# Vector-like quark doublets, weak-basis invariants and CP violation


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**[2412.21201]**


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# Motivation

- A fourth chiral generation of quarks is ruled out, but the **quark sector can be extended with VLQs**.

- VLQs **take part in many models** from GUTs, to solutions to the strong CP problem or the EW hierarchy problem. They have a **rich phenomenology** that can be used to try to explain several types of anomalies/ tensions.

- The introduction of VLQs allows for **extra Yukawa couplings and bare mass terms**. In principle, this means **more physical phases** which could lead to the **enhancement of CP violation** in the quark sector.

- Models with **VLQ iso-doublets** $(\mathbf{3}, \mathbf{2})_{1/6}$ are the **favored candidates in explaining the Cabibbo Angle Anomalies** (CAAs).

# The Cabibbo Angle Anomalies (CAAs)

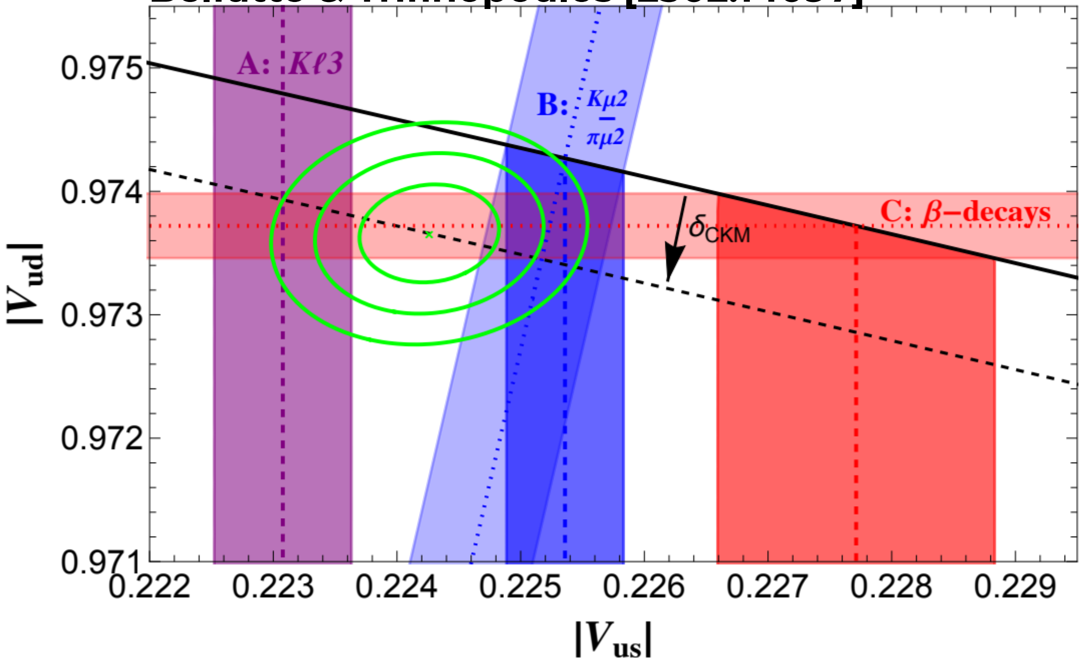
The independent determinations of **$|V_{us}|$**, the ratio **$|V_{us}/V_{ud}|$** and **$|V_{ud}|$** are **not in agreement with each other within the framework of the CKM unitary of SM** (discrepancy of ~3$\sigma$). Extensions with VLQs iso-singlets naturally introduce deviations to CKM unitarity.

**A** $K\ell 3$ : $\boxed{|V_{us}|_{\mathrm{A}} = 0.22308(55)}$

**B** $K\mu 2/\pi\mu 2$ : $\boxed{|V_{us}|/|V_{ud}|_B = 0.23126(48)}$

**C** $\beta -$ decays: $\boxed{|V_{ud}|_C = 0.97372(26)}$

Belfatto & Trifinopoulos [2302.14097]




(**A**+**B**) vs **C** at 3σ

CAA1: $\delta_{\mathrm{CKM}} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - \cancel{|V_{ub}|^2} \approx 1.60(0.53) \times 10^{-3}$

**A** vs **B** at 3.1σ

CAA2: $|V_{us}|_A = 0.22308(55)$ $|V_{us}|_B = 0.225313(47)$

# Extensions with VLQ iso-doublets

Introducing $N$ VLQ iso-doublets with q.n.s $(\mathbf{3},\mathbf{2})_{1/6}$

$$Q^0_{L\alpha} = \begin{pmatrix} U^0_{L\alpha} \\ D^0_{L\alpha} \end{pmatrix}, \quad Q^0_{R\alpha} = \begin{pmatrix} U^0_{R\alpha} \\ D^0_{R\alpha} \end{pmatrix} \qquad (\alpha = 1, \ldots, N)$$

leads to

Yukawa couplings

$$\begin{aligned} -\mathcal{L} = & \underbrace{(Y_u)_{ij}\, \overline{q^0_{Li}}\, \tilde{\Phi}\, u^0_{Rj} + (Y_d)_{ij}\, \overline{q^0_{Li}}\, \Phi\, d^0_{Rj} + \text{h.c.}}_{\text{SM}} \\ & \left.\begin{aligned} & + (Z_u)_{\alpha j}\, \overline{Q^0_{L\alpha}}\, \tilde{\Phi}\, u^0_{Rj} + (Z_d)_{\alpha j}\, \overline{Q^0_{L\alpha}}\, \Phi\, d^0_{Rj} + \text{h.c.} \\ & + (\overline{M})_{i\beta}\, \overline{q^0_{Li}}\, Q^0_{R\beta} + (M_Q)_{\alpha\beta}\, \overline{Q^0_{L\alpha}}\, Q^0_{R\beta} + \text{h.c.}\,, \end{aligned}\right\} \text{NP} \end{aligned}$$

Bare-mass terms (sector independent)

or using $\mathcal{Q}^0_L = \begin{pmatrix} q^0_L \\ Q^0_L \end{pmatrix}$

$$-\mathcal{L} = \boxed{\overline{\mathcal{Q}^0_L}\, \tilde{\Phi}\, \mathcal{Y}_u\, u^0_R + \overline{\mathcal{Q}^0_L}\, \Phi\, \mathcal{Y}_d\, d^0_R} + \boxed{\overline{\mathcal{Q}^0_L}\, M\, Q^0_R} + \text{h.c.}$$

# Extensions with VLQ iso-doublets

After SSB, one obtains

$$-\mathcal{L}_{\rm m} = \overline{\mathcal{U}_L^0}\mathcal{M}_u\mathcal{U}_R^0 + \overline{\mathcal{D}_L^0}\mathcal{M}_d\mathcal{D}_R^0 + \text{h.c.} \qquad \mathcal{U}_{L,R}^0 = \begin{pmatrix} u_{L,R}^0 \\ U_{L,R}^0 \end{pmatrix}, \qquad \mathcal{D}_{L,R}^0 = \begin{pmatrix} d_{L,R}^0 \\ D_{L,R}^0 \end{pmatrix},$$

with

$$\boxed{\mathcal{M}_q = \left.\begin{pmatrix} \overbrace{\quad m_q \quad}^{3} & \overbrace{\quad M \quad}^{N} \end{pmatrix}\right\}3{+}N} \qquad m_q \equiv v\,\mathcal{Y}_q = v\begin{pmatrix} Y_q \\ Z_q \end{pmatrix}, \qquad M \equiv \begin{pmatrix} \overline{M} \\ M_Q \end{pmatrix}$$

We can now diagonalize the mass matrices by transforming into the mass eigenstates

$$\mathcal{U}_{L,R}^0 = \mathcal{V}_{L,R}^u\,\mathcal{U}_{L,R} = \mathcal{V}_{L,R}^u\begin{pmatrix} u_{L,R} \\ U_{L,R} \end{pmatrix}, \qquad \mathcal{D}_{L,R}^0 = \mathcal{V}_{L,R}^d\,\mathcal{D}_{L,R} = \mathcal{V}_{L,R}^d\begin{pmatrix} d_{L,R} \\ D_{L,R} \end{pmatrix}$$

$$-\mathcal{L}_{\rm m} = \overline{\mathcal{U}}_L\,\mathcal{D}_u\,\mathcal{U}_R + \overline{\mathcal{D}}_L\,\mathcal{D}_d\,\mathcal{D}_R + \text{h.c.}$$

$$\mathcal{V}_L^{q\,\dagger}\,\mathcal{M}_q\,\mathcal{V}_R^q = \mathcal{D}_q \equiv \text{diag}(m_1^q,\, m_2^q,\, m_3^q,\, M_1^q,\, \ldots,\, M_N^q)$$

$$\boxed{\mathcal{V}_{L,R}^q = \underbrace{\begin{pmatrix} A_{L,R}^q \\ \hdashline B_{L,R}^q \end{pmatrix}}_{3+N} \begin{matrix} \}\,3 \\ \}\,N \end{matrix}}$$

# Extensions with VLQ iso-doublets

As for the gauge interactions we have

$$\left.\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \left[\overline{\mathcal{U}_L^0}\,\gamma^\mu\,\mathcal{D}_L^0 + \overline{U_R^0}\,\gamma^\mu\,D_R^0\right] + \text{h.c.}\right\}\ \textbf{flavor basis}$$

$$\left. = -\frac{g}{\sqrt{2}} W_\mu^+ \left[\overline{\mathcal{U}_L}\,\gamma^\mu\,V_L\,\mathcal{D}_L + \overline{\mathcal{U}_R}\,\gamma^\mu\,V_R\,\mathcal{D}_R\right] + \text{h.c.}\,,\right\}\ \textbf{mass basis}$$

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$$\left. = -\frac{g}{\sqrt{2}} W_\mu^+ \Big[\underbrace{\boxed{\overline{\mathcal{U}_L}\,\gamma^\mu\,V_L\,\mathcal{D}_L}}_{\text{LH currents}} + \overline{\mathcal{U}_R}\,\gamma^\mu\,V_R\,\mathcal{D}_R\Big] + \text{h.c.}\,,\right\}\ \textbf{mass basis}$$

$$\underbrace{\boxed{V_L = \mathcal{V}_L^{u\dagger}\mathcal{V}_L^d}}_{\text{Unitary}}$$

$$\mathcal{V}^q_{L,R} = \underbrace{\begin{pmatrix} A^q_{L,R} \\ \hdashline B^q_{L,R} \end{pmatrix}}_{3+N} \begin{matrix} \}\,3 \\ \}\,N \end{matrix}$$

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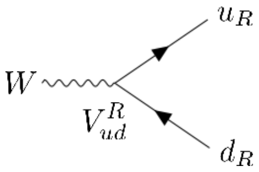
$$\left.\begin{aligned} &= -\frac{g}{\sqrt{2}}\, W_\mu^+ \overset{\text{LH currents}}{\boxed{\left[\overline{\mathcal{U}_L}\,\gamma^\mu\, V_L\, \mathcal{D}_L\right.}} + \overset{\text{RH currents}}{\boxed{\left.\overline{\mathcal{U}_R}\,\gamma^\mu\, V_R\, \mathcal{D}_R\right]}} + \text{h.c.}\,, \end{aligned}\right\}\ \textbf{mass basis}$$

$$\mathcal{V}_{L,R}^q = \left(\begin{array}{c} A_{L,R}^q \\ \hdashline B_{L,R}^q \end{array}\right)\begin{array}{l} \}\,3 \\ \}\,N \end{array}$$

3+*N*

$$\underset{\text{Unitary}}{\boxed{V_L = \mathcal{V}_L^{u\dagger}\mathcal{V}_L^d}} \qquad \underset{\text{Non-Unitary}}{\boxed{V_R = B_R^{u\dagger}B_R^d = \mathcal{V}_R^{u\dagger}\ \mathrm{diag}(0,0,0,1,\ldots,1)\,\mathcal{V}_R^d}}$$




RH Charged Currents

# Extensions with VLQ iso-doublets

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**LH currents** **RH currents**

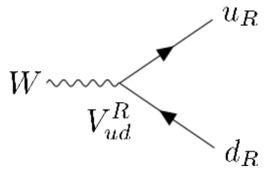
$$\mathcal{V}^q_{L,R} = \underbrace{\left(\begin{array}{c} A^q_{L,R} \\ \hdashline B^q_{L,R} \end{array}\right)}_{3+N}\begin{array}{l}\}\,3\\ \}\,N\end{array}$$

$$\boxed{V_L = \mathcal{V}^{u\dagger}_L \mathcal{V}^d_L}$$

**Unitary**

$$\boxed{V_R = B^{u\dagger}_R B^d_R = \mathcal{V}^{u\dagger}_R\ \text{diag}(0,0,0,1,\ldots,1)\,\mathcal{V}^d_R}$$

**Non-Unitary**




RH Charged Currents

$$\left.\mathcal{L}_Z = -\frac{g}{2c_W}\, Z_\mu \left[\overline{\mathcal{U}^0_L}\,\gamma^\mu\,\mathcal{U}^0_L - \overline{\mathcal{D}^0_L}\,\gamma^\mu\,\mathcal{D}^0_L + \overline{U^0_R}\,\gamma^\mu\,U^0_R - \overline{D^0_R}\,\gamma^\mu\,D^0_R - 2s^2_W J^\mu_{\text{em}}\right]\right\}\ \textbf{flavor basis}$$

$$\left. = -\frac{g}{2c_W}\, Z_\mu \left[\overline{\mathcal{U}_L}\,\gamma^\mu\,\mathcal{U}_L - \overline{\mathcal{D}_L}\,\gamma^\mu\,\mathcal{D}_L + \overline{\mathcal{U}_R}\,\gamma^\mu\,F_u\,\mathcal{U}_R - \overline{\mathcal{D}_R}\,\gamma^\mu\,F_d\,\mathcal{D}_R - 2s^2_W J^\mu_{\text{em}}\right]\right\}\ \textbf{mass basis}$$

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$$\left.= -\frac{g}{\sqrt{2}} W^+_\mu \underbrace{\boxed{\overline{\mathcal{U}_L}\,\gamma^\mu\,V_L\,\mathcal{D}_L}}_{\text{LH currents}} + \underbrace{\boxed{\overline{\mathcal{U}_R}\,\gamma^\mu\,V_R\,\mathcal{D}_R}}_{\text{RH currents}} + \text{h.c.}\,,\right\}\ \textbf{mass basis}$$

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3+*N*

$$\boxed{V_L = \mathcal{V}^{u\dagger}_L \mathcal{V}^d_L}$$

**Unitary**

$$\boxed{V_R = B^{u\dagger}_R B^d_R = \mathcal{V}^{u\dagger}_R\ \mathrm{diag}(0,0,0,1,\ldots,1)\,\mathcal{V}^d_R}$$

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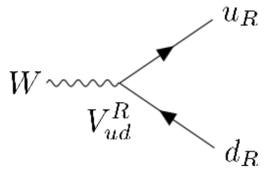
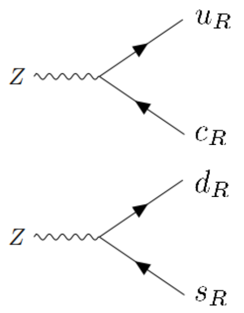



RH Charged Currents

$$\left.\mathcal{L}_Z = -\frac{g}{2c_W} Z_\mu \left[\overline{\mathcal{U}^0_L}\,\gamma^\mu\,\mathcal{U}^0_L - \overline{\mathcal{D}^0_L}\,\gamma^\mu\,\mathcal{D}^0_L + \overline{U^0_R}\,\gamma^\mu\,U^0_R - \overline{D^0_R}\,\gamma^\mu\,D^0_R - 2s^2_W J^\mu_{\text{em}}\right]\right\}\ \textbf{flavor basis}$$

$$\left.= -\frac{g}{2c_W} Z_\mu \left[\overline{\mathcal{U}_L}\,\gamma^\mu\,\mathcal{U}_L - \overline{\mathcal{D}_L}\,\gamma^\mu\,\mathcal{D}_L + \underbrace{\boxed{\overline{\mathcal{U}_R}\,\gamma^\mu\,F_u\,\mathcal{U}_R - \overline{\mathcal{D}_R}\,\gamma^\mu\,F_d\,\mathcal{D}_R}}_{\text{FCNCs}} - 2s^2_W J^\mu_{\text{em}}\right]\right\}\ \textbf{mass basis}$$

$$\boxed{\begin{aligned} F_u &= B^{u\dagger}_R B^u_R = V_R V^\dagger_R = \mathcal{V}^{u\dagger}_R\ \mathrm{diag}(0,0,0,1,\ldots,1)\,\mathcal{V}^u_R \\ F_d &= B^{d\dagger}_R B^d_R = V^\dagger_R V_R = \mathcal{V}^{d\dagger}_R\ \mathrm{diag}(0,0,0,1,\ldots,1)\,\mathcal{V}^d_R \end{aligned}}$$




Tree-Level FCNCs

# Approximate masses and mixing

Current bounds on VLQ masses **($M_Q$ > 1.15 TeV)** motivate expansions of observables on ***v/$M_Q$* << 1** and in terms of Lagrangian parameters.

For one doublet we can **decompose the mass matrices** as (analogous for the up sector):

$$\mathcal{M}_d = \begin{pmatrix} \hat{U}_L^{d\dagger} & \\ & 1 \end{pmatrix} \begin{pmatrix} v\hat{y}_d & & & \\ & v\hat{y}_s & & \\ & & v\hat{y}_b & \\ vz_d & vz_s & vz_b & M_Q \end{pmatrix} \begin{pmatrix} \hat{U}_R^{d} & \\ & 1 \end{pmatrix}$$

The **quark masses** are given by:

$$m_\alpha^2 = v^2 y_\alpha^2 = v^2 \hat{y}_\alpha^2 \left[1 - |z_\alpha|^2 \frac{v^2}{M_Q^2} + \mathcal{O}\left(\frac{v^4}{M_Q^4}\right)\right]$$

$$m_i^2 = v^2 y_i^2 = v^2 \hat{y}_i^2 \left[1 - |z_i|^2 \frac{v^2}{M_Q^2} + \mathcal{O}\left(\frac{v^4}{M_Q^4}\right)\right]$$

$$M_{T'}^2 = M_Q^2 \left[1 + |\boldsymbol{z}_u|^2 \frac{v^2}{M_Q^2} + \mathcal{O}\left(\frac{v^4}{M_Q^4}\right)\right]$$

$$M_{B'}^2 = M_Q^2 \left[1 + |\boldsymbol{z}_d|^2 \frac{v^2}{M_Q^2} + \mathcal{O}\left(\frac{v^4}{M_Q^4}\right)\right]$$

# Approximate masses and mixing

The RH mixings can be written as:

$$V_R = \begin{pmatrix} z_u^* z_d \frac{v^2}{M_Q^2} & z_u^* z_s \frac{v^2}{M_Q^2} & z_u^* z_b \frac{v^2}{M_Q^2} & -z_u^* \frac{v}{M_Q} \\ z_c^* z_d \frac{v^2}{M_Q^2} & z_c^* z_s \frac{v^2}{M_Q^2} & z_c^* z_b \frac{v^2}{M_Q^2} & -z_c^* \frac{v}{M_Q} \\ z_t^* z_d \frac{v^2}{M_Q^2} & z_t^* z_s \frac{v^2}{M_Q^2} & z_t^* z_b \frac{v^2}{M_Q^2} & -z_t^* \frac{v}{M_Q} \\ -z_d \frac{v}{M_Q} & -z_s \frac{v}{M_Q} & -z_b \frac{v}{M_Q} & 1 - \frac{1}{2}\left(|\boldsymbol{z}_u|^2 + |\boldsymbol{z}_d|^2\right) \frac{v^2}{M_Q^2} \end{pmatrix} + \mathcal{O}\left(\frac{v^3}{M_Q^3}\right)$$

$$\hat{V}^R_{\alpha i} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_i$$

For the 3x3 submatrices

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For the 3x3 submatrices

$$F_u = \begin{pmatrix} |z_u|^2 \frac{v^2}{M_Q^2} & z_u^* z_c \frac{v^2}{M_Q^2} & z_u^* z_t \frac{v^2}{M_Q^2} & -z_u^* \frac{v}{M_Q} \\ z_c^* z_u \frac{v^2}{M_Q^2} & |z_c|^2 \frac{v^2}{M_Q^2} & z_c^* z_t \frac{v^2}{M_Q^2} & -z_c^* \frac{v}{M_Q} \\ z_t^* z_u \frac{v^2}{M_Q^2} & z_t^* z_c \frac{v^2}{M_Q^2} & |z_t|^2 \frac{v^2}{M_Q^2} & -z_t^* \frac{v}{M_Q} \\ -z_u \frac{v}{M_Q} & -z_c \frac{v}{M_Q} & -z_t \frac{v}{M_Q} & 1 - |\boldsymbol{z}_u|^2 \frac{v^2}{M_Q^2} \end{pmatrix} + \mathcal{O}\left(\frac{v^3}{M_Q^3}\right)$$

$$\hat{F}^u_{\alpha i} \equiv \frac{v^2}{M_Q^2} z_\alpha^* z_\beta$$

$$\hat{F}^d_{ij} \equiv \frac{v^2}{M_Q^2} z_i^* z_j$$

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$$\boxed{\begin{aligned} \hat{F}^u_{\alpha i} &\equiv \frac{v^2}{M_Q^2} z_\alpha^* z_\beta \\ \hat{F}^d_{ij} &\equiv \frac{v^2}{M_Q^2} z_i^* z_j \end{aligned}}$$

# Approximate masses and mixing

While for the LH mixing we have:

$$V_L \simeq \begin{pmatrix} \mathbb{1}_{3\times 3} & -\hat{Y}_u \boldsymbol{z}_u^\dagger \frac{v^2}{M_Q^2} \\ \boldsymbol{z}_u \hat{Y}_u \frac{v^2}{M_Q^2} & 1 \end{pmatrix} \begin{pmatrix} \hat{V}_L & \\ & 1 \end{pmatrix} \begin{pmatrix} \mathbb{1}_{3\times 3} & \hat{Y}_d \boldsymbol{z}_d^\dagger \frac{v^2}{M_Q^2} \\ -\boldsymbol{z}_d \hat{Y}_d \frac{v^2}{M_Q^2} & 1 \end{pmatrix} + \mathcal{O}\left(\frac{v^4}{M_Q^4}\right)$$

$$\simeq \left(\begin{array}{c:c} \hat{V}_L \simeq \hat{U}_{uL}^\dagger U_{dL} & \mathcal{O}\left(yz\frac{v^2}{M_Q^2}\right) \\ \hdashline \mathcal{O}\left(yz\frac{v^2}{M_Q^2}\right) & 1 \end{array}\right)$$

For the 3x3 submatrix

$$V_{\alpha i}^L = \hat{V}_{\alpha i}^L + \mathcal{O}\left(y^2 z^2 \frac{v^4}{M_Q^4}\right)$$

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≈ the SM CKM mixing

Contrary to the VLQ iso-singlet case, here the LH mixing is unitary and the **couplings outside the 3x3 block are suppressed naturally**.

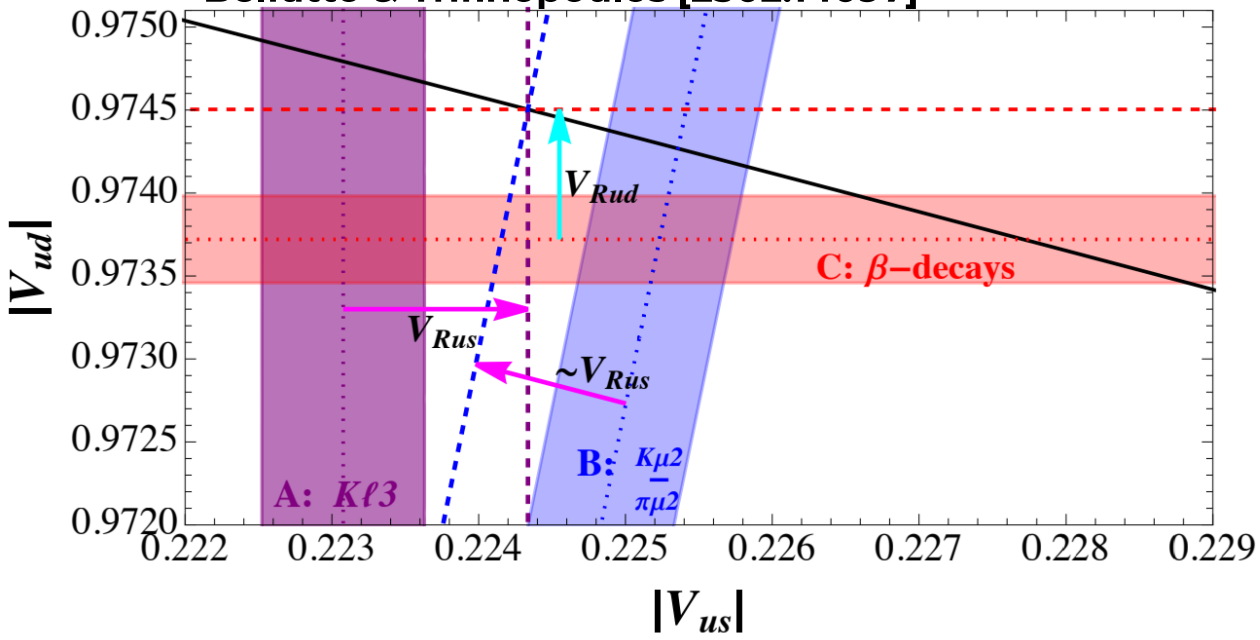
The **CAAs cannot be addressed** solely with the **LH currents**.

# Addressing the CAAs with VLQ doublets

The presence of **both LH and RH charged currents** makes it possible to **accommodate both CAA1 and CAA2**.

$$\mathcal{L}_{cc} \supset -\frac{g}{2\sqrt{2}} W_\mu^+ \overline{(u \quad c \quad t)} \left[\gamma^\mu \underbrace{\boxed{(\hat{V}_L + \hat{V}_R)}}_{\text{Vector coupling}} - \gamma^\mu\gamma^5 \underbrace{\boxed{(\hat{V}_L - \hat{V}_R)}}_{\text{Axial coupling}}\right] \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

**Belfatto & Trifinopoulos [2302.14097]**




Now **each measurement** is associated with a **distinct NP contribution** beyond the LH mixing:

Vector coupling $\boxed{K\ell3}$: $|V_{us}|_A = |\hat{V}^L_{us} + \hat{V}^R_{us}|$

Axial coupling $\boxed{K\mu2/\pi\mu2}$: $|V_{us}|/|V_{ud}|_B \approx \dfrac{|\hat{V}^L_{us} - \hat{V}^R_{us}|}{|\hat{V}^L_{ud} - \hat{V}^R_{ud}|}$

Vector coupling $\boxed{\beta - \text{decays}}$: $|V_{ud}|_C = |\hat{V}^L_{ud} + \hat{V}^R_{ud}|$

# Weak Basis Invariants

**Weak Basis Invariants** remain **unchanged under weak-basis transformations** (WBTs) which leave EW currents flavor-diagonal. **CP-odd WBIs point to sources of CPV**.

WBTs with VLQ iso-doublets:

$$\mathcal{U}_L^0 \to \boxed{\mathcal{W}_L}\,\mathcal{U}_L^0 \qquad \mathcal{D}_L^0 \to \boxed{\mathcal{W}_L}\,\mathcal{D}_L^0$$

(3+*N*)x(3+*N*) unitary

$$u_R^0 \to \boxed{W_R^u}\,u_R^0 \qquad d_R^0 \to \boxed{W_R^d}\,d_R^0$$

3x3 unitary

$$U_R^0 \to \boxed{W_R}\,U_R^0 \qquad D_R^0 \to \boxed{W_R}\,D_R^0$$

*N*x*N* unitary

Hermitian "building blocks" (all transforming as $X \to \mathcal{W}_L^\dagger X \mathcal{W}_L$)

$$H = MM^\dagger \qquad \mathcal{H}_q = \mathcal{M}_q\mathcal{M}_q^\dagger \qquad h_q = m_q m_q^\dagger = \mathcal{H}_q - H$$

$$\mathcal{M}_q = \left(\begin{array}{c:c} \overbrace{m_q}^{3} & \overbrace{M}^{N} \end{array}\right)\Bigg\}\,3+N$$

More **CP violating couplings imply more independent CP-odd WBIs**.
The WBI of lowest mass dimension is:

$$\mathrm{Im}\,\mathrm{Tr}\left(\mathcal{H}_u\mathcal{H}_d H\right) = 2i\sum_{\alpha,i} m_{u_\alpha}^3 m_{d_i}^3\,\mathrm{Im}\left(V_{\alpha i}^{L*}V_{\alpha i}^R\right)$$

# The Extreme Chiral Limit

With VLQs we can even obtain CP violation in the **limit of extremely high energies** (extreme chiral limit) where $\boldsymbol{m_u = m_c = m_d = m_s = 0}$ and there is **no CPV in the SM**.

With one VLQ doublet, in this limit the mass matrices reduce to the minimal form:

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{y}_t v & 0 \\ 0 & z_c v & z_t v & M_Q \end{pmatrix} \qquad \mathcal{M}_d = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & y_{23} v & 0 \\ 0 & 0 & y_{33} v & 0 \\ 0 & z_s v & \boxed{z_b v} & M_Q \end{pmatrix}$$

complex

CP violation may be present in this limit, as in general **one CP-odd WBI survives**

$$\frac{1}{M_Q^6} \mathrm{Im}\mathrm{Tr}\left[\mathcal{H}_u \mathcal{H}_d H\right] = \frac{v^4}{M_Q^4} \mathrm{Im}\left(\hat{y}_t y_{33} z_t z_b^*\right) \simeq \frac{m_t m_b}{M_Q^2} \mathrm{Im}\left(V_{tb}^L V_{tb}^{R*}\right)$$

# Weak Basis Invariants

We find a larger number and variety of independent CP-odd WBIs, signaling **many more sources of CP violation**.

In the SM, CP violation hinges solely on one WBI $\boxed{\mathrm{tr}\,[h_u, h_d]^3 \propto J}$

With **one VLQ doublet** alone we find a **much more complex situation**:

3-blocks: $\boxed{\mathrm{I}\,(n+m,\mathbf{3}) \equiv \mathrm{ImTr}\,(\mathcal{H}_u^n \mathcal{H}_d^m H)}$

4-blocks: $\boxed{\begin{array}{l}\mathrm{I}\,(n+m+p,\mathbf{4})_u \equiv \mathrm{ImTr}\,(\mathcal{H}_u^n \mathcal{H}_d^p \mathcal{H}_u^m H) \\ \mathrm{I}\,(n+m+p,\mathbf{4})_d \equiv \mathrm{ImTr}\,(\mathcal{H}_d^n \mathcal{H}_u^p \mathcal{H}_d^m H)\end{array}}$

5-blocks: $\boxed{\mathrm{I}\,(n+m+p+q,\mathbf{5}) \equiv \mathrm{ImTr}\,(\mathcal{H}_u^n \mathcal{H}_d^p \mathcal{H}_u^m \mathcal{H}_d^q H)}$

SM - like: $\boxed{\mathrm{I}\,(12,4) \equiv \mathrm{ImTr}\,(\mathcal{H}_u^2 \mathcal{H}_d^2 \mathcal{H}_u \mathcal{H}_d)}$

$(q, q' = u, d \quad q \neq q')$

| $\#z_{q_i}$ | $\#z_{q'_\alpha}$ | $\#\delta$ | $\#\mathrm{I}(\mathrm{M} \geq 6, 3)$ | $\#\mathrm{I}(\mathrm{M} \geq 8, 4)_q$ | $\#\mathrm{I}(\mathrm{M} \geq 8, 4)_{q'}$ | $\mathrm{I}(10, 5)$ | $\mathrm{I}^{\text{SM-like}}_{(12,4)}$ |
|---|---|---|---|---|---|---|---|
| 3 | 3 | 6 | 9 | | | | |
| 3 | 2 | 5 | 7 | | | | |
| 2 | 2 | 4 | 4 | | | | |
| 3 | 1 | 4 | 3 | 1 | | | |
| 2 | 1 | 3 | 2 | 1 | | | |
| 3 | 0 | 3 | $=0$ | 6 | $=0$ | $=0$ | |
| 2 | 0 | 2 | $=0$ | 2 | $=0$ | $=0$ | |
| 1 | 1 | 2 | 1 | | | 1 | |
| 1 | 0 | 1 | $=0$ | $=0$ | $=0$ | $=0$ | 1 |

Number and structure of **WBIs whose vanishing is needed for CPI** in each possible scenario

# Rephasing Invariants

Under **general rephasing transformations** $\mathcal{D}_i \to \mathcal{D}_i\, e^{i\varphi_i}$ and $\mathcal{U}_\alpha \to \mathcal{U}_\alpha\, e^{i\varphi_\alpha}$ we have

$$V^L_{\alpha i} \to e^{-i(\varphi_\alpha - \varphi_i)}\, V^L_{\alpha i} \qquad F^u_{\alpha\beta} \to e^{-i(\varphi_\alpha - \varphi_\beta)}\, F^u_{\alpha\beta}$$
$$V^R_{\alpha i} \to e^{-i(\varphi_\alpha - \varphi_i)}\, V^R_{\alpha i} \qquad F^d_{ij} \to e^{-i(\varphi_i - \varphi_j)}\, F^d_{ij}$$

and we can build the following **complex rephasing invariants (RIs)**

**Bilinears:** $\mathcal{B}_{\alpha i} = V^L_{\alpha i} V^{R*}_{\alpha i}$

**Trilinears:** $\mathcal{T}_{i,\alpha\beta} = V^{L*}_{\alpha i} V^{L*}_{\beta i} F^u_{\alpha\beta}$

$\mathcal{T}_{\alpha,ij} = V^{L*}_{\alpha i} V^{L*}_{\alpha j} F^d_{ij}$

**Quartets:** $\mathcal{Q}_{\alpha i\beta j} = V^L_{\alpha i} V^R_{\beta j} V^{L*}_{\alpha j} V^{L*}_{\beta i}$

$\boxed{Q_{\alpha i\beta j} = V^L_{\alpha i} V^L_{\beta j} V^{L*}_{\alpha j} V^{L*}_{\beta i}}$

**SM-Like**

For the 3x3 sector of SM

$\simeq$

*Effective RIs*

$$\hat{\mathcal{B}}_{\alpha i} \equiv \frac{v^2}{M_Q^2} z^*_\alpha z_i \hat{V}^{L*}_{\alpha i} \equiv \hat{V}^R_{\alpha i} \hat{V}^{L*}_{\alpha i}$$

$$\hat{\mathcal{T}}_{i,\alpha\beta} \equiv \frac{v^2}{M_Q^2} z^*_\alpha z_\beta \hat{V}^{L*}_{\alpha i} \hat{V}^L_{\beta i} \equiv \hat{F}^u_{\alpha\beta} \hat{V}^{L*}_{\alpha i} \hat{V}^L_{\beta i}$$

$$\hat{\mathcal{T}}_{\alpha,ij} \equiv \frac{v^2}{M_Q^2} z^*_i z_j \hat{V}^L_{\alpha i} \hat{V}^{L*}_{\alpha j} \equiv \hat{F}^d_{ij} \hat{V}^L_{\alpha i} \hat{V}^{L*}_{\alpha j}$$

$$\hat{\mathcal{Q}}_{\alpha i\beta j} \equiv \frac{v^2}{M_Q^2} z^*_\alpha z_i \hat{V}^L_{\beta j} \hat{V}^{L*}_{\alpha j} \hat{V}^{L*}_{\beta i} \equiv \hat{V}^R_{\alpha i} \hat{V}^L_{\beta j} \hat{V}^{L*}_{\alpha j} \hat{V}^{L*}_{\beta i}$$

$$\hat{Q}_{\alpha i\beta j} \equiv \hat{V}^L_{\alpha i} \hat{V}^L_{\beta j} \hat{V}^{L*}_{\alpha j} \hat{V}^{L*}_{\beta i}$$

# Connecting Rephasing Invariants and WBIs

We can establish unique connections between the **structure of CP-odd WBIs** and the **types of *effective* rephasing invariants**

**3-blocks:** $$\frac{1}{M_Q^6}\mathrm{Im}\mathrm{Tr}\left(\mathcal{H}_u\mathcal{H}_d H\right) \simeq \frac{v^2}{M_Q^2} y_\alpha y_i \boxed{\mathrm{Im}\hat{\mathcal{B}}_{\alpha i}}$$ **bilinear**

**4-blocks:** $$\frac{1}{M_Q^{10}}\mathrm{Im}\mathrm{Tr}\left(\mathcal{H}_d\mathcal{H}_u\mathcal{H}_d^2 H\right) \simeq \frac{v^4}{M_Q^4} y_\alpha y_i^3\, \mathrm{Im}\hat{\mathcal{B}}_{\alpha i} + \frac{v^6}{M_Q^6} y_\alpha y_\beta y_i^3 y_j\, \mathrm{Im}\left(\hat{\mathcal{B}}_{\alpha i}\hat{\mathcal{B}}_{\beta j}\right) + \frac{v^8}{M_Q^8} y_\alpha^4 y_i^3 y_j \boxed{\mathrm{Im}\hat{\mathcal{T}}_{\alpha,ij}}$$ **trilinear**

**5-blocks:** $$\frac{1}{M_Q^{10}}\mathrm{Im}\mathrm{Tr}\left(\mathcal{H}_d\mathcal{H}_u\mathcal{H}_d\mathcal{H}_u H\right) \simeq \frac{v^2}{M_Q^2} y_\alpha y_i\, \mathrm{Im}\hat{\mathcal{B}}_{\alpha i} + \frac{v^6}{M_Q^6} y_\alpha y_\beta^2 y_i y_j^2 \boxed{\mathrm{Im}\hat{\mathcal{Q}}^*_{\alpha i\beta j}}$$ ***exotic*-quartet**

**SM - like:** $$\frac{1}{M_Q^{12}}\mathrm{Im}\mathrm{Tr}\left(\mathcal{H}_d^2\mathcal{H}_u^2\mathcal{H}_d\mathcal{H}_u\right) \sim (\ldots)\mathrm{Im}\hat{\mathcal{B}}_{\alpha i} + (\ldots)\mathrm{Im}\hat{\mathcal{T}}_{\alpha,ij} + (\ldots)\mathrm{Im}\hat{\mathcal{T}}_{i,\alpha\beta} + (\ldots)\mathrm{Im}\hat{\mathcal{Q}}_{\alpha i\beta j} + \frac{v^{12}}{M_Q^{12}} \boxed{\mathrm{Im}\hat{\mathcal{Q}}_{\alpha i\beta j}}$$

***SM-like*-quartet**
$\approx J_{SM}$

**In the SM:** $$\boxed{\frac{1}{v^{12}}[h_u h_u^\dagger, h_d h_d^\dagger]^3 = [Y_u Y_u^\dagger, Y_d Y_d^\dagger]^3 \sim 10^{-25}}$$

# Bounds from Phenomenology

Bounds on **CP violating RIs** from flavor observables in the **limit where the mixing can address the CAAs**

$$\left|V_{ud}\right|_{\beta} = \left|\hat{V}^L_{ud} + \hat{V}^R_{ud}\right| \simeq \left|\hat{V}^L_{ud}\right| + \frac{\mathrm{Re}\hat{\mathcal{B}}_{ud}}{|\hat{V}^L_{ud}|}$$

$$\left|V_{us}\right|_{K\ell 3} = \left|\hat{V}^L_{us} + \hat{V}^R_{us}\right| \simeq \left|\hat{V}^L_{us}\right| + \frac{\mathrm{Re}\hat{\mathcal{B}}_{us}}{|\hat{V}^L_{us}|}$$

$$\left|\frac{V_{us}}{V_{ud}}\right|_{\frac{K\mu 2}{\pi\mu 2}} = \frac{\left|\hat{V}^L_{us} - \hat{V}^R_{us}\right|}{\left|\hat{V}^L_{ud} - \hat{V}^R_{ud}\right|} \simeq \frac{|\hat{V}^L_{us}|}{|\hat{V}^L_{ud}|}\left(1 - \frac{\mathrm{Re}\hat{\mathcal{B}}_{us}}{|\hat{V}^L_{us}|^2} + \frac{\mathrm{Re}\hat{\mathcal{B}}_{ud}}{|\hat{V}^L_{ud}|^2}\right)$$

$$\Downarrow$$

$$\frac{\mathrm{Re}\,\hat{\mathcal{B}}_{ud}}{|\hat{V}^L_{ud}|} = -0.79(27)\times 10^{-3}$$

$$\frac{\mathrm{Re}\,\hat{\mathcal{B}}_{us}}{|\hat{V}^L_{us}|} = -1.24(37)\times 10^{-3}$$

| Conventional name | Bounds on VLQ invariants |
|---|---|
| $\lvert\omega\rvert$ | $\frac{\lvert\hat{V}^L_{us}\rvert\,\mathrm{Re}\,\hat{\mathcal{B}}_{ud}}{\lvert\hat{V}^L_{ud}\rvert} - \frac{\lvert\hat{V}^L_{ud}\rvert\,\mathrm{Re}\,\hat{\mathcal{B}}_{us}}{\lvert\hat{V}^L_{us}\rvert} \lesssim 10^{-3}$ |
| $\mathrm{Re}\,\frac{\epsilon'}{\epsilon}$ | $\left\lvert\frac{\lvert\hat{V}^L_{us}\rvert\,\mathrm{Im}\,\hat{\mathcal{B}}_{ud}}{\lvert\hat{V}^L_{ud}\rvert} + \frac{\lvert\hat{V}^L_{ud}\rvert\,\mathrm{Im}\,\hat{\mathcal{B}}_{us}}{\lvert\hat{V}^L_{us}\rvert} - \frac{\mathrm{Im}\,\hat{\mathcal{T}}_{u,ds}\cos^2\theta_W}{\lvert\hat{V}^L_{us}\hat{V}^L_{ud}\rvert}\right\rvert \lesssim 6\times 10^{-7}$ |
| $d_{n,p}$ | $\frac{\mathrm{Im}\,\hat{\mathcal{B}}_{ud}}{\lvert\hat{V}^L_{ud}\rvert}, \frac{\mathrm{Im}\,\hat{\mathcal{B}}_{us}}{\lvert\hat{V}^L_{us}\rvert} \lesssim 3 - 6\times 10^{-6}$ |
| $\Gamma(Z\to \mathrm{had})$ | $\hat{F}^u_{\alpha\alpha}, \hat{F}^d_{ii} \lesssim 5\times 10^{-3}$ (for $\alpha \neq t$) |
| $\lvert\epsilon\rvert$, $\Delta m_K$ | $\frac{\lvert\hat{\mathcal{T}}_{u,ds}\rvert}{\lvert\hat{V}^L_{ud}\rvert\lvert\hat{V}^L_{us}\rvert} < 6\times 10^{-7} - 2\times 10^{-4}$ |
| $\mathrm{Br}(K_L\to\pi^0\nu\bar{\nu})$ | $\frac{\lvert\mathrm{Im}\,\hat{\mathcal{T}}_{u,ds}\rvert}{\lvert\hat{V}^L_{ud}\hat{V}^L_{us}\rvert} < 2\times 10^{-5}$ |
| $\mathrm{Br}(K^+\to\pi^+\nu\bar{\nu})$ | $\frac{\lvert\hat{\mathcal{T}}_{u,ds}\rvert}{\lvert\hat{V}^L_{ud}\hat{V}^L_{us}\rvert} < (3 - 8)\times 10^{-6}$ |

# Summary/Conclusions

- Extension with VLQs doublets provide the most promising solutions to the CAAs.
- A wide variety of possible new sources of CPV arise in these models. This is manifested via the appearance of new types of rephasing and weak basis invariants.
- The new sources can potentially be subject to much less suppression than the SM ones
- Some of these sources may allow for CPV at extremely high energies.
- We established a direct and unambiguous connection between WBIs and effective rephasing invariants and thus to flavor observables.

# Thank You!

Couplings to the Higgs:

$$\mathcal{L}_h = -\frac{h}{\sqrt{2}}\left[\overline{\mathcal{U}_L^0}\begin{pmatrix} Y_u & 0 \\ Z_u & 0 \end{pmatrix}\mathcal{U}_R^0 + \overline{\mathcal{D}_L^0}\begin{pmatrix} Y_d & 0 \\ Z_d & 0 \end{pmatrix}\mathcal{D}_R^0\right] + \text{h.c.}$$

$$= -\frac{h}{\sqrt{2}}\left[\overline{\mathcal{U}_L}\,\frac{\mathcal{D}_u}{v}\,(\mathbb{1} - F_u)\,\mathcal{U}_R + \overline{\mathcal{D}_L}\,\frac{\mathcal{D}_d}{v}\,(\mathbb{1} - F_d)\,\mathcal{D}_R\right] + \text{h.c.}\,,$$

Relation between LH, RH matrices and FCNCs:

$$\mathcal{V}_L^u\,\mathcal{D}_u\,B_R^{u\,\dagger} = \mathcal{V}_L^d\,\mathcal{D}_d\,B_R^{d\,\dagger}$$

$$\mathcal{D}_u\,V_R = V_L\,\mathcal{D}_d\,F_d\,, \qquad \mathcal{D}_d\,V_R^\dagger = V_L^\dagger\,\mathcal{D}_u\,F_u$$

$$\mathcal{D}_u\,F_u\,\mathcal{D}_u\,V_L = V_L\,\mathcal{D}_d\,F_d\,\mathcal{D}_d$$

$$d_n \simeq \mathrm{Im}\left[(1.4 \pm 0.7)\,\hat{V}^{L*}_{ud}\hat{V}^{R}_{ud} + (2.7 \pm 1.3)\,\hat{V}^{L*}_{us}\hat{V}^{R}_{us}\right] \times 10^{-7}\ \mathrm{e\ fm}\,,$$

$$d_p \simeq \mathrm{Im}\left[-\,(2.7 \pm 1.3)\,\hat{V}^{L*}_{ud}\hat{V}^{R}_{ud} - (3.6 \pm 1.5)\,\hat{V}^{L*}_{us}\hat{V}^{R}_{us}\right] \times 10^{-7}\ \mathrm{e\ fm}$$

$$\left|(2.3 \pm 1.1)\,\frac{\mathrm{Im}\left[\hat{V}^{L*}_{ud}\hat{V}^{R}_{ud}\right]}{|\hat{V}^{L}_{ud}|} + (1.0 \pm 0.5)\,\frac{\mathrm{Im}\left[\hat{V}^{L*}_{us}\hat{V}^{R}_{us}\right]}{|\hat{V}^{L}_{us}|}\right| \lesssim 3 \times 10^{-6}\,,$$

$$\left|(3.3 \pm 1.6)\,\frac{\mathrm{Im}\left[\hat{V}^{L*}_{ud}\hat{V}^{R}_{ud}\right]}{|\hat{V}^{L}_{ud}|} + (1.0 \pm 0.4)\,\frac{\mathrm{Im}\left[\hat{V}^{L*}_{us}\hat{V}^{R}_{us}\right]}{|\hat{V}^{L}_{us}|}\right| \lesssim 6 \times 10^{-6}\,,$$

$$M_{12}^{\mathrm{NP}} \simeq -\frac{1}{3} m_K f_K^2\, 0.43 \left\{ \frac{G_F}{\sqrt{2}}\, \hat{F}_{ds}^2 + \frac{G_F^2}{4\pi^2} \left[ \frac{1}{2} M_Q^2\, \hat{F}_{ds}^2 \right.\right.$$

$$\left.\left. - 3.1 \frac{m_K^2}{(m_d+m_s)^2}\, \hat{F}_{ds}\, \hat{V}_{td}^{L*} \hat{V}_{ts}^{L}\, m_W^2 f(x_Q, x_t) \right] \right\}$$

$$\frac{\mathrm{Im}\left[ M_{12}^{\mathrm{NP}} (\lambda_u)^2 \right]}{|\lambda_u|^2} = \frac{1}{3} m_K f_K^2\, 0.43\ \mathrm{Im} \left\{ \frac{G_F}{\sqrt{2}} \frac{\left( \hat{\mathcal{T}}_{u,ds} \right)^2}{|\lambda_u|^2} + \frac{G_F^2}{4\pi^2} \left[ \frac{1}{2} M_Q^2 \frac{\left( \hat{\mathcal{T}}_{u,ds} \right)^2}{|\lambda_u|^2} \right.\right.$$

$$\left.\left. - 3.1 \frac{m_K^2}{(m_d+m_s)^2} \frac{\hat{\mathcal{T}}_{u,ds}\, \hat{Q}_{tsud}}{|\lambda_u|^2}\, m_W^2 f(x_Q, x_t) \right] \right\}$$

$$\frac{\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})}{\mathrm{Br}(K_L \to \pi^0 \nu \bar{\nu})_{\mathrm{SM}}} \simeq \left| 1 - \frac{\frac{1}{2}\, \mathrm{Im}\left[ \hat{V}_{us}^{L} \hat{V}_{ud}^{L*}\, \hat{F}_{sd} \right]}{\mathrm{Im}\left[ \hat{V}_{us}^{L} \hat{V}_{ud}^{L*}\, \frac{\alpha}{2\pi \sin^2\theta_W} \left( \hat{V}_{cs}^{L*} \hat{V}_{cd}^{L} X(x_c) + \hat{V}_{ts}^{L*} \hat{V}_{td}^{L} X(x_t) \right) \right]} \right|^2$$

$$K^+ \to \pi^+ \nu \bar{\nu} \quad : \qquad 0.98 \lesssim \left| - \frac{\frac{1}{2} \left( \hat{V}_{us}^{L} \hat{V}_{ud}^{L*} \right) \hat{F}_{sd}}{\frac{\alpha}{2\pi \sin^2\theta_W} \left( \hat{V}_{us}^{L} \hat{V}_{ud}^{L*} \right) \left( \hat{V}_{cs}^{L*} \hat{V}_{cd}^{L} X(x_c) + \hat{V}_{ts}^{L*} \hat{V}_{td}^{L} X(x_t) \right)} + 1 \right| \lesssim 1.44$$