The μ–τ reflection symmetry of Majorana neutrinos in a cross seesaw system

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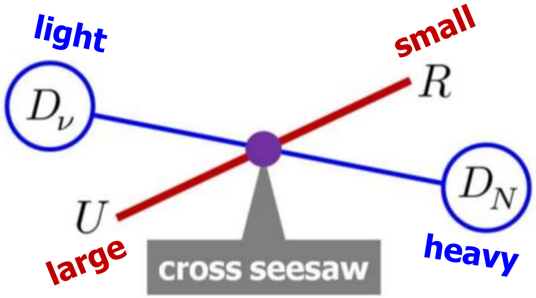


What’s somewhat different?

- in the mass basis
- with the exact seesaw
- a minimal flavor symmetry

FLASY 2025, 30.6—4.7.2025, Rome, Italy

OUTLINE

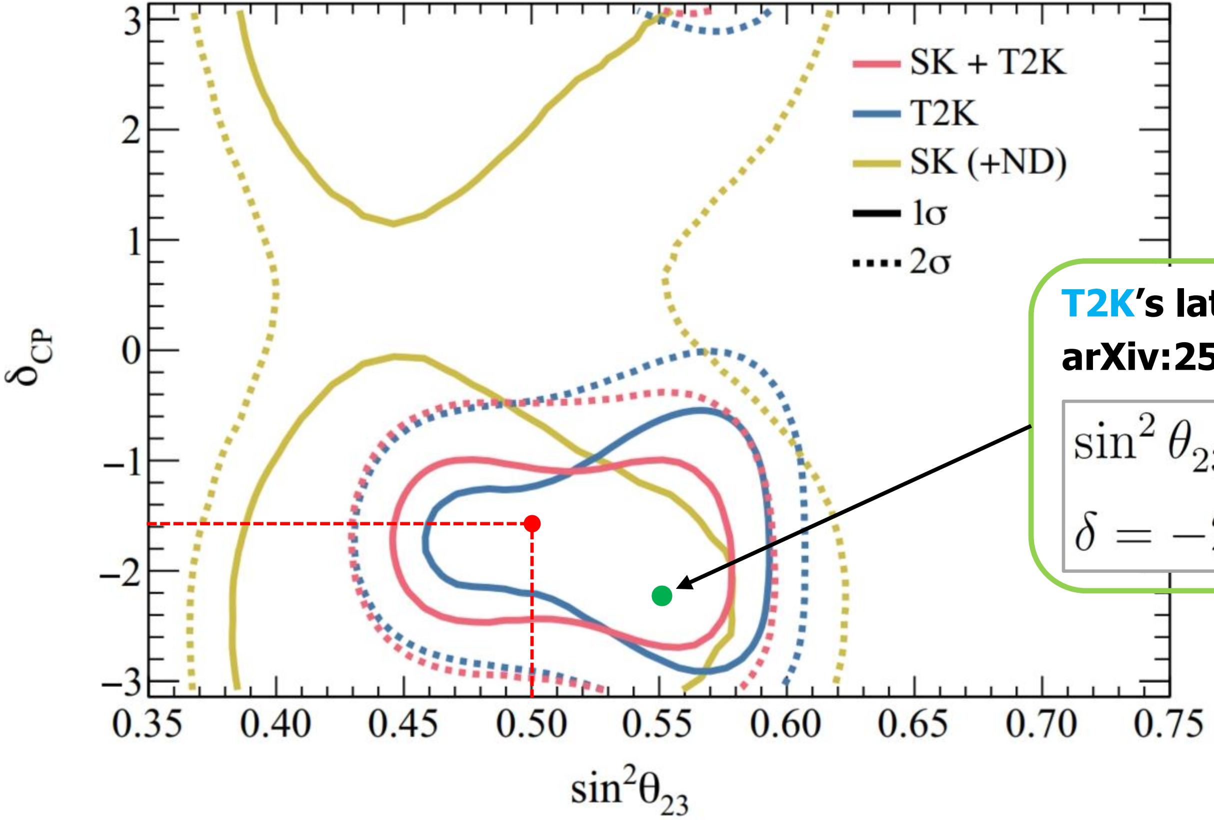
- ♦ **Hint for the mu-tau reflection symmetry**
- ♦ **The seesaw and the PMNS non-unitarity**
- ♦ **How the symmetry transmits on seesaw**
- ♦ **Application to LFV/LNV and CP violation**

Based on: ZZX

- **NPB 1013 (2025) 116853**; 2502.09286
- **RPP 86 (2023) 076201**; 2210.11922
- **JHEP 06 (2022) 034**; 2203.14185

Hint from the Super-K + T2K data

- The first joint oscillation analysis of **Super-K** and **T2K** data (2405.12488, PRL 134 (2025) 011801)



T2K's latest best-fit result
arXiv:2506.05889

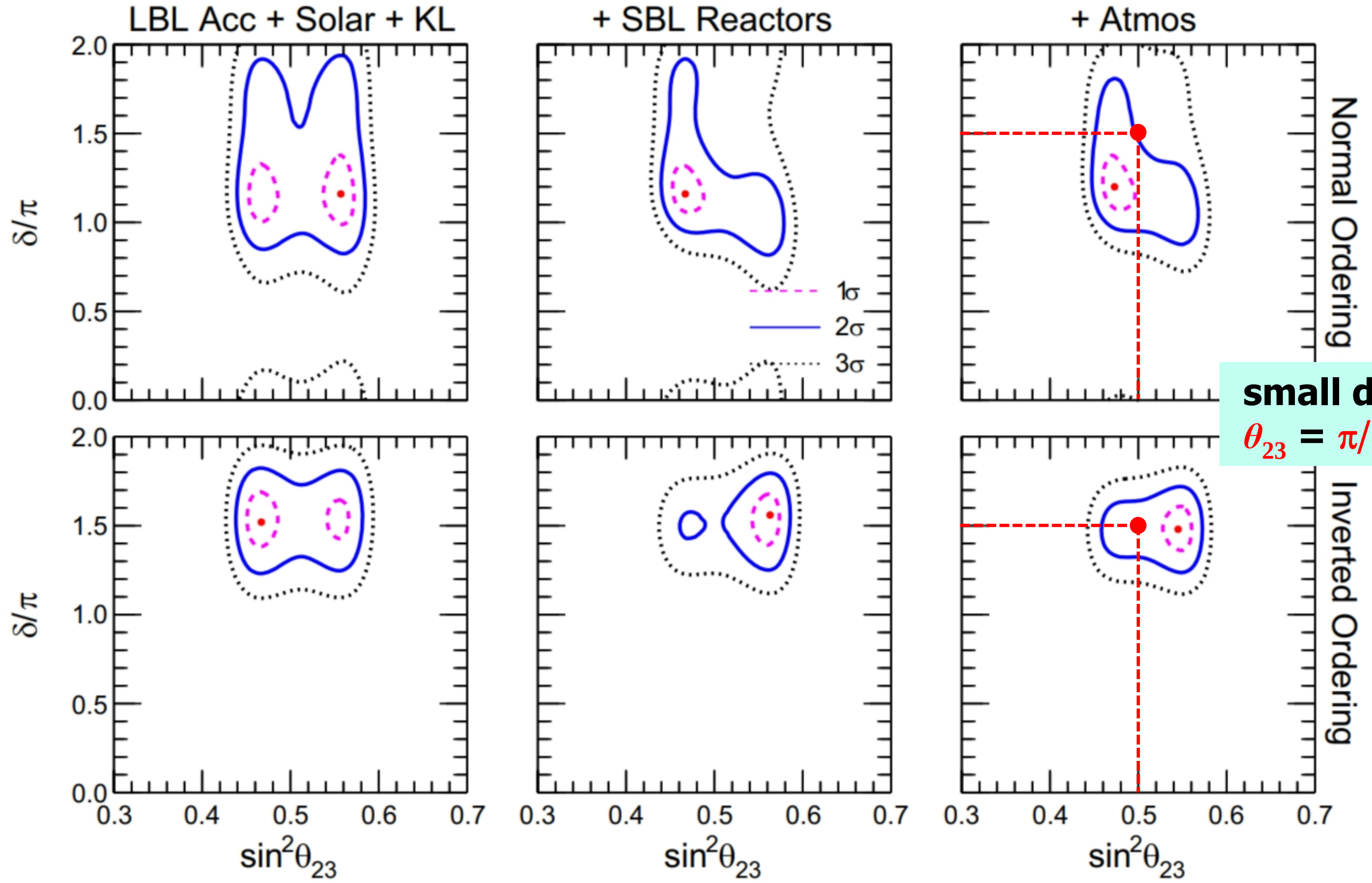
$$\sin^2\theta_{23} = 0.559^{+0.018}_{-0.078}$$

$$\delta = -2.18^{+1.22}_{-0.47}$$

small deviations from $\theta_{23} = \pi/4$ and $\delta = -\pi/2$?

Hint from the latest global analysis

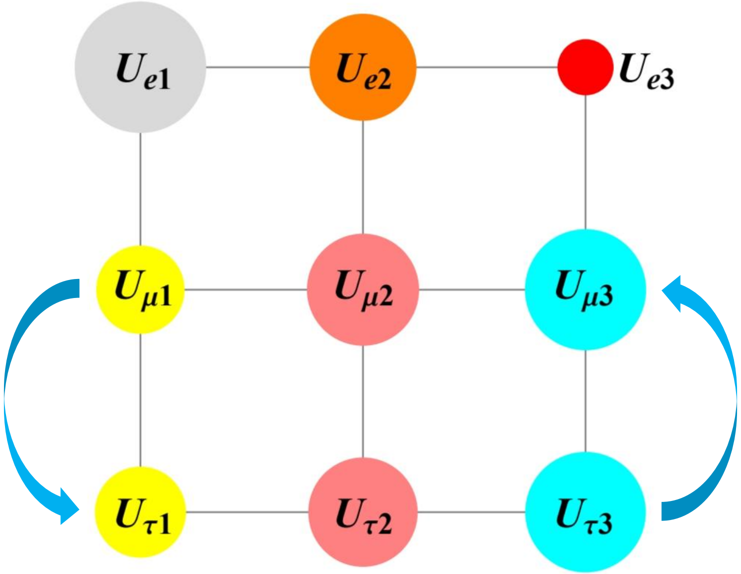
- ♦ **Eligio Lisi**'s team (**F. Capozzi** et al., 2503.07752, PRD 111 (2025) 093006; **Eligio**'s talk in **Flasy 25**)



small deviations from θ_{23} = π/4 and δ = −π/2?

Is there an approximate flavor symmetry?

♦ **9 moduli of the PMNS matrix elements constrained from data at the 3σ level:**



the area of each circle = an element's modulus

P. Harrison, W. Scott (2002):

mu-tau reflection symmetry

with both $\theta_{23} = \pi/4$ & $\delta = -\pi/2$

We are on the right track

♦ **The standard parametrization of the PMNS matrix with 3 Euler-like mixing angles and 3 CPV phases:**

PMNS =

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$\underline{\theta_{23} \sim \pi/4} \oplus \begin{cases} \underline{\theta_{13} \ll 1} \\ \text{or} \\ \underline{\delta \equiv \delta_{13} - \delta_{12} - \delta_{23} \sim \pm\pi/2} \end{cases}$$

$$|U_{\mu i}| \simeq |U_{\tau i}| \quad (i = 1, 2, 3)$$

What is the mu-tau reflection symmetry?

It is a working flavor symmetry requiring the effective Majorana neutrino mass term to be invariant under the transformations of left-handed neutrino fields [ZZX, Z.H. Zhao, 1512.04207 (RPP, 1996)]:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}}\, M_\nu \,(\nu_{\rm L})^c + {\rm h.c.}} \longleftarrow \boxed{\nu_{e{\rm L}} \to (\nu_{e{\rm L}})^c\,, \quad \nu_{\mu{\rm L}} \to (\nu_{\tau{\rm L}})^c\,, \quad \nu_{\tau{\rm L}} \to (\nu_{\mu{\rm L}})^c}$$

♦ **traditional CP transformation**

$$(t, \mathbf{x}) \longrightarrow (t, -\mathbf{x})$$

$$\left\{\begin{array}{l} \nu_{e{\rm L}} \longrightarrow (\nu_{e{\rm L}})^c \\ \nu_{\mu{\rm L}} \longrightarrow (\nu_{\mu{\rm L}})^c \\ \nu_{\tau{\rm L}} \longrightarrow (\nu_{\tau{\rm L}})^c \end{array}\right.$$

♦ **mu-tau-interchanging CP transformation**

$$(t, \mathbf{x}) \longrightarrow (t, -\mathbf{x})$$

$$\left\{\begin{array}{l} \nu_{e{\rm L}} \longrightarrow (\nu_{e{\rm L}})^c \\ \nu_{\mu{\rm L}} \longrightarrow (\nu_{\tau{\rm L}})^c \quad \longleftarrow \\ \nu_{\tau{\rm L}} \longrightarrow (\nu_{\mu{\rm L}})^c \quad \longleftarrow \end{array}\right.$$

Invariance: $\boxed{M_\nu = M_\nu^*}$ **CP conserving** $\boxed{M_\nu = \mathcal{P} M_\nu^* \mathcal{P}}$ **CP violating**

Constraints on the flavor structure of three Majorana neutrinos:

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$$\nu_e \quad \nu_\mu \leftrightarrow \nu_\tau^c$$

$$\mathcal{P} = \mathcal{P}^T = \mathcal{P}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

mu-tau permutation

$$\theta_{23} = \pi/4\,,\ \delta = \pm\,\pi/2$$

A most natural extension of the SM

♦ **Neutrinos surely have the *right* to be *right* (-handed) to keep a similar kind of left-right symmetry as charged leptons and quarks —— small animals' fair play?**

♦ **Then neutrinos are allowed to couple to the SM Higgs doublet —— the Yukawa interactions. Why not?**

♦ **But the gender of neutrinos (neutral) makes it very fair to add a Majorana mass term with *N* and *N^c^*, which is fully *harmless* to all the fundamental symmetries of the SM.**

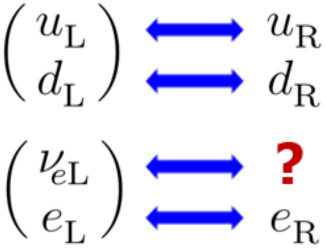
♦ **So we must be led to seesaw, plus leptogenesis as a big bonus —— kill two birds with one stone. (P. Minkowski 1977, …; M. Fukugita and T. Yanagida 1986; …)**

consistent with S. Weinberg's SMEFT (1979) → Seesaw EFT

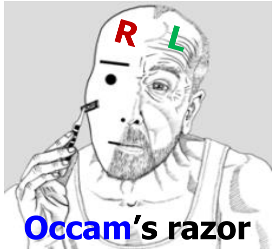
"unique"
d=5
operator

$$\mathcal{O}_{\rm W} = \frac{\overline{\ell_{\rm L}}\widetilde{H}\widetilde{H}^T\ell^c_{\rm L}}{\Lambda}$$

- **tiny neutrino masses !**
- **the Majorana nature ?**

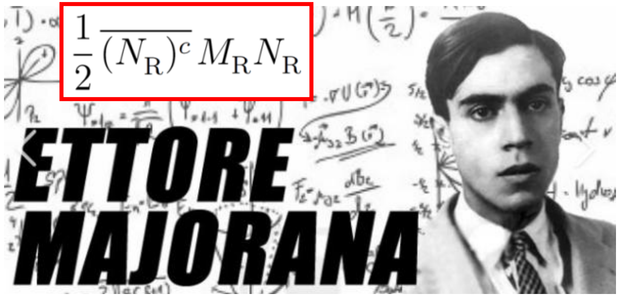


$$\begin{pmatrix} u_{\rm L} \\ d_{\rm L} \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} u_{\rm R} \\ d_{\rm R} \end{matrix} \qquad \begin{pmatrix} \nu_{e\rm L} \\ e_{\rm L} \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} ? \\ e_{\rm R} \end{matrix}$$



Occam's razor

I'm a good friend of all the fermions
Peter Higgs



$$\frac{1}{2}\overline{(N_{\rm R})^c}M_{\rm R}N_{\rm R}$$

Seesaw: an approximate form in the flavor basis

- ♦ **The seesaw mechanism (P. Minkowski 1977) formally works above the Fermi scale before SSB:**

$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}}\, Y_l H\, l_{\rm R} + \overline{\ell_{\rm L}}\, Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}}\, Y_l l_{\rm R} \underset{\uparrow}{\phi^0} + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \underline{\begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix}} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}}\, Y_l l_{\rm R} \underset{\uparrow}{\phi^+} - \overline{l_{\rm L}}\, Y_\nu N_{\rm R} \underset{\uparrow}{\phi^-} + {\rm h.c.}$$

- ♦ **A basis transformation to obtain the six Majorana neutrino masses:**

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

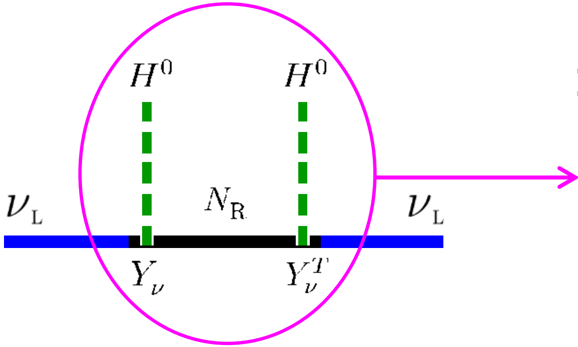
working masses:

$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} & \textbf{light} \\ D_N \equiv {\rm Diag}\{M_4, M_5, M_6\} & \textbf{heavy} \end{cases}$$

$\big\downarrow$ **SSB**

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}$$

6 × 6 mass matrix



Integrating out the heavy degrees of freedom:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^c + {\rm h.c.}}$$

$$M_\nu \simeq -Y_\nu \frac{\langle H\rangle^2}{M_{\rm R}} Y_\nu^T$$

Consistent with d=5 Weinberg operator

The seesaw relation in the flavor basis

Seesaw: an exact formula in the mass basis

- ◆ **In the mass basis of six Majorana neutrino fields, we have an exact seesaw relation:**

A block parameterization:
[ZZX, 1110.0083, PRD]

$$\mathbb{U} = \begin{pmatrix} I & 0 \\ 0 & U_0' \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} U_0 & 0 \\ 0 & I \end{pmatrix}$$

sterile (unitary) **Yukawa** (interplay) **active** (unitary)

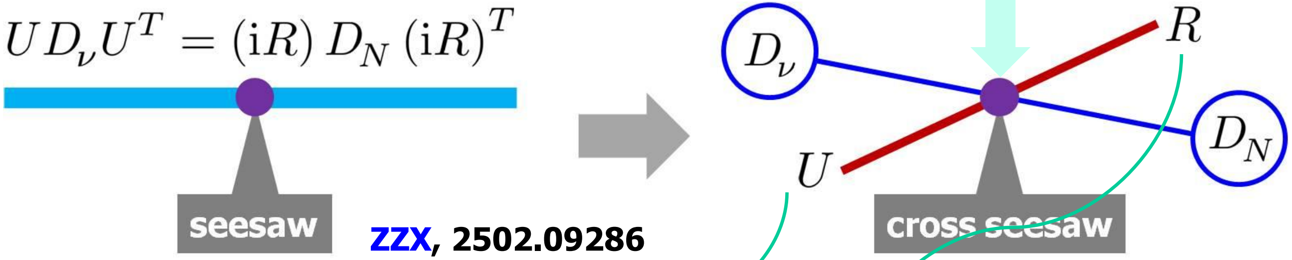
$U = AU_0$: the PMNS matrix
R : an analogue for heavy

Small masses are guaranteed!
Big flavor mixing is emergent?

- ◆ **The seesaw relation:**

exact seesaw:
cross seesaw:
in the mass basis

$$U D_\nu U^T = (\mathrm{i}R)\, D_N\, (\mathrm{i}R)^T$$



ZZX, 2502.09286

Leptonic weak cc interaction:

$$UU^\dagger + RR^\dagger = I$$

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\, \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}}\, \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

oscillations, LNV ← light **heavy → collider, LNV, LFV**

The full Euler-like parametrization

♦ **The 1st full Euler-like parametrization of $U = AU_0$ and R is useful for calculating flavor structures.**

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

derivable from the parameters of A and R

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36} \end{pmatrix}$$

ZZX
0709.2220/1110.0083

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{array} & c_{16}c_{26}\hat{s}^*_{36} \end{pmatrix}$$

The latest stringent *bounds* on possible PMNS nonunitarity.
M. Blennow et al. 2023

$$\begin{cases} \theta_{1j} < 2.92^\circ \\ \theta_{2j} < 0.27^\circ \\ \theta_{3j} < 2.56^\circ \\ [j = 4, 5, 6] \end{cases}$$

ZZX, J. Zhu, 2412.17698

Reliable analytical approximations

- The **PMNS** matrix $\underline{U = AU_0}$ in the seesaw mechanism is ***non-unitary***, but this effect is very small.

$$A = I - \frac{1}{2}\begin{pmatrix} s_{14}^2 + s_{15}^2 + s_{16}^2 & 0 & 0 \\ 2\hat{s}_{14}\hat{s}_{24}^* + 2\hat{s}_{15}\hat{s}_{25}^* + 2\hat{s}_{16}\hat{s}_{26}^* & s_{24}^2 + s_{25}^2 + s_{26}^2 & 0 \\ 2\hat{s}_{14}\hat{s}_{34}^* + 2\hat{s}_{15}\hat{s}_{35}^* + 2\hat{s}_{16}\hat{s}_{36}^* & 2\hat{s}_{24}\hat{s}_{34}^* + 2\hat{s}_{25}\hat{s}_{35}^* + 2\hat{s}_{26}\hat{s}_{36}^* & s_{34}^2 + s_{35}^2 + s_{36}^2 \end{pmatrix} + \mathcal{O}\left(s_{ij}^4\right)$$

So $U = AU_0 = U_0 + \text{nonunitarity corrections } (\lesssim 10^{-3})$.

- **Clarify a misconception: switching off non-unitarity effects leads us to** $U = U_0$ **—— It's wrong!**

Reason: switching off non-unitarity effects (i.e., $R = 0$ **and thus** $A = I$**) makes the seesaw collapse!**

- **non-unitarity of *U* is conceptually crucial but numerically negligible in most cases at low energies, since it arises from the Yukawa interactions between active and sterile sectors.**

$$U D_\nu U^T = (\mathrm{i}R)\, D_N\, (\mathrm{i}R)^T$$

$$R = \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* & \hat{s}_{16}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* & \hat{s}_{26}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* & \hat{s}_{36}^* \end{pmatrix} + \mathcal{O}\left(s_{ij}^3\right)$$

Reason: heavy Majorana neutrino decays and thus leptogenesis are fully determined by nonzero *R*.

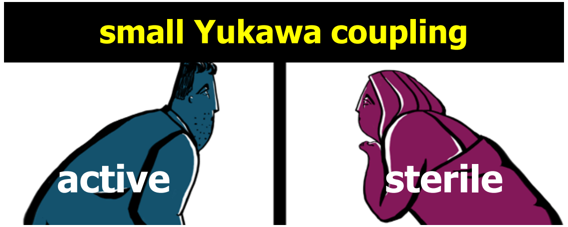
How to make masses tiny and flavor mixing big?

◆ In the canonical seesaw framework, it is technically natural to make ν-masses as tiny as possible:

$$U_0 D_\nu U_0^T = \left(\mathrm{i}A^{-1}R\right) D_N \left(\mathrm{i}A^{-1}R\right)^T \longrightarrow m_1 m_2 m_3 = M_4 M_5 M_6 \left[\det\left(\mathrm{i}A^{-1}R\right)\right]^2$$

determinants of the two sides:

tiny = huge × suppressor



$$A^{-1}R = \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix} + \mathcal{O}\left(s^3_{ij}\right)$$

◆ But how can we qualitatively see that large flavor mixing angles originate from the sterile sector?

Large flavor mixing of three active neutrinos is an *emergent* effect

active-sterile seesaw duality

The approximate mu-tau reflection symmetry may exist in the sterile sector

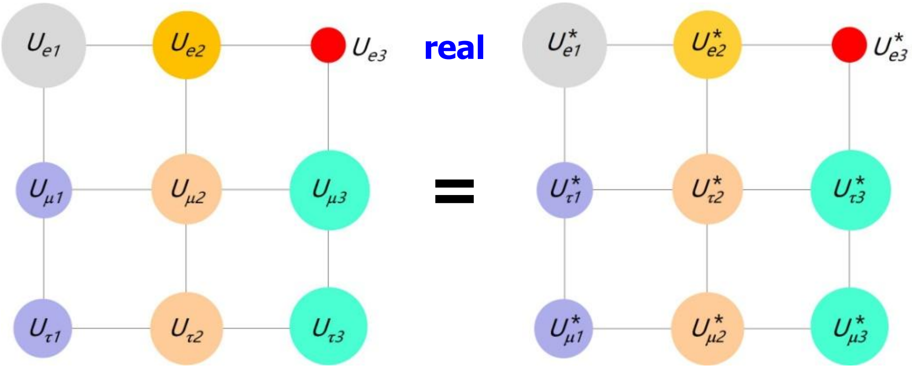
A bottom-up approach

- ◆ **Different from previous works, here let us start purely from the PMNS matrix constrained by data:**

A data-driven conjecture: $\boxed{U = \mathcal{P} U^{*} \zeta}$

$$\mathcal{P} = \mathcal{P}^{T} = \mathcal{P}^{\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\zeta = \mathrm{Diag}\{\eta_1, \eta_2, \eta_3\} \text{ with } \eta_i = \pm 1$$



- ◆ **In the basis where flavor states of charged leptons are identified with their mass states, we have**

the effective Majorana neutrino mass matrix

$$\boxed{M_{\nu} = U D_{\nu} U^{T} = \mathcal{P} U^{*} \zeta D_{\nu} \zeta U^{\dagger} \mathcal{P} = \mathcal{P} \left(U D_{\nu} U^{T} \right)^{*} \mathcal{P} = \mathcal{P} M_{\nu}^{*} \mathcal{P}}$$

$$D_{\nu} \equiv \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

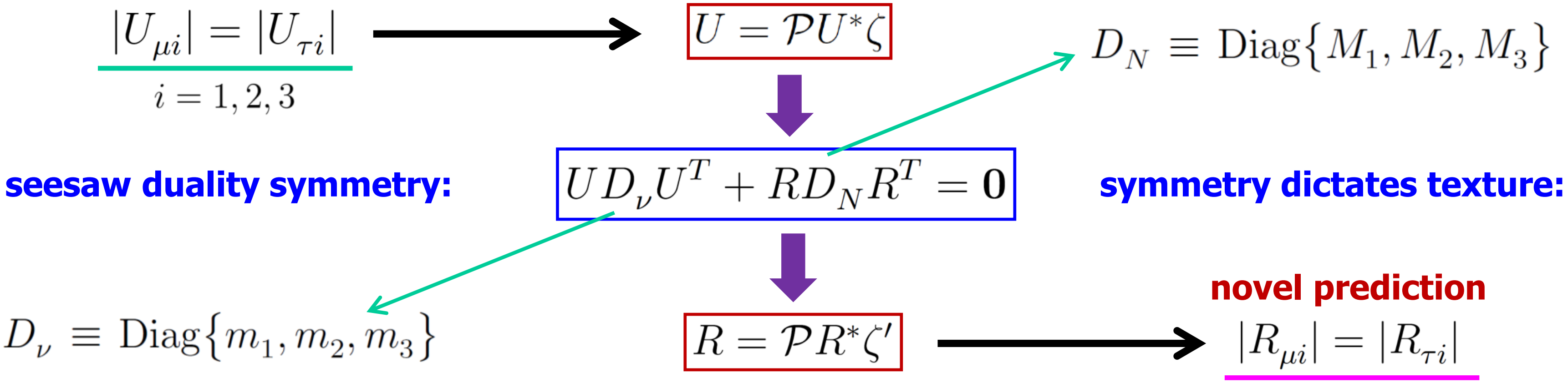
Substitute this into the mass term:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} \, M_{\nu} \, (\nu_{\rm L})^{c} + {\rm h.c.}} \Rightarrow \boxed{-\mathcal{L}^{\prime}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} \left(\mathcal{P} M_{\nu}^{*} \mathcal{P} \right) (\nu_{\rm L})^{c} + {\rm h.c.} = \frac{1}{2} \overline{[\mathcal{P}(\nu_{\rm L})^{c}]} \, M_{\nu} \, [\mathcal{P} \nu_{\rm L}] + {\rm h.c.}}$$

Then the invariance $\mathcal{L}^{\prime}_{\rm mass} = \mathcal{L}_{\rm mass}$ **leads us to the μ-τ reflection transformation** $\boxed{\nu_{\rm L} \to \mathcal{P}(\nu_{\rm L})^{c}}$. **QED**

Go across the seesaw bridge

♦ **Non-unitarity of the PMNS matrix has been constrained to be ≤ 0.1 %. So even in the presence of tiny unitarity violation, one may still make the conjecture:**



$$\underset{i = 1, 2, 3}{\underline{|U_{\mu i}| = |U_{\tau i}|}} \longrightarrow \boxed{U = \mathcal{P} U^* \zeta}$$

$$D_N \equiv \mathrm{Diag}\{M_1, M_2, M_3\}$$

seesaw duality symmetry:

$$\boxed{U D_\nu U^T + R D_N R^T = 0}$$

symmetry dictates texture:

$$D_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\}$$

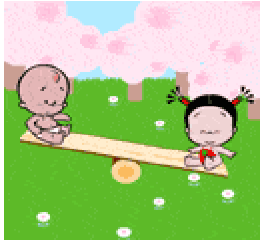
novel prediction

$$\boxed{R = \mathcal{P} R^* \zeta'} \longrightarrow \underline{|R_{\mu i}| = |R_{\tau i}|}$$

♦ **The top-down approach works in the same way —— the seesaw bridge helps *transmit* a potential μ-τ reflection symmetry of *R* to the active neutrino sector, leading to a μ-τ symmetry of *U*:**

Left: the active (light) sector

- **Naturally tiny neutrino masses**
- **Emergently large flavor mixing**

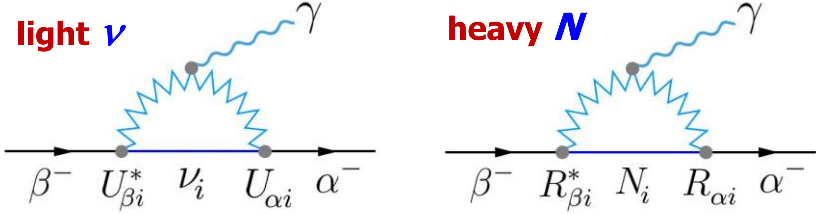


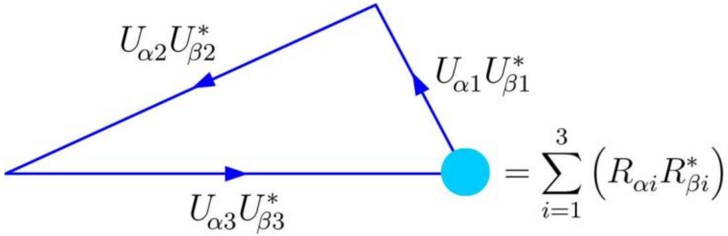
Right: the sterile (heavy) sector

- **Sufficiently tiny Yukawa couplings**
- **A potential μ-τ reflection symmetry**

Application (1): radiative cLFV

- **The mu-tau reflection symmetry of *R* constrains unitarity of the PMNS matrix via the cLFV decays**





In the full seesaw (ZZX, D. Zhang, 2009.09717) or its EFT with one-loop matching (D. Zhang, S. Zhou, 2107.12133):

$$\xi_{\alpha\beta} \equiv \frac{\Gamma(\beta^- \to \alpha^- + \gamma)}{\Gamma(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta)} \simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6}+\frac{1}{4}\cdot\frac{m_i^2}{M_W^2}\right) - \frac{1}{3}\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right|^2 \simeq \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right|^2$$

which allows us to constrain the unitarity hexagon using current experimental data on three radiative cLFV decays:

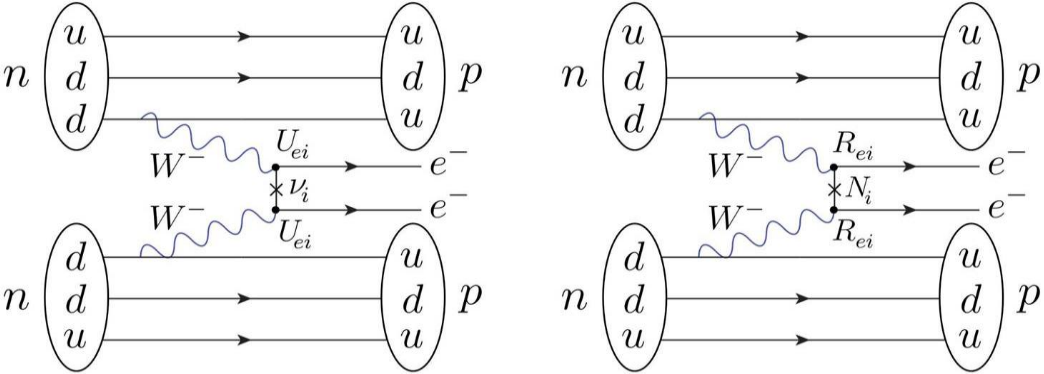
$$\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right| = \left|\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right| \simeq \sqrt{\frac{8\pi\xi_{\alpha\beta}}{3\alpha_{\rm em}}} \simeq 33.88\sqrt{\xi_{\alpha\beta}} \longrightarrow \left\{\begin{array}{l} \left|\sum_{i=1}^{3} U_{ei}U^*_{\mu i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\mu i}\right| < \underline{2.20\times 10^{-5}} \\ \left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < \underline{1.46\times 10^{-2}} \\ \left|\sum_{i=1}^{3} U_{\mu i}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{\mu i}R^*_{\tau i}\right| < 1.70\times 10^{-2} \end{array}\right.$$

Now imposing the mu-tau reflection symmetry, we have

$$\boxed{\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{j=4}^{6} R_{ej}R^*_{\tau j}\right| < 2.20\times 10^{-5}}$$

Application (2): LNV in 0ν2β

◆ **The contributions of light (left) and heavy (right) Majorana neutrinos to the 0ν2β decay channels:**



Seesaw + Unitarity:

$$\sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{j=1}^{3} M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{j=1}^{3} |R_{ej}|^2 = 1$$

◆ **Under the mu-tau reflection symmetry, there is no nontrivial phase in the effective mass of 0ν2β.**

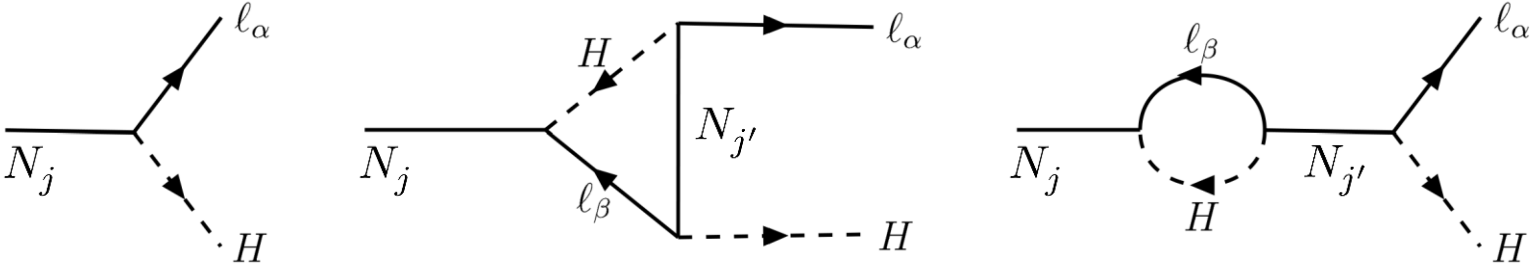
$$\Gamma_{0\nu 2\beta} \propto \left| \sum_{i=1}^{3} \underset{\substack{\uparrow \\ \textbf{real}}}{m_i U_{ei}^2} - M_A^2 \sum_{j=4}^{6} \frac{R_{ej}^2}{M_j} \mathcal{F}(A, M_j) \right|^2 = \left| \sum_{i=4}^{6} \left[M_j - \frac{M_A^2}{M_j} \mathcal{F}(A, M_j) \right] \underset{\substack{\uparrow \\ \textbf{real}}}{R_{ej}^2} \right|^2$$

If a signal of 0ν2β is seen, it will imply

- **a smoking gun for Majorana nature of massive neutrinos**
- **a support to the Weinberg operator and thus the seesaw**

Application (3): CP violation

- **The CP-violating asymmetries of heavy Majorana neutrino decays:**



M. Fukugita,
T. Yanagida 1986

$$\varepsilon_{j\alpha} \equiv \frac{\Gamma(N_j \to \ell_\alpha + H) - \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})}{\displaystyle\sum_\alpha \left[\Gamma(N_j \to \ell_\alpha + H) + \Gamma(N_j \to \overline{\ell_\alpha} + \overline{H})\right]}$$

$$D_j \equiv |R_{ej}|^2 + |R_{\mu j}|^2 + |R_{\tau j}|^2 \quad (j = 4, 5, 6)$$

$$\simeq \frac{1}{8\pi \langle H \rangle^2 D_j} \sum_{j'=4}^{6} \left\{ M_{j'}^2 \, \mathrm{Im}\left[\left(R^*_{\alpha j} R_{\alpha j'}\right) \sum_\beta \left[\left(R^*_{\beta j} R_{\beta j'}\right) \xi(x_{j'j}) + \left(R_{\beta j} R^*_{\beta j'}\right) \zeta(x_{j'j}) \right] \right] \right\}$$

- **Under the mu-tau reflection symmetry, the heavy and light CP asymmetries are both constrained:**

$$\boxed{R = \mathcal{P} R^* \zeta'}$$

$$\boxed{\begin{aligned} &\varepsilon_{je} = 0 \\ &\varepsilon_{j\tau} = -\varepsilon_{j\mu} \\ &\varepsilon_j \equiv \varepsilon_{je} + \varepsilon_{j\mu} + \varepsilon_{j\tau} = 0 \end{aligned}}$$

$$\longrightarrow \quad \eta \equiv \frac{n_{\mathrm{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10} \quad \Longleftrightarrow \quad |\mathcal{J}_\nu| = \frac{1}{2} \cdot |U_{e1}| \cdot |U_{e2}| \cdot |U_{e3}|$$

$$+ \text{tiny corrections}$$

Symmetry breaking helps!

How do right-handed neutrino fields transform?

♦ **Let us consider the neutrino mass term in the seesaw mechanism:**

$M_{\rm D} \equiv Y_\nu \langle H \rangle$

$$\boxed{-\mathcal{L}_\nu = \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}} \xrightarrow{\text{SSB}} \boxed{-\mathcal{L}'_\nu = \frac{1}{2}\overline{[\nu_{\rm L} \ \ (N_{\rm R})^c]} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + {\rm h.c.}}$$

Diagonalizing the 6 × 6 neutrino mass matrix:

$$\begin{pmatrix} U & R \\ S & Q \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} U & R \\ S & Q \end{pmatrix}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

exact seesaw

$$\boxed{U D_\nu U^T + R D_N R^T = \mathbf{0}}$$

Unitarity:

$$\begin{cases} UU^\dagger + RR^\dagger = SS^\dagger + QQ^\dagger = I \\ U^\dagger U + S^\dagger S = R^\dagger R + Q^\dagger Q = I \\ US^\dagger + RQ^\dagger = U^\dagger R + S^\dagger Q = \mathbf{0} \end{cases}$$

$$\boxed{\begin{array}{c} \underbrace{U = \mathcal{P} U^* \zeta \ \longrightarrow \ R = \mathcal{P} R^* \zeta'} \\ \Downarrow \\ \overbrace{S = \mathcal{T} S^* \zeta \qquad Q = \mathcal{T} Q^* \zeta'} \end{array}}$$

$\mathcal{T}$ = arbitrary unitary transformation

$$\boxed{M_{\rm D} = \mathcal{P} M_{\rm D}^* \mathcal{T} \ , \quad M_{\rm R} = \mathcal{T}^T M_{\rm R}^* \mathcal{T}}$$

♦ **Substitute these into the above neutrino mass term and require it to be invariant, we get transformations:**

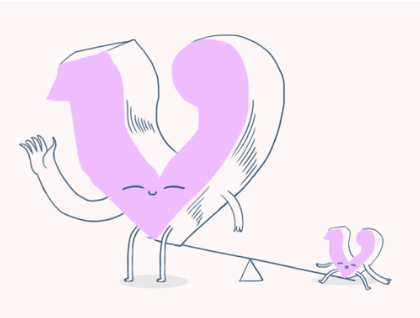
$$\boxed{\nu_{\rm L} \to \mathcal{P} (\nu_{\rm L})^c \ , \quad N_{\rm R} \to \mathcal{T}^* (N_{\rm R})^c}$$

Comments on model building

♦ Perhaps *1000* model-building exercises based on flavor symmetries have been done in the past *3* decades, to understand why lepton flavor mixing is as observed. Seesaws are needed in most cases.

S_3 , S_4 , A_4 , A_5 , D_4 , D_7 , T_7 , T′, Δ(27), Δ(48), …
$U(1)_F$, $SU(2)_F$, … modular, …

+



Big model?

Small model?

♦ In this way one often proceeds with

- a guiding principle (TH) or experimental hints (PH)
- a toolbox to make the model give something fine
- a dustbin to collect and hide some ugly things

♦ A symmetry implies that *behind it* there is something *unobservable* , but a flavor symmetry must be broken to makes something observable. Symmetry breaking is highly nontrivial in many cases.

The bottom line is to fit data —— a clear physical picture and not many free parameters?

The review papers since 2000: ZZX, 1909.09610 (PR 2020); F. Feruglio, A. Romanino, 1912.06028 (RMP 2021); ZZX, 2210.11922 (RPP 2023); G.J. Ding, S.F. King, 2311.09282 (RPP 2024); G.J. Ding, J.W.F. Valle, 2402.16963 (PR 2025)

Comments on the inverse seesaw

- ♦ **The inverse seesaw picture (D. Wyler, L. Wolfenstein 1983; R.N. Mohapatra, J.W.F. Valle, 1986):**

$$-\mathcal{L}'_{\rm ss} = \overline{\ell_{\rm L}}\,\widehat{Y}_l H l_{\rm R} + \overline{\ell_{\rm L}}\,Y_\nu \widetilde{H} N_{\rm R} + \overline{(N_{\rm R})^c}\,Y_S \Phi S_{\rm R} + \frac{1}{2}\overline{(S_{\rm R})^c}\,\hat{\mu} S_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}}\,\widehat{Y}_l l_{\rm R}\phi^0 + \frac{1}{2}\,\overline{\left[\nu_{\rm L} \quad (N_{\rm R})^c \quad (S_{\rm R})^c\right]} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} & \mathbf{0} \\ Y_\nu^T \phi^{0*} & \mathbf{0} & Y_S \Phi \\ \mathbf{0} & Y_S^T \Phi & \hat{\mu} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \\ S_{\rm R} \end{bmatrix}$$

$$+ \overline{\nu_{\rm L}}\,\widehat{Y}_l l_{\rm R}\phi^+ - \overline{l_{\rm L}}\,Y_\nu N_{\rm R}\phi^- + {\rm h.c.}\ ,$$

- **To lower the seesaw scale.**
- **Cost: many parameters.**
- **Gain: many papers?**

♦ **Diagonalization:**

$$\mathbb{U}'^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} & \mathbf{0} \\ Y_\nu^T \phi^{0*} & \mathbf{0} & Y_S \Phi \\ \mathbf{0} & Y_S^T \Phi & \hat{\mu} \end{pmatrix} \mathbb{U}'^* = \begin{pmatrix} D_\nu & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D_N & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & D_S \end{pmatrix}$$

$$D_\nu = \{m_1, m_2, m_3\}$$
$$D_N = \{M_4, M_5, M_6\}$$
$$D_S = \{M'_7, M'_8, M'_9\}$$

Weak CC interactions:

$$-\mathcal{L}'_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}}\,\gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_4 \\ N_5 \\ N_6 \end{pmatrix}_{\rm L} + R' \begin{pmatrix} N'_7 \\ N'_8 \\ N'_9 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

$$U \equiv A_2 A_1 U_0$$
$$R \equiv A_2 R_1$$
$$R' \equiv R_2$$

fine cancellation

♦ **The exact inverse seesaw relation:**

$$\boxed{U D_\nu U^T = ({\rm i}R)\, D_N \,({\rm i}R)^T + ({\rm i}R')\, D_S \,({\rm i}R')^T}$$

H.C. Han, ZZX, 2110.12705

Concluding remarks

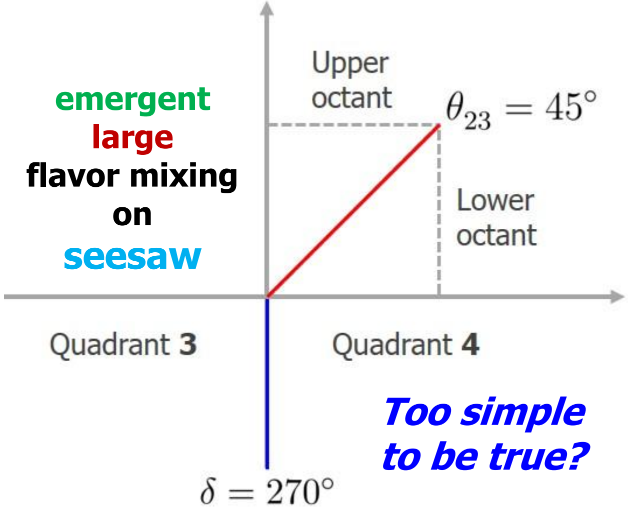
- ♦ 30 years ago, **H. Fritzsch** and **I** proposed an S(3)-symmetry-driven lepton mass ansatz, predicting the 1st (2 large + 1 small)-angle flavor mixing pattern (hep-ph/9509389, published in April 1996):

$$U = \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} + {\rm i}\sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{m_\mu}{m_\tau} \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} & \sqrt{\frac{1}{6}} \\ -\sqrt{\frac{1}{12}} & -\sqrt{\frac{1}{12}} & \sqrt{\frac{1}{3}} \end{pmatrix} \longrightarrow \boxed{\begin{array}{l} \theta_{12} \simeq 42^\circ \\ \theta_{13} \simeq 4^\circ \\ \theta_{23} \simeq 52^\circ \\ \delta \simeq \pm 90^\circ \end{array}}$$

In June 1998, the Super-K data on solar + atmospheric neutrinos hinted at $\underline{\theta_{12} \simeq \theta_{23} \simeq 45^\circ}$. **New Era**!

- ♦ Today, we consider a data-driven μ-τ reflection symmetry, or try many complicated flavor groups for model building.

- ♦ Although it is always fine to follow a bottom-up approach towards understanding the flavor structures of charged and neutral fermions, I believe that a true solution to the flavor issues must be top-down. *Theory is King in this regard.*



emergent
large
flavor mixing
on
seesaw

Too simple to be true?