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INSTITUT DE FÍSICA CORPUSCULAR

VNIVERSITAT DE VALÈNCIA

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

LFV decays in a realistic U(2) model of flavor

based on 2505.20281, A. Giarnetti, SM, D. Meloni, M. Rettaroli

FLASY
ROME 2025

Simone Marciano July 1st, 2025 Roma

Outline of the talk

➡ **Standard Model and its Drawbacks**: a (very short) overview

➡ **Lepton Flavor Violation**:

- why are we interested in (charged)LFV?

➡ **cLFV from heavy New Physics**:

- Bringing a Realistic Flavor Model Back to Life

➡ **Outlooks:**

- what if the NP is not so heavy?

The Standard Model

$$\mathscr{L}_{\mathrm{SM}} = \mathscr{L}_{\mathrm{g}} + \mathscr{L}_{\mathrm{y}} + \mathscr{L}_{\mathrm{h}}$$

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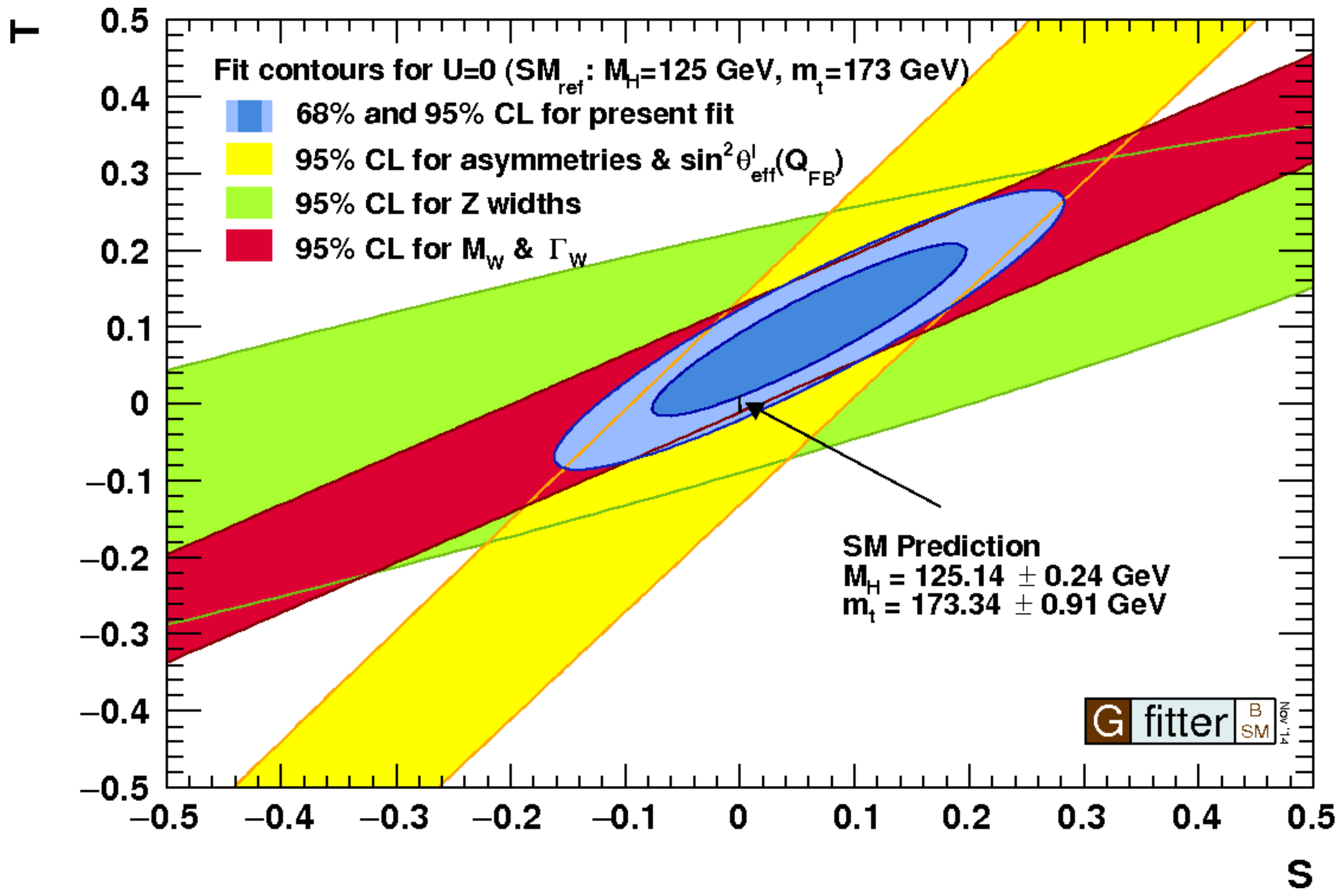
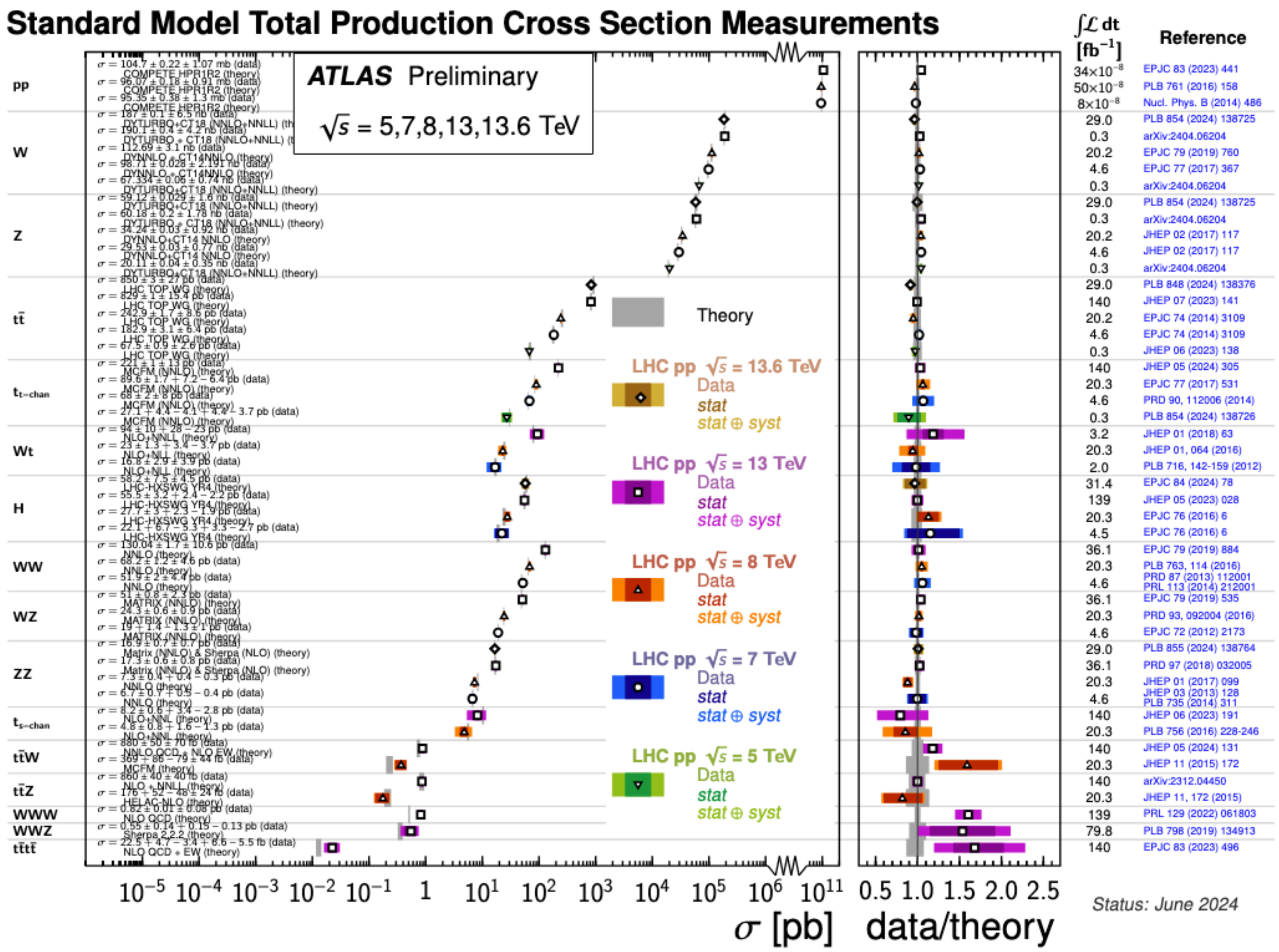
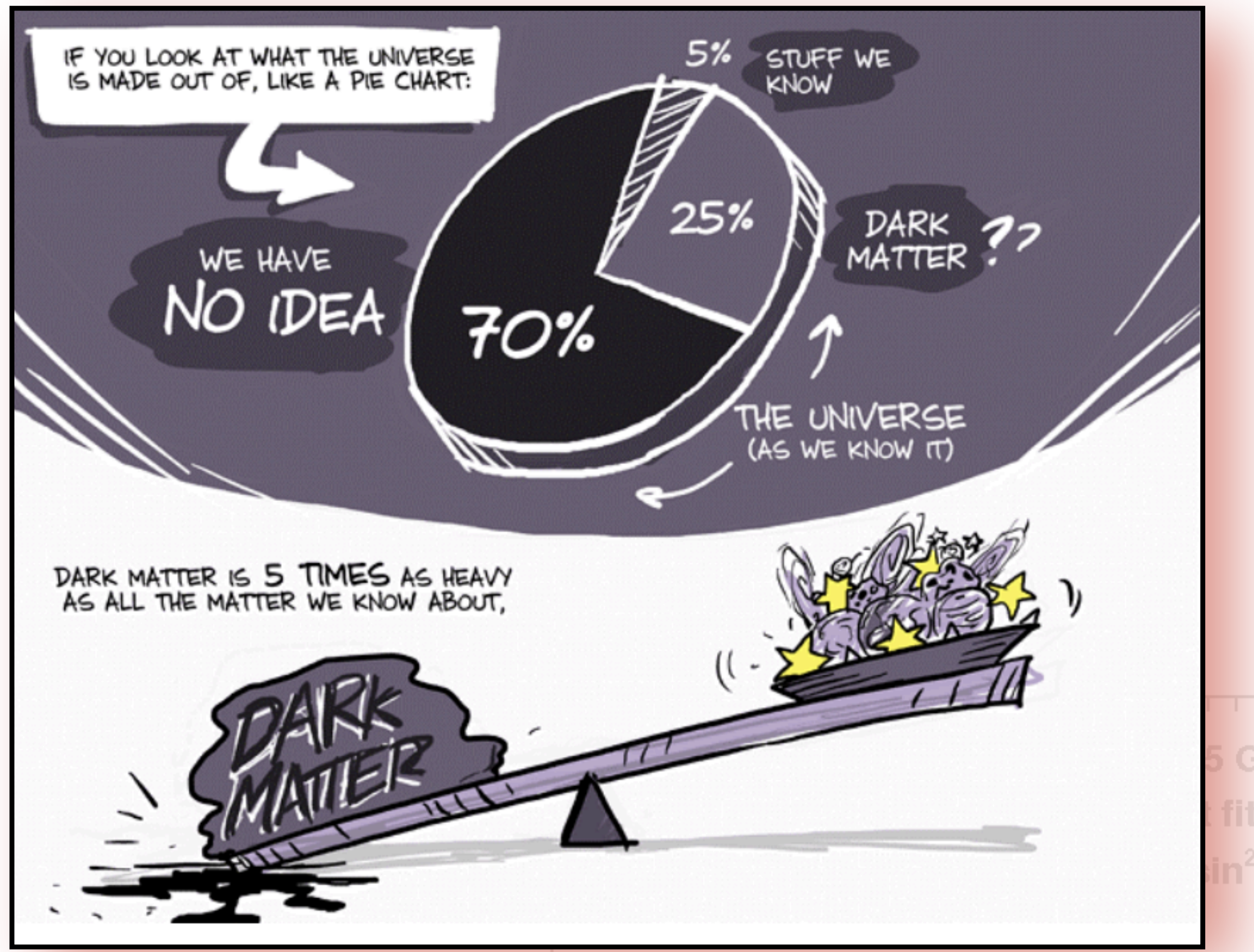
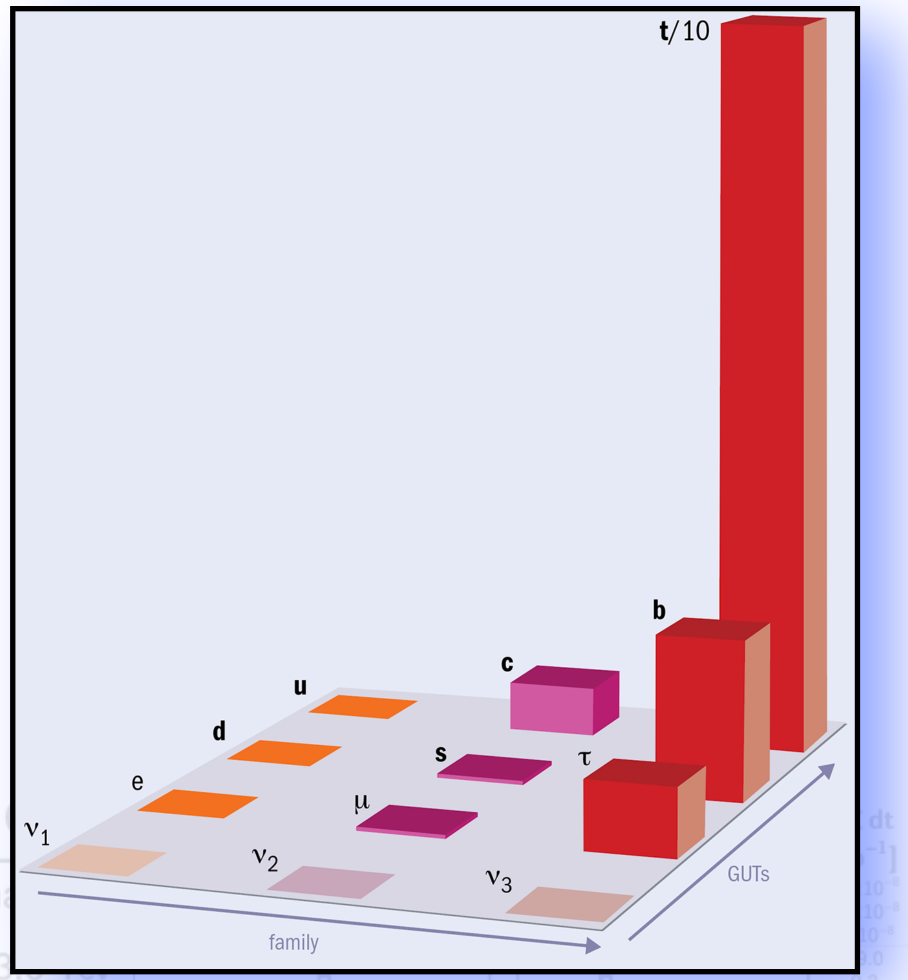




The Standard Model and its drawbacks





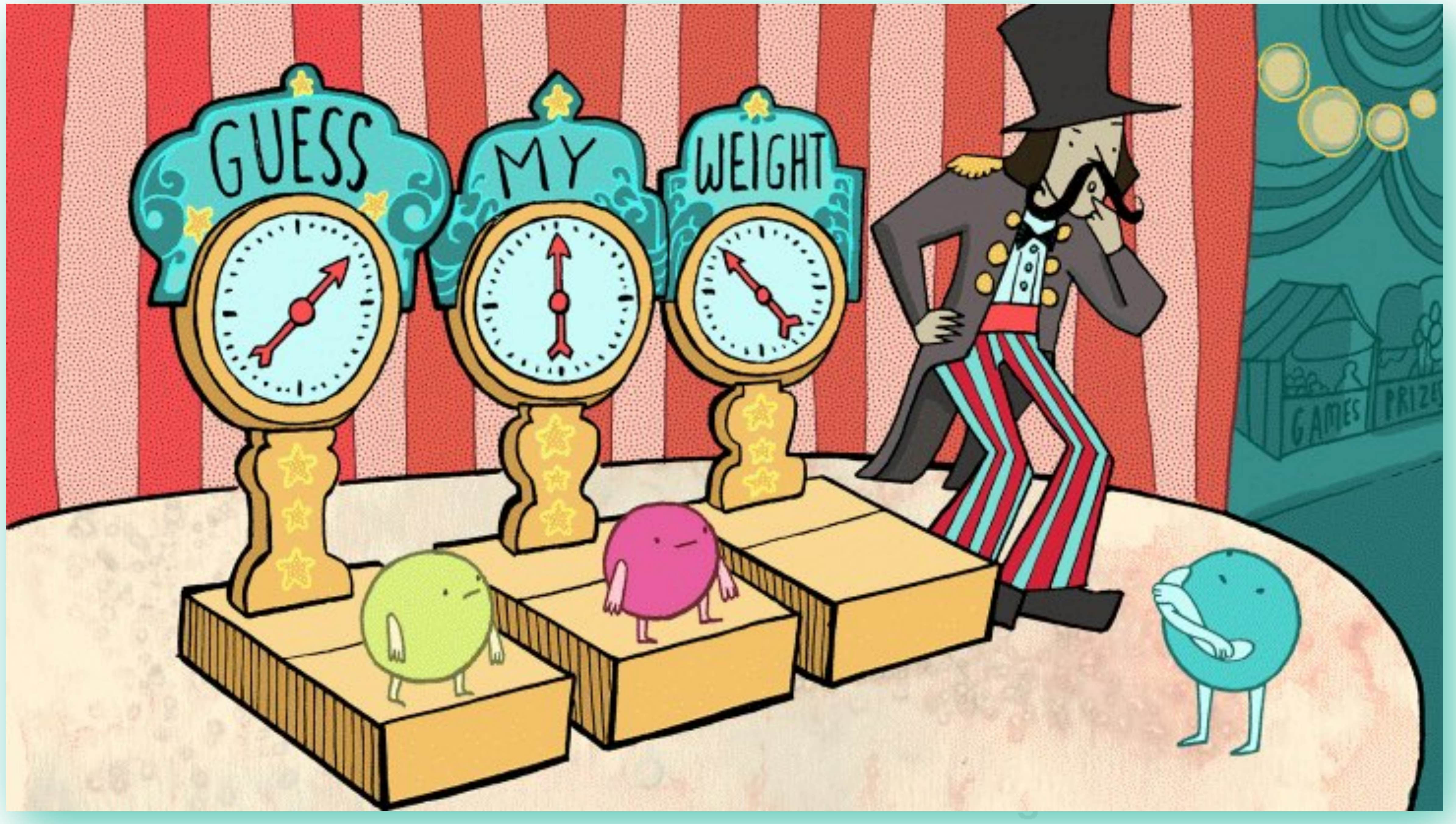
- Strong CP problem
- Flavor puzzle
- Dark Matter
- Baryon asymmetry of the Universe

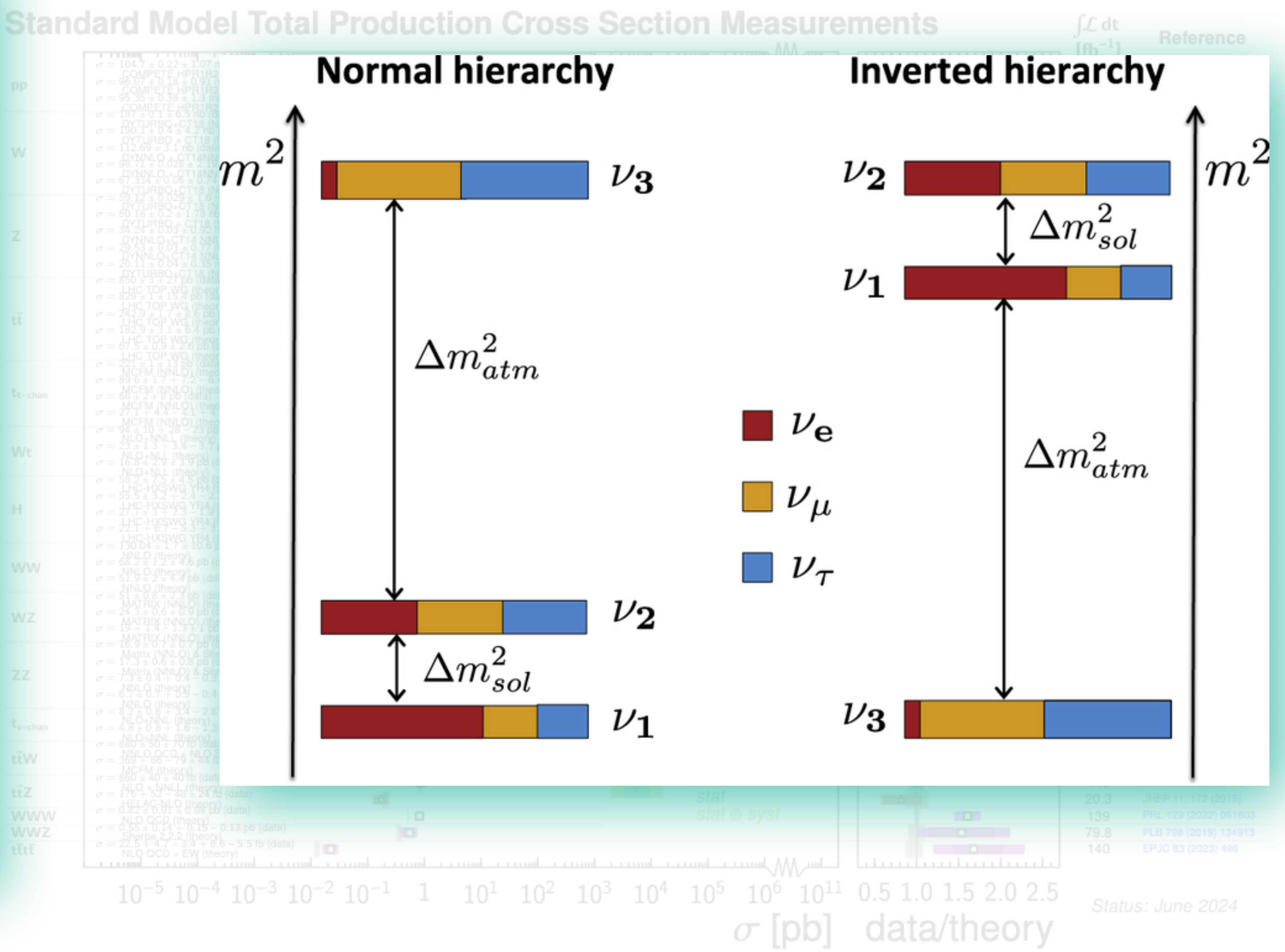
$$\eta_B \equiv \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 \approx 6.1 \cdot 10^{-10}$$

The Standard Model and its drawbacks

$\mathscr{L}_{SM} = \mathscr{L}_g + \mathscr{L}_y + \mathscr{L}_h$

- Neutrino masses: who ordered that?





Neutrinos in focus: entering the precision Era

DUNE
DEEP UNDERGROUND NEUTRINO EXPERIMENT

ハイパーカミオカンデ
Hyper-Kamiokande

ICECUBE GEN2

JUNO
江门中微子实验
Jiangmen Underground Neutrino Observatory

essvsb+
ESS neutrino Super Beam plus

Nu IT 6.0 (2024)

		Normal Ordering ($\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
IC19 without SK atmospheric data	$\sin^2\theta_{12}$	0.307 $^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$	0.308 $^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	33.68 $^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$	33.68 $^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	0.561 $^{0\ 012}_{0\ 015}$	$0.430 \to 0.596$	0.562 $^{0\ 012}_{0\ 015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	48.5 $^{0\ 7}_{0\ 9}$	$41.0 \to 50.5$	48.6 $^{0\ 7}_{0\ 9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	0.02195 $^{0\ 00054}_{0\ 00058}$	$0.02023 \to 0.02376$	0.02224 $^{0\ 00056}_{0\ 00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	8.52 $^{0\ 11}_{0\ 11}$	$8.18 \to 8.87$	8.58 $^{0\ 11}_{0\ 11}$	$8.24 \to 8.91$
	CP$/^\circ$	177 $^{19}_{20}$	$96 \to 422$	285 $^{25}_{28}$	$201 \to 348$
	$\frac{m^2_{21}}{10\ ^5\ \text{eV}^2}$	7.49 $^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$	7.49 $^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10\ ^3\ \text{eV}^2}$	+2.534 $^{0\ 025}_{0\ 023}$	$+2.463 \to +2.606$	2.510 $^{0\ 024}_{0\ 025}$	$2.584 \to \ \ 2.438$
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	$\sin^2\theta_{23}$	0.470 $^{0\ 017}_{0\ 013}$	$0.435 \to 0.585$	0.550 $^{0\ 012}_{0\ 015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	43.3 $^{1\ 0}_{0\ 8}$	$41.3 \to 49.9$	47.9 $^{0\ 7}_{0\ 9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	0.02215 $^{0\ 00056}_{0\ 00058}$	$0.02030 \to 0.02388$	0.02231 $^{0\ 00056}_{0\ 00056}$	$0.02060 \to 0.02409$
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JHEP 12 (2024) 216

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ESS neutrino Super Beam plus

+Great opportunity to probe also BSM scenarios!

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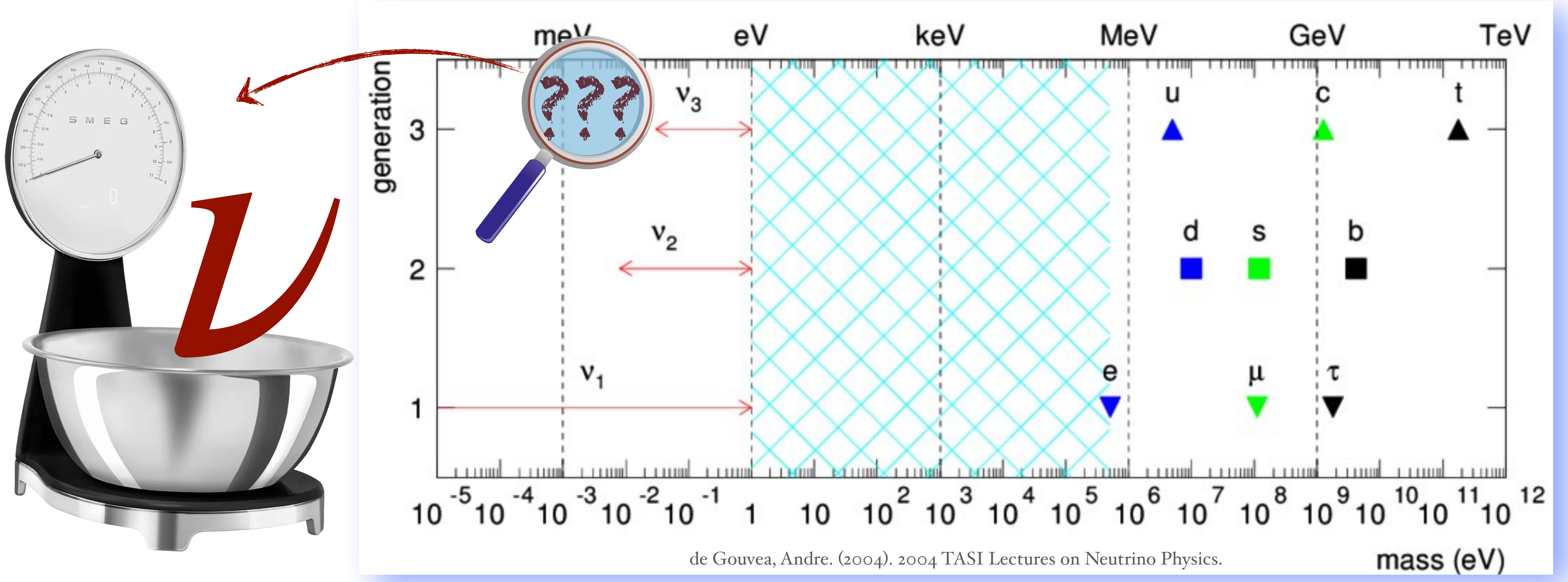
Neutrino masses and LFV



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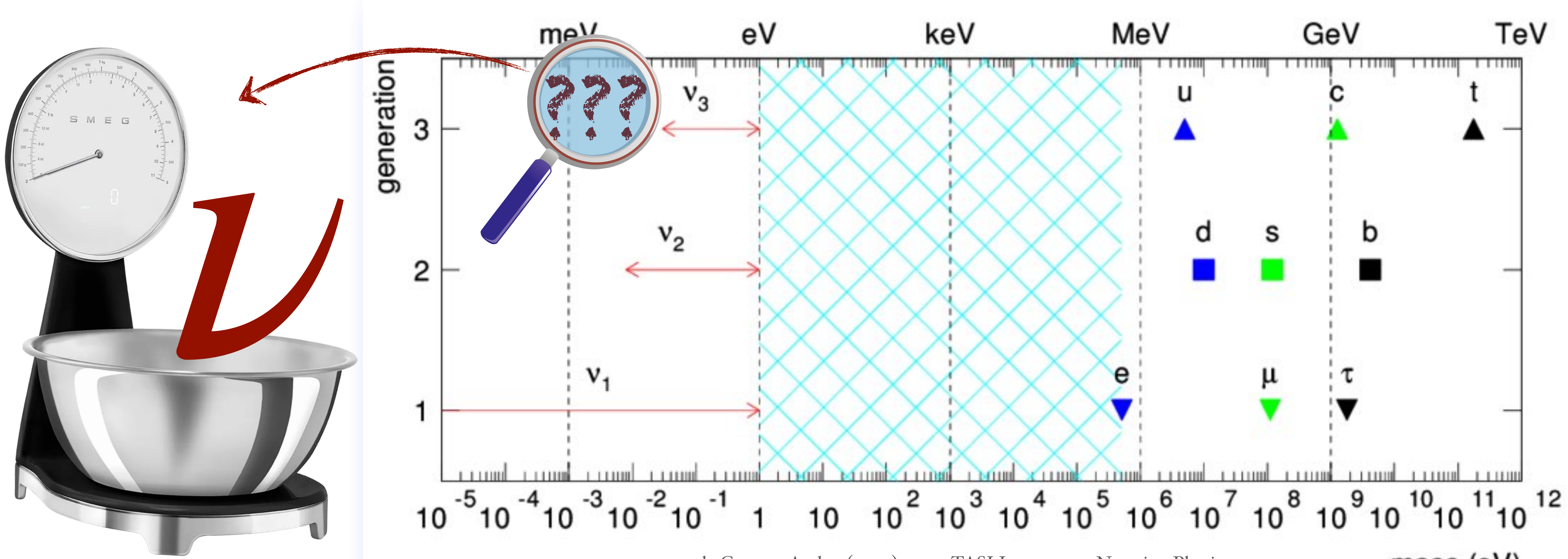


de Gouvea, Andre. (2004). 2004 TASI Lectures on Neutrino Physics.

Neutrino masses/mixing $\Longleftrightarrow \not{L}_e, \not{L}_\mu, \not{L}_\tau$

Lepton family numbers are not conserved

Neutrino masses/mixing $\Longleftrightarrow \not{L}_e, \not{L}_\mu, \not{L}_\tau$

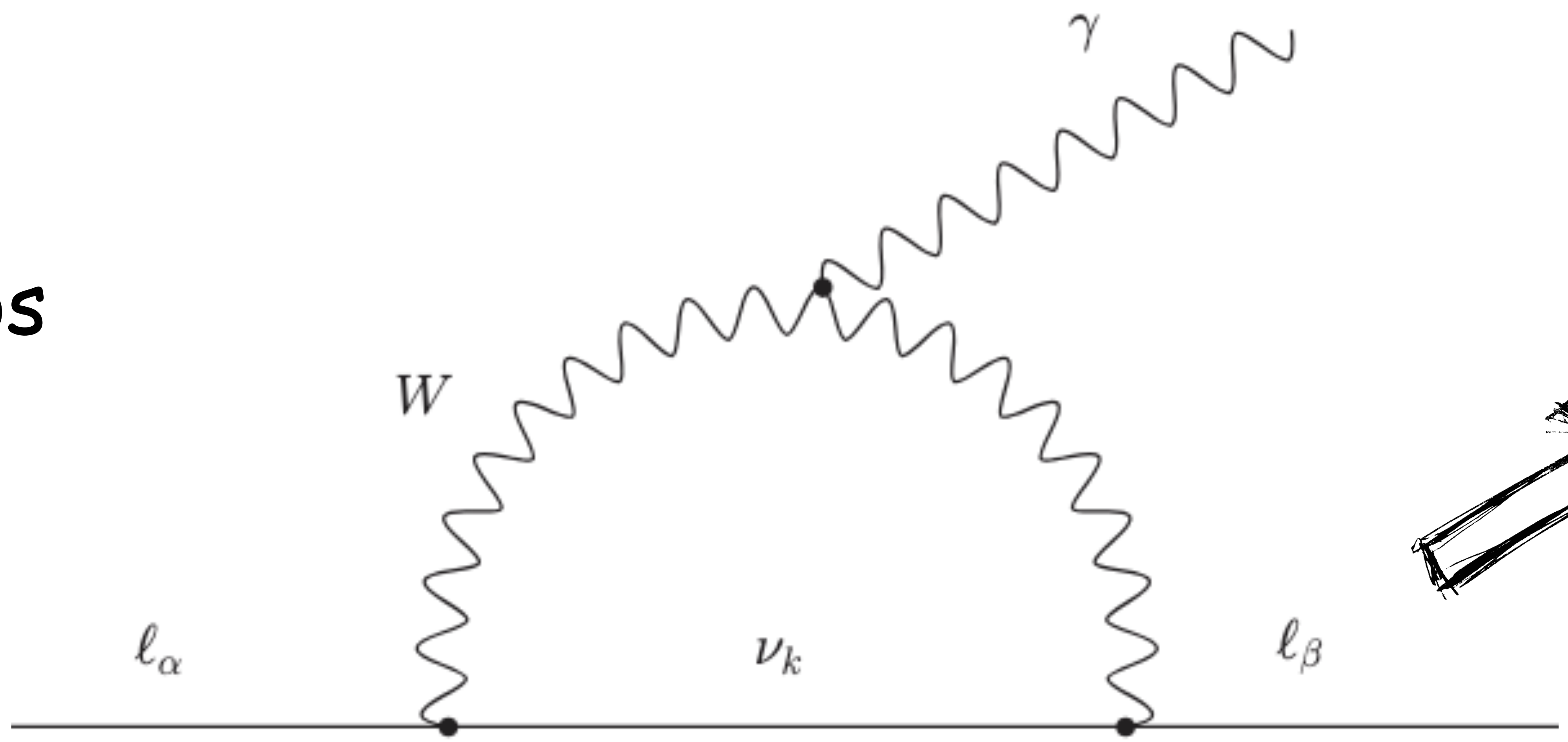
Lepton family numbers are not conserved

Thus, the lepton flavor violation in the charged sector is inevitable.

Why not cLFV?

$$\mu \to e\gamma\,, \quad \tau \to \mu\gamma\,, \quad \mu \to eee\,, \quad \ldots$$

SM+ RH-neutrinos



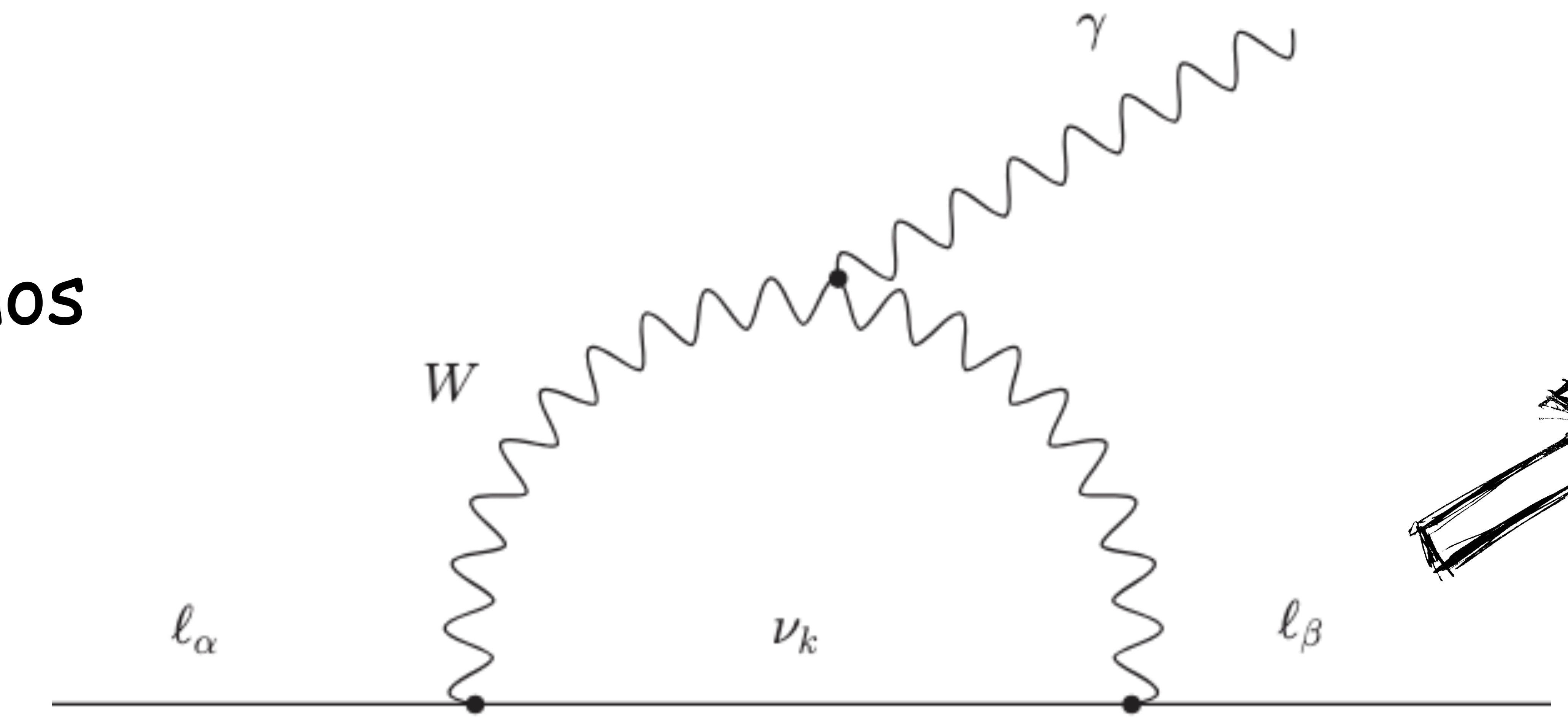
Cheng Li '77, '80; Petcov '77

The PMNS dictates how the RH-neutrinos couple to charged leptons of different flavors

$$\frac{\Gamma\left(\ell_\alpha \to \ell_\beta \gamma\right)}{\Gamma\left(\ell_\alpha \to \ell_\beta \bar{\nu}\nu\right)} = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3} U_{\alpha k} U^*_{\beta k} \frac{m^2_{\nu_k}}{M^2_W}\right|^2$$

cLFV in the Standard Model

SM+ RH-neutrinos



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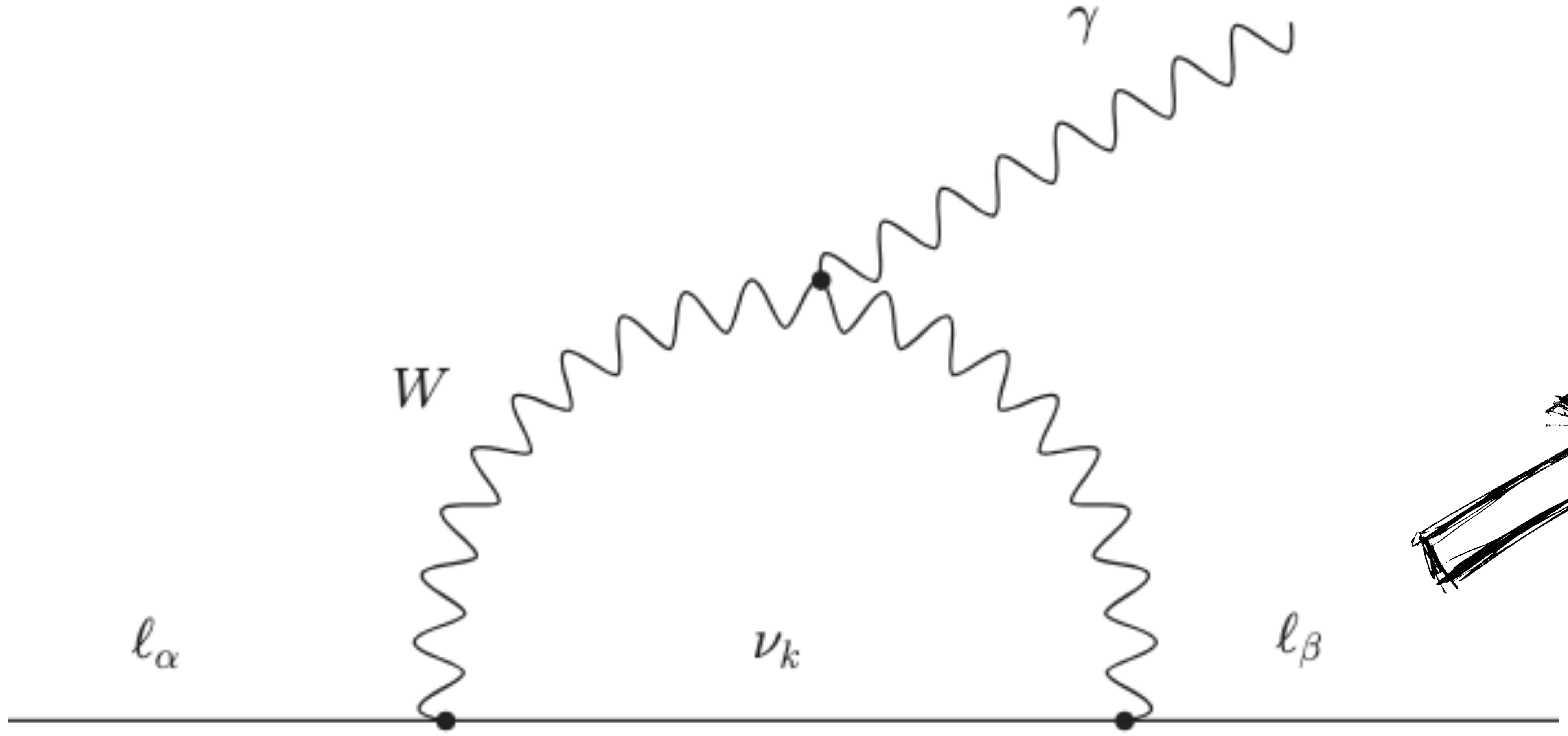
suppressed by tiny neutrino masses

$$\Rightarrow \quad \mathcal{B}(\mu \to e\gamma) \approx \mathcal{B}(\tau \to e\gamma) \approx \mathcal{B}(\tau \to \mu\gamma) = 10^{-55} \div 10^{-54}$$

Why are we interested in cLFV?



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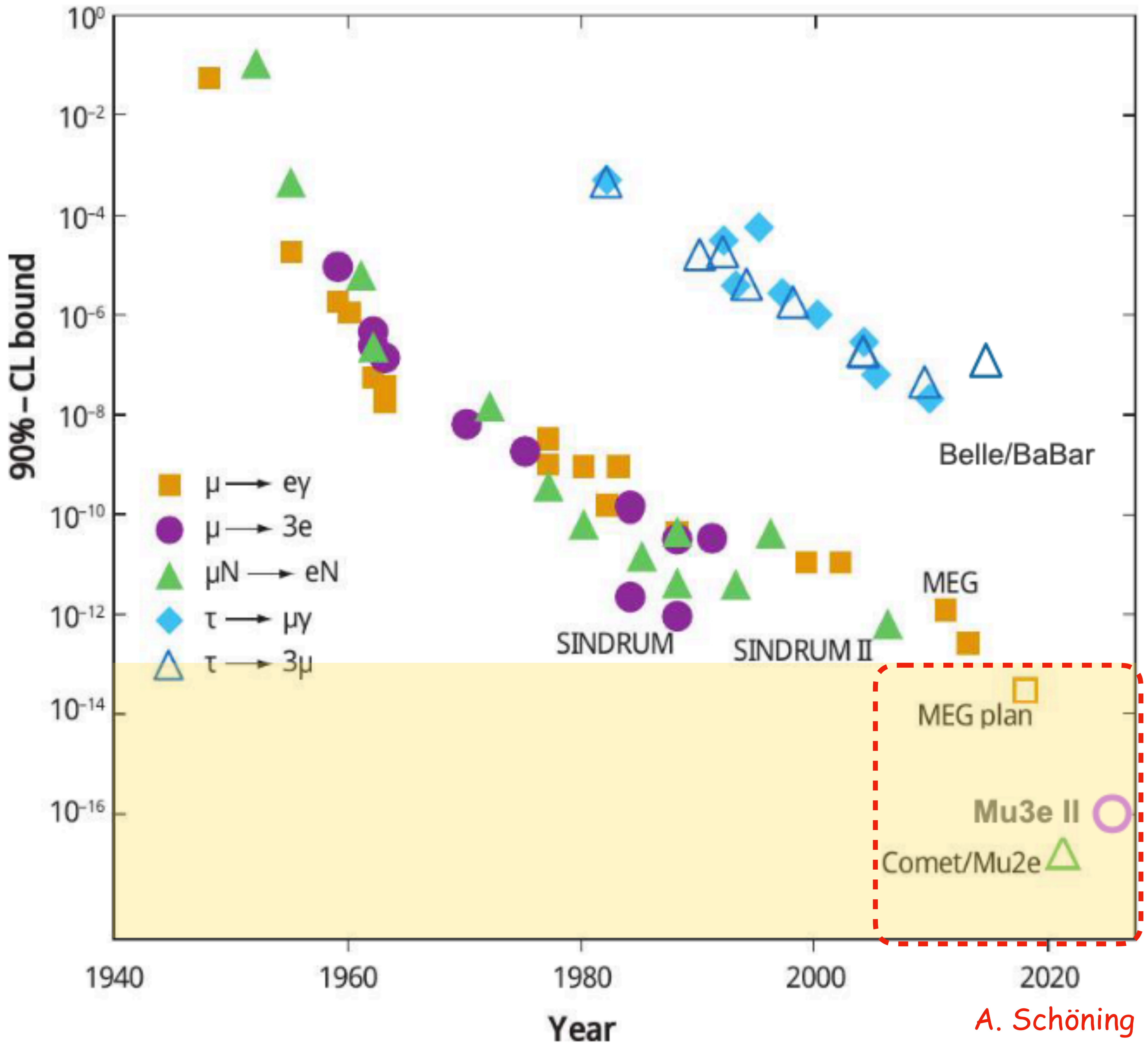
The PMNS dictates how the RH-neutrinos couple to charged leptons of different flavors

- An observation of LFV would be an unambigous signal of New Physics
- It could be related to the mechanism behind the neutrino masses
- Stringent bounds on NP coupling to leptons
- Probes scales beyond the LHC reach

We are interested, indeed… a long list of experiments



We are interested, indeed… a long list of experiments

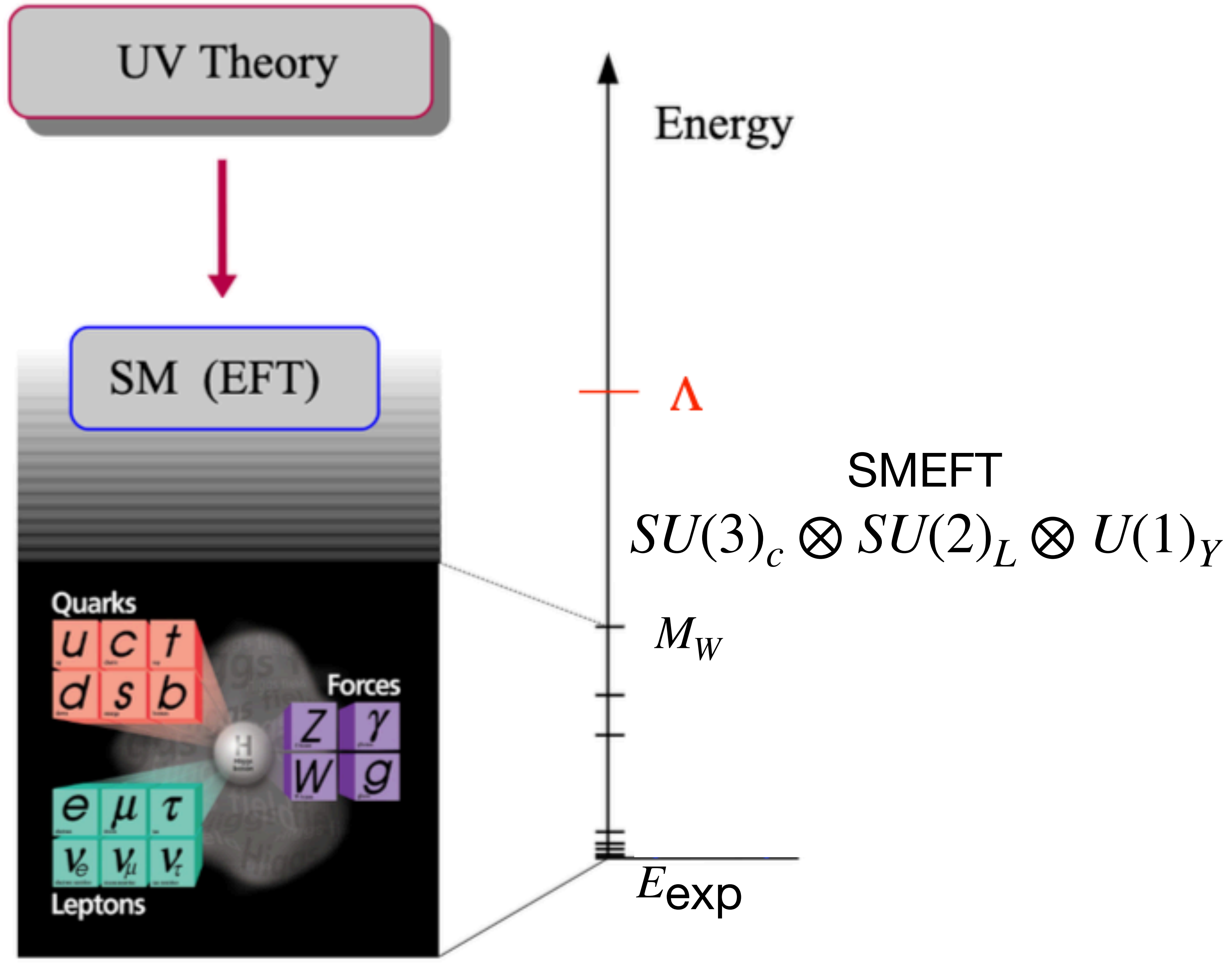
LFV obs.	Present bounds (90% CL)		Expected future limits	
$\mathrm{BR}(\mu \to e\gamma)$	4.2×10^{-13}	MEG (2016) [64]	6×10^{-14}	MEG-II [65]
$\mathrm{BR}(\mu \to eee)$	1.0×10^{-12}	SINDRUM (1988) [66]	10^{-16}	Mu3e [67]
$\mathrm{CR}(\mu \to e, \mathrm{Au})$	7.0×10^{-13}	SINDRUM II (2006) [68]	–	–
$\mathrm{CR}(\mu \to e, \mathrm{Al})$	–	–	6×10^{-17}	COMET/Mu2e [69, 70]
$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}	BaBar (2010) [71]	3×10^{-9}	Belle-II [72]
$\mathrm{BR}(\tau \to eee)$	2.7×10^{-8}	Belle (2010) [73]	5×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to e\mu\mu)$	2.7×10^{-8}	Belle (2010) [73]	5×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to \pi e)$	8.0×10^{-8}	Belle (2007) [74]	4×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to \rho e)$	1.8×10^{-8}	Belle (2011) [75]	3×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to \mu\gamma)$	4.2×10^{-8}	Belle (2021) [76]	10^{-9}	Belle-II [72]
$\mathrm{BR}(\tau \to \mu\mu\mu)$	2.1×10^{-8}	Belle (2010) [73]	4×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to \mu ee)$	1.8×10^{-8}	Belle (2010) [73]	3×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to \pi\mu)$	1.1×10^{-7}	Babar (2006) [77]	5×10^{-10}	Belle-II [72]
$\mathrm{BR}(\tau \to \rho\mu)$	1.2×10^{-8}	Belle (2011) [75]	2×10^{-10}	Belle-II [72]

Mode	LEP bound (95% CL)		LHC bound (95% CL)		CEPC/FCC-ee exp.
$\mathrm{BR}(Z \to \mu e)$	1.7×10^{-6}	[2]	7.5×10^{-7}	[3]	$10^{-8} – 10^{-10}$
$\mathrm{BR}(Z \to \tau e)$	9.8×10^{-6}	[2]	5.0×10^{-6}	[4, 5]	10^{-9}
$\mathrm{BR}(Z \to \tau\mu)$	1.2×10^{-5}	[6]	6.5×10^{-6}	[4, 5]	10^{-9}

L. Calibbi, X. Marcano, J. Roy (2021)

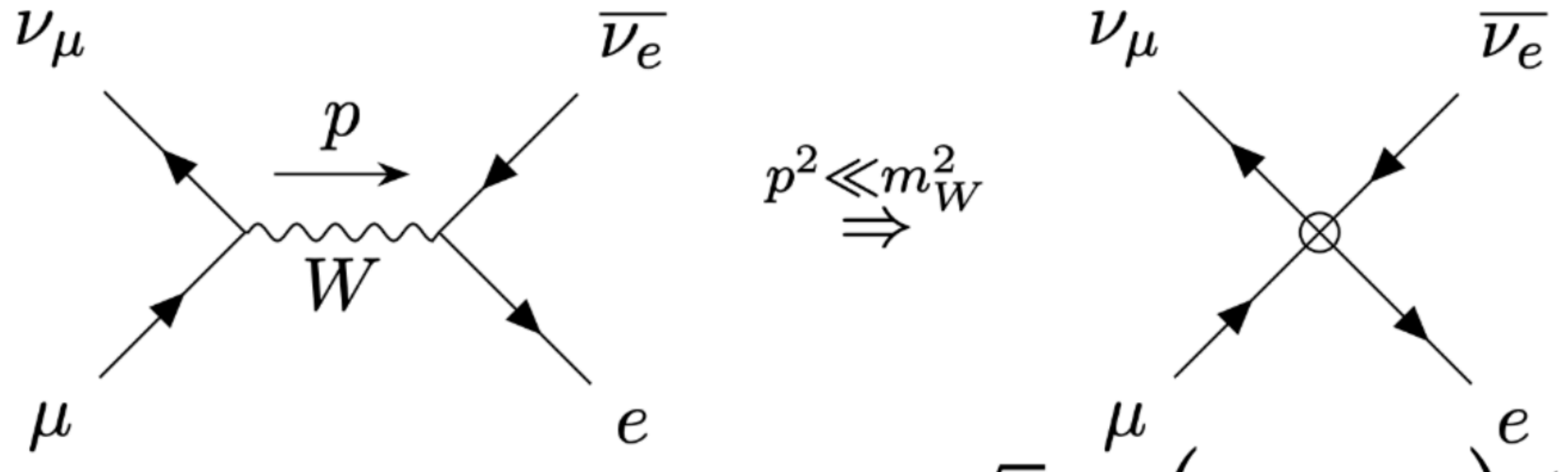
cLFV in SM effective field theory

The Effective Field Theory represents the low-energy approximation of an Ultraviolet (UV) theory, obtained by integrating out the heavy degrees of freedom



It the New Physics scale $\Lambda \gg M_W$,
the SM can be seen as the EFT of another UV theory

As the Fermi theory is the low energy limit of the SM



$$-2\sqrt{2}G_F\left(\overline{\nu_\mu}\gamma_\alpha P_L \mu\right)\left(\bar{e}\gamma^\alpha P_L \nu_e\right)$$

cLFV in SM effective field theory

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \left(\sum_{n=1}^{d} \sum_{i} \frac{\mathscr{C}_i^{(n)}}{\Lambda^{n-4}} \mathscr{O}_i^{(n)} + \text{H.c.} \right), \quad \text{for } d > 4,$$

Dimension-6 effective operators that can induce CLFV

4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

Grzadkowsi et al. '10; Crivellin Najjari Rosiek '13

cLFV in SM effective field theory

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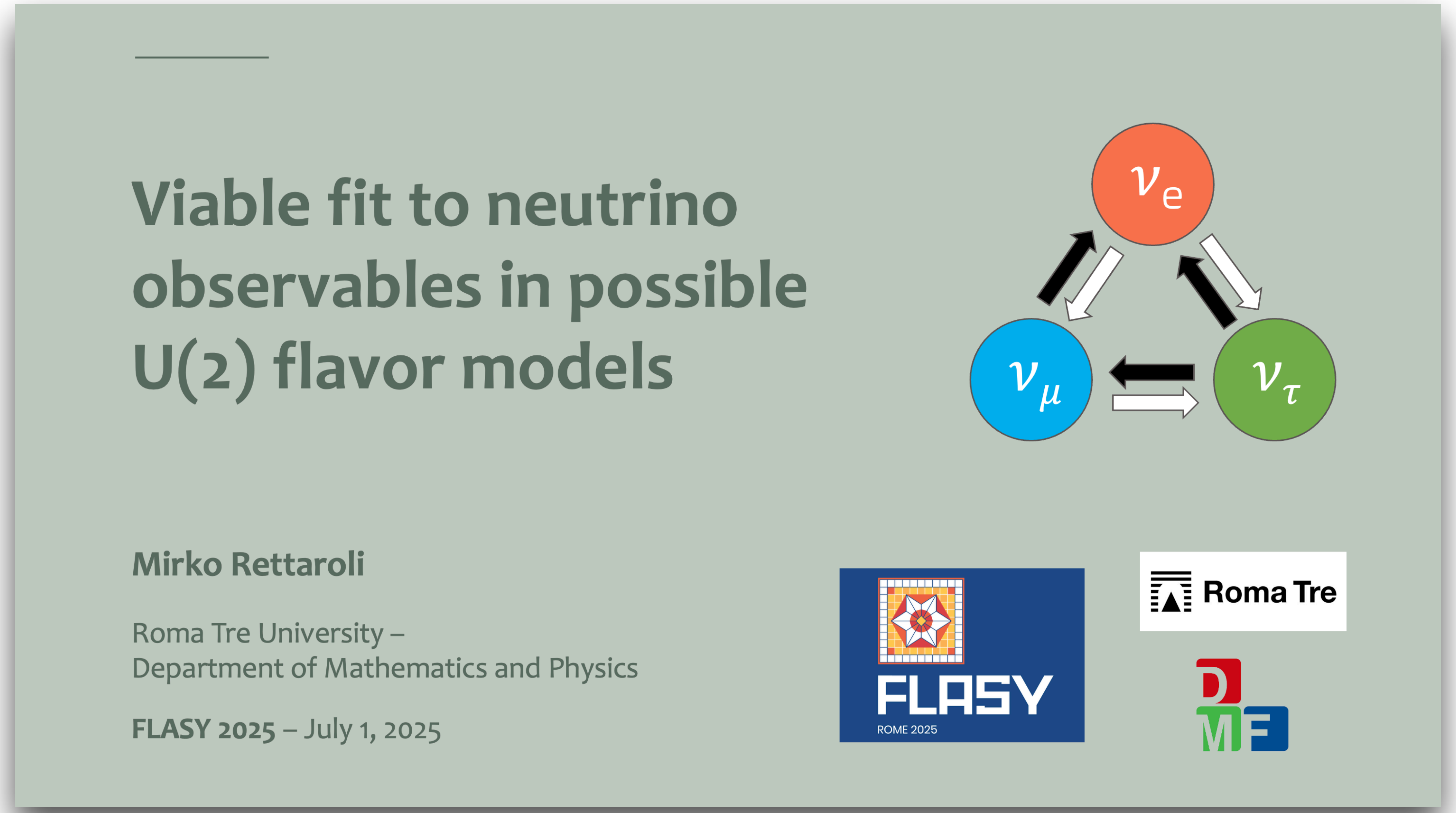
$$\mathscr{L}_{\text{dipole}} = \frac{1}{\Lambda^2} \left(\mathscr{C}'_{LR} \mathscr{O}_{LR}^{(6)} + \mathscr{C}'_{RL} \mathscr{O}_{RL}^{(6)} \right), \qquad \mathscr{C}'^{\dagger}_{RL} = \mathscr{C}'_{LR} = \begin{pmatrix} \mathscr{C}'_{ee} & \mathscr{C}'_{e\mu} & \mathscr{C}'_{e\tau} \\ \mathscr{C}'_{\mu e} & \mathscr{C}'_{\mu\mu} & \mathscr{C}'_{\mu\tau} \\ \mathscr{C}'_{\tau e} & \mathscr{C}'_{\tau\mu} & \mathscr{C}'_{\tau\tau} \end{pmatrix},$$

$$\mathscr{O}_{LR}^{(6)} = \frac{v}{\sqrt{2}} \bar{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}$$

U(2) applied to neutrinos: Version 2.0

Pattern	Charges	LO mass matrix in terms of λ
S1 A **S2** A **D1** A **D2** A	$(1,0,-2)$ $(1,1,-2)$ $(1,-2)$ $(2,-2)$	$\begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^4 & \lambda^4 \end{pmatrix}$
S3 A **S4** A	$(2,1,-2)$ $(2,2,-2)$	$\begin{pmatrix} \lambda^{12} & \lambda^8 & \lambda^8 \\ \lambda^8 & \lambda^4 & \lambda^4 \\ \lambda^8 & \lambda^4 & \lambda^4 \end{pmatrix}$
D3 A **D4** A	$(0,1)$ $(1,0)$	$\begin{pmatrix} \lambda^4 & 1 & \lambda^4 \\ 1 & 1/\lambda^4 & 1 \\ \lambda^4 & 1 & \lambda^4 \end{pmatrix}$
S1 B **S2** B	$(1,0,-2)$ $(1,1,-2)$	$\begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^5 \\ \lambda^4 & \lambda^4 & \lambda^5 \\ \lambda^5 & \lambda^5 & \lambda^6 \end{pmatrix}$
D1 B **D2** B	$(1,-2)$ $(2,-2)$	$\begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^7 \\ \lambda^6 & \lambda^4 & \lambda^5 \\ \lambda^7 & \lambda^5 & \lambda^6 \end{pmatrix}$
D5 B	$(0,0)$	$\begin{pmatrix} \lambda^8 & \lambda^2 & \lambda^7 \\ \lambda^2 & \lambda^4 & \lambda \\ \lambda^7 & \lambda & \lambda^6 \end{pmatrix}$

… how discussed in Rettaroli's talk



Input values and upper bounds

Anomalous magnetic moment of the lepton ℓ

$(\Delta a_\ell \equiv a_\ell^{exp} - a_\ell^{SM})$

$$\Delta a_\ell = \frac{4m_\ell}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\Re\left[\mathscr{C}'_{\ell\ell}\right]$$

Tree-level radiative LFV decays Branching Ratios

$$\mathscr{B}\left(\ell_\alpha \to \ell_\beta\gamma\right) = \frac{m_{\ell_\alpha}^3 v^2}{8\pi\Gamma_{\ell_\alpha}}\frac{1}{\Lambda^4}\left(|\mathscr{C}'_{\alpha\beta}|^2 + |\mathscr{C}'_{\beta\alpha}|^2\right)$$

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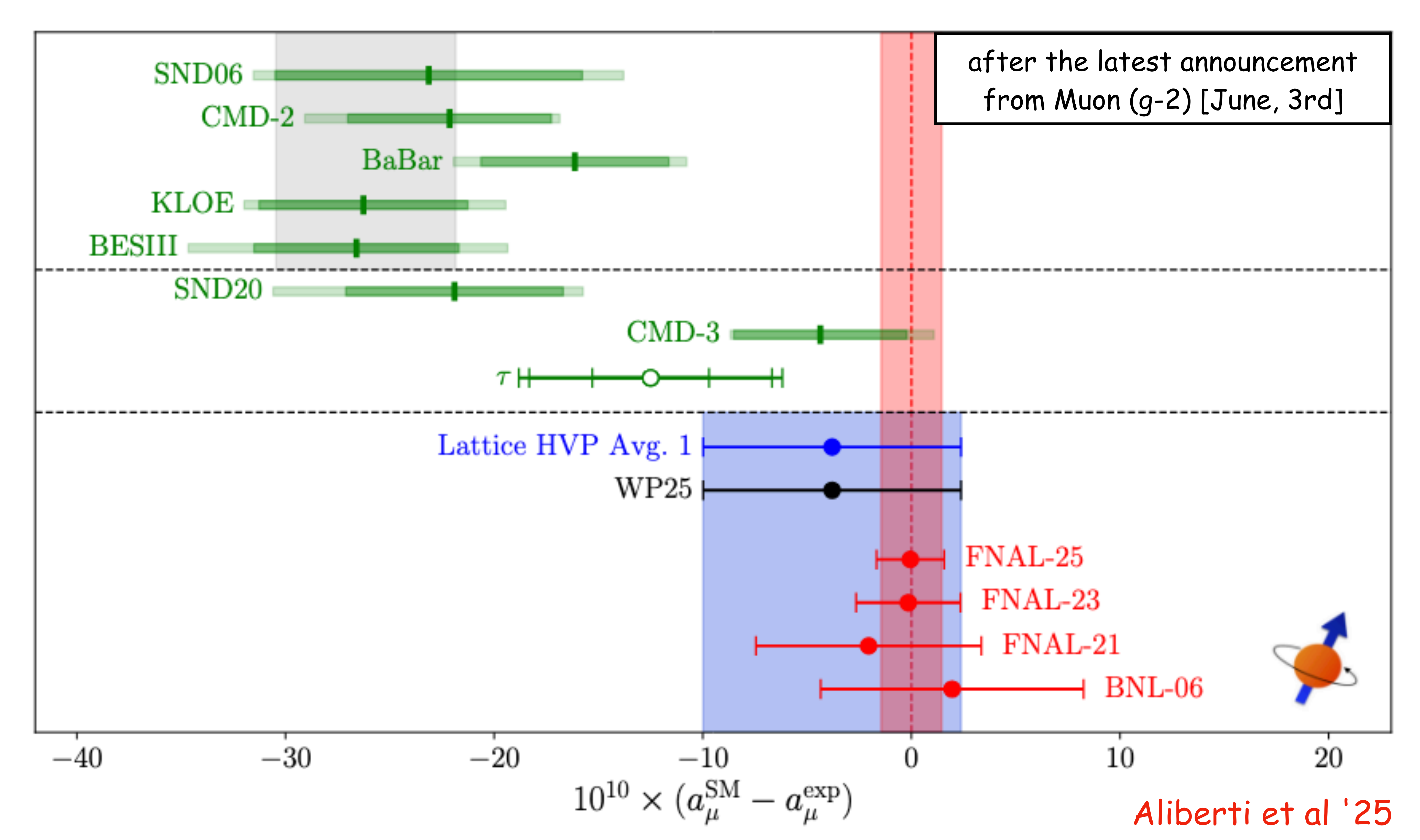
Observables	Exp.-SM/Bound		Wilson Coef. in $1/\Lambda^2$ [TeV^{-2}]
Δa_μ	249×10^{-11}	Muon g-2	$\Re\left[C'_{\mu\mu}\right] = 1.0 \times 10^{-5}$
$\mathcal{B}(\mu \to e\gamma)$	$< 4.2 \times 10^{-13}$	MEG	$\lvert C'_{e\mu\,(\mu e)}\rvert < 2.1 \times 10^{-10}$
$\mathcal{B}(\tau \to \mu\gamma)$	$< 4.2 \times 10^{-8}$	BaBar/BELLE	$\lvert C'_{\mu\tau\,(\tau\mu)}\rvert < 2.65 \times 10^{-6}$
$\mathcal{B}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$	BaBar/BELLE	$\lvert C'_{e\tau\,(\tau e)}\rvert < 2.35 \times 10^{-6}$

Input values and upper bounds

At the current level of precision there is not tension between the SM prediction and the experimental world average



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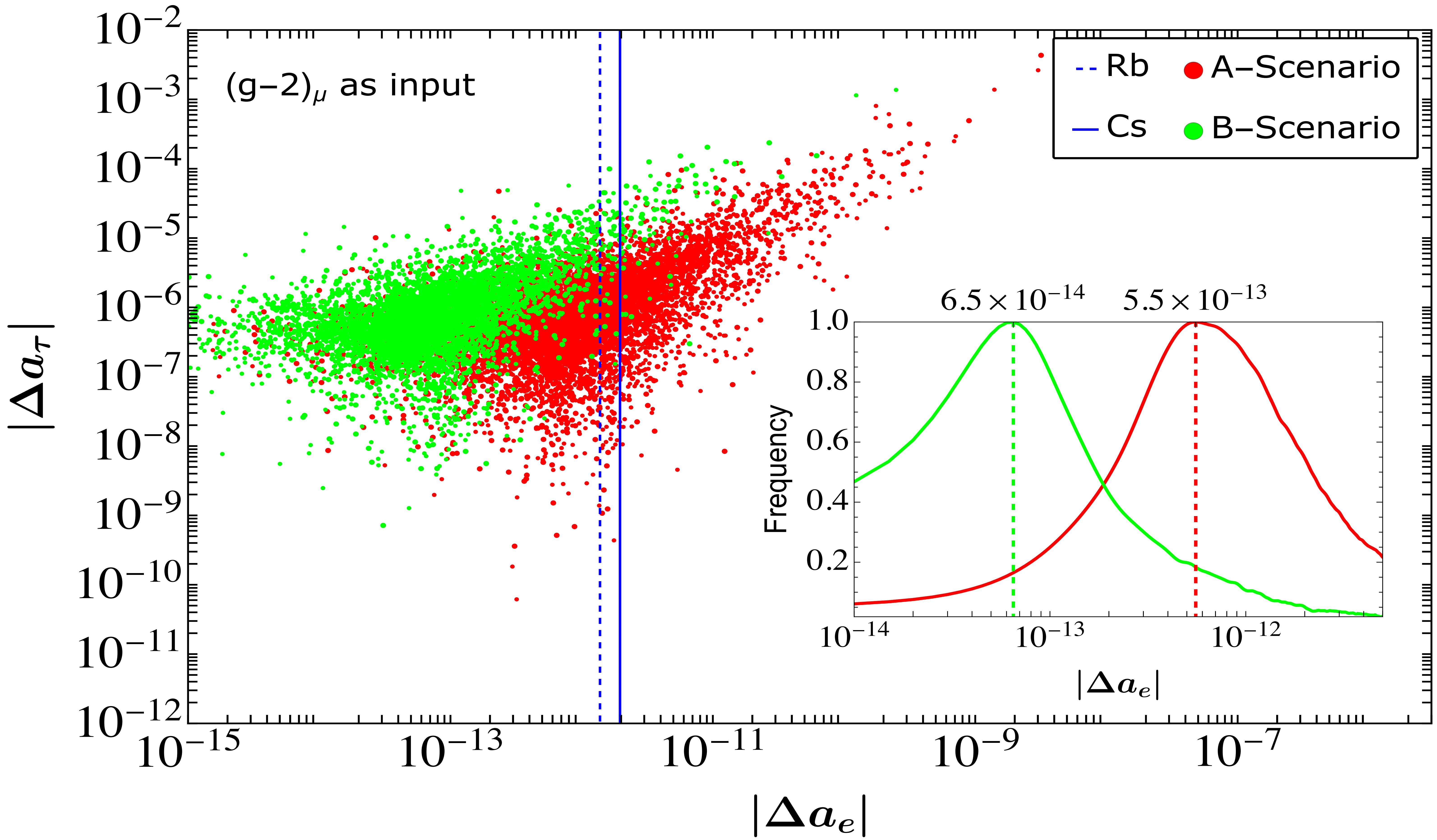
Predictions on the $(g-2)_{e,\tau}$

$$\Delta a_e^{\text{Rb}} = a_e^{\text{exp}} - a_e^{\text{SM,Rb}} = (+4.8 \pm 3.0) \times 10^{-13}\,,$$

L. Morel, Z. Yao, P. Cladé and S. Guellati-Khélifa [2020]

$$\Delta a_e^{\text{Cs}} = a_e^{\text{exp}} - a_e^{\text{SM,Cs}} = (-8.8 \pm 3.6) \times 10^{-13}\,.$$

R.H. Parker, C. Yu, W. Zhong, B. Estey and H. Müller [2018]



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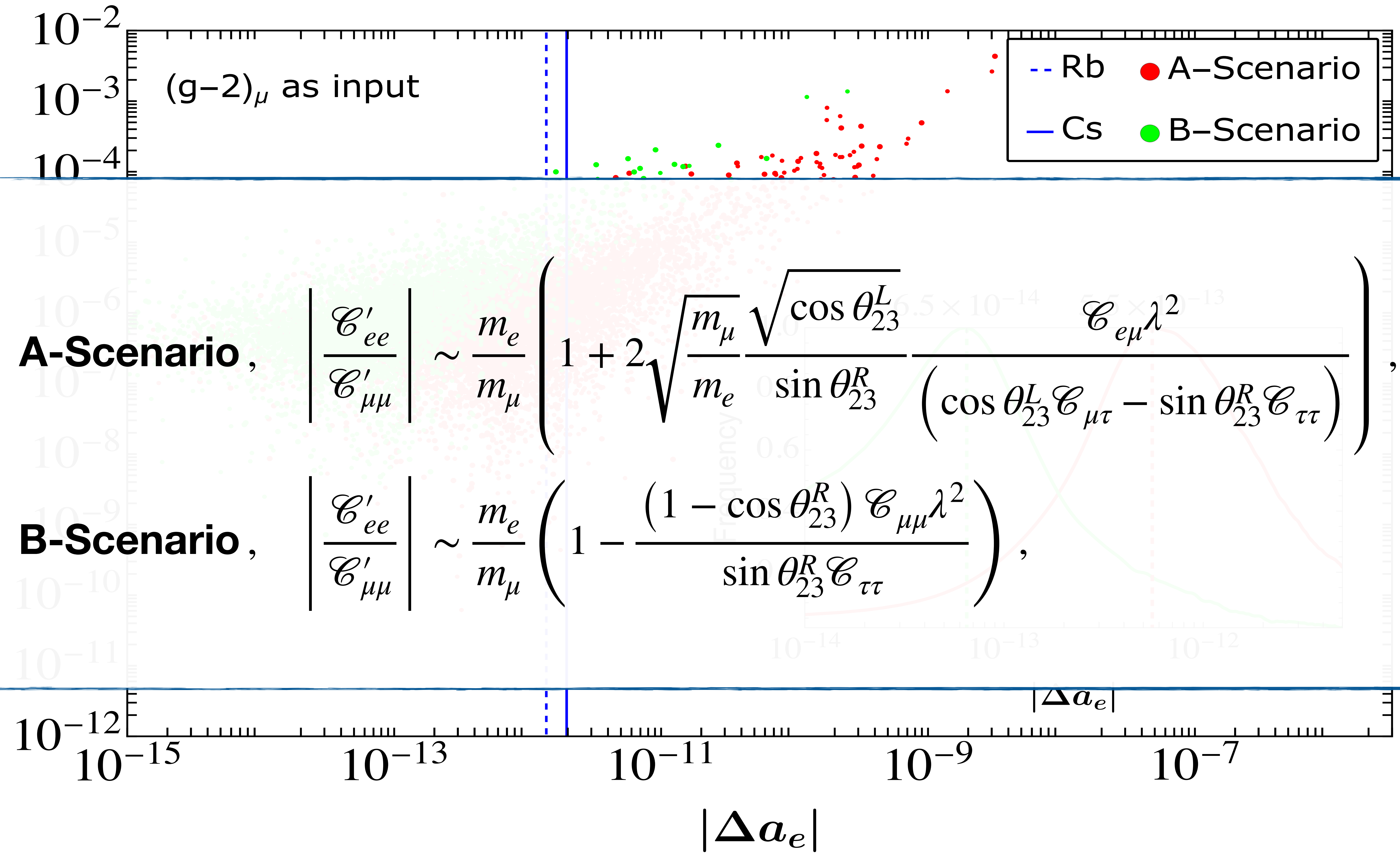

A-Scenario, $$\left|\frac{\mathcal{C}'_{ee}}{\mathcal{C}'_{\mu\mu}}\right| \sim \frac{m_e}{m_\mu}\left(1 + 2\sqrt{\frac{m_\mu}{m_e}}\frac{\sqrt{\cos\theta_{23}^L}}{\sin\theta_{23}^R}\frac{\mathcal{C}_{e\mu}\lambda^2}{\left(\cos\theta_{23}^L\mathcal{C}_{\mu\tau} - \sin\theta_{23}^R\mathcal{C}_{\tau\tau}\right)}\right),$$

B-Scenario, $$\left|\frac{\mathcal{C}'_{ee}}{\mathcal{C}'_{\mu\mu}}\right| \sim \frac{m_e}{m_\mu}\left(1 - \frac{\left(1-\cos\theta_{23}^R\right)\mathcal{C}_{\mu\mu}\lambda^2}{\sin\theta_{23}^R\mathcal{C}_{\tau\tau}}\right),$$

$\mu \to e\gamma$ in light of $(g-2)_\mu$

With $\mathcal{O}(1)$ WCs, $\mathcal{C}'_{e\mu(\mu e)}$ is suppressed by at most an $\mathcal{O}(10^{-2})$ with respect to $\mathcal{C}'_{\mu\mu}$

A-Scenario

$$\left|\frac{\mathcal{C}'_{e\mu}}{\mathcal{C}'_{\mu\mu}}\right| \sim \frac{1}{\sqrt{\cos\theta^L_{23}}}\sqrt{\frac{m_e}{m_\mu}} + \cot\theta^R_{23}\frac{\mathcal{C}_{e\mu}\lambda^2}{\cos\theta^L_{23}\mathcal{C}_{\mu\tau} - \sin\theta^L_{23}\mathcal{C}_{\tau\tau}} + \mathcal{O}(\lambda^4)\,,$$

$$\left|\frac{\mathcal{C}'_{\mu e}}{\mathcal{C}'_{\mu\mu}}\right| \sim \sqrt{\cos\theta^L_{23}}\sqrt{\frac{m_e}{m_\mu}} + \cot\theta^R_{23}\frac{\mathcal{C}_{e\mu}\lambda^2}{\cos\theta^L_{23}\mathcal{C}_{\mu\tau} - \sin\theta^L_{23}\mathcal{C}_{\tau\tau}} + \mathcal{O}(\lambda^2\sqrt{m_e/m_\mu})\,;$$

B-Scenario

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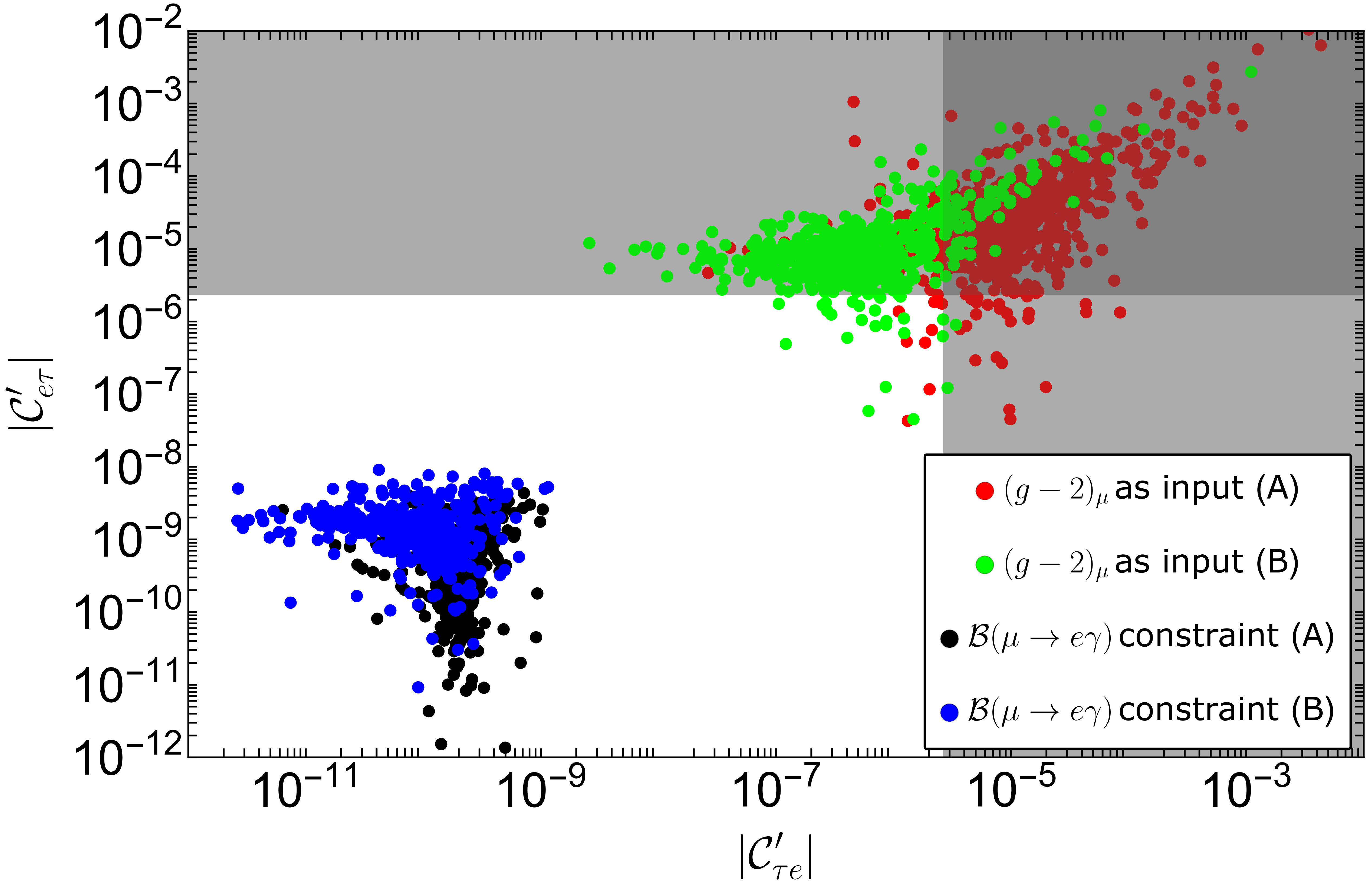
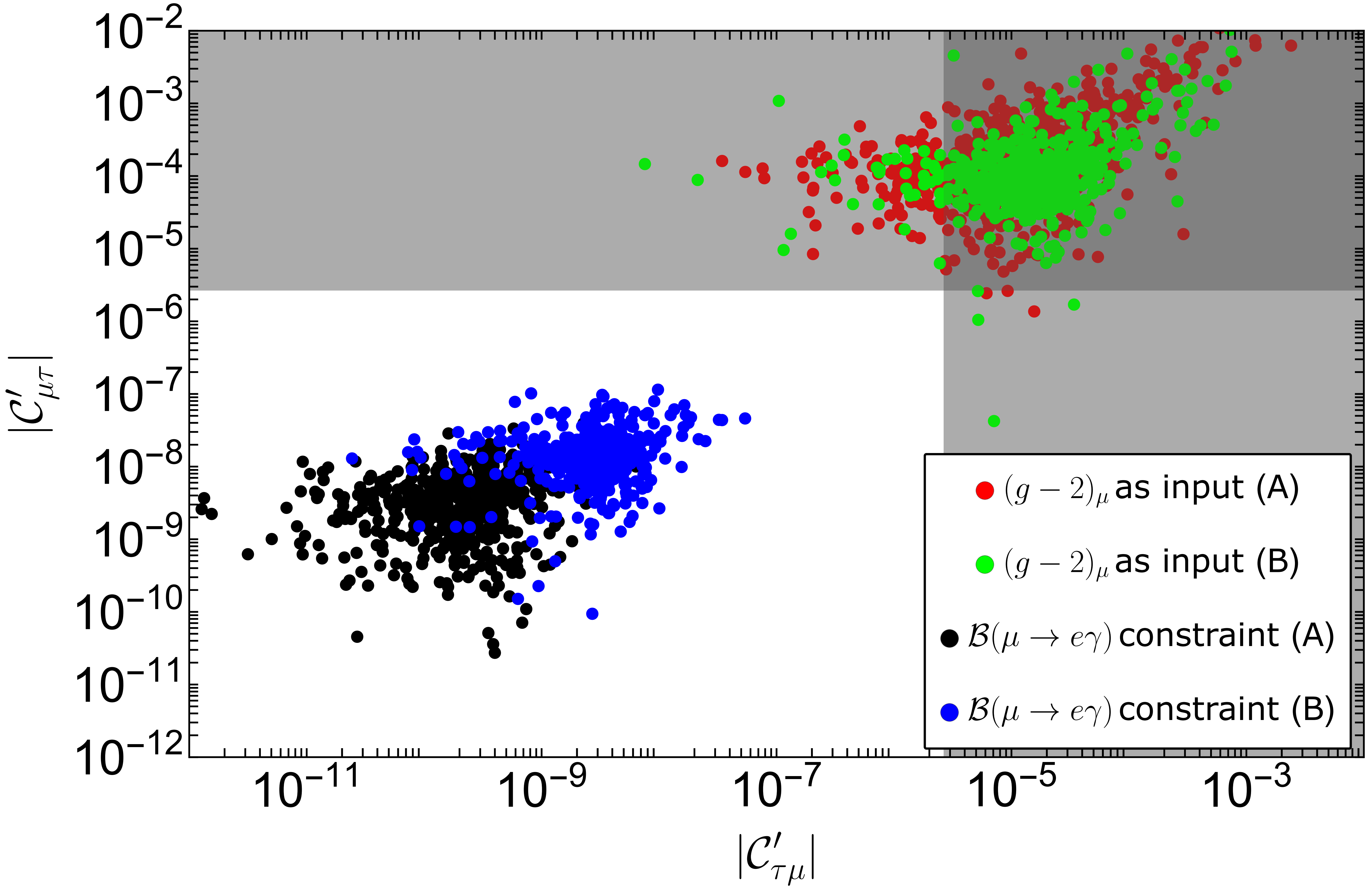
LFV decays in light of $(g-2)_\mu$





$\Rightarrow$ Impossible combined explanation of (old) AMM of muon and LFV constraints

LFV decays in light of $(g-2)_\mu$

After the latest announcement from the Muon (g-2) collaboration, our U(2) models are fully consistent with the data:

- Majorana neutrino mass term without invoking D_6 (Mirko's talk)
- Successfull fit to neutrino low-energy observables (Mirko's talk)
- In agreement with the latest measurements of $(g-2)_\mu$ and LFV
- Can we push this further?

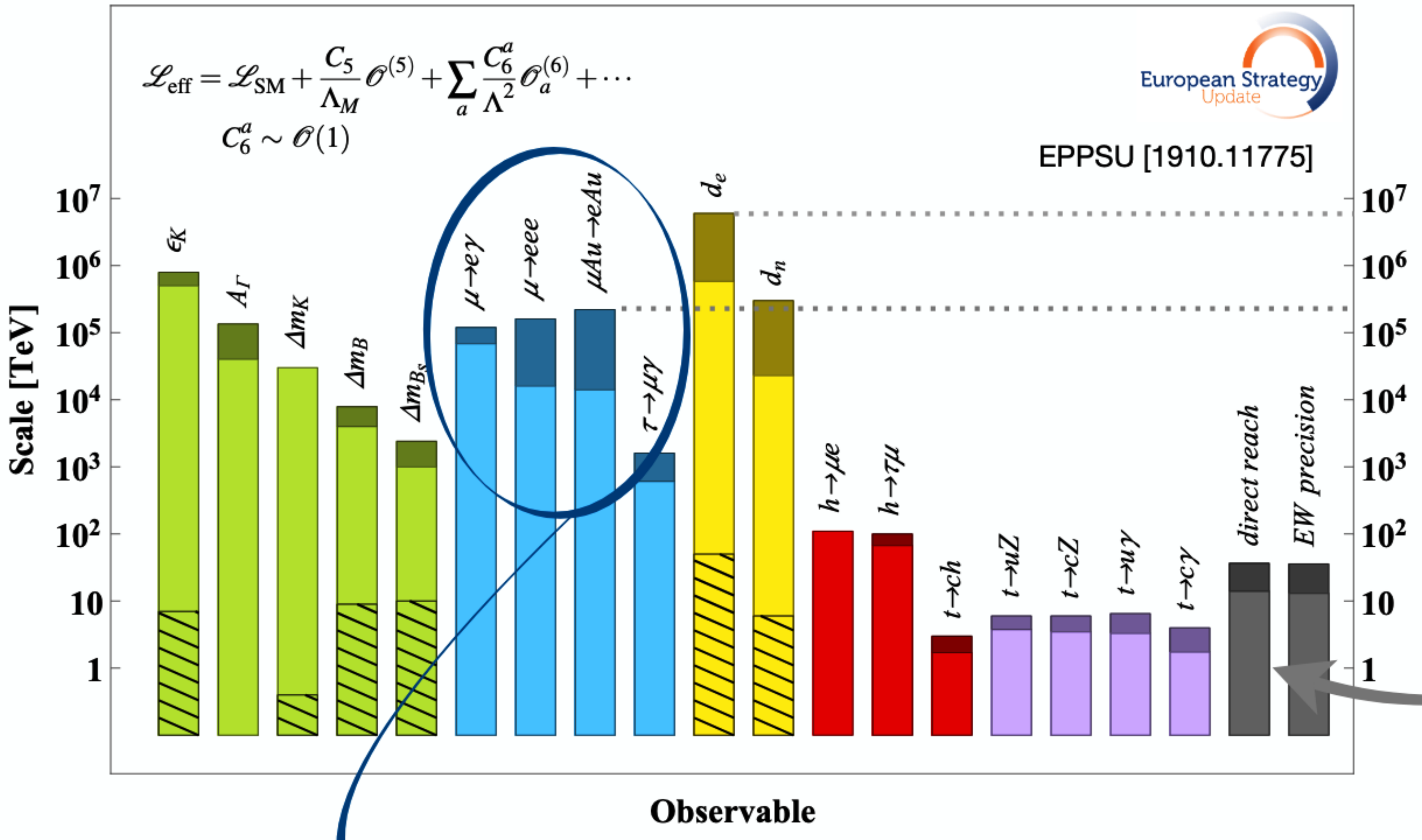
What if the flavon is not so heavy?

SM interpreted as a low-energy limit of a (complete, yet unknown) NP model

⇒ **Model-independent**, **effective approach (EFT)**

$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \sum_{n\geq 5} \frac{1}{\Lambda^{n-4}} \mathscr{C}^n(g, Y, \dots)\, \mathscr{O}^n(\ell, q, H, \gamma, \dots)$$

Cast **current data** in terms of $\mathscr{C}^6_{ij}$ and Λ_{NP} : $\mathscr{C}^6_{ij} \approx 1$ ⇒ **bounds on** Λ_{NP}



Flavour observables:
probes sensitive to very high NP **scales**

$\Lambda_{\text{NP}} \sim \mathscr{O}(10^5\ \text{TeV})$

well beyond collider's **reach!**

charged lepton flavour violating observables!

A.M. Teixeira, LPC Clermont

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$$\mathscr{L}^{\text{eff}} = \mathscr{L}^{\text{SM}} + \sum_{n\geq 5} \frac{1}{\Lambda^{n-4}} \mathscr{C}^n(g, Y, \dots)\, \mathscr{O}^n(\ell, q, H, \gamma, \dots)$$

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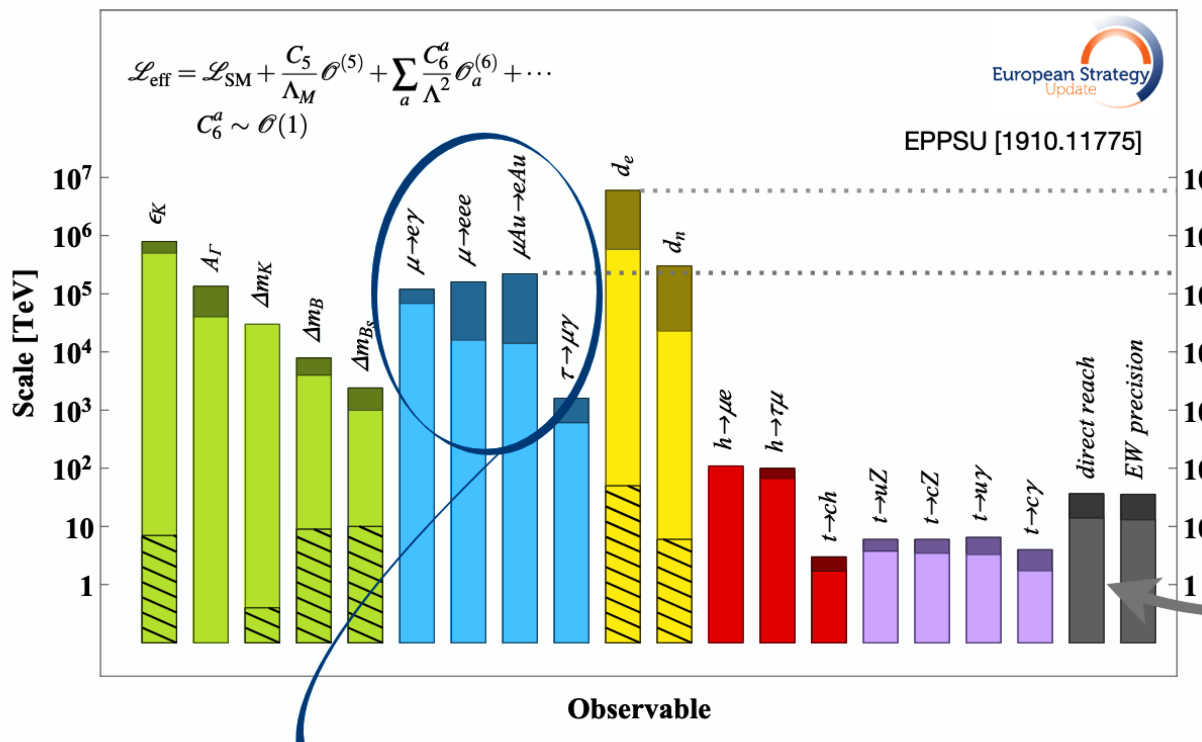

Flavour observables: **probes sensitive to very high NP scales** $\Lambda_{\text{NP}} \sim \mathscr{O}(10^5$ TeV) **well beyond collider's reach!**

charged lepton flavour violating observables!

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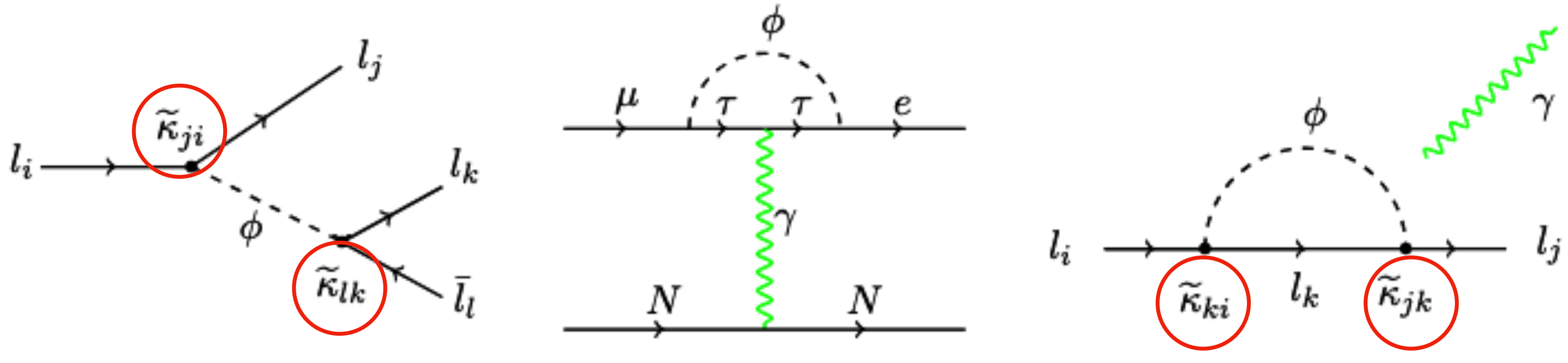

Huitu et al '16

$$\Gamma^{\text{tree}}(l_i \to l_j l_j l_j) = \frac{m_i^5}{4096\pi^3} \frac{(|\widetilde{\kappa}_{ji}|^2 + |\widetilde{\kappa}_{ij}|^2)|\widetilde{\kappa}_{jj}|^2}{m^4_{\text{Re}\phi}},$$

$$\Gamma^{1-\text{loop}}(\mu \to eee) = \frac{\alpha^2 m_\mu^3 m_\tau^2}{3072\pi^5} \frac{|\widetilde{\kappa}_{\tau e}|^2|\widetilde{\kappa}_{\mu\tau}|^2 + |\widetilde{\kappa}_{e\tau}|^2|\widetilde{\kappa}_{\tau\mu}|^2}{m^4_{\text{Re}\phi}} \left[\frac{3}{2} - \log\left(\frac{m^2_{\text{Re}\phi}}{m_\tau^2}\right)\right]^2 \left[\log\left(\frac{m_\mu^2}{m_e^2}\right) - \frac{11}{4}\right],$$

$$\Gamma^{1-\text{loop}}(\tau \to \mu\mu\mu) = \frac{\alpha^2 m_\tau^5}{3072\pi^5} \frac{(|\widetilde{\kappa}_{\tau\mu}|^2 + |\widetilde{\kappa}_{\mu\tau}|^2)|\widetilde{\kappa}_{\tau\tau}|^2}{m^4_{\text{Re}\phi}} \left[\frac{4}{3} - \log\left(\frac{m^2_{\text{Re}\phi}}{m_\tau^2}\right)\right]^2 \left[\log\left(\frac{m_\tau^2}{m_\mu^2}\right) - \frac{11}{4}\right],$$

$$\Gamma(\mu \to e\gamma) = \frac{\alpha m_\mu^3 m_\tau^2}{1024\pi^4} \frac{|\widetilde{\kappa}_{e\tau}|^2|\widetilde{\kappa}_{\tau\mu}|^2 + |\widetilde{\kappa}_{\tau e}|^2|\widetilde{\kappa}_{\mu\tau}|^2}{m^4_{\text{Re}\phi}} \left[\frac{3}{2} - \log\left(\frac{m^2_{\text{Re}\phi}}{m_\tau^2}\right)\right]^2,$$

$$\Gamma(\tau \to \mu\gamma) = \frac{\alpha m_\tau^5}{1024\pi^4} \frac{(|\widetilde{\kappa}_{\mu\tau}|^2 + |\widetilde{\kappa}_{\tau\mu}|^2)|\widetilde{\kappa}_{\tau\tau}|^2}{m^4_{\text{Re}\phi}} \left[\frac{4}{3} - \log\left(\frac{m^2_{\text{Re}\phi}}{m_\tau^2}\right)\right]^2.$$

$$\mathscr{C}_{ij} \ll 1$$

Flavor symmetry breaking $\sim \mathscr{O}$(TeV)

e.g. at high energy muon colliders resonances $\mu^+\mu^- \to \varphi^* \to \mu^+\mu^-$

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VNIVERSITAT DE VALÈNCIA

CSIC CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

Grazie - Thanks !

FLASY ROME 2025

Simone Marciano July 1st, 2025 Roma


BUS
TAXI

Backup slides

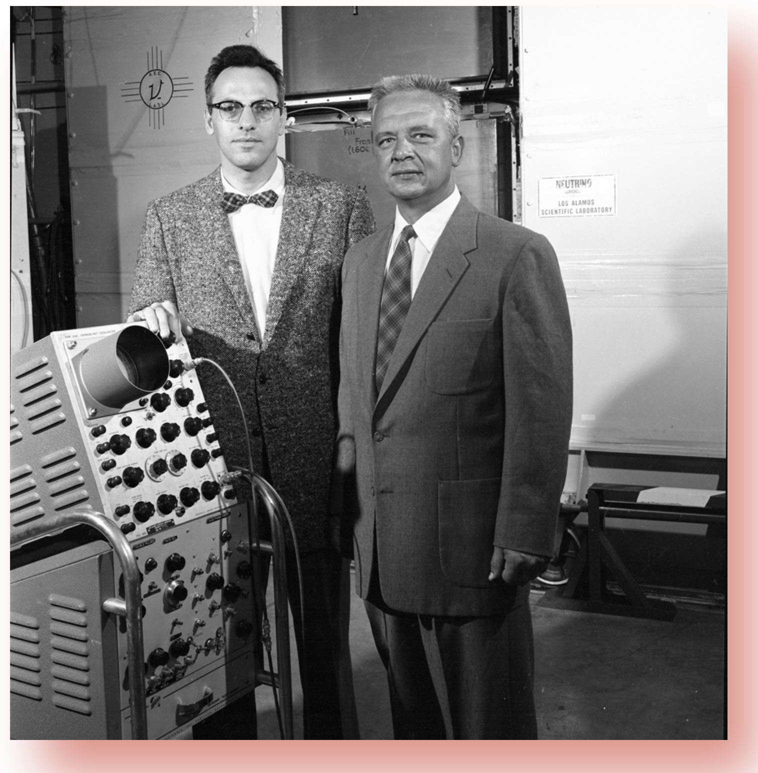
The curious case of Mr. Neutrino

1930

Pauli's hypotesis:
"what if the missing energy is carried off by an otherwise *invisible* particle?"

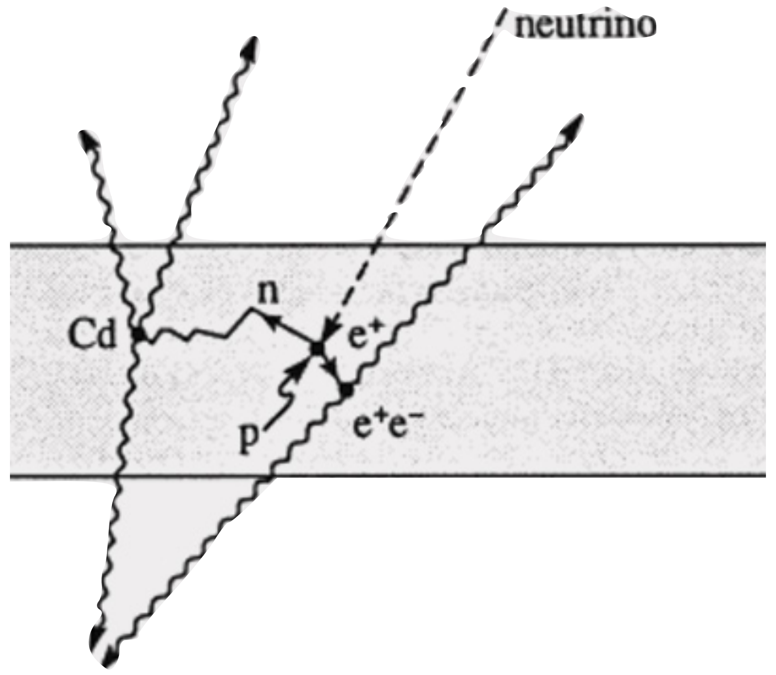
Fermi's theory of weak interactions and beta decay

1934



1956

Discovery of the neutrino by Cowan & Reines

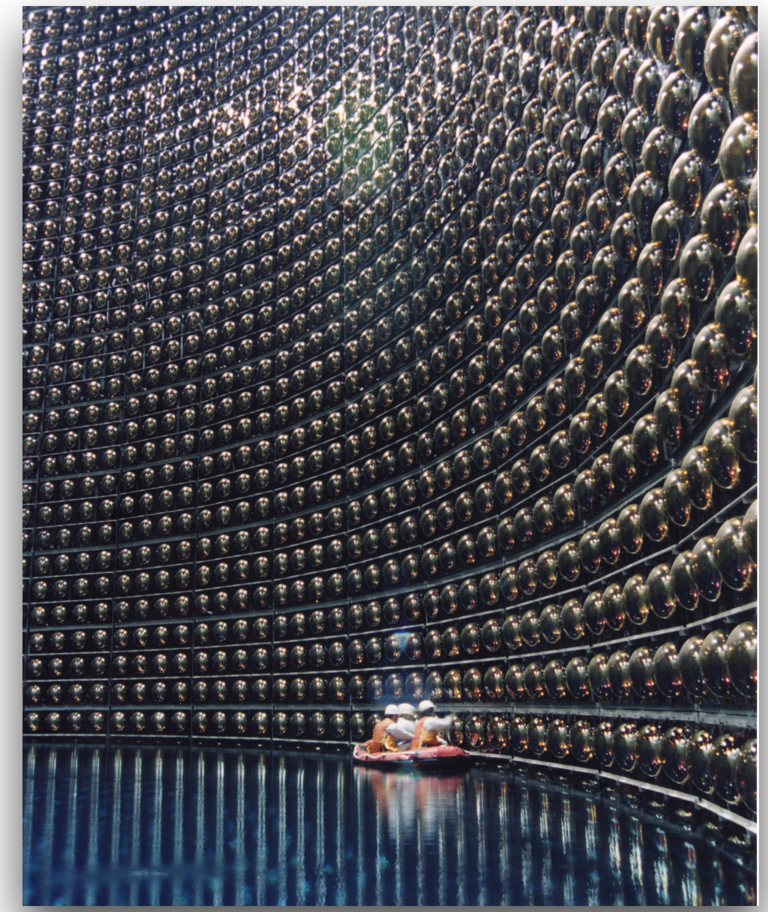
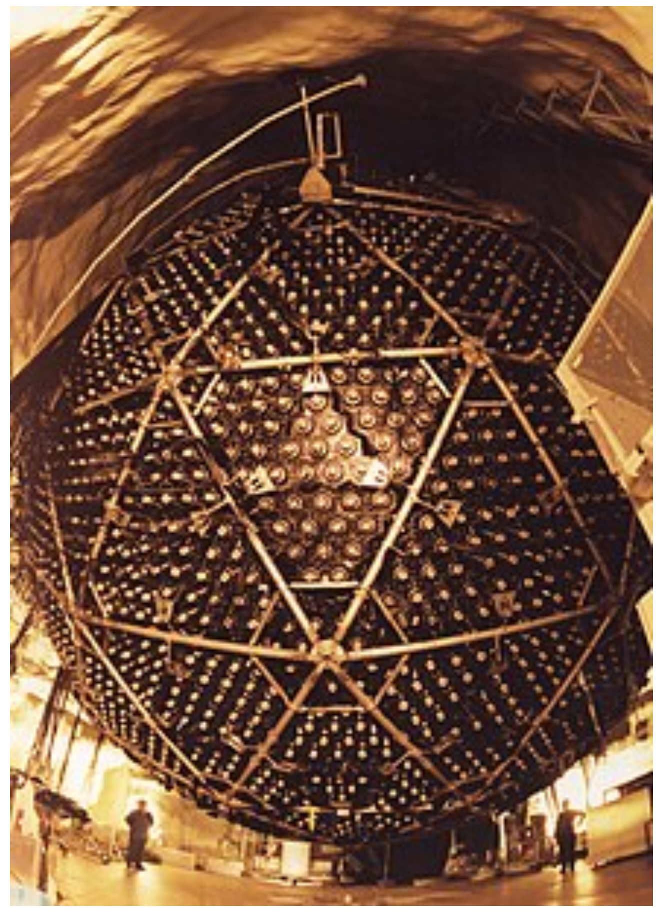


The Standard Model of Particle Physics

~ 1960/1970

- ✓ Neutrinos are assumed to be massless particles in the SM
  - No right-handed neutrino ν_R
  - Minimal lepton sector
  - Accidental lepton number conservation

- ✓ There are three different neutrino *flavors*: electron-, muon- and tauon- neutrinos



Physics Nobel 2015

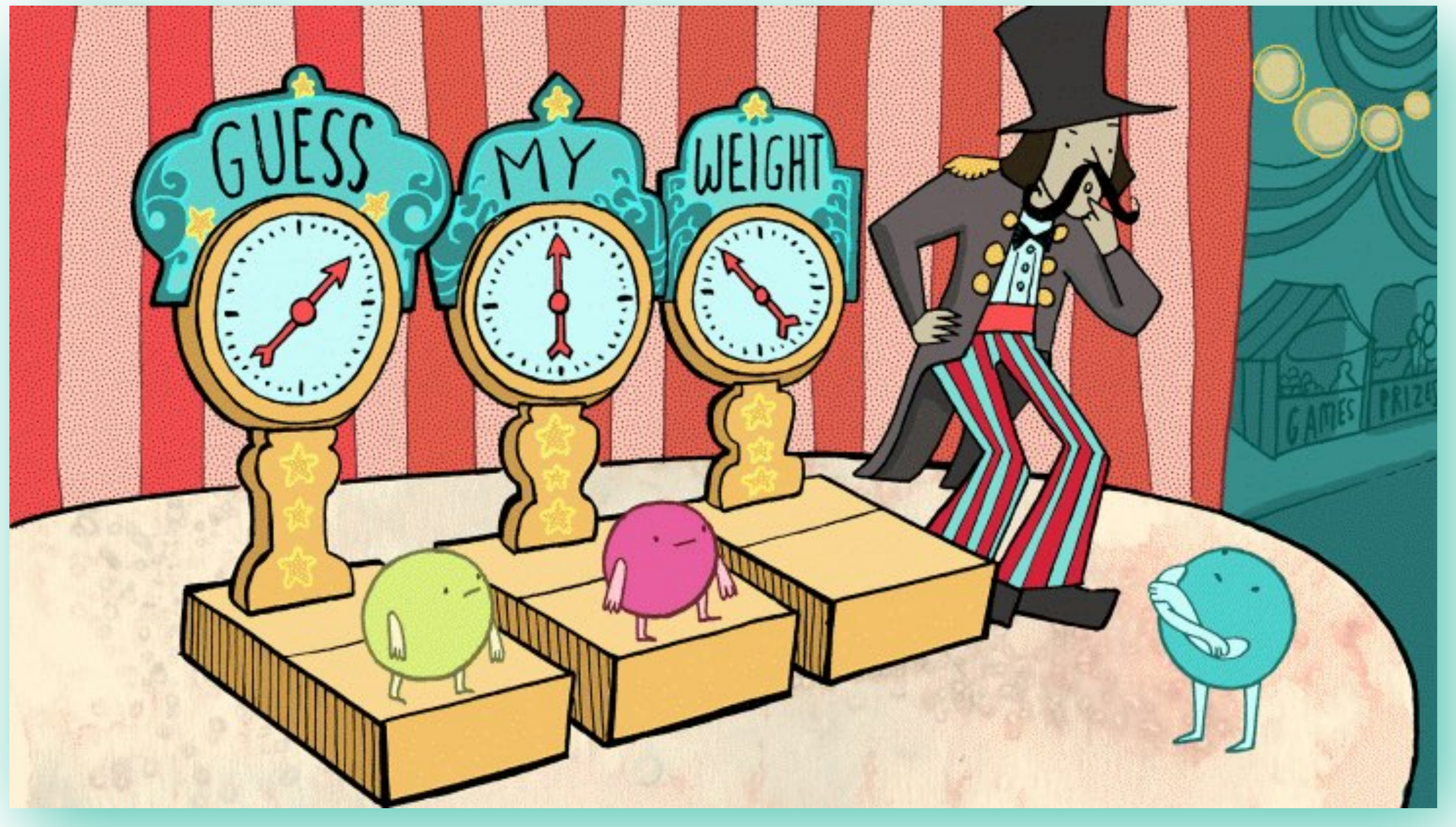
Neutrinos, an overview

✓ Neutrinos are assumed to be massless particles in the SM
- No right-handed neutrino ν_R
- Minimal lepton sector
- Accidental lepton number conservation

Quantum Mechanics at work!

✓ There are three different neutrino *flavors*: electron-, muon- and tauon- neutrinos

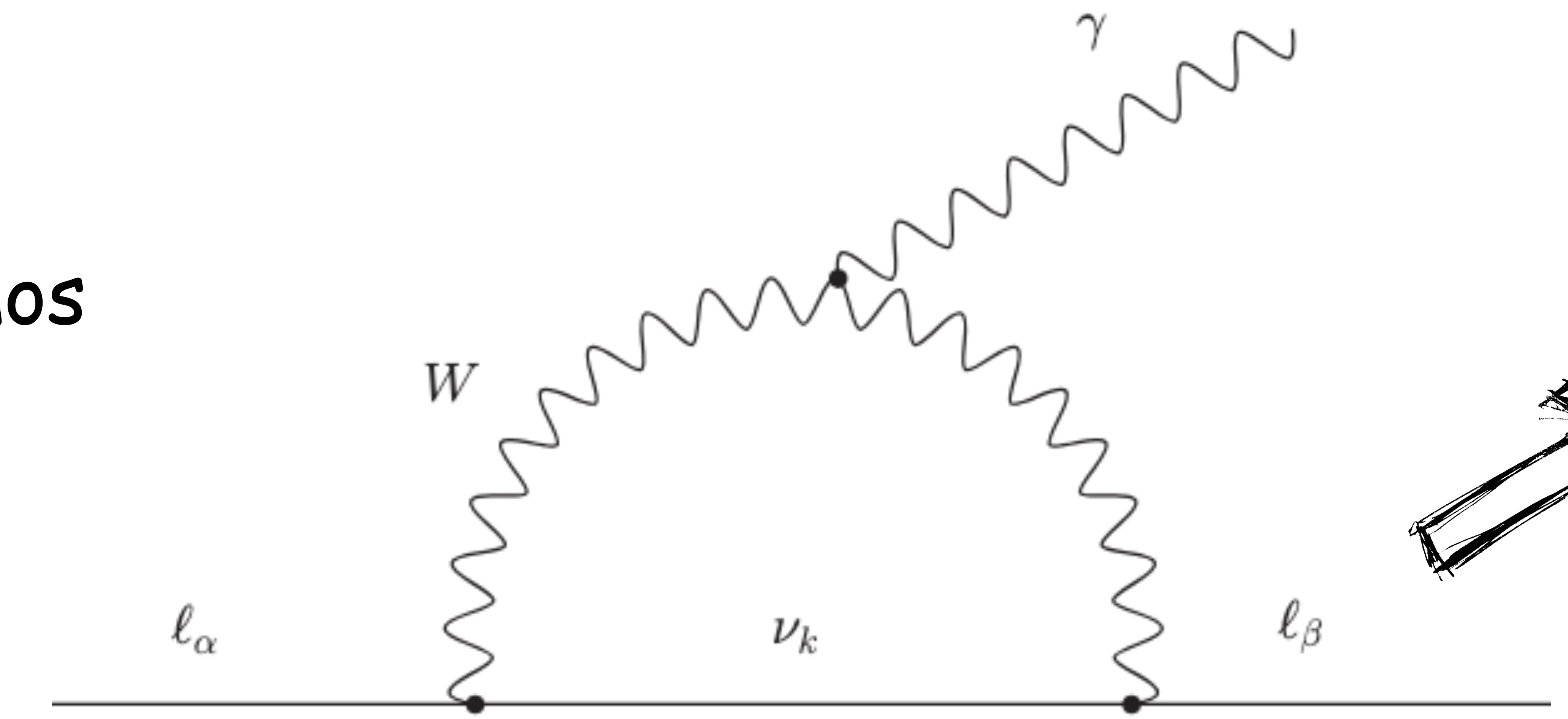
- Their mass eigenstates do not correspond to the flavor ones $|\nu_\alpha\rangle = U_{\alpha i}|\nu_i\rangle$
- Mass eigenstates evolve in time via the Schrödinger equation $|\nu_i(t)\rangle = e^{-im_i t}|\nu_i(0)\rangle$
- Flavor neutrino states **oscillate** $\langle\nu_\alpha(t)|\nu_\beta(0)\rangle \neq \delta_{\alpha\beta}$



➡ **Neutrinos must have a non zero mass!**

cLFV in the Standard Model

SM+ RH-neutrinos



Cheng Li '77, '80; Petcov '77

The PMNS dictates how the RH-neutrinos couple to charged leptons of different flavors

In presence of heavy (at least order TeV) New Physics we expect large effects

$$\frac{\Gamma\left(\ell_\alpha \to \ell_\beta \gamma\right)}{\Gamma\left(\ell_\alpha \to \ell_\beta \bar{\nu}\nu\right)} = \frac{3\alpha}{32\pi}\left|\sum_{k=1,3} U_{\alpha k} U^*_{\beta k} \frac{m^2_{\nu_k}}{M^2_W}\right|^2$$

suppressed by tiny neutrino masses

$$\mathcal{B}(\mu \to e\gamma) \approx \mathcal{B}(\tau \to e\gamma) \approx \mathcal{B}(\tau \to \mu\gamma) = 10^{-55} \div 10^{-54}$$

The Leptonic Dipole Operator

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \left(\sum_{n=1}^{d} \sum_{i} \frac{\mathscr{C}_i^{(n)}}{\Lambda^{n-4}} \mathscr{O}_i^{(n)} + \text{H.c.} \right) , \quad \text{for } d > 4 \, ,$$

$$\mathscr{L}_{\text{dipole}} = \frac{1}{\Lambda^2} \left(\mathscr{C}'_{LR} \mathscr{O}_{LR}^{(6)} + \mathscr{C}'_{RL} \mathscr{O}_{RL}^{(6)} \right) , \qquad \mathscr{C}'^{\dagger}_{RL} = \mathscr{C}'_{LR} = \begin{pmatrix} \mathscr{C}'_{ee} & \mathscr{C}'_{e\mu} & \mathscr{C}'_{e\tau} \\ \mathscr{C}'_{\mu e} & \mathscr{C}'_{\mu\mu} & \mathscr{C}'_{\mu\tau} \\ \mathscr{C}'_{\tau e} & \mathscr{C}'_{\tau\mu} & \mathscr{C}'_{\tau\tau} \end{pmatrix} ,$$

$$\mathscr{O}_{LR}^{(6)} = \frac{v}{\sqrt{2}} \bar{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu}$$

Neutrino mixing parameters

mixing matrix (PMNS)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

matter potential

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{3\ell} \end{pmatrix} U^\dagger + A_{CC} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

mass splittings

Nu IT 6.0 (2024)

		Normal Ordering ($\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC19 without SK atmospheric data	$\sin^2\theta_{12}$	$0.307\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$	$0.308\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561\ ^{0\ 012}_{0\ 015}$	$0.430 \to 0.596$	$0.562\ ^{0\ 012}_{0\ 015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	$48.5\ ^{0\ 7}_{0\ 9}$	$41.0 \to 50.5$	$48.6\ ^{0\ 7}_{0\ 9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	$0.02195\ ^{0\ 00054}_{0\ 00058}$	$0.02023 \to 0.02376$	$0.02224\ ^{0\ 00056}_{0\ 00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	$8.52\ ^{0\ 11}_{0\ 11}$	$8.18 \to 8.87$	$8.58\ ^{0\ 11}_{0\ 11}$	$8.24 \to 8.91$
	$_{\mathrm{CP}}/^\circ$	$177\ ^{19}_{20}$	$96 \to 422$	$285\ ^{25}_{28}$	$201 \to 348$
	$\frac{m^2_{21}}{10\ \ ^5\ \mathrm{eV}^2}$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10\ \ ^3\ \mathrm{eV}^2}$	$+2.534\ ^{0\ 025}_{0\ 023}$	$+2.463 \to +2.606$	$2.510\ ^{0\ 024}_{0\ 025}$	$2.584 \to\ \ 2.438$
		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC24 with SK atmospheric data	$\sin^2\theta_{12}$	$0.308\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$	$0.308\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470\ ^{0\ 017}_{0\ 013}$	$0.435 \to 0.585$	$0.550\ ^{0\ 012}_{0\ 015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3\ ^{1\ 0}_{0\ 8}$	$41.3 \to 49.9$	$47.9\ ^{0\ 7}_{0\ 9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215\ ^{0\ 00056}_{0\ 00058}$	$0.02030 \to 0.02388$	$0.02231\ ^{0\ 00056}_{0\ 00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56\ ^{0\ 11}_{0\ 11}$	$8.19 \to 8.89$	$8.59\ ^{0\ 11}_{0\ 11}$	$8.25 \to 8.93$
	$_{\mathrm{CP}}/^\circ$	$212\ ^{26}_{41}$	$124 \to 364$	$274\ ^{22}_{25}$	$201 \to 335$
	$\frac{m^2_{21}}{10\ \ ^5\ \mathrm{eV}^2}$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10\ \ ^3\ \mathrm{eV}^2}$	$+2.513\ ^{0\ 021}_{0\ 019}$	$+2.451 \to +2.578$	$2.484\ ^{0\ 020}_{0\ 020}$	$2.547 \to\ \ 2.421$

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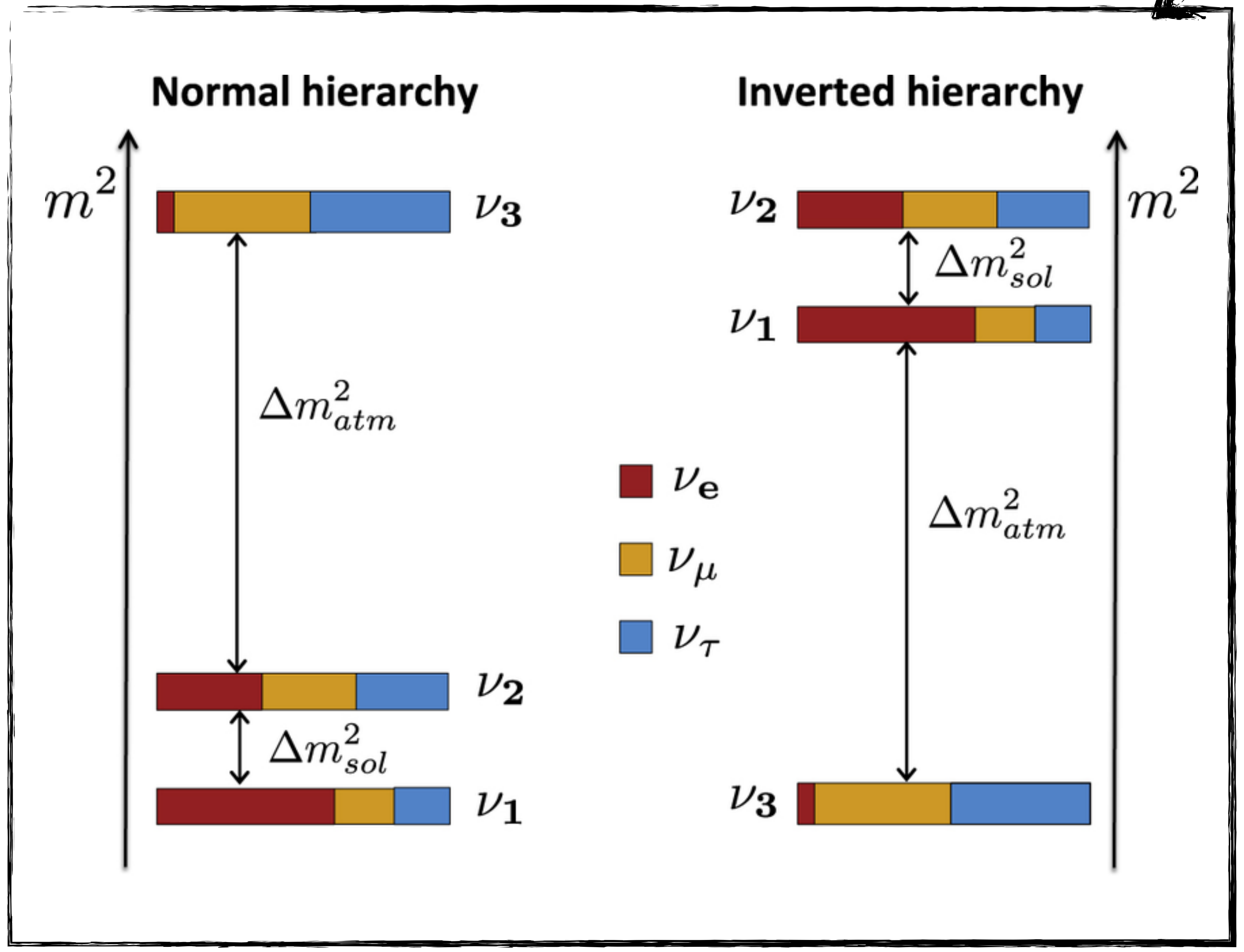

Nu IT 6.0 (2024)

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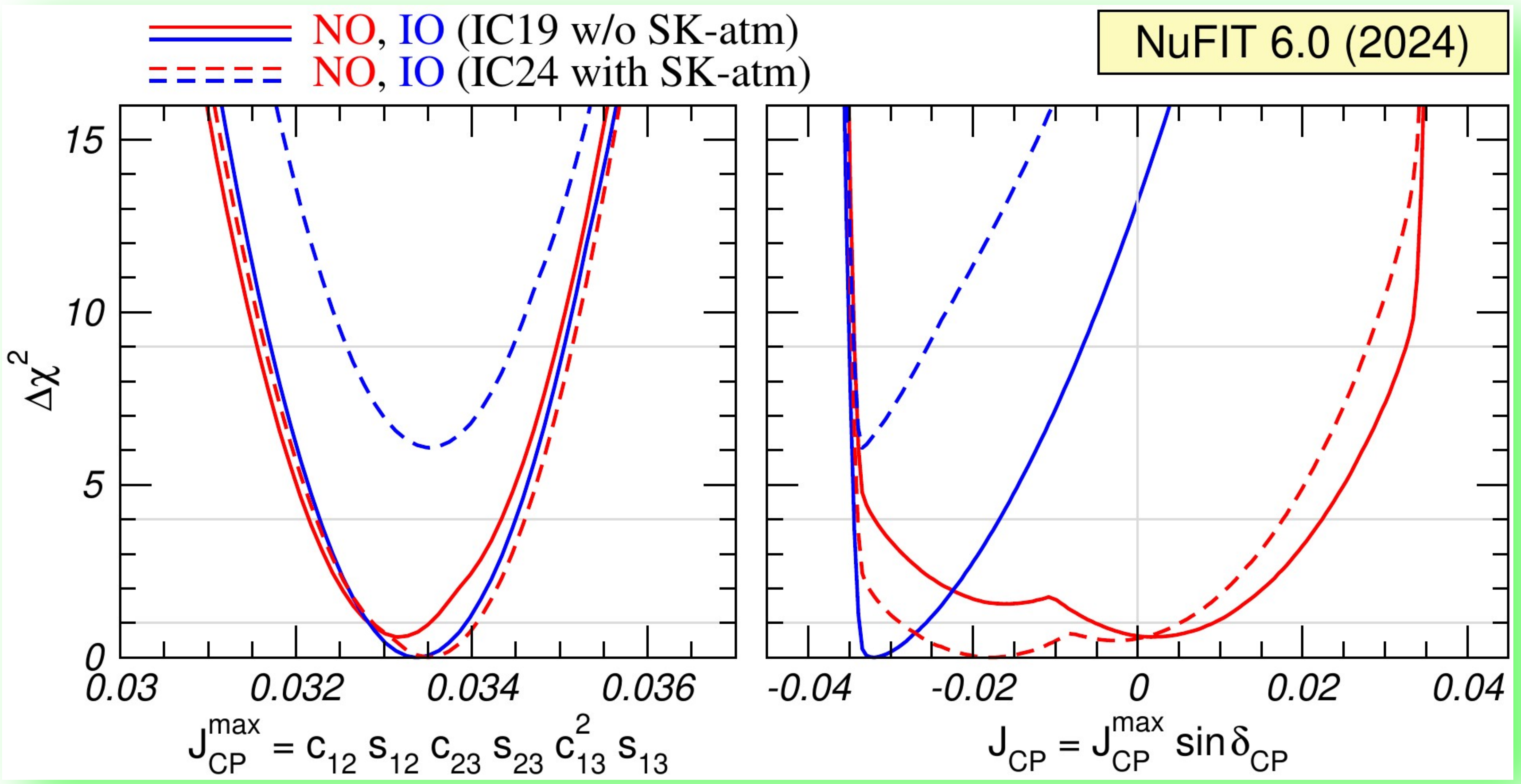
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.307\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$	$0.308\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561\ ^{0\ 012}_{0\ 015}$	$0.430 \to 0.596$	$0.562\ ^{0\ 012}_{0\ 015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	$48.5\ ^{0\ 7}_{0\ 9}$	$41.0 \to 50.5$	$48.6\ ^{0\ 7}_{0\ 9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	$0.02195\ ^{0\ 00054}_{0\ 00058}$	$0.02023 \to 0.02376$	$0.02224\ ^{0\ 00056}_{0\ 00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	$8.52\ ^{0\ 11}_{0\ 11}$	$8.18 \to 8.87$	$8.58\ ^{0\ 11}_{0\ 11}$	$8.24 \to 8.91$
	CP$/^\circ$	$177\ ^{19}_{20}$	$96 \to 422$	$285\ ^{25}_{28}$	$201 \to 348$
IC19	$\frac{m^2_{21}}{10^{\ 5}\ \text{eV}^2}$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10^{\ 3}\ \text{eV}^2}$	$+2.534\ ^{0\ 025}_{0\ 023}$	$+2.463 \to +2.606$	$2.510\ ^{0\ 024}_{0\ 025}$	$2.584 \to \ \ 2.438$
		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.308\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$	$0.308\ ^{0\ 012}_{0\ 011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$	$33.68\ ^{0\ 73}_{0\ 70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470\ ^{0\ 017}_{0\ 013}$	$0.435 \to 0.585$	$0.550\ ^{0\ 012}_{0\ 015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3\ ^{1\ 0}_{0\ 8}$	$41.3 \to 49.9$	$47.9\ ^{0\ 7}_{0\ 9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215\ ^{0\ 00056}_{0\ 00058}$	$0.02030 \to 0.02388$	$0.02231\ ^{0\ 00056}_{0\ 00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56\ ^{0\ 11}_{0\ 11}$	$8.19 \to 8.89$	$8.59\ ^{0\ 11}_{0\ 11}$	$8.25 \to 8.93$
	CP$/^\circ$	$212\ ^{26}_{41}$	$124 \to 364$	$274\ ^{22}_{25}$	$201 \to 335$
IC24	$\frac{m^2_{21}}{10^{\ 5}\ \text{eV}^2}$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$	$7.49\ ^{0\ 19}_{0\ 19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10^{\ 3}\ \text{eV}^2}$	$+2.513\ ^{0\ 021}_{0\ 019}$	$+2.451 \to +2.578$	$2.484\ ^{0\ 020}_{0\ 020}$	$2.547 \to \ \ 2.421$

Flavor puzzle and neutrino mixing

mixing matrix (PMNS)

reactor/interference

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Nu IT 6.0 (2024)

		Normal Ordering ($\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
IC19 without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.345$	$0.308^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{\,0\,73}_{\,0\,70}$	$31.63 \to 35.95$	$33.68^{\,0\,73}_{\,0\,70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561^{\,0\,012}_{\,0\,015}$	$0.430 \to 0.596$	$0.562^{\,0\,012}_{\,0\,015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	$48.5^{\,0\,7}_{\,0\,9}$	$41.0 \to 50.5$	$48.6^{\,0\,7}_{\,0\,9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	$0.02195^{\,0\,00054}_{\,0\,00058}$	$0.02023 \to 0.02376$	$0.02224^{\,0\,00056}_{\,0\,00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	$8.52^{\,0\,11}_{\,0\,11}$	$8.18 \to 8.87$	$8.58^{\,0\,11}_{\,0\,11}$	$8.24 \to 8.91$
	CP$/^\circ$	$177^{\,19}_{\,20}$	$96 \to 422$	$285^{\,25}_{\,28}$	$201 \to 348$
	$\frac{m^2_{21}}{10\ \ 5\ \mathrm{eV}^2}$	$7.49^{\,0\,19}_{\,0\,19}$	$6.92 \to 8.05$	$7.49^{\,0\,19}_{\,0\,19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10\ \ 3\ \mathrm{eV}^2}$	$+2.534^{\,0\,025}_{\,0\,023}$		$2.510^{\,0\,024}_{\,0\,025}$	$2.584 \to\ \ 2.438$
		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 6.1$)	
		bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
IC24 with SK atmospheric data	$\sin^2\theta_{12}$	$0.308^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.345$	$0.308^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{\,0\,73}_{\,0\,70}$	$31.63 \to 35.95$	$33.68^{\,0\,73}_{\,0\,70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470^{\,0\,017}_{\,0\,013}$	$0.435 \to 0.585$	$0.550^{\,0\,012}_{\,0\,015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3^{\,1\,0}_{\,0\,8}$	$41.3 \to 49.9$	$47.9^{\,0\,7}_{\,0\,9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215^{\,0\,00056}_{\,0\,00058}$	$0.02030 \to 0.02388$	$0.02231^{\,0\,00056}_{\,0\,00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56^{\,0\,11}_{\,0\,11}$	$8.19 \to 8.89$	$8.59^{\,0\,11}_{\,0\,11}$	$8.25 \to 8.93$
	CP$/^\circ$	$212^{\,26}_{\,41}$	$124 \to 364$	$274^{\,22}_{\,25}$	$201 \to 335$
	$\frac{m^2_{21}}{10\ \ 5\ \mathrm{eV}^2}$	$7.49^{\,0\,19}_{\,0\,19}$	$6.92 \to 8.05$	$7.49^{\,0\,19}_{\,0\,19}$	$6.92 \to 8.05$
	$\frac{m^2_{3\ell}}{10\ \ 3\ \mathrm{eV}^2}$	$+2.513^{\,0\,021}_{\,0\,019}$	$+2.451 \to +2.578$	$2.484^{\,0\,020}_{\,0\,020}$	$2.547 \to\ \ 2.421$

CP violation?

Flavor puzzle and neutrino mixing

mixing matrix (PMNS)

atmospheric

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

matter potential

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{3\ell} \end{pmatrix} U^\dagger + A_{CC} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

mass splittings

Nu IT 5.3 (2024)

without SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 2.3$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{\,0\,012}_{\,0\,011}$	[illegible]	$0.307^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.66^{\,0\,73}_{\,0\,70}$	[illegible]	$33.67^{\,0\,73}_{\,0\,71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.572^{\,0\,018}_{\,0\,023}$	$0.407 \to 0.620$	$0.578^{\,0\,016}_{\,0\,021}$	$0.412 \to 0.623$
$\theta_{23}/^\circ$	$49.1^{\,1\,0}_{\,1\,3}$	$39.6 \to 51.9$	$49.5^{\,0\,9}_{\,1\,2}$	$39.9 \to 52.1$
sin [illegible]	[illegible]	[illegible]	[illegible]	[illegible] 0.02396
θ_{13} [illegible]	[illegible]	[illegible]	[illegible]	[illegible] 8.90
CP /	[illegible]	[illegible]	[illegible]	[illegible] 360
$\frac{m^2_{21}}{10\ \ 5\ \mathrm{eV}^2}$	$7.41^{\,0\,21}_{\,0\,20}$	$6.81 \to 8.03$	$7.41^{\,0\,21}_{\,0\,20}$	$6.81 \to 8.03$
$\frac{m^2_{3\ell}}{10\ \ 3\ \mathrm{eV}^2}$	$+2.511^{\,0\,027}_{\,0\,027}$	$+2.428 \to +2.597$	$2.498^{\,0\,032}_{\,0\,024}$	$2.581 \to \ \ 2.409$

with SK atmospheric data

	Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 9.1$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.307^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.344$	$0.307^{\,0\,012}_{\,0\,011}$	$0.275 \to 0.344$
$\theta_{12}/^\circ$	$33.67^{\,0\,73}_{\,0\,71}$	$31.61 \to 35.94$	$33.67^{\,0\,73}_{\,0\,71}$	$31.61 \to 35.94$
$\sin^2\theta_{23}$	$0.454^{\,0\,019}_{\,0\,016}$	$0.411 \to 0.606$	$0.568^{\,0\,016}_{\,0\,021}$	$0.412 \to 0.611$
$\theta_{23}/^\circ$	$42.3^{\,1\,1}_{\,0\,9}$	$39.9 \to 51.1$	$48.9^{\,0\,9}_{\,1\,2}$	$39.9 \to 51.4$
$\sin^2\theta_{13}$	$0.02224^{\,0\,00056}_{\,0\,00057}$	$0.02047 \to 0.02397$	$0.02222^{\,0\,00069}_{\,0\,00057}$	$0.02049 \to 0.02420$
$\theta_{13}/^\circ$	$8.58^{\,0\,11}_{\,0\,11}$	$8.23 \to 8.91$	$8.57^{\,0\,13}_{\,0\,11}$	$8.23 \to 8.95$
CP $/^\circ$	$232^{\,39}_{\,25}$	$139 \to 350$	$273^{\,24}_{\,26}$	$195 \to 342$
$\frac{m^2_{21}}{10\ \ 5\ \mathrm{eV}^2}$	$7.41^{\,0\,21}_{\,0\,20}$	$6.81 \to 8.03$	$7.41^{\,0\,21}_{\,0\,20}$	$6.81 \to 8.03$
$\frac{m^2_{3\ell}}{10\ \ 3\ \mathrm{eV}^2}$	$+2.505^{\,0\,024}_{\,0\,026}$	$+2.426 \to +2.586$	$2.487^{\,0\,027}_{\,0\,024}$	$2.566 \to \ \ 2.407$

θ_{23} octant?

The atmospheric angle is almost maximal, but only small sensitivity to its octant

Flavor puzzle and neutrino mixing

mixing matrix (PMNS)

solar

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

matter potential

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \left[\frac{1}{2E_\nu} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{3\ell} \end{pmatrix} U^\dagger + A_{CC} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

mass splittings

Nu IT 5.3 (2024)

without SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.307\ ^{0\,012}_{0\,011}$	$0.275 \to 0.344$	$0.307\ ^{0\,012}_{0\,011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.66\ ^{0\,73}_{0\,70}$	$31.60 \to 35.94$	$33.67\ ^{0\,73}_{0\,71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.572\ ^{0\,018}_{0\,023}$	$0.407 \to 0.620$	$0.578\ ^{0\,016}_{0\,021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1\ ^{1\,0}_{1\,3}$	$39.6 \to 51.9$	$49.5\ ^{0\,9}_{1\,2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203\ ^{0\,00056}_{0\,00058}$	$0.02029 \to 0.02391$	$0.02219\ ^{0\,00059}_{0\,00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54\ ^{0\,11}_{0\,11}$	$8.19 \to 8.89$	$8.57\ ^{0\,11}_{0\,11}$	$8.23 \to 8.90$
	CP$/^\circ$	$197\ ^{41}_{25}$	$108 \to 404$	$286\ ^{27}_{32}$	$192 \to 360$
	$\frac{m^2_{21}}{10\ \ 5\ \mathrm{eV}^2}$	$7.41\ ^{0\,21}_{0\,20}$	$6.81 \to 8.03$	$7.41\ ^{0\,21}_{0\,20}$	$6.81 \to 8.03$
	$\frac{m^2_{3\ell}}{10\ \ 3\ \mathrm{eV}^2}$	$+2.511\ ^{0\,027}_{0\,027}$	$+2.428 \to +2.597$	$2.498\ ^{0\,032}_{0\,024}$	$2.581 \to \ \ 2.409$

with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\chi^2 = 9.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.307\ ^{0\,012}_{0\,011}$	$0.275 \to 0.344$	$0.307\ ^{0\,012}_{0\,011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.67\ ^{0\,73}_{0\,71}$	$31.61 \to 35.94$	$33.67\ ^{0\,73}_{0\,71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.454\ ^{0\,019}_{0\,016}$	$0.411 \to 0.606$	$0.568\ ^{0\,016}_{0\,021}$	$0.412 \to 0.611$
	$\theta_{23}/^\circ$	$42.3\ ^{1\,1}_{0\,9}$	$39.9 \to 51.1$	$48.9\ ^{0\,9}_{1\,2}$	$39.9 \to 51.4$
	$\sin^2\theta_{13}$	$0.02224\ ^{0\,00056}_{0\,00057}$	$0.02047 \to 0.02397$	$0.02222\ ^{0\,00069}_{0\,00057}$	$0.02049 \to 0.02420$
	$\theta_{13}/^\circ$	$8.58\ ^{0\,11}_{0\,11}$	$8.23 \to 8.91$	$8.57\ ^{0\,13}_{0\,11}$	$8.23 \to 8.95$
	CP$/^\circ$	$232\ ^{39}_{25}$	$139 \to 350$	$273\ ^{24}_{26}$	$195 \to 342$
	$\frac{m^2_{21}}{10\ \ 5\ \mathrm{eV}^2}$	$7.41\ ^{0\,21}_{0\,20}$	$6.81 \to 8.03$	$7.41\ ^{0\,21}_{0\,20}$	$6.81 \to 8.03$
	$\frac{m^2_{3\ell}}{10\ \ 3\ \mathrm{eV}^2}$	$+2.505\ ^{0\,024}_{0\,026}$	$+2.426 \to +2.586$	$2.487\ ^{0\,027}_{0\,024}$	$2.566 \to \ \ 2.407$

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\Re\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)\right] + 2\Im\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)\right]$$

ν_e appearance

$$P(\nu_\mu \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

ν_μ disappearance

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\Re\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)\right] + 2\Im\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)\right]$$

ν_e appearance

$$P(\nu_\mu \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2\theta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

ν_μ disappearance

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \cos^4\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

No dependence on the sign of the mass splitting

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\Re\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)\right] + 2\Im\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)\right]$$

ν_e appearance

$$P(\nu_\mu \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2\theta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

ν_μ disappearance

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \cos^4\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

The atmospheric angle θ_{23} is almost maximal,
but only small sensitivity to its octant

The three unknowns in neutrino oscillation

The oscillation probabilities (in vacuum) read:

$$\boldsymbol{P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\Re\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)\right] + 2\Im\left[\sum_{i>j}^{3} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)\right]}$$

ν_e appearance

$$P(\nu_\mu \to \nu_e) \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

ν_μ disappearance

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \sin^2 2\theta_{23} \cos^4 \theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

The CP - violating phase δ appears only in the subleading contribution

The AxiFlavon

$$\Phi = \frac{1}{\sqrt{2}} \left(V_\Phi + \phi\right) e^{ia/V_\Phi}$$

The CP-even flavon field ϕ has a mass $m_\phi \sim \mathcal{O}(V_\Phi)$, therefore it is not directly relevant for the low energy phenomenology and it can be integrated out

The CP-odd axiflavon is massless at the classical level, but receives a non-zero mass from the breaking of $U(1)$ by the QCD anomaly

The AxiFlavon

$$\mathscr{L} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{E}{N}\frac{\alpha_{em}}{8\pi}\frac{a}{f_a}F\tilde{F} \quad \Rightarrow \quad a = \sum_i \frac{X_i V_i a_i}{\sqrt{\sum X_j^2 V_j^2}}$$

$$\sqrt{2}Va = a_1 + a_2$$

$$\chi = \varepsilon_\chi \Lambda e^{-ia(x)/\sqrt{2}V}\,, \qquad \phi = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix} e^{-ia(x)/\sqrt{2}V}$$

Axion couplings

$$\delta_{ab} N = \sum_f \mathrm{Tr}\left(\lambda_a \lambda_b\right) Q_{PC}^f \, d\left(I_f\right)$$

$$E = \sum_f \left(Q_{em}^f\right)^2 d\left(C_f\right) Q_{pc}^f$$

The AxiFlavon

$$\mathscr{L} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{E}{N}\frac{\alpha_{em}}{8\pi}\frac{a}{f_a}F\tilde{F} \quad \Rightarrow \quad a = \sum_i \frac{X_i V_i a_i}{\sqrt{\sum X_j^2 V_j^2}}$$

$$\sqrt{2}Va = a_1 + a_2$$

$$\chi = \varepsilon_\chi \Lambda e^{-ia(x)/\sqrt{2}V}\,, \qquad \phi = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix} e^{-ia(x)/\sqrt{2}V}$$

$$N = \frac{1}{2}\left(4X_{10_a} + 2X_{10_3} + 2X_{10_a} + X_{10_3} + 2X_{\overline{5}_a} + X_{\overline{5}_3}\right) = 9/2\,,$$

$$E = \frac{5}{3}\left(2X_{10_a} + X_{10_3}\right) + \frac{4}{3}\left(2X_{10_a} + X_{10_3}\right) + \frac{1}{3}\left(2X_{\overline{5}_a} + X_{\overline{5}_3}\right) + \left(2X_{\overline{5}_a} + X_{\overline{5}_3}\right) + \left(2X_{10_a} + X_{10_3}\right) = 12\,.$$

The AxiFlavon

$$\mathscr{L} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G} + \frac{E}{N}\frac{\alpha_{em}}{8\pi}\frac{a}{f_a}F\tilde{F} \quad \Rightarrow \quad a = \sum_i \frac{X_i V_i a_i}{\sqrt{\sum X_j^2 V_j^2}}$$

$$\sqrt{2}Va = a_1 + a_2$$

$$\chi = \varepsilon_\chi \Lambda e^{-ia(x)/\sqrt{2}V}\,, \qquad \phi = \begin{pmatrix} \varepsilon_\phi \Lambda \\ 0 \end{pmatrix} e^{-ia(x)/\sqrt{2}V}$$

$$N = \frac{1}{2}\left(4X_{10_a} + 2X_{10_3} + 2X_{10_a} + X_{10_3} + 2X_{\bar{5}_a} + X_{\bar{5}_3}\right) = 9/2\,,$$

$$E = \frac{5}{3}\left(2X_{10_a} + X_{10_3} \text{ [illegible] }\right) + \frac{1}{3}\left(2X_{\bar{5}_a} + X_{\bar{5}_3}\right)$$
$$+ \left(2X_{\bar{5}_a} + X_{\bar{5}_3}\right) + \left(2X_{10_a} + X_{10_3}\right) = 12\,.$$

E/N=8/3

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$$\mathcal{L}_a = -\frac{\partial_\mu a}{\sqrt{2}V} \sum_{f=u,d,e} \left[g^L_{f_i f_j} f_i^\dagger \overline{\sigma}^\mu f_j + g^R_{f_i f_j} f_i^{c\dagger} \overline{\sigma}^\mu f_i^c \right]$$

$$g^L_{f_i f_j} = (V_{fL})_{ki} X_{f_k} (V_{fL})^*_{kj} = X_{f_a} \delta_{ij} + (X_{f_3} - X_{f_a})(V_{fL})_{3i} (V_{fL})^*_{3j} \,,$$

$$g^R_{f_i f_j} = (V_{fR})^*_{ki} X_{f^c_k} (V_{fR})_{kj} = X_{f^c_a} \delta_{ij} + (X_{f^c_3} - X_{f^c_a})(V_{fR})^*_{3i} (V_{fR})_{3j} \,.$$

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$$\varepsilon^f_{L,ij} \equiv (V^f_L)_{3i}(V^f_L)^*_{3j} \qquad \varepsilon^f_{R,ij} \equiv (V^f_R)_{3i}(V^f_R)^*_{3j}\,.$$

$$0 \leq \varepsilon^f_{L/R,ii} \leq 1 \qquad \sum_i \varepsilon^f_{L/R,ii} = 1$$

$$C^V_{f_i f_j} = \frac{-g^L_{f_i f_j} + g^R_{f_j f_i}}{2N} = \frac{X_{f^c_a} - X_{f_a}}{2N}\delta_{ij} + \frac{X_{f^c_3} - X_{f^c_a}}{2N}\varepsilon^f_{R,ij} - \frac{X_{f_3} - X_{f_a}}{2N}\varepsilon^f_{L,ij}\,,$$

$$C^A_{f_i f_j} = \frac{g^L_{f_i f_j} + g^R_{f_j f_i}}{2N} = \frac{X_{f^c_a} + X_{f_a}}{2N}\delta_{ij} + \frac{X_{f^c_3} - X_{f^c_a}}{2N}\varepsilon^f_{R,ij} + \frac{X_{f_3} - X_{f_a}}{2N}\varepsilon^f_{L,ij}\,,$$

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$$C^V_{f_i f_j} = \frac{-g^L_{f_i f_j} + g^R_{f_j f_i}}{2N} = \frac{X_{f^c_a} - X_{f_a}}{2N}\delta_{ij} + \frac{X_{f^c_3} - X_{f^c_a}}{2N}\varepsilon^f_{R,ij} - \frac{X_{f_3} - X_{f_a}}{2N}\varepsilon^f_{L,ij}\,,$$

$$C^A_{f_i f_j} = \frac{g^L_{f_i f_j} + g^R_{f_j f_i}}{2N} = \frac{X_{f^c_a} + X_{f_a}}{2N}\delta_{ij} + \frac{X_{f^c_3} - X_{f^c_a}}{2N}\varepsilon^f_{R,ij} + \frac{X_{f_3} - X_{f_a}}{2N}\varepsilon^f_{L,ij}\,,$$

$$\mathcal{L}_a = \frac{\partial_\mu a}{2 f_a}\overline{f}_i \gamma^\mu \left[C^V_{f_i f_j} + C^A_{f_i f_j}\gamma_5\right] f_j$$

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$$C^V_{u_i u_j} = \frac{\varepsilon^u_{L,ij} - \varepsilon^u_{R,ij}}{9}\,, \qquad C^A_{u_i u_j} = \frac{2\delta_{ij} - \varepsilon^u_{L,ij} - \varepsilon^u_{R,ij}}{9}$$

$$C^V_{d_i d_j} = \frac{\varepsilon^d_{L,ij}}{9}\,, \qquad C^A_{d_i d_j} = \frac{2\delta_{ij} - \varepsilon^d_{L,ij}}{9}\,,$$

$$C^V_{e_i e_j} = -\frac{\varepsilon^e_{R,ij}}{9}\,, \qquad C^A_{e_i e_j} = \frac{2\delta_{ij} - \varepsilon^e_{R,ij}}{9}\,.$$

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$$C^V_{u_i u_j} = \frac{\varepsilon^u_{L,ij} - \varepsilon^u_{R,ij}}{9}\,, \qquad C^A_{u_i u_j} = \frac{2\delta_{ij} - \varepsilon^u_{L,ij} - \varepsilon^u_{R,ij}}{9}$$

$$C^V_{d_i d_j} = \frac{\varepsilon^d_{L,ij}}{9}\,, \qquad C^A_{d_i d_j} = \frac{2\delta_{ij} - \varepsilon^d_{L,ij}}{9}\,,$$

$$C^V_{e_i e_j} = -\frac{\varepsilon^e_{R,ij}}{9}\,, \qquad C^A_{e_i e_j} = \frac{2\delta_{ij} - \varepsilon^e_{R,ij}}{9}\,.$$

$$\varepsilon^u_L \sim \varepsilon^u_R \sim \varepsilon^d_L \sim \varepsilon^e_R \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$C_u = C_d = C_e = C_c = C_s = C_\mu = \frac{2}{9}\,, \qquad C_t = 0\,, \qquad C_b = C_\tau = \frac{1}{9}$$

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Coupling	$m_a^{\max}/C$ [eV]	$m_a^{\max,\mathrm{U}(2)}$ [eV]	$f_a^{\min,\mathrm{U}(2)}$ [GeV]	Constraint
$C_{\mu e}$	$2.1 \cdot 10^{-3}$	78	$7.3 \cdot 10^{4}$	$\mu \to ea$ [30]
C_{bs}^{V}	$9.1 \cdot 10^{-2}$	16	$3.6 \cdot 10^{5}$	$B^+ \to K^+ a$ [28]
C_{sd}^{V}	$1.7 \cdot 10^{-5}$	0.58	$9.8 \cdot 10^{6}$	$K^+ \to \pi^+ a$ [29]
C_{ee}^{A}	$3.1 \cdot 10^{-3}$	0.014	$4.1 \cdot 10^{8}$	WD Cooling [33]
C_N	$3.5 \cdot 10^{-3}$	0.0092	$6.2 \cdot 10^{8}$	SN1987A [34]

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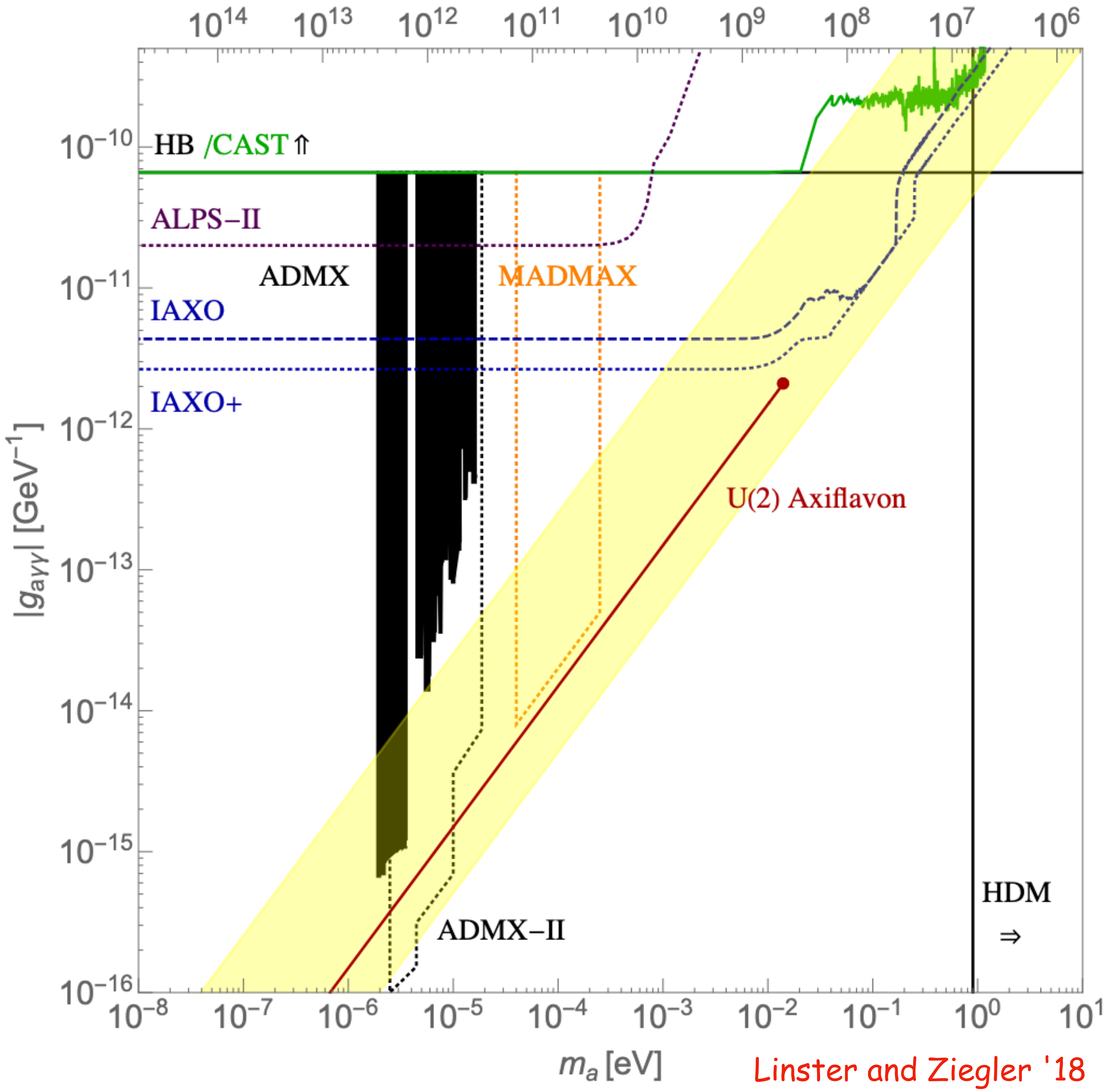

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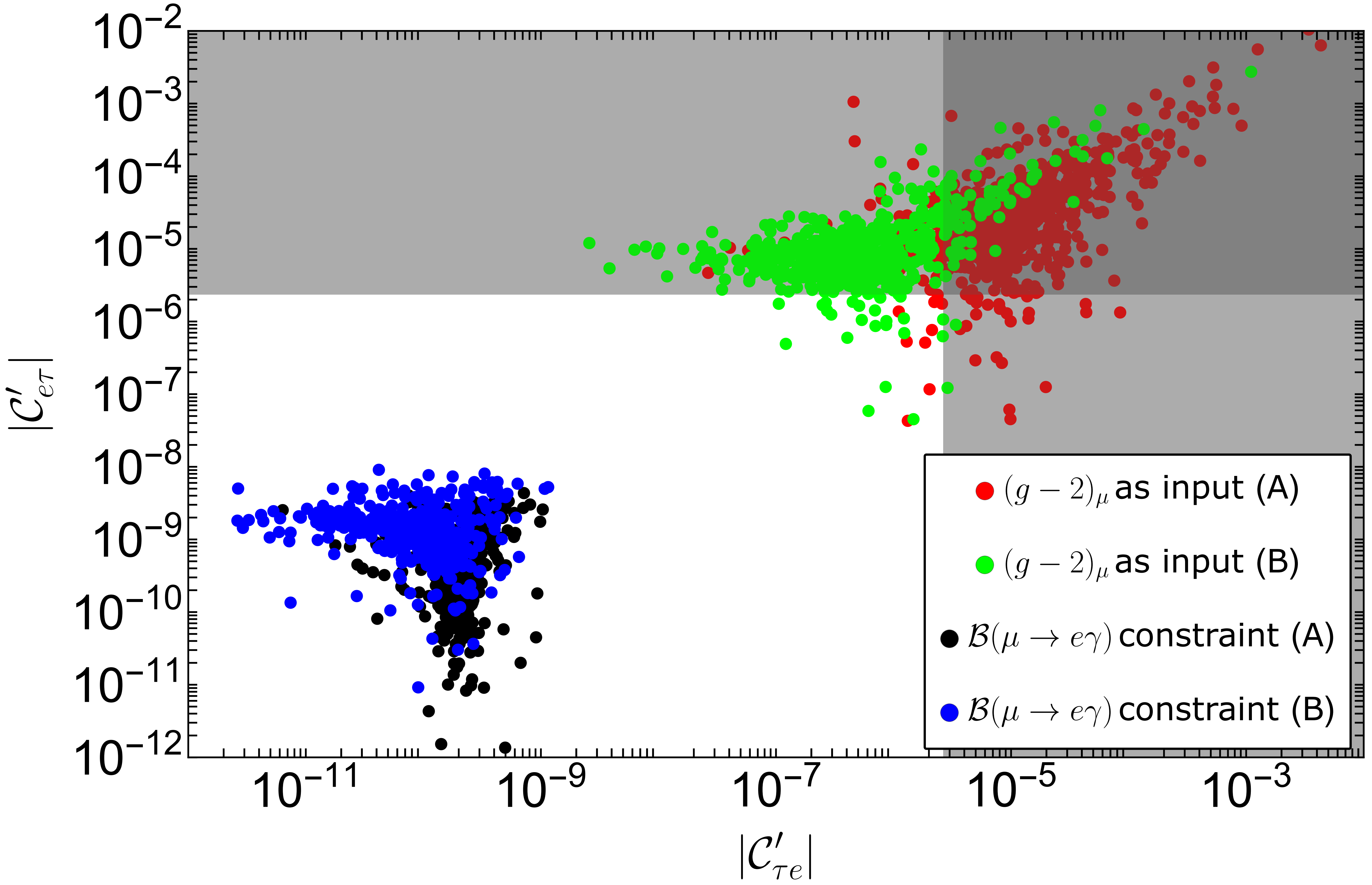
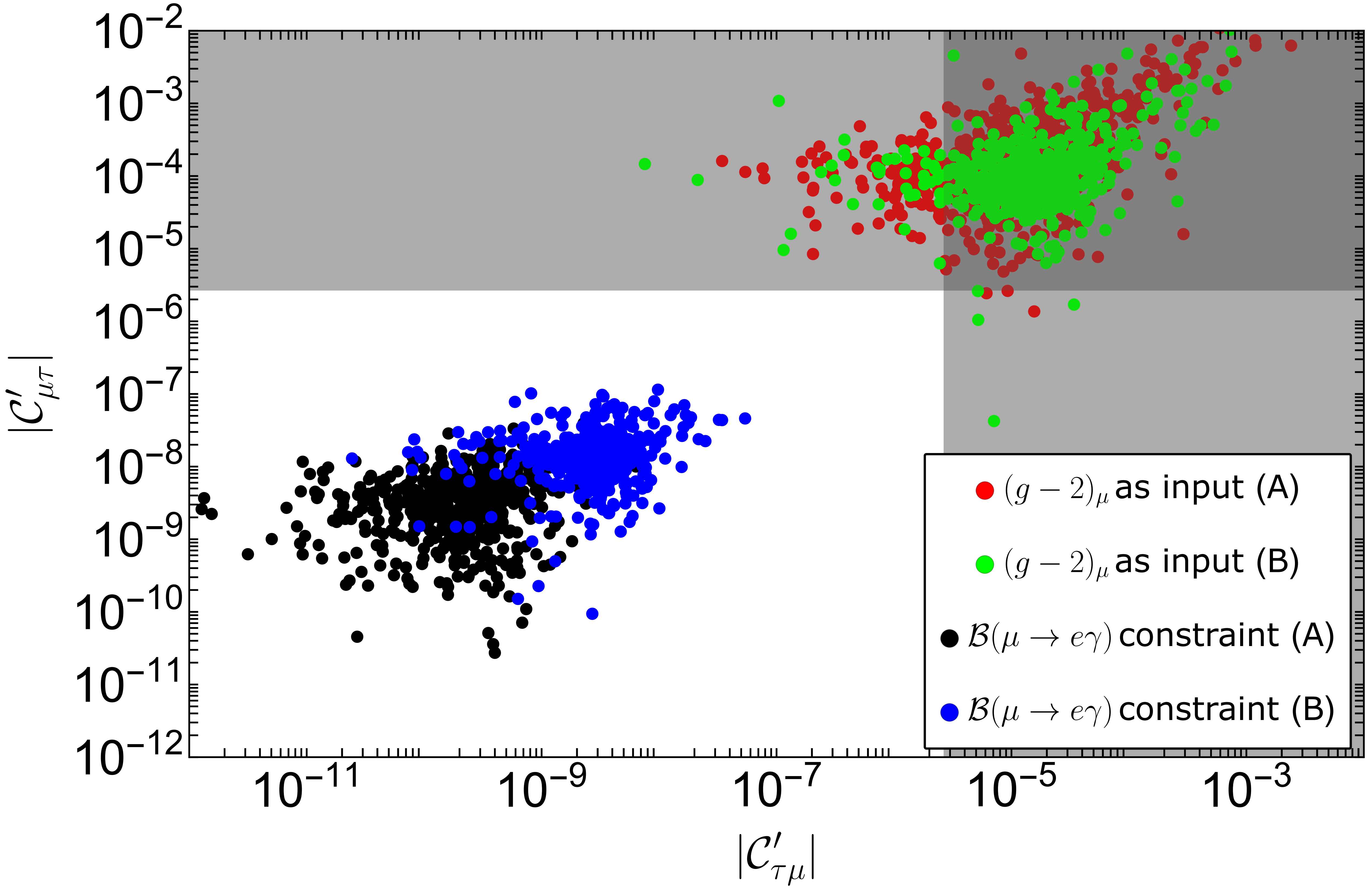
LFV decays in light of $(g-2)_\mu$





Within an $U(2)$ flavor model, the current experimental limits on the branching ratios of LFV decays are not compatible with the observed anomalous Δa_μ