BGL 3HDMs

The Flavour Puzzle meets Flavour Data

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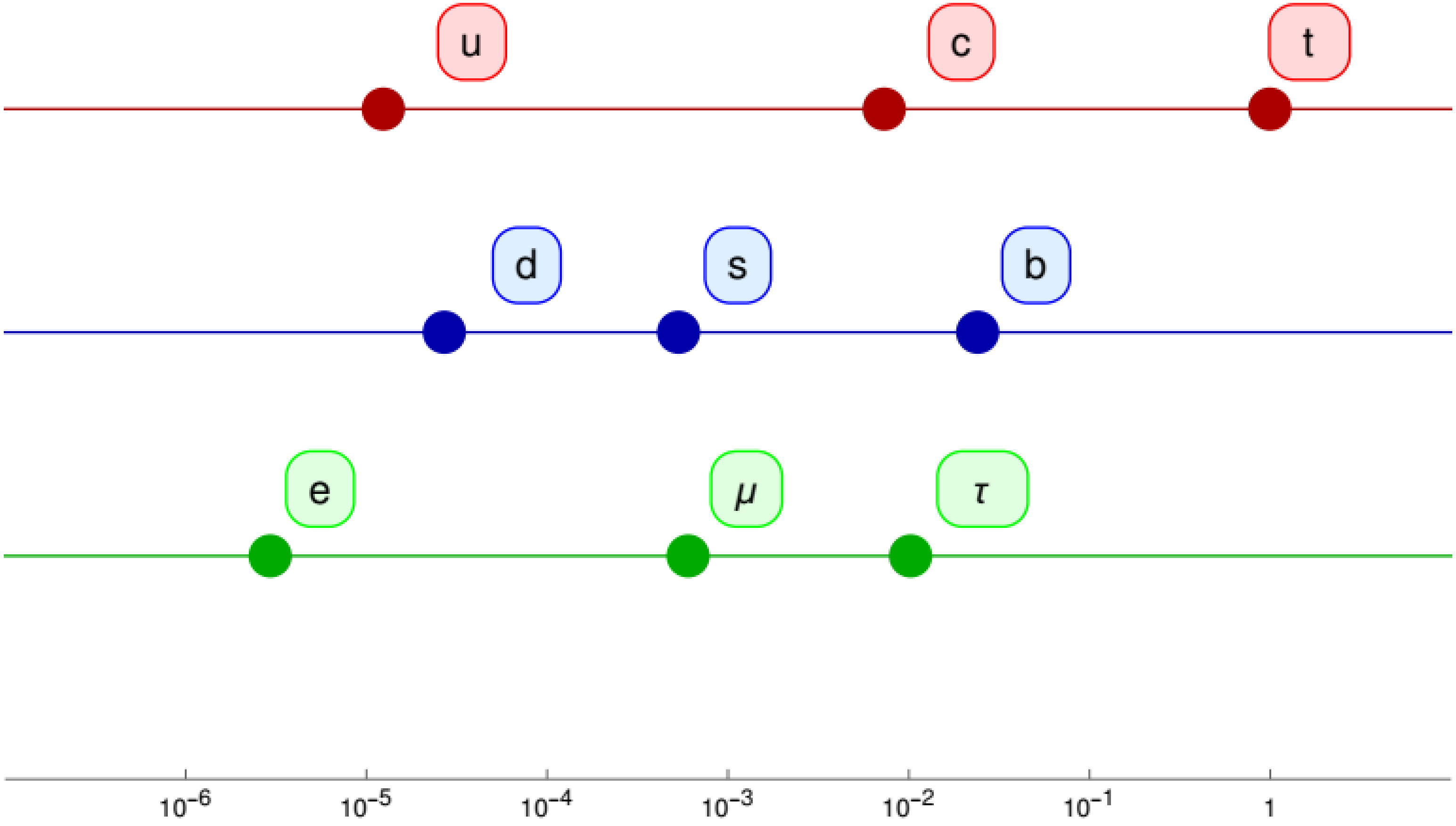
with Dipankar Das and Anugrah M. Prasad

Outline

- Motivation – (Half) The Flavour Puzzle
- Recipe
- Model
- Plots
- Conclusions

(Half) the Flavour Puzzle

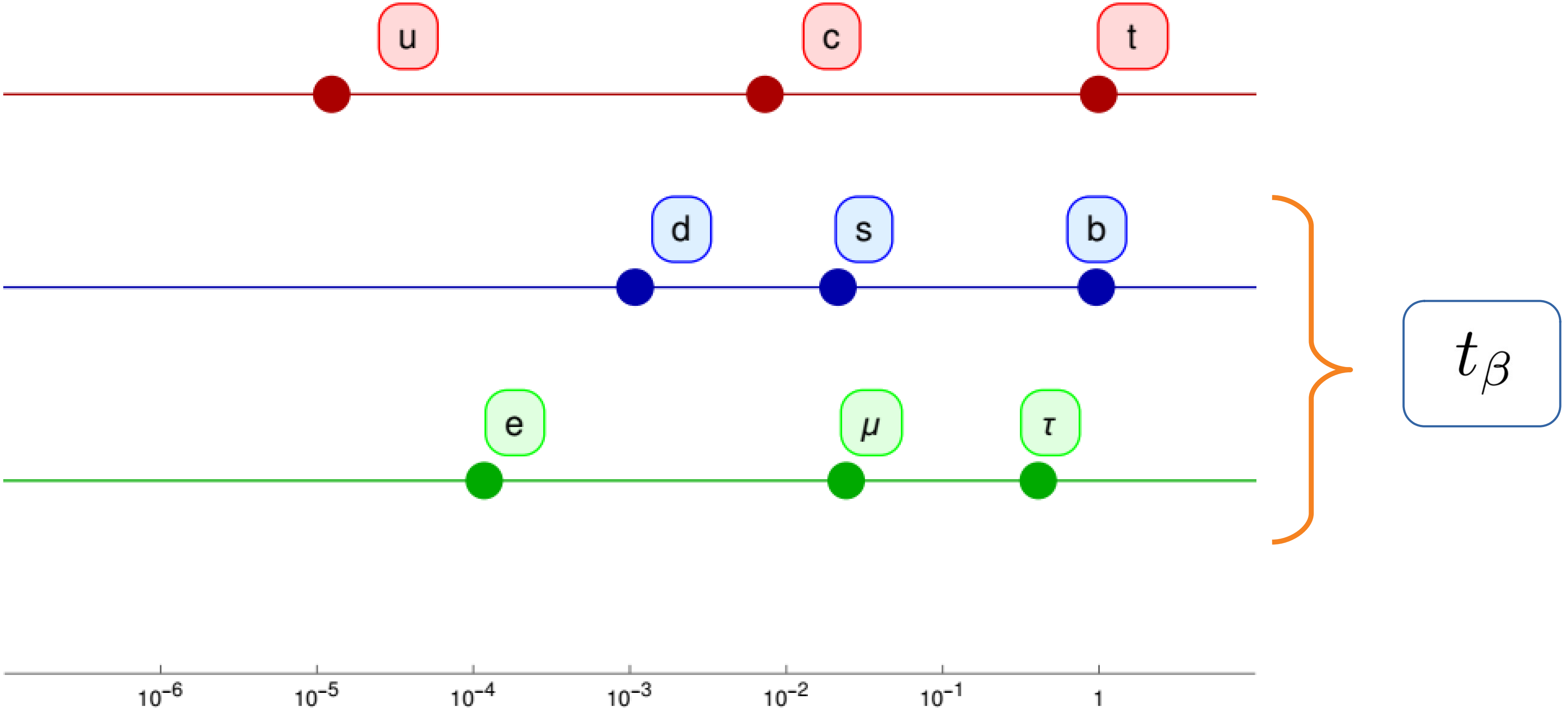
- SM



(Half) the Flavour Puzzle

- Type-II 2HDMs

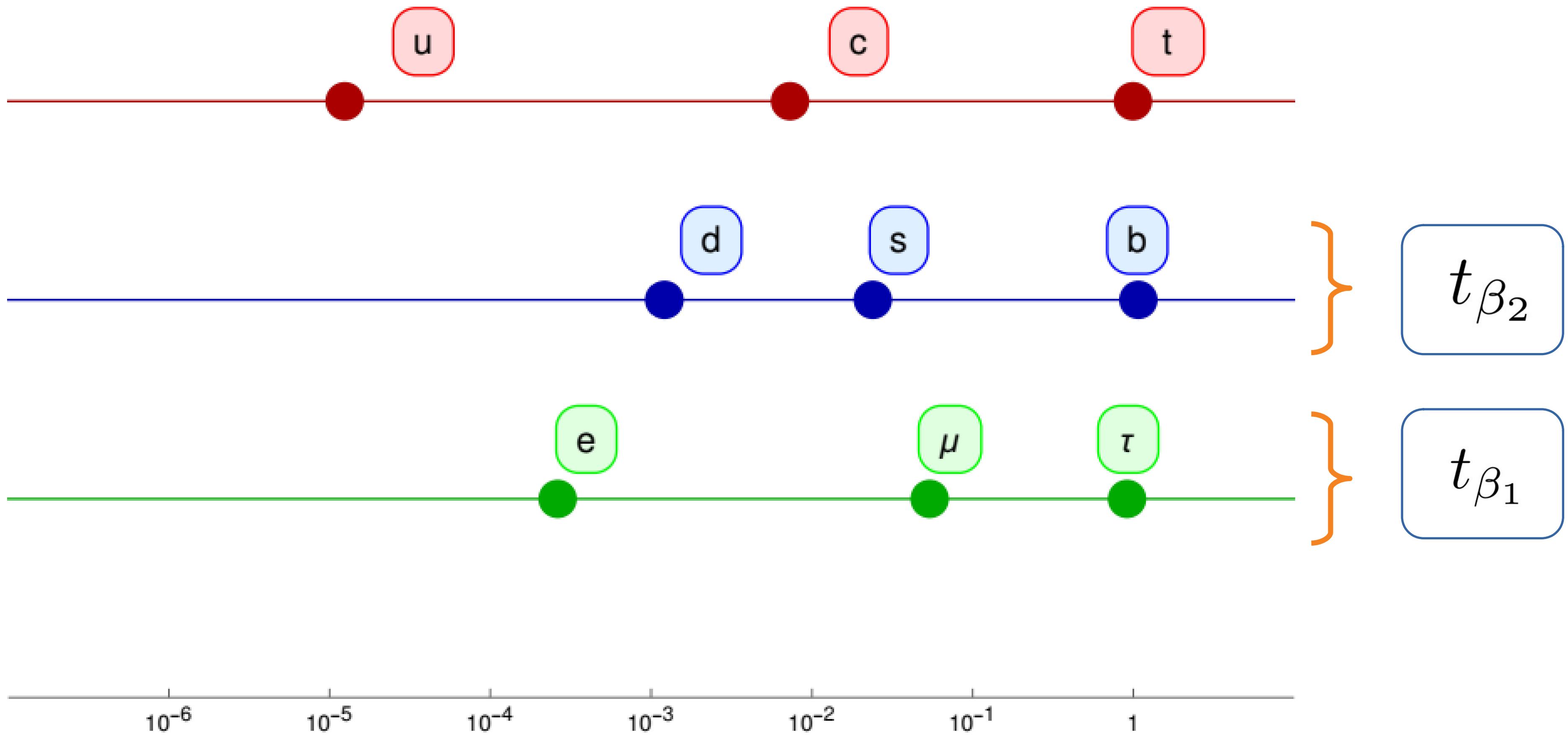
$t_\beta = 40$



(Half) the Flavour Puzzle

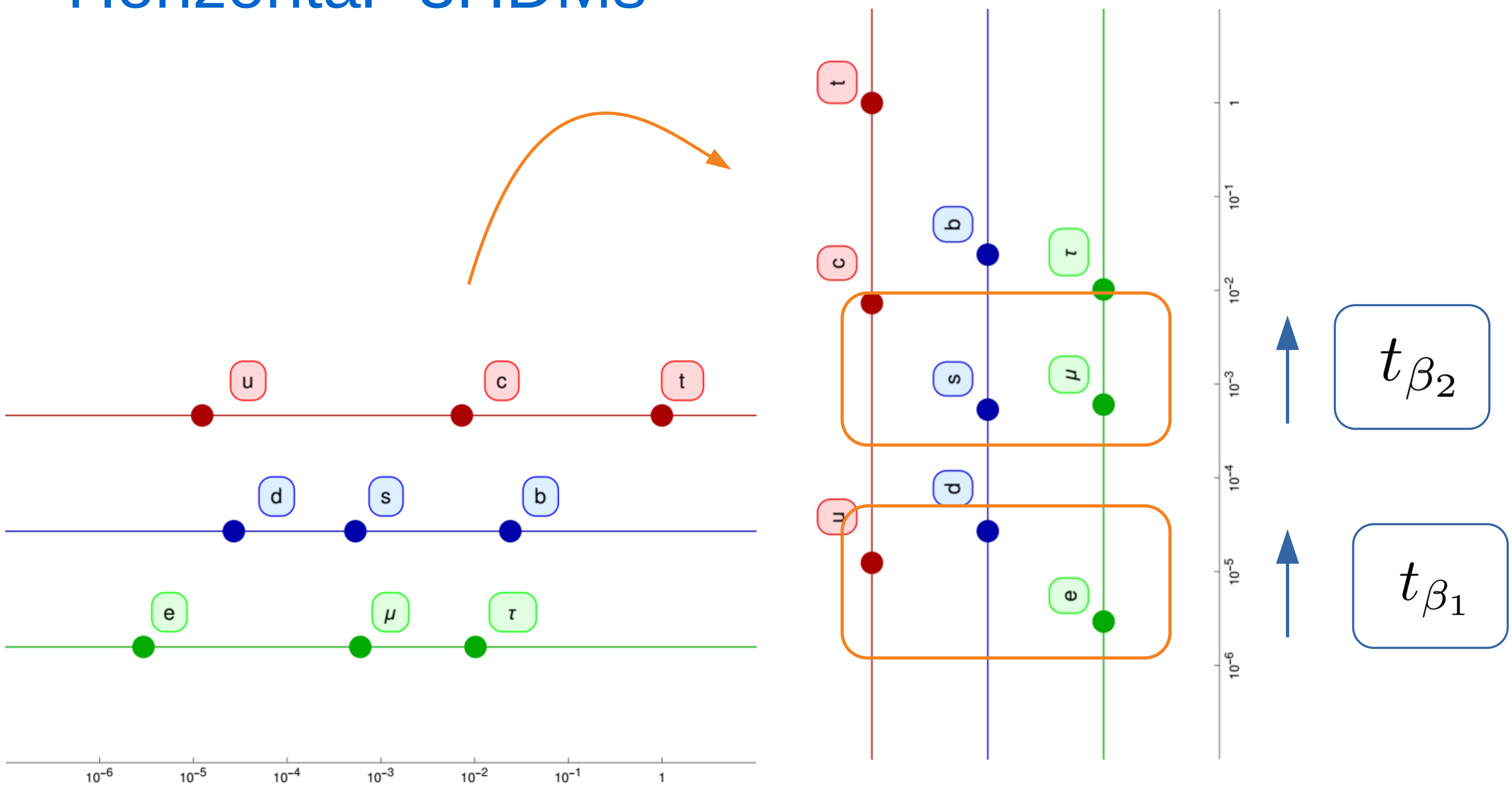
- Democratic (Type-Z) 3HDMs

$$t_{\beta_2} = 40$$
$$t_{\beta_1} = 2$$



(Half) the Flavour Puzzle

- “Horizontal” 3HDMs

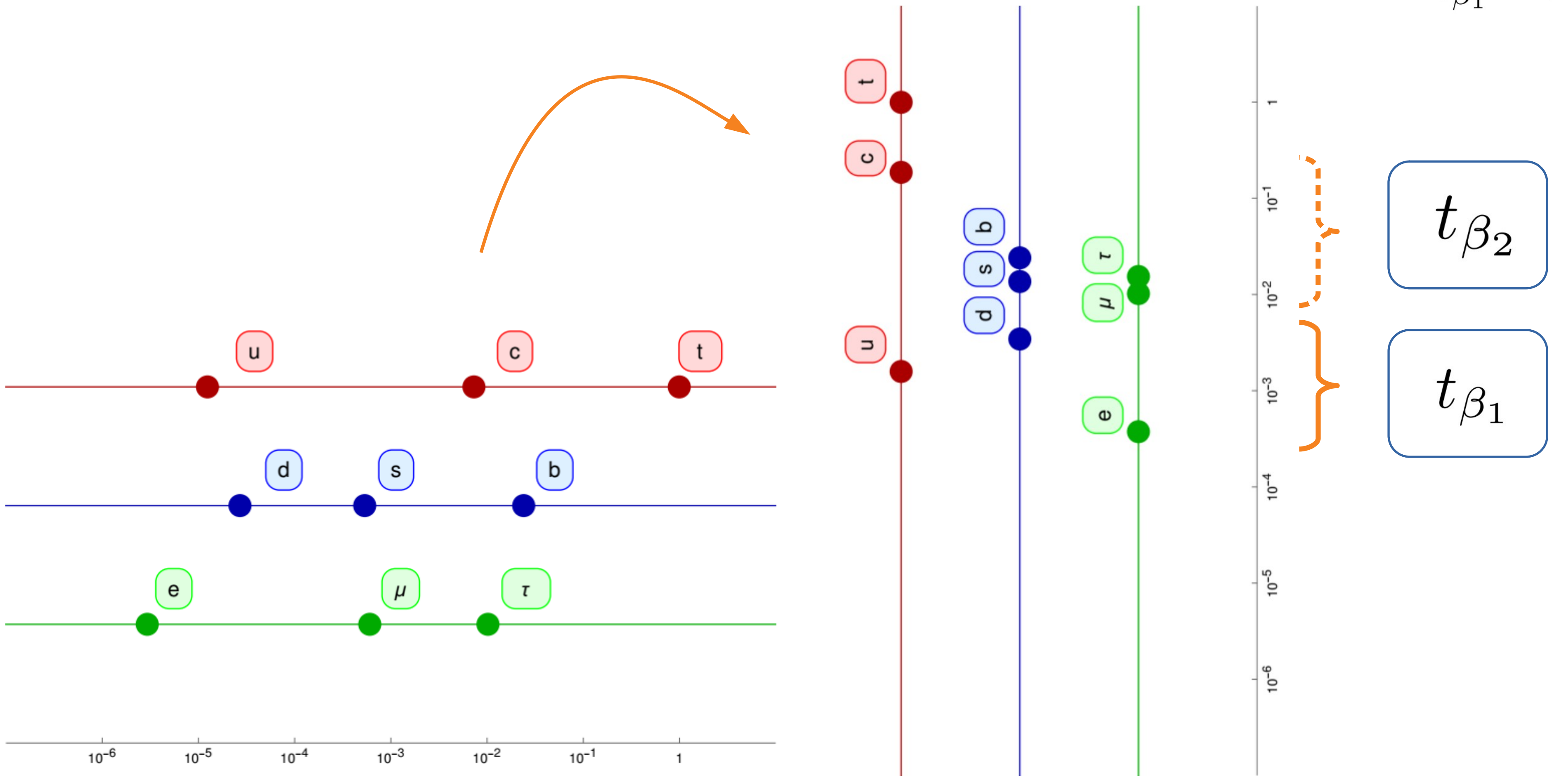


(Half) the Flavour Puzzle

- "Horizontal" 3HDMs

$t_{\beta_2} = 25$

$t_{\beta_1} = 5$



Recipe

Recipe

- Ingredients
  - 3 flavours of fermions
  - 3 flavours of Higgs
  - Flavour symmetry *to taste*
- Goal
  - Disentangling masses by flavour

(Ideally) $m_1 = y_1 v_1 \,, \qquad m_2 = y_2 v_2 \,, \qquad m_3 = y_3 v_3$

Recipe

- Instructions
  - Choose a sector to be diagonal

    (ups vs downs)
  - Ignore neutrinos

    (and charged-leptons?)
  - Rows *vs* Columns

Recipe

- Instructions
  - Choose a sector to be diagonal

(ups vs downs)

Non-diagonal Sector

$$M_u = \begin{pmatrix} \longleftarrow & v_1 & \longrightarrow \\ \longleftarrow & v_2 & \longrightarrow \\ \longleftarrow & v_3 & \longrightarrow \end{pmatrix}$$

Diagonal Sector

$$M_d = \begin{pmatrix} y_1 v_1 & & \\ & y_2 v_2 & \\ & & y_3 v_3 \end{pmatrix}$$

$$D_u^2 = V_{\mathrm{CKM}}^\dagger H_u V_{\mathrm{CKM}}$$

Recipe

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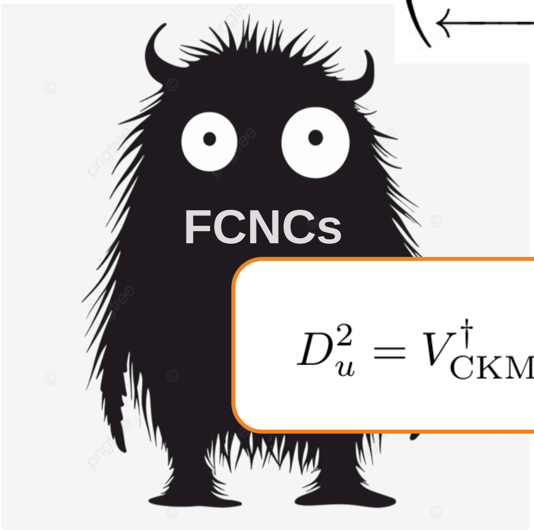

$$D_u^2 = V_{\rm CKM}^\dagger H_u V_{\rm CKM}$$

Recipe

- Instructions
  - Choose a sector to be diagonal

    (downs and charged-leptons)
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    ~~(and charged-leptons?)~~
  - Rows *vs* Columns

Recipe

- Instructions
  - Choose a sector to be diagonal ✓

    (downs and charged-leptons)
  - Ignore neutrinos ✓

    ~~(and charged-leptons?)~~
  - Rows *vs* Columns

$$M_u = \begin{pmatrix} \longleftarrow & v_1 & \longrightarrow \\ \longleftarrow & v_2 & \longrightarrow \\ \longleftarrow & v_3 & \longrightarrow \end{pmatrix} \rightarrow H_u = \begin{pmatrix} v_1^2 & v_1 v_2 & v_1 v_3 \\ v_1 v_2 & v_2^2 & v_2 v_3 \\ v_1 v_3 & v_2 v_3 & v_3^2 \end{pmatrix} \qquad M_u = \begin{pmatrix} \uparrow & \uparrow & \uparrow \\ v_1 & v_2 & v_3 \\ \downarrow & \downarrow & \downarrow \end{pmatrix} \rightarrow H_u = \begin{pmatrix} v^2 & v^2 & v^2 \\ v^2 & v^2 & v^2 \\ v^2 & v^2 & v^2 \end{pmatrix}$$

Recipe

- Instructions
  - Choose a sector to be diagonal ✓

    (downs and charged-leptons)
  - Ignore neutrinos ✓

    ~~(and charged-leptons?)~~ ✓
  - Rows *vs* Columns

$$M_u = \begin{pmatrix} \longleftarrow & v_1 & \longrightarrow \\ \longleftarrow & v_2 & \longrightarrow \\ \longleftarrow & v_3 & \longrightarrow \end{pmatrix} \rightarrow H_u = \begin{pmatrix} v_1^2 & v_1 v_2 & v_1 v_3 \\ v_1 v_2 & v_2^2 & v_2 v_3 \\ v_1 v_3 & v_2 v_3 & v_3^2 \end{pmatrix}$$

→

*

$$v_1 \ll v_2 \ll v_3$$

$$m_1 \approx v_1 \,, \quad m_2 \approx v_2 \,, \quad m_3 \approx v_3$$

* terms and conditions may apply

The Model

BGL-3HDMs

Flavour Symmetry

$$U(1)_1 \times U(1)_2$$

$$U(1)_1: \quad (Q_L)_1 \to e^{-i\psi_1}(Q_L)_1\,, \quad (n_R)_1 \to e^{-2i\psi_1}(n_R)_1\,, \quad \phi_1 \to e^{i\psi_1}\phi_1$$
$$U(1)_2: \quad (Q_L)_2 \to e^{-i\psi_2}(Q_L)_2\,, \quad (n_R)_2 \to e^{-2i\psi_2}(n_R)_2\,, \quad \phi_2 \to e^{i\psi_2}\phi_2$$

$$U(1)_1: \quad (L_L)_1 \to e^{-i\psi_1}(L_L)_1\,, \quad (\ell_R)_1 \to e^{-2i\psi_1}(\ell_R)_1$$
$$U(1)_2: \quad (L_L)_2 \to e^{-i\psi_2}(L_L)_2\,, \quad (\ell_R)_2 \to e^{-2i\psi_2}(\ell_R)_2$$

- Acts on LH and RH downs and charged-leptons
- Isolates generations
- Does not distinguish RH ups

BGL-3HDMs

Flavour Symmetry

$$U(1)_1 \times U(1)_2$$

$$\begin{aligned} U(1)_1: &\quad (Q_L)_1 \to e^{-i\psi_1}(Q_L)_1\,, \quad (n_R)_1 \to e^{-2i\psi_1}(n_R)_1\,, \quad \phi_1 \to e^{i\psi_1}\phi_1 \\ U(1)_2: &\quad (Q_L)_2 \to e^{-i\psi_2}(Q_L)_2\,, \quad (n_R)_2 \to e^{-2i\psi_2}(n_R)_2\,, \quad \phi_2 \to e^{i\psi_2}\phi_2 \end{aligned}$$

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- Acts on LH and RH downs and charged-leptons → Diagonal Mass Matrices
- Isolates generations + $M_u = \begin{pmatrix} \longleftarrow & v_1 & \longrightarrow \\ \longleftarrow & v_2 & \longrightarrow \\ \longleftarrow & v_3 & \longrightarrow \end{pmatrix}$ → Disentangles primary sources of masses
- Does not distinguish RH ups → As Economical as NFC versions

BGL-3HDMs

Flavour Symmetry

BGL

$$U(1)_1 \times U(1)_2$$

$$\begin{aligned} U(1)_1 : &\quad (Q_L)_1 \to e^{-i\psi_1}(Q_L)_1 \,, \quad (n_R)_1 \to e^{-2i\psi_1}(n_R)_1 \,, \quad \phi_1 \to e^{i\psi_1}\phi_1 \\ U(1)_2 : &\quad (Q_L)_2 \to e^{-i\psi_2}(Q_L)_2 \,, \quad (n_R)_2 \to e^{-2i\psi_2}(n_R)_2 \,, \quad \phi_2 \to e^{i\psi_2}\phi_2 \end{aligned}$$

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- Does not distinguish RH ups → As Economical as NFC versions

FCNCs only depend on Scalar Sector, quark masses, and the CKM

BGL-3HDMs

Yukawa Textures

$$v_1 = v\cos\beta_1\cos\beta_2$$
$$v_2 = v\sin\beta_1\cos\beta_2$$
$$v_3 = v\sin\beta_2$$

$$\Delta_1 = \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_2^d & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3^d \end{pmatrix}$$

similar for charged-leptons

FCNC matrices (Higgs Basis)

$$N_1^d = \mathrm{diag}\left(\frac{m_d(O_\beta)_{21}}{v_1}, \frac{m_s(O_\beta)_{22}}{v_2}, \frac{m_b(O_\beta)_{23}}{v_3}\right) = \mathrm{diag}\left(-\frac{m_d}{v}\frac{t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}, \frac{m_s}{v}\frac{t_{\beta_2}}{t_{\beta_1}s_{\beta_2}}, 0\right)$$

$$N_2^d = \mathrm{diag}\left(\frac{m_d(O_\beta)_{31}}{v_1}, \frac{m_s(O_\beta)_{32}}{v_2}, \frac{m_b(O_\beta)_{33}}{v_3}\right) = \mathrm{diag}\left(-\frac{m_d}{v}t_{\beta_2}, -\frac{m_s}{v}t_{\beta_2}, \frac{m_b}{v}\frac{1}{t_{\beta_2}}\right)$$

$$(N_1^u)_{ab} = \sum_{k=1}^{3}\frac{(O_\beta)_{2k}}{v_k}(V)_{ak}(V)^*_{bk}(D_u)_{bb} = \left(\frac{-t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}(V)_{a1}(V)^*_{b1} + \frac{t_{\beta_2}}{t_{\beta_1}s_{\beta_2}}(V)_{a2}(V)^*_{b2}\right)\frac{(D_u)_{bb}}{v}$$

$$(N_2^u)_{ab} = \sum_{k=1}^{3}\frac{(O_\beta)_{3k}}{v_k}(V)_{ak}(V)^*_{bk}(D_u)_{bb} = \left(-t_{\beta_2}(V)_{a1}(V)^*_{b1} - t_{\beta_2}(V)_{a2}(V)^*_{b2} + \frac{1}{t_{\beta_2}}(V)_{a3}(V)^*_{b3}\right)\frac{(D_u)_{bb}}{v}$$

BGL-3HDMs

Yukawa Textures

$$v_1 = v\cos\beta_1\cos\beta_2$$
$$v_2 = v\sin\beta_1\cos\beta_2$$
$$v_3 = v\sin\beta_2$$

$$\Delta_1 = \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_2^d & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3^d \end{pmatrix}$$

similar for charged-leptons

FCNC matrices (Higgs Basis)

$$N_1^d = \text{diag}\left(\frac{m_d(O_\beta)_{21}}{v_1}, \frac{m_s(O_\beta)_{22}}{v_2}, \frac{m_b(O_\beta)_{23}}{v_3}\right) = \text{diag}\left(-\frac{m_d}{v}\frac{t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}, \frac{m_s}{v}\frac{t_{\beta_2}}{t_{\beta_1}s_{\beta_2}}, 0\right)$$
$$N_2^d = \text{diag}\left(\frac{m_d(O_\beta)_{31}}{v_1}, \frac{m_s(O_\beta)_{32}}{v_2}, \frac{m_b(O_\beta)_{33}}{v_3}\right) = \text{diag}\left(-\frac{m_d}{v}t_{\beta_2}, -\frac{m_s}{v}t_{\beta_2}, \frac{m_b}{v}\frac{1}{t_{\beta_2}}\right)$$

Flavour Diagonal

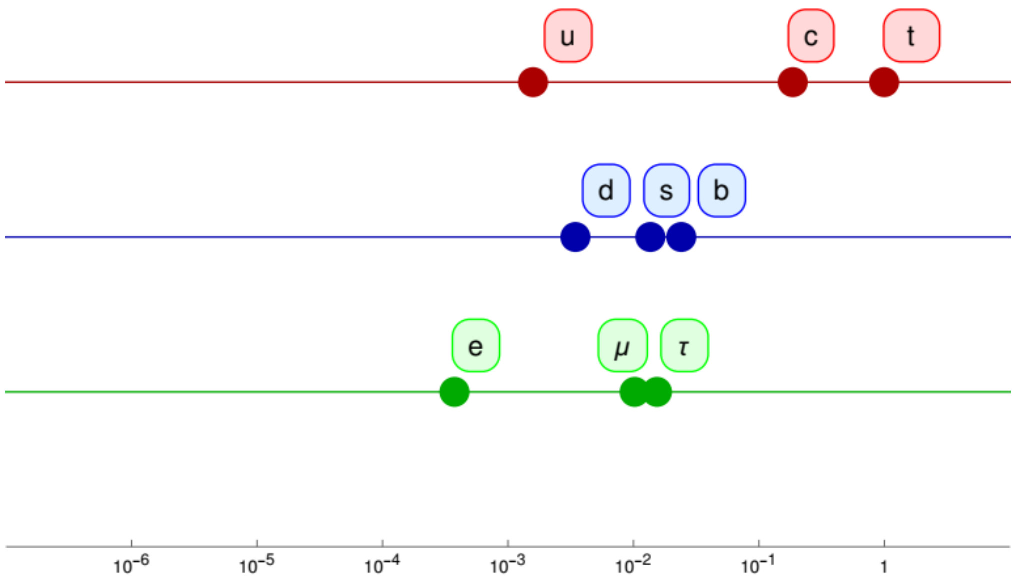
$$(N_1^u)_{ab} = \sum_{k=1}^{3}\frac{(O_\beta)_{2k}}{v_k}(V)_{ak}(V)^*_{bk}(D_u)_{bb} = \left(\frac{-t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}(V)_{a1}(V)^*_{b1} + \frac{t_{\beta_2}}{t_{\beta_1}s_{\beta_2}}(V)_{a2}(V)^*_{b2}\right)\frac{(D_u)_{bb}}{v}$$
$$(N_2^u)_{ab} = \sum_{k=1}^{3}\frac{(O_\beta)_{3k}}{v_k}(V)_{ak}(V)^*_{bk}(D_u)_{bb} = \left(-t_{\beta_2}(V)_{a1}(V)^*_{b1} - t_{\beta_2}(V)_{a2}(V)^*_{b2} + \frac{1}{t_{\beta_2}}(V)_{a3}(V)^*_{b3}\right)\frac{(D_u)_{bb}}{v}$$

Not Flavour Diagonal

Tree-level FCNCs
in the up sector

BGL-3HDMs

Perturbative Region



Not the Whole Picture

BGL-3HDMs

Perturbative Region

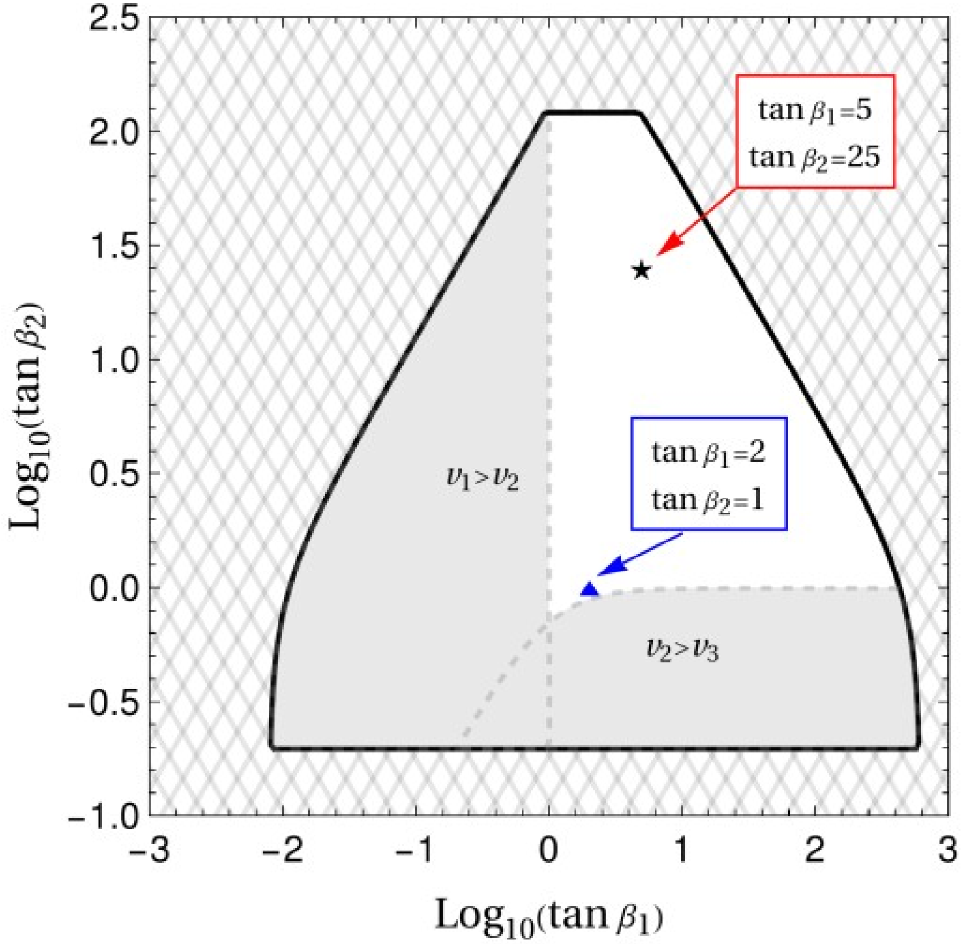
$$\left|N_{1,2}^{u,d}\right|_{ab} \leq \sqrt{4\pi}$$

- Large Hierarchies:
  Favoured by Flavour Puzzle

$\tan\beta_1=5$
$\tan\beta_2=25$

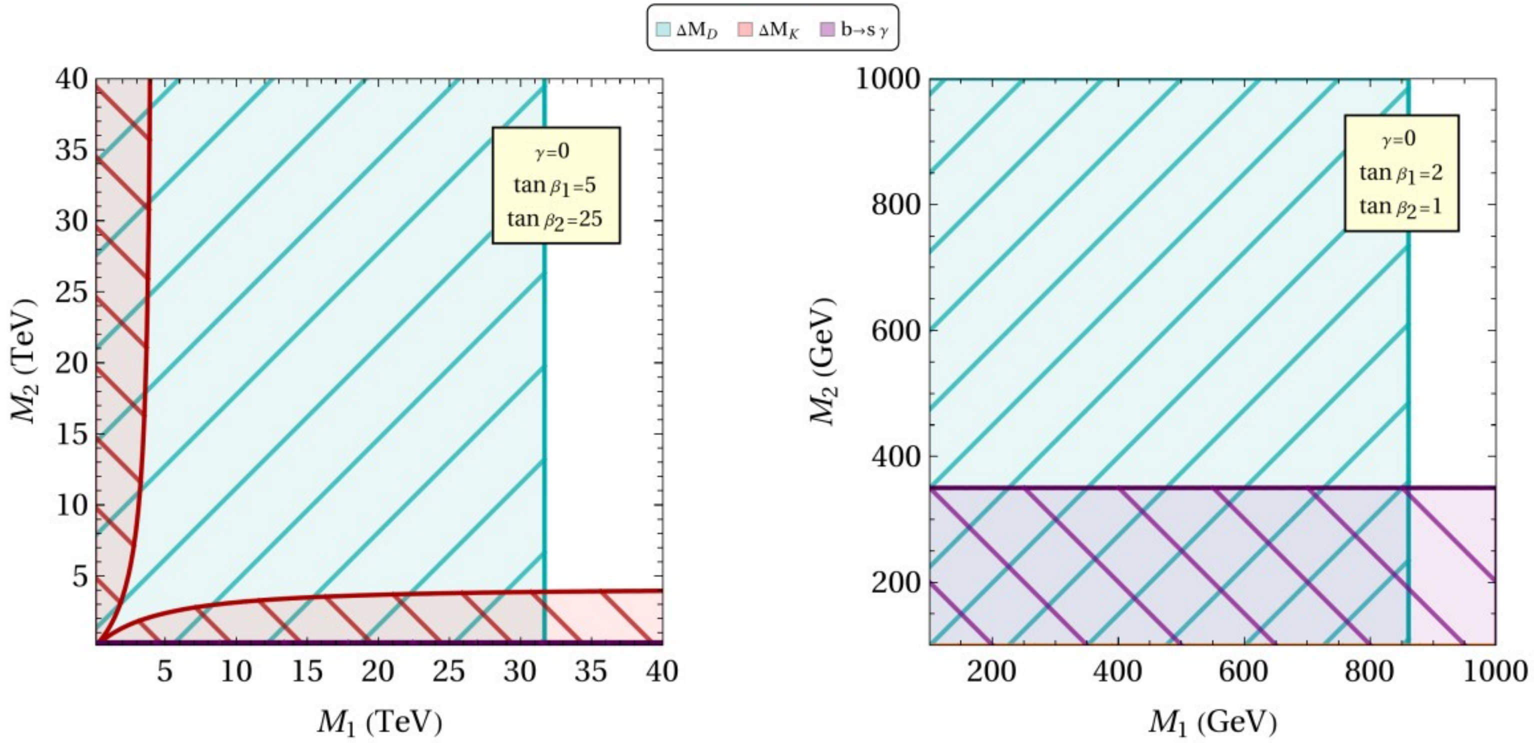
- Milder Hierarchies:
  Favoured by Flavour Data

$\tan\beta_1=2$
$\tan\beta_2=1$



BGL-3HDMs

Mass-Mass Plot



BGL-3HDMs

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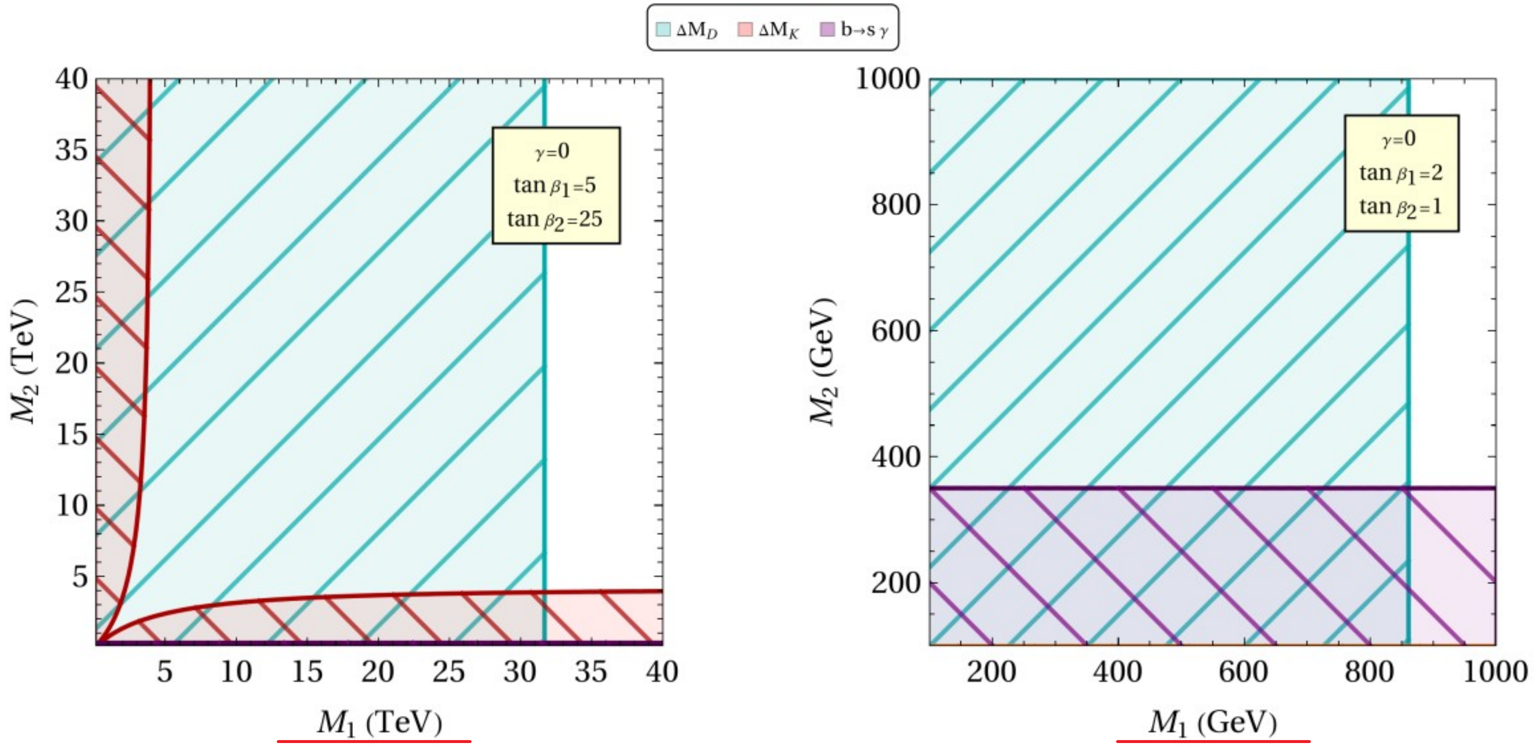


Favoured by
Flavour Puzzle

Favoured by
Flavour Data

BGL-3HDMs

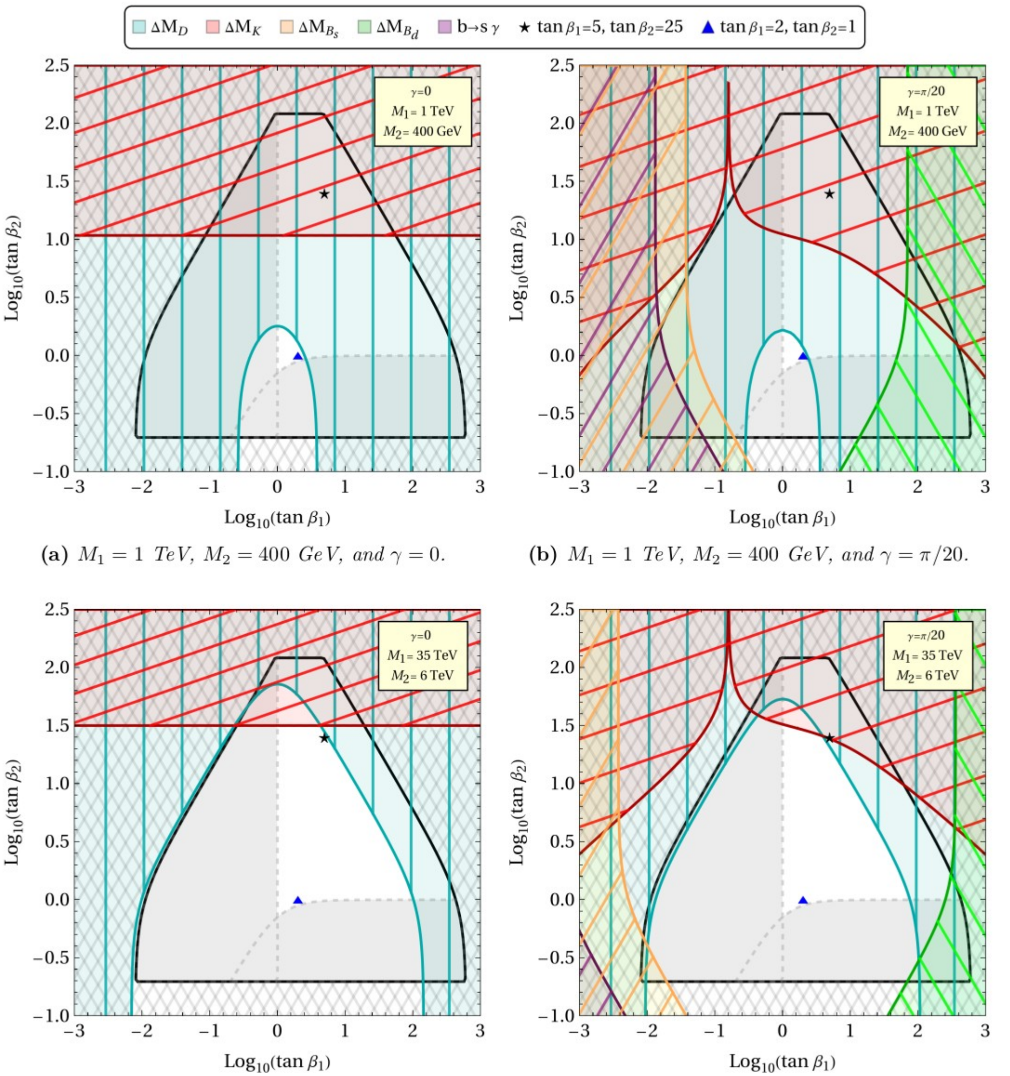
Benchmarks

- γ Controls the mixing between H_1 and H_2

- Cancellation in $(N_2^u)_{12}$

$$(N_2^u)_{12} = \boxed{\frac{m_c V_{ub} V_{cb}^*}{t_{\beta_2} v}} - \frac{m_c t_{\beta_2}}{v} \left(V_{ud} V_{cd}^* + V_{us} V_{cs}^*\right)$$

$$\left(V_{ud} V_{cd}^* + V_{us} V_{cs}^*\right) \to 0$$



(a) $M_1 = 1\ TeV$, $M_2 = 400\ GeV$, and $\gamma = 0$.

(b) $M_1 = 1\ TeV$, $M_2 = 400\ GeV$, and $\gamma = \pi/20$.

(c) $M_1 = 35\ TeV$, $M_2 = 6\ TeV$, and $\gamma = 0$.

(d) $M_1 = 35\ TeV$, $M_2 = 6\ TeV$, and $\gamma = \pi/20$.

Conclusions

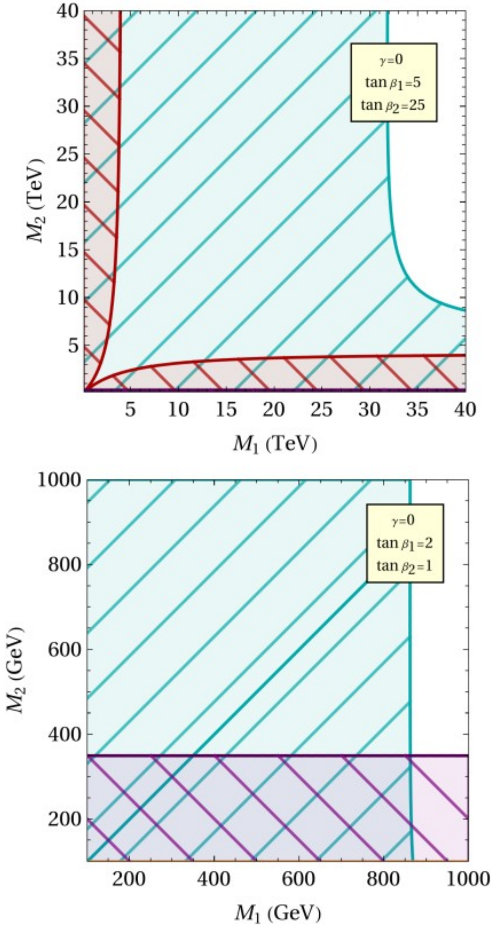
- Interesting Route for Flavour Puzzle
  - Generation-wise sources of mass
- Flavour Constraints in tension with Flavour Puzzle
- Flavour non-universal nonstandard Higgs Couplings as a side effect

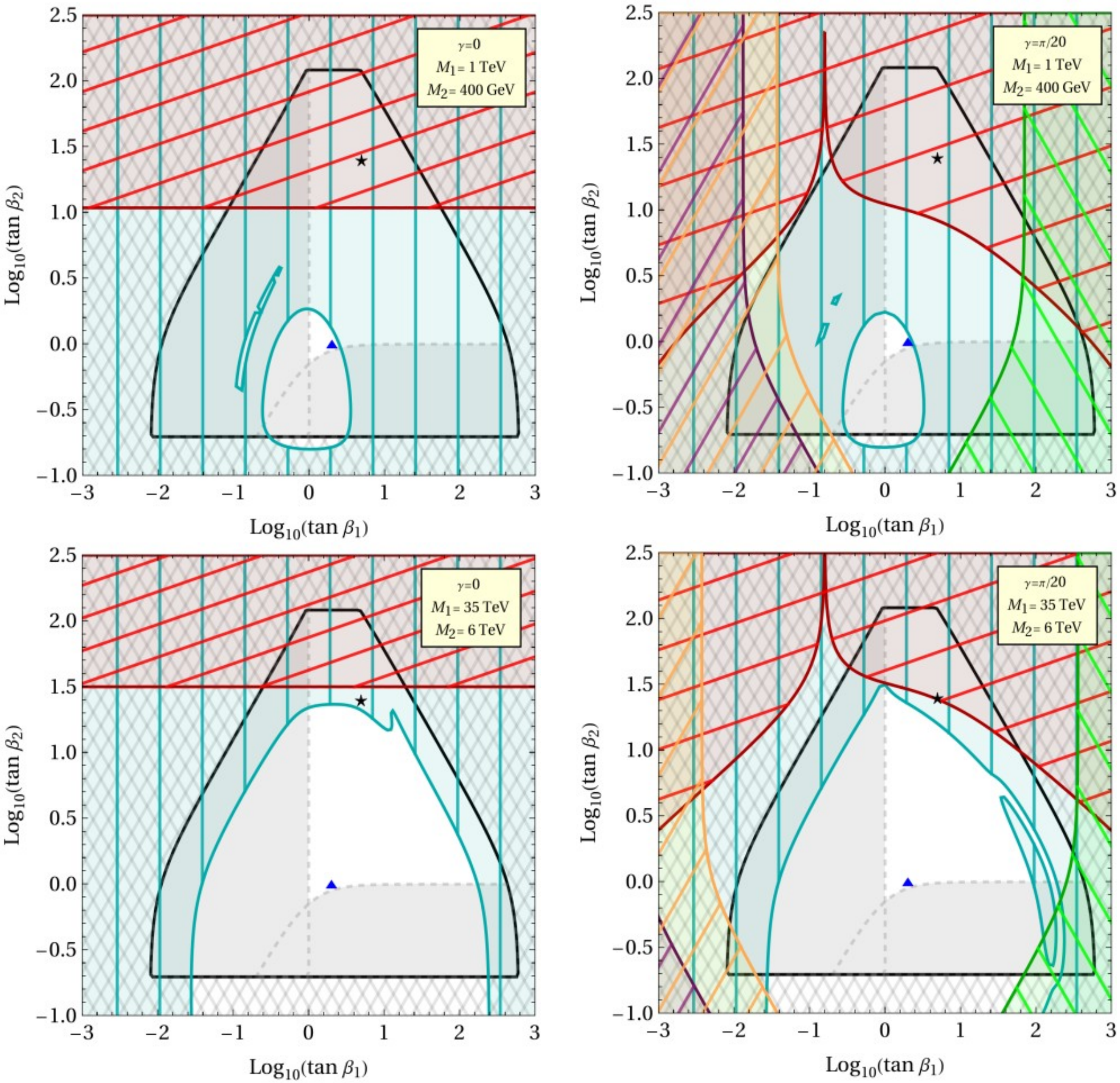
Thank You

Back Up

Back Up

Including 1-loop D mixing





Back Up

Scalar Sector

- Scalar Potential

$$V_{U(1)\times U(1)} = m_{11}^2\phi_1^\dagger\phi_1 + m_{22}^2\phi_2^\dagger\phi_2 + m_{33}^2\phi_3^\dagger\phi_3 - \left(m_{12}^2\phi_1^\dagger\phi_2 + m_{13}^2\phi_1^\dagger\phi_3 + m_{23}^2\phi_2^\dagger\phi_3 + \text{h.c.}\right)$$
$$+\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_3^\dagger\phi_3)^2 + \lambda_4(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_5(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3)$$
$$+\lambda_6(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) + \lambda_7(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda_8(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda_9(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2)\,,$$

- Higgs Basis

$$\begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix} = O_\beta \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A_1' \\ A_2' \end{pmatrix} = O_\beta \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ H_1'^\pm \\ H_2'^\pm \end{pmatrix} = O_\beta \begin{pmatrix} w_1^\pm \\ w_2^\pm \\ w_3^\pm \end{pmatrix}$$

$$O_\beta = \begin{pmatrix} \cos\beta_2\cos\beta_1 & \cos\beta_2\sin\beta_1 & \sin\beta_2 \\ -\sin\beta_1 & \cos\beta_1 & 0 \\ -\cos\beta_1\sin\beta_2 & -\sin\beta_1\sin\beta_2 & \cos\beta_2 \end{pmatrix}$$

- Mass Basis

$$\begin{pmatrix} G^\pm \\ H_1^\pm \\ H_2^\pm \end{pmatrix} = O_{\gamma_2} \begin{pmatrix} G^\pm \\ H_1'^\pm \\ H_2'^\pm \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A_1 \\ A_2 \end{pmatrix} = O_{\gamma_1} \begin{pmatrix} G^0 \\ A_1' \\ A_2' \end{pmatrix}, \qquad \begin{pmatrix} h \\ H_1 \\ H_2 \end{pmatrix} = O_\alpha O_\beta^T \begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix}$$

- Alignment Limit

$$\alpha_1 = \beta_1 \quad \text{and} \quad \alpha_2 = \beta_2$$

Custodial Limit

$$m_{H_1} = m_{A_1} = m_{C_1} \equiv M_1\,, \quad m_{H_2} = m_{A_2} = m_{C_2} \equiv M_2\,, \quad \alpha_3 = \gamma_1 = \gamma_2 \equiv \gamma$$

Back Up

Expressions for $b \to s\gamma$

- Typical Stuff

$$C_{7L} = A_\gamma^{\rm SM} + A_{\gamma L}^+\,, \qquad C_{7R} = \frac{m_s}{m_b} A_\gamma^{\rm SM} + A_{\gamma R}^+$$
$$C_{8L} = A_g^{\rm SM} + A_{gL}^+\,, \qquad C_{8R} = \frac{m_s}{m_b} A_g^{\rm SM} + A_{gR}^+$$

$$C_{7L}^{\rm eff} = \eta^{16/23} C_{7L} + \frac{8}{3}\left(\eta^{14/23} - \eta^{16/23}\right) C_{8L} + \sum_{i=1}^{8} h_i \eta^{a_i}$$
$$C_{7R}^{\rm eff} = \eta^{16/23} C_{7R} + \frac{8}{3}\left(\eta^{14/23} - \eta^{16/23}\right) C_{8R}\,,$$

$$A_{\gamma L,R}^+ = \frac{1}{V_{ts}^* V_{tb}} \sum_{q=u,c,t} V_{qs}^* V_{qb} \sum_{k=1,2} \left[C_{1L,R}^k(y_q^k) + \frac{2}{3} C_{2L,R}^k(y_q^k) \right]$$
$$A_{gL,R}^+ = \frac{1}{V_{ts}^* V_{tb}} \sum_{q=u,c,t} V_{qs}^* V_{qb} \sum_{k=1,2} C_{2L,R}^k(y_q^k)\,,$$

- Model-Specific

$$C_{1L,R}^{k,q}(y_q) = \frac{y_q}{4}\Bigg\{ \left[\overline{\mathcal{F}_2}(y_q) - \overline{\mathcal{F}_1}(y_q) \right] \left(\frac{m_{s,b}^2}{m_q^2} (Y_k)_{qb} (Y_k)_{qs}^* + (X_k)_{qb} (X_k)_{qs}^* \right) + \left[\overline{\mathcal{F}_1}(y_q) - \overline{\mathcal{F}_0}(y_q) \right] \left((X_k)_{qb} (Y_k)_{qs}^* + (X_k)_{qs}^* (Y_k)_{qb} \right) \Bigg\}\,,$$
$$C_{2L,R}^{k,q}(y_q) = \frac{y_q}{4}\Bigg\{ \left[\mathcal{F}_2(y_q) - \mathcal{F}_1(y_q) \right] \left(\frac{m_{s,b}^2}{m_q^2} (Y_k)_{qb} (Y_k)_{qs}^* + (X_k)_{qb} (X_k)_{qs}^* \right) - \mathcal{F}_1(y_q) \left((X_k)_{qb} (Y_k)_{qs}^* + (X_k)_{qs}^* (Y_k)_{qb} \right) \Bigg\}\,.$$

$$(X_k')_{q\beta} = \frac{v (N_k^{u\dagger} V)_{q\beta}}{m_q V_{q\beta}}$$
$$(Y_k')_{q\beta} = -\frac{v (V N_k^d)_{q\beta}}{m_\beta V_{q\beta}}$$

Back Up

Expressions for ΔM_P

- Tree-Level

$$C_2 = \sum_k \frac{-1}{2M^2_{S^0_k}} \left(\Gamma^L_{\overline{q}_1 q_2 S^0_k}\right)^2, \quad C_2' = \sum_k \frac{-1}{2M^2_{S^0_k}} \left(\Gamma^R_{\overline{q}_1 q_2 S^0_k}\right)^2, \quad C_4 = \sum_k \frac{-1}{M^2_{S^0_k}} \left(\Gamma^L_{\overline{q}_1 q_2 S^0_k} \Gamma^R_{\overline{q}_1 q_2 S^0_k}\right)$$

- Gauge-Charged

$$C_1 = \frac{1}{16\pi^2}\frac{\Gamma(4)}{6}\sum_i\sum_{a,b}\left(\Gamma^L_{\overline{q}_1 f_a W^\mp}\right)\left(\Gamma^L_{q_2\overline{f}_b W^\pm}\right)\left(\Gamma^L_{q_2\overline{f}_b H^\pm_i}\right)\left(\Gamma^R_{\overline{q}_1 f_a H^\mp_i}\right) m_a m_b f_1(M^2_W, M^2_{H^\pm_i}, m^2_a, m^2_b)$$

$$C_4 = \frac{-1}{16\pi^2}\frac{\Gamma(4)}{3}\sum_i\sum_{a,b}\left(\Gamma^L_{\overline{q}_1 f_a W^\mp}\right)\left(\Gamma^L_{q_2\overline{f}_b W^\pm}\right)\left(\Gamma^R_{q_2\overline{f}_b H^\pm_i}\right)\left(\Gamma^L_{\overline{q}_1 f_a H^\mp_i}\right) f_2(M^2_W, M^2_{H^\pm_i}, m^2_a, m^2_b)$$

- Charged-Charged

$$C_1 = \frac{1}{16\pi^2}\frac{\Gamma(4)}{24}\sum_{i,j}\sum_{a,b}\left(\Gamma^R_{\overline{q}_1 f_a H^\mp_i}\right)\left(\Gamma^L_{\overline{f}_a q_2 H^\pm_j}\right)\left(\Gamma^R_{\overline{q}_1 f_b H^\mp_j}\right)\left(\Gamma^L_{q_2\overline{f}_b H^\pm_i}\right) f_2(M^2_{H^\pm_j}, M^2_{H^\pm_j}, m^2_a, m^2_b),$$

$$C_2 = \frac{-1}{16\pi^2}\frac{\Gamma(4)}{12}\sum_{i,j}\sum_{a,b}\left(\Gamma^L_{\overline{q}_1 f_a H^\mp_i}\right)\left(\Gamma^L_{\overline{f}_a q_2 H^\pm_j}\right)\left(\Gamma^L_{\overline{q}_1 f_b H^\mp_j}\right)\left(\Gamma^L_{q_2\overline{f}_b H^\pm_i}\right) m_a m_b f_1(M^2_{H^\pm_i}, M^2_{H^\pm_j}, m^2_a, m^2_b),$$

$$C_4 = \frac{-1}{8\pi^2}\frac{\Gamma(4)}{12}\sum_{i,j}\sum_{a,b}\left(\Gamma^L_{\overline{q}_1 f_a H^\mp_i}\right)\left(\Gamma^L_{\overline{f}_a q_2 H^\pm_j}\right)\left(\Gamma^R_{\overline{q}_1 f_b H^\mp_j}\right)\left(\Gamma^R_{q_2\overline{f}_b H^\pm_i}\right) m_a m_b f_1(M^2_{H^\pm_i}, M^2_{H^\pm_j}, m^2_a, m^2_b),$$

$$C_5 = \frac{-1}{8\pi^2}\frac{\Gamma(4)}{12}\sum_{i,j}\sum_{a,b}\left(\Gamma^R_{\overline{q}_1 f_a H^\mp_i}\right)\left(\Gamma^L_{\overline{f}_a q_2 H^\pm_j}\right)\left(\Gamma^L_{\overline{q}_1 f_b H^\mp_j}\right)\left(\Gamma^R_{q_2\overline{f}_b H^\pm_i}\right) f_2(M^2_{H^\pm_i}, M^2_{H^\pm_j}, m^2_a, m^2_b).$$

- Neutral Boxes

$$C_1 = \frac{1}{16\pi^2}\frac{\Gamma(4)}{24}\sum_{k_1,k_2}\sum_{c,d}\left(\Gamma^R_{\overline{q}_1 f_c S^0_{k_1}}\right)\left(\Gamma^L_{q_2\overline{f}_c S^0_{k_2}}\right)\left(\Gamma^R_{\overline{q}_1 f_d S^0_{k_2}}\right)\left(\Gamma^L_{q_2\overline{f}_d S^0_{k_1}}\right) f_2(M^2_{S^0_{k_1}}, M^2_{S^0_{k_2}}, m^2_c, m^2_d),$$

$$C_2 = \frac{-1}{16\pi^2}\frac{\Gamma(4)}{12}\sum_{k_1,k_2}\sum_{c,d}\left(\Gamma^L_{\overline{q}_1 f_c S^0_{k_1}}\right)\left(\Gamma^L_{q_2\overline{f}_c S^0_{k_2}}\right)\left(\Gamma^L_{\overline{q}_1 f_d S^0_{k_2}}\right)\left(\Gamma^L_{q_2\overline{f}_d S^0_{k_1}}\right) m_c m_d f_1(M^2_{S^0_{k_1}}, M^2_{S^0_{k_2}}, m^2_c, m^2_d),$$

$$C_4 = \frac{-1}{8\pi^2}\frac{\Gamma(4)}{12}\sum_{k_1,k_2}\sum_{c,d}\left(\Gamma^L_{\overline{q}_1 f_c S^0_{k_1}}\right)\left(\Gamma^L_{q_2\overline{f}_c S^0_{k_2}}\right)\left(\Gamma^R_{\overline{q}_1 f_d S^0_{k_2}}\right)\left(\Gamma^R_{q_2\overline{f}_d S^0_{k_1}}\right) m_c m_d f_1(M^2_{S^0_{k_1}}, M^2_{S^0_{k_2}}, m^2_c, m^2_d),$$

$$C_5 = \frac{-1}{8\pi^2}\frac{\Gamma(4)}{12}\sum_{k_1,k_2}\sum_{c,d}\left(\Gamma^R_{\overline{q}_1 f_c S^0_{k_1}}\right)\left(\Gamma^L_{q_2\overline{f}_c S^0_{k_2}}\right)\left(\Gamma^L_{\overline{q}_1 f_d S^0_{k_2}}\right)\left(\Gamma^R_{q_2\overline{f}_d S^0_{k_1}}\right) f_2(M^2_{S^0_{k_1}}, M^2_{S^0_{k_2}}, m^2_c, m^2_d).$$

Back Up

Expressions for ΔM_P

- Mass and Higgs Bases

$$\begin{pmatrix} G^+ \\ H_k^+ \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & R_C \end{pmatrix} \begin{pmatrix} G^+ \\ H_k'^+ \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A_k \\ h \\ H_k \end{pmatrix} = \begin{pmatrix} 1 & 0 & \\ 0 & R_A & R_X \\ & R_X' & R_H \end{pmatrix} \begin{pmatrix} G^0 \\ A_k' \\ H_0 \\ R_k \end{pmatrix}$$

- SM-Higgs Couplings

$$\Gamma^L_{\bar{d}_\alpha d_\beta h} = -\left\{ \left(R_H^\dagger\right)_{k+1,1} - i\left[\left(R_X'\right)^\dagger\right]_{k,1} \right\} \left[\left(N_k^d\right)^\dagger\right]_{\alpha\beta} - \left(R_H^\dagger\right)_{11} \frac{(D_d)_{\alpha\beta}}{v},$$

$$\Gamma^R_{\bar{d}_\alpha d_\beta h} = -\left\{ \left(R_H^\dagger\right)_{k+1,1} + i\left[\left(R_X'\right)^\dagger\right]_{k,1} \right\} \left(N_k^d\right)_{\alpha\beta} - \left(R_H^\dagger\right)_{11} \frac{(D_d)_{\alpha\beta}}{v},$$

$$\Gamma^L_{\bar{u}_\alpha u_\beta h} = -\left\{ \left(R_H^\dagger\right)_{k+1,1} + i\left[\left(R_X'\right)^\dagger\right]_{k,1} \right\} \left[(N_k^u)^\dagger\right]_{\alpha\beta} - \left(R_H^\dagger\right)_{11} \frac{(D_u)_{\alpha\beta}}{v},$$

$$\Gamma^R_{\bar{u}_\alpha u_\beta h} = -\left\{ \left(R_H^\dagger\right)_{k+1,1} - i\left[\left(R_X'\right)^\dagger\right]_{k,1} \right\} (N_k^u)_{\alpha\beta} - \left(R_H^\dagger\right)_{11} \frac{(D_u)_{\alpha\beta}}{v}.$$

- Charged-Higgs Couplings

$$\Gamma^L_{\bar{d}_\alpha u_\beta H_j^-} = -\sqrt{2}\,(R_C)_{j,k} \left[\left(N_k^d\right)^\dagger V^\dagger\right]_{\alpha\beta}, \qquad \Gamma^R_{\bar{d}_\alpha u_\beta H_j^-} = \sqrt{2}\,(R_C)_{j,k} \left[V^\dagger N_k^u\right]_{\alpha\beta},$$

$$\Gamma^L_{\bar{u}_\alpha d_\beta H_j^+} = \sqrt{2}\left(R_C^\dagger\right)_{k,j} \left[(N_k^u)^\dagger V\right]_{\alpha\beta}, \qquad \Gamma^R_{\bar{d}_\alpha u_\beta H_j^-} = -\sqrt{2}\left(R_C^\dagger\right)_{k,j} \left[V N_k^d\right]_{\alpha\beta}$$

Back Up

Expressions for ΔM_P

- Heavy Higgs Couplings

$$\Gamma^L_{\bar{d}_\alpha d_\beta H_j} = -\left\{\left(R_H^\dagger\right)_{k+1,j+1} - i\left[(R'_X)^\dagger\right]_{k,j+1}\right\}\left[\left(N_k^d\right)^\dagger\right]_{\alpha\beta} - \left(R_H^\dagger\right)_{1,j+1}\frac{(D_d)_{\alpha\beta}}{v},$$
$$\Gamma^R_{\bar{d}_\alpha d_\beta H_j} = -\left\{\left(R_H^\dagger\right)_{k+1,j+1} + i\left[(R'_X)^\dagger\right]_{k,j+1}\right\}\left(N_k^d\right)_{\alpha\beta} - \left(R_H^\dagger\right)_{1,j+1}\frac{(D_d)_{\alpha\beta}}{v},$$
$$\Gamma^L_{\bar{u}_\alpha u_\beta H_j} = -\left\{\left(R_H^\dagger\right)_{k+1,j+1} + i\left[(R'_X)^\dagger\right]_{k,j+1}\right\}\left[(N_k^u)^\dagger\right]_{\alpha\beta} - \left(R_H^\dagger\right)_{1,j+1}\frac{(D_u)_{\alpha\beta}}{v},$$
$$\Gamma^R_{\bar{u}_\alpha u_\beta H_j} = -\left\{\left(R_H^\dagger\right)_{k+1,j+1} - i\left[(R'_X)^\dagger\right]_{k,j+1}\right\}(N_k^u)_{\alpha\beta} - \left(R_H^\dagger\right)_{1,j+1}\frac{(D_u)_{\alpha\beta}}{v},$$

$$\Gamma^L_{\bar{d}_\alpha d_\beta A_j} = -\left\{\left(R_X^\dagger\right)_{k+1,j} - i\left(R_A^\dagger\right)_{k,j}\right\}\left[\left(N_k^d\right)^\dagger\right]_{\alpha\beta} - \left(R_X^\dagger\right)_{1,j}\frac{(D_d)_{\alpha\beta}}{v},$$
$$\Gamma^R_{\bar{d}_\alpha d_\beta A_j} = -\left\{\left(R_X^\dagger\right)_{k+1,j} + i\left(R_A^\dagger\right)_{k,j}\right\}\left(N_k^d\right)_{\alpha\beta} - \left(R_X^\dagger\right)_{1,j}\frac{(D_d)_{\alpha\beta}}{v}$$
$$\Gamma^L_{\bar{u}_\alpha u_\beta A_j} = -\left\{\left(R_X^\dagger\right)_{k+1,j} + i\left(R_A^\dagger\right)_{k,j}\right\}\left[(N_k^u)^\dagger\right]_{\alpha\beta} - \left(R_X^\dagger\right)_{1,j}\frac{(D_u)_{\alpha\beta}}{v},$$
$$\Gamma^R_{\bar{u}_\alpha u_\beta A_j} = -\left\{\left(R_X^\dagger\right)_{k+1,j} - i\left(R_A^\dagger\right)_{k,j}\right\}(N_k^u)_{\alpha\beta} - \left(R_X^\dagger\right)_{1,j}\frac{(D_u)_{\alpha\beta}}{v}.$$

- Alignment Limit and CP-Conservation

$$\Gamma^L_{\bar{d}_\alpha d_\beta H_j} = -\left(R_H^\dagger\right)_{k+1,j+1}\left[\left(N_k^d\right)^\dagger\right]_{\alpha\beta},$$
$$\Gamma^R_{\bar{d}_\alpha d_\beta H_j} = -\left(R_H^\dagger\right)_{k+1,j+1}\left(N_k^d\right)_{\alpha\beta},$$
$$\Gamma^L_{\bar{u}_\alpha u_\beta H_j} = -\left(R_H^\dagger\right)_{k+1,j+1}\left[(N_k^u)^\dagger\right]_{\alpha\beta},$$
$$\Gamma^R_{\bar{u}_\alpha u_\beta H_j} = -\left(R_H^\dagger\right)_{k+1,j+1}(N_k^u)_{\alpha\beta},$$

$$\Gamma^L_{\bar{d}_\alpha d_\beta A_j} = i\left(R_A^\dagger\right)_{k,j}\left[\left(N_k^d\right)^\dagger\right]_{\alpha\beta},$$
$$\Gamma^R_{\bar{d}_\alpha d_\beta A_j} = -i\left(R_A^\dagger\right)_{k,j}\left(N_k^d\right)_{\alpha\beta}$$
$$\Gamma^L_{\bar{u}_\alpha u_\beta A_j} = -i\left(R_A^\dagger\right)_{k,j}\left[(N_k^u)^\dagger\right]_{\alpha\beta},$$
$$\Gamma^R_{\bar{u}_\alpha u_\beta A_j} = i\left(R_A^\dagger\right)_{k,j}(N_k^u)_{\alpha\beta}.$$