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Unitarity Triangle Angles Explained: a Predictive New Quark Mass Matrix Texture

Plus, un antipasto on the house:
A Brief Historical Perspective on TriBimaximal Mixing

Paul Harrison
University of Warwick

Talk at FLASY 2025
30th June 2025

arXiv: 2501.18508 with Bill Scott
(to be published in JHEP)

Outline of Talk(s)

- Historical Perspective on TriBimaximal Mixing (TBM) for leptons (a story of serendipity and following the data)
    - ν oscillation landscape pre-1994
    - Trimaximal Mixing (TMX)
    - CHOOZ, SNO and TBM
    - Legacy
- For a longer version:
  `https://indico.global/event/1732/contributions/30587/attachments/15624/24929/dhpTalk3.pdf`

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  - Confronting the data
  - Symmetries of the texture
  - Discussion and conclusions

ν Oscillation Landscape pre-1994

- Solar ν problem had been around since 1970s:
  - Early HOMESTAKE solar ν data 1970-1994
    $\Rightarrow Data/SSM = 0.30 \pm 0.04 \pm 0.09$

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- Atmospheric ν anomaly was known as a simple deficit of ν_μ c.f. ν_e: IMB (1985) and Kamiokande (1988) :
  - Was initially a simple deficit $\sim 50\%$, with no L/E dependence
  - Suggested large mixing angles

1994 Provided Step-change in Perspective

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Solar Data: P. Darriulat's ICHEP Conference Summary talk:

Experiment	Data/SSM (BP) %	Data/SSM (TCL) %
GALLEX	$60 \pm 8 \pm 5$	$64 \pm 8 \pm 5$
SAGE	$52 \pm 8 \pm 5$	$56 \pm 9 \pm 5$
Kamiokande	$51 \pm 4 \pm 6$	$66 \pm 5 \pm 8$
Homestake	(Pro memoria) $29 \pm 3 \pm 9$	

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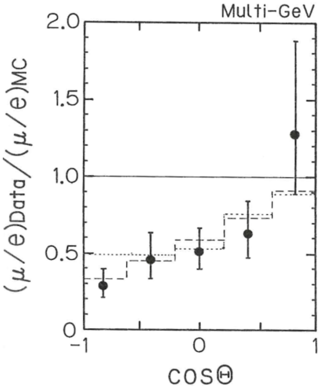
Atmospheric Neutrinos:

M. Nakahata, Glasgow ICHEP Conference 1994

Kamiokande Multi-Gev data

A clear zenith angle dependence!

Strongly favoured ν oscillations with large mixing angles



Meanwhile, also in 1994:

- Harrison and Scott propose TriMaximal mixing (TMX) for quarks(!):
  - "Generation permutation symmetry and the quark mixing matrix", PLB 333 (1994) 471-475, hep-ph/9406351

$$U = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} \\ \frac{\bar{\omega}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{bmatrix} \Rightarrow (|U_{i\alpha}|^2) = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

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- July '94, H&S realise that latest solar and atmospheric ν data are well-fitted by TMX!

At ICHEP 1994

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At ICHEP 1994

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- Following discussions with Don Perkins, we formed HPS Collaboration on TMX and ν oscillations
- Goals:

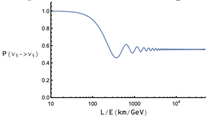
To fully embrace 3 flavours of neutrinos

To understand all the ν data in a single unified framework

To keep it as simple as possible (but not simpler)

3x3 mixing matrix
2 mass-squared differences

To interpret the theory visually



To highlight apparent symmetries, where plausible => Mass Matrix origins

$$\begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} \qquad \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix}$$

Threefold maximal lepton mixing (TMX) and the solar and atmospheric neutrino deficits (1995)

- PLB 349 (1995) 137 (“HPS1”)
- TMX $\Rightarrow$ universal survival (and appearance) probabilities (in vacuo) - eminently testable

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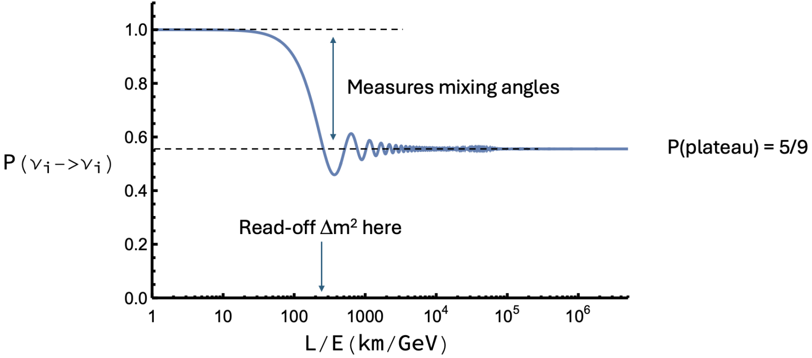

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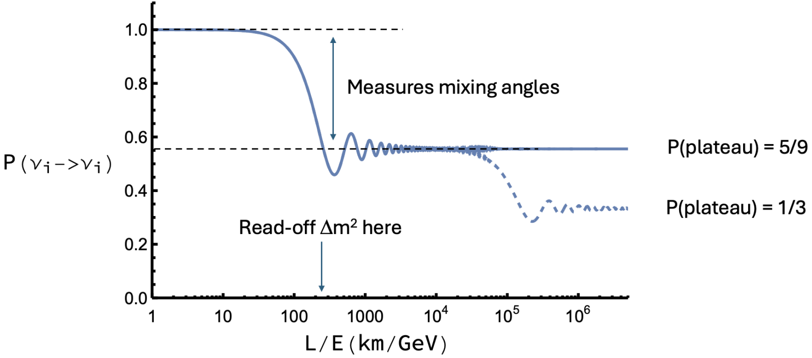

Plotted with Data (taken from HPS1)

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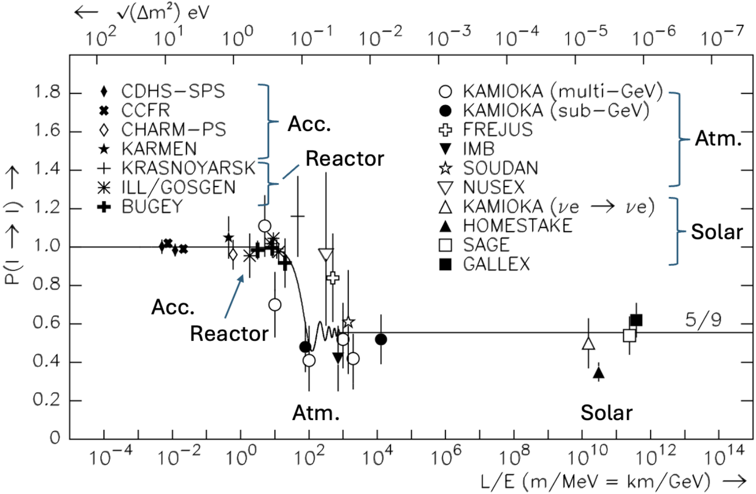


$\chi^2/dof = 20.5/28;$

$\Delta m^2_{31} = (7.8 \pm 1.2) \times 10^{-3}$ eV^2

Δm^2_{21} unresolved

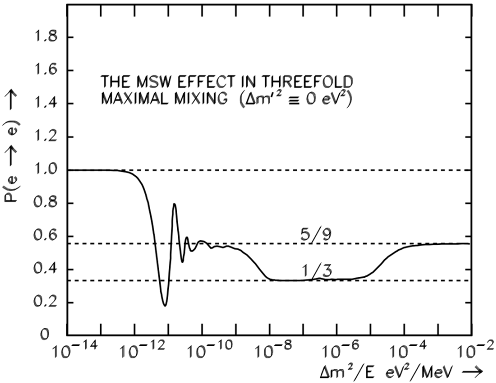
PLB 374 (1996) 111 (“HPS2”)

Included

A prediction:

The HPS “5/9-1/3-5/9” Bathtub

NB. required $\Delta m^2_{31} \sim 10^{-8} - 10^{-5}\,eV^2$



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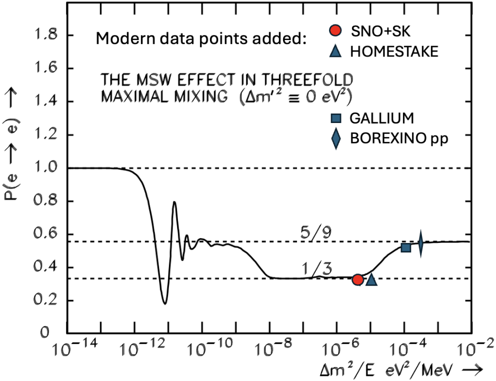

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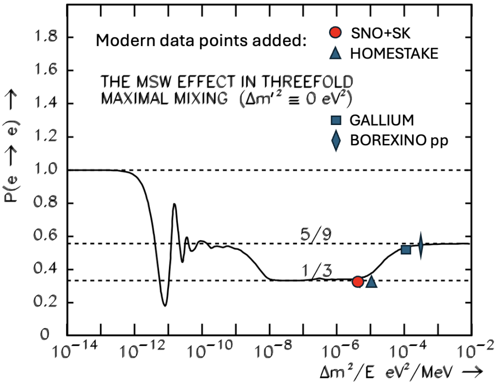

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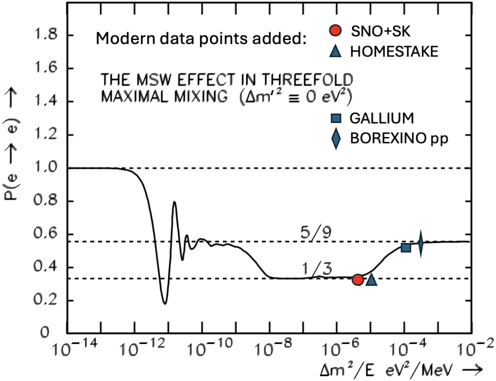


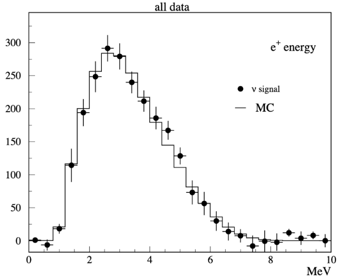
- HPS4 (PLB 458 (1999) 79) predicted for TMX "spectacular effects expected in long-baseline reactor and accelerator experiments"! viz. CHOOZ, PALO VERDE, MINOS etc.

BUT...

The CHOOZ Reactor result 1999:

$R = 1.01 \pm 0.03$ (stat) ± 0.03 (sys)
($L \sim 1$ km, $E \sim 3$ MeV)
No Spectacular Effect!

Conclude: **$|U_{e3}|^2$ small if $\Delta m^2 > 10^{-3}$ eV^2**



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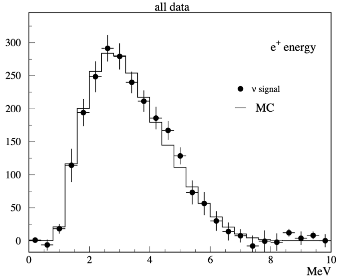


And…

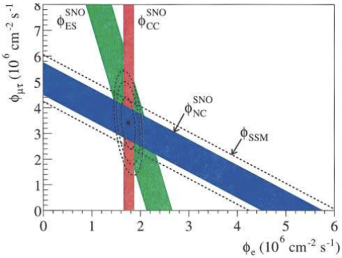
The SNO NC result 2002:

$\phi_{CC} / \phi_{NC} = 0.35 \pm 0.04 \sim 1/3$!

So…

Solar results energy-dependent

Trimaximal Mixing is Dead!



So, TriBiMaximal Mixing (TBM)

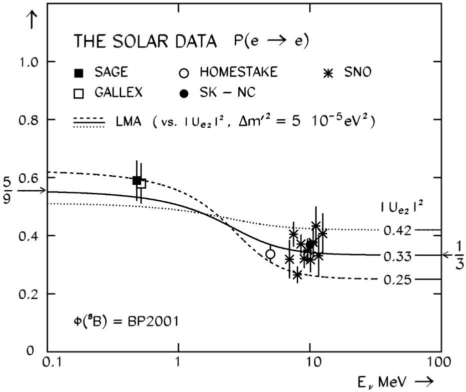
Long live Tri(Bi)Maximal Mixing!

Tri-bimaximal mixing and the neutrino oscillation data PLB 530 (2002) 167

"HPS5"

Return of the HPS "5/9-1/3-5/9" Bathtub! (this time at the second threshold)

Conclude: $|U_{e2}|^2 \sim 1/3$



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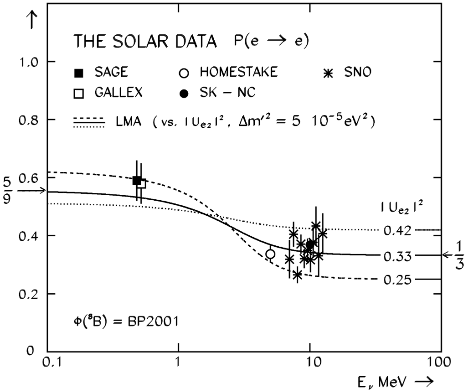
Also... **SUPERK Atmospheric neutrino results pointing strongly to twofold maximal $\nu_\mu - \nu_\tau$ mixing"**

Conclude: $|U_{\mu 3}|^2 \sim |U_{\tau 3}|^2 \sim 1/2$



HPS5 Concludes

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"The totality of the data clearly point to a particular form for the lepton mixing matrix"

$$U_{PMNS} = \begin{array}{c c} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\sqrt{\frac{1}{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \end{array}$$

Reactor Mixing Angle

Atmospheric Mixing Angle

Atmospheric Scale Δm^2_{32}

HPS5 Concludes

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Solar
Scale
Δm^2_{12}

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Solar
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Reactor
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Atmospheric
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Atmospheric
Scale
Δm^2_{32}

"Small non-zero values of U_{e3}, and/or somewhat different values of $|U_{\mu 3}|^2$… are more-or-less equally acceptable experimentally."

Legacy?

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TBM was finally excluded 10 years later since (as anticipated) $U_{e3} = 0$ was incompatible with new results from the Daya Bay and RENO reactor experiments (2012) and later the T2K accelerator experiment (2013)

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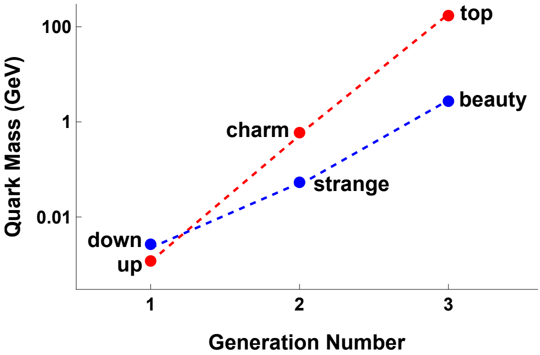
Currently, $|U_{e3}|^2 \sim 0.02$. Thus TBM remains a useful zeroth-order approximation to U_{PMNS}, while allowing the exciting prospect that leptonic CP violation may be accessible in the future

Mystery of Quark Mass Spectra

- Quark masses show marked hierarchical structure:



- Is quasi-"geometric":

$$\frac{m_c}{m_t} \simeq 0.0035$$

$$\frac{m_u}{m_c} \simeq 0.0020$$

$$\frac{m_s}{m_b} \simeq 0.020$$

$$\frac{m_d}{m_s} \simeq 0.050$$

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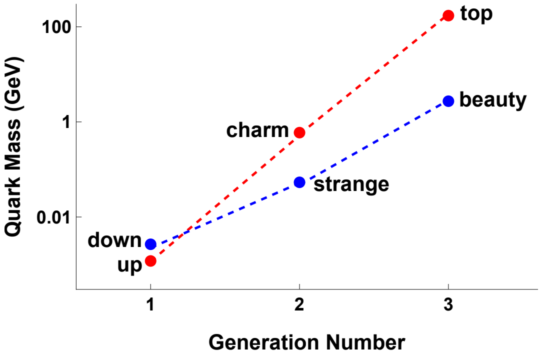

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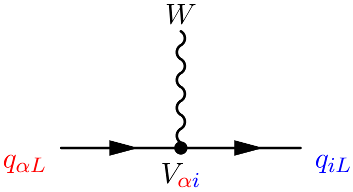
$$\frac{m_d}{m_s} \simeq 0.050$$

- Noted by very many authors
- Masses not predicted in the SM
- Hierarchy certainly not explained within SM
- BSM, Froggatt-Neilsen mechanism has had some success

Mystery of Quark Mixing Spectrum

- CKM quark mixing matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



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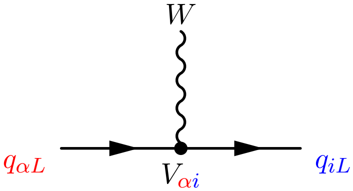


$$\sim \begin{pmatrix} 1 & \lambda & A\lambda^3(\overline{\rho} - i\overline{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \overline{\rho} - i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

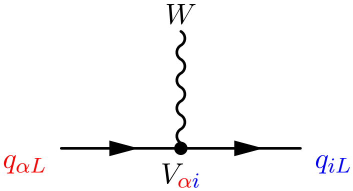
where $\lambda \equiv |V_{us}| \simeq 0.22$

$A, \ \overline{\rho}$ and $\overline{\eta} \lesssim \mathcal{O}(1)$

Mystery of Quark Mixing Spectrum

- CKM quark mixing matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$\sim \begin{pmatrix} 1 & \lambda & A\lambda^3(\overline{\rho} - i\overline{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \overline{\rho} - i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

where $\lambda \equiv |V_{us}| \simeq 0.22$

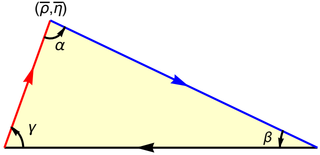
A, $\overline{\rho}$ and $\overline{\eta} \lesssim \mathcal{O}(1)$

- Elements not predicted by the SM
- Strong hierarchy certainly not explained within SM
- But masses and mixings *both* arise in the Yukawa/Mass matrices

Mysteries of the Unitarity Triangle

- Sides/Angles of UT are arbitrary in SM.
- But measured angles:

$$\alpha = (91.6 \pm 1.4)^\circ$$
$$\beta = (22.6 \pm 0.4)^\circ$$
$$\gamma = (65.7 \pm 1.3)^\circ$$



consistent with "special" values:

$$(\alpha, \beta, \gamma) \simeq (\tfrac{\pi}{2}, \tfrac{\pi}{8}, \tfrac{3\pi}{8}) \equiv (\alpha_0, \beta_0, \gamma_0).$$

- Seems striking!

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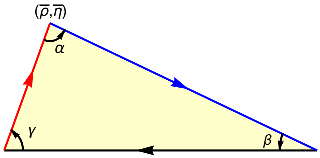

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- Seems striking!
- Coincidence or smoking gun?
- $\rightarrow$ Test as clue to what lies behind.

$\Rightarrow$ Build Special Angles into a Texture:

$$M_q^{HS} \equiv n_q \begin{pmatrix} c_q\lambda_q^4 & b\,\boldsymbol{\lambda_q^3} & 0 \\ b\,\boldsymbol{\lambda_q^{*3}} & b\lambda_q^2 & A_0\boldsymbol{\lambda_q^2} \\ 0 & A_0\boldsymbol{\lambda_q^{*2}} & 1 \end{pmatrix}, \qquad \begin{array}{c} q = u, d, \\ \boldsymbol{\lambda_q} \text{ complex} \\ \arg(\boldsymbol{\lambda_q}) \text{ unobservable} \end{array}$$

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- Complex ratio is *fixed constant:*

$$\frac{\boldsymbol{\lambda_u}}{\boldsymbol{\lambda_d}} \equiv -i\,\tan\frac{\pi}{8}.$$

  - $\arg \boldsymbol{\lambda_u}/\boldsymbol{\lambda_d} = -i$, is sole source of CP violation
  - $|\boldsymbol{\lambda_u}/\boldsymbol{\lambda_d}| \simeq 0.41$ controls relative strength of "u" and "d" mass hierarchies
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- Describes 10 observables with 7 real parameters

Leading-order Solution (Quark Masses)

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- Diagonalise $\rightarrow$ masses:

$$D_q = U_q M_q^{HS} U_q^\dagger = m_3^q \begin{pmatrix} (c_q - b)\lambda_q^4 & 0 & 0 \\ 0 & b\lambda_q^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad q = u, d,$$

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- Fits any m_u, m_d ✓ (no prediction here).

Leading-order Solution (Quark Mixing)

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- Diagonalised by 2×2 (complex) rotations in 23 and 12 spaces.
- Small entries induced in the 13 elements of U_q:

$$U_q \simeq \begin{pmatrix} 1 & \pm\boldsymbol{\lambda_q} & A_0\,\boldsymbol{\lambda_q^3} \\ \mp\boldsymbol{\lambda_q^*} & 1 & -A_0\,\boldsymbol{\lambda_q^2} \\ 0 & A_0\,\boldsymbol{\lambda_q^{*2}} & 1 \end{pmatrix}, \quad q = u, d.$$

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- C.f. Wolfenstein form:

$$V_{CKM} = \begin{pmatrix} 1 & \lambda & A\lambda^3(\overline{\rho} - i\overline{\eta}) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \overline{\rho} - i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} \Rightarrow \begin{cases} \lambda \simeq \lambda_0 \checkmark \\ A \simeq A_0 \checkmark \\ (\overline{\rho} + i\overline{\eta}) \simeq \dfrac{\boldsymbol{\lambda_u^*}}{\lambda_0} \end{cases}$$

The UT Angles

- Have deduced that:

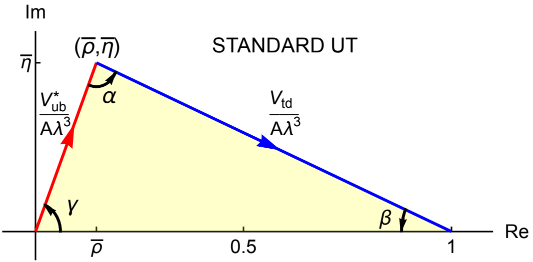
$$\lambda_u^* \simeq \lambda(\overline{\rho} + i\overline{\eta}) = \frac{V_{ub}^*}{A\lambda^2}$$

$$\lambda_d^* \simeq \lambda(1 - \overline{\rho} - i\overline{\eta}) = \frac{V_{td}}{A\lambda^2}$$

$$\Rightarrow \gamma \simeq \arg \lambda_u^*$$

$$\beta \simeq \arg \lambda_d$$

and $\alpha \simeq \arg(-\frac{\lambda_u}{\lambda_d})$



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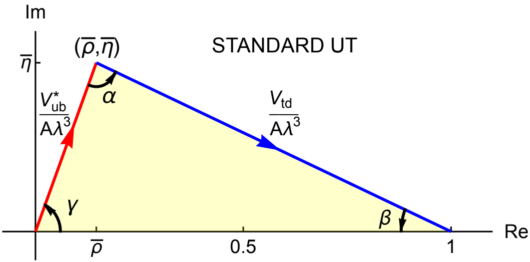

- Recall, HS texture asserts $\frac{\boldsymbol{\lambda_u}}{\boldsymbol{\lambda_d}} = -i \tan\frac{\pi}{8}$.
  - $\Rightarrow \alpha \simeq \frac{\pi}{2}$ ✓
  - $\Rightarrow \tan\beta \simeq |\frac{\boldsymbol{\lambda_u}}{\boldsymbol{\lambda_d}}|$ (see Figure).
  - $\Rightarrow \beta \simeq \frac{\pi}{8}$ ✓

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- Not necessarily!
- Because (exactly) these quantities run with renormalisation scale
- $\sim$ 14% from weak to GUT scales: $A(\uparrow), m_c/m_t(\uparrow)$ and $m_s/m_b(\downarrow)$.
- While λ, α, β, m_u/m_c and m_d/m_s are $\sim$ invariant.
- $\Rightarrow$ vary μ

Precision Fit to the Data

- Fit $\chi^2/\text{d.o.f} \simeq 1.01/2$

Observable	Input Renormalised to $\mu = 10^4$ TeV	Fitted Value at $\mu = 10^4$ TeV
$\lvert m_u/m_c\rvert\ (\times 10^3)$	2.00 ± 0.05	2.00
$\lvert m_d/m_s\rvert\ (\times 10^2)$	4.97 ± 0.06	4.97
$m_c/m_t\ (\times 10^3)$	3.46 ± 0.03	3.46
$m_s/m_b\ (\times 10^2)$	1.968 ± 0.008	1.968
λ	0.2250 ± 0.0007	0.2250
A	0.88 ± 0.02	0.88
$\overline{\rho}$	0.159 ± 0.009	0.152
$\overline{\eta}$	0.352 ± 0.007	0.348
UT Angles	–	Prediction from Fit
$\alpha\ (^\circ)$	91.6 ± 1.4	91.30 ± 0.02
$\beta\ (^\circ)$	22.6 ± 0.4	22.3 ± 0.1
$\gamma\ (^\circ)$	65.7 ± 1.3	66.4 ± 0.1

Some Details of the Fit

- Best fit renormalisation scale: $\mu \sim (0.3 \to 3) \times 10^4$ TeV.
- Fitted values of the free parameters:
  - $\lambda_0 = 0.22646$
  - $A_0 = 0.854$
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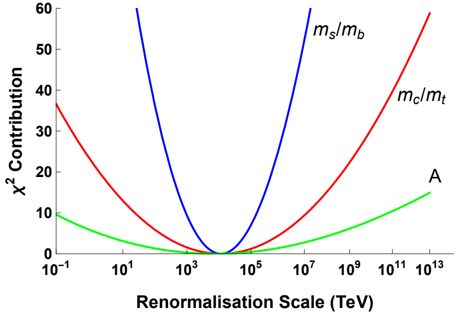
- Three curves minimise at common scale $\sim 10^4$ TeV
- Fitted values of observables in table represent predictions



The Leading Order UT (LO-UT)

- Define useful complex constants:

$$\boldsymbol{z_0} \equiv \boldsymbol{\lambda_u^*}/\lambda_0 = is_0\, e^{-i\beta_0} = \rho_0 + i\eta_0,$$
$$\overline{\boldsymbol{z}}_{\boldsymbol{0}} \equiv \boldsymbol{\lambda_d^*}/\lambda_0 = c_0\, e^{-i\beta_0} = 1 - \boldsymbol{z_0},$$

where
$s_0 \equiv \sin\frac{\pi}{8}$; $\quad c_0 \equiv \cos\frac{\pi}{8}$; $\quad \eta_0 = s_0 c_0 = \frac{1}{2\sqrt{2}}$ and $\quad \rho_0 = \sin^2\frac{\pi}{8}$.

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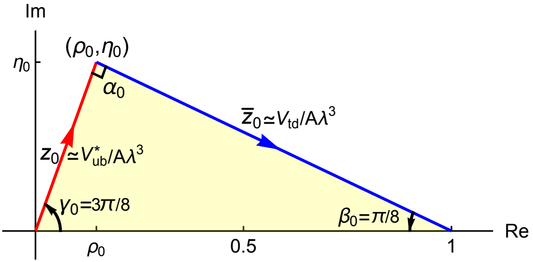
- Use to construct LO-UT

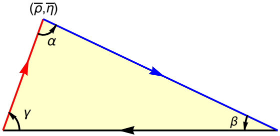


Symmetries of the MMs

- Properties of the *paired system* (M_u, M_d), rather than of either in isolation

Recall: CP Transformation and Rephasing

- CP:
- Under CP, all complex numbers in the MMs are complex-conjugated.



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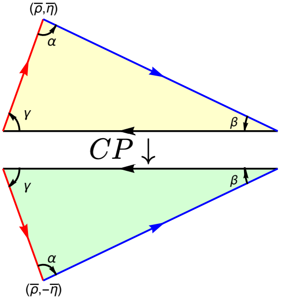

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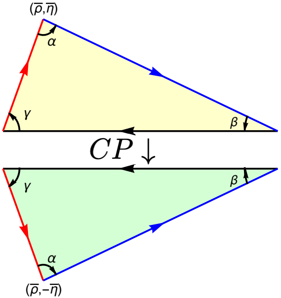

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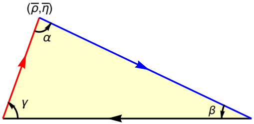

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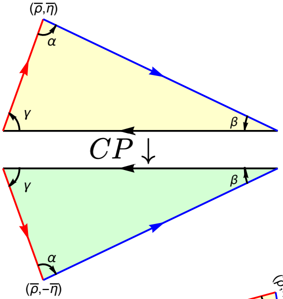
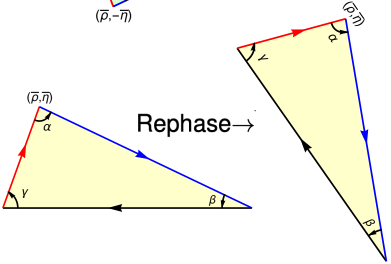
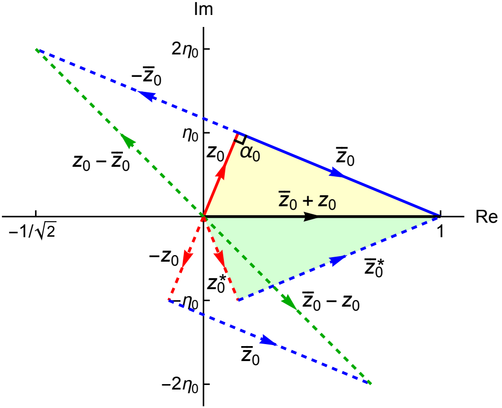


- Rephasing:
- Simultaneous *phase changes* of M_u and M_d unobservable
- UT simply rotates in complex plane
- (Physical) shape and size invariant



Symmetry for $\alpha_0 = \frac{\pi}{2}$

- In HS texture, simple sign change of z_0 (or of $\overline{z}_0$, but not both), flips orientation of the UT (see fig)
- Is only observable effect
- But iff $\alpha = \pm\frac{\pi}{2}$



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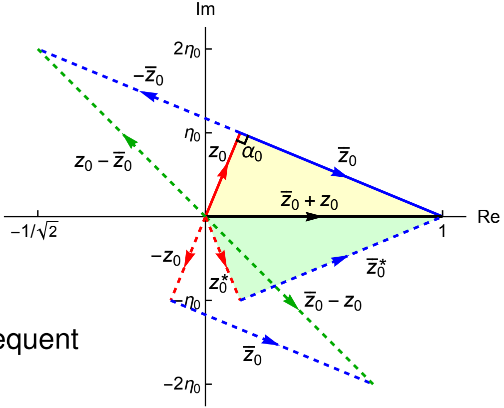
- It is equivalent to a CP transformation
- Can be reversed by a subsequent actual CP transformation
- Symmetry is good to all orders



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- First consider $\beta_0 = \tilde{\beta} \neq \frac{\pi}{8}$ (fig$\rightarrow$) keeping $\alpha = \frac{\pi}{2}$ and $\lambda_u + \lambda_d = \lambda_0$
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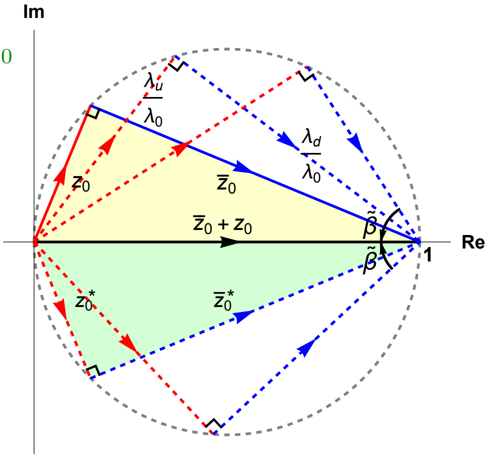
and $-\frac{\pi}{2} < \tilde{\beta} < \frac{\pi}{2}$



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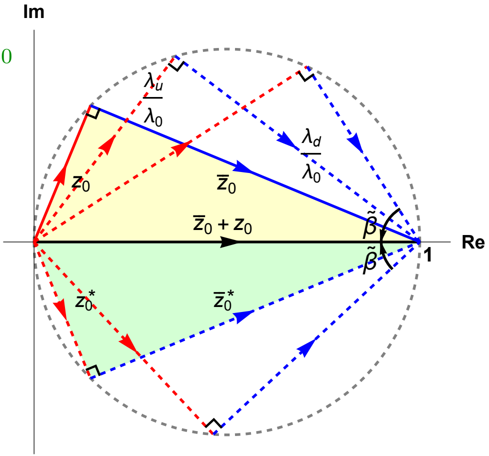

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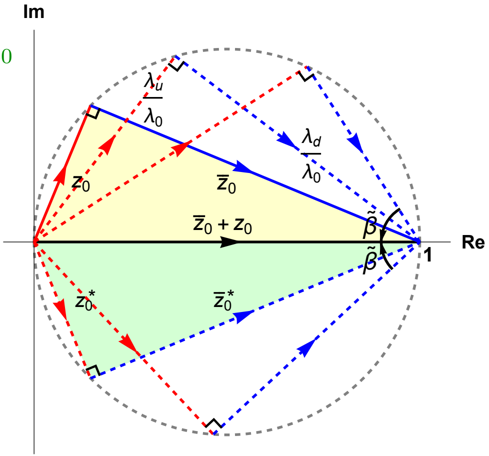

- $\Rightarrow$ to fix $\beta_0 = \frac{\pi}{8}$ require symmetry under transformation $(*)$ followed by CP flip

Discussion

- SM has no mechanism for constraining MMs
- $\Rightarrow$ Need to look BSM for models to do so

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- Much to be done to find concrete BSM model(s) to implement/embed these symmetries.

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$$\frac{m_c}{m_t}\frac{m_b}{m_s} = \left|\frac{\boldsymbol{\lambda_u}}{\boldsymbol{\lambda_d}}\right|^2 = \tan^2\frac{\pi}{8}(1+\mathcal{O}(\lambda_0^2)) = 0.176 \pm 0.001$$

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- Precise predictions of UT angles:
  - $\alpha - \frac{\pi}{2} = (1.30 \pm 0.02)^\circ$ c.f. $(1.6 \pm 1.4)^\circ$ (exp)
  - $\beta - \frac{\pi}{8} = (-0.2 \pm 0.1)^\circ$ c.f. $(0.1 \pm 0.4)^\circ$ (exp)
  - $\gamma - \frac{3\pi}{8} = (-1.1 \pm 0.1)^\circ$ c.f. $(-1.8 \pm 1.3)^\circ$ (exp)

Backup Slides

Isospin Reflection Symmetry?

- Can re-write texture:

$$M_q^{HS} \equiv n_q \begin{pmatrix} c' \lambda_q^4 & b' \boldsymbol{\lambda_q^3} & 0 \\ b' \boldsymbol{\lambda_q^{*3}} & b' \lambda_q^2 & A_0 \boldsymbol{\lambda_q^2} \\ 0 & A_0 \boldsymbol{\lambda_q^{*2}} & 1 \end{pmatrix} \pm d \, \lambda_q^4 I$$

  $b' \simeq b$. Still get good fit to data.

- First (leading) matrix solely responsible for quark mass differences and mixing parameters.
- Second (small) matrix is I_z-dependent "pedestal" on quark masses. Symmetric under a family-$SU(3)$ symmetry.
- All coefficients (λ_0, A_0, b', c', d) symmetric under isospin reflection operator $u \leftrightarrow d$.
- Symmetry broken (only) by $\boldsymbol{\lambda_q}$, n_q and the sign of d.

Analytic NLO Solutions: 1) Mixing Parameters

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- We give here the algebraic NLO solutions of the texture:

$$\begin{aligned}
\lambda &= \lambda_0 \left(1 + f_\lambda \lambda_0^2\right) + \mathcal{O}(\lambda_0^5) \\
A &= A_0 \left\{1 + \left[\tfrac{1}{4}(3b - 2\rho_0) - 2 f_\lambda\right] \lambda_0^2 \right\} + \mathcal{O}(\lambda_0^4) \\
\overline{\rho} &= \rho_0 \left(1 + c_0 f_\rho \lambda_0^2\right) + \mathcal{O}(\lambda_0^4) \\
\overline{\eta} &= \eta_0 \left\{1 + \left[s_0 f_\rho + \tfrac{1}{2}(1 - 5b)\right] \lambda_0^2 \right\} + \mathcal{O}(\lambda_0^4),
\end{aligned}$$

where $f_\lambda = \frac{1}{4}(3 f_A + 4\eta_0 f_c - 5)$,

$$f_A = \frac{1}{b}\left[A_0^2 + \frac{1}{2}(c_d + c_u)\right], \quad f_c = \frac{1}{b}(c_d - c_u)$$

and $f_\rho = s_0(1 + f_c) - \frac{1}{4s_0}[2 f_A + 2 f_c - 7b]$.

- NLO corrections above, as fractions of LO terms are respectively: -5.8×10^{-3}, $+2.6\%$, $+3.6\%$ and -1.8% (using fitted param values from table).

Analytic NLO Solutions: 2) Mass Ratios

- For the quark mass ratios, we find:

$$\frac{m_1^q}{m_2^q} = -\lambda_q^2(1-r_q)\left\{1+\left[r_A\frac{(2-r_q)}{(1-r_q)}-2\right]\lambda_q^2\right\}+\mathcal{O}(\lambda_q^6)$$
$$\frac{m_2^q}{m_3^q} = b\lambda_q^2\left[1+(1-r_A)\lambda_q^2\right]+\mathcal{O}(\lambda_q^6),$$

  where $r_q = \frac{c_q}{b}$ and $r_A = \frac{A_0^2}{b}$.
- NLO corrections to mass ratios m_c/m_t, m_s/m_b, m_u/m_c, m_d/m_s as fractions of LO terms are (resp.) -4.3×10^{-3}, -2.5%, $+4.3\%$, and $+4.5\%$ (using fitted param values from table).
- All results compatible with full numerical results reported in table.