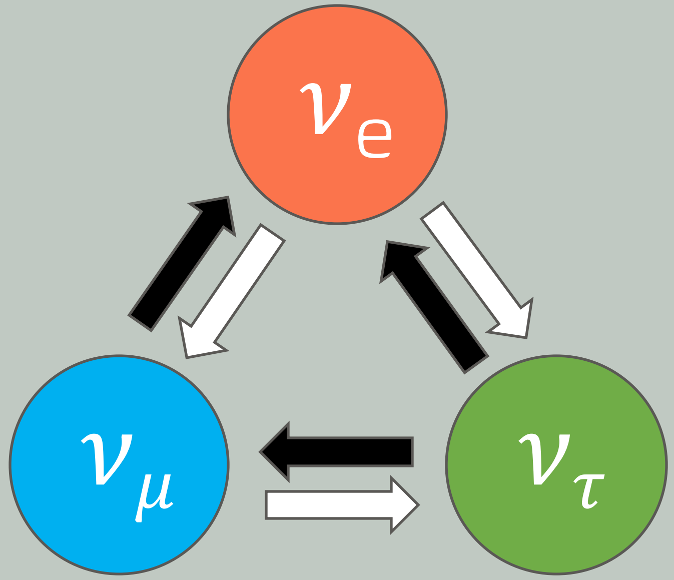
Viable fit to neutrino observables in possible U(2) flavor models



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Roma Tre

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ROME 2025

Roma Tre Neutrino Theory Group

Reference paper:

A Roadmap for neutrino charge assignments in $U(2)_F$ Flavor Models: Implications for LFV processes and leptonic anomalous magnetic moments

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A Roadmap for neutrino charge assignments in $U(2)_F$ Flavor Models: Implications for LFV processes and leptonic anomalous magnetic moments

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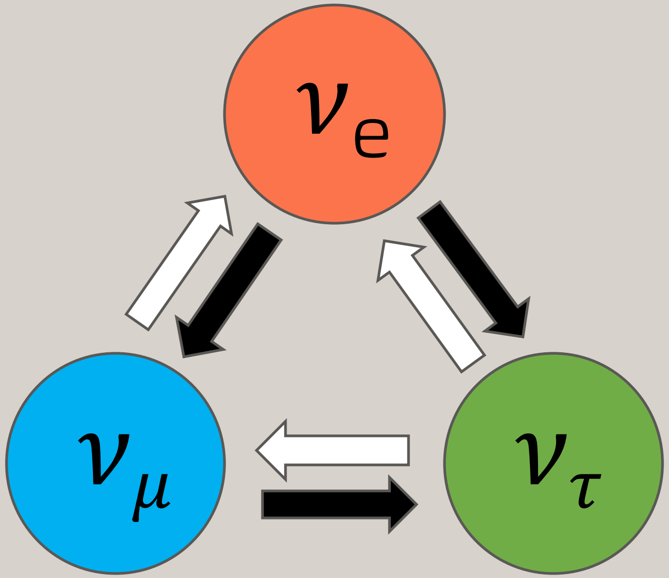
Abstract: We build upon a simple $U(2)_F$ model of flavor, in which all fermion masses and mixing hierarchies arise from powers of two small parameters controlling $U(2)_F$ breaking. In the original formulation, an isomorphism to the discrete $D_6 \times U(1)_F$ symmetry was invoked to generate a Majorana neutrino mass term. Here, we retain the successful features of that model for the charged leptons and quarks, while exploring alternative neutrino charge assignments within the $U(2)_F$ framework. This approach allows us to generate Majorana neutrino masses via the seesaw mechanism without introducing any additional symmetries nor invoking any isomorphism. We further examine the implications of our models for Lepton Flavor Violating (LFV) decays, analyzing the processes $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ and their connection with the leptonic anomalous magnetic moments. We show that within the Standard Model Effective Field Theory (SMEFT) approach the current limits on the branching ratios of LFV decays obtained in our $U(2)_F$ models are not compatible with the anomaly observed for $(g-2)_\mu$, thereby suggesting that either $(g-2)_\mu$ must be very close to the Standard Model predictions or the invoked flavor symmetry is not appropriate to describe the current anomalies.

Keywords: Neutrino Masses, See-saw Models, Lepton Number Violation, Model of flavor, Beyond the Standard Model

01 Neutrino masses and mixing



PMNS parameterization

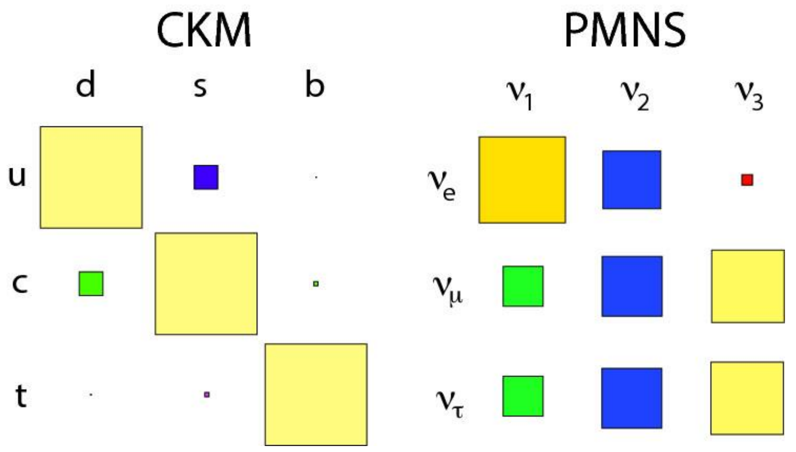
Neutrino oscillations are described by the **PMNS matrix:**

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\mathrm{CP}}} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}(e^{-i\alpha_1}, e^{-i\alpha_2}, 1)$$

$c_{ij}, s_{ij} \equiv \cos\theta_{ij}, \sin\theta_{ij}$

δ_{CP} $\rightarrow$ CP violating phase

α_1, α_2 $\rightarrow$ Majorana phases



Open problems:

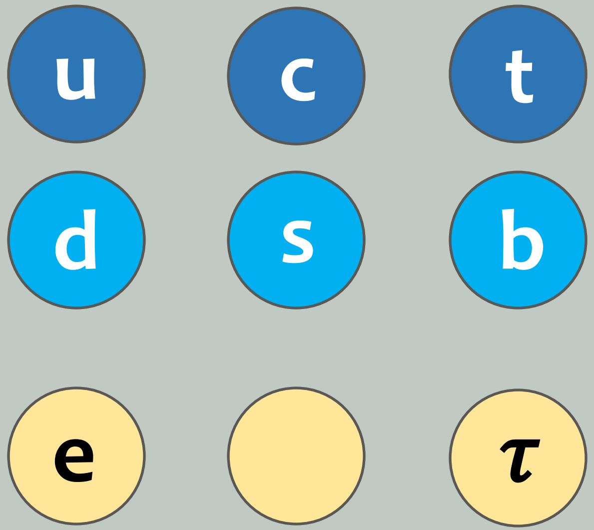
- Why neutrino masses are so small?
- Which is the ordering of neutrino masses (NO or IO)?
- Which is the absolute scale of neutrino masses?
- Why do neutrinos exhibit this mixing structure?
- Neutrinos are Dirac or Majorana?

…

→ **"SM flavor puzzle"**

02

U(2) and application to quarks and charged leptons



A possible solution to the problems of the SM flavor puzzle is the introduction of

FLAVOR SYMMETRIES

In our case:

$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{\text{SM}} \times U(2)_F$$

$U(2)_F$ → Spontaneously broken by the VEVs of two **flavons**: ϕ and χ.

$U(2)_F$ is locally isomorphic to $SU(2)_F \times U(1)_F$, under which SM fermions are charged.

The $SU(2)_F$ and $U(1)_F$ quantum numbers of the fermionic representations and the flavon fields are:

Fields / Representations	$\mathbf{SU(2)_F}$	$\mathbf{U(1)_F}$
L_a , D_a , Q_a , U_a , E_a $(a = 1, 2)$	$\mathbf{2}$	1
Q_3 , U_3 , E_3	$\mathbf{1}$	0
L_3 , D_3	$\mathbf{1}$	1
H	$\mathbf{1}$	0
ϕ_a $(a = 1, 2)$	$\mathbf{2}$	-1
χ	$\mathbf{1}$	-1

$\boldsymbol{L, Q}$ → LH lepton and quark doublets

$\boldsymbol{E, U, D}$ → RH electron, up and down quarks

3 generations for each representation (= 3 flavors)

$U(2)_F$ is locally isomorphic to $SU(2)_F \times U(1)_F$, under which SM fermions are charged.

The $SU(2)_F$ and $U(1)_F$ quantum numbers of the fermionic representations and the flavon fields are:

Fields / Representations	$\mathbf{SU(2)_F}$	$\mathbf{U(1)_F}$
$L_a\ ,\ D_a\ ,\ Q_a\ ,\ U_a\ ,\ E_a \quad (a=1,2)$	$\mathbf{2}$	1
$Q_3\ ,\ U_3\ ,\ E_3$	$\mathbf{1}$	0
$L_3\ ,\ D_3$	$\mathbf{1}$	1
H	$\mathbf{1}$	0
$\phi_a \quad (a=1,2)$	$\mathbf{2}$	-1
χ	$\mathbf{1}$	-1

The choice of the $U(1)_F$ charges for the SM fermions is the one that reproduces the physical values of masses and mixings for quarks and charged leptons.

Source: M. Linster, R. Ziegler. A realistic U(2) model of flavor – *JHEP* 08 (2018) 058

We have to construct the $U(2)_F$ invariant Lagrangian for the quark and charged lepton sectors.

After inserting the flavon VEVs

$$\langle\phi\rangle = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix} \qquad \langle\chi\rangle = \varepsilon_\chi \Lambda$$

$(\varepsilon_\phi, \varepsilon_\chi \sim \mathcal{O}(0.01))$

$$Y_u = \begin{pmatrix} \lambda^u_{11}\varepsilon_\phi^2\varepsilon_\chi^4 & \lambda^u_{12}\varepsilon_\chi^2 & \lambda^u_{13}\varepsilon_\phi\varepsilon_\chi^2 \\ -\lambda^u_{12}\varepsilon_\chi^2 & \lambda^u_{22}\varepsilon_\phi^2 & \lambda^u_{23}\varepsilon_\phi \\ \lambda^u_{31}\varepsilon_\phi\varepsilon_\chi^2 & \lambda^u_{32}\varepsilon_\phi & \lambda^u_{33} \end{pmatrix},$$

$$Y_d = \begin{pmatrix} \lambda^d_{11}\varepsilon_\phi^2\varepsilon_\chi^4 & \lambda^d_{12}\varepsilon_\chi^2 & \lambda^d_{13}\varepsilon_\phi\varepsilon_\chi^3 \\ -\lambda^d_{12}\varepsilon_\chi^2 & \lambda^d_{22}\varepsilon_\phi^2 & \lambda^d_{23}\varepsilon_\phi\varepsilon_\chi \\ \lambda^d_{31}\varepsilon_\phi\varepsilon_\chi^2 & \lambda^d_{32}\varepsilon_\phi & \lambda^d_{33}\varepsilon_\chi \end{pmatrix},$$

$$Y_e = \begin{pmatrix} e_{11}\varepsilon_\phi^2\varepsilon_\chi^4 & e_{12}\varepsilon_\chi^2 & e_{13}\varepsilon_\phi\varepsilon_\chi^2 \\ -e_{12}\varepsilon_\chi^2 & e_{22}\varepsilon_\phi^2 & e_{23}\varepsilon_\phi \\ e_{31}\varepsilon_\phi\varepsilon_\chi^3 & e_{32}\varepsilon_\phi\varepsilon_\chi & e_{33}\varepsilon_\chi \end{pmatrix}.$$

Assuming that $\lambda_{ij}^{u,d}, e_{ij} \sim \mathcal{O}(1)$, we can diagonalize the three Yukawa matrices obtaining:

$$y_u \sim \frac{\varepsilon_\chi^4}{\varepsilon_\phi^2}\,, \qquad y_d \sim y_e \sim \frac{\varepsilon_\chi^4}{\varepsilon_\phi^2}\,, \qquad V_{ub} \sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi}\,,$$

$$y_c \sim \varepsilon_\phi^2\,, \qquad y_s \sim y_\mu \sim \frac{\varepsilon_\phi^2 \varepsilon_\chi}{\sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}}\,, \qquad V_{cb} \sim \varepsilon_\phi\,,$$

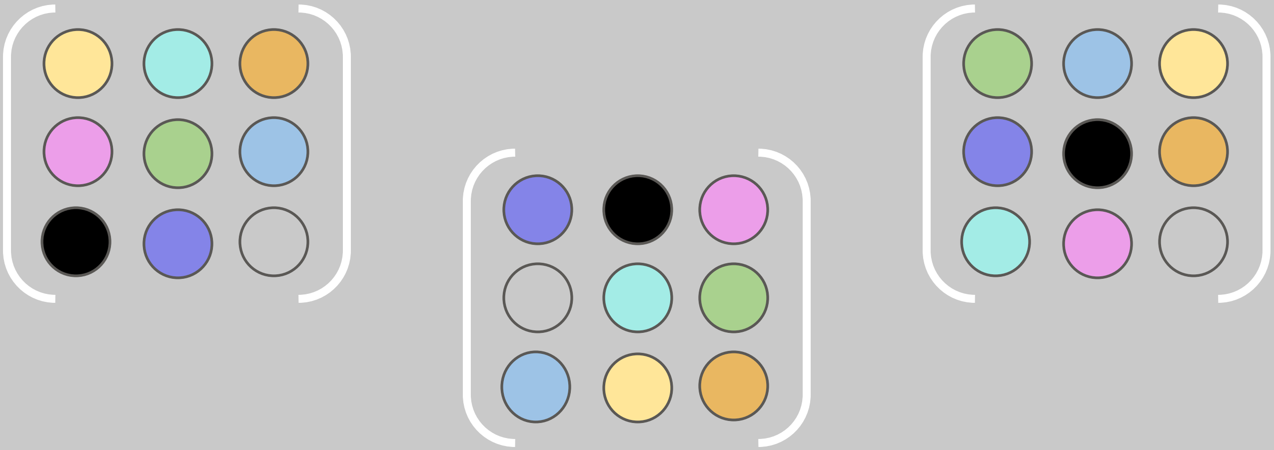
$$y_t \sim 1\,, \qquad y_b \sim y_\tau \sim \sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}\,, \qquad V_{us} \sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi^2}\,.$$

These predictions can reproduce the experimental values by taking

$\varepsilon_\phi \sim \lambda^2$ and $\varepsilon_\chi \sim \lambda^{2\div3}$

$\lambda = 0{,}2$ is roughly the Wolfenstein parameter ($\approx \sin\theta_c$)

03 U(2) to neutrinos: the possible patterns



We will assume **Majorana neutrinos**.

It follows that the Lagrangian contains a neutrino Yukawa coupling term and a Majorana mass term:

$$\mathcal{L}_\nu = L^T Y_\nu N H + \frac{1}{2} N^T M_\nu N + h.c.$$

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$$\mathcal{L}_\nu = L^T Y_\nu N H + \frac{1}{2} N^T M_\nu N + h.c.$$

RH neutrino (3 generations)

U(2) to neutrinos: the possible patterns

Linster and Ziegler's hypotheses:

Source: M. Linster, R. Ziegler. A realistic U(2) model of flavor – *JHEP* 08 (2018) 058

- $(N_1\ , N_2)$ is an $SU(2)_F$ doublet
- N_3 is a singlet
- $U(1)_F$ charges X_D and X_3 are positive

With these hypotheses we obtain:

$$Y_\nu = \begin{pmatrix} y_{11}\varepsilon_\phi^2\varepsilon_\chi^{|3+X_D|} & y_{12}\varepsilon_\chi^{|1+X_D|} & y_{13}\varepsilon_\phi\varepsilon_\chi^{|2+X_3|} \\ -y_{12}\varepsilon_\chi^{|1+X_D|} & y_{22}\varepsilon_\phi^2\varepsilon_\chi^{|X_D-1|} & y_{23}\varepsilon_\phi\varepsilon_\chi^{|X_3|} \\ y_{31}\varepsilon_\phi\varepsilon_\chi^{|2+X_D|} & y_{32}\varepsilon_\phi\varepsilon_\chi^{|X_D|} & y_{33}\varepsilon_\chi^{|1+X_3|} \end{pmatrix},$$

$$M_\nu = M\begin{pmatrix} k_{11}\varepsilon_\phi^2\varepsilon_\chi^{|2+2X_D|} & k_{12}\varepsilon_\phi^2\varepsilon_\chi^{|2X_D|} & k_{13}\varepsilon_\phi\varepsilon_\chi^{|1+X_D+X_3|} \\ k_{12}\varepsilon_\phi^2\varepsilon_\chi^{|2X_D|} & k_{22}\varepsilon_\phi^2\varepsilon_\chi^{|2X_D-2|} & k_{23}\varepsilon_\phi\varepsilon_\chi^{|X_D+X_3-1|} \\ k_{13}\varepsilon_\phi\varepsilon_\chi^{|1+X_D+X_3|} & k_{23}\varepsilon_\phi\varepsilon_\chi^{|X_D+X_3-1|} & k_{33}\varepsilon_\chi^{|2X_3|} \end{pmatrix}.$$

The Majorana neutrino mass matrix is obtained using the **type-I see-saw** mechanism:

$$m_\nu^M = v^2\, Y_\nu\, M_\nu^{-1}\, Y_\nu^T$$

In Linster and Ziegler's hypotheses we arrive to (neglecting $\mathcal{O}(1)$ coefficients):

$$m_\nu^M \sim \frac{v^2}{M}\begin{pmatrix} \varepsilon_\chi^4/\varepsilon_\phi^2 & \varepsilon_\chi^2/\varepsilon_\phi^2 & \varepsilon_\chi^3/\varepsilon_\phi \\ \varepsilon_\chi^2/\varepsilon_\phi^2 & 1/\varepsilon_\phi^2 & \varepsilon_\chi/\varepsilon_\phi \\ \varepsilon_\chi^3/\varepsilon_\phi & \varepsilon_\chi/\varepsilon_\phi & \varepsilon_\chi^2 \end{pmatrix} \sim \begin{pmatrix} \varepsilon^2 & 1 & \varepsilon^2 \\ 1 & 1/\varepsilon^2 & 1 \\ \varepsilon^2 & 1 & \varepsilon^2 \end{pmatrix}$$

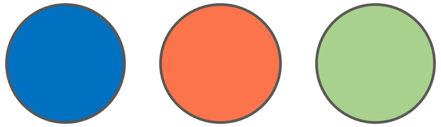
This structure is ruled out by the authors because it implies NO with $\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \sim \varepsilon^8$.

RH neutrinos are not SM particles, so we have total freedom in the assignment of the $SU(2)_F$ and $U(1)_F$ quantum numbers.

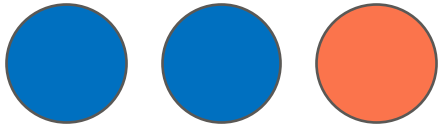
In our approach, there are **3 factors of arbitrariness:**

1 The freedom to choose how the 3 components of N transform under $SU(2)_F$:

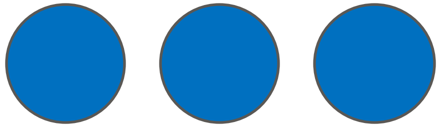
Model S



Model D



Model T



2 The freedom in assigning the values of the $U(1)_F$ charges:

Model S

$\boldsymbol{X_1}$, $\boldsymbol{X_2}$, $\boldsymbol{X_3}$

Model D

$\boldsymbol{X_D}$, $\boldsymbol{X_3}$

Model T

$\boldsymbol{X_T}$

3 The two possible choices of ε_ϕ and ε_χ:

A - Scenario

$$\varepsilon_\phi = \varepsilon_\chi = \lambda^2$$

B - Scenario

$$\varepsilon_\phi = \lambda^2 \, , \; \varepsilon_\chi = \lambda^3$$

$$(\lambda = 0{,}2)$$

From this arbitrariness we obtain **104** different mass matrix **PATTERNS**, which differ from each other in the:

- **powers of λ in the leading order (LO) structure**
- **coefficients (combinations of y_{ij} and k_{ij})**

$$\begin{pmatrix} a\,\lambda^8 & b\,\lambda^6 & c\,\lambda^7 \\ b\,\lambda^6 & d\,\lambda^4 & e\,\lambda^5 \\ c\,\lambda^7 & e\,\lambda^5 & f\,\lambda^6 \end{pmatrix}$$

The 104 patterns are so distributed:

- 48 for Model **S**
- 46 for Model **D**
- 10 for Model **T**

Numerical analysis

We test whether each pattern is able to reproduce these six dimensionless low-energy observables:

Parameter	bfp $\pm 1\sigma$ NO	bfp $\pm 1\sigma$ IO
$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$33.68^{+0.73}_{-0.70}$
$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$47.9^{+0.7}_{-0.9}$
$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.59^{+0.11}_{-0.11}$
$\alpha \equiv \Delta m^2_{\rm sol}/\lvert\Delta m^2_{\rm atm}\rvert$	0.0298 ± 0.0008	0.0302 ± 0.0008
m_e/m_μ	0.0048 ± 0.0002	
m_μ/m_τ	0.0565 ± 0.0045	

For each pattern, we test separately for the hypothesis of NO and IO.

U(2) to neutrinos: the possible patterns

Parameter set:

$p_i = \{\boldsymbol{e_{ij}}, \boldsymbol{y_{ij}}, \boldsymbol{k_{ij}}\}$

CHARGED LEPTONS

NEUTRINOS

Predicted observables:

$q_j(p_i)$

Constraints on p_i:

- y_{ij}, k_{ij} complex parameters
- e_{ij} real parameters

modulus within the range $[\lambda, \lambda^{-1}]$

A numerical function creates random samples of the set p_i and calculates the six $q_j(p_i)$.

Fit procedure

We want to minimize these two functions:

$$\chi^2(p_i) = \sum_{j=1}^{6} \left(\frac{q_j(p_i) - q_j^{\text{b-f}}}{\sigma_j} \right)^2$$

$$\mathrm{P_{MG}} = \sum_j \log^2(|\gamma_j^{p_i}|)$$

$q_j^{\text{b-f}}$: best fit values

σ_j : 1σ uncertainties

Parameter of Metric Goodness **:**

If it is low, Yukawa hierarchy arises solely by the $U(2)_F$ breaking

($\gamma_j^{p_i}$ is any parameter belonging to p_i)

The fit is considered as satisfactory if there is at least one set p_i for which:

$$\chi^2 < 20$$

$$\mathrm{P_{MG}} < 30$$

↓

13 VIABLE PATTERNS

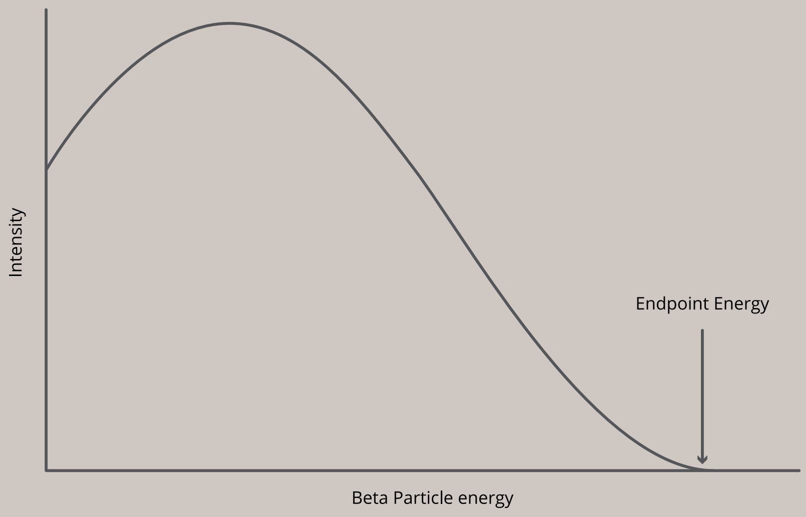
(6 for Model **S** and 7 for Model **D**, all valid only in **NO** hypothesis)

U(2) to neutrinos: the possible patterns

Pattern	Charges	LO mass matrix in terms of λ
S1 A **S**2 A **D**1 A **D**2 A	$(1,0,-2)$ $(1,1,-2)$ $(1,-2)$ $(2,-2)$	$\begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^4\\ \lambda^4 & \lambda^4 & \lambda^4\\ \lambda^4 & \lambda^4 & \lambda^4\end{pmatrix}$
S3 A **S**4 A	$(2,1,-2)$ $(2,2,-2)$	$\begin{pmatrix}\lambda^{12} & \lambda^8 & \lambda^8\\ \lambda^8 & \lambda^4 & \lambda^4\\ \lambda^8 & \lambda^4 & \lambda^4\end{pmatrix}$
D3 A **D**4 A	$(0,1)$ $(1,0)$	$\begin{pmatrix}\lambda^4 & 1 & \lambda^4\\ 1 & 1/\lambda^4 & 1\\ \lambda^4 & 1 & \lambda^4\end{pmatrix}$
S1 B **S**2 B	$(1,0,-2)$ $(1,1,-2)$	$\begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^5\\ \lambda^4 & \lambda^4 & \lambda^5\\ \lambda^5 & \lambda^5 & \lambda^6\end{pmatrix}$
D1 B **D**2 B	$(1,-2)$ $(2,-2)$	$\begin{pmatrix}\lambda^8 & \lambda^6 & \lambda^7\\ \lambda^6 & \lambda^4 & \lambda^5\\ \lambda^7 & \lambda^5 & \lambda^6\end{pmatrix}$
D5 B	$(0,0)$	$\begin{pmatrix}\lambda^8 & \lambda^2 & \lambda^7\\ \lambda^2 & \lambda^4 & \lambda\\ \lambda^7 & \lambda & \lambda^6\end{pmatrix}$

ANARCHICAL PATTERN

04 Predictions on neutrino observables

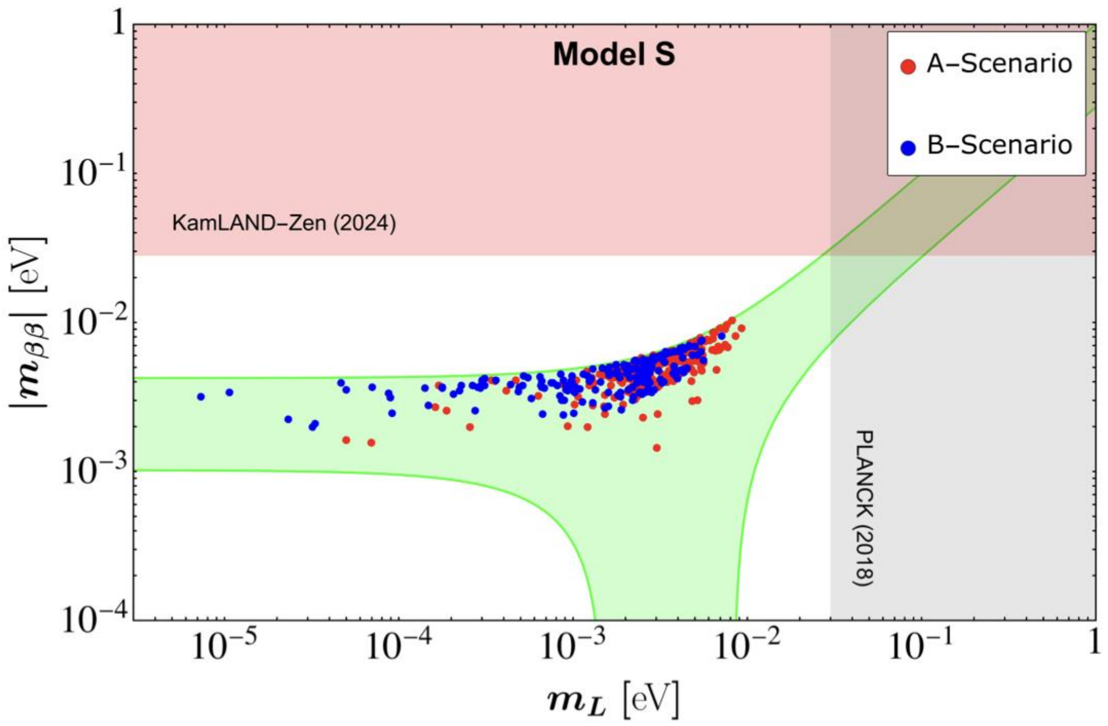


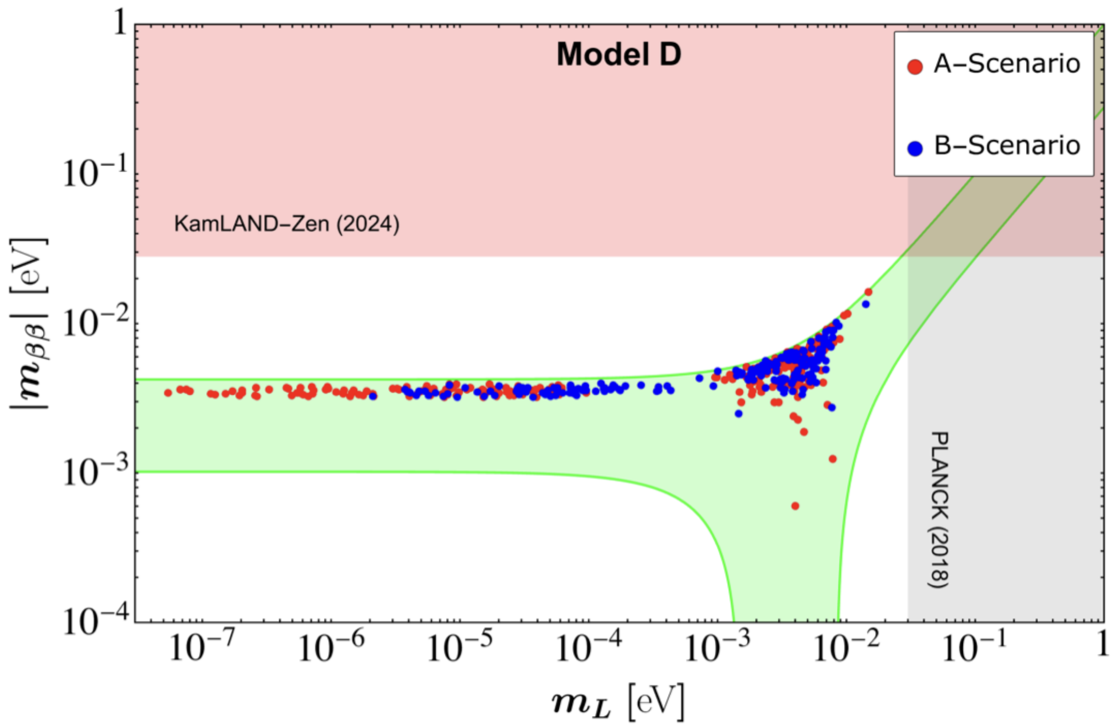
Effective Majorana mass parameter ($m_{\beta\beta}$)

$$|m_{\beta\beta}| = \left|\sum_i (U_{ei})^2 m_i\right|$$

The rate of the $0\nu2\beta$ decay would depend on this parameter.

Effective Majorana mass parameter ($m_{\beta\beta}$)





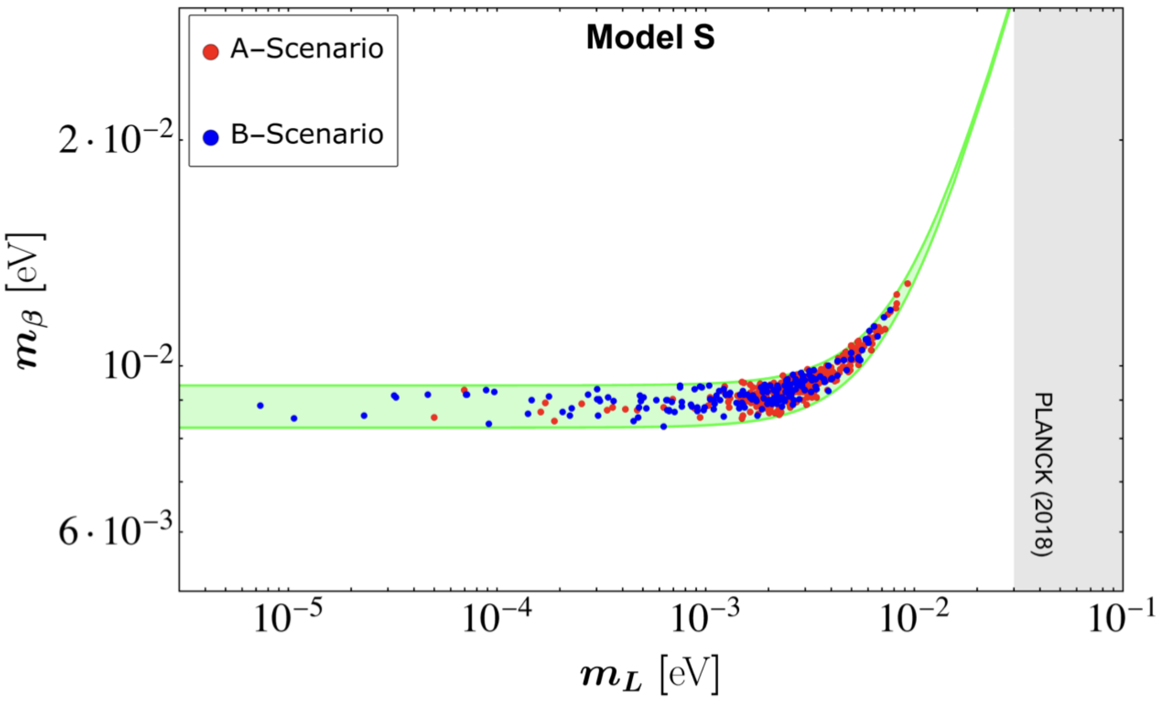
Each point is a valid representation, *i.e.* a set $\{e_{ij}, y_{ij}, k_{ij}\}$ which reproduces all the 6 fit observables within the 3σ range.

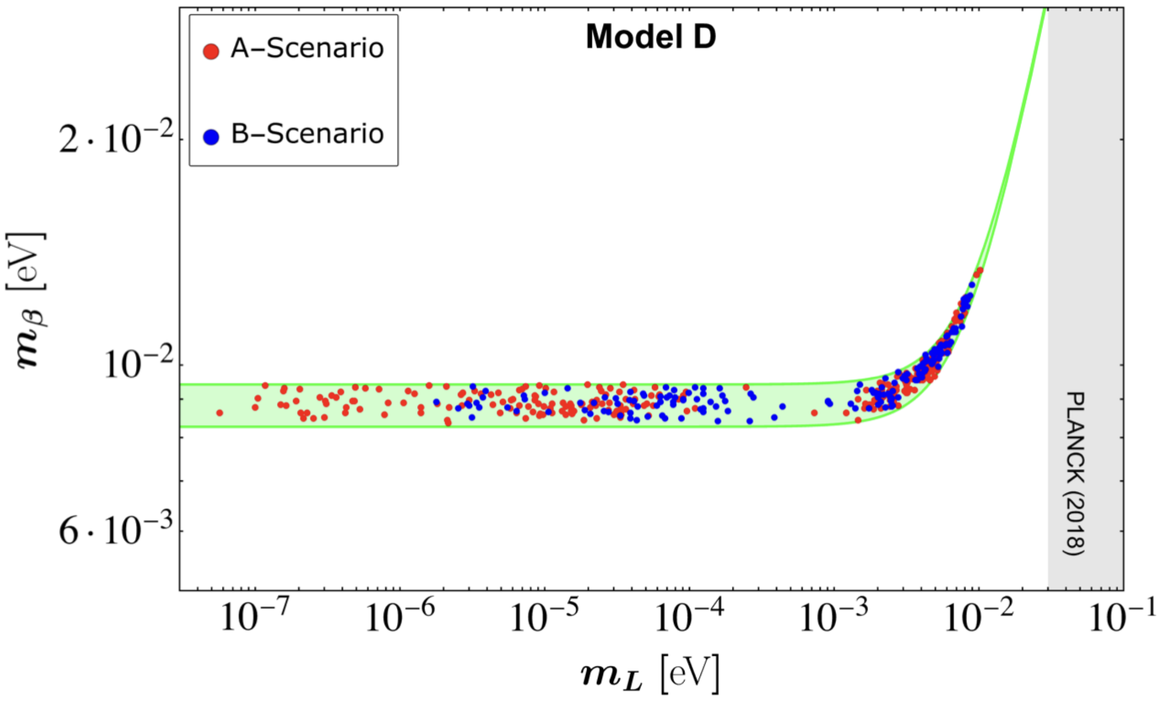
Effective electron neutrino mass (m_β)

$$m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

It determines the endpoint of the beta decay spectrum.

Effective electron neutrino mass (m_β)





Further developments of the model

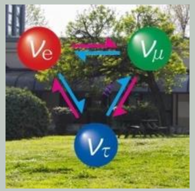
From the viable patterns we can make predictions on Lepton Flavor Violating (LFV) decays in connection with the leptonic anomalous magnetic moments.

For further details, see the talk by **Simone Marciano**

The talk will begin in a few minutes – STAY TUNED !

Thank you!

Contact: mirko. rettaroli@uniroma3.it



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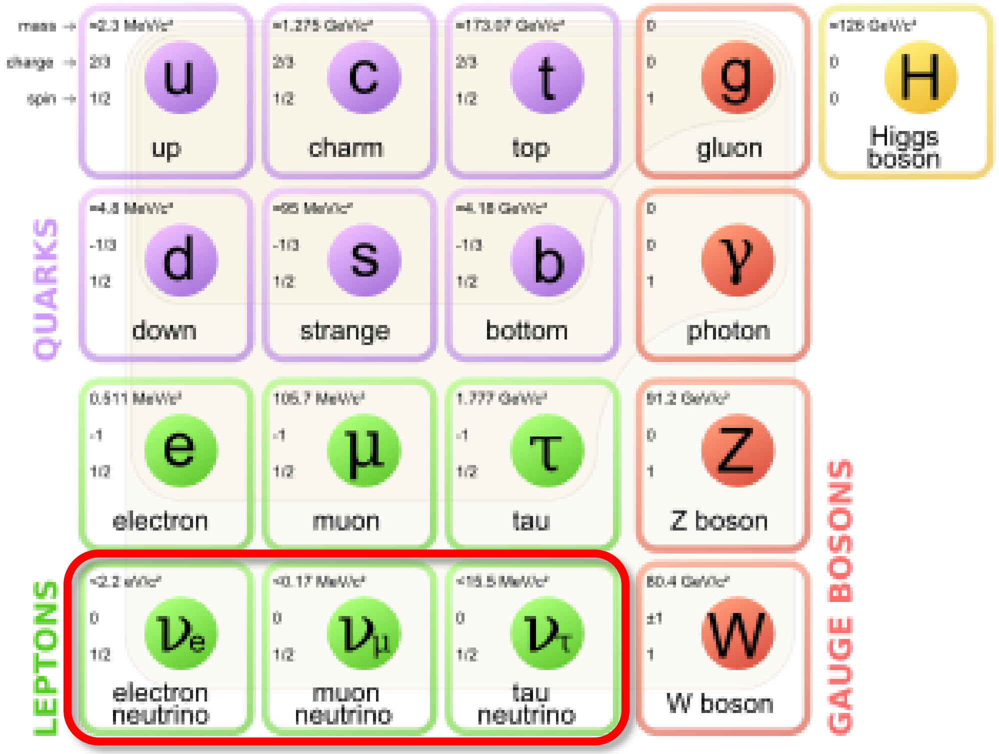
A Appendix slides

The elementary particles of the SM are divided in:

- Fermions → Quarks
  → **Leptons**
  ↓
  Neutrinos

- Bosons



Experimental parameters

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC24 with SK atmospheric data	$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
	$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

NuFIT 6.0 (September 2024)

Lagrangians

$$\mathcal{L}_u = \frac{\lambda^u_{11}}{\Lambda^6}\chi^4(\phi^*_a Q_a)(\phi^*_b U_b)H + \frac{\lambda^u_{12}}{\Lambda^2}\chi^2\epsilon_{ab}Q_a U_b H + \frac{\lambda^u_{13}}{\Lambda^3}\chi^2(\phi^*_a Q_a)U_3 H$$
$$+ \frac{\lambda^u_{22}}{\Lambda^2}(\epsilon_{ab}\phi_a Q_b)(\epsilon_{cd}\phi_c U_d)H + \frac{\lambda^u_{23}}{\Lambda}(\epsilon_{ab}\phi_a Q_b)U_3 H + \frac{\lambda^u_{31}}{\Lambda^3}\chi^2 Q_3(\phi^*_a U_a)H$$
$$+ \frac{\lambda^u_{32}}{\Lambda}Q_3(\epsilon_{ab}\phi_a U_b)H + \lambda^u_{33}Q_3 U_3 H\ ,$$

$$\mathcal{L}_d = \frac{\lambda^d_{11}}{\Lambda^6}\chi^4(\widetilde{\phi}\cdot Q)(\widetilde{\phi}\cdot D)H + \frac{\lambda^d_{12}}{\Lambda^2}\chi^2(Q\cdot D)H + \frac{\lambda^d_{13}}{\Lambda^4}\chi^3(\widetilde{\phi}\cdot Q)D_3 H$$
$$+ \frac{\lambda^d_{22}}{\Lambda^2}(\phi\cdot Q)(\phi\cdot D)H + \frac{\lambda^d_{23}}{\Lambda^2}\chi(\phi\cdot Q)D_3 H + \frac{\lambda^d_{31}}{\Lambda^3}\chi^2 Q_3(\widetilde{\phi}\cdot D)H$$
$$+ \frac{\lambda^d_{32}}{\Lambda}Q_3(\phi\cdot D)H + \frac{\lambda^d_{33}}{\Lambda}\chi Q_3 D_3 H\ .$$

$$\mathcal{L}_e = \frac{\lambda^e_{11}}{\Lambda^6}\chi^4(\widetilde{\phi}\cdot L)(\widetilde{\phi}\cdot E)H + \frac{\lambda^e_{12}}{\Lambda^2}\chi^2(L\cdot E)H + \frac{\lambda^e_{13}}{\Lambda^3}\chi^2(\widetilde{\phi}\cdot L)E_3 H + \\ + \frac{\lambda^e_{22}}{\Lambda^2}(\phi\cdot L)(\phi\cdot E)H + \frac{\lambda^e_{23}}{\Lambda}(\phi\cdot L)E_3 H + \frac{\lambda^e_{31}}{\Lambda^4}\chi^3 L_3(\widetilde{\phi}\cdot E)H + \\ + \frac{\lambda^e_{32}}{\Lambda^2}\chi L_3(\phi\cdot E)H + \frac{\lambda^e_{33}}{\Lambda}\chi L_3 E_3 H \ .$$

Comparison between Yukawas and exp. mass ratios

$$y_u \sim \frac{\varepsilon_\chi^4}{\varepsilon_\phi^2}\,, \qquad y_d \sim y_e \sim \frac{\varepsilon_\chi^4}{\varepsilon_\phi^2}\,, \qquad V_{ub} \sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi}\,,$$

$$y_c \sim \varepsilon_\phi^2\,, \qquad y_s \sim y_\mu \sim \frac{\varepsilon_\phi^2 \varepsilon_\chi}{\sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}}\,, \qquad V_{cb} \sim \varepsilon_\phi\,,$$

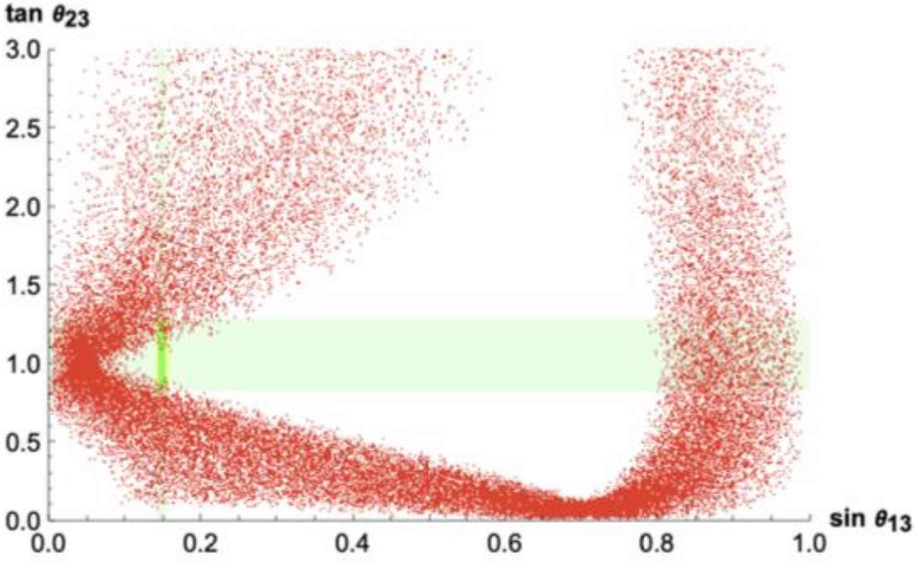
$$y_t \sim 1\,, \qquad y_b \sim y_\tau \sim \sqrt{\varepsilon_\phi^2 + \varepsilon_\chi^2}\,, \qquad V_{us} \sim \frac{\varepsilon_\chi^2}{\varepsilon_\phi^2}\,.$$

$$\frac{m_u}{m_t} \approx \lambda^{(7.1 \div 7.7)}\,, \qquad \frac{m_d}{m_b} \approx \lambda^{(4.2 \div 4.4)}\,, \qquad \frac{m_e}{m_\tau} \approx \lambda^{5.1}\,,$$

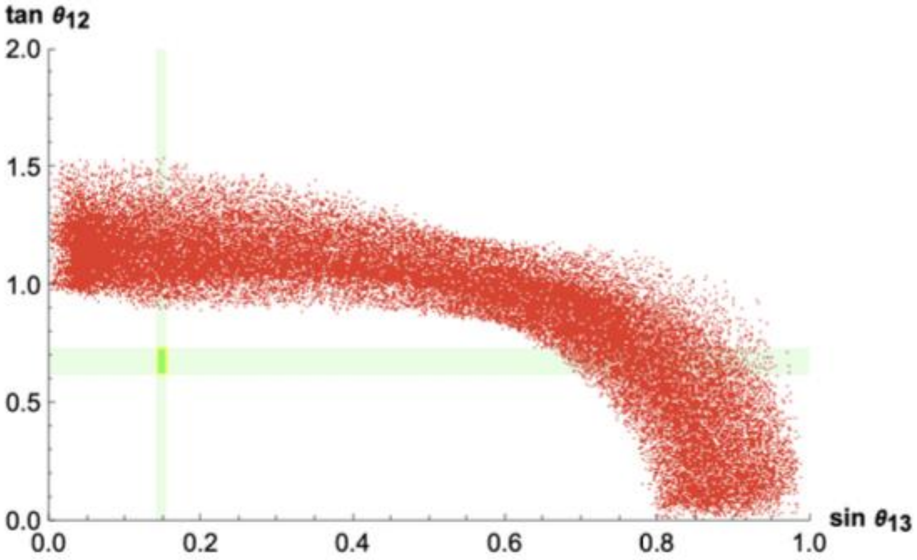
$$\frac{m_c}{m_t} \approx \lambda^{3.5}\,, \qquad \frac{m_s}{m_b} \approx \lambda^{(2.4 \div 2.5)}\,, \qquad \frac{m_\mu}{m_\tau} \approx \lambda^{1.8}\,,$$

$$V_{ub} \approx \lambda^3\,, \qquad V_{cb} \approx \lambda^2\,, \qquad V_{us} \approx \lambda\,,$$

T2 A - scatter plots



(a) Scatter plot $\tan\theta_{23}$ / $\sin\theta_{13}$



(b) Scatter plot $\tan\theta_{12}$ / $\sin\theta_{13}$

U(2) symmetry breaking model

QUARKS AND CHARGED LEPTONS

Masses and mixings are successfully reproduced

$\varepsilon_\phi \sim \lambda^2$

$\varepsilon_\chi \sim \lambda^{2\div 3}$

MAJORANA NEUTRINOS

~~Linster and Ziegler's hypothesis~~

Complete arbitrariness on $U(2)_F$ quantum numbers

104 patterns

NUMERICAL FIT

13 VIABLE PATTERNS

Appendix – Fit results

Pattern	S1 A	S2 A	S3 A	S4 A	S1 B	S2 B
e_{11}	-0.33	-3.25	2.98	-2.02	3.01	-4.34
e_{12}	-0.44	-0.88	-0.41	-0.59	-2.15	-3.27
e_{13}	4.09	3.81	-2.20	4.13	0.33	-1.04
e_{22}	4.11	-0.53	2.76	2.53	1.86	-2.96
e_{23}	-3.74	3.78	2.72	3.03	-0.93	-1.41
e_{31}	1.23	3.63	0.60	0.91	3.71	2.29
e_{32}	2.76	-3.90	3.99	3.57	1.07	-1.51
e_{33}	1.86	-0.28	0.82	0.39	2.52	3.27
y_{11}	1.31	-1.33	-3.31	-3.07	3.22	3.35
y_{12}	1.73	-3.55	-1.60	-2.02	-3.68	-3.95
y_{13}	1.03	2.15	-4.20	-4.04	-0.56	0.48
y_{21}	3.06	4.31	-3.39	-3.27	0.95	-3.71
y_{22}	-2.41	0.48	1.40	-0.38	3.85	1.53
y_{23}	2.93	1.15	-1.85	-2.96	-0.65	-2.78
y_{31}	-1.19	-3.00	-2.90	-2.13	-4.27	-4.24
y_{32}	3.66	-3.01	-0.52	0.41	2.77	0.36
y_{33}	0.34	-1.80	1.65	4.06	2.19	1.98
k_{11}	-1.21	1.09	-4.12	-1.66	4.19	0.29
k_{12}	1.81	1.33	0.42	4.06	-1.57	-1.71
k_{13}	-3.89	1.94	-0.24	0.78	1.66	-3.52
k_{22}	3.94	3.37	-1.57	-1.95	1.29	-2.85
k_{23}	-2.75	-3.58	4.01	0.38	2.60	0.73
k_{33}	1.09	1.55	-0.54	-1.35	-1.82	-2.89
$\sin^2\theta_{12}$	0.302	0.302	0.302	0.308	0.307	0.305
$\sin^2\theta_{13}$	0.0222	0.0225	0.0225	0.0219	0.0224	0.0222
$\sin^2\theta_{23}$	0.454	0.457	0.454	0.458	0.458	0.458
α	0.0294	0.0292	0.0296	0.0294	0.0296	0.0295
r_{12}	0.0048	0.0048	0.0047	0.0048	0.0048	0.0048
r_{23}	0.0547	0.0561	0.0567	0.0565	0.0571	0.0563
χ^2	0.24	0.45	0.64	0.70	0.34	0.23
d_{FT}	7.12	4.19	12.2	6.02	5.53	5.72
$\chi^2 + P_{MG}$	27.00	26.62	26.22	25.97	24.19	29.04

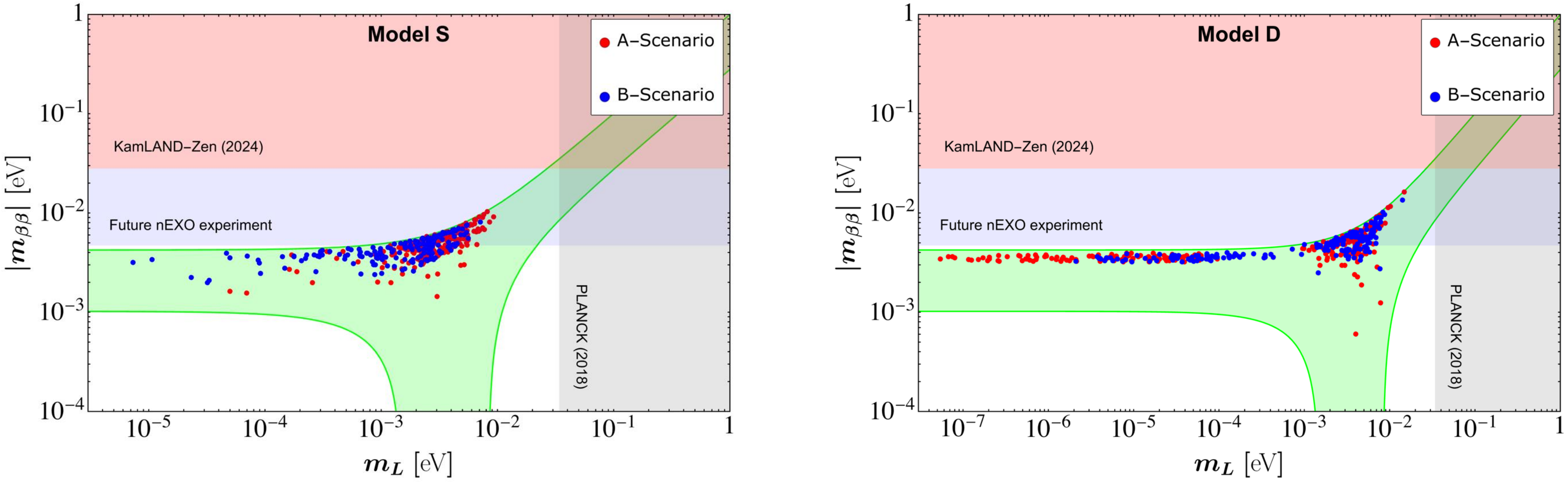
Pattern	D1 A	D2 A	D3 A	D4 A	D1 B	D2 B	D5 B
e_{11}	1.00	-1.47	0.92	-3.18	1.69	0.94	-0.74
e_{12}	-0.32	-0.53	0.34	-0.31	-2.69	3.38	-1.28
e_{13}	0.62	-1.76	-2.15	3.60	2.15	0.89	1.95
e_{22}	-3.30	-3.01	3.18	0.92	-3.81	-2.33	-1.83
e_{23}	-2.67	3.10	-2.24	-2.40	1.19	-1.40	0.50
e_{31}	-2.06	-1.63	3.39	-3.94	-1.65	4.01	3.82
e_{32}	2.11	4.13	-3.88	2.65	-2.64	3.30	-3.68
e_{33}	-2.23	-0.65	-3.97	1.06	3.20	-2.77	2.61
y_{11}	2.36	-1.76	3.30	3.74	1.64	3.02	2.97
y_{12}	-1.02	0.76	2.86	-2.87	-2.72	3.29	-1.13
y_{13}	2.15	3.37	3.54	-3.98	0.36	0.42	-2.24
y_{22}	-3.05	3.89	-1.14	2.47	-1.61	-0.75	3.20
y_{23}	-1.11	-1.78	-2.58	-3.15	-2.56	2.08	3.28
y_{31}	-3.49	0.72	2.82	-0.38	-1.41	-3.40	3.32
y_{32}	3.44	-1.59	3.79	2.50	0.86	4.32	0.88
y_{33}	2.92	2.39	-0.42	-1.15	-3.64	1.02	1.26
k_{11}	1.61	3.55	-1.40	-1.61	3.21	4.05	2.10
k_{12}	-3.40	3.37	-3.92	-3.44	1.44	1.10	-3.50
k_{13}	-2.55	-3.41	2.31	3.76	0.39	-1.80	1.60
k_{22}	2.20	4.00	3.23	-1.59	1.86	3.67	-0.34
k_{23}	-0.95	3.05	0.28	-2.29	-0.62	0.77	1.12
k_{33}	3.65	0.56	1.82	-3.49	2.16	-2.11	-0.25
$\sin^2\theta_{12}$	0.297	0.305	0.305	0.297	0.300	0.305	0.303
$\sin^2\theta_{13}$	0.0221	0.0222	0.0221	0.0219	0.0223	0.0222	0.0222
$\sin^2\theta_{23}$	0.452	0.447	0.451	0.455	0.444	0.457	0.450
α	0.0298	0.0295	0.0296	0.0304	0.0296	0.0298	0.0295
r_{12}	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048
r_{23}	0.0571	0.0577	0.0546	0.0569	0.0553	0.0570	0.0572
χ^2	0.39	0.24	0.35	1.96	0.38	0.30	0.06
d_{FT}	17.1	7.03	37.6	4.26	8.41	3.31	45.0
$\chi^2 + P_{MG}$	24.44	27.71	28.10	28.43	19.97	22.33	22.07

A Appendix – Fit results

Pattern	S1 A	S2 A	S3 A	S4 A	S1 B	S2 B
e_{11}	-0.33	-3.25	2.98	-2.02	3.01	-4.34
e_{12}	-0.44	-0.88	-0.41	-0.59	-2.15	-3.27
e_{13}	4.09	3.81	-2.20	4.13	0.33	-1.04
e_{22}	4.11	-0.53	2.76	2.53	1.86	-2.96
e_{23}	-3.74	3.78	2.72	3.03	-0.93	-1.41
e_{31}	1.23	3.63	0.60	0.91	3.71	2.29
e_{32}	2.76	-3.90	3.99	3.57	1.07	-1.51
e_{33}	1.86	-0.28	0.82	0.39	2.52	3.27
y_{11}	1.31	-1.33	-3.31	-3.07	3.22	3.35
y_{12}	1.73	-3.55	-1.60	-2.02	-3.68	-3.95
y_{13}	1.03	2.15	-4.20	-4.04	-0.56	0.48
y_{21}	3.06	4.31	-3.39	-3.27	0.95	-3.71
y_{22}	-2.41	0.48	1.40	-0.38	3.85	1.53
y_{23}	2.93	1.15	-1.85	-2.96	-0.65	-2.78
y_{31}	-1.19	-3.00	-2.90	-2.13	-4.27	-4.24
y_{32}	3.66	-3.01	-0.52	0.41	2.77	0.36
y_{33}	0.34	-1.80	1.65	4.06	2.19	1.98
k_{11}	-1.21	1.09	-4.12	-1.66	4.19	0.29
k_{12}	1.81	1.33	0.42	4.06	-1.57	-1.71
k_{13}	-3.89	1.94	-0.24	0.78	1.66	-3.52
k_{22}	3.94	3.37	-1.57	-1.95	1.29	-2.85
k_{23}	-2.75	-3.58	4.01	0.38	2.60	0.73
k_{33}	1.09	1.55	-0.54	-1.35	-1.82	-2.89
$\sin^2\theta_{12}$	0.302	0.302	0.302	0.308	0.307	0.305
$\sin^2\theta_{13}$	0.0222	0.0225	0.0225	0.0219	0.0224	0.0222
$\sin^2\theta_{23}$	0.454	0.457	0.454	0.458	0.458	0.458
α	0.0294	0.0292	0.0296	0.0294	0.0296	0.0295
r_{12}	0.0048	0.0048	0.0047	0.0048	0.0048	0.0048
r_{23}	0.0547	0.0561	0.0567	0.0565	0.0571	0.0563
χ^2	0.24	0.45	0.64	0.70	0.34	0.23
d_{FT}	7.12	4.19	12.2	6.02	5.53	5.72
$\chi^2 + P_{MG}$	27.00	26.62	26.22	25.97	24.19	29.04

Pattern	D1 A	D2 A	D3 A	D4 A	D1 B	D2 B	D5 B
e_{11}	1.00	-1.47	0.92	-3.18	1.69	0.94	-0.74
e_{12}	-0.32	-0.53	0.34	-0.31	-2.69	3.38	-1.28
e_{13}	0.62	-1.76	-2.15	3.60	2.15	0.89	1.95
e_{22}	-3.30	-3.01	3.18	0.92	-3.81	-2.33	-1.83
e_{23}	-2.67	3.10	-2.24	-2.40	1.19	-1.40	0.50
e_{31}	-2.06	-1.63	3.39	-3.94	-1.65	4.01	3.82
e_{32}	2.11	4.13	-3.88	2.65	-2.64	3.30	-3.68
e_{33}	-2.23	-0.65	-3.97	1.06	3.20	-2.77	2.61
y_{11}	2.36	-1.76	3.30	3.74	1.64	3.02	2.97
y_{12}	-1.02	0.76	2.86	-2.87	-2.72	3.29	-1.13
y_{13}	2.15	3.37	3.54	-3.98	0.36	0.42	-2.24
y_{22}	-3.05	3.89	-1.14	2.47	-1.61	-0.75	3.20
y_{23}	-1.11	-1.78	-2.58	-3.15	-2.56	2.08	3.28
y_{31}	-3.49	0.72	2.82	-0.38	-1.41	-3.40	3.32
y_{32}	3.44	-1.59	3.79	2.50	0.86	4.32	0.88
y_{33}	2.92	2.39	-0.42	-1.15	-3.64	1.02	1.26
k_{11}	1.61	3.55	-1.40	-1.61	3.21	4.05	2.10
k_{12}	-3.40	3.37	-3.92	-3.44	1.44	1.10	-3.50
k_{13}	-2.55	-3.41	2.31	3.76	0.39	-1.80	1.60
k_{22}	2.20	4.00	3.23	-1.59	1.86	3.67	-0.34
k_{23}	-0.95	3.05	0.28	-2.29	-0.62	0.77	1.12
k_{33}	3.65	0.56	1.82	-3.49	2.16	-2.11	-0.25
$\sin^2\theta_{12}$	0.297	0.305	0.305	0.297	0.300	0.305	0.303
$\sin^2\theta_{13}$	0.0221	0.0222	0.0221	0.0219	0.0223	0.0222	0.0222
$\sin^2\theta_{23}$	0.452	0.447	0.451	0.455	0.444	0.457	0.450
α	0.0298	0.0295	0.0296	0.0304	0.0296	0.0298	0.0295
r_{12}	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048	0.0048
r_{23}	0.0571	0.0577	0.0546	0.0569	0.0553	0.0570	0.0572
χ^2	0.39	0.24	0.35	1.96	0.38	0.30	0.06
d_{FT}	17.1	7.03	37.6	4.26	8.41	3.31	45.0
$\chi^2 + P_{MG}$	24.44	27.71	28.10	28.43	19.97	22.33	22.07

$m_{\beta\beta}$ plots with the next gen experiments



Parameters

	e_{ij}	y_{ij}	k_{ij}	total
Model S	8	9	6	**23**
Model D	8	8	6	**22**
Model T	8	7	5	**20**