Effective description of Nelson-Barr models and the θ parameter

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Based on 2506.03257 with Gustavo Alves

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The strong CP problem

- Source of CP breaking in the SM: $\bar{\theta}\, G \cdot \tilde{G} \qquad \sim \mathbf{E}_a \cdot \mathbf{B}_a$
- Experimentally $\quad \bar{\theta} \lesssim 10^{-10}$ from neutron EDM
- Theoretically $\quad \bar{\theta}$ has two contributions: quark Yukawa + QCD
- Both could be oder one, e.g., $\delta_{\rm CKM} \sim 66^\circ$, $\theta_{QCD} \in [0, 2\pi)$.
- Strong CP problem: Why $\bar{\theta}$ is so small?
- Technical naturalness does not apply: CP is violated in Nature.
- However, if small, radiatively stable (7 loop β within the SM).

The strong CP problem

How to solve it?

1. Massless u quark (disfavored by lattice)
2. Promote $\bar{\theta}$ to a field $a(x)/f \to$ QCD potential $\to \langle\bar{\theta}\rangle \approx 0$ + axion
3. CP or P is a symmetry which is only spontaneously broken

The strong CP problem

3. CP or P is a symmetry which is only spontaneously broken

- Since P or CP is only spontaneously broken, $\theta_{\mathrm{QCD}} = 0$
- P is a symmetry $\rightarrow$ Left-right models

  Beg,Tsao,'78; Mohapatra,Senjanovic,'78,...

- CP is a symmetry
  - Nelson-Barr mechanism — Nelson,'84; Barr,'84
  - Others
    - susy non-renormalization theorems — Hiller,Schmaltz,'01
    - modular symmetries — Feruglio's talk
    - texture-zeros — Tanimoto's talk.
    - ...

The strong CP problem

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Nelson-Barr theories

Nelson-Barr UV

Λ_{cp}

VLQ-EFT

M_{VLQ}

SMEFT

v

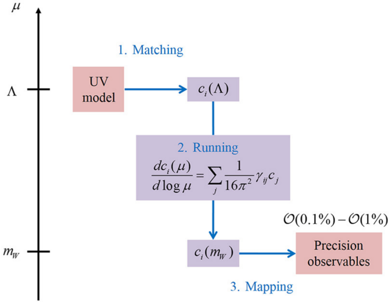
SMEFT broken phase

- Nelson '84. Barr '84.
- CP spontaneously broken at $\Lambda_{\rm cp}$
- VLQs transmit CP violation to the SM
- Arranges $\bar{\theta} = 0$ at tree level

$$\bar{\theta}_{tree} = \theta_{\rm QCD} + \arg\det Y_d + \arg\det Y_u$$

- Large $\delta_{\rm CKM}$ should be generated
- Radiative corrections are calculable and should be tiny

Effective theories and matching



- Matching between theories (EFTs)
- Running within the EFTs
- Functional methods at 1-loop

  Chan,'86. Gaillard,'86. Cheyettte,'88.

  Henning,Lu,Murayama,'14,'16. Drozd,Ellis,Quevillon,You,'16.

  Fuentes-Martins,Portoles,Ruiz-Femenia,'16. Cohen,Zhang,'21. · · ·

Effective theories and matching

Nelson-Barr UV

Λ_{cp}

VLQ-EFT

M_{VLQ}

SMEFT

v

SMEFT broken phase

Our task

- Integrating out CP breaking scalars at Λ_{cp}
- Integrating out VLQs at M_{VLQ}
- Running between Λ_{cp} and M_{VLQ}

Nelson-Barr theories

CP basis

$$\mathscr{L}_{\rm UV} = (\text{kinetic}) - \Big(\bar{q}_L \mathscr{Y}^u \tilde{H} u_R + \bar{q}_L \mathscr{Y}^d H d_R + h.c.\Big)$$
$$- \Big(\bar{B}_L \mathscr{M}^{Bd} d_R + \bar{B}_L (\mathscr{F}_i s_i) d_R + \bar{B}_L \mathscr{M}^B B_R + h.c.\Big) - V(|H|^2, s_i)$$

Arbitrary number of scalars s_i and VLQs B_{rR}, B_{rL}

$$\mathscr{M}^{Bd} = \mathscr{F}_i u_i$$

$$V_S = \tfrac{1}{2} M_{ij}^2 s_i s_j + \tfrac{1}{3!} \lambda'_{ijk} s_i s_j s_k + \tfrac{1}{4!} \lambda_{ijkl} s_i s_j s_k s_l$$
$$V_{HS} = H^\dagger H(\tfrac{1}{2} \gamma_{ij} s_i s_j + \gamma'_i s_i)\,.$$

VLQ mass basis $\rightarrow$ generic case

$$\mathscr{L}_{\rm UV} = (\text{kinetic}) - \Big(\bar{q}_L Y^u \tilde{H} u_R + \bar{q}_L Y^d H d_R + \bar{q}_L Y^B H B_R + h.c.\Big)$$
$$- \Big(\bar{B}_L M^B B_R + \bar{B}_L (F_i s_i) d_R + \bar{B}_L (G_i s_i) B_R + h.c.\Big) - V(|H|^2, s_i)$$

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VLQs of Nelson-Barr type (NB-VLQs)

Tree-level matching up to dimension 4

Cherchiglia,Nishi,JHEP'20. Cherchiglia,Conto,Nishi,JHEP'21. Alves,Cherchiglia,Nishi,JHEP'23.

One parameter less

	# of param.	# of CP-odd
SM	$3+3+3+1=10$	1
generic VLQ	16	3
NB-VLQ	15	1

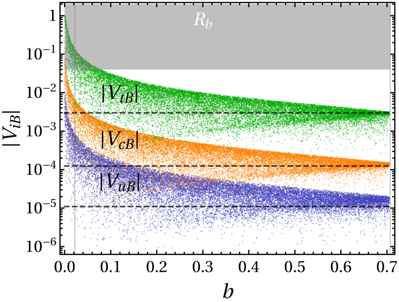
For down type NB-VLQs

$$|Y_1^B| : |Y_2^B| : |Y_3^B| \sim |V_{ub}| : |V_{cb}| : |V_{tb}|$$
$$\sim 0.0036 : 0.04 : 1$$

Largely flavor safe

See Martinelli's talk

$M_B = 1.4\,\mathrm{TeV}$



VLQs of Nelson-Barr type (NB-VLQs)

In leading seesaw

$$Y^d Y^{d\dagger} = V_{\text{CKM}}\,\text{diag}(y_d^2, y_s^2, y_b^2)\,V_{\text{CKM}}^\dagger$$

CP phase in the CKM should come from the complex part in

$$\boxed{Y^d Y^{d\dagger} = \mathscr{Y}^d \left(\mathbb{1}_3 - w w^\dagger\right) \mathscr{Y}^{d\mathsf{T}}}$$

Then

$$\tilde{w}^\dagger = \mathscr{M}^{B^{-1}} \mathscr{M}^{Bd} \sim w^\dagger = M^{B^{-1}} \mathscr{M}^{Bd} \sim O(1)$$

NB-VLQs cannot decouple

Correlations

$$\boxed{Y^B = Y^d \tilde{w}}$$

VLQ Yukawa inherit the hierarchy of SM Yukawa/CKM

Tree-level matching at $\Lambda_{\rm cp}$ up to dimension 6

Alves,Nishi,2506.03257

	Operator	Wilson coefficient	Op. dim.
$\mathcal{O}^{ara'r'}_{BdBd}$	$\bar{B}^a_L d^r_R \bar{B}^{a'}_L d^{r'}_R$	$\frac{1}{2}M^{-2}_{ik}[F_i]^{ar}[F_k]^{a'r'}$	6
$\mathcal{O}^{raa'b'}_{dBBd}$	$\bar{d}^r_R B^a_L \bar{B}^{a'}_L d^{r'}_R$	$M^{-2}_{ik}[F^\dagger_i]^{ra}[F_k]^{a'r'}$	6
$\mathcal{O}^{aba'b'}_{B_LB_RB_LB_R}$	$\bar{B}^a_L B^b_R \bar{B}^{a'}_L B^{b'}_R$	$\frac{1}{2}M^{-2}_{ik}[G_i]^{ab}[G_k]^{a'b'}$	6
$\mathcal{O}^{aba'b'}_{B_RB_LB_LB_R}$	$\bar{B}^a_R B^b_L \bar{B}^{a'}_L B^{b'}_R$	$M^{-2}_{ik}[G^\dagger_i]^{ab}[G_k]^{a'b'}$	6
$\mathcal{O}^{raa'b'}_{dBBB}$	$\bar{d}^r_R B^a_L \bar{B}^{a'}_L B^{b'}_R$	$\frac{1}{2}M^{-2}_{ik}[F^\dagger_i]^{ra}[G_k]^{a'b'}$	6
$\mathcal{O}^{bra'b'}_{BBBd}$	$\bar{B}^b_L d^r_R \bar{B}^{a'}_L B^{b'}_R$	$\frac{1}{2}M^{-2}_{ik}[F_i]^{br}[G_k]^{a'b'}$	6
$\mathcal{O}^{ar}_{BdHH}$	$\bar{B}^a_L d^r_R (H^\dagger H)$	$M^{-2}_{ik}[F_i]^{ar}\gamma'_k$	5
$\mathcal{O}^{ab}_{BBHH}$	$\bar{B}^a_L B^b_R (H^\dagger H)$	$M^{-2}_{ik}[G_i]^{ab}\gamma'_k$	5
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$-\frac{1}{2}\gamma'_i M^{-4}_{ik}\gamma'_k$	6
$\mathcal{O}_H$	$(H^\dagger H)^3$	$-\frac{1}{2}\gamma'_i M^{-2}_{ij}\gamma_{jl}M^{-2}_{lk}\gamma'_k + \frac{1}{6}\lambda'_{ijk}M^{-2}_{il}\gamma'_l M^{-2}_{jm}\gamma'_m M^{-2}_{kn}\gamma'_n$	6
$\mathcal{O}_{\delta\lambda_H}$	$(H^\dagger H)^2$	$\frac{1}{2}\gamma'_i M^{-2}_{ik}\gamma'_k$	4

One-loop matching at $\Lambda_{\rm cp}$ up to dimension 5

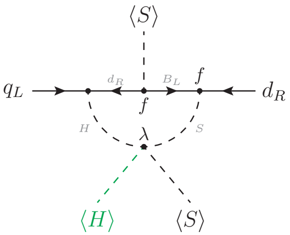
$$\mathscr{L}^{1\text{-}\ell}_{\rm EFT}\Big|_{\rm new} = +\bar{B}_L C^{\sigma\rm eff}_{BB}\,\sigma\cdot F B_R + \bar{B}_L C^{\sigma\rm eff}_{Bd}\,\sigma\cdot F d_R + h.c.$$

$$\begin{aligned}16\pi^2[C^{\sigma\rm eff}_{BB}]^{ab} &= \tfrac{1}{2}M^{-2}_{ik}\Big\{(\tfrac{3}{2}\delta_{kj}+L_{kj})[G_i M^{B\dagger} G_j]^{ab}\\ &\quad -\tfrac{1}{6}[(G_i G_k^\dagger + F_i F_k^\dagger)M^B + M^B G_i^\dagger G_k]^{ab}\Big\}\,,\\ 16\pi^2[C^{\sigma\rm eff}_{Bd}]^{ar} &= \tfrac{1}{2}M^{-2}_{ik}\Big\{(\tfrac{3}{2}\delta_{kj}+L_{kj})[G_i M^{B\dagger} F_j]^{ar} - \tfrac{1}{6}[M^B G_i^\dagger F_k]^{ar}\Big\}\end{aligned}$$

Matching at $M_{\rm VLQ}$

- Tree-level matching
- One-loop matching only tracking contributions to $\bar{\theta}$
- Relevant one-loop RGE between $\Lambda_{\rm cp}$ and $M_{\rm VLQ}$

Final formula for $\bar{\theta}$: Nelson-Barr case



Estimate

$$\delta\bar{\theta} \quad \sim \quad \frac{f^2 \lambda_{HS}}{16\pi^2}$$

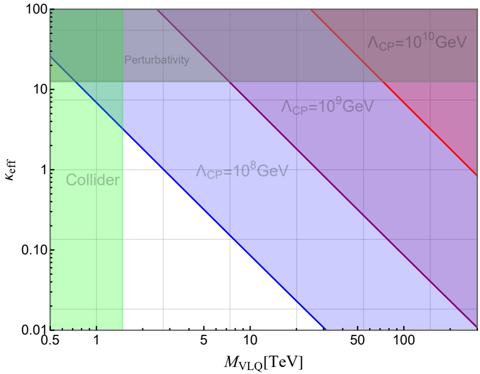
Exact formula with log enhancement

$$\bar{\theta}(M_{\mathrm{VLQ}}) = -\frac{1}{8\pi^2} u_l \gamma_{lj} M_{ji}^{-2} u_k \,\mathrm{Im}\,\mathrm{tr}\left[\mathscr{F}_i \mathscr{F}_k^\dagger\right] \log \frac{M_{\mathrm{VLQ}}}{\Lambda_{\mathrm{cp}}}$$

$$\langle S_i \rangle = u_i$$

Running from $\Lambda_{\mathrm{cp}} \to M_{\mathrm{VLQ}}$

Final formula for $\bar{\theta}$: Nelson-Barr case



$\bar{\theta} < 10^{-10}$

$F_i \sim G_i \sim M_{\rm VLQ}/\Lambda_{\rm cp}$

This suppression is enough for $\Lambda_{\rm cp} \sim 10^8\,\rm GeV$ and $M_{\rm VLQ} \sim 2\,\rm TeV$

Small $M_{\rm VLQ}$ is techinically natural

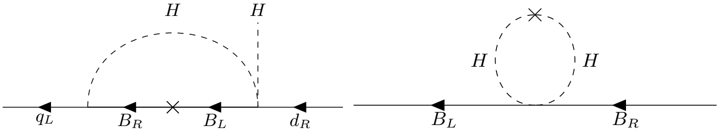
$$\bar{\theta}(M_{\rm VLQ}) = -\frac{1}{8\pi^2}\kappa_{\rm eff}\left(\frac{M_{\rm VLQ}}{\Lambda_{\rm cp}}\right)^2 \log\frac{M_{\rm VLQ}}{\Lambda_{\rm cp}}$$

$$\kappa_{\rm eff} \equiv \Lambda_{\rm cp}(M^{-2}\gamma')_i\, \mathrm{Im\,tr}\left[\frac{M^{B\dagger}}{M_{\rm VLQ}} G_i \frac{\Lambda_{\rm cp}}{M_{\rm VLQ}}\right]$$

Final formula for $\bar{\theta}$: effective case

Dimension 5 operators

$$\mathcal{O}_{BdHH} = \bar{B}_L d_R |H|^2 \,, \quad \mathcal{O}_{BBHH} = \bar{B}_L B_R |H|^2$$



$$8\pi^2 \delta\theta(M_{\rm VLQ}) = -\Big(\mathrm{Im\,tr}\left[Y^{d\,-1} Y^B M^{B\dagger} C_{BdHH} \right] - m_H^2\, \mathrm{Im\,tr}\left[M^{B\,-1} \left(C_{BBHH} - C_{BdHH} Y^{d\,-1} Y^B \right) \right] \Big) \log \frac{M_{\rm VLQ}}{\Lambda}$$

Barring cancellations, this leads generically to very strong bounds.

Matching contribution to $\theta_{\rm QCD}$

In the SMEFT $\mathcal{O}_{dH} = \bar{q}_L H d_R |H|^2$ contributes additionally to quark masses

$$[M^d]^{rs} = \frac{v_T}{\sqrt{2}}\left([Y^d]^{rs} - \tfrac{1}{2}v^2[C_{dH}]^{rs}\right)$$

For SM+VLQ

$$\mathscr{M}^{d+B} = \begin{pmatrix} \frac{1}{\sqrt{2}}Y^d v & \frac{1}{\sqrt{2}}Y^B v \\ -\frac{1}{2}C_{BdHH}v^2 & M^B - \frac{1}{2}C_{BBHH}v^2 \end{pmatrix}$$

In Nelson-Barr, $\arg\det\mathscr{M}^{d+B} = 0$ at tree-level.

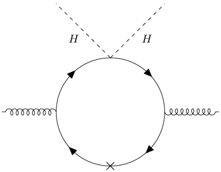
But we if we integrate out the VLQ only at tree-level, C_{BBHH} would not enter and we would get $\bar{\theta} \neq 0$ at tree-level from $C_{BdHH} \to C_{dH}$.

This contradicts the NB construction.

Final formula for $\bar{\theta}$: effective case

The resolution is that at one-loop,

$$\boxed{\mathrm{Im}\,\mathrm{Tr}[M^{B^{-1}} C_{BBHH}]|H|^2 \frac{g_s^2}{64\pi^2} G \cdot \tilde{G}}$$



leads to the tree-level equivalent

$$\boxed{\theta = \frac{v^2}{2}\,\mathrm{Im}\,\mathrm{Tr}\left[M^{B^{-1}} C_{BBHH}\right]}$$

This contribution cancels the tree-level contribution from C_{BdHH} in $\mathcal{O}_{dH} = \bar{q}_L H d_R |H|^2$

Needs updating in 2506.03257

CP4 model

Cherchiglia, Nishi, JHEP 1903 (2019) 040

- We can improve on the Bento-Parada-Branco (BBP) model by using a nonconventional CP

Ivanov, Silva, PRD'16

- Two VQLs B_1, B_2 (more fields)
- $\delta\bar{\theta} = 0$ at one-loop
- Dead-duck 2-loop graph also vanishes
- No ad hoc $\mathbb{Z}_2$ is needed (embedded) compared to BBP
- 2-loop estimate:

$$\boxed{\delta\bar{\theta} \sim \frac{f^4\lambda}{(16\pi^2)^2}}$$

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CP4 model

In the effective calculation, the dominant contribution goes as

$$u_l \gamma_{lj} M^{-2}_{ji} u_k \,\mathrm{Im}\,\mathrm{tr}\left[\mathscr{F}_i \mathscr{F}_k^\dagger\right]$$

But there is an approximate residual $\mathbb{Z}_2$ in the potential

$$M^2 = \mathrm{diag}(M_1^2, M_2^2)\,, \quad [u_i] = (u_S, 0)^\mathsf{T}$$

$$[\gamma_{ij}] = \gamma_S \mathbb{1}_2\,, \quad [\gamma'_i] = \gamma_S u_S (1, 0)^\mathsf{T}\,.$$

Irreducible contriutions to $\bar{\theta}$

There are irreducible contributions to $\bar{\theta}$ arising from the NB-VLQs.

The non-decoupling contribution arises first at 3-loops.

They are relevant for $n_B \geq 2$ and $n_T \geq 2$. Valenti,Vecchi,JHEP'21

For $n_B = 1$

$$\delta\bar{\theta} \sim \left(\frac{1}{16\pi^2}\right)^3 Y^{B\dagger}[X_d, X_u]Y^B$$

$$X_u \equiv Y^u Y^{u\dagger}\,, \quad X_d \equiv Y^d Y^{d\dagger}$$

Incidentally, this flavor invariant is $\langle [X_u, X_d] YY^\dagger \rangle \sim$ degree 6 instead of 12 (Jarlskog)

For SM+(1 singlet VLQ), deLima,Nishi,JHEP24

$$\boxed{\text{invariant} = \text{polynomial(20 CP even, 9 CP odd)}}$$

$$\mathrm{tr}\,\{[A, B]C\}^2$$

Necessity of these 9 CP odd invariants was not known before

delAguila,Aguilar-Saavedra,Branco,'98 found 7 invariants

Conclusions

- Exact one-loop contribution to $\bar{\theta}$ in Nelson-Barr theories with log enhancement
- In the VLQ-EFT, one-loop running of $\bar{\theta}$ is induced by dimension 5

$$\mathcal{O}_{BdHH} = \bar{B}_L d_R |H|^2 , \quad \mathcal{O}_{BBHH} = \bar{B}_L B_R |H|^2$$

- In the SMEFT, one-loop running of $\bar{\theta}$ is induced by $\bar{q}_L H d_R |H|^2$, $|H|^2 G\tilde{G}$
- Subtlety in the matching of θ ...

More details

- G. H. S. Alves and C. C. Nishi, "Effective description of Nelson-Barr models and the theta parameter," [2506.03257 [hep-ph]].
- E. L. F. de Lima and C. C. Nishi, "Flavor invariants for the SM with one singlet vector-like quark," JHEP **11** (2024), 157 [2408.10325 [hep-ph]].
- G. H. S. Alves, A. L. Cherchiglia and C. C. Nishi, Phys. Rev. D **108** (2023) no.3, 035049 [2304.06078 [hep-ph]].
- A. L. Cherchiglia, G. De Conto and C. C. Nishi, JHEP **11** (2021), 093 [2103.04798 [hep-ph]].
- A. L. Cherchiglia and C. C. Nishi, "Consequences of vector-like quarks of Nelson-Barr type," JHEP **2008** (2020) 104 [2004.11318 [hep-ph]].
- A. L. Cherchiglia and C. C. Nishi, "Solving the strong CP problem with non-conventional CP," JHEP **1903** (2019) 040 [1901.02024 [hep-ph]].

Thank you!

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Backup slides

Flavor invariants

- CP violation in the SM can be quantified with a single flavor invariant

$$
\begin{aligned}
I_{66} &= \det([X_u, X_d]) \sim i \times 3 \times 10^{-13} \\
&= 2i(y_u^2 - y_c^2)\cdots(y_b^2 - y_d^2)J \\
J &= \mathrm{Im}[V_{us}V_{cb}V_{ub}^* V_{cs}^*]
\end{aligned}
$$

- As spurions under $G_F = U(3)_q \otimes U(3)_u \otimes U(3)_d$:

$$
Y^u \sim (\mathbf{\bar{3}}, \mathbf{3}, \mathbf{1})\,, \qquad Y^d \sim (\mathbf{\bar{3}}, \mathbf{1}, \mathbf{3})
$$

- All invariants can be written in terms of

$$
X^u \equiv Y^u Y^{u\dagger}\,, \qquad X^d \equiv Y^d Y^{d\dagger}
$$

which are $\mathbf{\bar{3}} \otimes \mathbf{3}$ of $SU(3)_q$

Flavor invariants

- The set of all of polynomial invariants forms a ring which is finitely generated
    1. Basic invariants = generating set = {primary} + {secondary}
    2. Primary invariants = algebraically independent
- Example: I_{66} in the SM is basic but not primary because

$$\boxed{(\text{CP odd})^2 = (\text{CP even})}$$

This is a syzygy

Bento,Silva,Trautner,'23. Jenkins,Manohar,'09.

$$\boxed{\text{invariant} = (\text{CP even}) + I_{66} \times (\text{CP even})}$$

Flavor invariants

- The number and degree of polynomial invariants can be given by the Hilbert series calculable with the Molien-Weyl formula

  Applied to HEP in Jenkins,Manohar,'09. Lehman,Martin,'15. ...

- Very useful for listing invariant operators in SMEFT

  2+84+$\cdots$, Henning,Melia,Murayama,'16. Lehman,Martin,'16....

- For the SM (Plethystic logarithm=PL)

$$\mathrm{PL}[H(X_u = X_d = q^2)] = 2q^2 + 3q^4 + 4q^6 + q^8 + q^{12} - q^{24}$$

  counts the number of basic invariants (may fail)

- For SM+(1 singlet VLQ) deLima,Nishi,JHEP24

$$\mathrm{PL}[H(X_u = X_d = q^2, Y = q)] = 3q^2 + 5q^4 + 8q^6 + \cdots + 2q^{12} - 4q^{14}$$

- Total of 29=20 CP even + 9 CP odd

$$\langle [X_u, X_d] YY^\dagger \rangle \sim \text{degree } 6$$

  The necessity of these 9 CP odd invariants was not known before

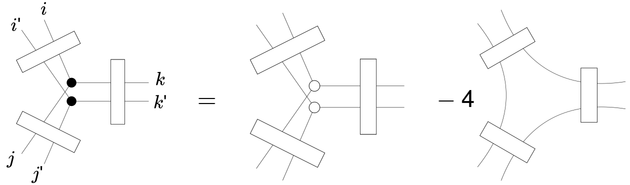
  delAguila,Aguilar-Saavedra,Branco,'98 found 7 invariants

Flavor invariants: syzygy identity

For hermitean 3×3 matrices

$$\langle [A, B] C \rangle^2 = \text{CP even}$$

$(f_{ijk})^2 \sim$



$+$ 6 tensors

See complete identity in deLima,Nishi,JHEP24, 2408.10325