Spontaneous CP Violation in an Axion Model:

implication for lepton flavor and minimal seesaw

Sin Kyu Kang

(Seoul Tech)

(In collaboration with Yang Hwan Ahn, Jongkuk Kim)

FLASY

ROME 2025

11TH WORKSHOP

Flavor Symmetries and Consequences in Accelerators and Cosmology

Motivation & Background

- While CP violation has been observed in the SM through the complex phase of the CKM matrix, its fundamental origin remains unknown.
- One intriguing possibility is that CP is spontaneously broken when scalar fields acquire complex VEVs.
- At the same time, the strong CP problem remains one of the outstanding puzzle in particle physics, which can be elegantly resolved by introducing PQ symmetry.
- Axion models, such as the DFSZ model, naturally implement the PQ mechanism, providing compelling solution to the strong CP problem.
- The goal of this work is to investigate whether spontaneous CP violation (SCPV) can be realized within a DFSZ-type axion model.
- We will show that the CP phase generated by SCPV can simultaneously account for the complex phase in the CKM and PMNS mixing matrices.

  → PQ breaking scale is linked to seesaw scale

DFSZ Axion Model

Dine, Fischler, Srednicki, PLB104(1981)199;
Zhitnitsky, SJNP 31 (1980)260

- Scalar sectors:
  - Two Higgs doublets Φ_1, Φ_2 and a singlet S.
  - Impose PQ symmetry: global U(1) symmetry.
  - Axion: pseudo Nambu-Goldstone boson of PQ symmetry breaking.
  - Scalar potential :

$$V(\Phi_1,\Phi_2,S) = \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \kappa_1(\Phi_1^\dagger\Phi_1)|S|^2 + \kappa_2(\Phi_2^\dagger\Phi_2)|S|^2 + (\kappa_3(\Phi_1^\dagger\Phi_2)S^2 + h.c.) + V(S),$$

  - $V(S)$: scalar potential for S whose form is supposed to spontaneously break PQ sym.
  - Spontaneous breaking of EW symmetry and PQ symmetry give rise to VEVs, v_1, v_2, v_σ
  - Neutral parts are decomposed into

$$\phi_1^0 = \frac{1}{\sqrt{2}} e^{i\xi\frac{r}{v}}(v_1 + h_1)\,, \quad \phi_2^0 = \frac{1}{\sqrt{2}} e^{-i\frac{\xi}{vr}}(v_2 + h_2)\,, \quad S = \frac{1}{\sqrt{2}} e^{i\frac{A_\sigma}{v_\sigma}}(v_\sigma + h_\sigma)$$

$$\left(r = \frac{v_2}{v_1},\ v_1^2 + v_2^2 = v^2 = 246^2 \right)$$

Extension of DFSZ Axion Model

- Fermion sectors & PQ charges
  - SM fermions + two singlet heavy neutrinos N_1 , N_2
  - The charge assignment to fields:

Field	Q_L	u_R	d_R	L_L	e_R	Φ_1	Φ_2	S	N_1	N_1
$SU(2)_L$ rep.	**2**	**1**	**1**	**2**	**1**	**2**	**2**	**1**	**1**	**1**
$U(1)_Y$	$+1/6$	$+2/3$	$-1/3$	$-1/2$	-1	$+1/2$	$+1/2$	0	0	0
PQ Charge(X_i)	0	0	-2	-1	-3	2	0	1	1	-1

- X-current for PQ $U(1)_X$: $J_\mu^X = f_A \partial_\mu A + \frac{1}{2} \sum_\psi X_\psi \bar{\psi} \gamma_\mu \gamma_5 \psi$ with $f_A = \sqrt{v_\sigma^2 + \left(\frac{2 v_1 v_2}{v}\right)^2}$
- $\psi \rightarrow$ all X-charged Dirac fermions and J_μ^X is anomalous (violated at 1-loop by anomaly)
- this current creates a QCD axion

Extension of DFSZ Axion Model

- After PQ breaking, $V(\Phi_1, \Phi_2, S)$ at low energy goes to $V_{\rm 2HD}(\Phi_1, \Phi_2)$

$$V_{\rm 2HD} = m_{11}^2(\Phi_1^\dagger\Phi_1) + m_{22}^2(\Phi_2^\dagger\Phi_2) - m_{12}^2(\Phi_1^\dagger\Phi_2) + h.c.$$
$$+ \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4|\Phi_1^\dagger\Phi_2|^2,$$

$$m_{11}^2 = \kappa_1 v_\sigma^2/2, \; m_{22}^2 = \kappa_2 v_\sigma^2/2, \;\; m_{12}^2 = -\kappa_3 v_\sigma^2/2.$$

(Here, we assume real v_σ so that all parameters in $V_{\rm 2HD}$ are real)

- Since the axion solution to strong CP problem requires v_σ to be order 10^{12}GeV, the couplings κ_i must be extremely small in order to achieve EW sym. breaking.
- Such small couplings are natural as they are protected by the underlying shift symmetry of S in the limit of vanishing couplings.
- In case that κ_1 and κ_2 are negative, EW symmetry can spontaneously be broken via non-trivial vacua of neutral components of Φ_1 and Φ_2.
- V_{2HD} can not induce spontaneous CP violation.

Spontaneous CP Violation

- Spontaneous CPV can be induced from the form of 2HDM scalar potential

$$V = V_{2\mathrm{HD}} + (\Phi_1^\dagger \Phi_2)\left(\lambda_5 \Phi_1^\dagger \Phi_2 + \lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2 + h.c.\right).$$

- Taking $\langle \phi_1^0 \rangle = \frac{v_1 e^{i\delta}}{\sqrt{2}}$, $\langle \phi_2^0 \rangle = \frac{v_2}{\sqrt{2}}$, the necessary and sufficient conditions for a stationary point with $\sin\delta \neq 0$:

[Liu & Wolfenstein, NPB289(1987), Maekawa, PLB282 (1992)]

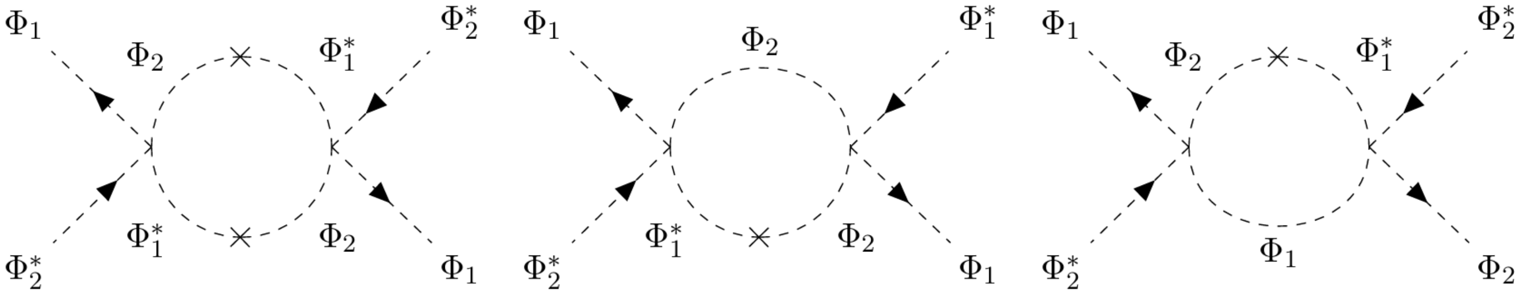
$$\lambda_5 > 0 \text{ and } \qquad -1 < \cos\delta = \frac{2m_{12}^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2} < 1.$$

- The potential at the minimum :

$$V_{\min} = \frac{1}{2} m_{11}^2 v_1^2 + \frac{1}{2} m_{22}^2 v_2^2 - \lambda_5 v_1^2 v_2^2 \cos^2\delta + \frac{1}{4}\left[\lambda_1 v_1^4 + \lambda_2 v_2^4 + (\lambda_3 + \lambda_4 - 2\lambda_5)\right].$$

Spontaneous CP Violation

- 1-loop generation of $\lambda_{5,6,7}$



(a) 1-loop generation of λ_5. (b) 1-loop generation of λ_6. (c) 1-loop generation of λ_7.

$$\lambda_5 = -\frac{3\lambda_3^2}{256\pi^2}\frac{m_{12}^4}{m_{11}^2 m_{22}^2}K(m_{11}^2, m_{22}^2),$$

$$\lambda_6 = -\frac{3\lambda_3^2}{256\pi^2}\frac{m_{12}^2}{m_{11}^2}J(m_{11}^2, m_{22}^2),$$

$$\lambda_7 = -\frac{3\lambda_3^2}{256\pi^2}\frac{m_{12}^2}{m_{22}^2}J(m_{22}^2, m_{11}^2),$$

$$K(x,y) = \frac{xy}{(x-y)^2}\left(\frac{x+y}{x-y}\ln\frac{x}{y} - 2\right),$$

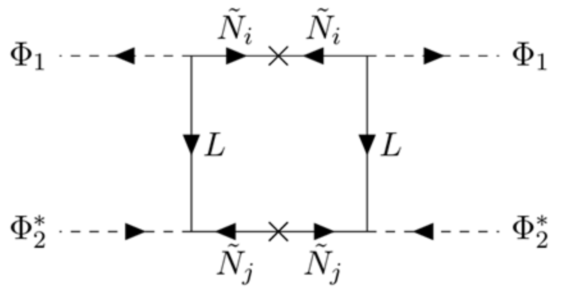
$$J(x,y) = \frac{x(x-y) - xy\ln\frac{x}{y}}{(x-y)^2}.$$

λ_5 is always negative, so SCPV is not achieved.

Spontaneous CP Violation

- Additional 1-loop generation of λ_5
  - Leptonic Yukawa Lagrangian constrained by PQ symmetry :

$$-\mathcal{L}_L^Y = \bar{L}_L Y_\ell \Phi_1 \ell_R + \bar{L}_L Y_{N_1} \tilde{\Phi}_1 N_1 + \bar{L}_L Y_{N_2} \tilde{\Phi}_2 N_2 + \frac{Y_1}{2\Lambda} N_1^c S^* S^* N_1 + \frac{Y_2}{2\Lambda} N_2^c S S N_2 + + \frac{Y_3}{\Lambda} N_1^c S^* S N_2 + \text{h.c.}$$



$$\lambda_5^{(f)} = -\frac{Y_{N_1}^2 Y_{N_2}^2}{16\pi^2} \left(2\cos^2\theta_N \sin^2\theta_N + (\cos^4\theta_N + \sin^4\theta_N) \frac{M_1 M_2}{M_1^2 - M_2^2} \ln \frac{M_1^2}{M_2^2} \right)$$

- The positive sign can be achieved when M_1 and M_2 are of opposite signs.
- Assuming $\lambda_5^{(f)}$ is dominant over λ_5 from boson-loop, SCPV becomes achievable under the right parameter conditions.

Spontaneous CP Violation

- Parameter set satisfying the condition for SCPV

Input : $M_1 = -10^6$ GeV, $M_2 = 1.06 \times 10^6$ GeV
- $\theta_N = \arccos(0.01)$, $Y_{N_1} \sim Y_{N_2} \sim 1$
- $\tan\beta = \frac{v_2}{v_1} = 1$

→ $\lambda_5 \sim \lambda_5^{(f)} \sim 0.006$

$$\cos\delta \sim \frac{m_{12}^2}{2\lambda_5 v_1 v_2}$$

$\delta = 0.19\pi$ (required for CP phase in CKM)

↓

$m_{12}^2 \sim 298\ \mathrm{GeV}^2$ → m_{A^0}~20 GeV

CP Violation in CKM Matrix

- Given the EW and PQ symmetries, quark Yukawa Lagrangian is

$$-\mathcal{L}_Y = \bar{Q}_L Y_d^{(1)} \Phi_1 d_R + \bar{Q}_L Y_u^{(2)} \tilde{\Phi}_2 u_R + \bar{Q}_L Y_d^{(2)} \Phi_2 d_R \frac{S^2}{\Lambda^2} + \bar{Q}_L Y_u^{(1)} \tilde{\Phi}_1 u_R \frac{S^2}{\Lambda^2} + \text{h.c.}$$

- After PQ sym. is broken at high E, S gets a VEV v_σ , which is taken to be the same order of Λ so that dim=6 op. can sizably contribute to quark masses
- We assume the Yukawa matrices to have the forms:

[BGL texture : Branco, Grimus, Lavoura, PLB380 (1996) 119]

$$Y_{u,d}^{(1)} \sim \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_{u,d}^{(2)} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

X : non-zero

CP Violation in CKM Matrix

- In Higgs basis, the Yukawa interactions become

$$\bar{Q}_L \frac{\sqrt{2}}{v} \left(M_d^0 H_1 + N_d^0 H_2 \right) d_R + \bar{Q}_L \frac{\sqrt{2}}{v} \left(M_u^0 H_1 + N_u^0 H_2 \right) d_R .$$

$$\begin{aligned} M_d^0 &= \frac{v}{2} \left[c_\beta Y_1^d + e^{i\delta} s_\beta Y_2^d \right] , & N_d^0 &= \frac{v}{2} \left[s_\beta Y_1^d - e^{i\delta} c_\beta Y_2^d \right] , \\ M_u^0 &= \frac{v}{2} \left[c_\beta Y_1^u + e^{-i\delta} s_\beta Y_2^u \right] , & N_u^0 &= \frac{v}{2} \left[s_\beta Y_1^d - e^{-i\delta} c_\beta Y_2^d \right] , \end{aligned}$$

$$M_f^0 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta_f} \end{pmatrix} \hat{M}_f^0 \equiv P_{\delta_f} \hat{M}_f^0$$

$$O_{f_L}^T \hat{M}_f^0 \hat{M}_f^{0T} O_{f_L} = \mathrm{Diag}(m_{f_1}^2, m_{f_2}^2, m_{f_3}^2)$$

$$\Rightarrow \quad U_{f_L} = P_{\delta_f} O_{f_L}$$

$$U_{\mathrm{CKM}} = U_{u_L}^\dagger U_{d_L} = O_{u_L}^T P_{\delta_u}^* P_{\theta_d} O_{d_L} = O_{u_L}^T \mathrm{Diag.}(1, 1, e^{2i\delta}) O_{d_L} .$$

CP Violation in CKM Matrix

- The rotation matrices are taken to be

$$O_{u_L} = R_{23}(\theta_2)\,, \qquad O_{d_L} = R_{12}(\theta_1)R_{13}(\theta_3)$$

$$\Rightarrow \quad \widehat{M}_u^0 \widehat{M}_u^{oT} = \begin{pmatrix} m_U^2 & 0 & 0 \\ 0 & A_U & B_U \\ 0 & B_U & C_U \end{pmatrix} \qquad \widehat{M}_d^0 \widehat{M}_d^{oT} = \begin{pmatrix} A_d & B_d & C_d \\ B_d & D_d & E_d \\ C_d & E_d & F_d \end{pmatrix}$$

where $A_\alpha, B_\alpha, C_\alpha$ ….. functions of m_f^2 and three mixing angles θ_i

- The CKM matrix is parametrized by

$$\begin{aligned} U_{\mathrm{CKM}} &= U_{u_L}^\dagger U_{d_L} = R_{23}(\theta_2)^T \cdot P_{2\delta} \cdot R_{12}(\theta_1) R_{13}(\theta_3), \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{2i\delta} \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{pmatrix} \end{aligned}$$

- Jarlskog invariant : $J = \frac{1}{2} \sin 2\theta_2 \cos\theta_1 \sin\theta_1 \cos^2\theta_2 \sin 2\delta.$

CP Violation in CKM Matrix

- To test the viability of our model, we perform a numerical fit of the parameters $\theta_{1,2,3}, \delta$ to 9 magnitudes of the CKM matrix elements and J_{CP}.
- The fit minimizes a χ^2 function defined by

$$\chi^2 = \sum_i \left(\frac{|V_{ij}^{\text{model}}| - |V_{ij}^{\text{exp}}|}{\sigma_{ij}} \right)^2 + \left(\frac{J^{\text{model}} - J^{\text{exp}}}{\sigma_J} \right)^2 ,$$

- Using the latest PDG values for the CKM magnitudes and uncertainties, the best- fit parameters are found to be

$$\sin\theta_1 = 0.2285,$$
$$\sin\theta_2 = 0.0410,$$
$$\sin\theta_3 = 0.0039,$$
$$\delta = 24.37^\circ,$$

$$J = 3.18 \times 10^{-5},$$

$$\chi^2_{\text{total}} = 3.77$$

$$p = 0.583$$

Indicating an excellent fit and confirming that the parametrization is statistically consistent with experimental observations at the 1σ level

Scalar mediated FCNC

- The Yukawa matrices N_f^0 can not simultaneously be diagonalized by U_{f_L}

  → Flavor changing neutral currents

$$U_{f_L}^\dagger N_f^0 U_{f_R} = U_{f_L}^\dagger N_f^0 O_{f_R} = U_{f_L}^\dagger \begin{pmatrix} t_\beta & & \\ & t_\beta & \\ & & -\frac{1}{t_\beta} \end{pmatrix} U_{f_L} U_{f_L}^\dagger M_f^0 O_{f_R}$$

$$U_{f_L}^\dagger \begin{pmatrix} t_\beta & & \\ & t_\beta & \\ & & -\frac{e^{i\delta}}{t_\beta} \end{pmatrix} U_{f_L} = t_\beta I - \left(t_\beta + \frac{1}{t_\beta}\right) U_{f_L}^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} U_{f_L}$$

- FCNC is proportional to $(U_{f_L})_{3i}^*(U_{f_L})_{3j}$.
- This problem can be avoided if $(U_{f_L})_{31}$ *and* $(U_{f_L})_{32}$ are either zero or so small that can be safe from the current constraints from experiments.
- Our predictions : $[(O_{u_L})_{32} \cdot (O_{u_L})_{33}]^2 \approx 1.64 \times 10^{-3}$ $\rightarrow Br(t \rightarrow ch)$

  $[(O_{d_L})_{31} \cdot (O_{d_L})_{33}]^2 \approx 1.43 \times 10^{-5}$ $\rightarrow B_d^0 \bar{B}_d^0$ osc.

CP Violation in PMNS Matrix

- The model encompasses the minimal seesaw (MS)

Frampton, Glashow Yanagida PLB548(2002),
Endoh, Kaneko, **SKK**, Morozumi, **Tanimoto** PRL(2002),
Raidal, Strumia PLB533 (2003),
Ibarra, Ross PLB 575 (2003),
Guo, **Xing** PLB583(2004) …

$$-\mathcal{L}_L^Y = \bar{L}_L Y_\ell \Phi_1 \ell_R + \bar{L}_L Y_{N_1} \tilde{\Phi}_1 N_1 + \bar{L}_L Y_{N_2} \tilde{\Phi}_2 N_2 + \frac{Y_1}{2\Lambda} N_1^c S^* S^* N_1 + \frac{Y_2}{2\Lambda} N_2^c S S N_2 + + \frac{Y_3}{\Lambda} N_1^c S^* S N_2 + \text{h.c.}$$

- Mass terms for the lepton sector of the MS reads,

$$-\mathcal{L}_m = \overline{l_{iL}} m_{l_i} l_{iR} + \overline{\nu_{Li}} m_{D_{ij}} N_j + \frac{1}{2} \overline{(N_i)^c} M_{ij} N_j$$

- The Dirac mass term m_D is 3 x 2 complex matrix :

$$(m_D)_{l1} = \frac{1}{\sqrt{2}} v_1 (Y_{N_1})_{1l} e^{i\delta} \equiv a_l \quad (m_D)_{l2} = \frac{1}{\sqrt{2}} v_2 (Y_{N_2})_{l2} \equiv b_i$$

- Heavy neutrino mass matrix

$$M = \frac{v_\sigma^2}{2\Lambda} \begin{pmatrix} Y_1 & Y_2 \\ Y_2 & Y_3 \end{pmatrix}$$

When $Y_2^2 > Y_1 Y_3$, one of mass eigenvalues is (-)

CP Violation in PMNS Matrix

- Type-I seesaw mechanism yields the effective neutrino masses given by

$$m_{eff} = -m_D U_N \frac{1}{M^d} U_N^* m_D^T = U^*_{\text{PMNS}} m_\nu^d U^\dagger_{\text{PMNS}}$$

- Using Casas-Ibarra parameterization,

$$\boxed{m_D U_N \frac{1}{\sqrt{M^d}} O^T = i U^*_{\text{PMNS}} \sqrt{m_\nu^d},}$$

$$\sqrt{m_\nu^d} = \text{Diag.}[\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3}],$$
$$1/\sqrt{M^d} = \text{Diag.}[1/\sqrt{M_1}, 1/\sqrt{M_2}]$$

- $\begin{cases} O \text{ is 2 by 2 orthogonal complex matrix} \\ U_N \text{ is 2 by 2 rotation matrix (paramterized by } \theta_N) \end{cases}$

- One light neutrino is massless in MS → $m_1 = 0$ (for NH)
- PMNS mixing matrix may have one Majorana phase (α)

CP Violation in PMNS Matrix

- Performing numerical Analysis under the SCPV constraint
- Input

LH side of CI
- Heavy neutrino masses : $M_1 = -10^6$ GeV,
  $M_2 = 1.06 \times 10^6$ GeV
- Mixing angle :$\theta_N = \arccos(0.01)$
- SCPV phase : $\delta = 0.19\,\pi$ (from quark sector)

RH side of CI (from NuFIT5.3)
- Light neutrino masses (NH) : $m_1 = 0$,
  $m_2 = \sqrt{\Delta m_{21}^2} = 0.0086$ eV,
  $m_3 = \sqrt{\Delta m_{31}^2}$ =0.05 eV,
- PMNS angles : $\sin^2\theta_{12} = 0.303$,
  $\sin^2\theta_{23} = 0.451, \sin^2\theta_{13} = 0.022$,
- Majorana phase : $\alpha = 0.5\,\pi$

- Free parameters :

- δ_{CP} (CP phase in U_{PMNS}), a_i, b_i, O_{ij} (9 independent)

⇨ Predicted parameter set satisfying CI relation

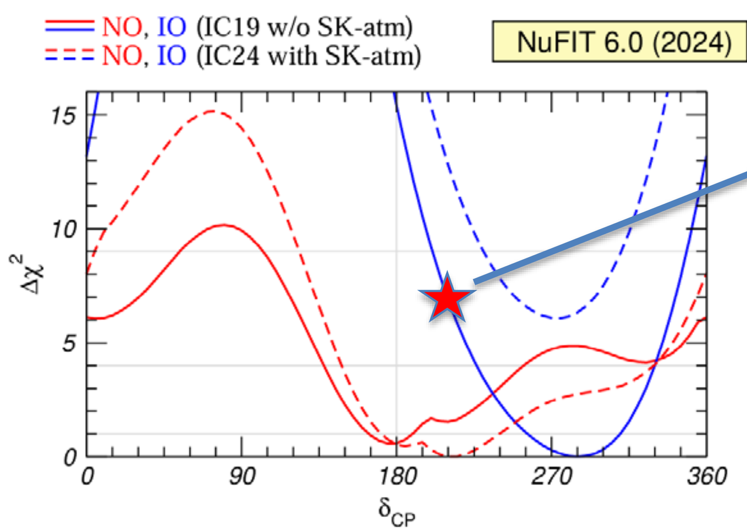
$$m_D U_N \frac{1}{\sqrt{M^d}} O^T = i U^*_{\text{PMNS}} \sqrt{m_\nu^d}$$

CP Violation in PMNS Matrix

- We perform a parameter scan to find the optimal value of δ_{CP}, approaching the global fit value within 1σ, while ensuring that the second column of the Dirac mass matrix becomes real, based on the Casas-Ibarra parametrization.

$$m_D = 10^{-5}\begin{pmatrix} 0.27\ e^{i0.19\pi} & 0.79 \\ -1.34\ e^{i0.19\pi} & -2.40 \\ -1.57\ e^{i0.19\pi} & -3.10 \end{pmatrix}\text{GeV} \quad O \approx \begin{pmatrix} -0.997 & -0.042+0.069i \\ 0.042+0.069i & -0.997 \end{pmatrix}$$

$\delta_{CP} \cong 194°$ → $J_{\rm CP} \approx 8\times 10^{-3}$.



Our prediction:
~ 1.6σ away from BF

Conclusion

- We have demonstrated that CP can be spontaneously broken within a DFSZ-type axion model
- We have shown that CP-violating phase in the CKM matrix can arise from the spontaneous breaking of CP.
- Furthermore, we have demonstrated that the Dirac-type CP phase in the PMNS matrix can also be generated from the same source of SCPV responsible for quark sector CP violation.
- PQ breaking is connected to seesaw scale.
- This framework for lepton sector may provide a natural setup for realizing baryogenesis via leptogenesis.

CP Violation in CKM Matrix

- The rotation matrices are taken to be

$$O_{u_L} = R_{12}(\theta_1)R_{23}(\theta_2)\ , \qquad O_{d_L} = R_{13}(\theta_3)$$

- The CKM matrix is parametrized by

$$\begin{aligned} &= U_{u_L}^\dagger U_{d_L} = (R_{12}(\theta_1)R_{23}(\theta_2))^T P_{2\delta} \cdot R_{13}(\theta_3), \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{2i\delta} \end{pmatrix} \begin{pmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{pmatrix} \end{aligned}$$

- Absorbing the phase matrix $P_{2\delta}$ into LH quarks, the final form of CKM matrix :

$$V = (R_{12}(\theta_1)R_{23}(\theta_2))^T \cdot R_{13}(\theta_3, \delta)$$

$$R_{13}(\theta_3, \delta) = \begin{pmatrix} \cos\theta_3 & 0 & \sin\theta_3\, e^{-i2\delta} \\ 0 & 1 & 0 \\ -\sin\theta_3\, e^{i2\delta} & 0 & \cos\theta_3 \end{pmatrix}.$$

CP Violation in CKM Matrix

- To test the viability of our model, we perform a numerical fit of the parameters $\theta_{1,2,3}, \delta$ to 9 magnitudes of the CKM matrix elements and J_{CP}.
- The fit minimizes a χ^2 function defined by

$$\chi^2 = \sum_i \left(\frac{|V_{ij}^{\text{model}}| - |V_{ij}^{\text{exp}}|}{\sigma_{ij}} \right)^2 + \left(\frac{J^{\text{model}} - J^{\text{exp}}}{\sigma_J} \right)^2 ,$$

- Using the latest PDG values for the CKM magnitudes and uncertainties, the best- fit parameters are found to be

$$\sin\theta_1 = 0.2276, \quad \sin\theta_2 = 0.0413, \quad \sin\theta_3 = 0.0037, \quad 2\delta = 68.93^\circ,$$

$$J = 3.04 \times 10^{-5}$$

$$\chi^2_{\text{mod}} = 5.41, \quad \chi^2_{\text{CP}} = 0.018, \quad \chi^2_{\text{total}} = 5.43.$$

$$p = 0.49,$$

Indicating an excellent fit and confirming that the parametrization is statistically consistent with experimental observations at the 1σ level