J. High Energy Phys. 03 (2005) 024

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r_{e\mu}, r_{\mu\tau}, m_\tau & i = 6,7,8 \qquad \text{charged lepton masses}, \\
\theta^q_{12}, \theta^q_{23}, \theta^q_{13}, \delta^q_{CP} & i = 9,10,11,12 \quad \text{quark mixing}, \\
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S. Antusch and V. Maurer, J. High Energy Phys. 11 (2013) 115.

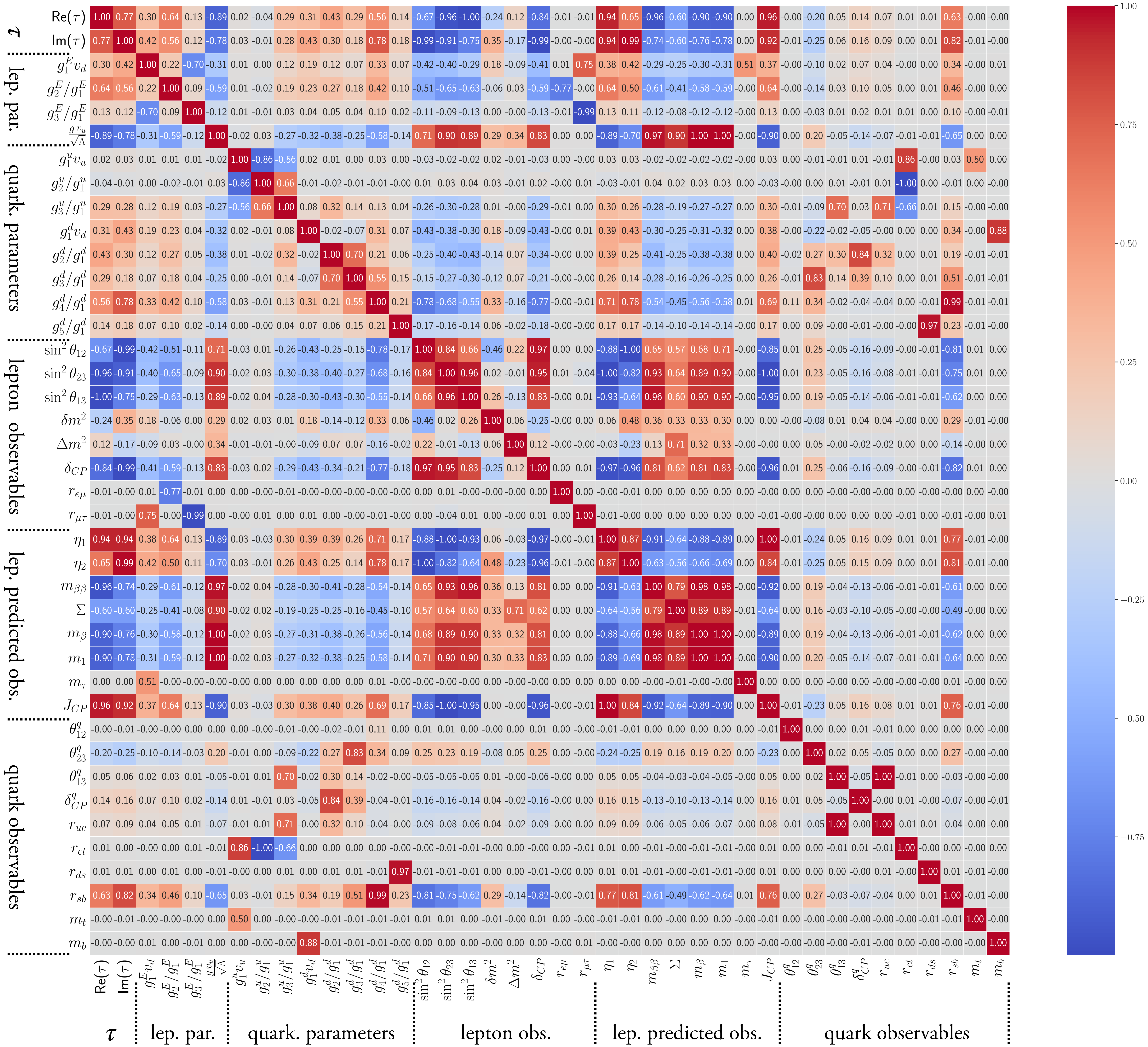
The Lagrangian of the model contains 14 real parameters, 6 for leptons, 10 for quarks, with τ in common

$$
\mathbf{P}_{\text{leptons}} = \left(\tau, g_1^E\, v_d, \frac{g_2^E}{g_1^E}, \frac{g_3^E}{g_1^E}, \frac{g\, v_u}{\sqrt{\Lambda}}\right) ,
$$

$$
\mathbf{P}_{\text{quarks}} = \left(\tau, g_1^u\, v_u, \frac{g_2^u}{g_1^u}, \frac{g_3^u}{g_1^u}, g_1^d\, v_d, \frac{g_2^d}{g_1^d}, \frac{g_3^d}{g_1^d}, \frac{g_4^d}{g_1^d}, \frac{g_5^d}{g_1^d}\right)
$$

Complete correlations matrix

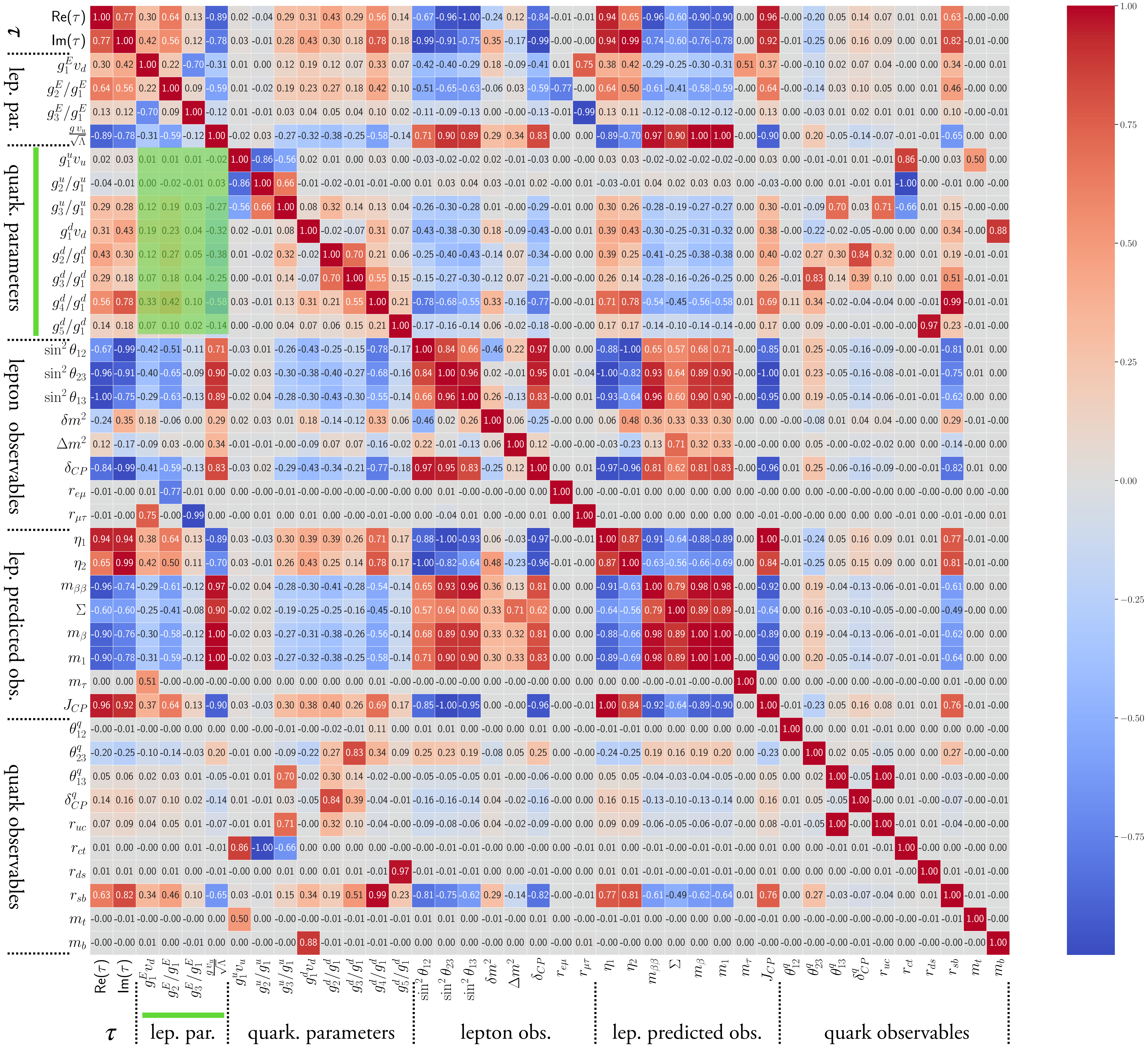
Complete correlations matrix



	Re(τ)	Im(τ)	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$	η_1	η_2	$m_{\beta\beta}$	Σ	m_β	m_1	m_τ	J_{CP}	θ_{12}^q	θ_{23}^q	θ_{13}^q	δ_{CP}^q	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
Re(τ)	1.00	0.77	0.30	0.64	0.13	-0.89	0.02	-0.04	0.29	0.31	0.43	0.29	0.56	0.14	-0.67	-0.96	-1.00	-0.24	0.12	-0.84	-0.01	-0.01	0.94	0.65	-0.96	-0.60	-0.90	-0.90	0.00	0.96	-0.00	-0.20	0.05	0.14	0.07	0.01	0.01	0.63	-0.00	-0.00
Im(τ)	0.77	1.00	0.42	0.56	0.12	-0.78	0.03	-0.01	0.28	0.43	0.30	0.18	0.78	0.18	-0.99	-0.91	-0.75	0.35	-0.17	-0.99	-0.00	-0.00	0.94	0.99	-0.74	-0.60	-0.76	-0.78	0.00	0.92	-0.01	-0.25	0.06	0.16	0.09	0.00	0.01	0.82	-0.01	-0.00
$g_1^E v_d$	0.30	0.42	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07	-0.42	-0.40	-0.29	0.18	-0.09	-0.41	0.01	0.75	0.38	0.42	-0.29	-0.25	-0.30	-0.31	0.51	0.37	-0.00	-0.10	0.02	0.07	0.04	-0.00	0.00	0.34	-0.00	0.01
g_2^E/g_1^E	0.64	0.56	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10	-0.51	-0.65	-0.63	-0.06	0.03	-0.59	-0.77	-0.00	0.64	0.50	-0.61	-0.41	-0.58	-0.59	-0.00	0.64	-0.00	-0.14	0.03	0.10	0.05	0.00	0.01	0.46	-0.00	0.00
g_3^E/g_1^E	0.13	0.12	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02	-0.11	-0.09	-0.13	0.00	-0.00	-0.13	-0.01	-0.99	0.13	0.11	-0.12	-0.08	-0.12	-0.12	-0.00	0.13	0.00	-0.03	0.01	0.02	0.01	0.01	0.00	0.10	-0.00	-0.01
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.89	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14	0.71	0.90	0.89	0.29	0.34	0.83	0.00	0.00	-0.89	-0.70	0.97	0.90	1.00	1.00	-0.00	-0.90	0.00	0.20	-0.05	-0.14	-0.07	-0.01	-0.01	-0.65	0.00	0.00
$g_1^u v_u$	0.02	0.03	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00	-0.03	-0.02	-0.02	0.02	-0.01	-0.03	0.00	-0.00	0.03	0.03	-0.02	-0.02	-0.02	-0.02	-0.00	0.03	0.00	-0.01	-0.01	0.01	-0.01	0.86	-0.00	0.03	0.50	0.00
g_2^u/g_1^u	-0.04	-0.01	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00	0.01	0.03	0.04	0.03	-0.01	0.02	-0.00	0.01	-0.03	-0.01	0.04	0.02	0.03	0.03	0.00	-0.03	-0.00	0.00	0.01	-0.01	0.01	-1.00	0.00	-0.01	0.00	-0.00
g_3^u/g_1^u	0.29	0.28	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04	-0.26	-0.30	-0.28	0.01	-0.00	-0.29	-0.01	0.00	0.30	0.26	-0.28	-0.19	-0.27	-0.27	0.00	0.30	-0.01	-0.09	0.70	0.03	0.71	-0.66	0.01	0.15	-0.00	-0.00
$g_1^d v_d$	0.31	0.43	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07	-0.43	-0.38	-0.30	0.18	-0.09	-0.43	-0.00	0.01	0.39	0.43	-0.30	-0.25	-0.31	-0.32	0.00	0.38	-0.00	-0.22	-0.02	-0.05	-0.00	0.00	0.00	0.34	-0.00	0.88
g_2^d/g_1^d	0.43	0.30	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06	-0.25	-0.40	-0.43	-0.14	0.07	-0.34	-0.00	0.00	0.39	0.25	-0.41	-0.25	-0.38	-0.38	0.00	0.40	-0.02	0.27	0.30	0.84	0.32	0.00	0.01	0.19	-0.01	-0.01
g_3^d/g_1^d	0.29	0.18	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15	-0.15	-0.27	-0.30	-0.12	0.07	-0.21	-0.00	-0.00	0.26	0.14	-0.28	-0.16	-0.26	-0.25	0.00	0.26	-0.01	0.83	0.14	0.39	0.10	0.00	0.01	0.51	-0.01	-0.01
g_4^d/g_1^d	0.56	0.78	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21	-0.78	-0.68	-0.55	0.33	-0.16	-0.77	-0.00	-0.01	0.71	0.78	-0.54	-0.45	-0.56	-0.58	0.01	0.69	0.11	0.34	-0.02	-0.04	-0.04	0.00	-0.01	0.99	-0.01	-0.01
g_5^d/g_1^d	0.14	0.18	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00	-0.17	-0.16	-0.14	0.06	-0.02	-0.18	-0.00	-0.00	0.17	0.17	-0.14	-0.10	-0.14	-0.14	-0.00	0.17	0.00	0.09	-0.00	-0.01	-0.00	-0.00	0.97	0.23	-0.01	-0.00
$\sin^2\theta_{12}$	-0.67	-0.99	-0.42	-0.51	-0.11	0.71	-0.03	0.01	-0.26	-0.43	-0.25	-0.15	-0.78	-0.17	1.00	0.84	0.66	-0.46	0.22	0.97	0.00	0.00	-0.88	-1.00	0.65	0.57	0.68	0.71	-0.00	-0.85	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	-0.96	-0.91	-0.40	-0.65	-0.09	0.90	-0.02	0.03	-0.30	-0.38	-0.40	-0.27	-0.68	-0.16	0.84	1.00	0.96	0.02	-0.01	0.95	0.01	-0.04	-1.00	-0.82	0.93	0.64	0.89	0.90	-0.00	-1.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	-1.00	-0.75	-0.29	-0.63	-0.13	0.89	-0.02	0.04	-0.28	-0.30	-0.43	-0.30	-0.55	-0.14	0.66	0.96	1.00	0.26	-0.13	0.83	-0.00	0.01	-0.93	-0.64	0.96	0.60	0.90	0.90	-0.00	-0.95	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.24	0.35	0.18	-0.06	0.00	0.29	0.02	0.03	0.01	0.18	-0.14	-0.12	0.33	0.06	-0.46	0.02	0.26	1.00	0.06	-0.25	-0.00	0.00	0.06	0.48	0.36	0.33	0.33	0.30	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.12	-0.17	-0.09	0.03	-0.00	0.34	-0.01	-0.01	-0.00	-0.09	0.07	0.07	-0.16	-0.02	0.22	-0.01	-0.13	0.06	1.00	0.12	-0.00	-0.00	-0.03	-0.23	0.13	0.71	0.32	0.33	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
δ_{CP}	-0.84	-0.99	-0.41	-0.59	-0.13	0.83	-0.03	0.02	-0.29	-0.43	-0.34	-0.21	-0.77	-0.18	0.97	0.95	0.83	-0.25	0.12	1.00	0.00	0.01	-0.97	-0.96	0.81	0.62	0.81	0.83	-0.00	-0.96	0.01	0.25	-0.06	-0.16	-0.09	-0.00	-0.01	-0.82	0.01	0.00
$r_{e\mu}$	-0.01	-0.00	0.01	-0.77	-0.01	0.00	0.00	-0.00	-0.01	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.01	-0.00	-0.00	-0.00	0.00	1.00	0.00	-0.00	-0.01	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	-0.01	-0.00	0.75	-0.00	-0.99	0.00	-0.00	0.01	0.00	0.01	0.00	-0.00	-0.01	-0.00	0.00	-0.04	0.01	0.00	-0.00	0.01	0.00	1.00	-0.01	-0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
η_1	0.94	0.94	0.38	0.64	0.13	-0.89	0.03	-0.03	0.30	0.39	0.39	0.26	0.71	0.17	-0.88	-1.00	-0.93	0.06	-0.03	-0.97	-0.00	-0.01	1.00	0.87	-0.91	-0.64	-0.88	-0.89	0.00	1.00	-0.01	-0.24	0.05	0.16	0.09	0.01	0.01	0.77	-0.01	-0.00
η_2	0.65	0.99	0.42	0.50	0.11	-0.70	0.03	-0.01	0.26	0.43	0.25	0.14	0.78	0.17	-1.00	-0.82	-0.64	0.48	-0.23	-0.96	-0.01	-0.00	0.87	1.00	-0.63	-0.56	-0.66	-0.69	0.00	0.84	-0.01	-0.25	0.05	0.15	0.09	0.00	0.01	0.81	-0.01	-0.00
$m_{\beta\beta}$	-0.96	-0.74	-0.29	-0.61	-0.12	0.97	-0.02	0.04	-0.28	-0.30	-0.41	-0.28	-0.54	-0.14	0.65	0.93	0.96	0.36	0.13	0.81	0.00	0.00	-0.91	-0.63	1.00	0.79	0.98	0.98	-0.00	-0.92	0.00	0.19	-0.04	-0.13	-0.06	-0.01	-0.01	-0.61	0.00	0.00
Σ	-0.60	-0.60	-0.25	-0.41	-0.08	0.90	-0.02	0.02	-0.19	-0.25	-0.25	-0.16	-0.45	-0.10	0.57	0.64	0.60	0.33	0.71	0.62	0.00	0.00	-0.64	-0.56	0.79	1.00	0.89	0.89	-0.01	-0.64	0.00	0.16	-0.03	-0.10	-0.05	-0.00	-0.00	-0.49	-0.00	0.00
m_β	-0.90	-0.76	-0.30	-0.58	-0.12	1.00	-0.02	0.03	-0.27	-0.31	-0.38	-0.26	-0.56	-0.14	0.68	0.89	0.90	0.33	0.32	0.81	0.00	0.00	-0.88	-0.66	0.98	0.89	1.00	1.00	-0.00	-0.89	0.00	0.19	-0.04	-0.13	-0.06	-0.01	-0.01	-0.62	0.00	0.00
m_1	-0.90	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14	0.71	0.90	0.90	0.30	0.33	0.83	0.00	0.00	-0.89	-0.69	0.98	0.89	1.00	1.00	-0.00	-0.90	0.00	0.20	-0.05	-0.14	-0.07	-0.01	-0.01	-0.64	0.00	0.00
m_τ	0.00	0.00	0.51	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.01	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.00	1.00	0.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
J_{CP}	0.96	0.92	0.37	0.64	0.13	-0.90	0.03	-0.03	0.30	0.38	0.40	0.26	0.69	0.17	-0.85	-1.00	-0.95	0.00	-0.00	-0.96	-0.00	-0.01	1.00	0.84	-0.92	-0.64	-0.89	-0.90	0.00	1.00	-0.01	-0.23	0.05	0.16	0.08	0.01	0.01	0.76	-0.01	-0.00
θ_{12}^q	-0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.02	-0.01	0.11	0.00	0.01	0.01	0.00	-0.00	0.00	0.01	0.00	-0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	-0.01	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ_{23}^q	-0.20	-0.25	-0.10	-0.14	-0.03	0.20	-0.01	0.00	-0.09	-0.22	0.27	0.83	0.34	0.09	0.25	0.23	0.19	-0.08	0.05	0.25	0.00	-0.00	-0.24	-0.25	0.19	0.16	0.19	0.20	0.00	-0.23	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ_{13}^q	0.05	0.06	0.02	0.03	0.01	-0.05	-0.01	0.01	0.70	-0.02	0.30	0.14	-0.02	-0.00	-0.05	-0.05	-0.05	0.01	-0.00	-0.06	-0.00	0.00	0.05	0.05	-0.04	-0.03	-0.04	-0.05	-0.00	0.05	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ_{CP}^q	0.14	0.16	0.07	0.10	0.02	-0.14	0.01	-0.01	0.03	-0.05	0.84	0.39	-0.04	-0.01	-0.16	-0.16	-0.14	0.04	-0.02	-0.16	-0.00	0.00	0.16	0.15	-0.13	-0.10	-0.13	-0.14	0.00	0.16	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	0.07	0.09	0.04	0.05	0.01	-0.07	-0.01	0.01	0.71	-0.00	0.32	0.10	-0.04	-0.00	-0.09	-0.08	-0.06	0.04	-0.02	-0.09	-0.00	0.00	0.09	0.09	-0.06	-0.05	-0.06	-0.07	-0.00	0.08	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	0.01	0.00	-0.00	0.00	0.01	-0.01	0.86	-1.00	-0.66	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.01	-0.00	0.00	-0.00	0.00	-0.01	0.01	0.00	-0.01	-0.00	-0.01	-0.01	-0.00	0.01	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	0.01	0.01	0.00	0.01	0.00	-0.01	-0.00	0.00	0.01	0.00	0.01	0.01	-0.01	0.97	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.00	0.00	0.01	0.01	-0.01	-0.00	-0.01	-0.01	-0.00	0.01	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	0.63	0.82	0.34	0.46	0.10	-0.65	0.03	-0.01	0.15	0.34	0.19	0.51	0.99	0.23	-0.81	-0.75	-0.62	0.29	-0.14	-0.82	-0.00	-0.01	0.77	0.81	-0.61	-0.49	-0.62	-0.64	0.01	0.76	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	-0.00	-0.01	-0.00	-0.00	-0.00	0.00	0.50	0.00	-0.00	-0.00	-0.01	-0.01	-0.01	-0.01	0.01	0.01	0.00	-0.01	-0.00	0.01	0.00	-0.00	-0.01	-0.01	0.00	-0.00	0.00	0.00	-0.00	-0.01	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	-0.00	-0.00	0.01	0.00	-0.01	0.00	0.00	-0.00	-0.00	0.88	-0.01	-0.01	-0.01	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	0.01	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

Complete correlations matrix

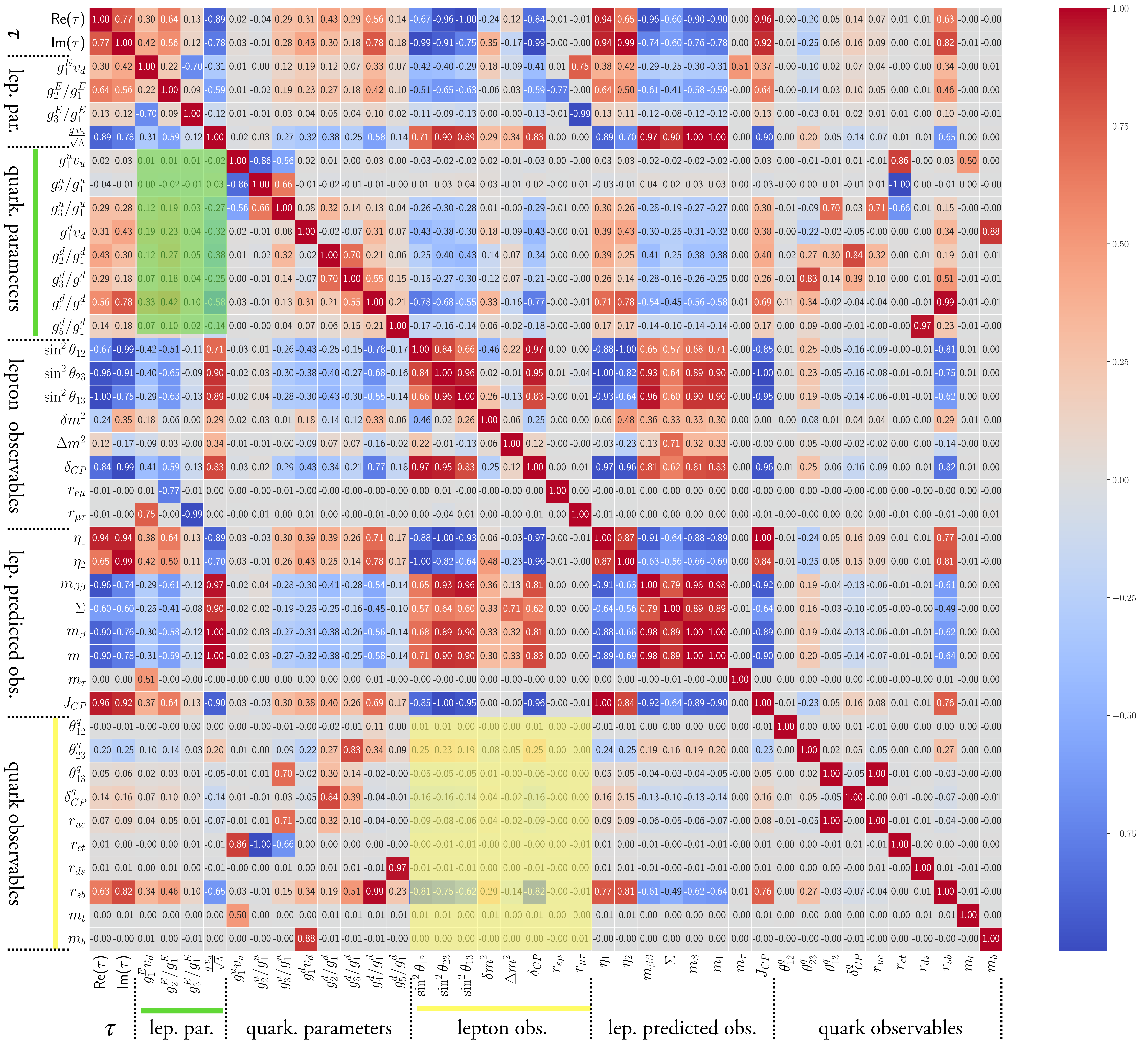
Correlations between lepton and quark parameters



Complete correlations matrix

Correlations between lepton and quark parameters

Correlations between lepton and quark observables



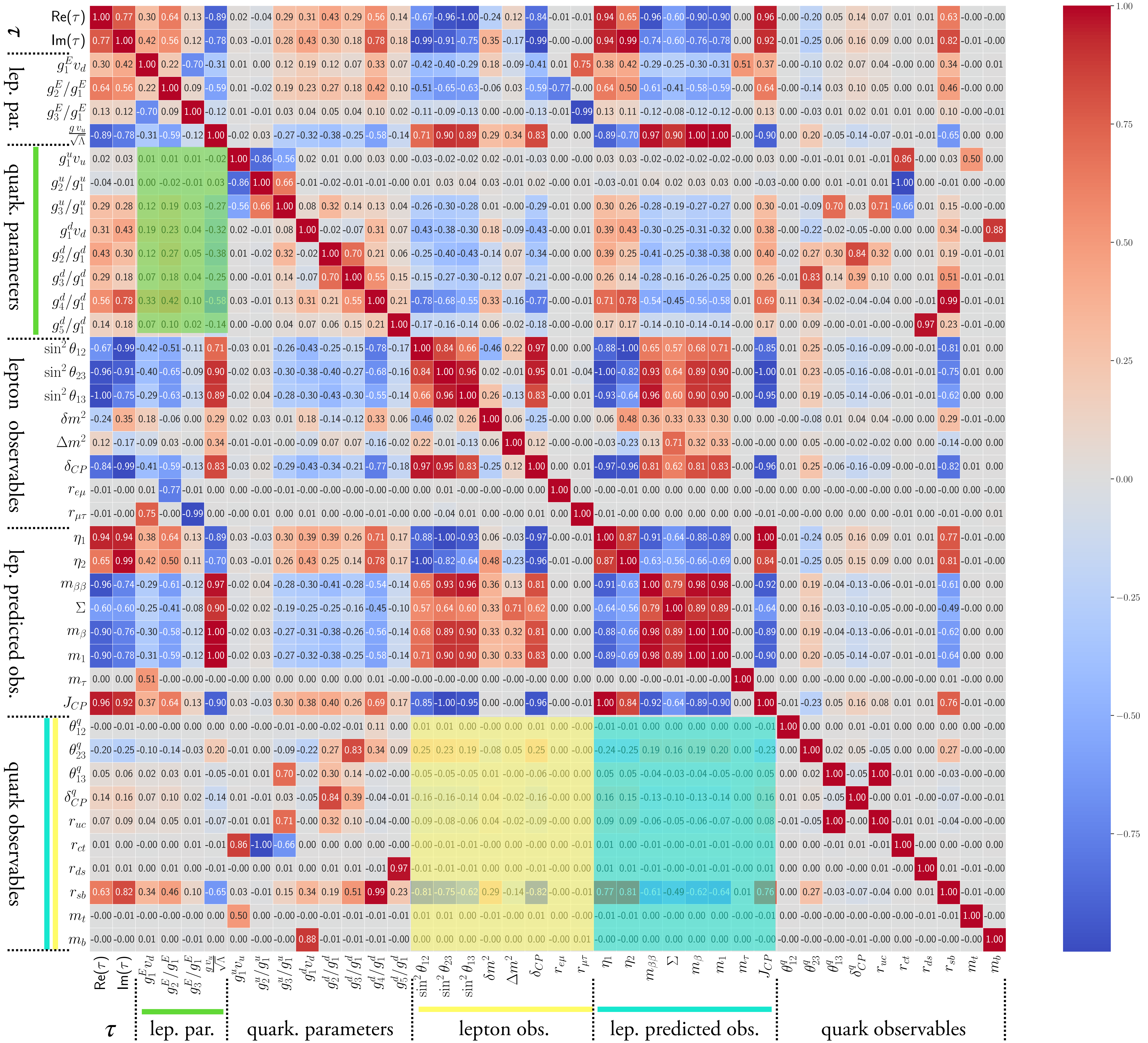
	Re(τ)	Im(τ)	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$	η_1	η_2	$m_{\beta\beta}$	Σ	m_β	m_1	m_τ	J_{CP}	θ_{12}^q	θ_{23}^q	θ_{13}^q	δ_{CP}^q	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
Re(τ)	1.00	0.77	0.30	0.64	0.13	-0.89	0.02	-0.04	0.29	0.31	0.43	0.29	0.56	0.14	-0.67	-0.96	-1.00	-0.24	0.12	-0.84	-0.01	-0.01	0.94	0.65	-0.96	-0.60	-0.90	-0.90	0.00	0.96	-0.00	-0.20	0.05	0.14	0.07	0.01	0.01	0.63	-0.00	-0.00
Im(τ)	0.77	1.00	0.42	0.56	0.12	-0.78	0.03	-0.01	0.28	0.43	0.30	0.18	0.78	0.18	-0.99	-0.91	-0.75	0.35	-0.17	-0.99	-0.00	-0.00	0.94	0.99	-0.74	-0.60	-0.76	-0.78	0.00	0.92	-0.01	-0.25	0.06	0.16	0.09	0.00	0.01	0.82	-0.01	-0.00
$g_1^E v_d$	0.30	0.42	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07	-0.42	-0.40	-0.29	0.18	-0.09	-0.41	0.01	0.75	0.38	0.42	-0.29	-0.25	-0.30	-0.31	0.51	0.37	-0.00	-0.10	0.02	0.07	0.04	-0.00	0.00	0.34	-0.00	0.01
g_2^E/g_1^E	0.64	0.56	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10	-0.51	-0.65	-0.63	-0.06	0.03	-0.59	-0.77	-0.00	0.64	0.50	-0.61	-0.41	-0.58	-0.59	-0.00	0.64	-0.00	-0.14	0.03	0.10	0.05	0.00	0.01	0.46	-0.00	0.00
g_3^E/g_1^E	0.13	0.12	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02	-0.11	-0.09	-0.13	0.00	-0.00	-0.13	-0.01	-0.99	0.13	0.11	-0.12	-0.08	-0.12	-0.12	-0.00	0.13	0.00	-0.03	0.01	0.02	0.01	0.01	0.00	0.10	-0.00	-0.01
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.89	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14	0.71	0.90	0.89	0.29	0.34	0.83	0.00	0.00	-0.89	-0.70	0.97	0.90	1.00	1.00	-0.00	-0.90	0.00	0.20	-0.05	-0.14	-0.07	-0.01	-0.01	-0.65	0.00	0.00
$g_1^u v_u$	0.02	0.03	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00	-0.03	-0.02	-0.02	0.02	-0.01	-0.03	0.00	-0.00	0.03	0.03	-0.02	-0.02	-0.02	-0.02	-0.00	0.03	0.00	-0.01	-0.01	0.01	-0.01	0.86	-0.00	0.03	0.50	0.00
g_2^u/g_1^u	-0.04	-0.01	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00	0.01	0.03	0.04	0.03	-0.01	0.02	-0.00	0.01	-0.03	-0.01	0.04	0.02	0.03	0.03	0.00	-0.03	-0.00	0.00	0.01	-0.01	0.01	-1.00	0.00	-0.01	0.00	-0.00
g_3^u/g_1^u	0.29	0.28	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04	-0.26	-0.30	-0.28	0.01	-0.00	-0.29	-0.01	0.00	0.30	0.26	-0.28	-0.19	-0.27	-0.27	0.00	0.30	-0.01	-0.09	0.70	0.03	0.71	-0.66	0.01	0.15	-0.00	-0.00
$g_1^d v_d$	0.31	0.43	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07	-0.43	-0.38	-0.30	0.18	-0.09	-0.43	-0.00	0.01	0.39	0.43	-0.30	-0.25	-0.31	-0.32	0.00	0.38	-0.00	-0.22	-0.02	-0.05	-0.00	0.00	0.00	0.34	-0.00	0.88
g_2^d/g_1^d	0.43	0.30	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06	-0.25	-0.40	-0.43	-0.14	0.07	-0.34	-0.00	0.00	0.39	0.25	-0.41	-0.25	-0.38	-0.38	0.00	0.40	-0.02	0.27	0.30	0.84	0.32	0.00	0.01	0.19	-0.01	-0.01
g_3^d/g_1^d	0.29	0.18	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15	-0.15	-0.27	-0.30	-0.12	0.07	-0.21	-0.00	-0.00	0.26	0.14	-0.28	-0.16	-0.26	-0.25	0.00	0.26	-0.01	0.83	0.14	0.39	0.10	0.00	0.01	0.51	-0.01	-0.01
g_4^d/g_1^d	0.56	0.78	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21	-0.78	-0.68	-0.55	0.33	-0.16	-0.77	-0.00	-0.01	0.71	0.78	-0.54	-0.45	-0.56	-0.58	0.01	0.69	0.11	0.34	-0.02	-0.04	-0.04	0.00	-0.01	0.99	-0.01	-0.01
g_5^d/g_1^d	0.14	0.18	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00	-0.17	-0.16	-0.14	0.06	-0.02	-0.18	-0.00	-0.00	0.17	0.17	-0.14	-0.10	-0.14	-0.14	-0.00	0.17	0.00	0.09	-0.00	-0.01	-0.00	-0.00	0.97	0.23	-0.01	-0.00
$\sin^2\theta_{12}$	-0.67	-0.99	-0.42	-0.51	-0.11	0.71	-0.03	0.01	-0.26	-0.43	-0.25	-0.15	-0.78	-0.17	1.00	0.84	0.66	-0.46	0.22	0.97	0.00	0.00	-0.88	-1.00	0.65	0.57	0.68	0.71	-0.00	-0.85	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	-0.96	-0.91	-0.40	-0.65	-0.09	0.90	-0.02	0.03	-0.30	-0.38	-0.40	-0.27	-0.68	-0.16	0.84	1.00	0.96	0.02	-0.01	0.95	0.01	-0.04	-1.00	-0.82	0.93	0.64	0.89	0.90	-0.00	-1.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	-1.00	-0.75	-0.29	-0.63	-0.13	0.89	-0.02	0.04	-0.28	-0.30	-0.43	-0.30	-0.55	-0.14	0.66	0.96	1.00	0.26	-0.13	0.83	-0.00	0.01	-0.93	-0.64	0.96	0.60	0.90	0.90	-0.00	-0.95	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.24	0.35	0.18	-0.06	0.00	0.29	0.02	0.03	0.01	0.18	-0.14	-0.12	0.33	0.06	-0.46	0.02	0.26	1.00	0.06	-0.25	-0.00	0.00	0.06	0.48	0.36	0.33	0.33	0.30	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.12	-0.17	-0.09	0.03	-0.00	0.34	-0.01	-0.01	-0.00	-0.09	0.07	0.07	-0.16	-0.02	0.22	-0.01	-0.13	0.06	1.00	0.12	-0.00	-0.00	-0.03	-0.23	0.13	0.71	0.32	0.33	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
δ_{CP}	-0.84	-0.99	-0.41	-0.59	-0.13	0.83	-0.03	0.02	-0.29	-0.43	-0.34	-0.21	-0.77	-0.18	0.97	0.95	0.83	-0.25	0.12	1.00	0.00	0.01	-0.97	-0.96	0.81	0.62	0.81	0.83	-0.00	-0.96	0.01	0.25	-0.06	-0.16	-0.09	-0.00	-0.01	-0.82	0.01	0.00
$r_{e\mu}$	-0.01	-0.00	0.01	-0.77	-0.01	0.00	0.00	-0.00	-0.01	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.01	-0.00	-0.00	-0.00	0.00	1.00	0.00	-0.00	-0.01	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	-0.01	-0.00	0.75	-0.00	-0.99	0.00	-0.00	0.01	0.00	0.01	0.00	-0.00	-0.01	-0.00	0.00	-0.04	0.01	0.00	-0.00	0.01	0.00	1.00	-0.01	-0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
η_1	0.94	0.94	0.38	0.64	0.13	-0.89	0.03	-0.03	0.30	0.39	0.39	0.26	0.71	0.17	-0.88	-1.00	-0.93	0.06	-0.03	-0.97	-0.00	-0.01	1.00	0.87	-0.91	-0.64	-0.88	-0.89	0.00	1.00	-0.01	-0.24	0.05	0.16	0.09	0.01	0.01	0.77	-0.01	-0.00
η_2	0.65	0.99	0.42	0.50	0.11	-0.70	0.03	-0.01	0.26	0.43	0.25	0.14	0.78	0.17	-1.00	-0.82	-0.64	0.48	-0.23	-0.96	-0.01	-0.00	0.87	1.00	-0.63	-0.56	-0.66	-0.69	0.00	0.84	-0.01	-0.25	0.05	0.15	0.09	0.00	0.01	0.81	-0.01	-0.00
$m_{\beta\beta}$	-0.96	-0.74	-0.29	-0.61	-0.12	0.97	-0.02	0.04	-0.28	-0.30	-0.41	-0.28	-0.54	-0.14	0.65	0.93	0.96	0.36	0.13	0.81	0.00	0.00	-0.91	-0.63	1.00	0.79	0.98	0.98	-0.00	-0.92	0.00	0.19	-0.04	-0.13	-0.06	-0.01	-0.01	-0.61	0.00	0.00
Σ	-0.60	-0.60	-0.25	-0.41	-0.08	0.90	-0.02	0.02	-0.19	-0.25	-0.25	-0.16	-0.45	-0.10	0.57	0.64	0.60	0.33	0.71	0.62	0.00	0.00	-0.64	-0.56	0.79	1.00	0.89	0.89	-0.01	-0.64	0.00	0.16	-0.03	-0.10	-0.05	-0.00	-0.00	-0.49	-0.00	0.00
m_β	-0.90	-0.76	-0.30	-0.58	-0.12	1.00	-0.02	0.03	-0.27	-0.31	-0.38	-0.26	-0.56	-0.14	0.68	0.89	0.90	0.33	0.32	0.81	0.00	0.00	-0.88	-0.66	0.98	0.89	1.00	1.00	-0.00	-0.89	0.00	0.19	-0.04	-0.13	-0.06	-0.01	-0.01	-0.62	0.00	0.00
m_1	-0.90	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14	0.71	0.90	0.90	0.30	0.33	0.83	0.00	0.00	-0.89	-0.69	0.98	0.89	1.00	1.00	-0.00	-0.90	0.00	0.20	-0.05	-0.14	-0.07	-0.01	-0.01	-0.64	0.00	0.00
m_τ	0.00	0.00	0.51	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.01	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.00	1.00	0.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
J_{CP}	0.96	0.92	0.37	0.64	0.13	-0.90	0.03	-0.03	0.30	0.38	0.40	0.26	0.69	0.17	-0.85	-1.00	-0.95	0.00	-0.00	-0.96	-0.00	-0.01	1.00	0.84	-0.92	-0.64	-0.89	-0.90	0.00	1.00	-0.01	-0.23	0.05	0.16	0.08	0.01	0.01	0.76	-0.01	-0.00
θ_{12}^q	-0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.02	-0.01	0.11	0.00	0.01	0.01	0.00	-0.00	0.00	0.01	0.00	-0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	-0.01	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ_{23}^q	-0.20	-0.25	-0.10	-0.14	-0.03	0.20	-0.01	0.00	-0.09	-0.22	0.27	0.83	0.34	0.09	0.25	0.23	0.19	-0.08	0.05	0.25	0.00	-0.00	-0.24	-0.25	0.19	0.16	0.19	0.20	0.00	-0.23	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ_{13}^q	0.05	0.06	0.02	0.03	0.01	-0.05	-0.01	0.01	0.70	-0.02	0.30	0.14	-0.02	-0.00	-0.05	-0.05	-0.05	0.01	-0.00	-0.06	-0.00	0.00	0.05	0.05	-0.04	-0.03	-0.04	-0.05	-0.00	0.05	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ_{CP}^q	0.14	0.16	0.07	0.10	0.02	-0.14	0.01	-0.01	0.03	-0.05	0.84	0.39	-0.04	-0.01	-0.16	-0.16	-0.14	0.04	-0.02	-0.16	-0.00	0.00	0.16	0.15	-0.13	-0.10	-0.13	-0.14	0.00	0.16	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	0.07	0.09	0.04	0.05	0.01	-0.07	-0.01	0.01	0.71	-0.00	0.32	0.10	-0.04	-0.00	-0.09	-0.08	-0.06	0.04	-0.02	-0.09	-0.00	0.00	0.09	0.09	-0.06	-0.05	-0.06	-0.07	-0.00	0.08	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	0.01	0.00	-0.00	0.00	0.01	-0.01	0.86	-1.00	-0.66	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.01	-0.00	0.00	-0.00	0.00	-0.01	0.01	0.00	-0.01	-0.00	-0.01	-0.01	-0.00	0.01	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	0.01	0.01	0.00	0.01	0.00	-0.01	-0.00	0.00	0.01	0.00	0.01	0.01	-0.01	0.97	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.00	0.00	0.01	0.01	-0.01	-0.00	-0.01	-0.01	-0.00	0.01	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	0.63	0.82	0.34	0.46	0.10	-0.65	0.03	-0.01	0.15	0.34	0.19	0.51	0.99	0.23	-0.81	-0.75	-0.62	0.29	-0.14	-0.82	-0.00	-0.01	0.77	0.81	-0.61	-0.49	-0.62	-0.64	0.01	0.76	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	-0.00	-0.01	-0.00	-0.00	-0.00	0.00	0.50	0.00	-0.00	-0.00	-0.01	-0.01	-0.01	-0.01	0.01	0.01	0.00	-0.01	-0.00	0.01	0.00	-0.00	-0.01	-0.01	0.00	-0.00	0.00	0.00	-0.00	-0.01	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	-0.00	-0.00	0.01	0.00	-0.01	0.00	0.00	-0.00	-0.00	0.88	-0.01	-0.01	-0.01	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	0.01	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

Complete correlations matrix

Correlations between lepton and quark parameters

Correlations between lepton and quark observables

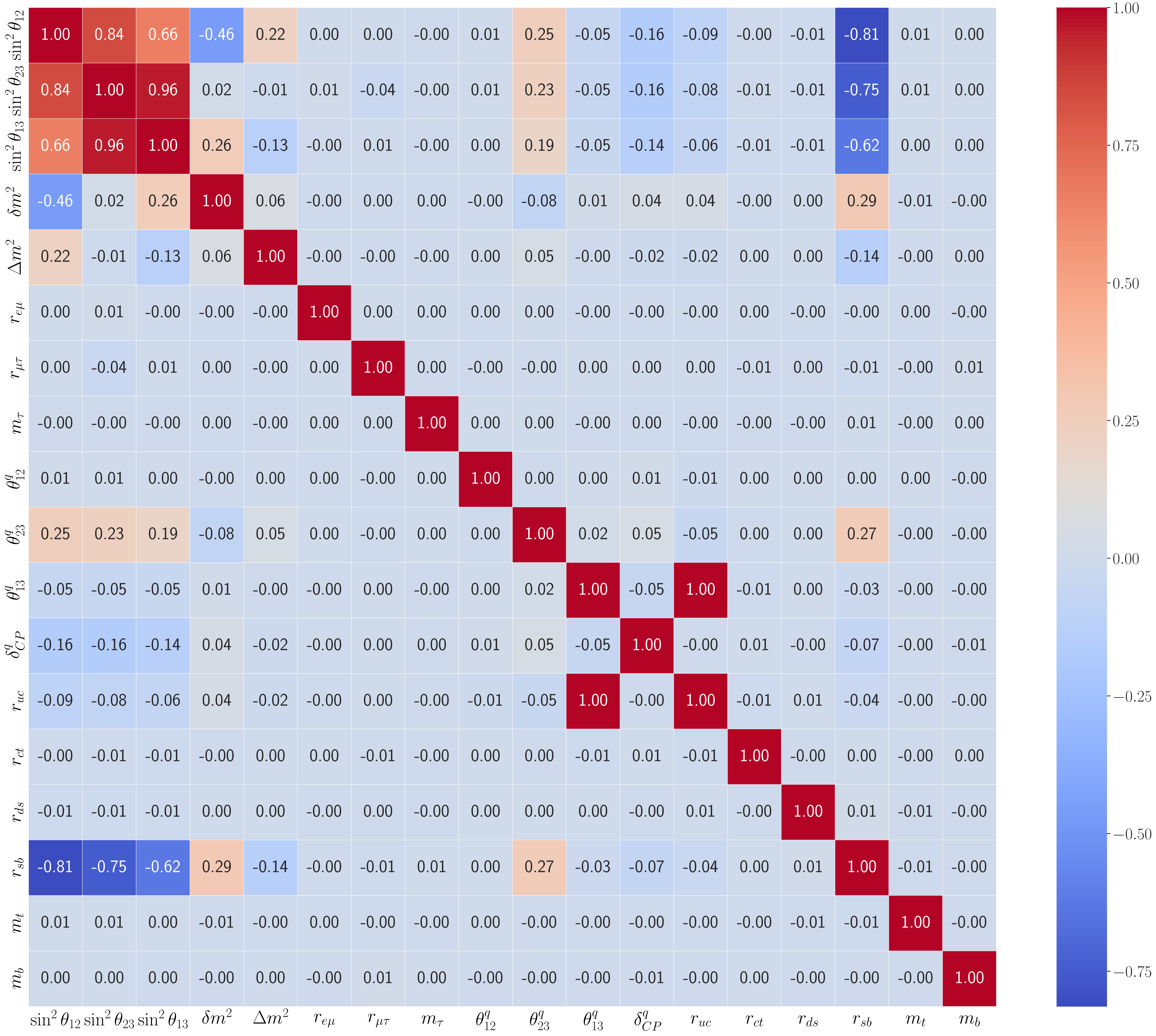
Correlations between predicted neutrino unknowns and quark observables



	Re(τ)	Im(τ)	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$	η_1	η_2	$m_{\beta\beta}$	Σ	m_β	m_1	m_τ	J_{CP}	θ_{12}^q	θ_{23}^q	θ_{13}^q	δ_{CP}^q	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
Re(τ)	1.00	0.77	0.30	0.64	0.13	-0.89	0.02	-0.04	0.29	0.31	0.43	0.29	0.56	0.14	-0.67	-0.96	-1.00	-0.24	0.12	-0.84	-0.01	-0.01	0.94	0.65	-0.96	-0.60	-0.90	-0.90	0.00	0.96	-0.00	-0.20	0.05	0.14	0.07	0.01	0.01	0.63	-0.00	-0.00
Im(τ)	0.77	1.00	0.42	0.56	0.12	-0.78	0.03	-0.01	0.28	0.43	0.30	0.18	0.78	0.18	-0.99	-0.91	-0.75	0.35	-0.17	-0.99	-0.00	-0.00	0.94	0.99	-0.74	-0.60	-0.76	-0.78	0.00	0.92	-0.01	-0.25	0.06	0.16	0.09	0.00	0.01	0.82	-0.01	-0.00
$g_1^E v_d$	0.30	0.42	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07	-0.42	-0.40	-0.29	0.18	-0.09	-0.41	0.01	0.75	0.38	0.42	-0.29	-0.25	-0.30	-0.31	0.51	0.37	-0.00	-0.10	0.02	0.07	0.04	-0.00	0.00	0.34	-0.00	0.01
g_2^E/g_1^E	0.64	0.56	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10	-0.51	-0.65	-0.63	-0.06	0.03	-0.59	-0.77	-0.00	0.64	0.50	-0.61	-0.41	-0.58	-0.59	-0.00	0.64	-0.00	-0.14	0.03	0.10	0.05	0.00	0.01	0.46	-0.00	0.00
g_3^E/g_1^E	0.13	0.12	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02	-0.11	-0.09	-0.13	0.00	-0.00	-0.13	-0.01	-0.99	0.13	0.11	-0.12	-0.08	-0.12	-0.12	-0.00	0.13	0.00	-0.03	0.01	0.02	0.01	0.01	0.00	0.10	-0.00	-0.01
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.89	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14	0.71	0.90	0.89	0.29	0.34	0.83	0.00	0.00	-0.89	-0.70	0.97	0.90	1.00	1.00	-0.00	-0.90	0.00	0.20	-0.05	-0.14	-0.07	-0.01	-0.01	-0.65	0.00	0.00
$g_1^u v_u$	0.02	0.03	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00	-0.03	-0.02	-0.02	0.02	-0.01	-0.03	0.00	-0.00	0.03	0.03	-0.02	-0.02	-0.02	-0.02	-0.00	0.03	0.00	-0.01	-0.01	0.01	-0.01	0.86	-0.00	0.03	0.50	0.00
g_2^u/g_1^u	-0.04	-0.01	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00	0.01	0.03	0.04	0.03	-0.01	0.02	-0.00	0.01	-0.03	-0.01	0.04	0.02	0.03	0.03	0.00	-0.03	-0.00	0.00	0.01	-0.01	0.01	-1.00	0.00	-0.01	0.00	-0.00
g_3^u/g_1^u	0.29	0.28	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04	-0.26	-0.30	-0.28	0.01	-0.00	-0.29	-0.01	0.00	0.30	0.26	-0.28	-0.19	-0.27	-0.27	0.00	0.30	-0.01	-0.09	0.70	0.03	0.71	-0.66	0.01	0.15	-0.00	-0.00
$g_1^d v_d$	0.31	0.43	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07	-0.43	-0.38	-0.30	0.18	-0.09	-0.43	-0.00	0.01	0.39	0.43	-0.30	-0.25	-0.31	-0.32	0.00	0.38	-0.00	-0.22	-0.02	-0.05	-0.00	0.00	0.00	0.34	-0.00	0.88
g_2^d/g_1^d	0.43	0.30	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06	-0.25	-0.40	-0.43	-0.14	0.07	-0.34	-0.00	0.00	0.39	0.25	-0.41	-0.25	-0.38	-0.38	0.00	0.40	-0.02	0.27	0.30	0.84	0.32	0.00	0.01	0.19	-0.01	-0.01
g_3^d/g_1^d	0.29	0.18	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15	-0.15	-0.27	-0.30	-0.12	0.07	-0.21	-0.00	-0.00	0.26	0.14	-0.28	-0.16	-0.26	-0.25	0.00	0.26	-0.01	0.83	0.14	0.39	0.10	0.00	0.01	0.51	-0.01	-0.01
g_4^d/g_1^d	0.56	0.78	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21	-0.78	-0.68	-0.55	0.33	-0.16	-0.77	-0.00	-0.01	0.71	0.78	-0.54	-0.45	-0.56	-0.58	0.01	0.69	0.11	0.34	-0.02	-0.04	-0.04	0.00	-0.01	0.99	-0.01	-0.01
g_5^d/g_1^d	0.14	0.18	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00	-0.17	-0.16	-0.14	0.06	-0.02	-0.18	-0.00	-0.00	0.17	0.17	-0.14	-0.10	-0.14	-0.14	-0.00	0.17	0.00	0.09	-0.00	-0.01	-0.00	-0.00	0.97	0.23	-0.01	-0.00
$\sin^2\theta_{12}$	-0.67	-0.99	-0.42	-0.51	-0.11	0.71	-0.03	0.01	-0.26	-0.43	-0.25	-0.15	-0.78	-0.17	1.00	0.84	0.66	-0.46	0.22	0.97	0.00	0.00	-0.88	-1.00	0.65	0.57	0.68	0.71	-0.00	-0.85	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	-0.96	-0.91	-0.40	-0.65	-0.09	0.90	-0.02	0.03	-0.30	-0.38	-0.40	-0.27	-0.68	-0.16	0.84	1.00	0.96	0.02	-0.01	0.95	0.01	-0.04	-1.00	-0.82	0.93	0.64	0.89	0.90	-0.00	-1.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	-1.00	-0.75	-0.29	-0.63	-0.13	0.89	-0.02	0.04	-0.28	-0.30	-0.43	-0.30	-0.55	-0.14	0.66	0.96	1.00	0.26	-0.13	0.83	-0.00	0.01	-0.93	-0.64	0.96	0.60	0.90	0.90	-0.00	-0.95	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.24	0.35	0.18	-0.06	0.00	0.29	0.02	0.03	0.01	0.18	-0.14	-0.12	0.33	0.06	-0.46	0.02	0.26	1.00	0.06	-0.25	-0.00	0.00	0.06	0.48	0.36	0.33	0.33	0.30	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.12	-0.17	-0.09	0.03	-0.00	0.34	-0.01	-0.01	-0.00	-0.09	0.07	0.07	-0.16	-0.02	0.22	-0.01	-0.13	0.06	1.00	0.12	-0.00	-0.00	-0.03	-0.23	0.13	0.71	0.32	0.33	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
δ_{CP}	-0.84	-0.99	-0.41	-0.59	-0.13	0.83	-0.03	0.02	-0.29	-0.43	-0.34	-0.21	-0.77	-0.18	0.97	0.95	0.83	-0.25	0.12	1.00	0.00	0.01	-0.97	-0.96	0.81	0.62	0.81	0.83	-0.00	-0.96	0.01	0.25	-0.06	-0.16	-0.09	-0.00	-0.01	-0.82	0.01	0.00
$r_{e\mu}$	-0.01	-0.00	0.01	-0.77	-0.01	0.00	0.00	-0.00	-0.01	-0.00	-0.00	-0.00	-0.00	-0.00	0.00	0.01	-0.00	-0.00	-0.00	0.00	1.00	0.00	-0.00	-0.01	0.00	0.00	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	-0.01	-0.00	0.75	-0.00	-0.99	0.00	-0.00	0.01	0.00	0.01	0.00	-0.00	-0.01	-0.00	0.00	-0.04	0.01	0.00	-0.00	0.01	0.00	1.00	-0.01	-0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
η_1	0.94	0.94	0.38	0.64	0.13	-0.89	0.03	-0.03	0.30	0.39	0.39	0.26	0.71	0.17	-0.88	-1.00	-0.93	0.06	-0.03	-0.97	-0.00	-0.01	1.00	0.87	-0.91	-0.64	-0.88	-0.89	0.00	1.00	-0.01	-0.24	0.05	0.16	0.09	0.01	0.01	0.77	-0.01	-0.00
η_2	0.65	0.99	0.42	0.50	0.11	-0.70	0.03	-0.01	0.26	0.43	0.25	0.14	0.78	0.17	-1.00	-0.82	-0.64	0.48	-0.23	-0.96	-0.01	-0.00	0.87	1.00	-0.63	-0.56	-0.66	-0.69	0.00	0.84	-0.01	-0.25	0.05	0.15	0.09	0.00	0.01	0.81	-0.01	-0.00
$m_{\beta\beta}$	-0.96	-0.74	-0.29	-0.61	-0.12	0.97	-0.02	0.04	-0.28	-0.30	-0.41	-0.28	-0.54	-0.14	0.65	0.93	0.96	0.36	0.13	0.81	0.00	0.00	-0.91	-0.63	1.00	0.79	0.98	0.98	-0.00	-0.92	0.00	0.19	-0.04	-0.13	-0.06	-0.01	-0.01	-0.61	0.00	0.00
Σ	-0.60	-0.60	-0.25	-0.41	-0.08	0.90	-0.02	0.02	-0.19	-0.25	-0.25	-0.16	-0.45	-0.10	0.57	0.64	0.60	0.33	0.71	0.62	0.00	0.00	-0.64	-0.56	0.79	1.00	0.89	0.89	-0.01	-0.64	0.00	0.16	-0.03	-0.10	-0.05	-0.00	-0.00	-0.49	-0.00	0.00
m_β	-0.90	-0.76	-0.30	-0.58	-0.12	1.00	-0.02	0.03	-0.27	-0.31	-0.38	-0.26	-0.56	-0.14	0.68	0.89	0.90	0.33	0.32	0.81	0.00	0.00	-0.88	-0.66	0.98	0.89	1.00	1.00	-0.00	-0.89	0.00	0.19	-0.04	-0.13	-0.06	-0.01	-0.01	-0.62	0.00	0.00
m_1	-0.90	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14	0.71	0.90	0.90	0.30	0.33	0.83	0.00	0.00	-0.89	-0.69	0.98	0.89	1.00	1.00	-0.00	-0.90	0.00	0.20	-0.05	-0.14	-0.07	-0.01	-0.01	-0.64	0.00	0.00
m_τ	0.00	0.00	0.51	-0.00	-0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	0.01	-0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.00	1.00	0.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
J_{CP}	0.96	0.92	0.37	0.64	0.13	-0.90	0.03	-0.03	0.30	0.38	0.40	0.26	0.69	0.17	-0.85	-1.00	-0.95	0.00	-0.00	-0.96	-0.00	-0.01	1.00	0.84	-0.92	-0.64	-0.89	-0.90	0.00	1.00	-0.01	-0.23	0.05	0.16	0.08	0.01	0.01	0.76	-0.01	-0.00
θ_{12}^q	-0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00	-0.00	-0.01	-0.00	-0.02	-0.01	0.11	0.00	0.01	0.01	0.00	-0.00	0.00	0.01	0.00	-0.00	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	-0.01	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ_{23}^q	-0.20	-0.25	-0.10	-0.14	-0.03	0.20	-0.01	0.00	-0.09	-0.22	0.27	0.83	0.34	0.09	0.25	0.23	0.19	-0.08	0.05	0.25	0.00	-0.00	-0.24	-0.25	0.19	0.16	0.19	0.20	0.00	-0.23	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ_{13}^q	0.05	0.06	0.02	0.03	0.01	-0.05	-0.01	0.01	0.70	-0.02	0.30	0.14	-0.02	-0.00	-0.05	-0.05	-0.05	0.01	-0.00	-0.06	-0.00	0.00	0.05	0.05	-0.04	-0.03	-0.04	-0.05	-0.00	0.05	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ_{CP}^q	0.14	0.16	0.07	0.10	0.02	-0.14	0.01	-0.01	0.03	-0.05	0.84	0.39	-0.04	-0.01	-0.16	-0.16	-0.14	0.04	-0.02	-0.16	-0.00	0.00	0.16	0.15	-0.13	-0.10	-0.13	-0.14	0.00	0.16	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	0.07	0.09	0.04	0.05	0.01	-0.07	-0.01	0.01	0.71	-0.00	0.32	0.10	-0.04	-0.00	-0.09	-0.08	-0.06	0.04	-0.02	-0.09	-0.00	0.00	0.09	0.09	-0.06	-0.05	-0.06	-0.07	-0.00	0.08	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	0.01	0.00	-0.00	0.00	0.01	-0.01	0.86	-1.00	-0.66	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.01	-0.01	-0.00	0.00	-0.00	0.00	-0.01	0.01	0.00	-0.01	-0.00	-0.01	-0.01	-0.00	0.01	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	0.01	0.01	0.00	0.01	0.00	-0.01	-0.00	0.00	0.01	0.00	0.01	0.01	-0.01	0.97	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.00	0.00	0.01	0.01	-0.01	-0.00	-0.01	-0.01	-0.00	0.01	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	0.63	0.82	0.34	0.46	0.10	-0.65	0.03	-0.01	0.15	0.34	0.19	0.51	0.99	0.23	-0.81	-0.75	-0.62	0.29	-0.14	-0.82	-0.00	-0.01	0.77	0.81	-0.61	-0.49	-0.62	-0.64	0.01	0.76	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	-0.00	-0.01	-0.00	-0.00	-0.00	0.00	0.50	0.00	-0.00	-0.00	-0.01	-0.01	-0.01	-0.01	0.01	0.01	0.00	-0.01	-0.00	0.01	0.00	-0.00	-0.01	-0.01	0.00	-0.00	0.00	0.00	-0.00	-0.01	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	-0.00	-0.00	0.01	0.00	-0.01	0.00	0.00	-0.00	-0.00	0.88	-0.01	-0.01	-0.01	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	0.01	-0.00	-0.00	0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

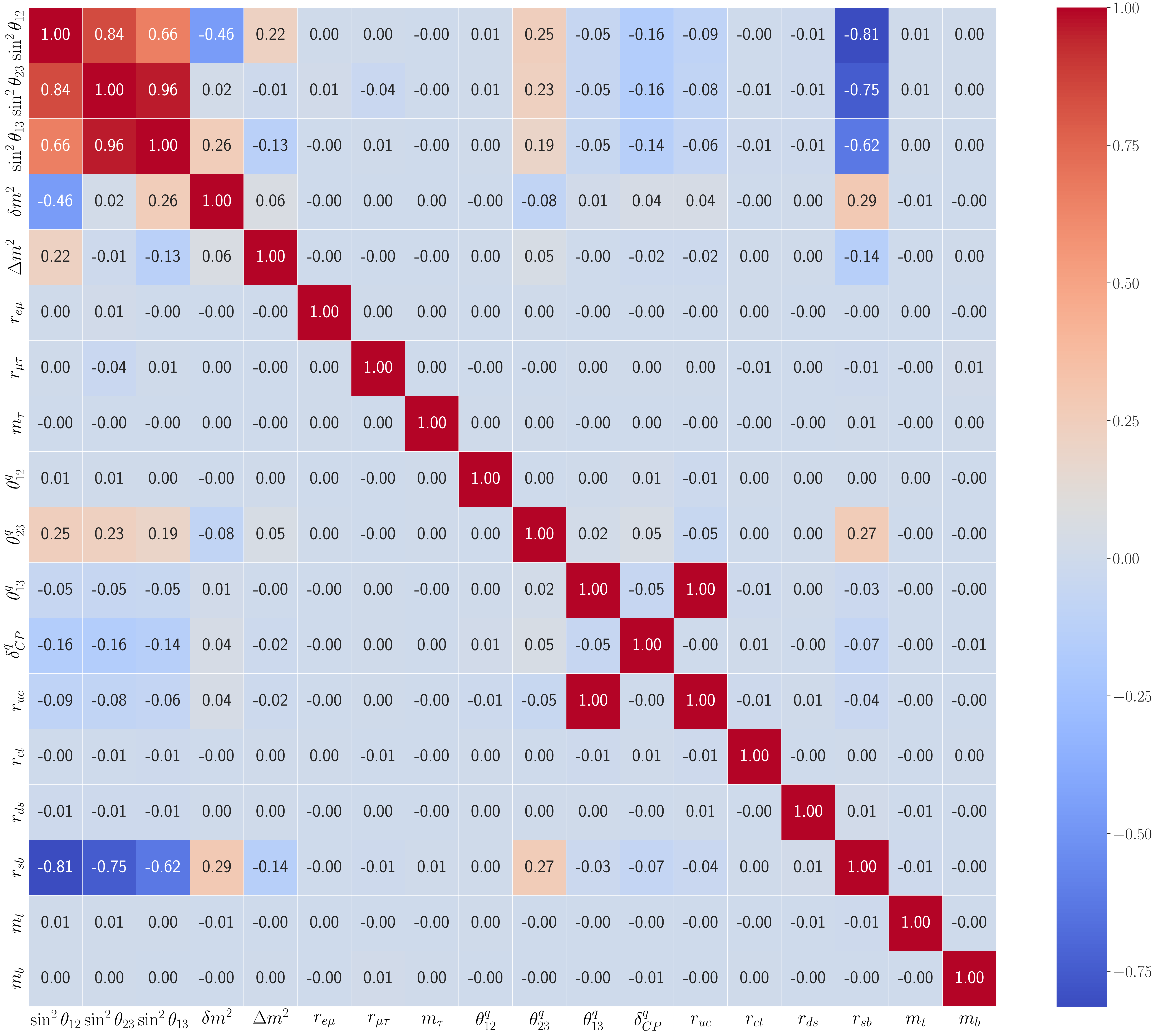
Correlations among Observables

Correlations among Observables



	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	$r_{e\mu}$	$r_{\mu\tau}$	m_τ	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.84	0.66	-0.46	0.22	0.00	0.00	-0.00	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	0.84	1.00	0.96	0.02	-0.01	0.01	-0.04	-0.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	0.66	0.96	1.00	0.26	-0.13	-0.00	0.01	-0.00	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.46	0.02	0.26	1.00	0.06	-0.00	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.22	-0.01	-0.13	0.06	1.00	-0.00	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
$r_{e\mu}$	0.00	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	0.00	-0.04	0.01	0.00	-0.00	0.00	1.00	0.00	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
m_τ	-0.00	-0.00	-0.00	0.00	-0.00	0.00	0.00	1.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.00	-0.00	0.00	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.00	-0.00	0.00	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ^q_{13}	-0.05	-0.05	-0.05	0.01	-0.00	-0.00	0.00	-0.00	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ^q_{CP}	-0.16	-0.16	-0.14	0.04	-0.02	-0.00	0.00	0.00	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	-0.09	-0.08	-0.06	0.04	-0.02	-0.00	0.00	-0.00	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	-0.00	-0.01	-0.01	-0.00	0.00	0.00	-0.01	-0.00	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	-0.01	-0.01	-0.01	0.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	-0.81	-0.75	-0.62	0.29	-0.14	-0.00	-0.01	0.01	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	0.01	0.01	0.00	-0.01	-0.00	0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	0.00	0.00	0.00	-0.00	0.00	-0.00	0.01	0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

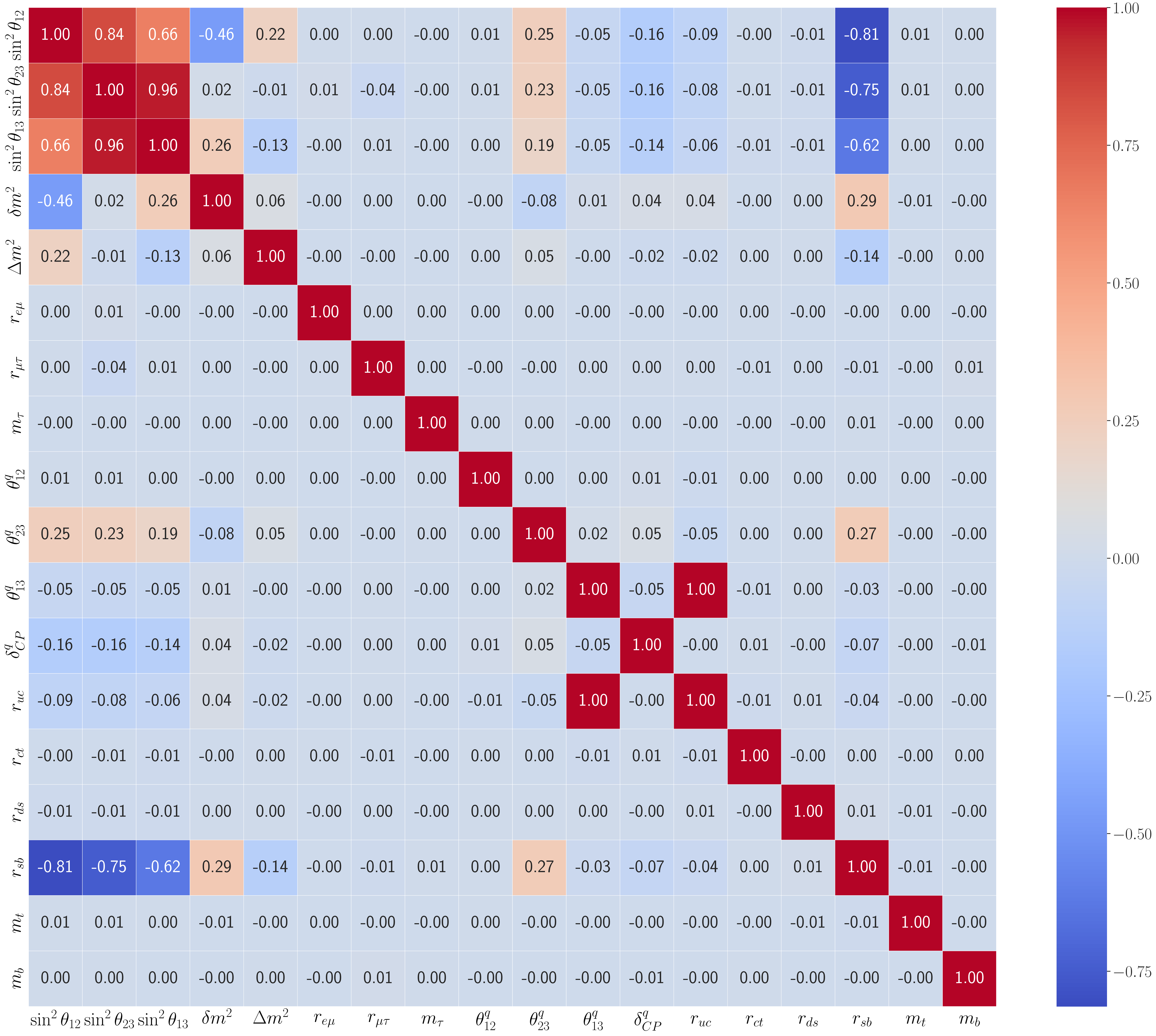
Correlations among Observables



	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	$r_{e\mu}$	$r_{\mu\tau}$	m_τ	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.84	0.66	-0.46	0.22	0.00	0.00	-0.00	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	0.84	1.00	0.96	0.02	-0.01	0.01	-0.04	-0.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	0.66	0.96	1.00	0.26	-0.13	-0.00	0.01	-0.00	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.46	0.02	0.26	1.00	0.06	-0.00	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.22	-0.01	-0.13	0.06	1.00	-0.00	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
$r_{e\mu}$	0.00	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	0.00	-0.04	0.01	0.00	-0.00	0.00	1.00	0.00	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
m_τ	-0.00	-0.00	-0.00	0.00	-0.00	0.00	0.00	1.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.00	-0.00	0.00	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.00	-0.00	0.00	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ^q_{13}	-0.05	-0.05	-0.05	0.01	-0.00	-0.00	0.00	-0.00	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ^q_{CP}	-0.16	-0.16	-0.14	0.04	-0.02	-0.00	0.00	0.00	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	-0.09	-0.08	-0.06	0.04	-0.02	-0.00	0.00	-0.00	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	-0.00	-0.01	-0.01	-0.00	0.00	0.00	-0.01	-0.00	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	-0.01	-0.01	-0.01	0.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	-0.81	-0.75	-0.62	0.29	-0.14	-0.00	-0.01	0.01	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	0.01	0.01	0.00	-0.01	-0.00	0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	0.00	0.00	0.00	-0.00	0.00	-0.00	0.01	0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

Observables show a more complex and rich pattern of correlations in this case, $N_{\text{dof}} = 4 > 0$, than in the two previous cases, $N_{\text{dof}} \leq 0$

Correlations among Observables

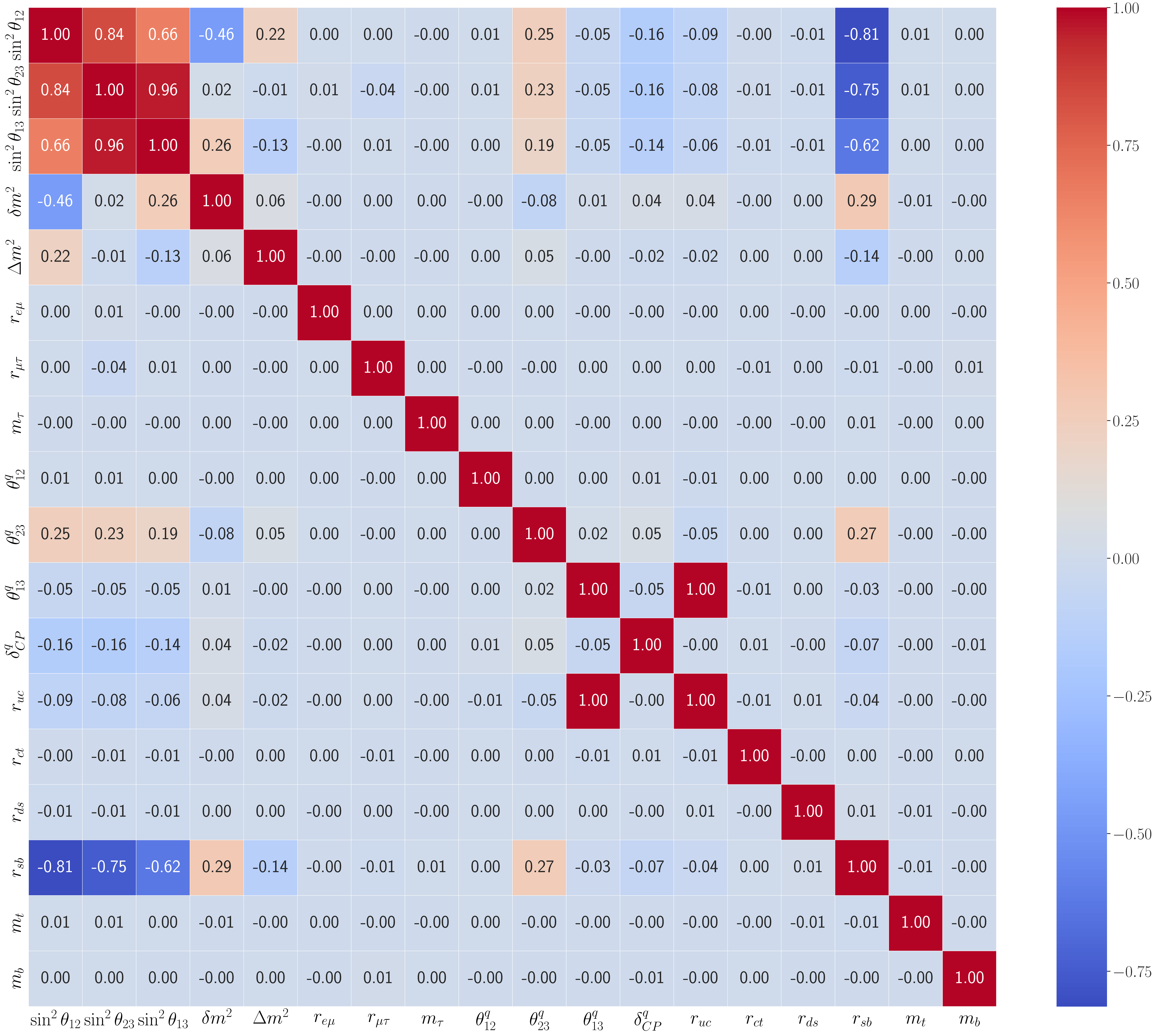


	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	$r_{e\mu}$	$r_{\mu\tau}$	m_τ	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.84	0.66	-0.46	0.22	0.00	0.00	-0.00	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	0.84	1.00	0.96	0.02	-0.01	0.01	-0.04	-0.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	0.66	0.96	1.00	0.26	-0.13	-0.00	0.01	-0.00	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.46	0.02	0.26	1.00	0.06	-0.00	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.22	-0.01	-0.13	0.06	1.00	-0.00	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
$r_{e\mu}$	0.00	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	0.00	-0.04	0.01	0.00	-0.00	0.00	1.00	0.00	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
m_τ	-0.00	-0.00	-0.00	0.00	-0.00	0.00	0.00	1.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.00	-0.00	0.00	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.00	-0.00	0.00	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ^q_{13}	-0.05	-0.05	-0.05	0.01	-0.00	-0.00	0.00	-0.00	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ^q_{CP}	-0.16	-0.16	-0.14	0.04	-0.02	-0.00	0.00	0.00	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	-0.09	-0.08	-0.06	0.04	-0.02	-0.00	0.00	-0.00	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	-0.00	-0.01	-0.01	-0.00	0.00	0.00	-0.01	-0.00	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	-0.01	-0.01	-0.01	0.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	-0.81	-0.75	-0.62	0.29	-0.14	-0.00	-0.01	0.01	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	0.01	0.01	0.00	-0.01	-0.00	0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	0.00	0.00	0.00	-0.00	0.00	-0.00	0.01	0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

Observables show a more complex and rich pattern of correlations in this case, $N_{\text{dof}} = 4 > 0$, than in the two previous cases, $N_{\text{dof}} \leq 0$

In most of the cases, correlations seem to be governed by the modulus τ

Correlations among Observables



	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	$r_{e\mu}$	$r_{\mu\tau}$	m_τ	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
$\sin^2\theta_{12}$	1.00	0.84	0.66	-0.46	0.22	0.00	0.00	-0.00	0.01	0.25	-0.05	-0.16	-0.09	-0.00	-0.01	-0.81	0.01	0.00
$\sin^2\theta_{23}$	0.84	1.00	0.96	0.02	-0.01	0.01	-0.04	-0.00	0.01	0.23	-0.05	-0.16	-0.08	-0.01	-0.01	-0.75	0.01	0.00
$\sin^2\theta_{13}$	0.66	0.96	1.00	0.26	-0.13	-0.00	0.01	-0.00	0.00	0.19	-0.05	-0.14	-0.06	-0.01	-0.01	-0.62	0.00	0.00
δm^2	-0.46	0.02	0.26	1.00	0.06	-0.00	0.00	0.00	-0.00	-0.08	0.01	0.04	0.04	-0.00	0.00	0.29	-0.01	-0.00
Δm^2	0.22	-0.01	-0.13	0.06	1.00	-0.00	-0.00	-0.00	0.00	0.05	-0.00	-0.02	-0.02	0.00	0.00	-0.14	-0.00	0.00
$r_{e\mu}$	0.00	0.01	-0.00	-0.00	-0.00	1.00	0.00	0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	0.00	-0.00
$r_{\mu\tau}$	0.00	-0.04	0.01	0.00	-0.00	0.00	1.00	0.00	-0.00	-0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.00	0.01
m_τ	-0.00	-0.00	-0.00	0.00	-0.00	0.00	0.00	1.00	0.00	0.00	-0.00	0.00	-0.00	-0.00	-0.00	0.01	-0.00	0.00
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.00	-0.00	0.00	1.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.00	-0.00	0.00	0.00	1.00	0.02	0.05	-0.05	0.00	0.00	0.27	-0.00	-0.00
θ^q_{13}	-0.05	-0.05	-0.05	0.01	-0.00	-0.00	0.00	-0.00	0.00	0.02	1.00	-0.05	1.00	-0.01	0.00	-0.03	-0.00	-0.00
δ^q_{CP}	-0.16	-0.16	-0.14	0.04	-0.02	-0.00	0.00	0.00	0.01	0.05	-0.05	1.00	-0.00	0.01	-0.00	-0.07	-0.00	-0.01
r_{uc}	-0.09	-0.08	-0.06	0.04	-0.02	-0.00	0.00	-0.00	-0.01	-0.05	1.00	-0.00	1.00	-0.01	0.01	-0.04	-0.00	-0.00
r_{ct}	-0.00	-0.01	-0.01	-0.00	0.00	0.00	-0.01	-0.00	0.00	0.00	-0.01	0.01	-0.01	1.00	-0.00	0.00	-0.00	0.00
r_{ds}	-0.01	-0.01	-0.01	0.00	0.00	-0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.01	-0.00	1.00	0.01	-0.01	-0.00
r_{sb}	-0.81	-0.75	-0.62	0.29	-0.14	-0.00	-0.01	0.01	0.00	0.27	-0.03	-0.07	-0.04	0.00	0.01	1.00	-0.01	-0.00
m_t	0.01	0.01	0.00	-0.01	-0.00	0.00	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	1.00	-0.00
m_b	0.00	0.00	0.00	-0.00	0.00	-0.00	0.01	0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.00	-0.00	-0.00	1.00

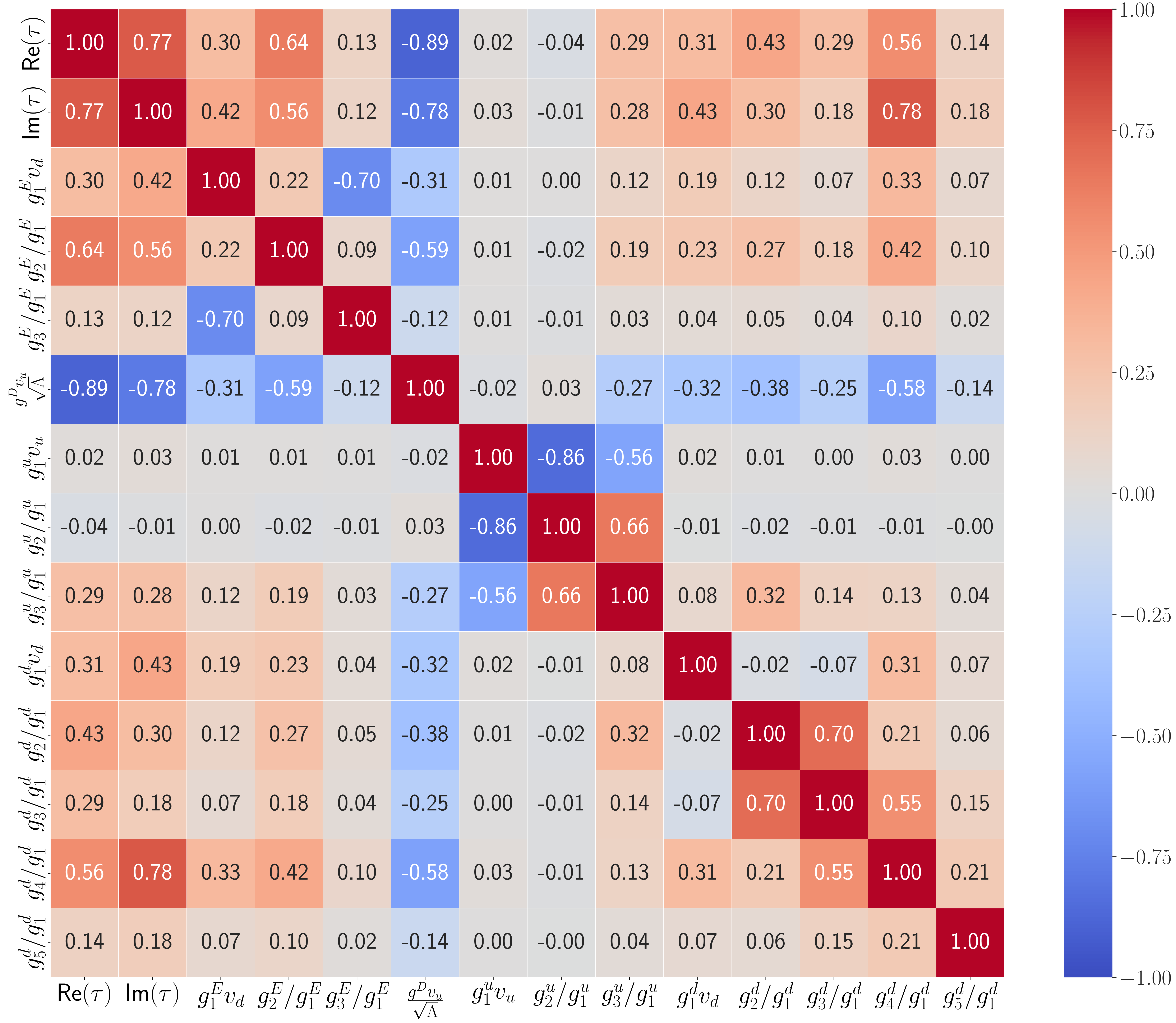
Observables show a more complex and rich pattern of correlations in this case, $N_{\text{dof}} = 4 > 0$, than in the two previous cases, $N_{\text{dof}} \leq 0$

In most of the cases, correlations seem to be governed by the modulus τ

Interesting case of the (θ^q_{13}, r_{uc}) pair, for which the correlation is ~1 and is not related to τ

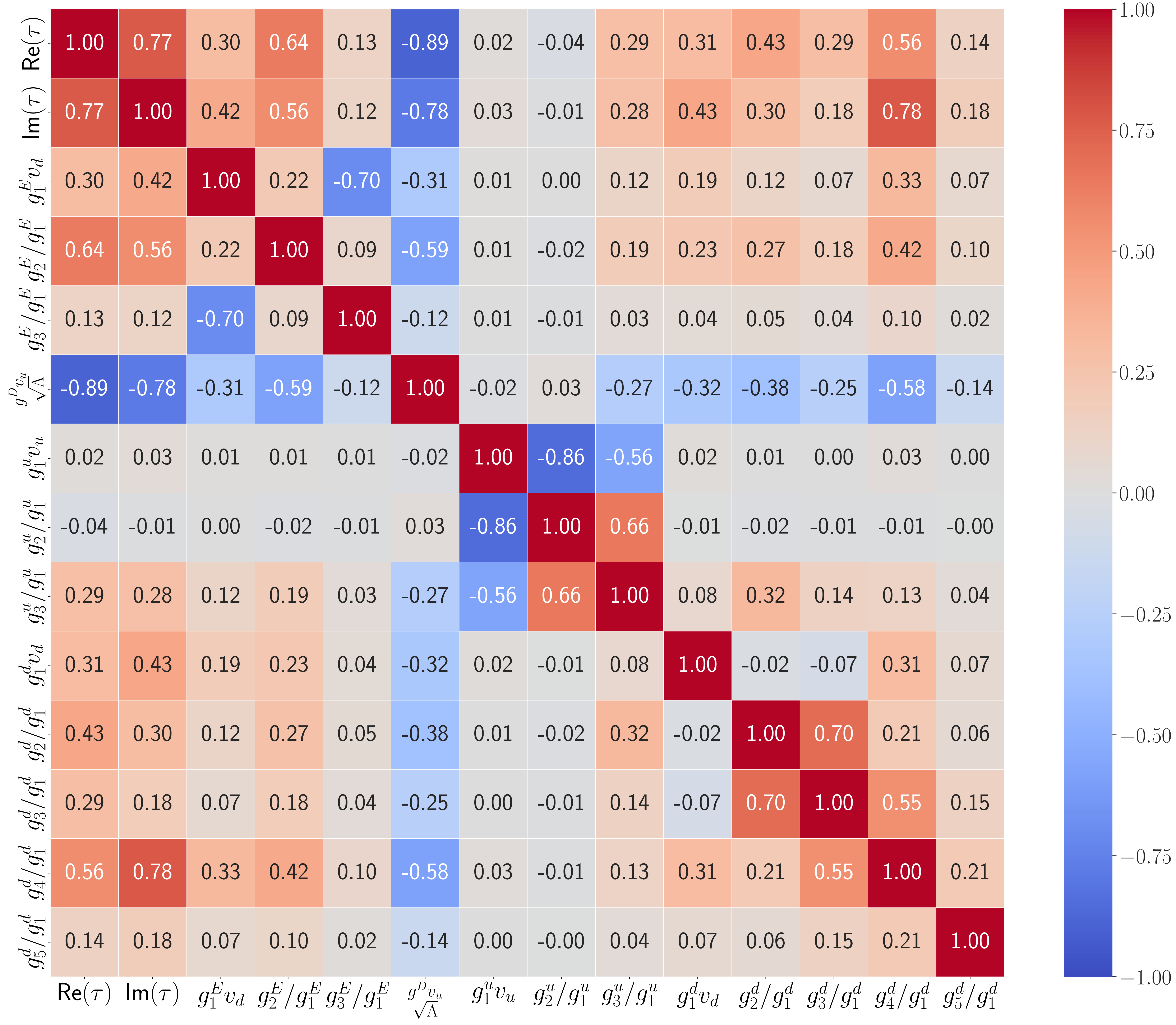
Correlations between Parameters

Correlations between Parameters



	Re(τ)	Im(τ)	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g^D v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
Re(τ)	1.00	0.77	0.30	0.64	0.13	-0.89	0.02	-0.04	0.29	0.31	0.43	0.29	0.56	0.14
Im(τ)	0.77	1.00	0.42	0.56	0.12	-0.78	0.03	-0.01	0.28	0.43	0.30	0.18	0.78	0.18
$g_1^E v_d$	0.30	0.42	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.64	0.56	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	0.13	0.12	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g^D v_u}{\sqrt{\Lambda}}$	-0.89	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.02	0.03	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	-0.04	-0.01	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.29	0.28	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.31	0.43	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.43	0.30	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.29	0.18	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.56	0.78	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.14	0.18	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

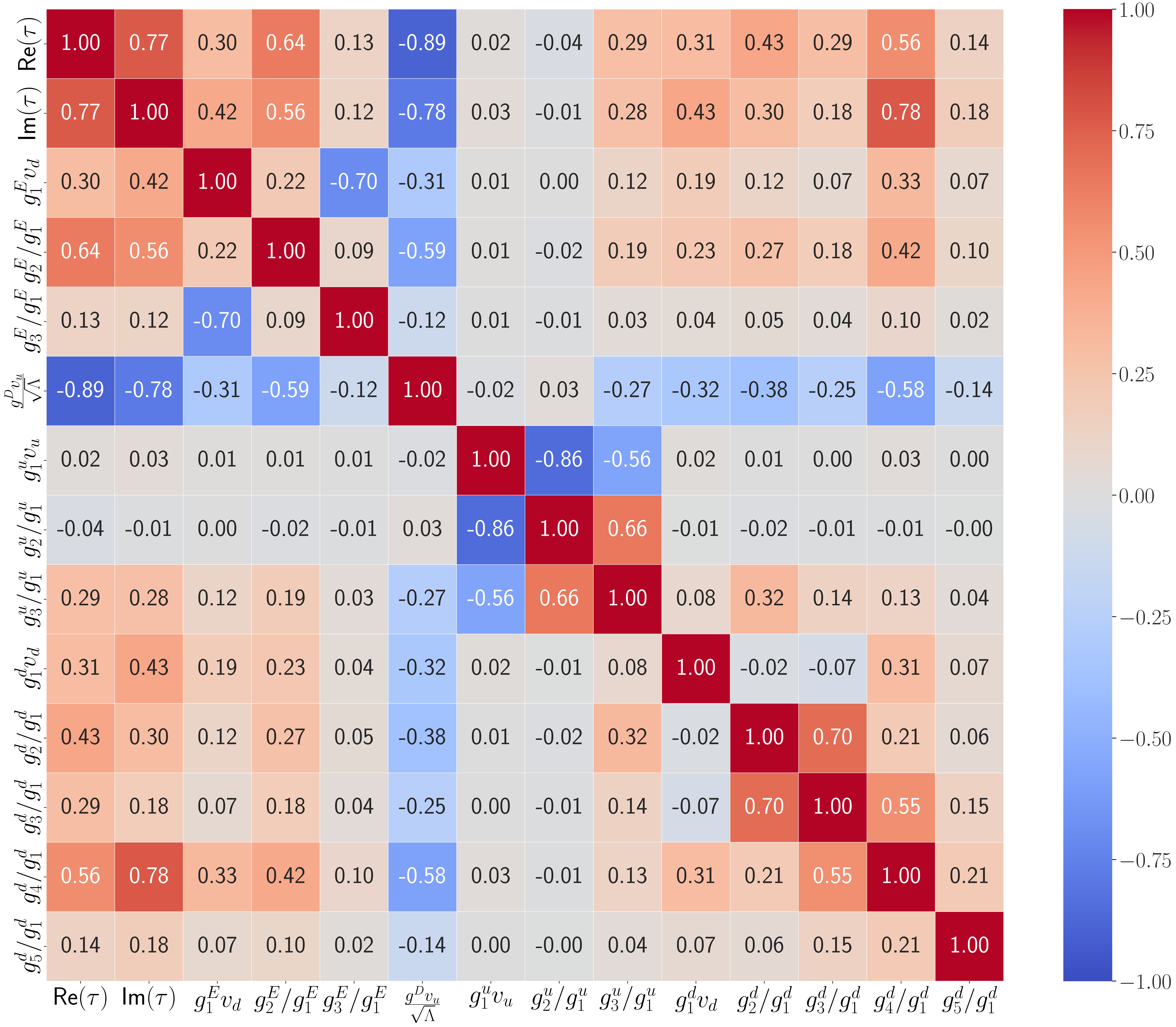
Correlations between Parameters



	Re(τ)	Im(τ)	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g^D v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
Re(τ)	1.00	0.77	0.30	0.64	0.13	-0.89	0.02	-0.04	0.29	0.31	0.43	0.29	0.56	0.14
Im(τ)	0.77	1.00	0.42	0.56	0.12	-0.78	0.03	-0.01	0.28	0.43	0.30	0.18	0.78	0.18
$g_1^E v_d$	0.30	0.42	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.64	0.56	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	0.13	0.12	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g^D v_u}{\sqrt{\Lambda}}$	-0.89	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.02	0.03	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	-0.04	-0.01	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.29	0.28	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.31	0.43	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.43	0.30	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.29	0.18	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.56	0.78	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.14	0.18	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

As for the other two other examples, significant correlations among parameters naturally arise due to symmetry-imposed structure of the theory

Correlations between Parameters



	Re(τ)	Im(τ)	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g^D v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
Re(τ)	1.00	0.77	0.30	0.64	0.13	-0.89	0.02	-0.04	0.29	0.31	0.43	0.29	0.56	0.14
Im(τ)	0.77	1.00	0.42	0.56	0.12	-0.78	0.03	-0.01	0.28	0.43	0.30	0.18	0.78	0.18
$g_1^E v_d$	0.30	0.42	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.64	0.56	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	0.13	0.12	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g^D v_u}{\sqrt{\Lambda}}$	-0.89	-0.78	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.02	0.03	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	-0.04	-0.01	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.29	0.28	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.31	0.43	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.43	0.30	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.29	0.18	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.56	0.78	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.14	0.18	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

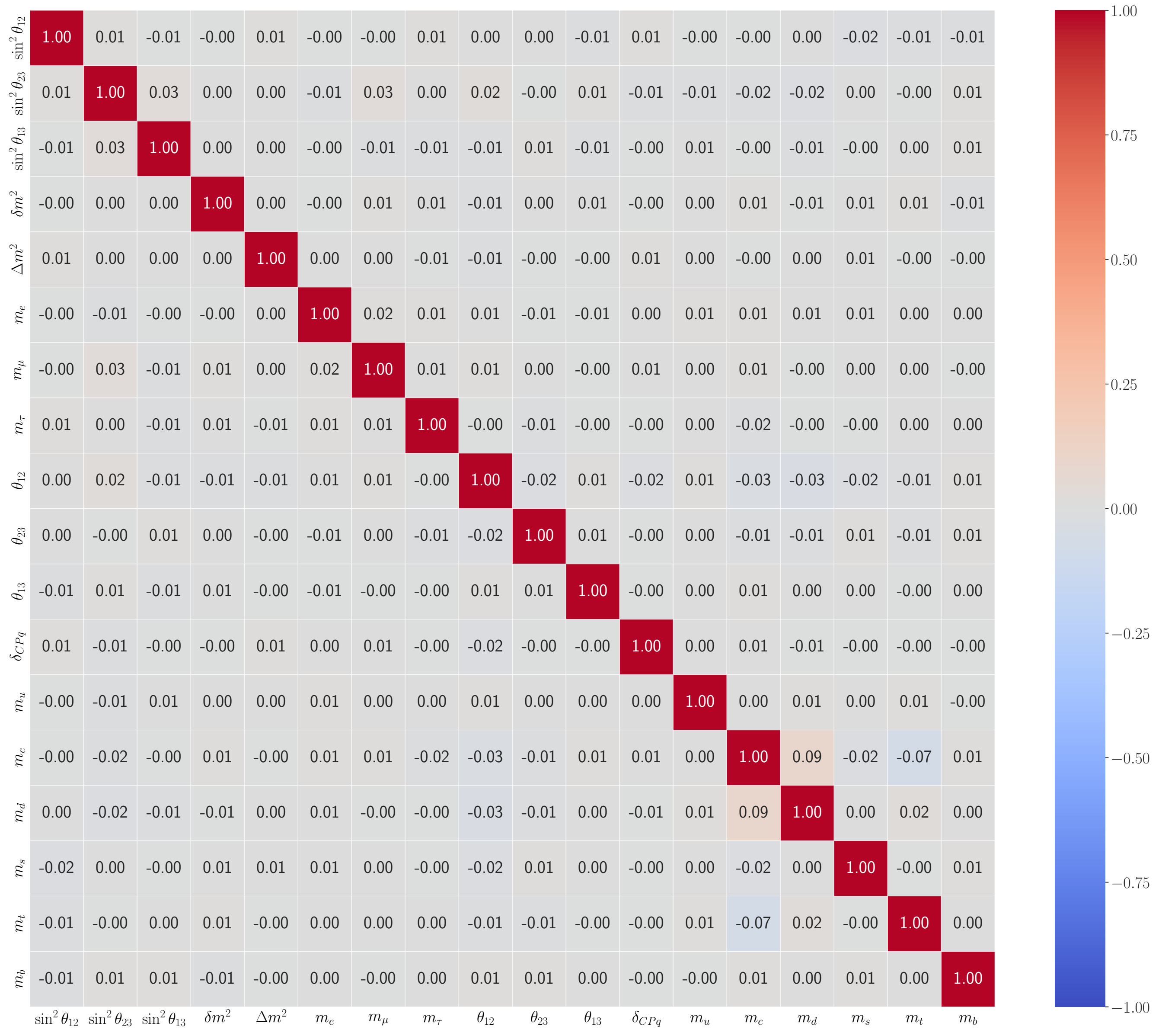
As for the other two other examples, significant correlations among parameters naturally arise due to symmetry-imposed structure of the theory

Apparently, there is not a significant difference with respect to the case of overfitting models

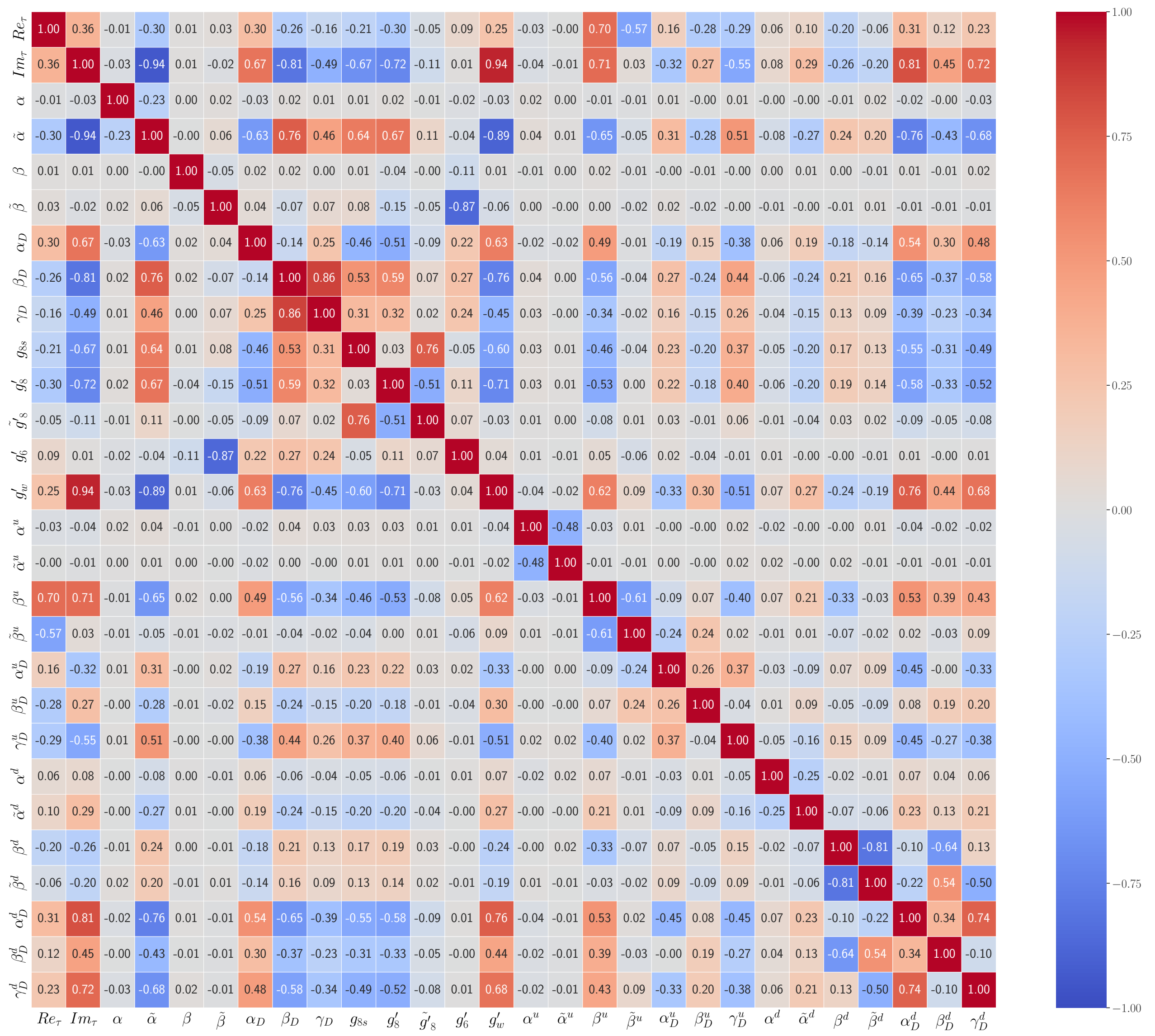
Observables

Parameters

$$N_{\mathrm{obs}} < N_{\mathrm{par}} \Rightarrow N_{\mathrm{dof}} < 0$$



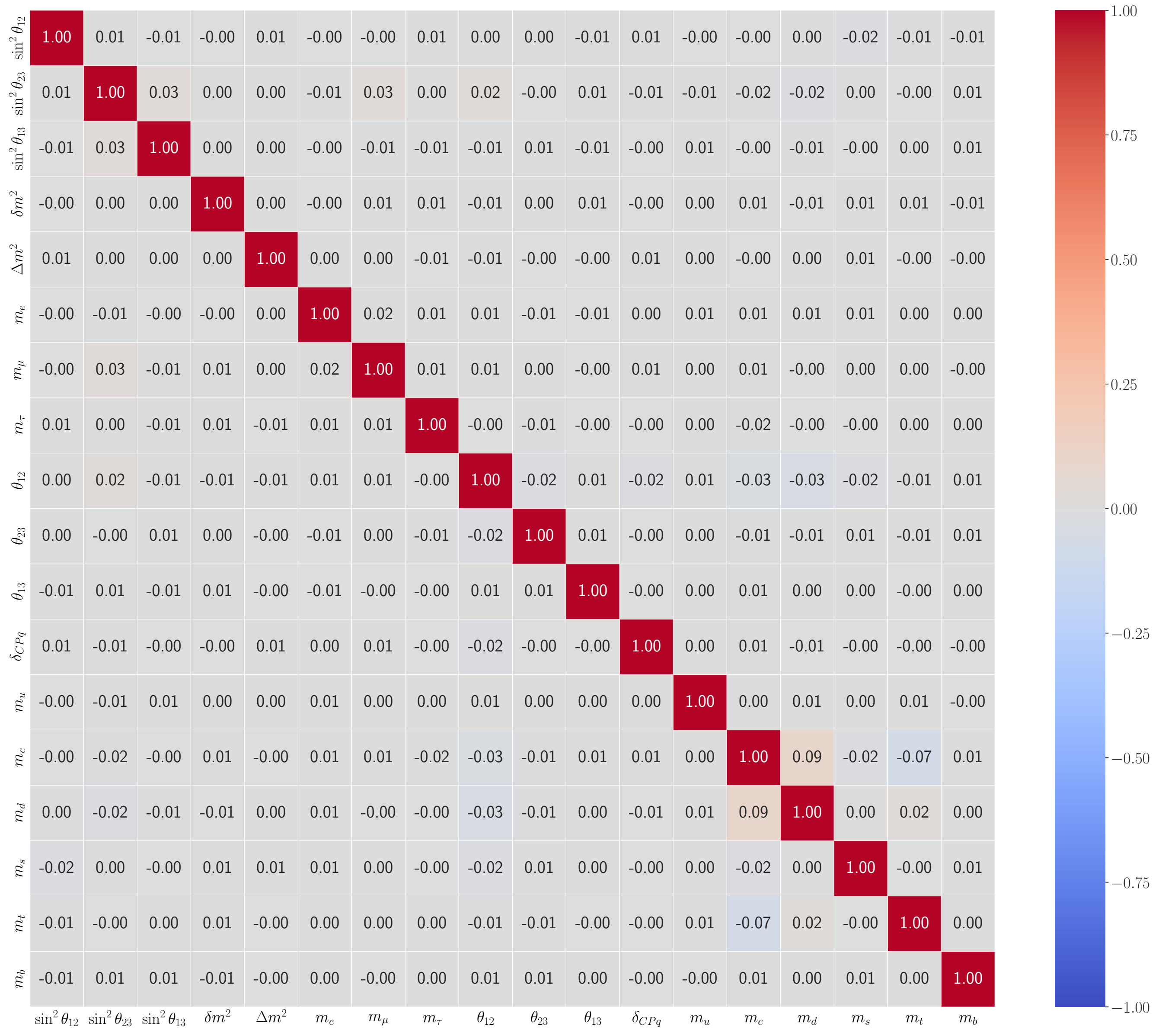
Observables

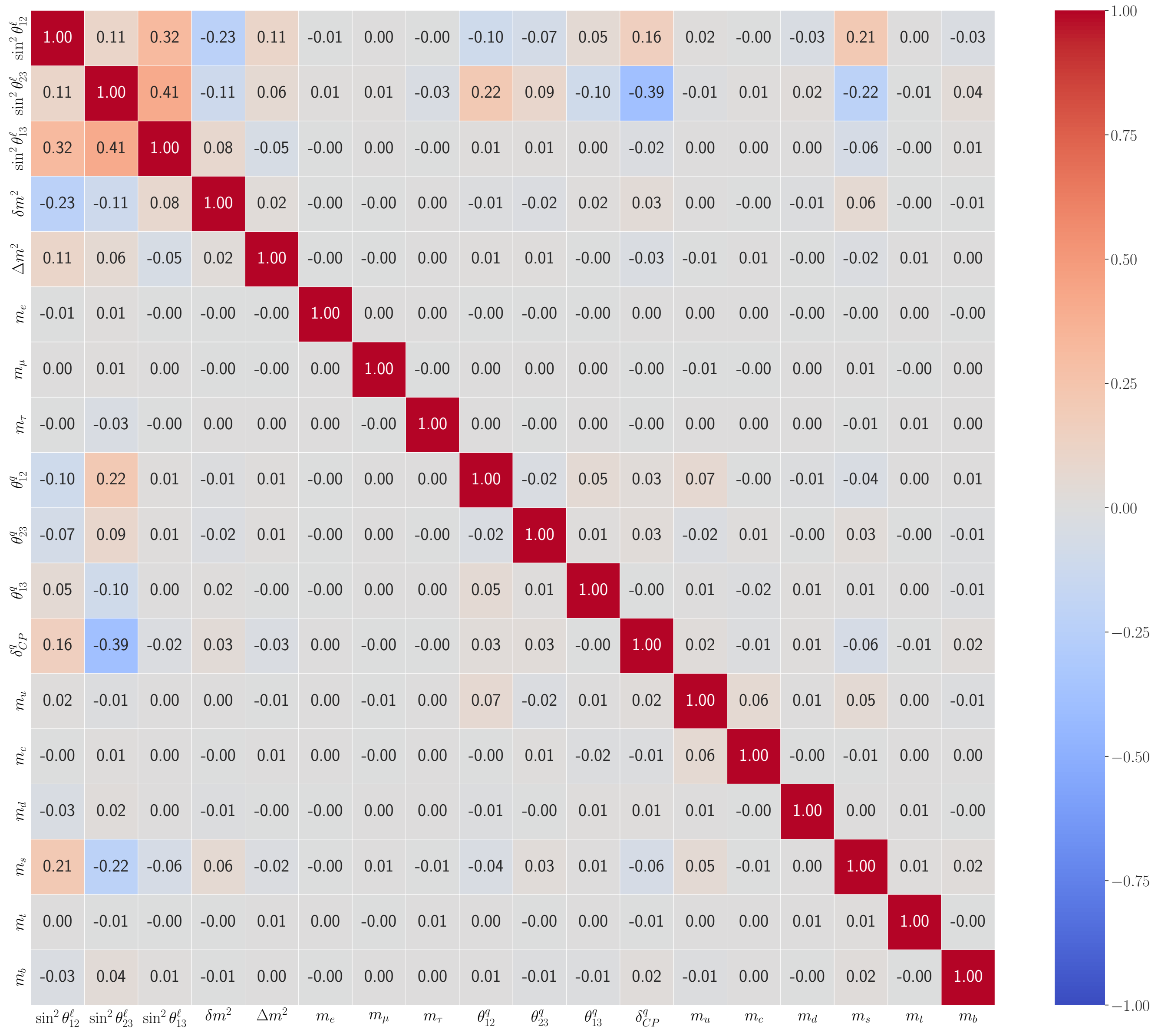


Parameters

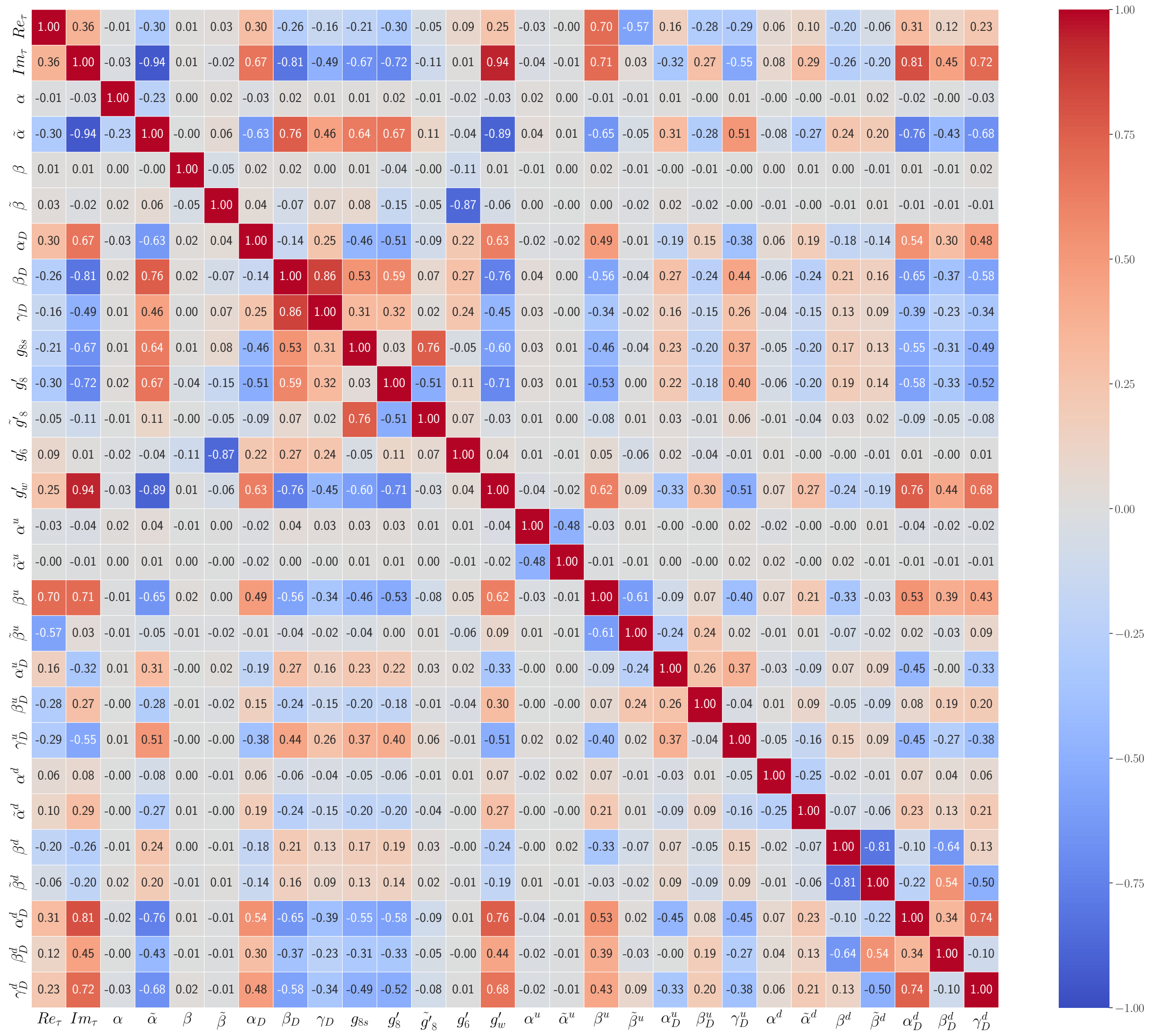
$N_{\text{obs}} < N_{\text{par}} \Rightarrow N_{\text{dof}} < 0$

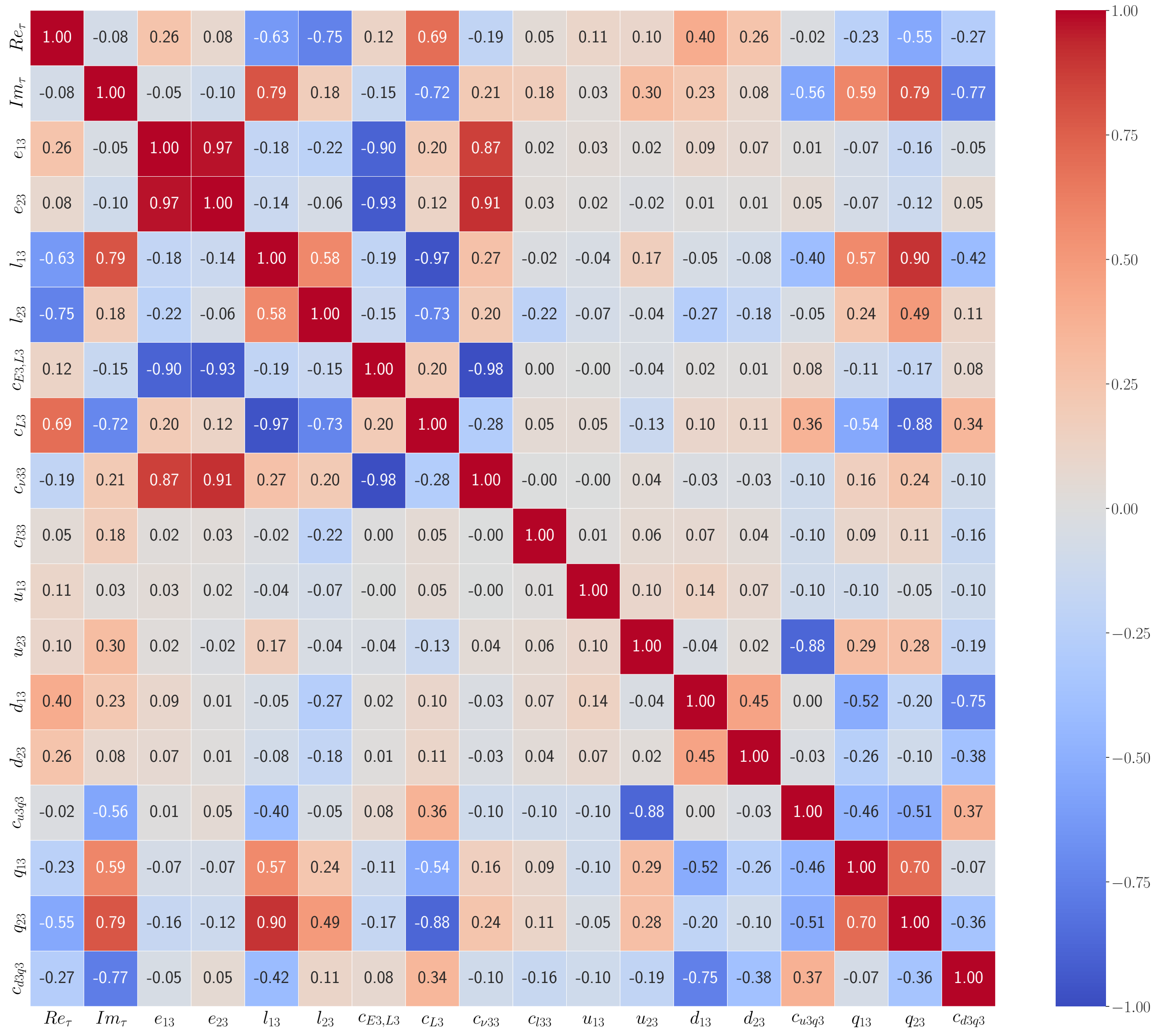
$N_{\text{obs}} = N_{\text{par}} \Rightarrow N_{\text{dof}} = 0$





Observables



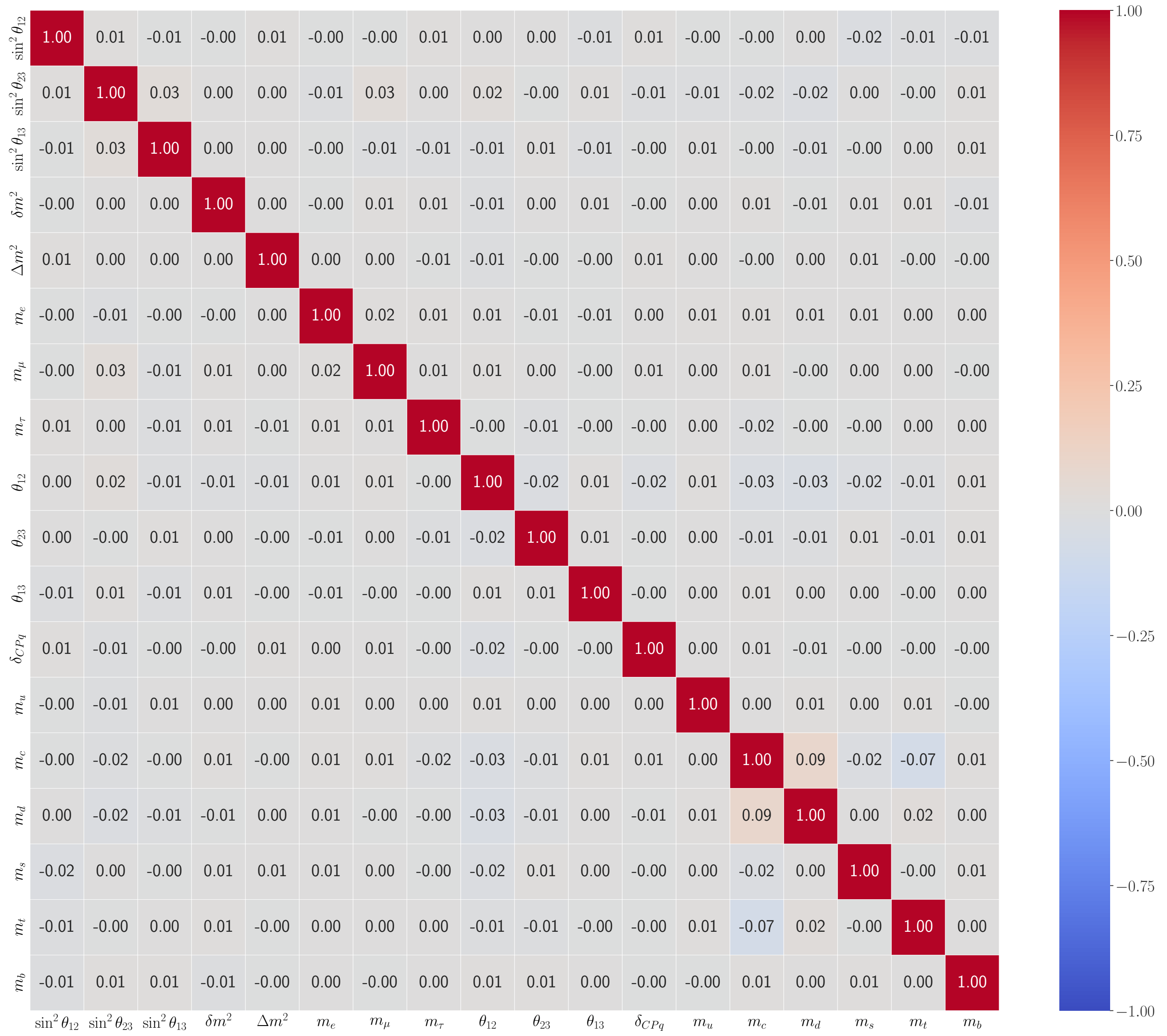


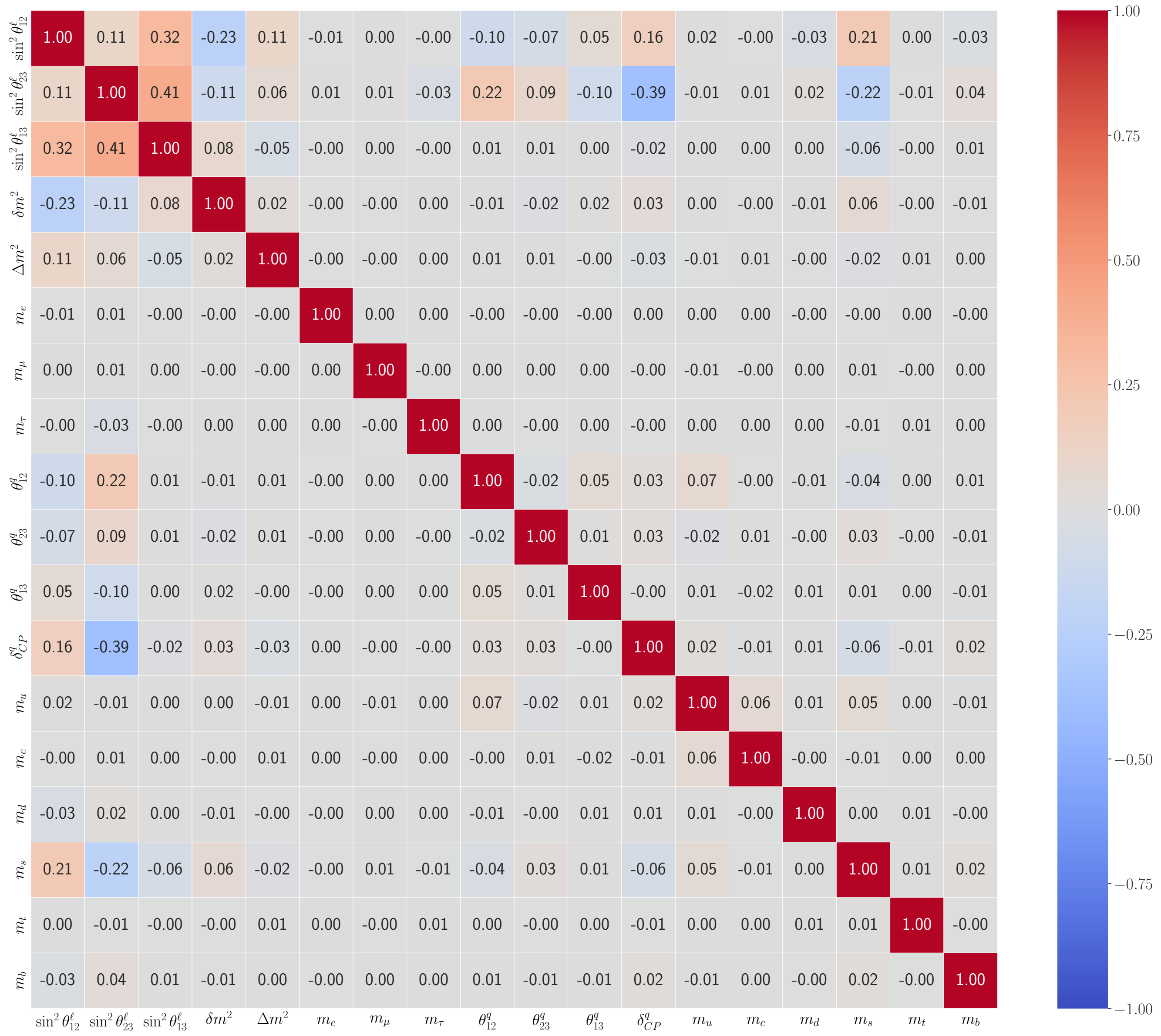
Parameters

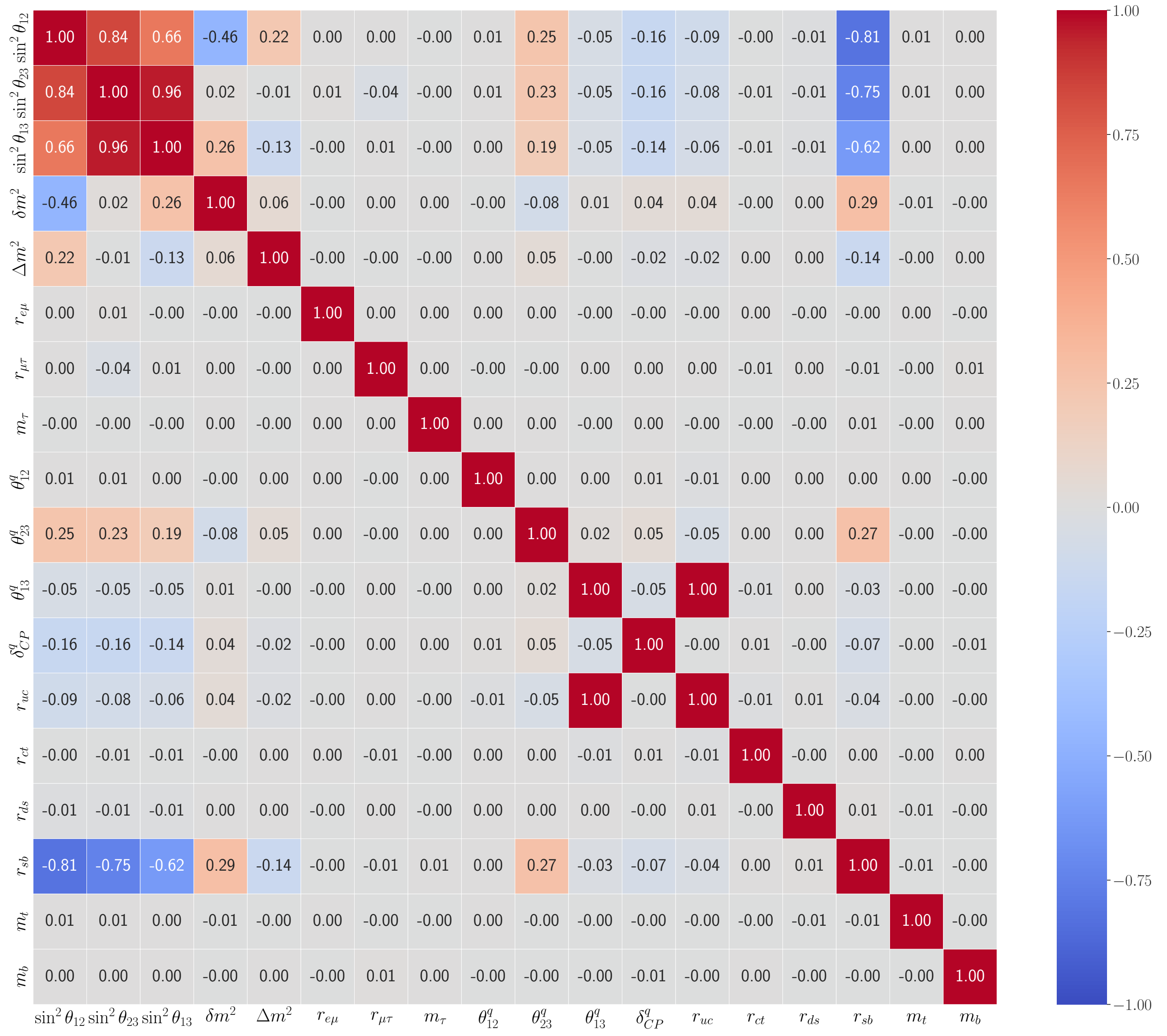
$N_{\mathrm{obs}} < N_{\mathrm{par}} \Rightarrow N_{\mathrm{dof}} < 0$

$N_{\mathrm{obs}} = N_{\mathrm{par}} \Rightarrow N_{\mathrm{dof}} = 0$

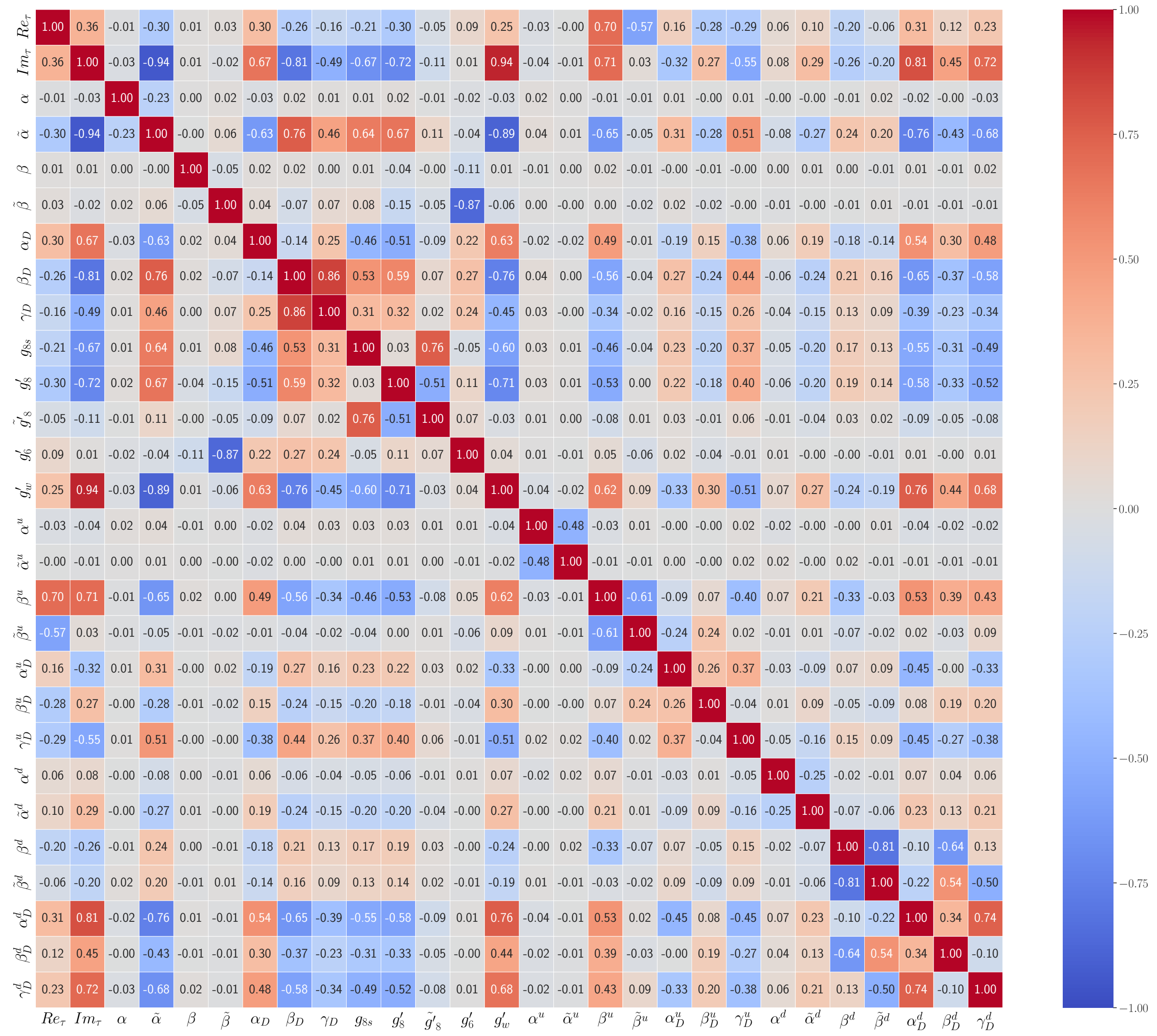
$N_{\mathrm{obs}} > N_{\mathrm{par}} \Rightarrow N_{\mathrm{dof}} > 0$

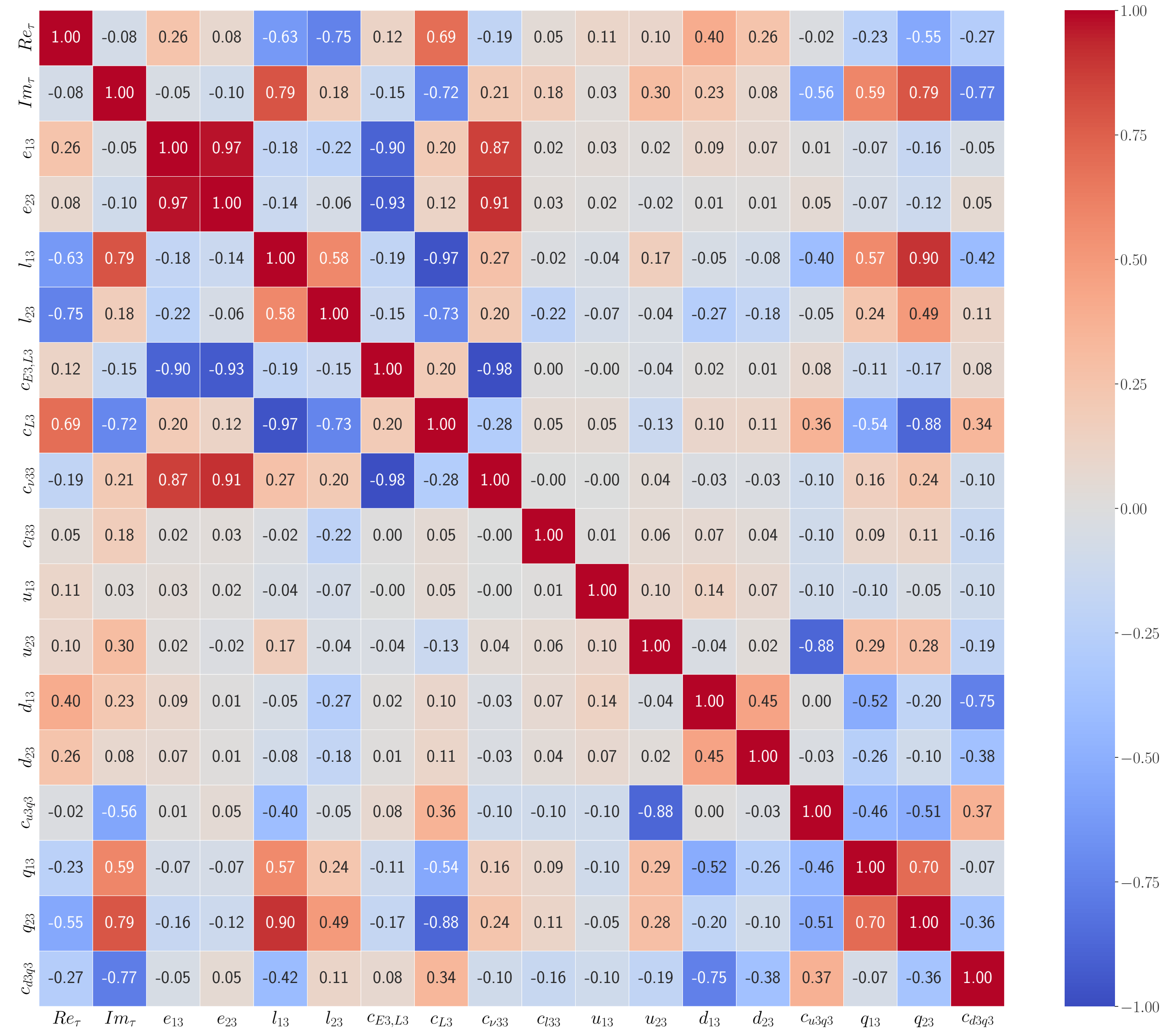


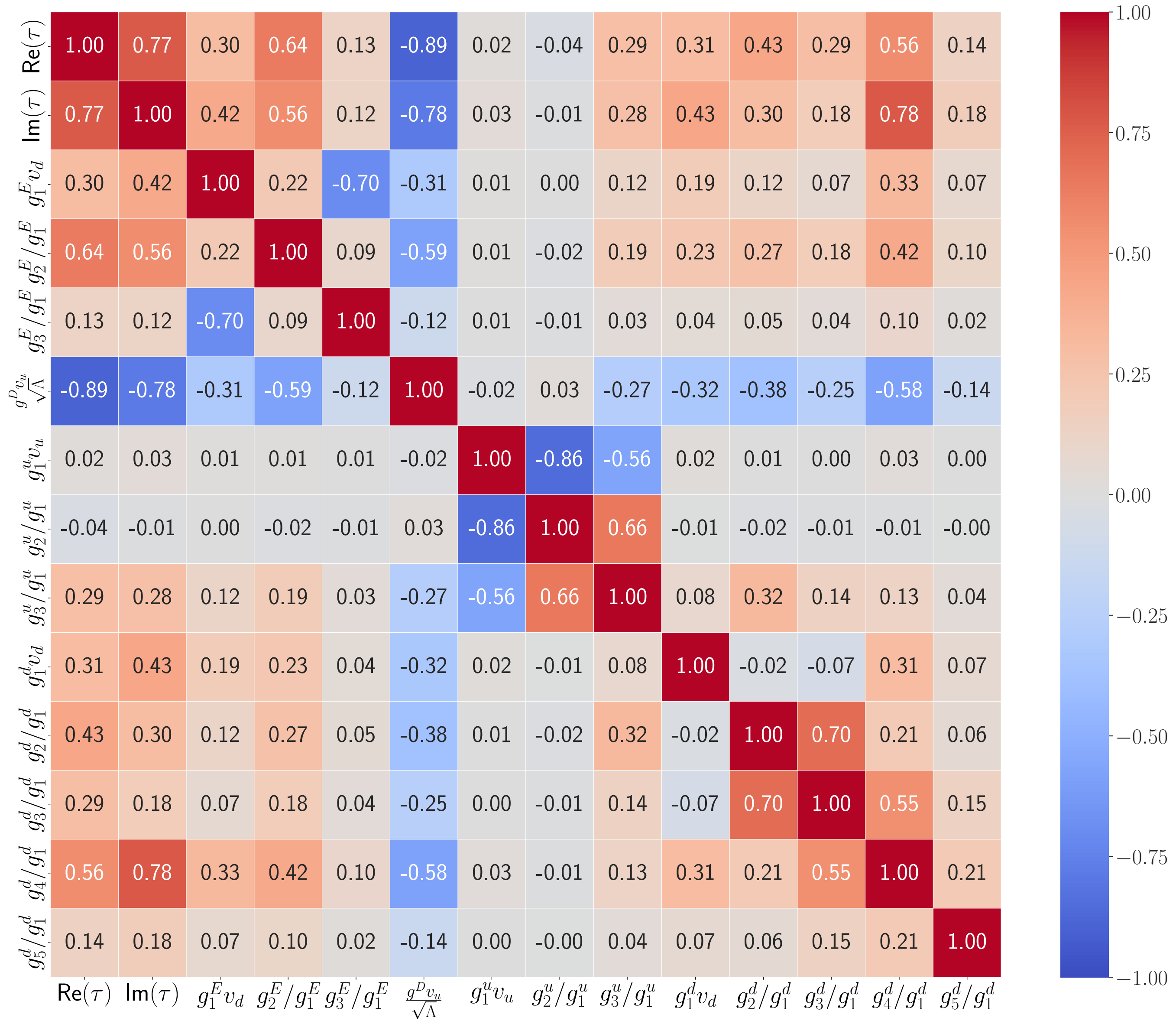




Observables







Parameters

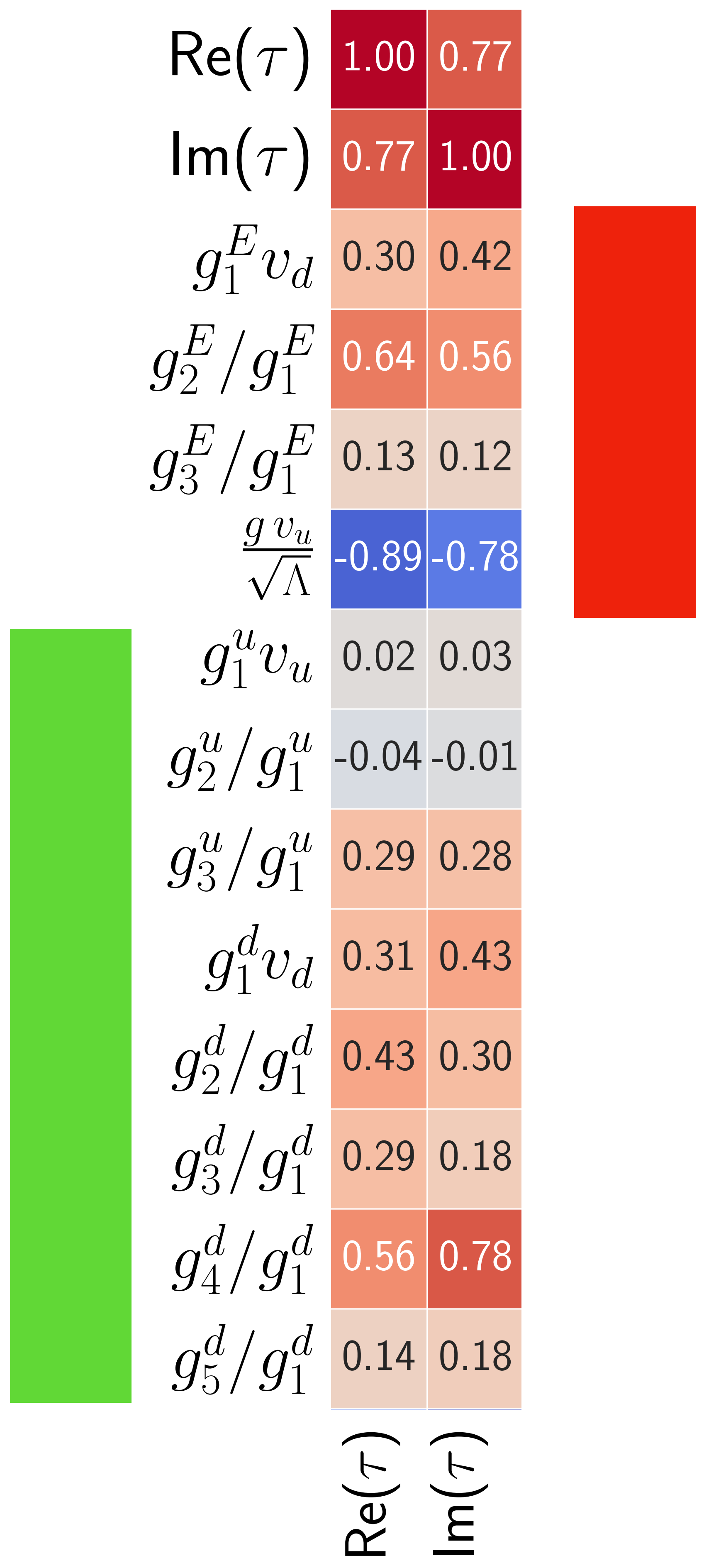
Example of correlations

Example of correlations

For instance → model parameters most correlated to τ in the two sectors

Example of correlations

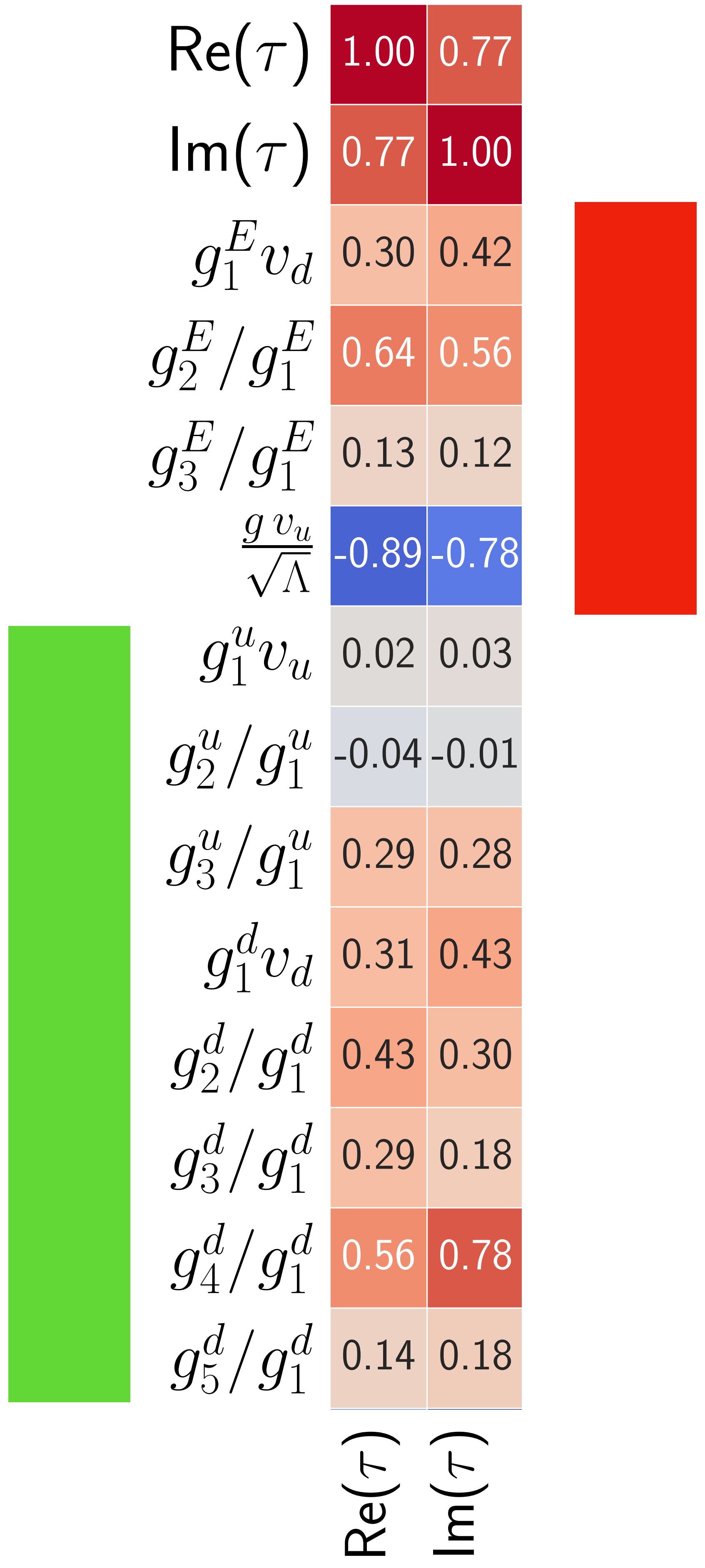
For instance → model parameters most correlated to τ in the two sectors



	Re(τ)	Im(τ)
Re(τ)	1.00	0.77
Im(τ)	0.77	1.00
$g_1^E v_d$	0.30	0.42
g_2^E/g_1^E	0.64	0.56
g_3^E/g_1^E	0.13	0.12
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.89	-0.78
$g_1^u v_u$	0.02	0.03
g_2^u/g_1^u	-0.04	-0.01
g_3^u/g_1^u	0.29	0.28
$g_1^d v_d$	0.31	0.43
g_2^d/g_1^d	0.43	0.30
g_3^d/g_1^d	0.29	0.18
g_4^d/g_1^d	0.56	0.78
g_5^d/g_1^d	0.14	0.18

Example of correlations

For instance → model parameters most correlated to τ in the two sectors



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$g_1^d v_d$	0.31	0.43
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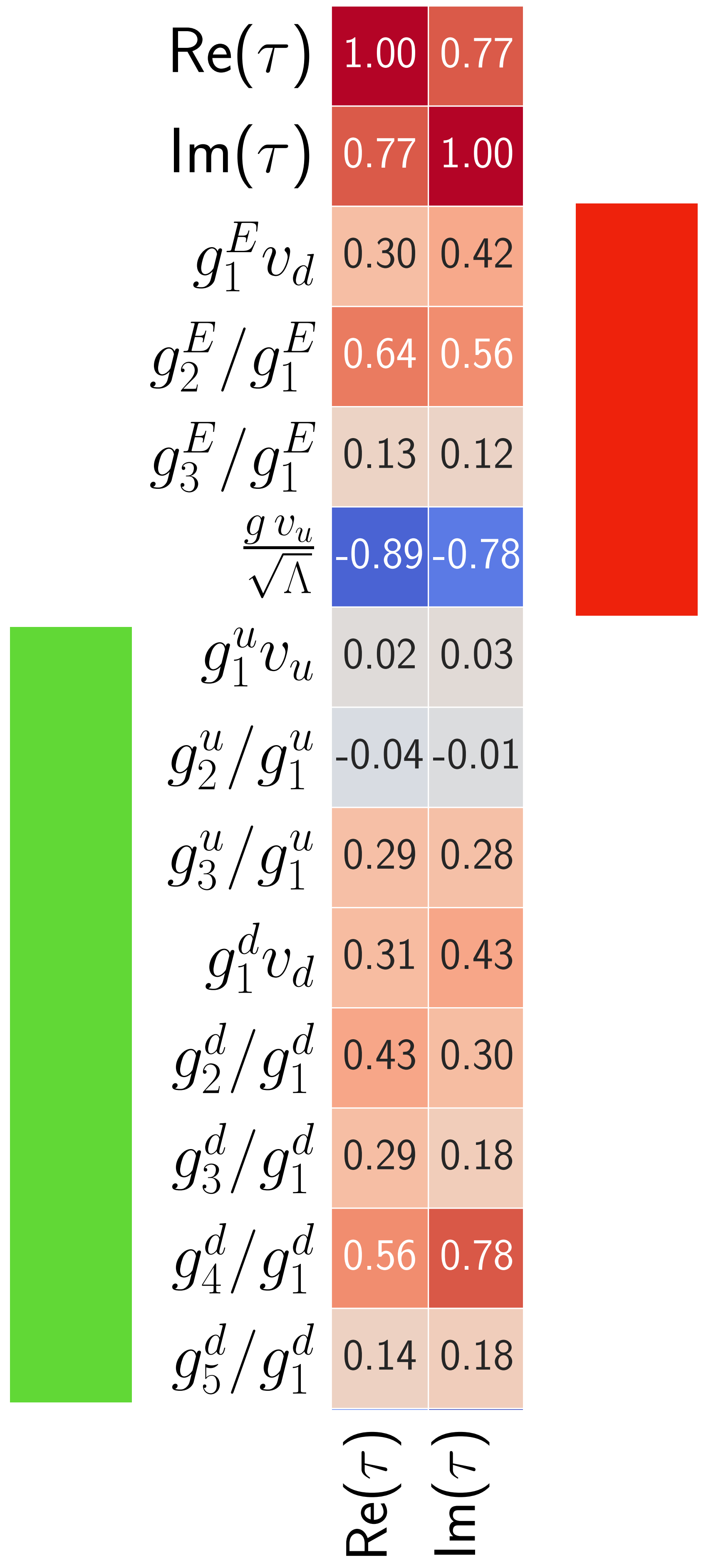
Leptons → $\frac{g v_u}{\sqrt{\Lambda}}$ (negative correlation)

Example of correlations

For instance → model parameters most correlated to τ in the two sectors



	$\mathrm{Re}(\tau)$	$\mathrm{Im}(\tau)$
$\mathrm{Re}(\tau)$	1.00	0.77
$\mathrm{Im}(\tau)$	0.77	1.00
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g_2^E/g_1^E	0.64	0.56
g_3^E/g_1^E	0.13	0.12
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.89	-0.78
$g_1^u v_u$	0.02	0.03
g_2^u/g_1^u	-0.04	-0.01
g_3^u/g_1^u	0.29	0.28
$g_1^d v_d$	0.31	0.43
g_2^d/g_1^d	0.43	0.30
g_3^d/g_1^d	0.29	0.18
g_4^d/g_1^d	0.56	0.78
g_5^d/g_1^d	0.14	0.18

Leptons → $\frac{g v_u}{\sqrt{\Lambda}}$ (negative correlation)

Quarks → $\frac{g_4^d}{g_1^d}$ (positive correlation)

Example of correlations

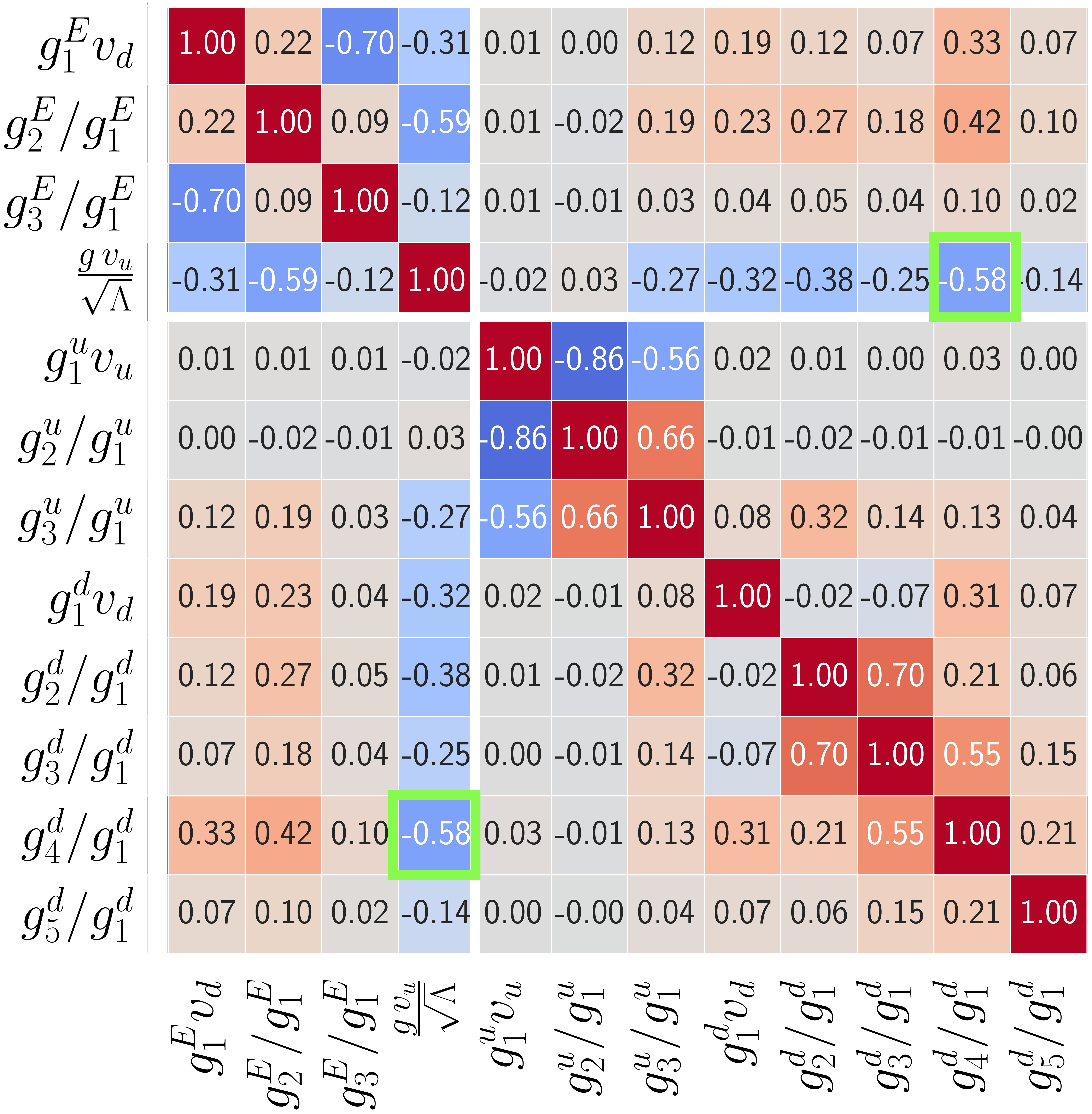
Example of correlations

We can directly verify the correlation among quark and lepton parameters

Example of correlations

We can directly verify the correlation among quark and lepton parameters

$\frac{g_4^d}{g_1^d}$ and $\frac{g v_u}{\sqrt{\Lambda}}$ are, indeed the most (anti)correlated parameters among the two different sectors



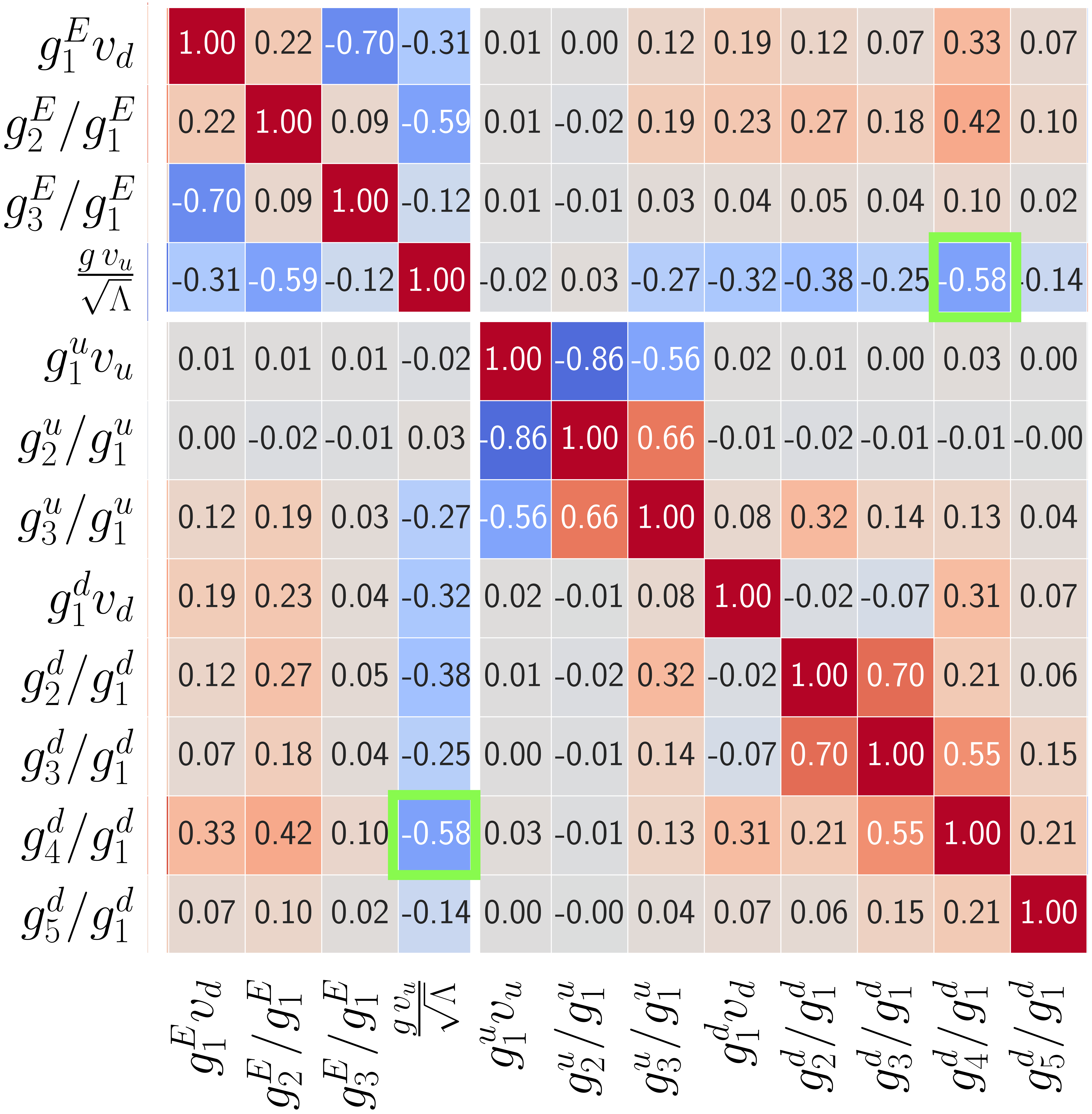
	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
$g_1^E v_d$	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

Example of correlations

We can directly verify the correlation among quark and lepton parameters

$\frac{g_4^d}{g_1^d}$ and $\frac{g v_u}{\sqrt{\Lambda}}$ are, indeed the most (anti)correlated parameters among the two different sectors

We trace back their anti correlation going back to their correlation to the real and imaginary parts of τ

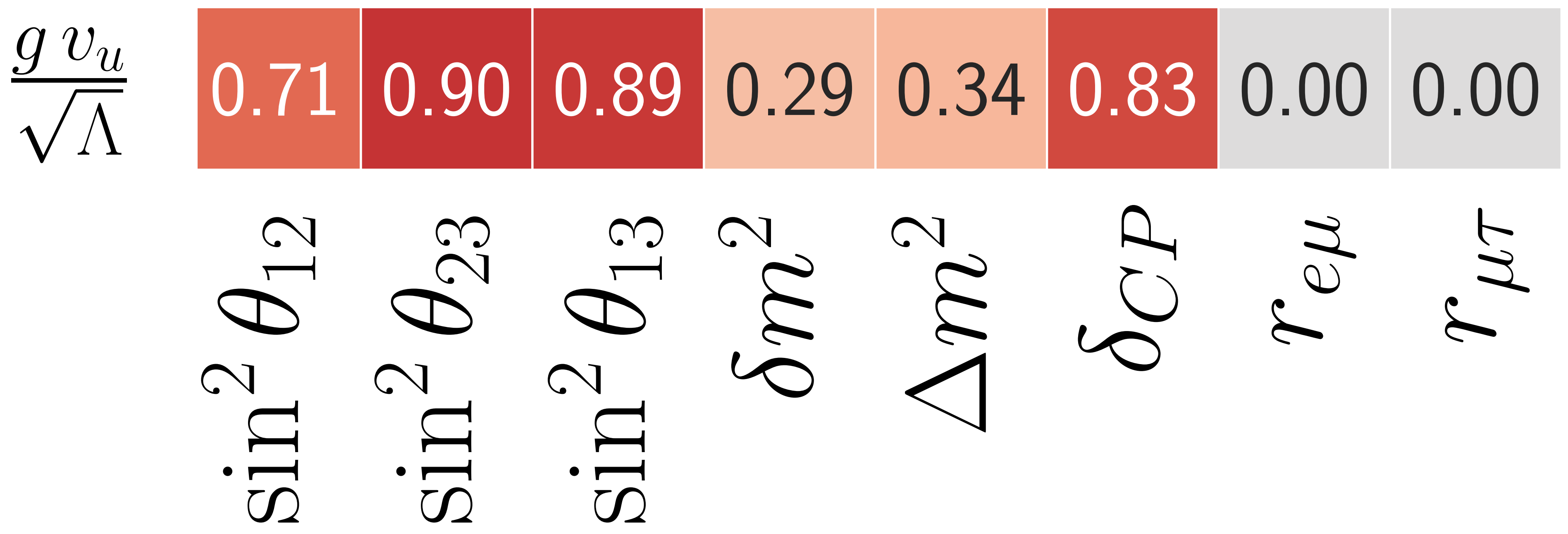


	$g_1^E v_d$	g_2^E/g_1^E	g_3^E/g_1^E	$\frac{g\,v_u}{\sqrt{\Lambda}}$	$g_1^u v_u$	g_2^u/g_1^u	g_3^u/g_1^u	$g_1^d v_d$	g_2^d/g_1^d	g_3^d/g_1^d	g_4^d/g_1^d	g_5^d/g_1^d
$g_1^E v_d$	1.00	0.22	-0.70	-0.31	0.01	0.00	0.12	0.19	0.12	0.07	0.33	0.07
g_2^E/g_1^E	0.22	1.00	0.09	-0.59	0.01	-0.02	0.19	0.23	0.27	0.18	0.42	0.10
g_3^E/g_1^E	-0.70	0.09	1.00	-0.12	0.01	-0.01	0.03	0.04	0.05	0.04	0.10	0.02
$\frac{g\,v_u}{\sqrt{\Lambda}}$	-0.31	-0.59	-0.12	1.00	-0.02	0.03	-0.27	-0.32	-0.38	-0.25	-0.58	-0.14
$g_1^u v_u$	0.01	0.01	0.01	-0.02	1.00	-0.86	-0.56	0.02	0.01	0.00	0.03	0.00
g_2^u/g_1^u	0.00	-0.02	-0.01	0.03	-0.86	1.00	0.66	-0.01	-0.02	-0.01	-0.01	-0.00
g_3^u/g_1^u	0.12	0.19	0.03	-0.27	-0.56	0.66	1.00	0.08	0.32	0.14	0.13	0.04
$g_1^d v_d$	0.19	0.23	0.04	-0.32	0.02	-0.01	0.08	1.00	-0.02	-0.07	0.31	0.07
g_2^d/g_1^d	0.12	0.27	0.05	-0.38	0.01	-0.02	0.32	-0.02	1.00	0.70	0.21	0.06
g_3^d/g_1^d	0.07	0.18	0.04	-0.25	0.00	-0.01	0.14	-0.07	0.70	1.00	0.55	0.15
g_4^d/g_1^d	0.33	0.42	0.10	-0.58	0.03	-0.01	0.13	0.31	0.21	0.55	1.00	0.21
g_5^d/g_1^d	0.07	0.10	0.02	-0.14	0.00	-0.00	0.04	0.07	0.06	0.15	0.21	1.00

Correlation between Parameters and Observables

Correlation between Parameters and Observables

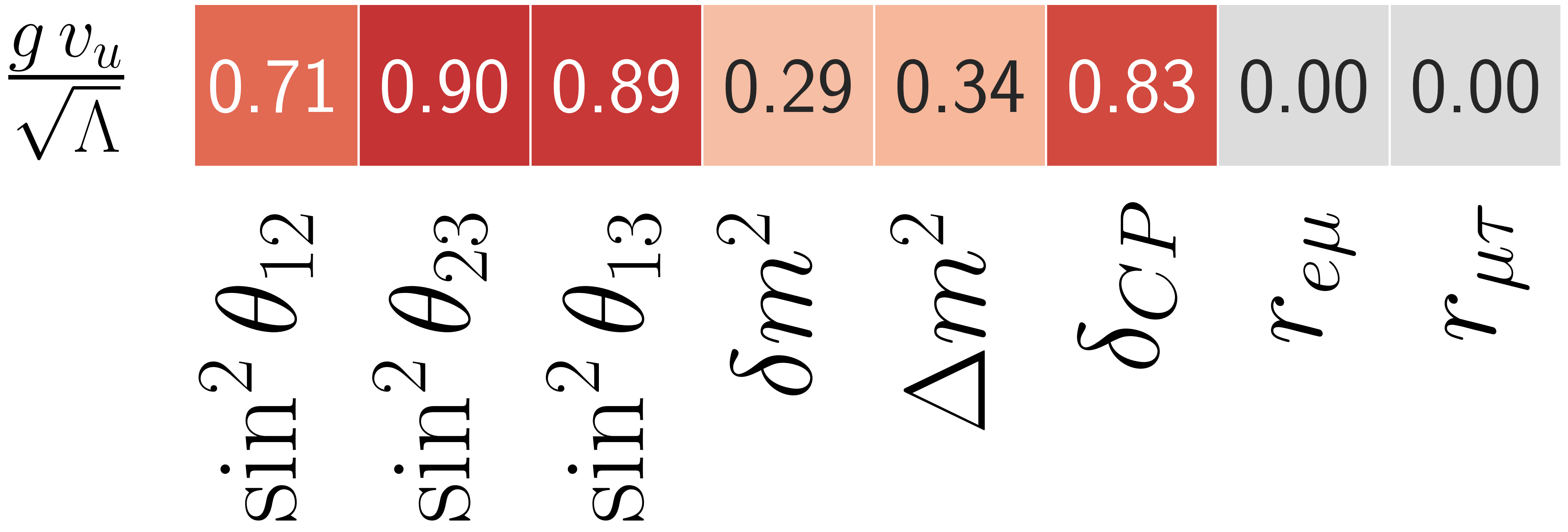
All neutrino observables are positively correlated to $\frac{gv_u}{\sqrt{\Lambda}}$, (in particular $\sin^2\theta_{12}$ and the other two mixing angles)



$\frac{g\,v_u}{\sqrt{\Lambda}}$	0.71	0.90	0.89	0.29	0.34	0.83	0.00	0.00
	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$

Correlation between Parameters and Observables

All neutrino observables are positively correlated to $\frac{gv_u}{\sqrt{\Lambda}}$, (in particular $\sin^2\theta_{12}$ and the other two mixing angles)



	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$
$\frac{g\,v_u}{\sqrt{\Lambda}}$	0.71	0.90	0.89	0.29	0.34	0.83	0.00	0.00

The quark parameter most correlated to quark observables is $\frac{g_4^d}{g_1^d}$

	θ^q_{12}	θ^q_{23}	θ^q_{13}	δ^q_{CP}	r_{uc}	r_{ct}	r_{ds}	r_{sb}	m_t	m_b
g^d_4/g^d_1	0.11	0.34	-0.02	-0.04	-0.04	0.00	-0.01	0.99	-0.01	-0.01

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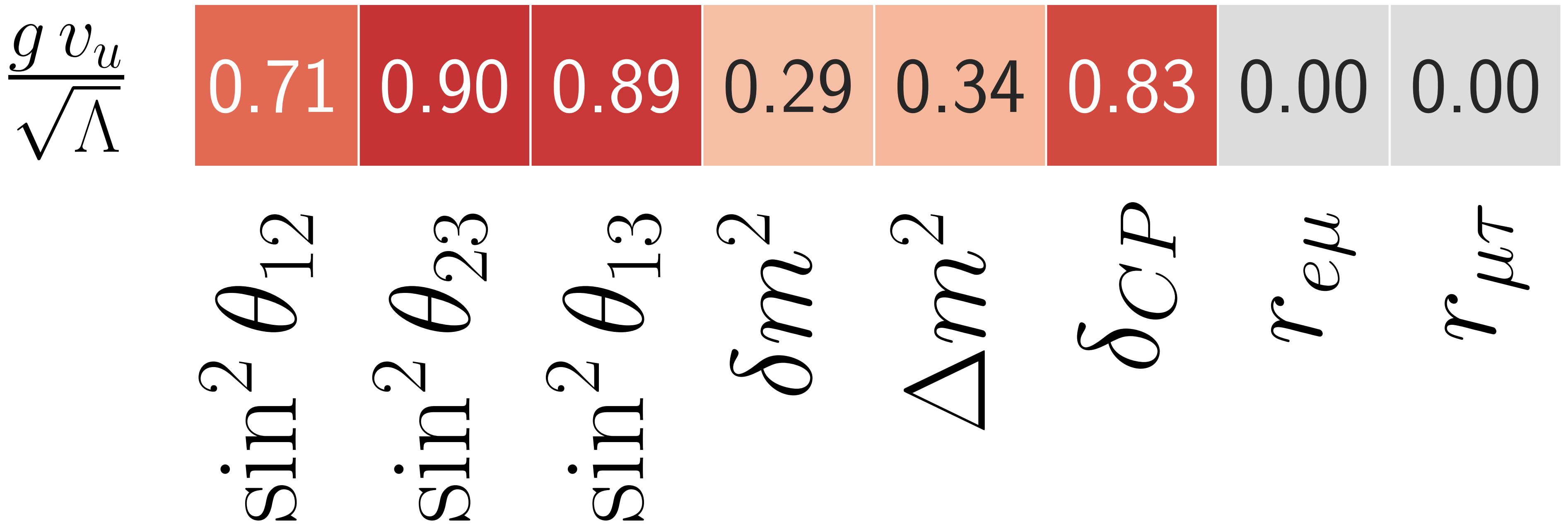

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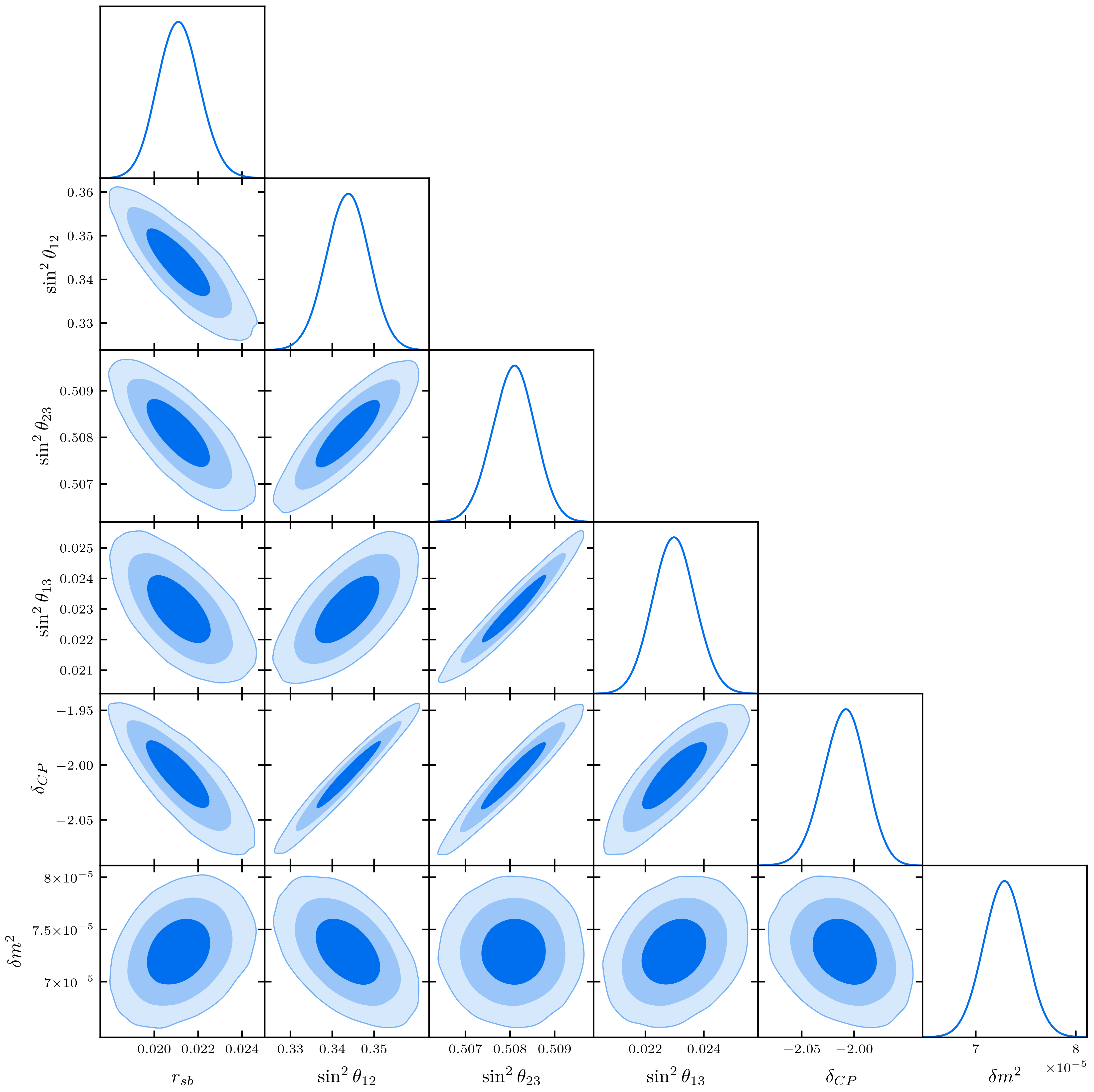
We can presume the presence of a significant anticorrelation between r_{sb} and all neutrino mixing angles

Correlations between quark and lepton observables



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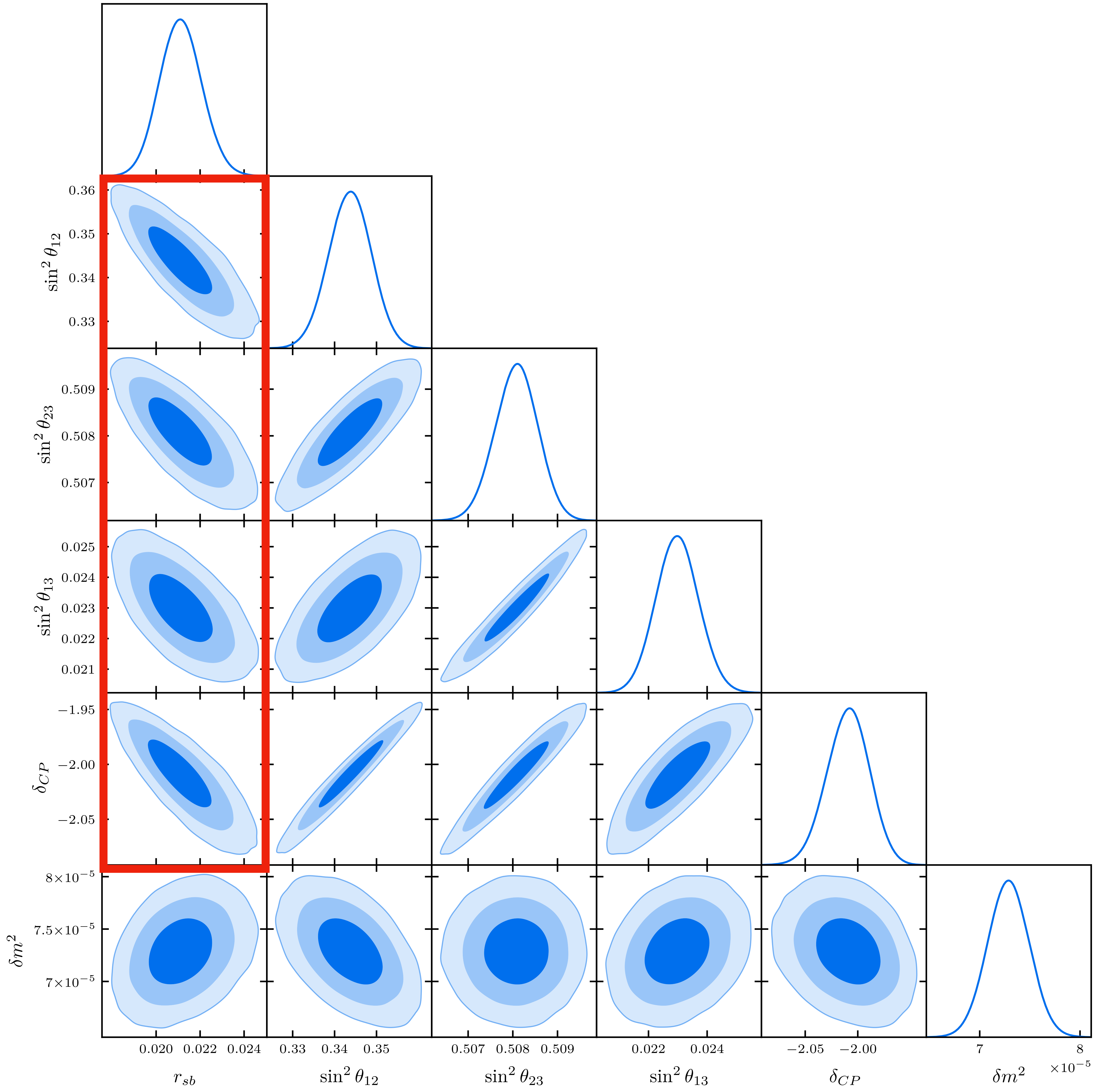

Significative anticorrelation between r_{sb} and $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}, \delta_{CP}$

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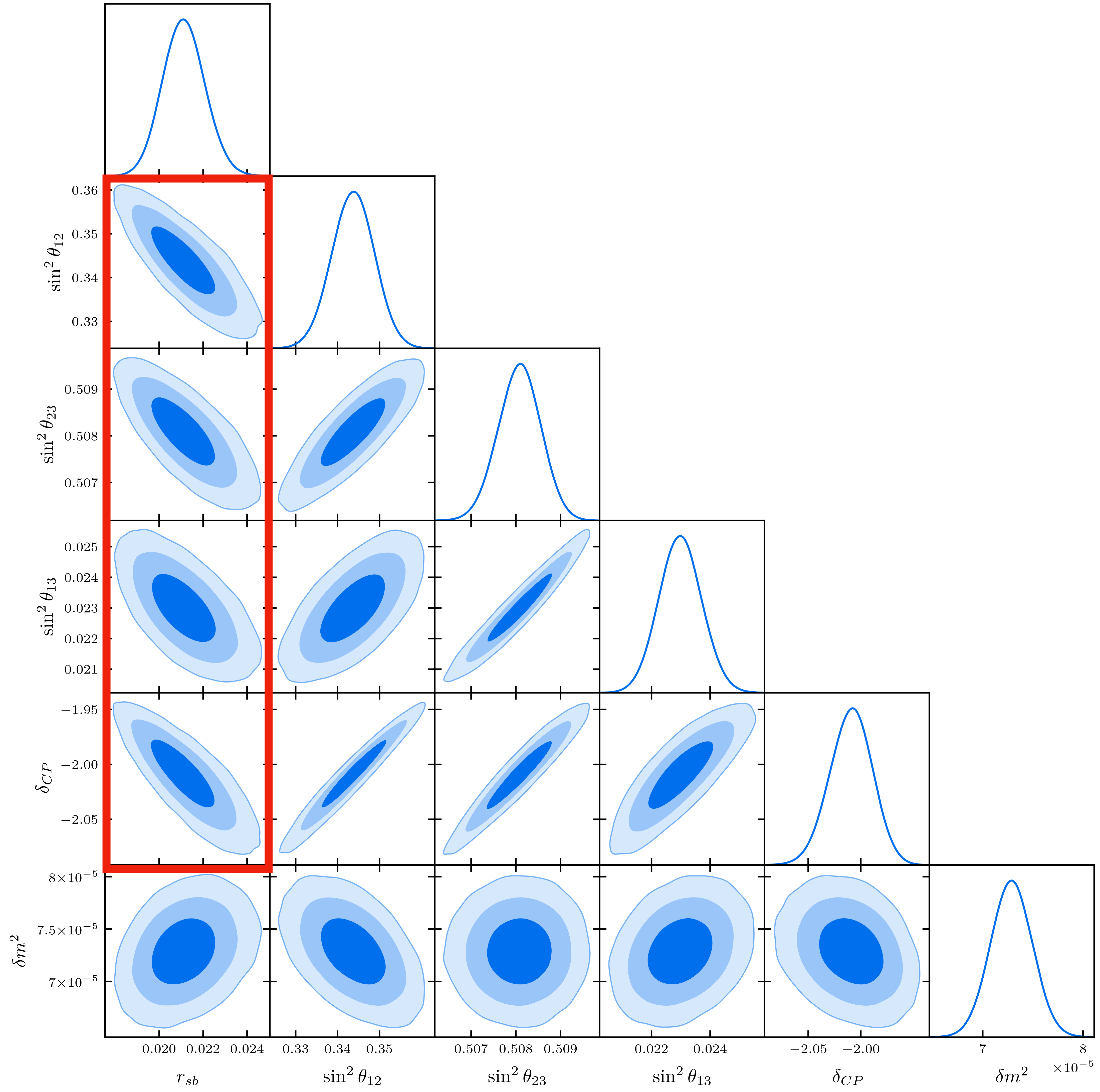

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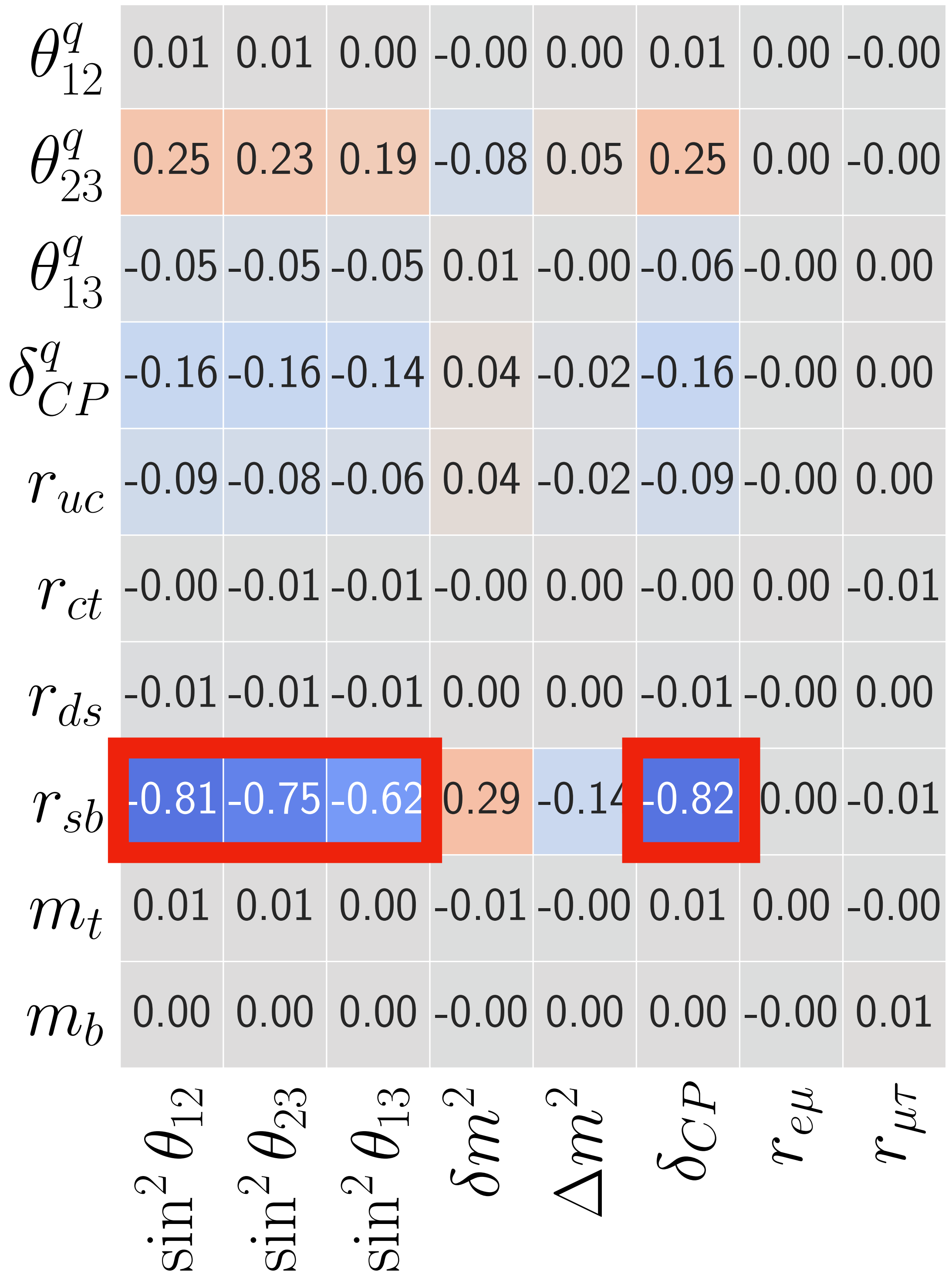
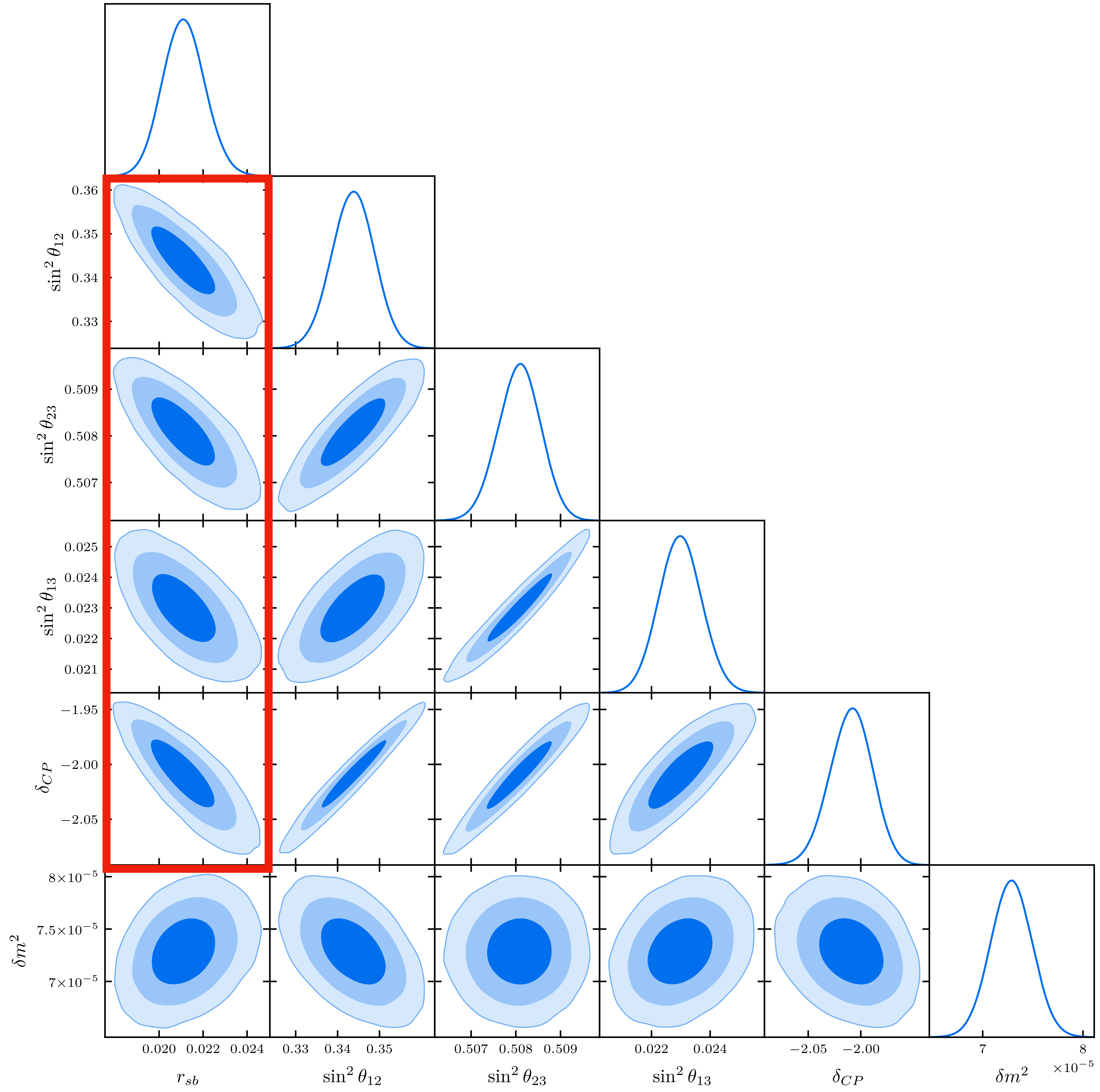


	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δm^2	Δm^2	δ_{CP}	$r_{e\mu}$	$r_{\mu\tau}$
θ^q_{12}	0.01	0.01	0.00	-0.00	0.00	0.01	0.00	-0.00
θ^q_{23}	0.25	0.23	0.19	-0.08	0.05	0.25	0.00	-0.00
θ^q_{13}	-0.05	-0.05	-0.05	0.01	-0.00	-0.06	-0.00	0.00
δ^q_{CP}	-0.16	-0.16	-0.14	0.04	-0.02	-0.16	-0.00	0.00
r_{uc}	-0.09	-0.08	-0.06	0.04	-0.02	-0.09	-0.00	0.00
r_{ct}	-0.00	-0.01	-0.01	-0.00	0.00	-0.00	0.00	-0.01
r_{ds}	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.00	0.00
r_{sb}	-0.81	-0.75	-0.62	0.29	-0.14	-0.82	0.00	-0.01
m_t	0.01	0.01	0.00	-0.01	-0.00	0.01	0.00	-0.00
m_b	0.00	0.00	0.00	-0.00	0.00	0.00	-0.00	0.01

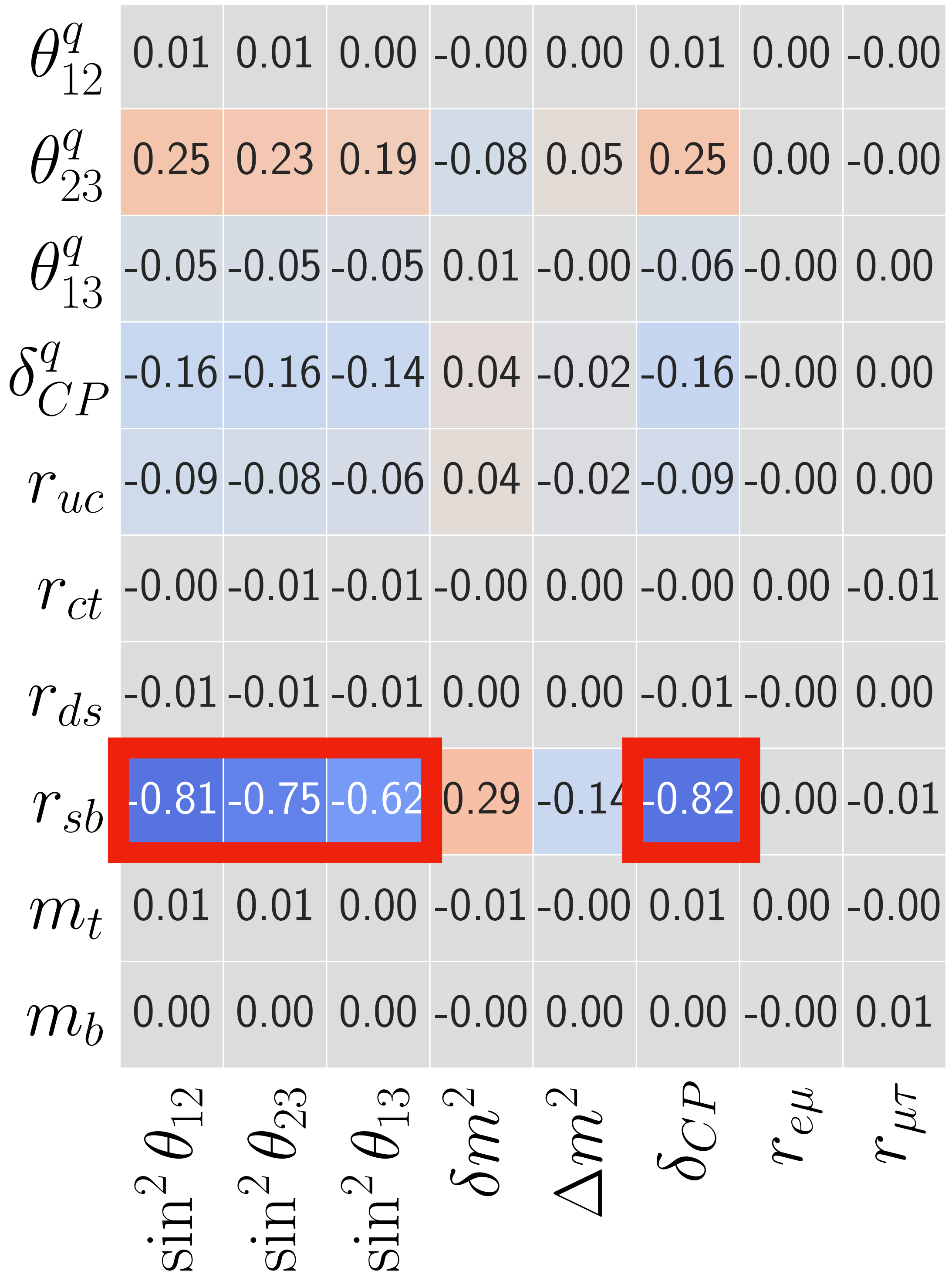
Going back to the results of the fit we understand how the 3σ tension on the values of $\sin^2\theta_{12}$ predicted by the model (0.344) and the measured one (0.303) induces a new tension on r_{sb} in the combined fit

Overestimating $\sin^2\theta_{12}$ at 3σ causes an underestimation of r_{sb} at $\sim 2.7\sigma$

Correlations between quark and lepton observables



Significative anticorrelation between r_{sb} and $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, δ_{CP}



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→ The existence of such correlations between the lepton and quark observables could be a strong indicator of an underlying modular flavour symmetry in particle physics

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Future Outlook: The tension observed between the predicted value of $\sin^2\theta_{12}$ and its experimental value in the 2O model highlights how these frameworks can be concretely tested and potentially falsified by future precision measurements. This demonstrates that modular symmetry models are not just theoretical curiosities but are becoming increasingly constrained by experimental data