Neutrino Mass Sum Rules from Modular A_4 Symmetry

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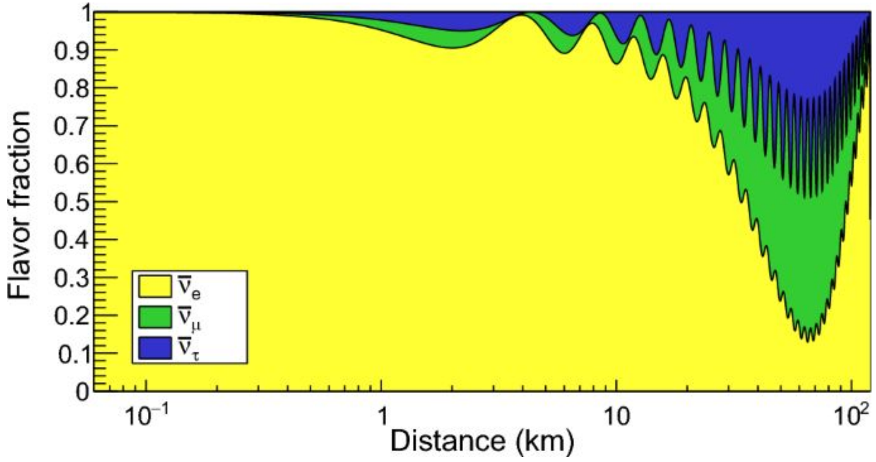
11TH WORKSHOP
Flavor Symmetries and Consequences in Accelerators and Cosmology

Neutrino Oscillations

- Neutrino oscillations suggest that neutrinos must be massive !



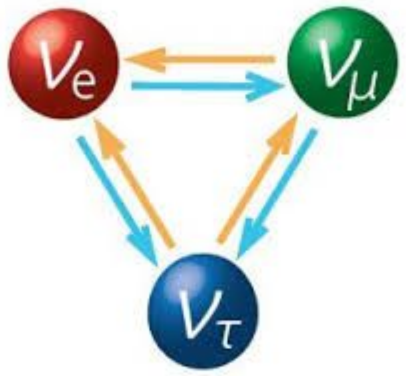


Fig. 1: Neutrino Oscillations

Source: S.Jang, et al. . νOscillation: a software package for computation and simulation of neutrino oscillation and detection

Neutrino Mass Ordering

Neutrino Oscillation: Mass Ordering ?

Patterns of mixing angles: $\theta_{23} \sim 45^o$, $\theta_{12} \sim 33^o$, $\theta_{13} \sim 8^o$

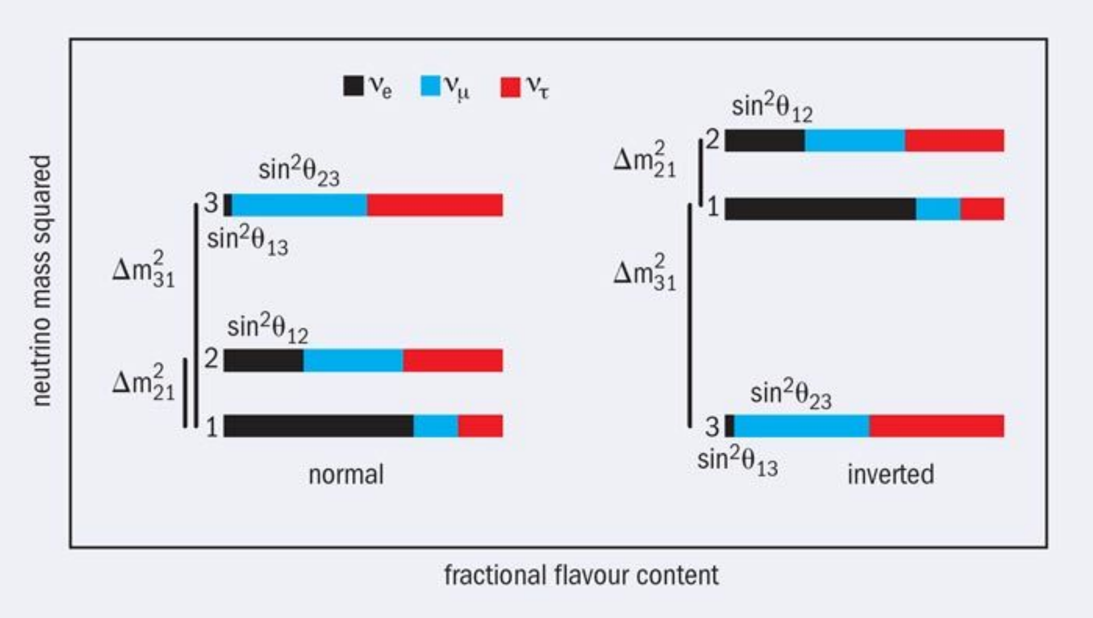


Fig. 2: Neutrinos mass ordering

Absolute Neutrino Mass Observables (m_e , m_{ee} , Σ)

Beta Decay : Sensitive to the effective electron neutrino mass

$$m_e = \sqrt{\sum_i |U_{ei}|^2 m_i^2} = \left[c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2\right]^{\frac{1}{2}}$$

0vee decay (Only for Majorana Neutrino) : Effective Majorana mass

Lisi's talk

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{2i\phi_{12}} m_2 + s_{13}^2 e^{2i\phi_{13}} m_3|$$

Cosmology : Sum of Neutrino Mass

$$\Sigma = m_1 + m_2 + m_3$$

Flavor Structure
+
Absolute Mass Scale

The Symmetry Approach to Flavor Structure

- Symmetries play an important role in particle physics, for example, the SM is built on gauge symmetries (strong, weak, and electromagnetic).

- Discrete symmetries are often utilized to control the allowed couplings (e.g., Abelian symmetries like Z_N) and to explain the origin of flavor structures (typically using non-Abelian symmetries).

- Non-Abelian discrete symmetries are commonly referred to as flavor symmetries, such as S_3, A_4, S_4, etc.

$$\begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}$$

- Usually a flavon field φ is introduced for the breaking pattern.

E. Ma and G. Rajasekaran, hep-ph/0106291, K. S. Babu, et al., hep-ph/0206292, G. Altarelli and F. Feruglio, hep-ph/0512103

Modular Symmetry

- Modular symmetry provides an elegant framework for understanding fermion masses and mixing without introducing flavon fields.
- It replaces traditional discrete flavor symmetries by promoting Yukawa couplings to modular forms, where $y \equiv y(\tau)$.
- Finite modular groups like Γ_N (e.g., $\Gamma_3 \simeq A_4$, $\Gamma_4 \simeq S_4$) play the role of effective flavor symmetries.
- Fermion fields are assigned modular weights (k) and representations, leading to flavor structures without flavons.

F. Feruglio, 1706.08749

Neutrino Mass Sum Rules

- Mass sum rules among the neutrino masses.

$$m_1 + m_2 = m_3$$

$$\frac{1}{m_1} + \frac{1}{m_2} = \frac{1}{m_3}$$

$$\frac{2}{m_2} + \frac{1}{m_3} = \frac{1}{m_1}$$

These sum rules constraint neutrino masses and predict the absolute neutrino mass observables.

G. Altarelli et al., JHEP 03 (2008), 052, M. Hirsch et al., Phys. Rev. D 78 (2008), 093007, F. Bazzocchi et al. Phys. Rev. D 80 (2009), 053003

Our Model

- We employ a modular A_4 group.
- ‘k’ denotes the modular weight.
- Neutrino Masses and Mixing will be generated from the type-II Seesaw Mechanism.

Fields	$SU(2)_L$	$U(1)_Y$	$\Gamma_3 \simeq \mathcal{A}_4$	$-k$
L_i	2	$-\frac{1}{2}$	**3**	-3
E_i^c	1	1	**1, 1′, 1″**	-1
H_u	2	$\frac{1}{2}$	**1**	0
H_d	2	$-\frac{1}{2}$	**1**	0
Δ	3	1	**1**	0

Tab 1: Model particle contents

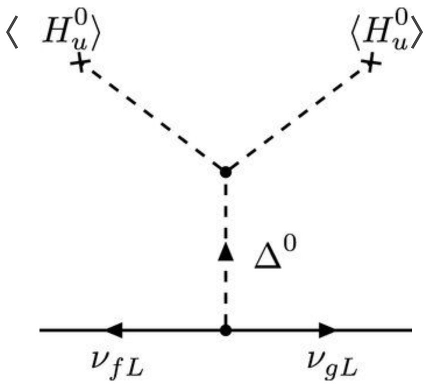


Fig. 3: Type-II seesaw mass diagram

Charged Lepton and Neutrino Mass Matrix

The superpotential of the model is given by:

$$\mathcal{W} = \alpha_1 (\boldsymbol{Y_e} L)_1 E_1^c H_d + \alpha_2 (\boldsymbol{Y_e} L)_{1''} E_2^c H_d + \alpha_3 (\boldsymbol{Y_e} L)_{1'} E_3^c H_d + \alpha \left(\boldsymbol{Y_{\nu,1}} (LL)_{3_S}\right)_1 \Delta + \beta \left(\boldsymbol{Y_{\nu,2}} (LL)_{3_S}\right)_1 \Delta + \mu H_u H_d + \mu_\Delta H_d H_d \Delta$$

Yukawas	$\Gamma_3 \simeq \mathcal{A}_4$	$\boldsymbol{k}$
$\boldsymbol{Y_e = Y_3^{(4)}}$	**3**	4
$\boldsymbol{Y_{\nu,1} = Y_{3a}^{(6)}}$	**3**	6
$\boldsymbol{Y_{\nu,2} = Y_{3b}^{(6)}}$	**3**	6

Tab 2: Yukawa transformation

$$M_\ell = v_H \begin{pmatrix} Y_{3,1}^{(4)} & Y_{3,2}^{(4)} & Y_{3,3}^{(4)} \\ Y_{3,3}^{(4)} & Y_{3,1}^{(4)} & Y_{3,2}^{(4)} \\ Y_{3,2}^{(4)} & Y_{3,3}^{(4)} & Y_{3,1}^{(4)} \end{pmatrix} \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix} \qquad M_\nu = v_\Delta \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ * & 2Y_2 & -Y_1 \\ * & * & 2Y_3 \end{pmatrix}$$

where $Y_i \equiv \alpha \boldsymbol{Y}_{3a,i}^{(6)} + \beta \boldsymbol{Y}_{3b,i}^{(6)}$ with $i \in \{1, 2, 3\}$

Proof of Sum Rule

$$r^2 \equiv |Y_1|^2 + |Y_2|^2 + |Y_3|^2$$

$$\mathrm{Tr}(M_\nu^\dagger M_\nu) = 6r^2$$

$$\mathrm{Tr}(M_\nu^\dagger M_\nu M_\nu^\dagger M_\nu) = 18r^4$$

$$\Rightarrow \quad \tfrac{1}{2}\mathrm{Tr}(M_\nu^\dagger M_\nu)^2 = \mathrm{Tr}(M_\nu^\dagger M_\nu M_\nu^\dagger M_\nu)$$

$$\frac{1}{2}(m_1^2 + m_2^2 + m_3^2)^2 = m_1^4 + m_2^4 + m_3^4$$

$$\Rightarrow \quad m_3^2 = (m_1 \pm m_2)^2$$

$$m_3^{\mathrm{NO}} = m_1^{\mathrm{NO}} + m_2^{\mathrm{NO}}, \qquad m_3^{\mathrm{NO}} > m_2^{\mathrm{NO}} > m_1^{\mathrm{NO}}$$

$$m_2^{\mathrm{IO}} = m_1^{\mathrm{IO}} + m_3^{\mathrm{IO}}, \qquad m_2^{\mathrm{IO}} > m_1^{\mathrm{IO}} > m_3^{\mathrm{IO}}$$

$$m_{\mathrm{heaviest}} = \frac{1}{2}\sum_i m_i$$

Sum Rule Predictions

A fascinating aspect of this neutrino mass matrix is the sum rule, which is invariant irrespective of the ordering of neutrino masses.

$$m_{\text{heaviest}} = \frac{1}{2}\sum_i m_i$$

The sum rule, together with the two mass-squared differences, allows definite neutrino masses.

NO:

$$m_3 = m_1 + m_2$$

$$\Delta m_{21}^2 = 7.5 \times 10^{-5}\,\text{eV}^2, \quad \Delta m_{31}^2 = 2.55 \times 10^{-3}\,\text{eV}^2$$

$$m_1 = 0.0282\,\text{eV}, \quad m_2 = 0.0295\,\text{eV}, \quad m_3 = 0.0578\,\text{eV}$$

IO:

$$m_2 = m_1 + m_3$$

$$\Delta m_{21}^2 = 7.5 \times 10^{-5}\,\text{eV}^2, \quad \Delta m_{31}^2 = -2.45 \times 10^{-3}\,\text{eV}^2$$

$$m_3 = 7.5 \times 10^{-4}\,\text{eV}, \quad m_1 = 0.049\,\text{eV}, \quad m_2 = 0.050\,\text{eV}$$

Sum Rule Implications

- This sum rule leads to precise prediction for neutrinoless double beta decay.

$$|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right| = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{2i\phi_{12}} m_2 + s_{13}^2 e^{2i\phi_{13}} m_3|$$

→ *Effective mass for neutrinoless double beta decay*

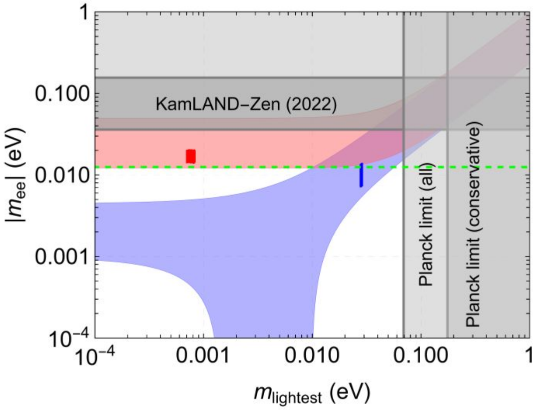


Fig. 4: Neutrinoless double beta decay

Dashed green line is for nEXO future sensitivity.

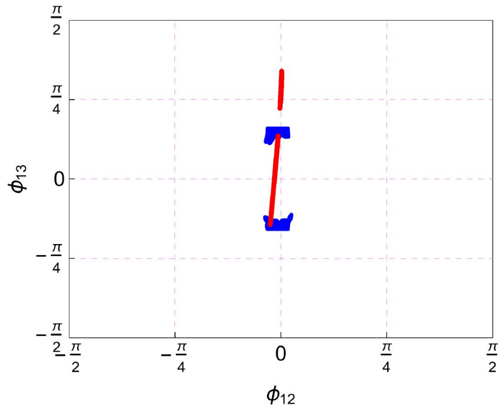


Fig. 5: Correlation of Majorana phases

- Majorana phases are strongly correlated with each other.
- Effective mass of beta decay and sum of neutrino masses are precisely determined.

P. F. de Salas, et al., 2006.11237

Neutrino Oscillations Predictions

- Correlation between atmospheric angle and imaginary part of complex parameter τ.

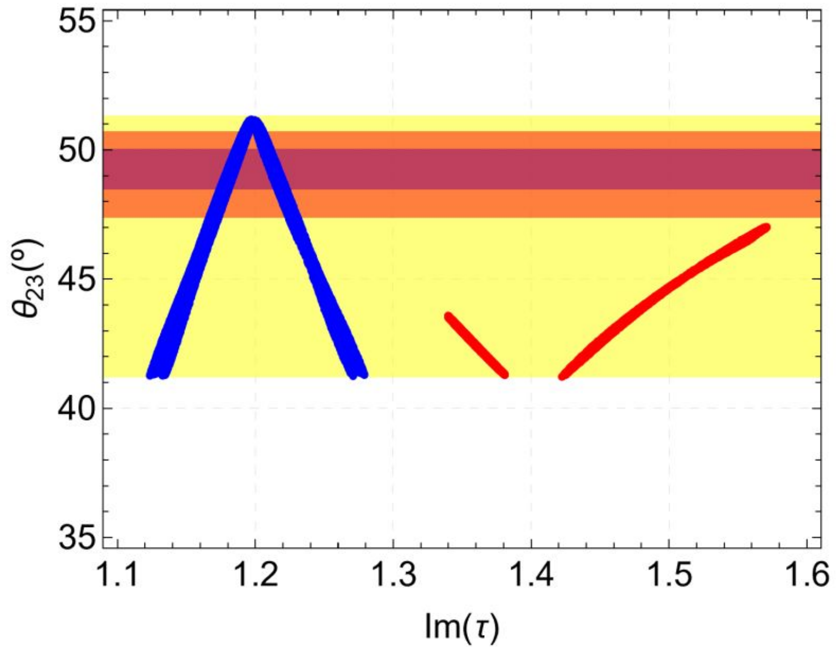


Fig. 6: Model predictions for atmospheric mixing angle vs Im(τ).

Neutrino Oscillations Predictions: NO

- Correlation of mixing angles and CP phase for NO.
- Lower bound on mixing angle $\theta_{13} > 8.36^o$.

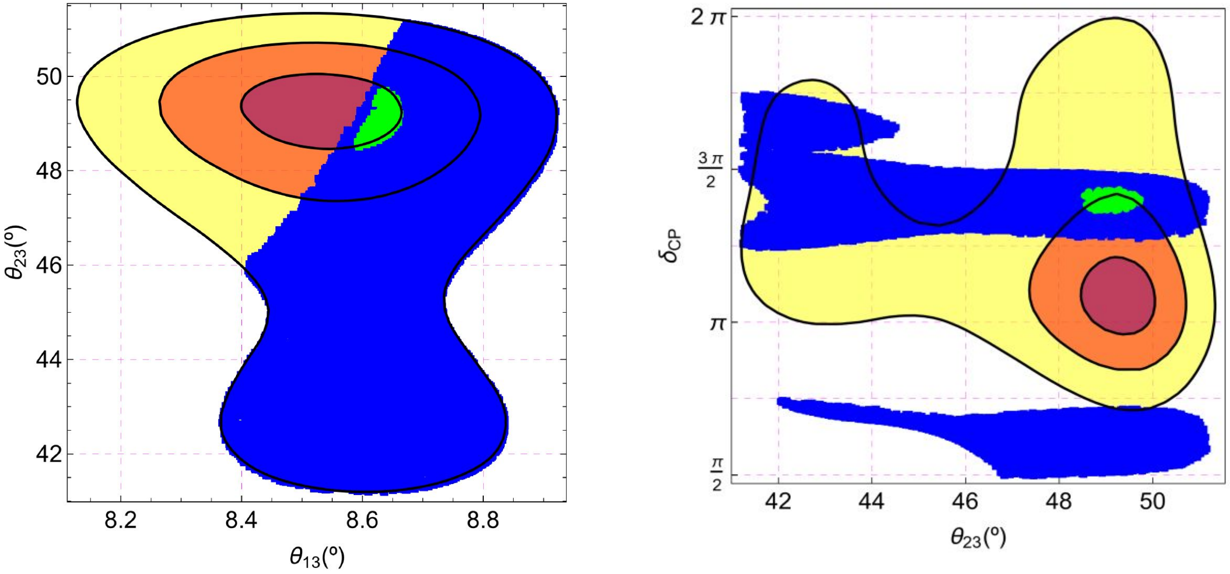
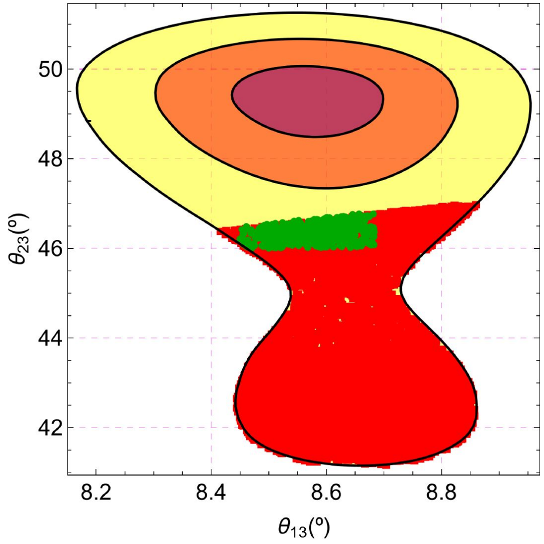


Fig. 7: Model predictions for mixing angles and CP phase in NO

Neutrino Oscillations Predictions: IO

- Correlation of mixing angles and CP phase for IO.
- Upper bound on mixing angle $\theta_{23} < 46.8^{o}$.



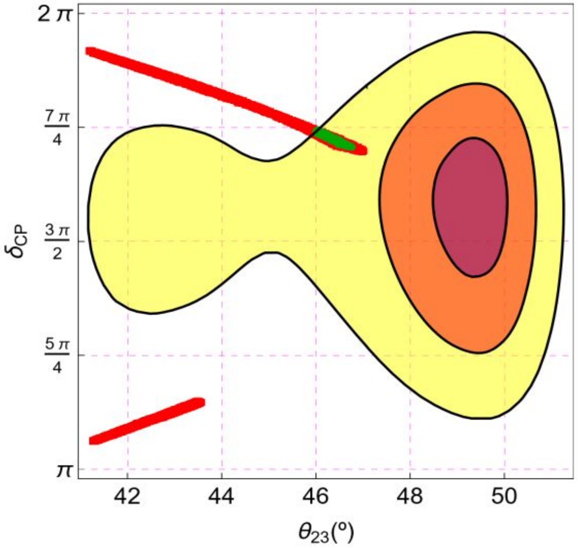


Fig. 8: Model predictions for mixing angle and CP phase in IO

Conclusions

- Modular A_4 symmetry has been employed in type-II seesaw mechanism.
- Neutrino mass structure leads to a sum rule for physical neutrino masses valid for both NO and IO.
- Sum rule fixes neutrino mass and provides a testable prediction for the sum of neutrino mass, neutrinoless double beta decay and beta decay
- Mixing angles have strong correlations with the complex modulus parameter τ.
- The mixing angle θ_{13} has lower bound in NO while upper bound on mixing angle θ_{23} for IO.
- CP phase in both NO and IO has correlation that can be tested in future experiments like DUNE.


RAHMET
OBRIGADO
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谢谢
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DANK JE
THANK YOU
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ευχαριστώ
GRACIAS
BARKA
MERCI
СПАСИБО
ARIGATO
TAKK
TACK

Back Up

A_4 Representation for Modular Weight k

TABLE III. $\mathcal{A}_4$ representations for different weight k.

Weight (k)	d_k	$\mathcal{A}_4$ representations
2	3	$\mathbf{3}$
4	5	$\mathbf{3}+\mathbf{1}+\mathbf{1}'$
6	7	$\mathbf{3}+\mathbf{3}+\mathbf{1}$
8	9	$\mathbf{3}+\mathbf{3}+\mathbf{1}+\mathbf{1}'+\mathbf{1}''$
10	11	$\mathbf{3}+\mathbf{3}+\mathbf{3}+\mathbf{1}+\mathbf{1}'$