Light states in real multi-Higgs models with spontaneous CP violation

Miguel Nebot

U. of Valencia – IFIC

Motivation – Generic

- Theoretical arguments to constrain masses of (new) particles:
  vacuum stability, triviality, perturbativity, ...
  [short of experimental evidence]
  Weinberg ☞ PRL36 (1976)
  Politzer & Wolfram ☞ PLB82 (1979)
  Cabibbo, Maiani, Parisi & Petronzio ☞ NPB158 (1979)
  Dashen & Neuberger ☞ PRL50 (1983)
  Callaway ☞ NPB233 (1984)
- Perturbative unitarity, high energy scattering of bosons
  Lee, Quigg & Thacker ☞ PRL38 (1977), ☞ PRD16 (1977)
  also Dicus & Mathur ☞ PRD7 (1973)
  Langacker & Weldon ☞ PRL52 (1984)
  Weldon ☞ PLB146 (1984)

Motivation – Specific

- 2HDM with SCPV sourcing all CP violation
  - phenomenologically viable, including realistic CKM and SFCNC under control
  - masses of the new scalars all bounded (from above) owing to perturbativity requirements on the quartic couplings in the scalar potential

  MN, Botella & Branco, ☞ arXiv:1808.00493, EPJC79 (2019)
- General real* 2HDM with SCPV and bounded masses

  MN, ☞ arXiv:1911.02266, PRD102 (2020)
- Is some of this carried over to the real nHDM with SCPV?

*Invariant lagrangian under $\Phi \mapsto \Phi^*$.

Motivation – Specific

- In the 2HDM the point is that the stationarity conditions allow to trade all 3 quadratic couplings in the potential for quartics ($\times$ vacuum expectation values), which are bounded.
- Pessimistic prospects: for nHDM, "free" quadratic couplings can drive large masses*.
- In fact the number of quadratic couplings scales with n^2 while the number of stationarity conditions scales with n: is that the end of it? No, as I will try to show in the following.

*Except for "the Higgs"

Outline

Work done in collaboration with:

Carlos Miró & Daniel Queiroz

☞ `arXiv:2411.00084`, PRD111 (2025)

Real 2HDM with SCPV

The scalar potential

$$V(\Phi_1,\Phi_2) = \mu_1^2\Phi_1^\dagger\Phi_1 + \mu_2^2\Phi_2^\dagger\Phi_2 + \mu_{12}^2\mathcal{H}_{12} + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2$$
$$+ \lambda_{1,2}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_{1,12}(\Phi_1^\dagger\Phi_1)\mathcal{H}_{12} + \lambda_{2,12}(\Phi_2^\dagger\Phi_2)\mathcal{H}_{12}$$
$$+ \lambda_{12,12}\mathcal{H}_{12}^2 + \lambda_{12,12}^{\mathcal{A}}\mathcal{A}_{12}^2$$

$$\mathcal{H}_{12} = \frac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) \qquad \mathcal{A}_{12} = \frac{1}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right)$$

All μ_a^2, μ_{12}^2, λ_a, $\lambda_{1,2}$, $\lambda_{a,12}$, $\lambda_{12,12}$, $\lambda_{12,12}^{\mathcal{A}}$ real

Field expansions

$$\Phi_a = \frac{e^{i\theta_a}}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\mathrm{C}_a^+ \\ v_a + \mathrm{R}_a + i\mathrm{I}_a\end{pmatrix}, \quad \langle\Phi_a\rangle = \frac{e^{i\theta_a}v_a}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$$

Real 2HDM with SCPV

Scalar potential

$$\mathrm{V}(v_a, \theta_a) = V(\langle\Phi_1\rangle, \langle\Phi_2\rangle)$$

with $\langle\Phi_1\rangle$, $\langle\Phi_2\rangle$:

$$\Phi_a^\dagger\Phi_a \to \frac{v_a^2}{2}, \quad \mathcal{H}_{12} \to \frac{c_{12}v_1v_2}{2}, \quad \mathcal{A}_{12} \to -i\frac{s_{12}v_1v_2}{2}$$

where $c_{12} \equiv \cos(\theta_1 - \theta_2)$ and $s_{12} \equiv \sin(\theta_1 - \theta_2)$

$$\mathrm{V}(v_a, \theta_a) = \mu_1^2\frac{v_1^2}{2} + \mu_2^2\frac{v_2^2}{2} + \mu_{12}^2\frac{c_{12}v_1v_2}{2} + \lambda_1\frac{v_1^4}{4} + \lambda_2\frac{v_2^4}{4} + \lambda_{1,2}\frac{v_1^2v_2^2}{4}$$
$$+ \lambda_{1,12}\frac{c_{12}v_1^3v_2}{4} + \lambda_{2,12}\frac{c_{12}v_1v_2^3}{4} + \lambda_{12,12}\frac{c_{12}^2v_1^2v_2^2}{4} - \lambda_{12,12}^{\mathcal{A}}\frac{s_{12}^2v_1^2v_2^2}{4}$$

And now stationarity conditions $\partial_{v_a}\mathrm{V} = \partial_{\theta_a}\mathrm{V} = 0$

Real 2HDM with SCPV

Stationarity conditions $\partial_{v_a}\mathrm{V} = \partial_{\theta_a}\mathrm{V} = 0$

$$\partial_{v_1}\mathrm{V} = \mu_1^2 v_1 + \mu_{12}^2\frac{c_{12}v_2}{2} + \lambda_1 v_1^3 + \lambda_{1,2}\frac{v_1 v_2^2}{2}$$
$$+ \lambda_{1,12}\frac{3c_{12}v_1^2 v_2}{4} + \lambda_{2,12}\frac{c_{12}v_2^3}{4} + \lambda_{12,12}\frac{c_{12}^2 v_1 v_2^2}{2} - \lambda_{12,12}^{\mathcal{A}}\frac{s_{12}^2 v_1 v_2^2}{2}$$

$$\partial_{v_2}\mathrm{V} = \mu_2^2 v_2 + \mu_{12}^2\frac{c_{12}v_1}{2} + \lambda_2 v_2^3 + \lambda_{1,2}\frac{v_1^2 v_2}{2}$$
$$+ \lambda_{1,12}\frac{c_{12}v_1^3}{4} + \lambda_{2,12}\frac{3c_{12}v_1 v_2^2}{4} + \lambda_{12,12}\frac{c_{12}^2 v_1^2 v_2}{2} - \lambda_{12,12}^{\mathcal{A}}\frac{s_{12}^2 v_1^2 v_2}{2}$$

$$\partial_{\theta_2}\mathrm{V} = \mu_{12}^2\frac{s_{12}v_1 v_2}{2} + \lambda_{1,12}\frac{s_{12}v_1^3 v_2}{4} + \lambda_{2,12}\frac{s_{12}v_1 v_2^3}{4}$$
$$+ \lambda_{12,12}\frac{c_{12}s_{12}v_1^2 v_2^2}{2} + \lambda_{12,12}^{\mathcal{A}}\frac{c_{12}s_{12}v_1^2 v_2^2}{2} = -\partial_{\theta_1}\mathrm{V}$$

Solve $\partial_{\theta_2}\mathrm{V} = 0$ for μ_{12}^2, $\partial_{v_1}\mathrm{V} = 0$ for μ_1^2 and $\partial_{v_2}\mathrm{V} = 0$ for μ_2^2

. . . *no quadratic couplings left*!

Real 2HDM with SCPV

Mass matrices

$$(M_{\pm}^2)_{a,b} = \left[\frac{\partial^2 V}{\partial \mathrm{C}_a^+ \partial \mathrm{C}_b^-}\right],$$

$$(M_0^2)_{a,b} = \left[\frac{\partial^2 V}{\partial \mathrm{R}_a \partial \mathrm{R}_b}\right], \quad (M_0^2)_{n+a,n+b} = \left[\frac{\partial^2 V}{\partial \mathrm{I}_a \partial \mathrm{I}_b}\right],$$

$$(M_0^2)_{a,n+b} = (M_0^2)_{n+b,a} = \left[\frac{\partial^2 V}{\partial \mathrm{R}_a \partial \mathrm{I}_b}\right]$$

$[f]$: f evaluated at vanishing fields $\mathrm{C}_a^{\pm}, \mathrm{R}_a, \mathrm{I}_a \to 0$

Real 2HDM with SCPV

The mass matrix of the charged scalars is amiable

$$M_{\pm}^2 = \frac{1}{2}\lambda_{12,12}^{\mathcal{A}}\begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}$$

Rotation into "the" Higgs basis ($v = \sqrt{v_1^2 + v_2^2}$)

$$\mathcal{R}_{\mathrm{Ch}} = \frac{1}{v}\begin{pmatrix} v_1 & v_2 \\ -v_2 & v_1 \end{pmatrix},$$

$$\mathcal{R}_{\mathrm{Ch}}\, M_{\pm}^2\, \mathcal{R}_{\mathrm{Ch}}^T = \frac{1}{2}\lambda_{12,12}^{\mathcal{A}} v^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

As expected

- one massless Goldstone $G^{\pm}$
- and one charged scalar with mass$^2 = \frac{1}{2}\lambda_{12,12}^{\mathcal{A}} v^2$,

  which is bounded by perturbativity constraints on $\lambda_{12,12}^{\mathcal{A}}$

Real 2HDM with SCPV

The mass matrix of the neutral scalars is less amiable
Rotation into "the" Higgs basis ($v = \sqrt{v_1^2 + v_2^2}$)

$$\mathcal{R}_{\rm N} = \begin{pmatrix} \mathcal{R}_{\rm Ch} & \mathbf{0} \\ \mathbf{0} & \mathcal{R}_{\rm Ch} \end{pmatrix},$$

$$M^2_{0,\rm HB} = \mathcal{R}_{\rm N}\, M^2_0\, \mathcal{R}_{\rm N}^T$$

with

$$M^2_{0,\rm HB} = \begin{pmatrix} \times & \times & 0 & \times \\ \times & \times & 0 & \times \\ 0 & 0 & 0 & 0 \\ \times & \times & 0 & \times \end{pmatrix}$$

As expected

- one massless Goldstone G^0
- 3 neutral scalars with bounded masses from bounded λ's

Real 2HDM with SCPV

The mass matrix of the neutral scalars is less amiable

$$(M^2_{0,\mathrm{HB}})_{11} = \frac{2}{v^2}\begin{pmatrix} \lambda_1 v_1^4 + \lambda_2 v_2^4 + \lambda_{1,2} v_1^2 v_2^2 + c_{12}(\lambda_{1,12} v_1^3 v_2 + \lambda_{2,12} v_1 v_2^3) \\ +\lambda_{12,12} c_{12}^2 v_1^2 v_2^2 - \lambda^{\mathcal{A}}_{12,12} s_{12}^2 v_1^2 v_2^2 \end{pmatrix}$$

$$(M^2_{0,\mathrm{HB}})_{22} = \frac{2}{v^2}\begin{pmatrix} (\lambda_1 + \lambda_2 - \lambda_{1,2}) v_1^2 v_2^2 + c_{12}(\lambda_{2,12} - \lambda_{1,12}) v_1 v_2 (v_1^2 - v_2^2) \\ +\lambda_{12,12} c_{12}^2 (v_1^2 - v_2^2)^2 + \lambda^{\mathcal{A}}_{12,12}\left(\frac{c_{12}^2}{4}(v_1^2 - v_2^2)^2 + v_1^2 v_2^2\right) \end{pmatrix}$$

$$(M^2_{0,\mathrm{HB}})_{12} = \frac{1}{v^2}\begin{pmatrix} (2\lambda_2 v_2^2 - 2\lambda_1 v_1^2 + \lambda_{1,2}(v_1^2 - v_2^2)) v_1 v_2 \\ +\frac{c_{12}}{2}(\lambda_{1,12} v_1^2 (v_1^2 - 3v_2^2) - \lambda_{2,12} v_2^2 (v_2^2 - 3v_1^2)) \\ +\lambda_{12,12} c_{12}^2 v_1 v_2 (v_1^2 - v_2^2) - \lambda^{\mathcal{A}}_{12,12} s_{12}^2 v_1 v_2 (v_1^2 - v_2^2) \end{pmatrix}$$

$$(M^2_{0,\mathrm{HB}})_{14} = \frac{s_{12}}{2}(\lambda_{1,12} v_1^2 + \lambda_{2,12} v_2^2 + 2(\lambda_{12,12} + \lambda^{\mathcal{A}}_{12,12}) c_{12} v_1 v_2)$$

$$(M^2_{0,\mathrm{HB}})_{24} = \frac{s_{12}}{2}((\lambda_{2,12} - \lambda_{1,12}) v_1 v_2 + (\lambda_{12,12} + \lambda^{\mathcal{A}}_{12,12}) c_{12}(v_1^2 - v_2^2))$$

$$(M^2_{0,\mathrm{HB}})_{44} = v^2 \frac{s_{12}^2}{2}(\lambda_{12,12} + \lambda^{\mathcal{A}}_{12,12})$$

Real 2HDM with SCPV

Recap

- Through stationarity conditions all 3 quadratic couplings in the potential traded for quartics (and vevs)
- $\Rightarrow$ new scalars have bounded masses through perturbativity bounds on quartic couplings

Real nHDM with SCPV

Real nHDM scalar potential

$$
\begin{aligned}
V(\Phi_a) = &\sum_{a=1}^{n} \mu_a^2 \Phi_a^\dagger \Phi_a + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \mu_{ab}^2 \mathcal{H}_{ab} + \sum_{a=1}^{n} \lambda_a (\Phi_a^\dagger \Phi_a)^2 \\
&+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \lambda_{a,b} (\Phi_a^\dagger \Phi_a)(\Phi_b^\dagger \Phi_b) + \sum_{a=1}^{n}\sum_{b=1}^{n-1}\sum_{c=b+1}^{n} \lambda_{a,bc} (\Phi_a^\dagger \Phi_a)\mathcal{H}_{bc} \\
&+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd} \mathcal{H}_{ab}\mathcal{H}_{cd} \\
&+ \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda^{\mathcal{A}}_{ab,cd} \mathcal{A}_{ab}\mathcal{A}_{cd}
\end{aligned}
$$

$$
\mathcal{H}_{ab} = \frac{1}{2}\left(\Phi_a^\dagger \Phi_b + \Phi_b^\dagger \Phi_a\right) \qquad \mathcal{A}_{ab} = \frac{1}{2}\left(\Phi_a^\dagger \Phi_b - \Phi_b^\dagger \Phi_a\right)
$$

μ_a^2, μ_{ab}^2, λ_a, $\lambda_{a,b}$, $\lambda_{a,bc}$, $\lambda_{ab,cd}$, $\lambda^{\mathcal{A}}_{ab,cd}$ real, CP invariant ($\Phi_a \mapsto \Phi_a^*$)

Real nHDM with SCPV

- Quadratic couplings, $n\ \mu_a^2 + n(n-1)/2\ \mu_{ab}^2$: $n(n+1)/2$
- Stationarity conditions: $2n-1$
- Omitting Goldstones, $n-1$ charged and $2n-1$ neutral scalars

n	2	3	4	5	6	7
$n(n+1)/2$	3	6	10	15	21	28
$2n-1$	3	5	7	9	11	13
$(n-1)(n-2)/2$	0	1	3	6	10	15

- Quadratic couplings in excess of the number of stationarity conditions + the number of (neutral) scalars
- Can they make all (new) scalar masses $\gg v$?
- Let us do some numerical exercise

Real nHDM with SCPV, numerical phenomenology

Starting with the scalar potential for the real nHDM, with a given n:

- Compute $2n-1$ stationarity conditions
- Trade $2n-1$ quadratic couplings for quartic and other quadratic couplings
- Compute mass matrices
- Generate random numerical quartics, free quadratics, v_a and θ_a
- Compute eigenvalues of mass matrices

Real nHDM with SCPV

Real nHDM scalar potential, $\mathrm{V}(v_a,\theta_a) = V(\langle\Phi_a\rangle)$

$$
\begin{aligned}
4\mathrm{V}(v_a,\theta_a) = {} & 2\sum_{a=1}^{n}\mu_a^2 v_a^2 + 2\sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\mu_{ab}^2 c_{ab} v_a v_b + \sum_{a=1}^{n}\lambda_a v_a^4 \\
& + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\lambda_{a,b} v_a^2 v_b^2 + \sum_{a=1}^{n}\sum_{b=1}^{n-1}\sum_{c=b+1}^{n}\lambda_{a,bc} c_{bc} v_a^2 v_b v_c \\
& + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd} c_{ab} c_{cd} v_a v_b v_c v_d \\
& + \sum_{a=1}^{n-1}\sum_{b=a+1}^{n}\sum_{c=1}^{n-1}\sum_{d=c+1}^{n}\Bigg|_{(a,b)\leq(c,d)} \lambda_{ab,cd}^{\mathcal{A}} s_{ab} s_{cd} v_a v_b v_c v_d
\end{aligned}
$$

where $c_{ab} = \cos(\theta_a - \theta_b)$, $s_{ab} = \sin(\theta_a - \theta_b)$

Real nHDM with SCPV

Stationarity conditions (focus on quadratics)

$$\partial_{\theta_1} \mathrm{V} = -\frac{1}{2}\sum_{b=2}^{n} \mu^2_{1b} s_{1b} v_1 v_b + \text{Quartics}$$

$$\partial_{\theta_j} \mathrm{V} = \frac{1}{2}\sum_{a=1}^{j-1} \mu^2_{aj} s_{aj} v_a v_j - \frac{1}{2}\sum_{b=j+1}^{n} \mu^2_{jb} s_{jb} v_j v_b + \text{Quartics}$$

$$\partial_{\theta_n} \mathrm{V} = \frac{1}{2}\sum_{a=1}^{n-1} \mu^2_{an} s_{an} v_a v_n + \text{Quartics}$$

$$\text{N.B.} \ \sum_{j=1}^{n} \partial_{\theta_j} \mathrm{V} = 0$$

Trade all μ^2_{1j} for other quadratics and quartics

Real nHDM with SCPV

Stationarity conditions (focus on quadratics)

$$\partial_{v_1} \mathrm{V} = \mu_1^2 v_1 + \frac{1}{2}\sum_{b=2}^{n} \mu_{1b}^2 c_{1b} v_b + \text{Quartics}$$

$$\partial_{v_j} \mathrm{V} = \mu_j^2 v_j + \frac{1}{2}\sum_{a=1}^{j-1} \mu_{aj}^2 c_{aj} v_a + \frac{1}{2}\sum_{b=j+1}^{n} \mu_{jb}^2 c_{jb} v_b + \text{Quartics}$$

$$\partial_{v_n} \mathrm{V} = \mu_n^2 v_n + \frac{1}{2}\sum_{a=1}^{n-1} \mu_{an}^2 c_{an} v_a + \text{Quartics}$$

Trade all μ_j^2 for other quadratics and quartics
$\Rightarrow$ all n μ_a^2's and all $n-1$ μ_{1j}^2 quadratics removed, we are left with
$(n-1)(n-2)/2$ quadratics μ_{ab}^2 $a \geq 2$, $b > a$

Real nHDM with SCPV, numerical phenomenology

Numerical generation

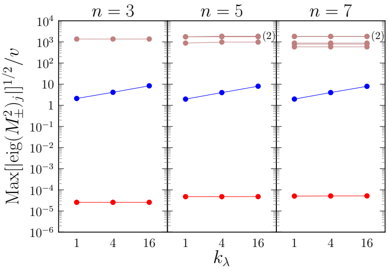
- Random $\mu^2_{ab} \in [-1;+1] \times k_\mu \times 10^{10}$ GeV^2 $(a \geq 2,\ b > a)$
- Random $\lambda_a, \lambda_{a,b}, \lambda_{a,bc}, \lambda_{ab,cd}, \lambda^{\mathcal{A}}_{ab,cd} \in [-1;+1] \times k_\lambda$
- Random v_a $(v_1^2 + \ldots + v_n^2 = v^2 = 246^2$ GeV$^2)$
- Random $\theta_a \in [-\pi;+\pi]$
- Discard cases in which the stationarity conditions yield quadratics outside $[-1;+1] \times k_\mu \times 10^{10}$ GeV^2
- Compute the resulting "mass^2 matrices"
- Order eigenvalues according to their absolute values
- No requirement on positivity of the eigenvalues (local minimum)
- No requirement on boundedness from below of the potential*
- Repeat and keep the largest value of each |eigenvalue|
- Results in the following plots

*No need to sound the alarm because of the absence of these two requirements

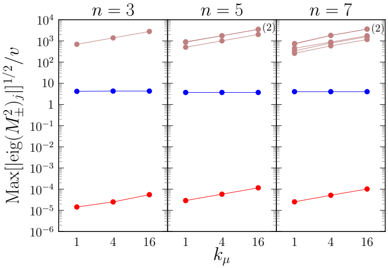
Real nHDM with SCPV, numerical phenomenology

Charged mass matrix

$k_\mu = 4$



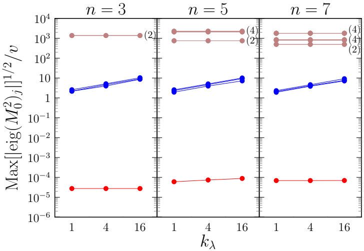
$k_\lambda = 4$



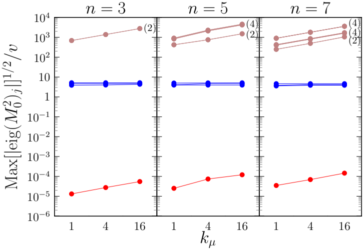
- Numerical zero Goldstone
- One light $\mathcal{O}(v)$ state, sensitive to k_λ, insensitive to k_μ
- $n-2$ heavy states, insensitive to k_λ, sensitive to k_μ

Neutral mass matrix

$k_\mu = 4$



$k_\lambda = 4$



- Numerical zero Goldstone
- Three light $\mathcal{O}(v)$ states, sensitive to k_λ, insensitive to k_μ
- $2n-4$ heavy states, insensitive to k_λ, sensitive to k_μ

Real nHDM with SCPV, numerical phenomenology

Recap

- As expected, "numerical massless" Goldstone.
- As expected, heavy states, insensitive to k_λ, sensitive to k_μ.
- Unexpected, light $\mathcal{O}(v)$ states, sensitive to k_λ, insensitive to k_μ.

How can they ignore $\mu_{ab}^2 \gg v^2$?

Real nHDM with SCPV – no quartics

Short of analytic black sorcery☆,

how do we gain understanding of what is at work?

- Consider the limit where all the quartic couplings are negligible with respect to the quadratic ones

$$V(\Phi_a) \to V_2(\Phi_a) = \sum_{a=1}^{n} \mu_a^2 \Phi_a^\dagger \Phi_a + \sum_{a=1}^{n-1} \sum_{b=a+1}^{n} \mu_{ab}^2 \mathcal{H}_{ab}$$

- In particular: what about null eigenvectors of the mass matrices in that regime? (since they ignore $\mu_{ab}^2 \gg v^2$)
- Then, treat quartic couplings as a perturbation

☆Obtain the eigenvalues and eigenvectors of the mass matrices for generic n

Real nHDM with SCPV, analysis – no quartics

Stationarity conditions (again, need them soon)

$$\partial_{\theta_1} \mathrm{V} = -\frac{1}{2}\sum_{b=2}^{n} \mu^2_{1b} s_{1b} v_1 v_b$$

$$\partial_{\theta_j} \mathrm{V} = \frac{1}{2}\sum_{a=1}^{j-1} \mu^2_{aj} s_{aj} v_a v_j - \frac{1}{2}\sum_{b=j+1}^{n} \mu^2_{jb} s_{jb} v_j v_b$$

$$\partial_{\theta_n} \mathrm{V} = \frac{1}{2}\sum_{a=1}^{n-1} \mu^2_{an} s_{an} v_a v_n \qquad \text{N.B. } \sum_{j=1}^{n} \partial_{\theta_j} \mathrm{V} = 0$$

$$\partial_{v_1} \mathrm{V} = \mu^2_1 v_1 + \frac{1}{2}\sum_{b=2}^{n} \mu^2_{1b} c_{1b} v_b$$

$$\partial_{v_j} \mathrm{V} = \mu^2_j v_j + \frac{1}{2}\sum_{a=1}^{j-1} \mu^2_{aj} c_{aj} v_a + \frac{1}{2}\sum_{b=j+1}^{n} \mu^2_{jb} c_{jb} v_b$$

$$\partial_{v_n} \mathrm{V} = \mu^2_n v_n + \frac{1}{2}\sum_{a=1}^{n-1} \mu^2_{an} c_{an} v_a$$

Real nHDM with SCPV, analysis – no quartics

Read out mass terms

$$
\begin{aligned}
V_2(\Phi_a)\big|_{\text{dim}=2} = &\sum_{a=1}^{n} \mu_a^2 \left(\mathrm{C}_a^- \mathrm{C}_a^+ + \frac{1}{2}\left[\mathrm{R}_a^2 + \mathrm{I}_a^2\right]\right) \\
&+ \frac{1}{2}\sum_{a=1}^{n-1}\sum_{b=a+1}^{n} \mu_{ab}^2 \begin{pmatrix} c_{ab}\left[\mathrm{C}_a^-\mathrm{C}_b^+ + \mathrm{C}_b^-\mathrm{C}_a^+\right] - i s_{ab}\left[\mathrm{C}_a^-\mathrm{C}_b^+ - \mathrm{C}_b^-\mathrm{C}_a^+\right] \\ c_{ab}\left[\mathrm{R}_a\mathrm{R}_b + \mathrm{I}_a\mathrm{I}_b\right] + s_{ab}\left[\mathrm{R}_a\mathrm{I}_b - \mathrm{R}_b\mathrm{I}_a\right] \end{pmatrix}
\end{aligned}
$$

$$
V_2(\Phi_a)\big|_{\text{dim}=2} = \begin{pmatrix} \mathrm{C}_1^- \dots \mathrm{C}_n^- \end{pmatrix} M_\pm^2 \begin{pmatrix} \mathrm{C}_1^+ \\ \vdots \\ \mathrm{C}_n^+ \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \mathrm{R}_1 \dots \mathrm{R}_n & \mathrm{I}_1 \dots \mathrm{I}_n \end{pmatrix} M_0^2 \begin{pmatrix} \mathrm{R}_1 \\ \vdots \\ \mathrm{R}_n \\ \mathrm{I}_1 \\ \vdots \\ \mathrm{I}_n \end{pmatrix}
$$

Real nHDM with SCPV, analysis – no quartics

Mass matrices, $\theta_{ab} \equiv \theta_a - \theta_b$

$$M^2_{\pm} = \begin{pmatrix} \mu_1^2 & \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 & \mu_2^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{2n}}\mu_{2n}^2 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \mu_{n-1}^2 & \frac{1}{2}e^{i\theta_{n-1n}}\mu_{n-1n}^2 \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 & \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 & \cdots & \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 & \mu_n^2 \end{pmatrix}$$

$$M^2_0 = \begin{pmatrix} \mathrm{Re}\left(M^2_\pm\right) & \mathrm{Im}\left(M^2_\pm\right) \\ -\mathrm{Im}\left(M^2_\pm\right) & \mathrm{Re}\left(M^2_\pm\right) \end{pmatrix}, \qquad \begin{cases} \mathrm{Re}\left(M^2_\pm\right)^T = \mathrm{Re}\left(M^2_\pm\right) \\ \mathrm{Im}\left(M^2_\pm\right)^T = -\mathrm{Im}\left(M^2_\pm\right) \end{cases}$$

Null eigenvector $\vec{u} \in \mathbb{C}^n$ of $M^2_\pm$

$$M^2_\pm \vec{u} = \vec{0}_n$$

Real nHDM with SCPV – no quartics

One can read $M_{\pm}^2\,\vec{u}=\vec{0}_n$ as

$$\left(\mathrm{Re}\left(M_{\pm}^2\right)+i\mathrm{Im}\left(M_{\pm}^2\right)\right)\left(\mathrm{Re}\left(\vec{u}\right)+i\mathrm{Im}\left(\vec{u}\right)\right)=$$
$$\mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right)-\mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right)$$
$$+\,i\left(\mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right)+\mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right)\right)=\vec{0}_n$$

that is

$$\mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right)-\mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right)=\vec{0}_n$$
$$\mathrm{Im}\left(M_{\pm}^2\right)\mathrm{Re}\left(\vec{u}\right)+\mathrm{Re}\left(M_{\pm}^2\right)\mathrm{Im}\left(\vec{u}\right)=\vec{0}_n$$

which means

$$\begin{pmatrix}\mathrm{Re}\left(M_{\pm}^2\right) & \mathrm{Im}\left(M_{\pm}^2\right)\\ -\mathrm{Im}\left(M_{\pm}^2\right) & \mathrm{Re}\left(M_{\pm}^2\right)\end{pmatrix}\begin{pmatrix}\mathrm{Re}\left(\vec{u}\right)\\ -\mathrm{Im}\left(\vec{u}\right)\end{pmatrix}=\begin{pmatrix}\vec{0}_n\\ \vec{0}_n\end{pmatrix}$$
$$\begin{pmatrix}\mathrm{Re}\left(M_{\pm}^2\right) & \mathrm{Im}\left(M_{\pm}^2\right)\\ -\mathrm{Im}\left(M_{\pm}^2\right) & \mathrm{Re}\left(M_{\pm}^2\right)\end{pmatrix}\begin{pmatrix}\mathrm{Im}\left(\vec{u}\right)\\ \mathrm{Re}\left(\vec{u}\right)\end{pmatrix}=\begin{pmatrix}\vec{0}_n\\ \vec{0}_n\end{pmatrix}$$

Real nHDM with SCPV, analysis – no quartics

- If there is a null eigenvector $\vec{u} \in \mathbb{C}^n$ of $M^2_{\pm}$
  $\Rightarrow$ two null eigenvectors $\begin{pmatrix} \mathrm{Re}(\vec{u}) \\ -\mathrm{Im}(\vec{u}) \end{pmatrix}$, $\begin{pmatrix} \mathrm{Im}(\vec{u}) \\ \mathrm{Re}(\vec{u}) \end{pmatrix} \in \mathbb{R}^{2n}$ of M^2_0
- We already know a null eigenvector $\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ of $M^2_{\pm}$,
  corresponding to the charged Goldstone since

$$\begin{pmatrix} \mu_1^2 & \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 & \mu_2^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{2n}}\mu_{2n}^2 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \mu_{n-1}^2 & \frac{1}{2}e^{i\theta_{n-1n}}\mu_{n-1n}^2 \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 & \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 & \cdots & \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 & \mu_n^2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_{n-1} \\ v_n \end{pmatrix} =$$

$$\begin{pmatrix} \mu_1^2 v_1 + \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 v_2 + \ldots + \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 v_n \\ \vdots \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 v_1 + \ldots + \frac{1}{2}e^{-i\theta_{12}}\mu_{1j-1}^2 v_{j-1} + \mu_2^2 v_j + \frac{1}{2}e^{i\theta_{jj+1}}\mu_{jj+1}^2 v_{j+1} + \ldots + \frac{1}{2}e^{i\theta_{jn}}\mu_{2n}^2 v_n \\ \vdots \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 v_1 + \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 v_2 + \ldots + \frac{1}{2}e^{-i\theta_{n-1n}}\mu_{n-1n}^2 v_{n-1} + \mu_n^2 v_n \end{pmatrix}$$

Real nHDM with SCPV, analysis – no quartics

which, of course, looks suspiciously familiar

$$M_{\pm}^2 \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = \begin{pmatrix} \partial_{v_1} \mathrm{V}_2 - \frac{i}{v_1}\partial_{\theta_1} \mathrm{V}_2 \\ \vdots \\ \partial_{v_j} \mathrm{V}_2 - \frac{i}{v_j}\partial_{\theta_j} \mathrm{V}_2 \\ \vdots \\ \partial_{v_n} \mathrm{V}_2 - \frac{i}{v_n}\partial_{\theta_n} \mathrm{V}_2 \end{pmatrix} = \vec{0}_n$$

…but we already knew about the charged Goldstone

- In the neutral sector it gives the neutral Goldstone

and "the Higgs"

$$\vec{r}_G^{\,T} = (\vec{0}_n, v_1, \ldots, v_n), \qquad \vec{r}_h^{\,T} = (v_1, \ldots, v_n, \vec{0}_n)$$

- Can we find another null eigenvector?
- Stare intensely at $M_{\pm}^2$ …

⚡⚡⚡ there is another simple null eigenvector!

Real nHDM with SCPV, analysis – no quartics

- The *other* null eigenvector

$$\vec{c}_0^{\,T} = (e^{i2\theta_1}v_1, \ldots, e^{i2\theta_j}v_j, \ldots, e^{i2\theta_n}v_n)$$

- Check:

$$\begin{pmatrix} \mu_1^2 & \frac{1}{2}e^{i\theta_{12}}\mu_{12}^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{1n}}\mu_{1n}^2 \\ \frac{1}{2}e^{-i\theta_{12}}\mu_{12}^2 & \mu_2^2 & \cdots & \cdots & \frac{1}{2}e^{i\theta_{2n}}\mu_{2n}^2 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \mu_{n-1}^2 & \frac{1}{2}e^{i\theta_{n-1\,n}}\mu_{n-1\,n}^2 \\ \frac{1}{2}e^{-i\theta_{1n}}\mu_{1n}^2 & \frac{1}{2}e^{-i\theta_{2n}}\mu_{2n}^2 & \cdots & \frac{1}{2}e^{-i\theta_{n-1\,n}}\mu_{n-1\,n}^2 & \mu_n^2 \end{pmatrix} \begin{pmatrix} e^{i2\theta_1}v_1 \\ e^{i2\theta_2}v_2 \\ \vdots \\ e^{i2\theta_{n-1}}v_{n-1} \\ e^{i2\theta_n}v_n \end{pmatrix}$$

$$M_\pm^2\,\vec{c}_0 = \begin{pmatrix} e^{i2\theta_1}(\partial_{v_1}\mathrm{V}_2 + \frac{i}{v_1}\partial_{\theta_1}\mathrm{V}_2) \\ \vdots \\ e^{i2\theta_j}(\partial_{v_j}\mathrm{V}_2 + \frac{i}{v_j}\partial_{\theta_j}\mathrm{V}_2) \\ \vdots \\ e^{i2\theta_n}(\partial_{v_n}\mathrm{V}_2 + \frac{i}{v_n}\partial_{\theta_n}\mathrm{V}_2) \end{pmatrix} = \vec{0}_n \qquad \checkmark$$

Real nHDM with SCPV, analysis – no quartics

Null eigenvectors of mass matrices with no quartics in V

- Charged

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}, \qquad \begin{pmatrix} e^{i2\theta_1} v_1 \\ \vdots \\ e^{i2\theta_n} v_n \end{pmatrix}$$

- Neutral

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \qquad \begin{pmatrix} 0 \\ \vdots \\ 0 \\ v_1 \\ \vdots \\ v_n \end{pmatrix}, \qquad \begin{pmatrix} v_1 \cos 2\theta_1 \\ \vdots \\ v_n \cos 2\theta_n \\ -v_1 \sin 2\theta_1 \\ \vdots \\ -v_n \sin 2\theta_n \end{pmatrix}, \qquad \begin{pmatrix} v_1 \sin 2\theta_1 \\ \vdots \\ v_n \sin 2\theta_n \\ v_1 \cos 2\theta_1 \\ \vdots \\ v_n \cos 2\theta_n \end{pmatrix}$$

- Not orthogonal! But no problem, one can always orthonormalize

Real nHDM with SCPV, analysis

- For the complete problem, reintroduce quartics as a perturbation (degenerate perturbation theory)
- Goldstones remain Goldstones
- One charged and three neutral scalars get masses $\mathcal{O}(v)$ (as the numerical exercise hinted)

Real nHDM with SCPV, analysis

Is there a symmetry interpretation of this result?

- Notice that for the CP conjugate vevs $\langle \Phi_a^* \rangle = \langle \Phi_a \rangle^*$

$$V(\langle \Phi_1 \rangle^*, \ldots, \langle \Phi_n \rangle^*) = V(\langle \Phi_1 \rangle, \ldots, \langle \Phi_n \rangle)$$

  $\Rightarrow \langle \Phi_a \rangle^*$ give a different candidate vacuum
- With quartic couplings ignored,
  mass terms in V_2 *do not involve* vevs
- $\Rightarrow$ both the Goldstone corresponding to the vacuum and the Goldstone that corresponds to the CP transformed vacuum
  yield zero eigenvalues
- The latter are not, however, true Goldstones
  when V_4 is reintroduced

Conclusions

- Real 2HDM with SCPV is peculiar: bounded spectrum
  - stationarity conditions remove all quadratic couplings in V
  - (quartics bounded by perturbativity considerations)
  - (Generic property in symmetry-shaped potentials)
- Real nHDM with SCPV
  - stationarity conditions cannot remove all quadratic couplings in V
  - "overabundance" of free quadratic couplings
  - one could have expected that besides "the Higgs" (+ Goldstones), all scalars could have large masses
  - ...but that is not the case: (at least) one charged and two new neutral scalars have $\mathcal{O}(v)$ masses
    - analysis in the absence of quartic couplings
    - null eigenvectors of the mass matrices in that situation
- Open ends
  - Generic phenomenological prospects related to the light states?
  - Similar situation for other potentials?
  - ...

Grazie mille!

Thank you!