Inflation and QCD axion from Modular Flavor Symmetry

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work in progress
collaboration with
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Problems to be solved in this work

- **Moduli Stabilization**

  moduli must be stabilized with heavy mass $\begin{cases} \text{to have determined value} \\ \text{not to be observed} \\ \text{over DM abondance} \end{cases}$

- **Strong CP problem**

  why θ-angle is fine tuned to almost zero $\rightarrow$ QCD axion = moduli ?

- **Inflation – the birth of this universe**

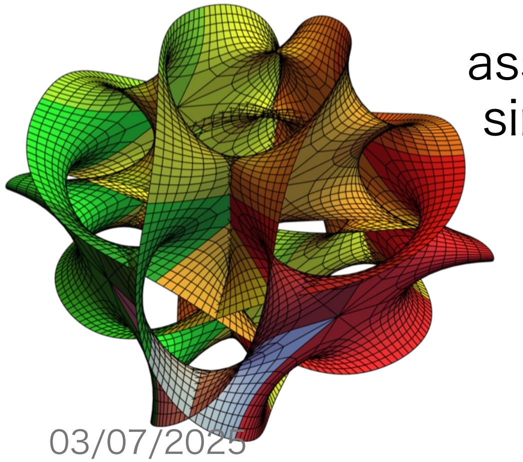
  what is the inflaton potential satisfying slow-roll condition

Modular Symmetry as a Guide

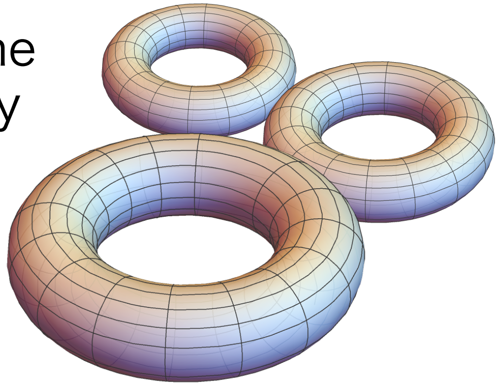
1. Is a symmetry naturally which arises in String Theory

String theory demand small extra-dimensions.
Moduli determine their shape and size, and modular symmetry is its symmetry.

2. Has same algebra with discrete flavor symmetry



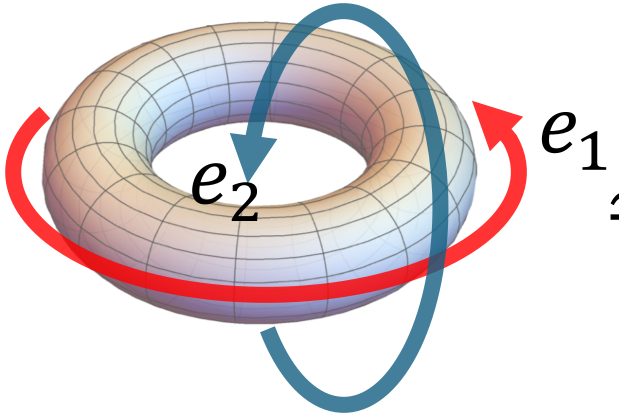
assume
simply
~



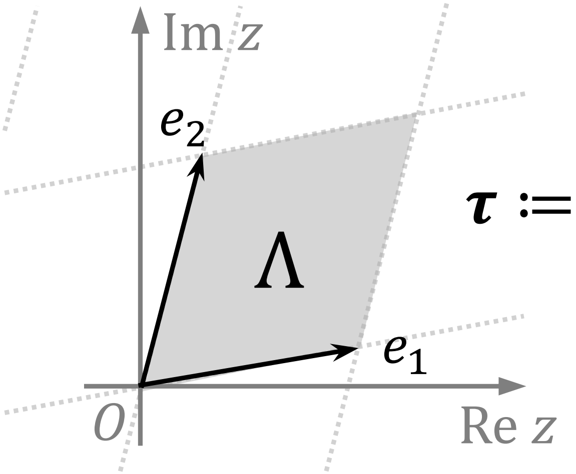
such as S_3, A_4, S_4 …

ex. [E. Ma and G. Rajasekaran, PRD (2001)]

Modular Symmetry SL(2,Z)



$\simeq$



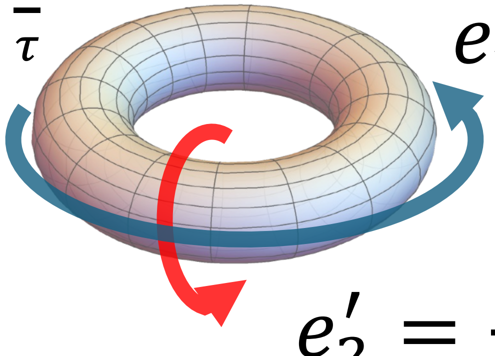
$\boldsymbol{\tau} \coloneqq e_2/e_1$

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbb{Z})$$

corresponds to redefinitions of periods

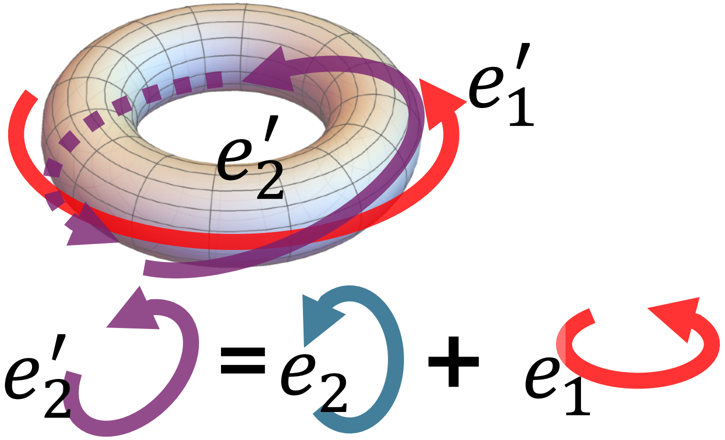
$S : \tau \to -\frac{1}{\tau}$



$e_1' = e_2$

$e_2' = -e_1$

$T : \tau \to \tau + 1$



Modular Flavor Symmetry

Modular trans. : $S: \tau \rightarrow -\frac{1}{\tau}$ $T: \tau \rightarrow \tau + 1$

Finite Modular Group $S^4 = (ST)^2 = 1 = \boldsymbol{T^N}$

Level N	2	3	4	5
Discrete Symmetry	S_3	A_4	S_4	A_5

Modular form $Y(\tau) \xrightarrow{\gamma} Y(\tau') = (c\tau + d)^k \underline{\rho(\gamma)}\, Y(\tau)$

Modular Weight (k)

$\rho(\gamma)$: unitary matrix for modular trans. γ

[S. Ferrara, D. Lust, A. D. Shapere and S. Theisen, PRL (1989)]
[F. Feruglio, arXiv:1706.08749]
[T. Kobayashi, N. Omoto, Y. Shimizu and K. Takagi, JHEP (2018)]

Level N and weight k is determined by hand for models

Model : VLQ 1-loop correction

$$K = -h\log(-i\tau + i\tau^\dagger) + \sum_i \left(\frac{Q_i^\dagger Q_i}{(-i\tau + i\tau^\dagger)^{k_{Q_i}}} + \frac{\overline{Q}_i^\dagger \overline{Q}_i}{(-i\tau + i\tau^\dagger)^{k_{\overline{Q}_i}}} \right) + K_{\rm SM}$$

$$W = W_{\rm MSSM} + M_Q \sum_{i,j} Y^{k_Y}(\tau) Q_i \bar{Q}_j, \qquad k_Y = k_{Q_i} + k_{\bar{Q}_j} - h,$$

Consider Vector Like Quarks living in High scale for generating moduli potential

Distribute modular weights to be modular symmetric model

In this model we regard: $\tau = a + i\,\phi$

Moduli Stabilization and QCD Axion

T. Higaki, J. Kawamura and T. Kobayashi, JHEP (2024)

Model : Coleman-Weinberg Potential

$m_i \equiv m_0$: soft mass. set $m_0 = 10^{-5} M_Q$

$$V_{\rm CW} = \frac{1}{32\pi^2}\sum_i \left[\left(m_i^2 + m_{Q_i}^2(\tau)\right)^2 \left(\log\left(\frac{m_i^2 + m_{Q_i}^2(\tau)}{\mu^2}\right) - \frac{3}{2}\right) - (m_{Q_i}^2(\tau))^2 \left(\log\left(\frac{m_{Q_i}^2(\tau)}{\mu^2}\right) - \frac{3}{2}\right)\right],$$

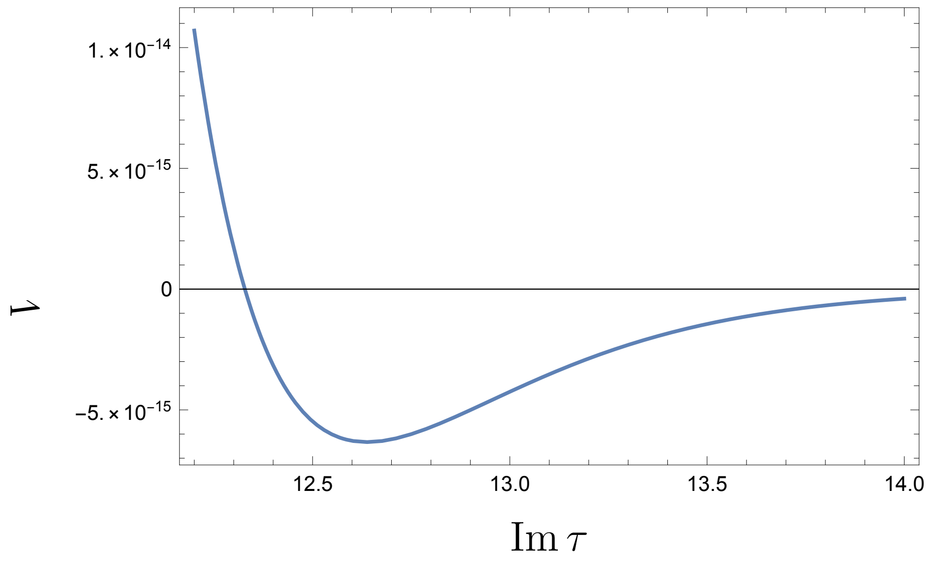
$$m_{Q_i}^2(\tau) := M_{Q_i}^2 \left(-i\tau + i\tau^\dagger\right)^{k_i} \left|Y_{r_i}^{(k_i)}(\tau)\right|^2 .$$

$$K = -h\log(-i\tau + i\tau^\dagger) + \sum_i \left(\frac{Q_i^\dagger Q_i}{(-i\tau + i\tau^\dagger)^{k_{Q_i}}} + \frac{\overline{Q}_i^\dagger \overline{Q}_i}{(-i\tau + i\tau^\dagger)^{k_{\overline{Q}_i}}}\right),$$

$$W = \sum_i M_{Q_i} Y_{r_i}^{(k_i)}(\tau) \overline{Q}_i Q_i,$$

take weight 12 non-trivial singlet

Im τ can be stabilized at large value



Applicability of **Re τ** to QCD axion

$$|\Delta\theta| \simeq 2 \times 10^{-10} \times \sin\theta_0 \left(\frac{|\Omega|}{10^4}\right)\left(\frac{m_0}{10^7\ {\rm GeV}}\right)^2 \left(\frac{M_Q}{M_p}\right)^2 \left(\frac{x_0}{28}\right)^{12} \left(\frac{\epsilon}{10^{-12}}\right)^7 .$$

General analysis

T. Higaki, J. Kawamura and T. Kobayashi, JHEP (2024)
Y. Abe, **KG**, T. Higaki, J. Kawamura and T. Kobayashi, work in progress

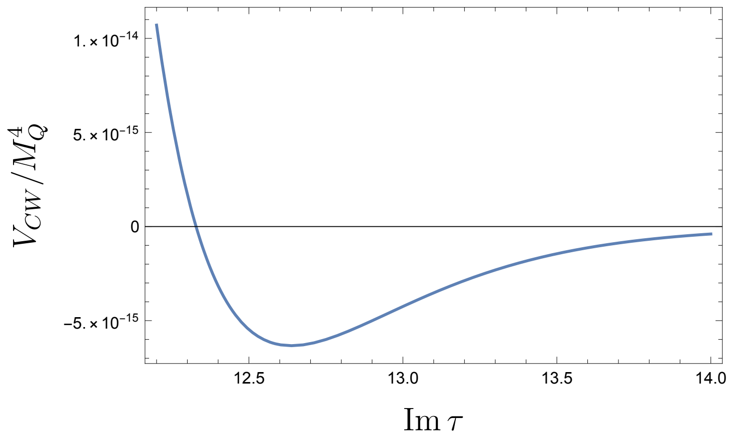
$$Y^{(k)}_{\mathbf{1}_t}(\tau) = q^{t/N}\sum_{n=0}^{\infty} c_n q^n$$ for trivial $t = 0$, non-trivial $t = 1,2,\ldots$

For Large $x = 2\mathrm{Im}\,\tau \gg 1$

potential goes 0 for $x \gg 1$

$$V_{\mathrm{CW}} = \frac{m_0^2 \tilde{M}_Q^2}{16\pi^2} x^k \mathrm{e}^{-px} (A(x) - 1) + \mathcal{O}(|q|, m_0^4) \qquad p = \frac{2\pi t}{N}$$

$$A(x) := \log\frac{\tilde{M}_Q^2}{\mu^2} + k\log x - px$$



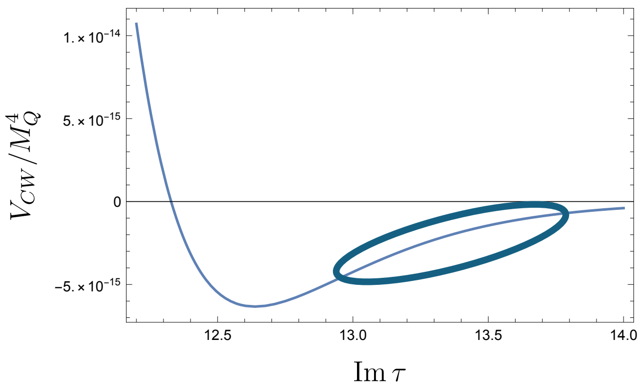
Minimum of this potential $$\frac{\mathrm{d}V_{\mathrm{CW}}}{\mathrm{d}x} \simeq \frac{m_0^2 \tilde{M}_Q^2}{16\pi^2} x^{k-1} \mathrm{e}^{-px} (k - px)\, A(x). \qquad A(x_0) = 0, \quad \therefore\ x_0^k \mathrm{e}^{-px_0} = \frac{\mu^2}{\tilde{M}_Q^2}$$

$$V_{\mathrm{CW}}\big|_{\mathrm{min.}} = -\frac{m_0^2\mu^2}{16\pi^2}$$ Indep. of Level and weight.

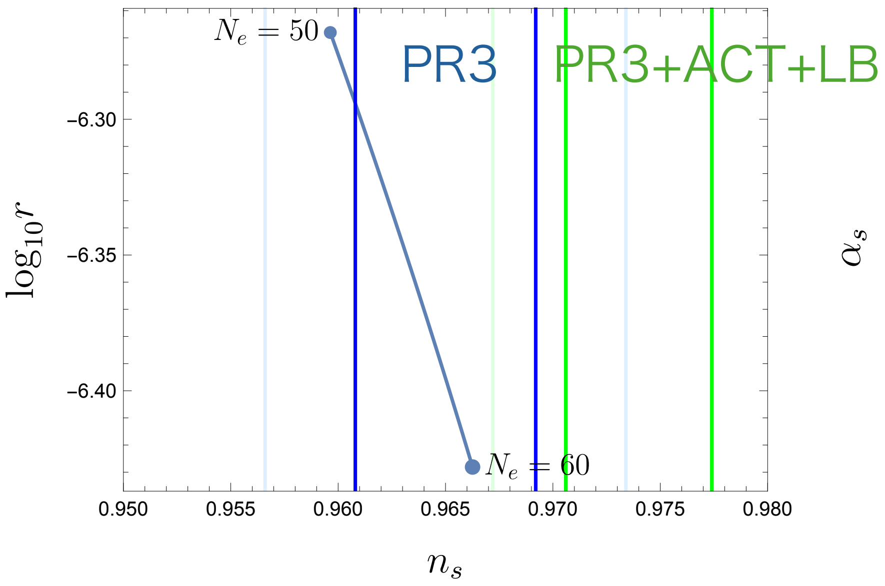
Potential goes 0 value from $V < 0$
for **any non-trivial singlet**.

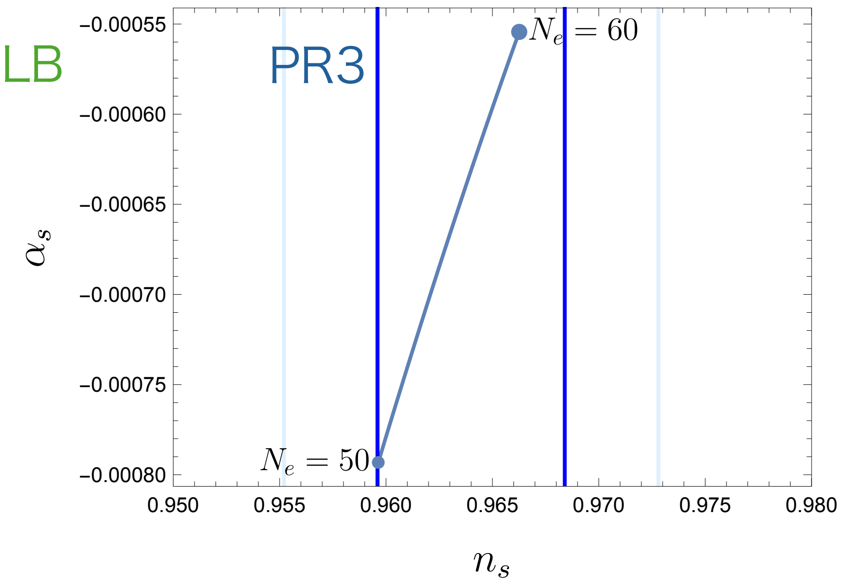
Application to Inflation

work in progress



slow roll condition
$\epsilon, \eta < 1$





consistent with observations

normalize M_Q by powerspectrum

N_e	n_s	r	$\mathcal{P}_{\mathcal{R}}/(M_Q/M_{\rm Pl})^4$	$M_Q/M_{\rm Pl}$
50	0.960(0.9596)	5.73×10^{-7}	7.23×10^{-8}	0.413
60	0.966	3.97×10^{-7}	1.05×10^{-7}	0.376

nearly $\mathcal{O}(1)$

Preliminary Result 1

work in progress

Axion quality is hardly realized considering moduli mass

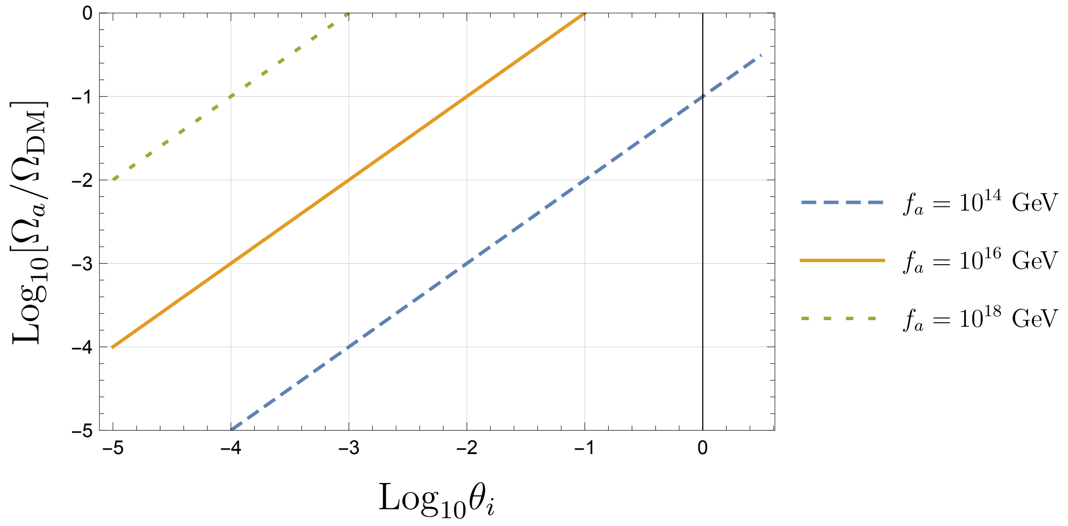
$$|\Delta\theta| \lesssim 4.5 \times 10^{-8} \sin 2\phi_0 \left(\frac{m_0}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{M_Q}{10^{18}\ \mathrm{GeV}}\right)^2 \left(\frac{x_0}{25}\right)^{12} \left(\frac{\epsilon}{10^{-12}}\right)^8$$

$$m_X \sim 0.8\ \mathrm{TeV} \times \sqrt{\left(\frac{|\Delta\theta|}{10^{-10}}\right)\frac{1}{h \sin 2\phi_0}} \left(\frac{25}{x_0}\right)^5 \left(\frac{10^{-12}}{\epsilon}\right)^3$$

lower than general bound 10 TeV

Bound for ALP from iso curvature : $\beta_{\mathrm{iso}} \lesssim 0.035$

$$\left(\frac{\Omega_a}{\Omega_d}\right)^2 \left(\frac{H_I}{10^{10}\ \mathrm{GeV}}\right)^2 \left(\frac{10^{16}\mathrm{GeV}}{f_a}\right)^2 \left(\frac{1}{\theta_i}\right)^2 \lesssim 10^2$$



In this model ALP can be exist.

line shape seems trivial from the def. of axion-DM ratio
area lower than the lines is allowed

Preliminary Result 2

work in progress

Reheating Temperature Calculation : Lagrangian. Here $T \coloneqq -i\tau$.

$$K = -h \log\left(T + T^\dagger - \frac{z}{h}|\eta(T)|^{-4}|H_u + H_d^\dagger|^2\right)$$
$$+ \sum_i \left(\frac{Q_i^\dagger Q_i}{(T+T^\dagger)^{k_{Q_i}}} + \frac{\bar{Q}_i^\dagger \bar{Q}_i}{(T+T^\dagger)^{k_{\bar{Q}_i}}} \right) + \sum_{i \in \mathrm{MSSM}} \frac{|\Phi_i|^2}{(T+T^\dagger)^{k_{\Phi_i}}}$$

GM-term with Modular sym.

G. F. Giudice and A. Masiero, PRL (1988)

$$W = W_{\mathrm{MSSM}} + M_Q \sum_{i,j} Y^{k_Y}(T) Q_i \bar{Q}_j, \quad k_Y = k_{Q_i} + k_{\bar{Q}_j} - h,$$

$$f_G(T) = \frac{\mathrm{i}}{16\pi^2} \log \eta(T)^{2k_G}$$

SM Gauge kinetic function with Modular sym.

$\boldsymbol{k_G}$ is a sum of weight of particles participating in the interaction.

F. Feruglio, A. Strumia and A. Titov, JHEP (2023)

F. Feruglio, M.Parriciatu, A. Strumia and A. Titov, JHEP (2024)

F. Feruglio, A. Marrone, A. Strumia and A. Titov, 2505.20395

Preliminary Result 2

work in progress

Decay ratio: We assume high energy SUSY

and inflaton ϕ decay only to axion a, gauge bosons A, SM Higgs H

$$\Gamma_{\text{tot}} = \Gamma(\phi \to aa) + \Gamma(\phi \to A_\mu A_\mu) + \Gamma(\phi \to HH)$$

$$\Gamma \propto \frac{m_\phi^3}{M_{Pl}^2}$$

the ratio to …

- SM particles is suppressed since they are EW scale
- VLQ is forbidden kinematically since its mass is around M_{Pl}

$$\Gamma_{\text{SM}} \propto \frac{m_{\text{EW}}^2}{m_\phi^2}\frac{m_\phi^3}{M_{Pl}^2}$$

$$T_D = 15\ \text{MeV}\left(\frac{15}{g_\star(T_D)}\right)^{1/4}\left(\frac{m_\phi}{180\ \text{TeV}}\right)^{3/2}$$ $\gg$ 4.5 MeV for BBN

Summary

Problems: moduli stabilization, QCD axion and Inflation
can be explained by **Modular Symmetric Model**.

We analyzed **the Coleman-Weinberg potential**
as an inflaton potential.

It seems **inflation can occur for non-trivial singlets**.

This inflation model clear various constraints from observation.
But we need more discussion about **the moduli problem and reheating**.

Back Up

Measurement Result

Planck Collaboration 2018
M. Tristram, et al., PRD(2022)
T. Louis, et al., arXive:2503.14452

$$n_s = 0.965 \pm 0.0042 \quad \text{(PR3)},$$
$$n_s = 0.974 \pm 0.0034 \quad \text{(PR3+ACT+LB)},$$

$$n_s = 0.964 \pm 0.0044, \quad \alpha_s = -0.0045 \pm 0.0067 \quad \text{(PR3)},$$

$$r < 0.032 \quad \text{(PR4+BK18+BAO+lensing)}$$

$$\mathcal{P}_{\mathcal{R}} = 2.10 \times 10^{-9}$$

Potential and normalization

For $\mathrm{Im}\,\tau \gg 1$

$$V_{\mathrm{CW}} = \frac{m_0^2 \tilde{M}_Q^2}{16\pi^2} x^k \mathrm{e}^{-px} \left(A(x) - 1\right) + \mathcal{O}(|q|, m_0^4),$$

$$A(x) \coloneqq \log \frac{\tilde{M}_Q^2}{\mu^2} + k \log x - px$$

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 N_{,\phi}^{\,2} \;\; = \frac{m_0^2 \tilde{M}_Q^2}{12\pi^2 \cdot 16\pi^2} \frac{\left(x^k \mathrm{e}^{-px}(A(x) - 1) + \mu^2/\tilde{M}_Q^2\right)^3}{\left(x^k \mathrm{e}^{-px}(k - px) A(x)\right)^2}$$

$$\mathcal{P}_{\mathcal{R}} \propto m_0^2 M_Q^2 \left(\frac{\mu}{M_Q}\right)^6$$

Axion Quality Detailed Calculation

$$\bar{\theta}(\tau) = \theta_0 + \arg Y_{\mathbf{1}_t}^{(k)}(\tau) = \theta_0 + \frac{t}{N}\phi + \mathcal{O}(|q|), \quad \phi := 2\pi \mathrm{Re}\,\tau$$

$$V_a = -\Lambda_{\mathrm{QCD}}^4 \cos\left(\bar{\theta}(\phi)\right) + \Delta V$$

$$\Delta\theta = \frac{1}{\Lambda_{\mathrm{QCD}}^4} \frac{\partial \Delta V}{\partial \bar{\theta}} = -\frac{c_1^2 m_0^2 M_Q^2}{8\pi^2 \Lambda_{\mathrm{QCD}}^4} \frac{N}{t} x_0^k \epsilon^{2(t+N)} \sin 2\phi_0. \qquad \epsilon = \mathrm{e}^{-\pi x/N}$$

Dacay Ratio

S. Nakamura and M. Yamaguchi, PRL (2006)
T. Higaki, K. Nakayama and F. Takahashi, JHEP (2013)

Axion

$$\mathcal{L}_{\phi\to aa} = K_{TT}(\partial \mathrm{Im}\, T)^2 \ni \frac{1}{\sqrt{2}} \frac{K_{TTT}}{K_{TT}^{3/2}} \phi (\partial a)^2$$

$$\Gamma(\phi \to aa) = \frac{1}{64\pi} \frac{K_{TTT}^2}{K_{TT}^3} m_\phi^3$$

Dacay Ratio

S. Nakamura and M. Yamaguchi, PRL (2006)
T. Higaki, K. Nakayama and F. Takahashi, JHEP (2013)

Gauge Bosons

$$\mathcal{L} = -\frac{1}{4\sqrt{2}}\left(\mathrm{Re}(\partial_T f_{\mathrm{vis}})\tau F^a_{\mu\nu}F^{\mu\nu a} - \mathrm{Im}(\partial_T f_{\mathrm{vis}})\tau F^a_{\mu\nu}\tilde{F}^{\mu\nu a}\right)$$
$$= -\frac{1}{4\sqrt{2K_{TT}}(\mathrm{Re} f_{\mathrm{vis}})}\left(\mathrm{Re}(\partial_T f_{\mathrm{vis}})\hat{\tau}\hat{F}^a_{\mu\nu}\hat{F}^{\mu\nu a} - \mathrm{Im}(\partial_T f_{\mathrm{vis}})\hat{\tau}\hat{F}^a_{\mu\nu}\hat{\tilde{F}}^{\mu\nu a}\right)$$

$$\Gamma(\phi \to A_\mu A_\mu) = \frac{N_G}{128\pi}\frac{|\partial_T f_G|^2}{(\mathrm{Re}\, f_G)^2}\frac{m_\phi^3}{K_{TT}},$$

Dacay Ratio

S. Nakamura and M. Yamaguchi, PRL (2006)
T. Higaki, K. Nakayama and F. Takahashi, JHEP (2013)

Higgs

$$\mathcal{L} = \frac{1}{\sqrt{2}}\left[(\partial_T Z_u)\tau(H_u \partial^2 H_u^\dagger + \text{h.c.}) + (\partial_T Z_d)\tau(H_d \partial^2 H_d^\dagger + \text{h.c.})\right] + \frac{g_T}{\sqrt{2}}(\partial^2 \tau)(H_u H_d + \text{h.c.}).$$

$$= \sqrt{\frac{2}{K_{TT}}}\hat{\tau}\left[\left(\frac{\partial_T Z_u}{Z_u}\right) m_{H_u}^2 |\hat{H}_u|^2 + \left(\frac{\partial_T Z_d}{Z_d}\right) m_{H_d}^2 |\hat{H}_d|^2\right] + \frac{g_T}{\sqrt{2K_{TT}Z_u Z_d}} m_\tau^2 \hat{\tau}(\hat{H}_u \hat{H}_d + \text{h.c.}).$$

supressed supressed leading

$$\Gamma(\phi \to HH) \simeq \frac{1}{8\pi}\frac{2\partial_T Z_H}{Z_H^2}\frac{m_\phi^3}{K_{TT}}, \qquad Z_H = z(T+T^\dagger)^{-1}|\eta(\tau)|^{-4}$$

Dacay Ratio

S. Nakamura and M. Yamaguchi, PRL (2006)
T. Higaki, K. Nakayama and F. Takahashi, JHEP (2013)

Total

$$T = -i\tau$$

$$\begin{aligned}\Gamma_{\rm tot} &= \Gamma(\phi \to aa) + \Gamma(\phi \to A_\mu A_\mu) + \Gamma(\phi \to HH) \\ &= \frac{1}{64\pi}\frac{K_{TTT}^2}{K_{TT}^3} m_\phi^3 + \sum_G \frac{N_G}{128\pi}\frac{|\partial_T f_G|^2}{(\mathrm{Re}\, f_G)^2}\frac{m_\phi^3}{K_{TT}} + \frac{1}{8\pi}\frac{2\partial_T Z_H}{Z_H^2}\frac{m_\phi^3}{K_{TT}} \\ &= \frac{1}{64\pi}\left(\frac{K_{TTT}^2}{K_{TT}^2} + 6\left|\frac{\partial_T \log\eta(\langle T\rangle)}{\mathrm{Re}\log\eta(\langle T\rangle)}\right|^2 + 16\frac{\partial_T Z_H}{Z_H^2}\right)\frac{m_\phi^3}{K_{TT}}\end{aligned}$$

For Re T ≫ 1

$$= \frac{m_\phi^3}{64\pi h M_{\rm Pl}^2}\left\{25 + \frac{128}{z}\mathrm{e}^{-\pi\langle\mathrm{Re}\,\tau\rangle/3}\left(\frac{\pi}{3}\langle\mathrm{Re}\,T\rangle^3 - \langle\mathrm{Re}\,T\rangle^2\right)\right\}$$