Charged Lepton Flavour Violating Meson Decays in Seesaw Models

(arXiv:2410.10490)

Pravesh C. Awasthi

Indian Inst. Tech. Bombay, India

In Collaboration with

Jai More, Akhila Kumar Pradhan, Kumar Rao, Purushottam Sahu, and S. Uma Sankar

FLASY 2025, Rome

11th Workshop on Flavour Symmetries and Consequences in Accelerators and Cosmology

Content

Motivation

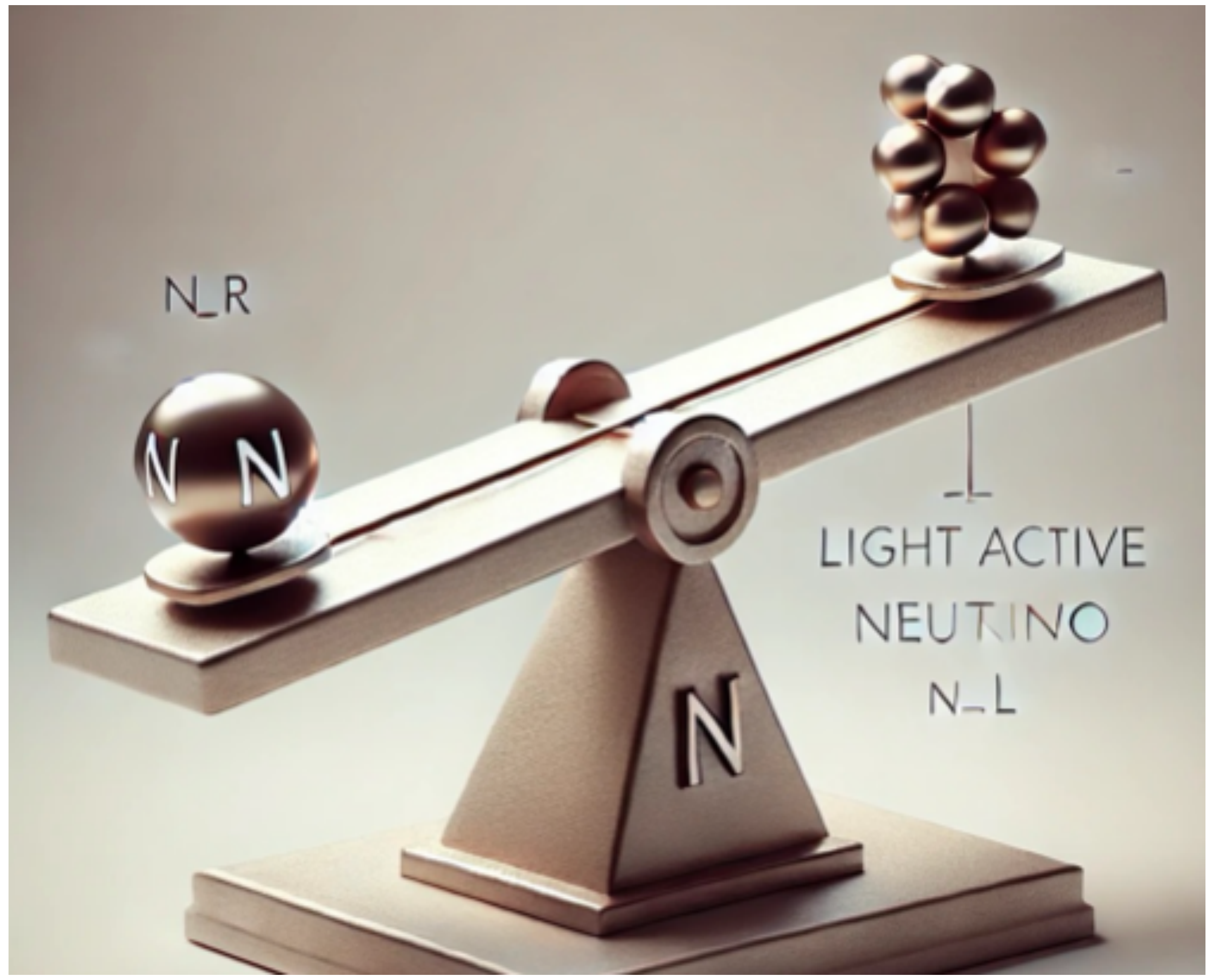
Neutrino Oscillations ⟷ **Massive neutrinos**

$$\underbrace{\nu_\alpha}_{\text{flavour}} = U_{\alpha i} \underbrace{\nu_i}_{\text{mass}}$$

$i = 1,2,3.$

$\alpha = e, \mu, \tau\,.$

- Neutrino masses via standard Higgs mechanism requires extreme fine-tuning ($\sim 10^{-12}$) of neutrino Yukawa couplings
- To avoid this extreme fine-tuning, many neutrino mass mechanism have been proposed
- Of these, see-saw models are the most popular



Motivation

Neutrino Oscillation → **Lepton flavour violation In neutrino sector**

- This necessarily leads to **Charged lepton flavour violation** **(CLFV)**

$$l_\alpha \overset{W}{\rightarrow} \nu_\alpha \xrightarrow{\text{oscillation}} \nu_\beta \overset{W}{\rightarrow} l_\beta$$

- Amount of **CLFV** depends on the details of the neutrino mass model

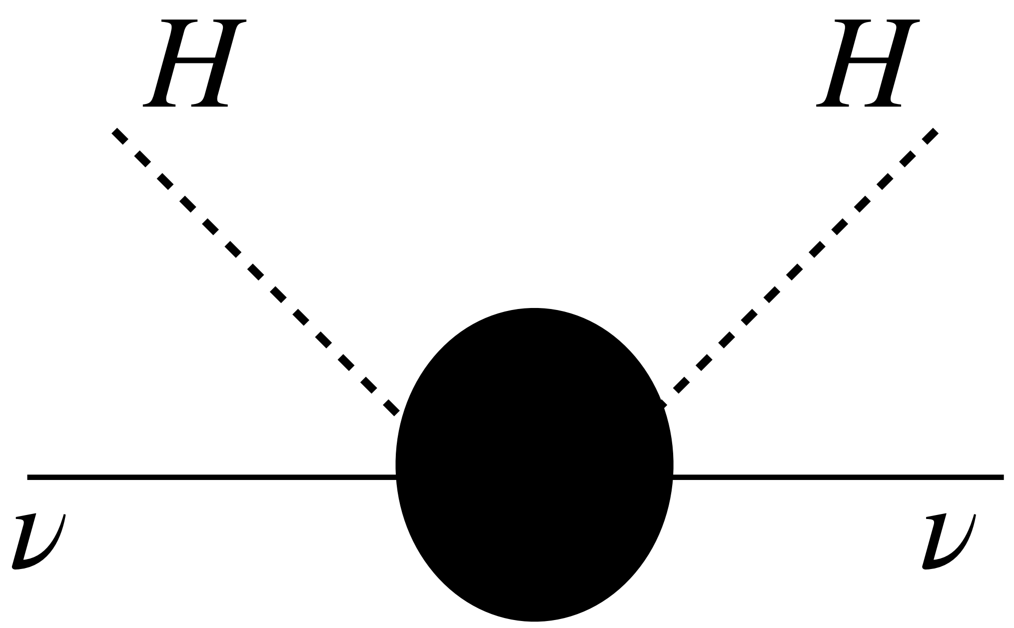
- In this work, we establish relations between **Branching Ratios of radiative CLFV and meson CLFV decays in seesaw models.**

Massive Neutrinos : Seesaw models

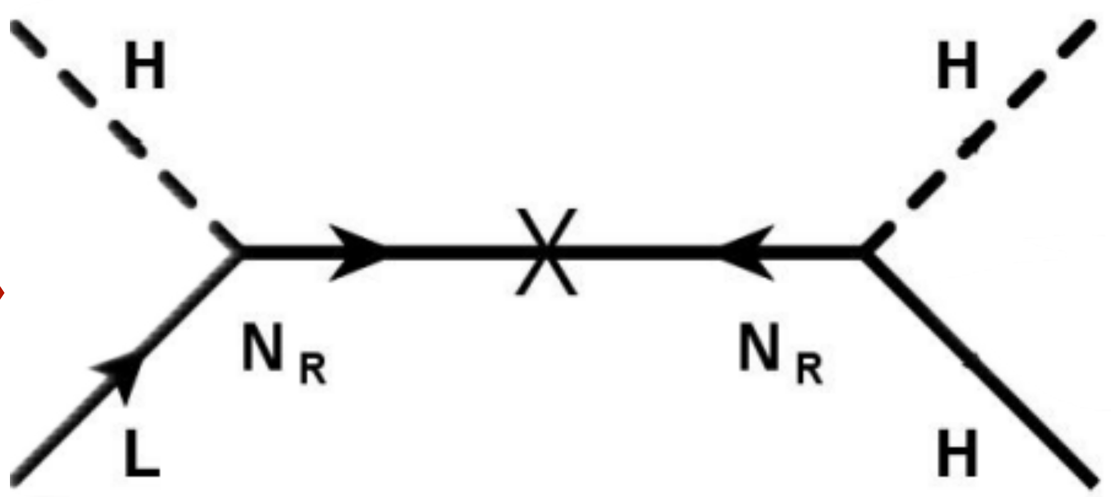
- **Dimension-5 operator in SM fields → Neutrino mass**

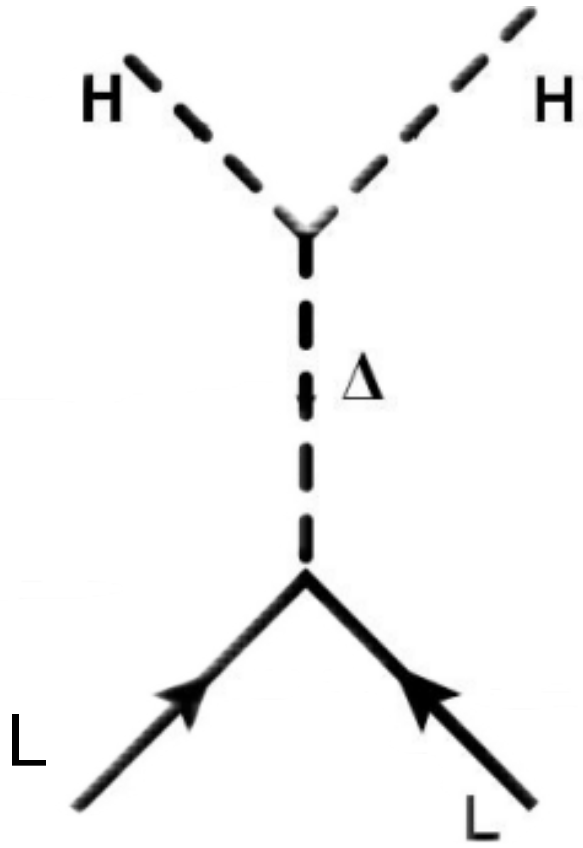
$$\mathscr{L}_5 = \frac{g}{\Lambda}(L^T\sigma_2 H)C^\dagger(H^T\sigma_2 L) \xrightarrow{\text{EW -SSB}} \frac{gv^2}{2\Lambda}\nu_L^T C^\dagger \nu_L + \text{H.c.}$$

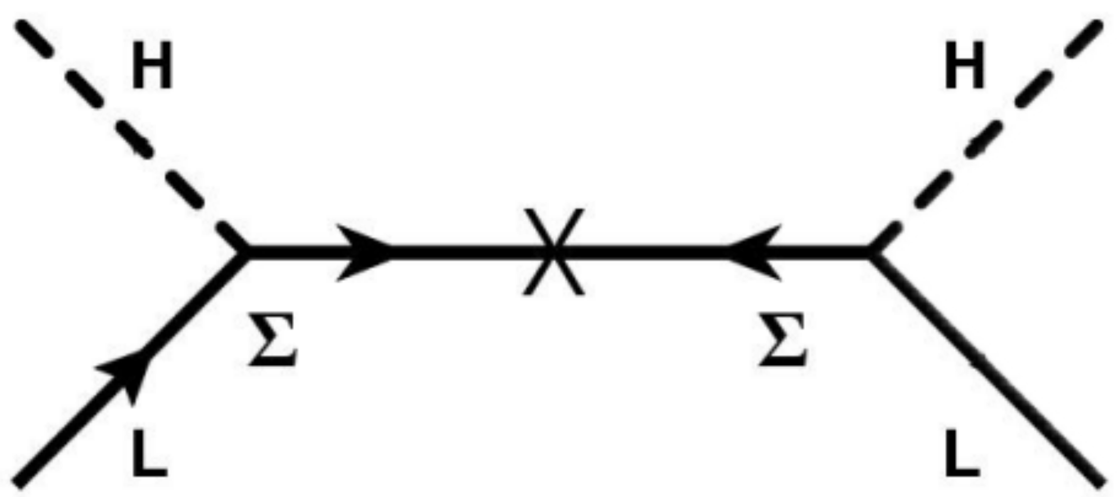
(Weinberg, PRD 22 (1980) 1694)



UV Completion →







Type-1 **Type-II** **Type-III**

A Seesaw Description : Type-I

$$\mathscr{L}_{Type-1} = -\, L Y_D \tilde{\phi} N_R - \frac{1}{2} N_R^C M_R N_R + h.c.$$

(Minkowski,1977, Mohapatra, Senjanovic (1980))

- **Complete set of mass terms-** $\mathscr{L}_M = \frac{1}{2} \overline{n_L} M n_L$; $n_L = \begin{bmatrix} \nu_L \\ N_R^c \end{bmatrix}$

with $M = \begin{bmatrix} 0 & M_D \\ M_D^T & M_R \end{bmatrix}$, $M_D = \frac{Y_D v}{\sqrt{2}}$, $M_R \rightarrow 2 \times 2$ matrix

- Can be diagonalised by an **unitary transformation** : $M_R >> M_D$

$m_\nu^I \approx -\, M_D M_R^{-1} M_D^T$ **and** $M_N \approx M_R$

Type-I seesaw

- **Complete diagonalising matrix -** $R = M_D^* M_R^{-1}$; $R \to 3 \times 2$ matrix

$$U = U_I \times W = \begin{bmatrix} U_{\nu\nu} & U_{\nu N} \\ U_{N\nu} & U_{NN} \end{bmatrix} \approx \begin{bmatrix} (1 - \frac{1}{2}RR^\dagger)U_\nu & RU_N \\ -R^\dagger U_\nu & (1 - \frac{1}{2}R^\dagger R)U_N \end{bmatrix}$$

(W. Grimus and L. Lavoura (2000))

With $W = \begin{bmatrix} \sqrt{1 - RR^\dagger} & R \\ -R^\dagger & \sqrt{1 - R^\dagger R} \end{bmatrix}$; $U_I = \begin{bmatrix} U_\nu & 0 \\ 0 & U_N \end{bmatrix}$

- **Non-unitarity disrupts exact GIM Cancellation : Enhance Flavour violation**

$$U_{PMNS} = U_\nu(1 - \frac{1}{2}RR^\dagger) = U_\nu(1 + \underbrace{\eta})$$

Flavour Changing currents

$$\nu_\alpha = [U_\nu(1+\eta)]_{\alpha i}\nu_i + \underbrace{R_{\alpha K}}N_K]$$

Active flavour

Light-heavy mixing in Type-I and Type-III

- **Charged current**

$$\mathscr{L}^{\nu}_{cc} = -\frac{g}{2\sqrt{2}}\bar{l}_\alpha\gamma_\mu(1-\gamma_5)\underbrace{[(1+\eta)U_\nu]_{\alpha i}}\nu_i W^\mu + h.c.$$

$$\mathscr{L}^{N}_{cc} = -\frac{g}{2\sqrt{2}}\bar{l}_\alpha\gamma_\mu(1-\gamma_5)\underbrace{R_{\alpha K}}N_K W^\mu + h.c$$

- Flavor changing **neutral current** **in Type-III**

$$\mathscr{L}^{l}_{FCNC} = -\frac{g}{2\cos\Theta_W}\underbrace{(R_{\alpha K}R^\dagger_{K\beta})}\bar{l}_\alpha\gamma_\mu(1-\gamma_5)l_\beta Z^\mu + h.c.$$

Light-heavy mixing:Constraint on R (Theoretical)

- **Casas Ibarra** parameterisation

(Casas, Ibarra (2001))

$$R = M_D^* M_R^{-1} \quad = \sqrt{m_\nu^{dia}}\, U O^* \sqrt{M_R^{-1}}$$

- There is a lot of freedom in parameterizing the unphyscial matrix O

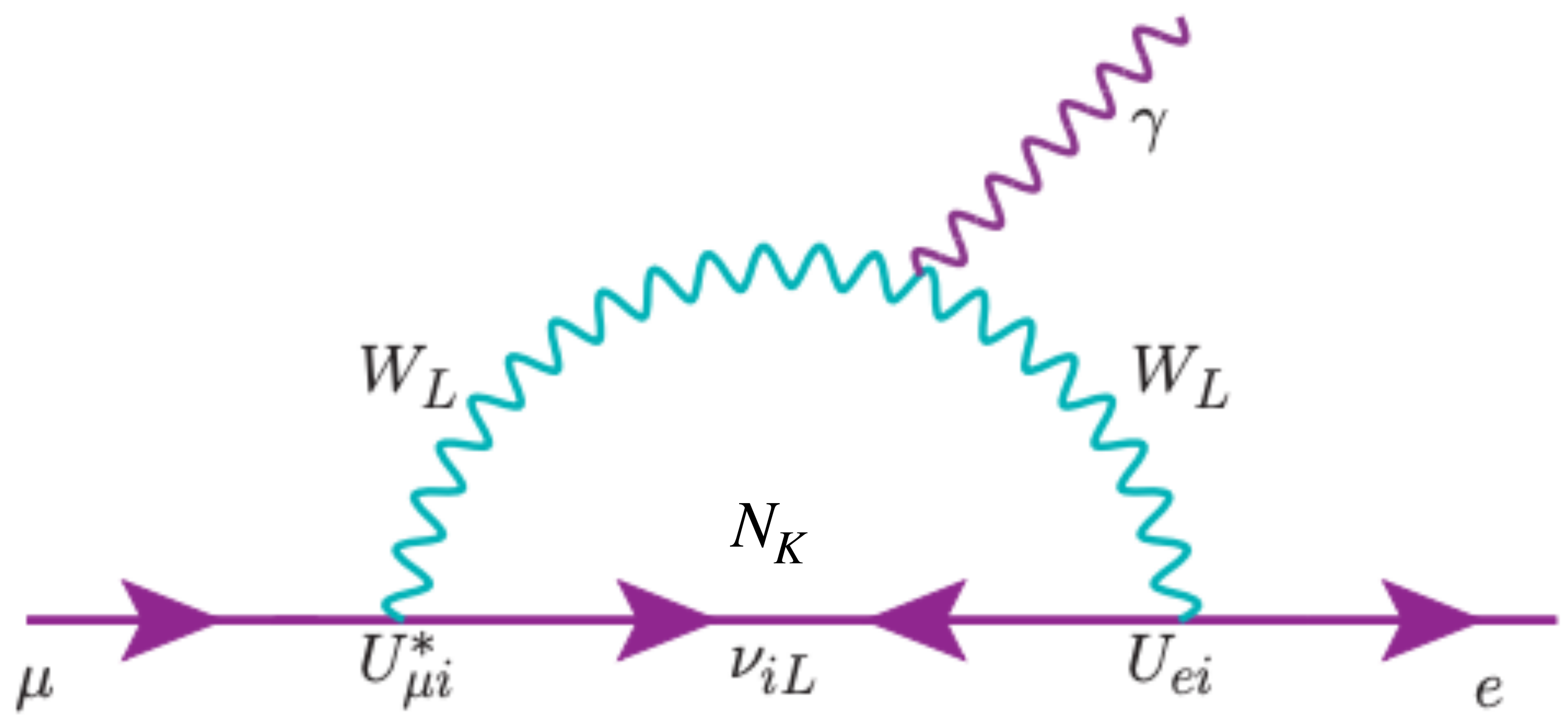
- Due to this freedom, **the light-heavy mixing matrix will only have a mild dependence on** M_R

- The strongest constraints on R come from experiments

Constraint on R from $\mu \to e\gamma$

- For **K=2, nearly degenerate** case : **Type-I**

$$M_2 = M_1(1+Z), Z << 1$$



$$B.R.(\mu \to e\gamma) \approx \frac{3\alpha}{32\pi}\left(\frac{2+Z}{1+Z}\right)^2 \underbrace{|R_{e1}R^*_{\mu 1}|}^2 [F(X_1) - F(0)]^2$$

(Ibarra, Molinaro, Petcov(2011))

with

$$F(X_K) = \frac{10 - 43X_K + 78X_K^2 - 49X_K^3 + 4X_K^4 + 18X_K^3 \log X_K}{3(X_K-1)^4};$$

$$X_K = \left(\frac{M_K}{M_W}\right)^2$$

Constraint on R from $\mu \to e\gamma$

- **Constraint of light-heavy mixing parameters** $(Z = 10^{-3})$

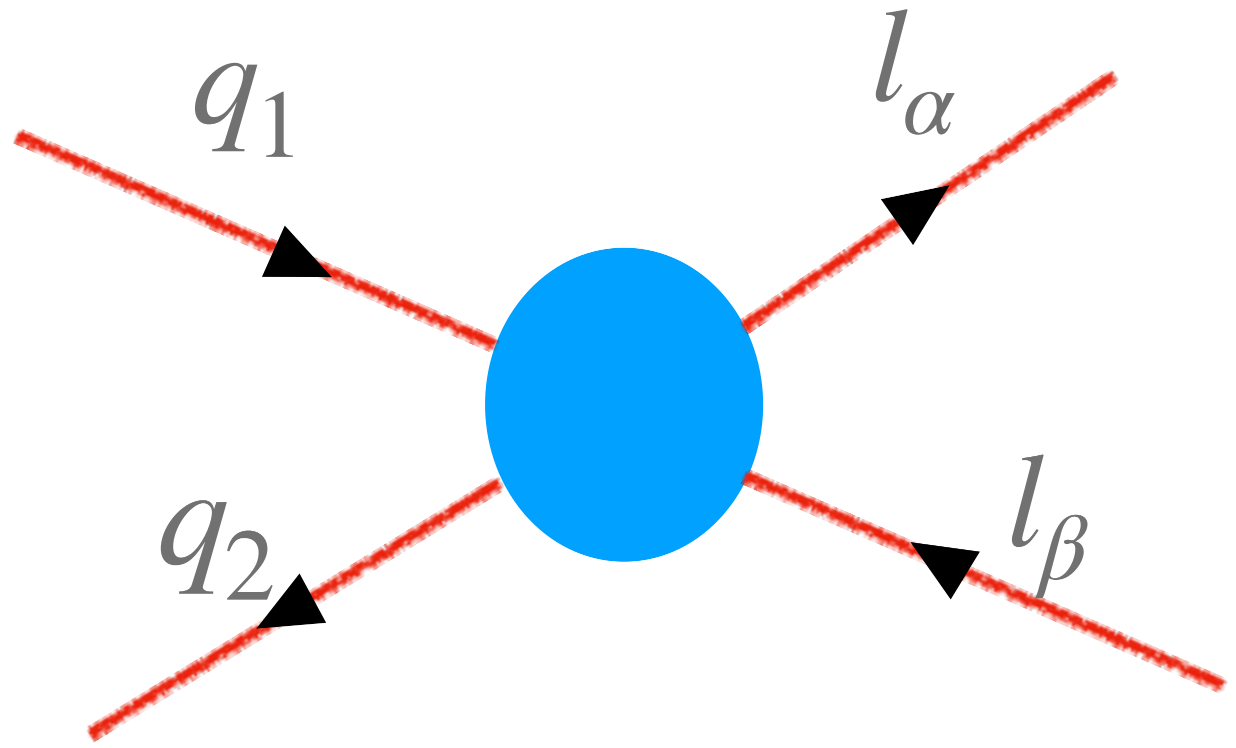
Type-I	$M_1 = 100GeV$	$M_1 = 1TeV$
$\|R_{e1}R^*_{\mu 1}\|$	3.43×10^{-5}	1.17×10^{-5}
$\|R_{e1}R^*_{\tau 1}\|$	9.62×10^{-3}	3.28×10^{-3}
$\|R_{\tau 1}R^*_{\mu 1}\|$	11.1×10^{-3}	3.79×10^{-3}

Type-III	$M_{\Sigma_1} = 100GeV$	$M_{\Sigma_1} = 1TeV$
$\|R_{e1}R^*_{\mu 1}\|$	3.69×10^{-6}	5.39×10^{-6}
$\|R_{e1}R^*_{\tau 1}\|$	1.03×10^{-3}	1.51×10^{-3}
$\|R_{\tau 1}R^*_{\mu 1}\|$	1.19×10^{-3}	1.74×10^{-3}

CLFV Decay of Mesons

- **For the transition** $q_1 \rightarrow q_2 l_\beta^+ l_\alpha^-$ (Becirevic, Jaffredo, Pinheiro, Sumnesari(2024))

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{j=u,c,t} V_{jq_1}^* V_{jq_2}[C_9 O_9 + C_{10} O_{10}]$$



With

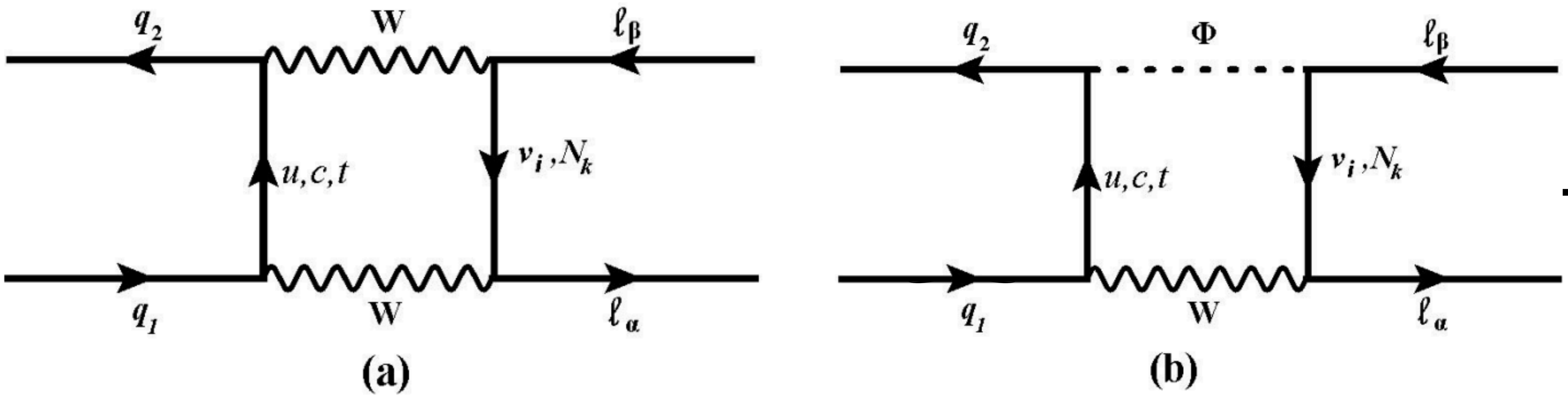
$$O_9 = \frac{\alpha}{8\pi}[\overline{q_2}\gamma^\mu(1-\gamma_5)q_1][l_\alpha \gamma_\mu l_\beta]$$

$$O_{10} = \frac{\alpha}{8\pi}[\overline{q_2}\gamma^\mu(1-\gamma_5)q_1][l_\alpha \gamma_\mu \gamma_5 l_\beta]$$

$\Rightarrow$ $C_9 = -C_{10}$

CLFV Decay of Mesons

- **Type-I**

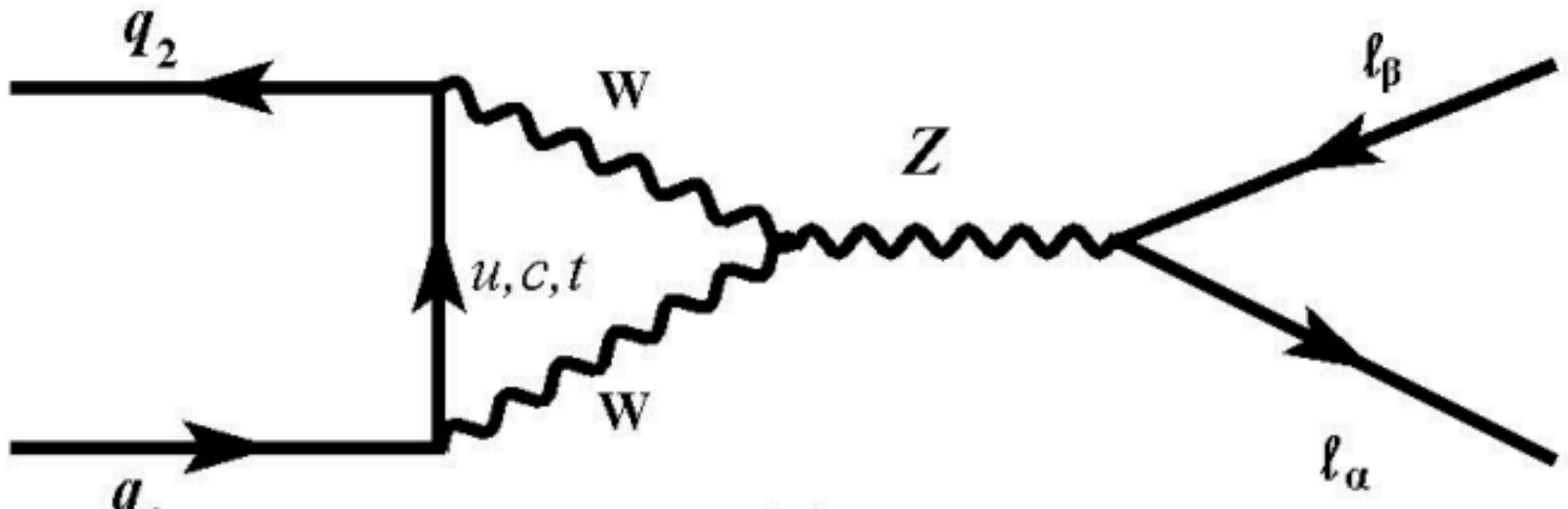


+ W ⟷ Φ

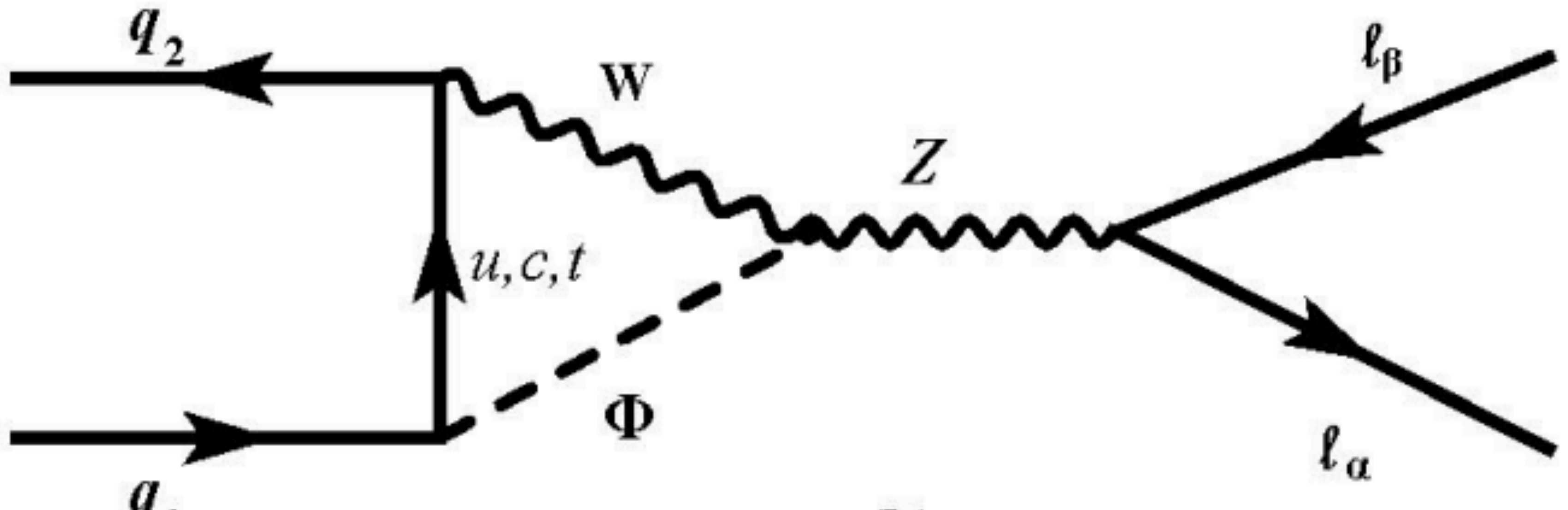
$$C_{9(I)} \approxeq X_t \underbrace{[R_{\alpha 1} R^*_{\beta 1}]} \left[\frac{1}{2\sin^2\theta_W} I_1(X_t, X_1) \right]$$

CLFV Decay of Mesons

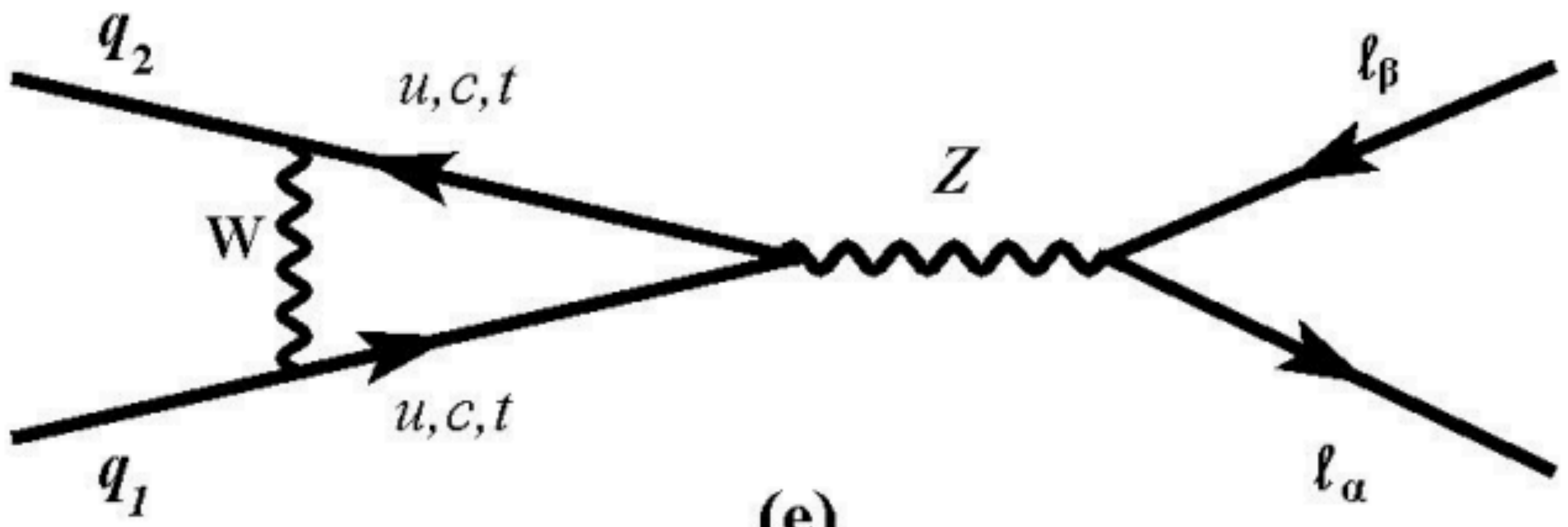
- **Additional contribution in Type-III:**



(a)



(b)



(e)

$+ \mathbf{W} \longleftrightarrow \Phi$

$$C_{9(III)} \approxeq X_t[\underbrace{R_{\alpha 1} R^*_{\beta 1}}]I_{tot}(X_t, X_1)$$

Bounds on CLFV K-meson and B-meson Decays

- **Purely leptonic decay :**

$$Br(K_L \to \mu^+ e^-) = 2\tau_K \frac{G_F^2 \alpha^2}{32\pi^3} f_K^2 \left(1 - \frac{\mu^2}{m_K^2}\right)^2 |V_{ts} V_{td}^*|^2 \underbrace{|C_9|^2}$$

- **Semileptonic decays:**

$$Br(K^+ \to \pi^+ \mu^+ e^-) = a_9^K \underbrace{|C_9|^2}$$

$$a_9^K = 1.94 \times 10^{-13}$$

$$Br(B \to M' l_\beta^+ l_\alpha^-) = 2 a_9^B \underbrace{|C_9|^2} \times 10^{-9}$$

(Carrasco, Lami et al. (2016))

- **Can use CLFV parameter from radiative $\mu \to e\gamma$ to predict the upper bounds on CLFV Meson decays.**

Results: Upper Bounds on CLFV Meson Decays

- Similarly, we can relate B.R. of other **radiative and meson CLFV decays.**

Decay	Exp. limit	Type-I $M_1 = 100(1000)GeV$	Type-III $M_{\Sigma_1} = 100(1000)GeV$
$Br(K_L \to \mu e)$	6.3×10^{-12}	$4.32(1.11) \times 10^{-20}$	$1.67(3.37) \times 10^{-19}$
$Br(K^+ \to \pi^+ \mu^+ e^-)$	1.1×10^{-10}	$5.48(1.42) \times 10^{-22}$	$2.12(4.28) \times 10^{-21}$
$Br(B_S \to \mu e)$	5.4×10^{-9}	$2.64(0.68) \times 10^{-19}$	$1.01(2.06) \times 10^{-18}$
$Br(B_S \to \mu\tau)$	3.4×10^{-5}	$6.19(0.16) \times 10^{-12}$	$2.37(4.81) \times 10^{-11}$
$Br(B^+ \to K^+ \mu\tau)$	3.1×10^{-5}	$6.76(1.75) \times 10^{-12}$	$2.60(5.25) \times 10^{-11}$
$Br(B^+ \to \pi^+ \mu\tau)$	7.2×10^{-5}	$2.61(0.67) \times 10^{-13}$	$1.00(2.02) \times 10^{-12}$
$Br(B_S \to \phi\mu\tau)$	2.0×10^{-5}	$1.18(0.30) \times 10^{-11}$	$4.53(9.18) \times 10^{-11}$

Prediction of CLFV Meson decays in seesaw models

Results

- **Type-I :** Meson CLFV B.R. $\sim 10^{-3} \times$ Radiative CLFV B.R.

- **Type-II :** Meson CLFV B.R. is negligibly small ($\sim 10^{-50}$) due to exact GIM cancellation

- **Type-III :** Similar to type-I with additional contribution from FCNC of Z-boson to charged lepton

Meson CLFV B.R. $\sim 10^{-2} \times$ Radiative CLFV B.R.

Conclusions

- Models addressing neutrino masses also predict CLFV process, which are experimentally testable
- In three basic **seesaw models** -

Meson CLFV B.R. < **Radiative CLFV B.R.**

- Can distinguish between different types of seesaw
- If experiments find **meson CLFV B.R. is greater than radiative CLFV decay** → **neutrino mass generatuion is not simple seesaw**
- Studying a model where the **radiative CLFV** is vanishingly small but **meson CLFV** saturates experimental upper bounds

Thank You !

The calculation of all the diagrams in 3, leads to the following effective Hamiltonian for type-I seesaw mechanism,

$$\begin{aligned}\mathcal{H}_{\text{eff}} &= f_I \left[\overline{q_2}\gamma^\mu(1-\gamma_5)q_1\right]\left[\bar{\ell}_\alpha\gamma^\mu(1-\gamma_5)\ell_\beta\right] \\ &= f_I\Big[\big(\overline{q_2}\,\gamma^\mu(1-\gamma_5)q_1\big)\big(\bar{\ell}_\alpha\,\gamma_\mu\,\ell_\beta\big) - \big(\overline{q_2}\gamma^\mu(1-\gamma_5)q_1\big)\big(\bar{\ell}_\alpha\gamma_\mu\gamma_5\;\ell_\beta\big)\Big].\end{aligned} \tag{A.1}$$

The overall factor f_I for type-I seesaw is given by

$$f_I = f_I^a + f_I^{b+c} + f_I^d, \tag{A.2}$$

where

$$\begin{aligned}f_I^a &= \frac{G_F^2 M_W^2}{8\pi^2}\sum_j V_{j\,q_1}^* V_{j\,q_2} x_j \sum_k R_{\alpha\,k} R_{\beta\,k}^* \mathcal{I}_1(x_j\,,x_k), \\ f_I^{b+c} &= \frac{G_F^2 M_W^2}{8\pi^2}\sum_j V_{j\,q_1}^* V_{j\,q_2} x_j \sum_k R_{\alpha\,k} R_{\beta\,k}^* \left(R_{\beta\,k} + R_{\alpha\,k}^\star\right)\mathcal{I}_2(x_j\,,x_k), \\ f_I^d &= \frac{G_F^2 M_W^2}{8\pi^2}\sum_j V_{j\,q_1}^* V_{j\,q_2} x_j \sum_k R_{\alpha\,k} R_{\beta\,k}^* R_{\alpha\,k} R_{\beta\,k}^* \mathcal{I}_2(x_j\,,x_k).\end{aligned} \tag{A.3}$$

Loop function in Type-1 seesaw