Phase and Amplitude Noise Measurements

Fundamentals and Best Practices

Maximilian Schütte
LLRF Workshop 2024 – INFN-LNF, Frascati
29th October 2024

HELMHOLTZ

European XFEL

DESY.

Talk Overview

- Measurement techniques & analyser settings

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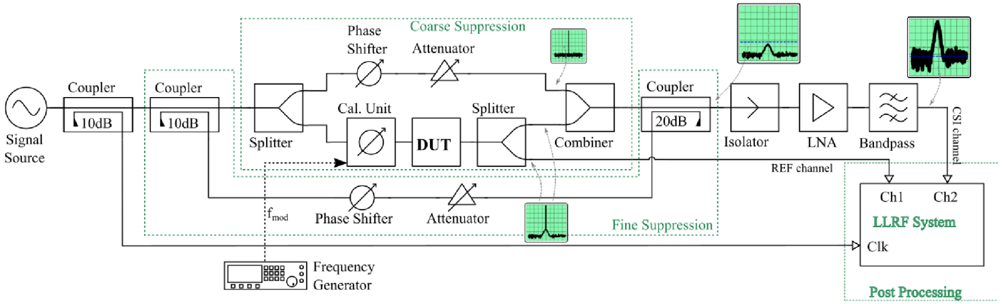
- Common mistakes and limitations

© nagwa

- Advanced techniques



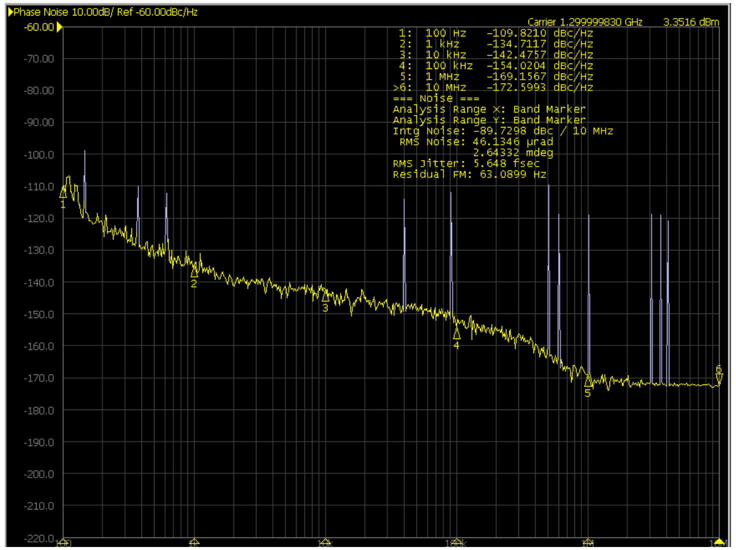
Courtesy F. Ludwig, M. Hoffmann

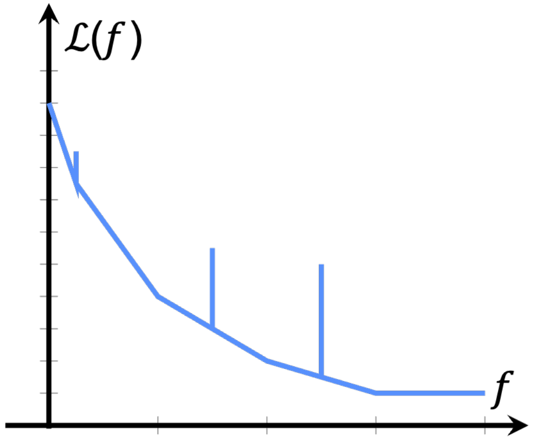
The goal of this talk is to understand …

- Phase & amplitude noise

$$\mathcal{L}(f) = \frac{1}{2} S'_\varphi(f), \qquad S_\varphi(f) = \mathbb{E}\left[\left|\int_{-\infty}^{\infty} R_\varphi(\tau)\, e^{-i2\pi f\tau}\, \mathrm{d}\tau\right|\right]$$

- Graphical representations





Perspective

An Overview: Reference Distribution and Synchronization Systems
From Sub-picoseconds to Sub-femtoseconds Stability

Marie Kristin Czwalinna
(DESY)

Tomorrow – 09:05

Basics of RF Reference Signal Generation and Synchronization Systems

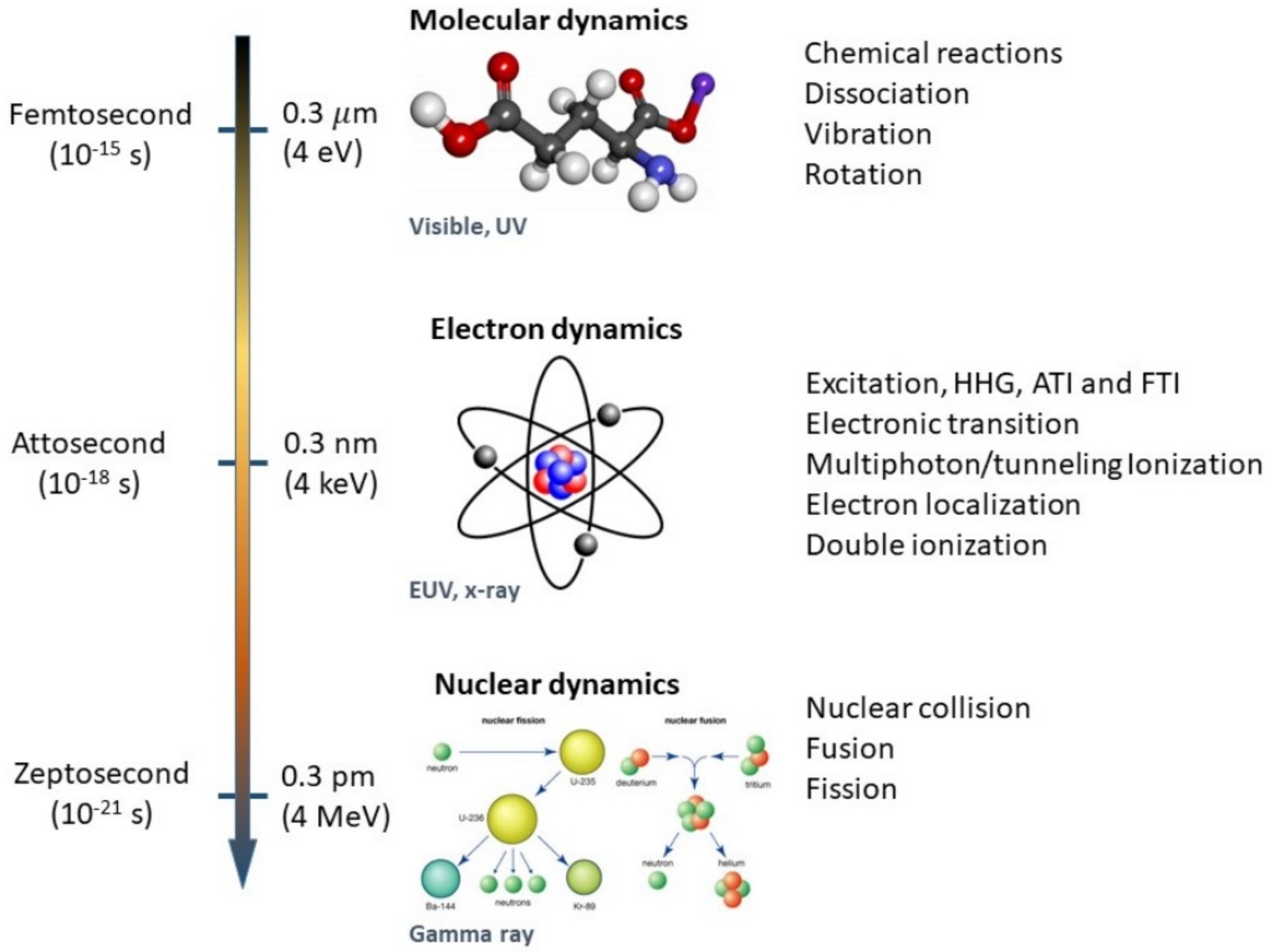
Krzysztof Czuba
(Warsaw University of Technology)

Tomorrow – 16:35

Phase & Amplitude Noise Theory

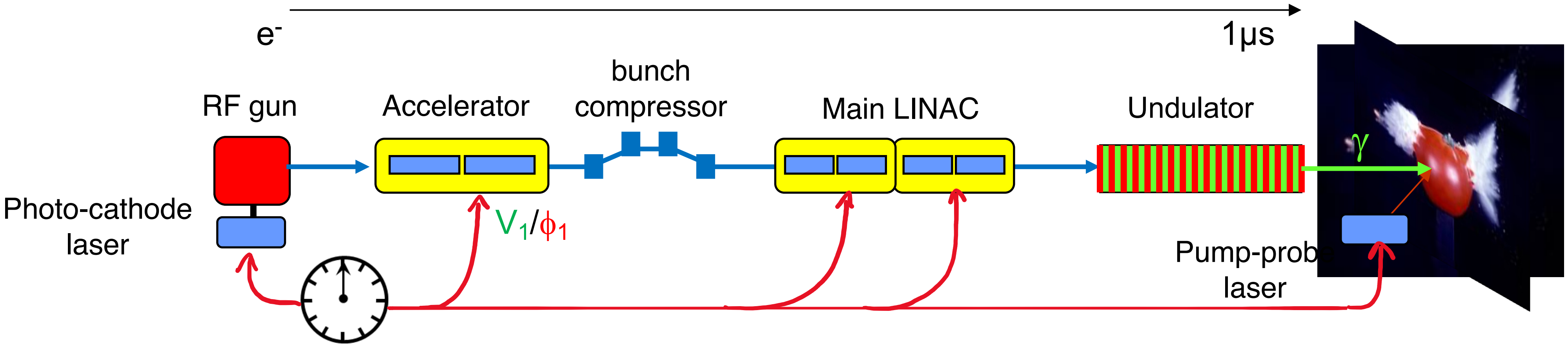
The Importance of Phase Noise

An example related to fs pump-probe experimentation with particle accelerators



The Importance of Phase Noise

An example related to fs pump-probe experimentation with particle accelerators



Timing jitter behind BC
C ~5 … 20 typically

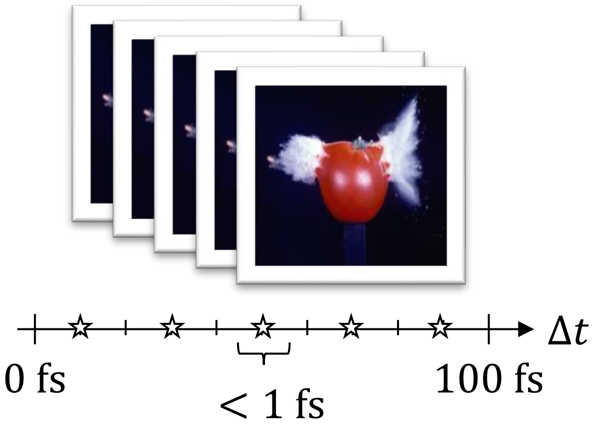
Voltage | **Phase** | **Init. arrival**

$$\Sigma_{t,f}^2 = \left(\frac{R_{56}}{c_0}\right)^2 \cdot \frac{\sigma_{V_1}^2}{V_1^2} + \left(\frac{C-1}{C}\right)^2 \cdot \frac{\sigma_{\phi_1}^2}{\omega_{rf}^2} + \left(\frac{1}{C}\right)^2 \cdot \Sigma_{t,i}^2$$

XFEL: 1.5ps/%
FLASH: 7.0ps/%

2 ps/deg
L-band

0.05 ps/ps
C=20



- Bunch propagation is **absolute** measure of time.
- Phase noise accumulated over this time causes wrong ETA → $\frac{1\ \text{fs}}{1\ \mu\text{s}} = 10^{-9}$.
- Can estimate RF field stability requirements: **0.01% amplitude, 0.01° phase**.

courtesy of H. Schlarb

The Harmonic Oscillator

One model fits (almost) all

Amplitude

Carrier frequency

Phase fluctuation dominates!

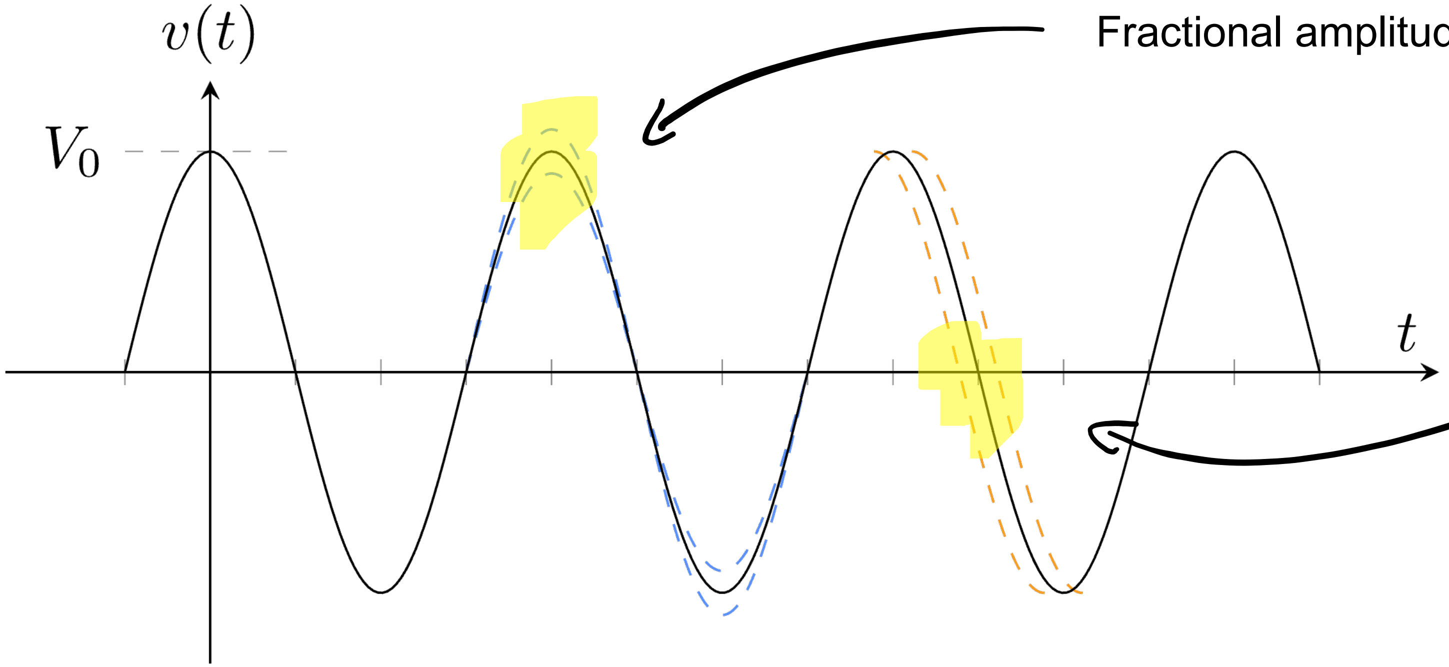
Ideal harmonic oscillation $v(t) = V_0 \cos(2\pi\nu_0 t)$

Real harmonic oscillation $v(t) = V_0\big(1 + a(t)\big)\cos\big(2\pi\nu_0 t + \varphi(t)\big)$

$a(t), \varphi(t) \ll 1$

Fractional amplitude fluctuation

Phase fluctuation

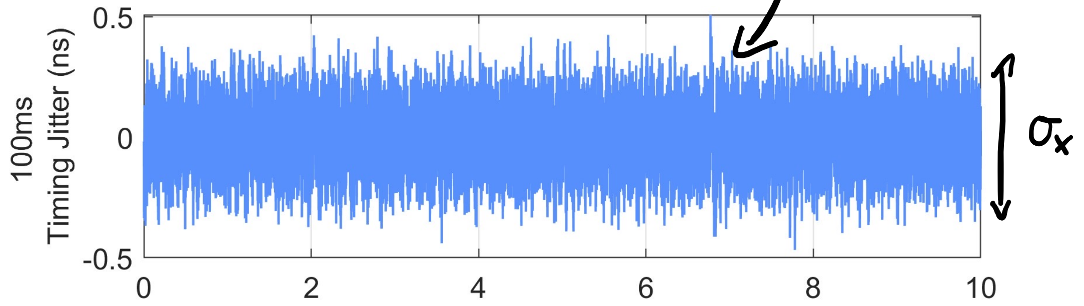


Timing fluctuation: $x(t) = \frac{\varphi(t)}{2\pi\nu_0}$

Phase & Amplitude Noise Calculations

Timing jitter & drift

Residual jitter after subtracting 100ms mov. avg.



Statistical Properties – Time Domain

- RMS phase / timing fluctuations

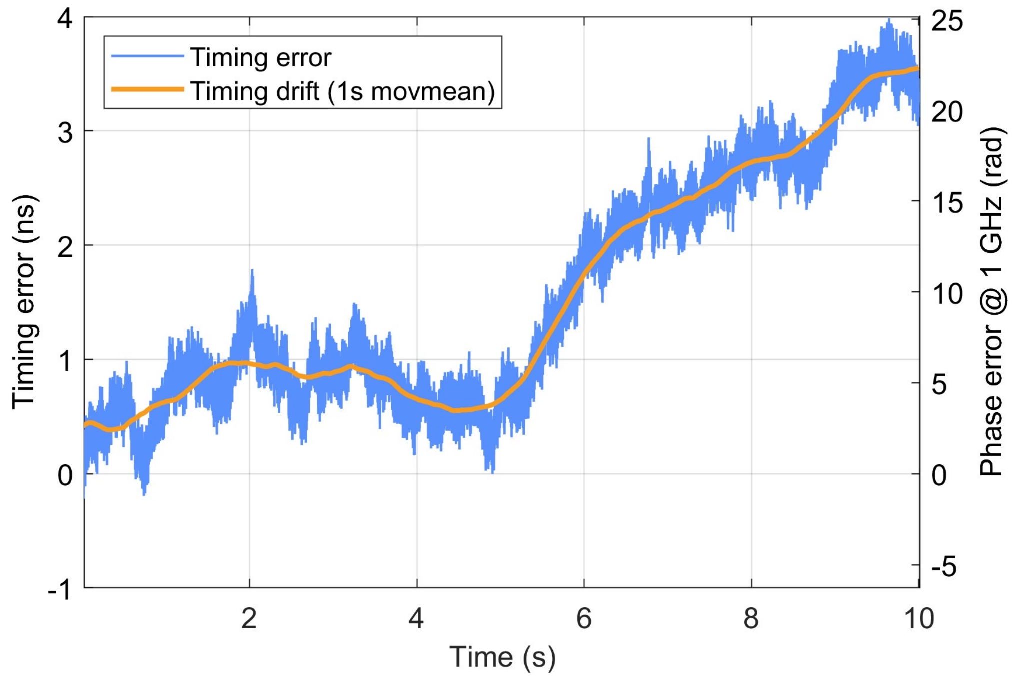
Often ignored

$$x_{\mathrm{RMS}} = \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} x(t)^2 \, \mathrm{d}\tau} = \sqrt{\sigma_x^2 + \bar{x}^2}$$

- Phase fluctuations are unbounded random walks.

$$\lim_{T\to\infty} x_{\mathrm{RMS}} = \infty$$

- **Averaging Time / Bandwidth is essential!**
- Peak-to-Peak drift → “12fs pk-pk over 24h”.
- ITU definition: “Drift / Wander” > ~0.1s > “Jitter”.



Phase & Amplitude Noise Calculations

Spectral representations

Statistical Properties – Frequency Domain

- Spectral analysis via Fourier Theory
  … but Fourier Transform is only meaningful for deterministic signals.
- **(Two-sided) power spectral density (PSD)**

$$S_\varphi(f) = \int_{-\infty}^{\infty} R_\varphi(\tau)\, e^{-i2\pi\tau}\, \mathrm{d}\tau\,, \qquad [S_\varphi(f)] = \frac{\mathrm{rad}^2}{\mathrm{Hz}}$$

- Auto-correlation function $R_\varphi(\tau)$ not accessible
  … estimation via Wiener-Khinchin **Periodogram**:

$$S_\varphi(f) = \lim_{T\to\infty} \frac{1}{T}\, |\hat{\varphi}_T(f)|^2$$

- Real signal → one-sided spectrum $S'_\varphi(f)$ is sufficient.
  - "Single-Sideband" only applies to carrier signals.

Mathematical Rigor

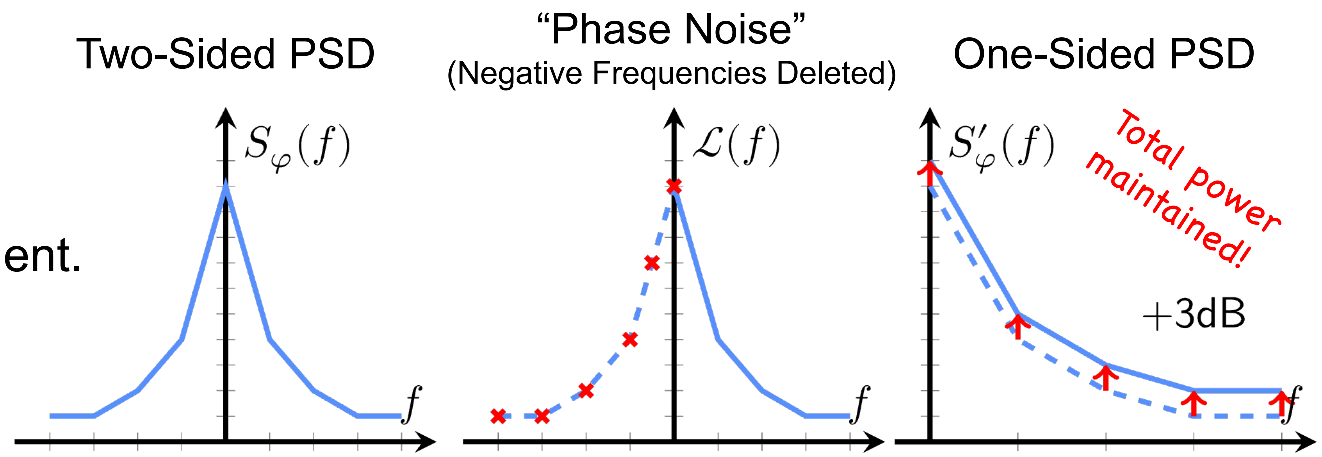
"Does Wiener-Khinchin apply?"

Process $\boldsymbol{\varphi}(t)$ must be:

- Wide-sense stationary
- Ergodic

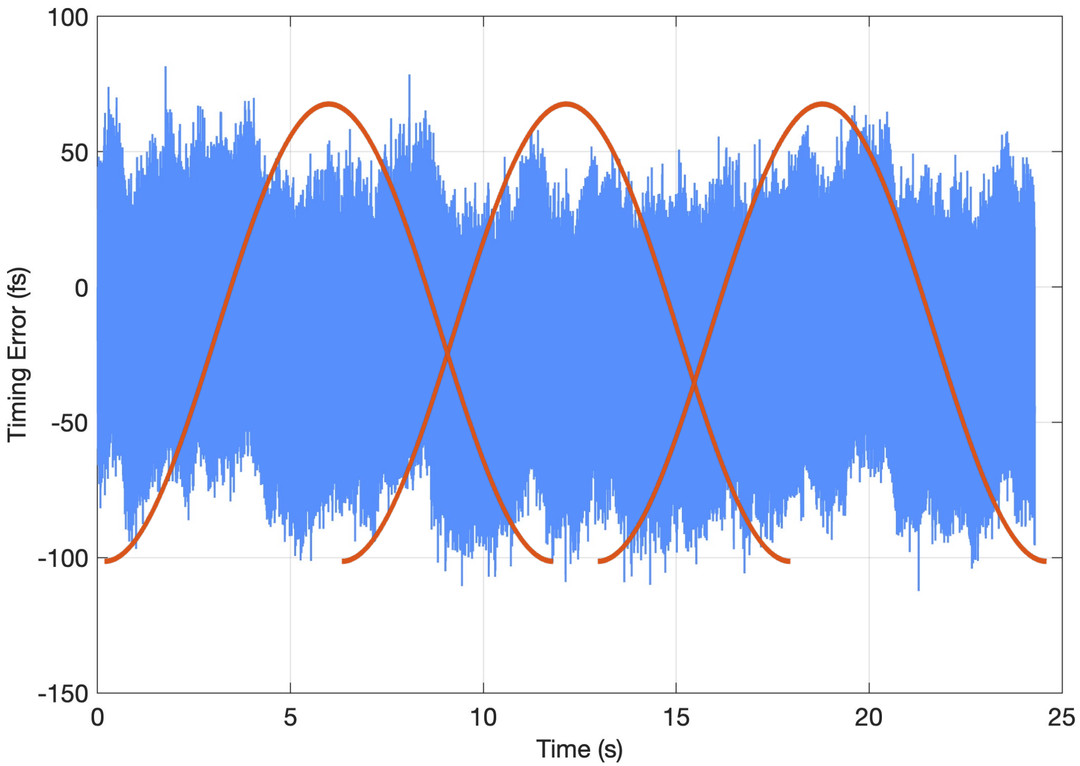
Solution:

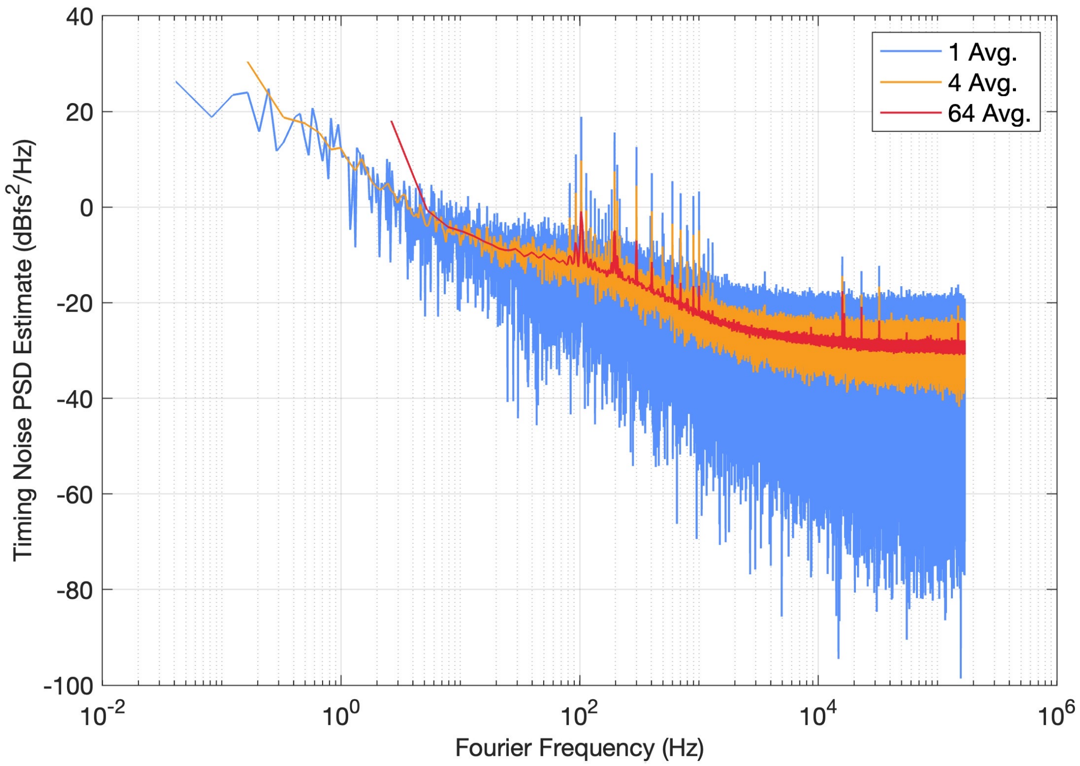
- Only observe for a finite time and assume stationarity
- Remove phase offset at each measurement and assume ergodicity.



Averaging in Estimation – Bartlett and Welch's Method

Improving the estimate, reducing sample variance





Phase & Amplitude Noise Definitions (IEEE Standard 1139)

Old ~~habits~~ definitions die hard

Phase Noise

- Definition since '99

$$\mathcal{L}(f) = \frac{1}{2}\ S'_{\varphi}(f), \qquad [10\log_{10}\mathcal{L}(f)] = \frac{\text{dBc}}{\text{Hz}}$$

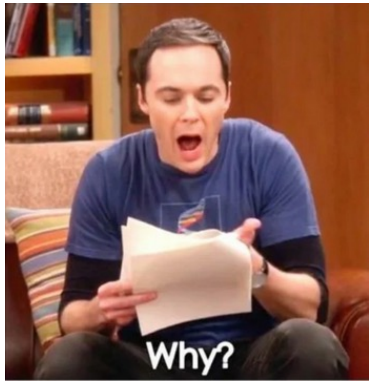
- Preferred in practice

Amplitude Noise

- Definition since '99

$$\frac{1}{2} S'_{a}(f), \qquad \left[\frac{1}{2} S'_{a}(f)\right] = \frac{1}{\text{Hz}}$$

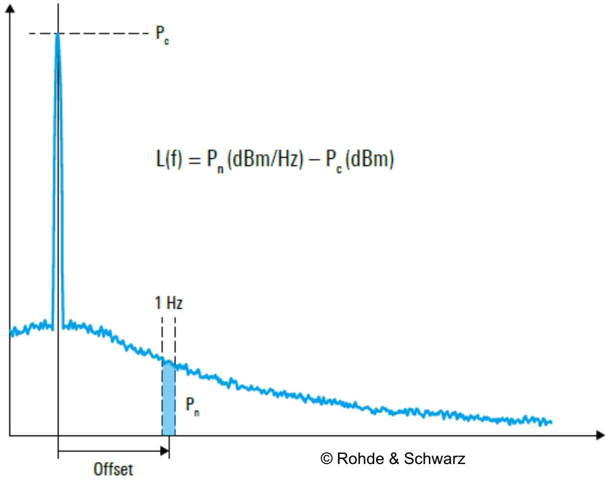
- Often also in dB / Hz

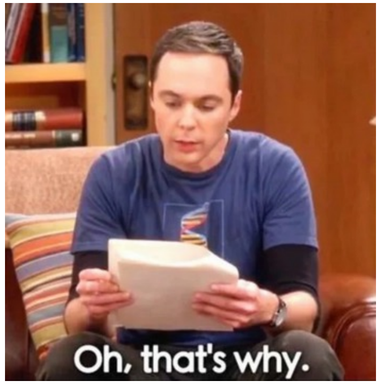


Historical Context

Phase noise **used** to be derived from the carrier's side-lobes on an RF analyser.

- Not accurate close to the carrier.
- Does not discriminate phase and amplitude fluctuations.





Phase & Amplitude Noise Calculations

Integrated Jitter

Linking Time & Frequency Domain

- Using the Plancherel Theorem, we find

$$x_{\mathrm{RMS}} = \frac{1}{2\pi\nu_0}\sqrt{\int_0^{\infty} S'_{\varphi}(f)\,\mathrm{d}f} = \infty$$

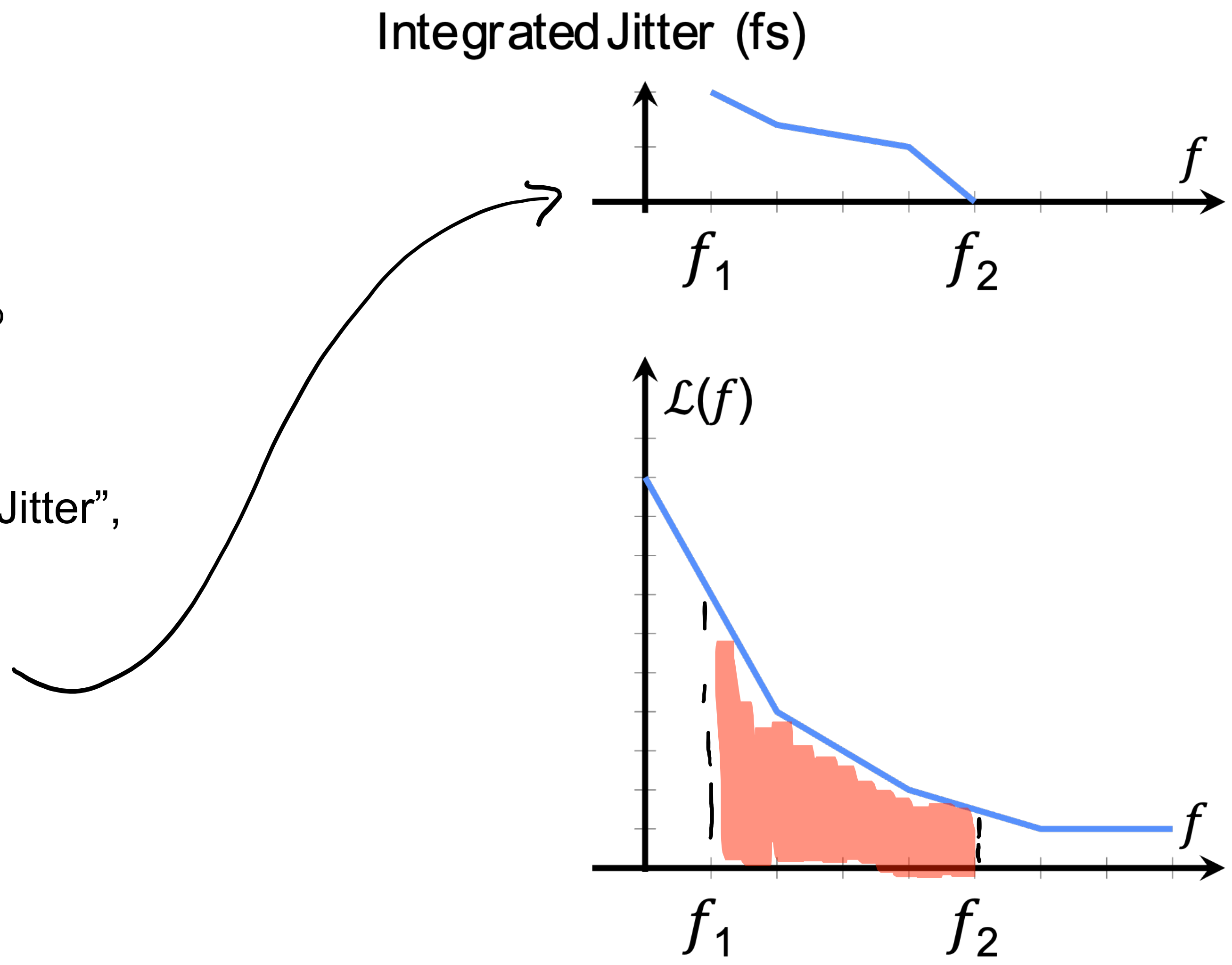
- Perhaps more useful: “Integrated (Timing) Jitter”,

$$J_t = \frac{1}{2\pi\nu_0}\sqrt{\int_{f_1}^{f_2} S'_{\varphi}(f)\,\mathrm{d}f}$$

- Cumulative integrated jitter curve

$$f_1 \to f_2, \qquad \text{or,} \qquad f_2 \to f_1$$

- Always state integration bounds!



Phase & Amplitude Noise Calculations

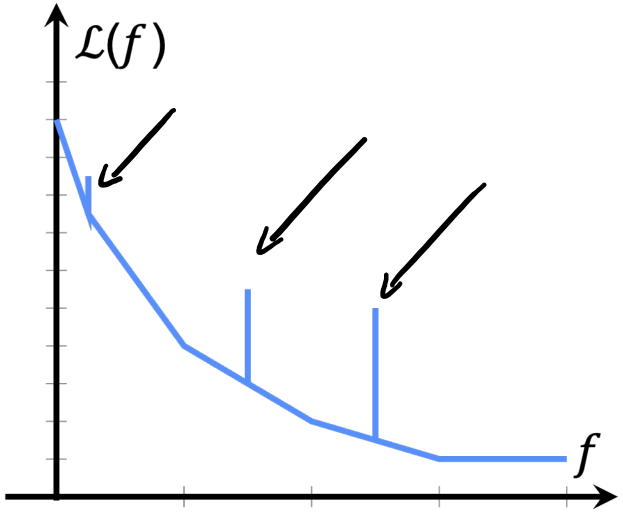
Spurious Content

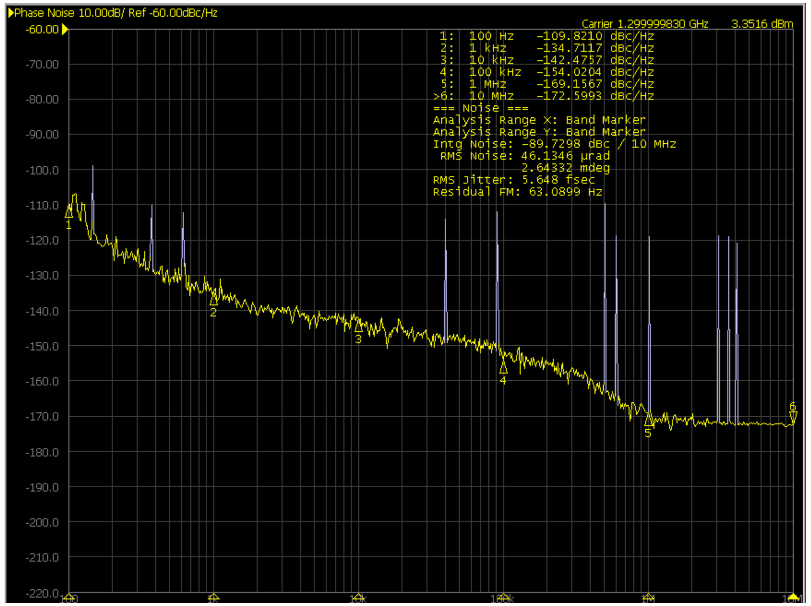
Handling the discrete & deterministic

- Non-random disturbances cause narrow spikes = “Spurs”.
- Jitter adds in root-sum-square.

$$J_t = \sqrt{J_{t,\text{smooth}}^2 + J_{t,\text{spur1}}^2 + \cdots + J_{t,\text{spur}N}^2}$$

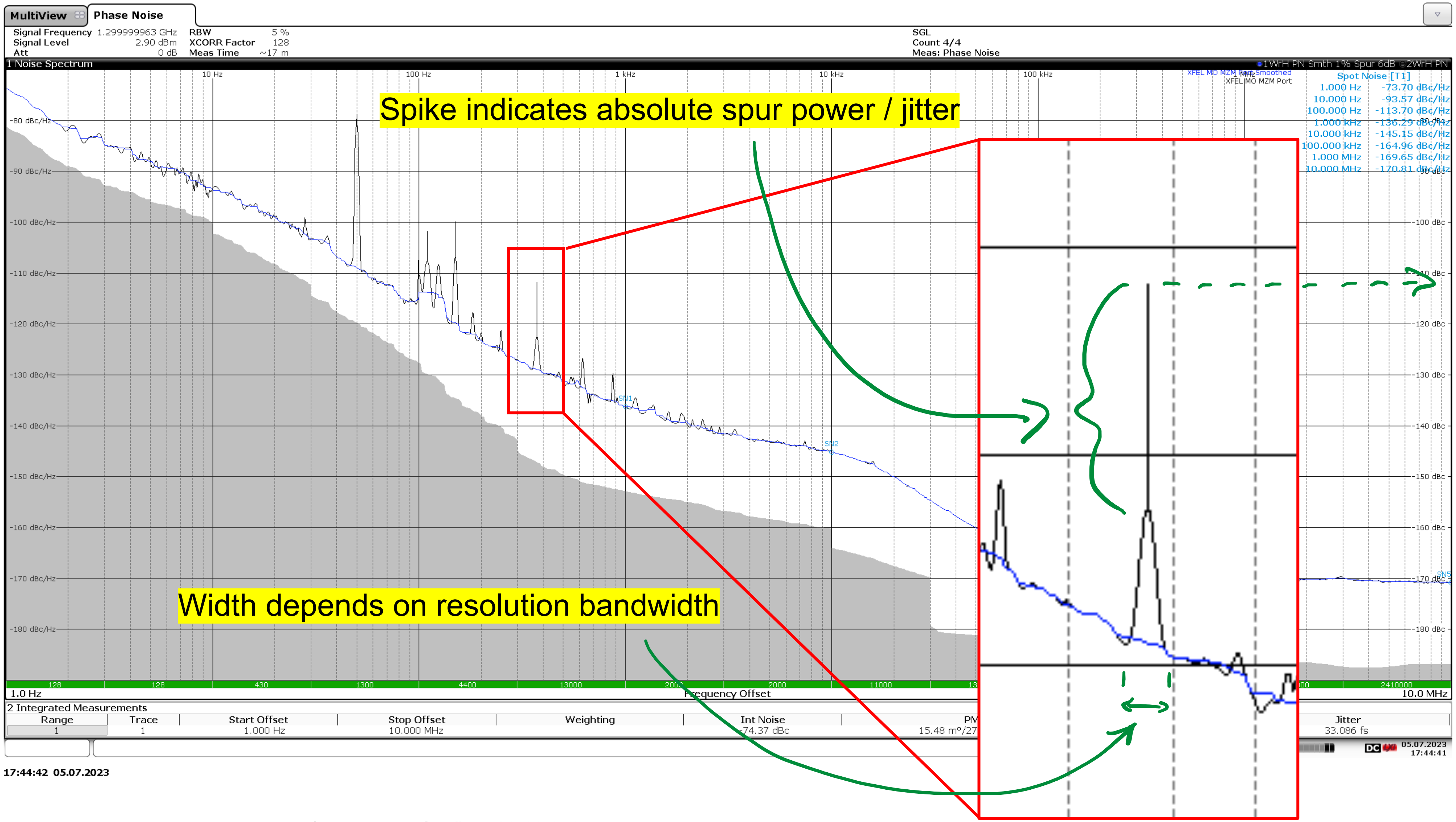
- Same as integrating over “Spurs in dBc / Hz” trace.
- Beware: Display on analysers varies!





Spurious Content

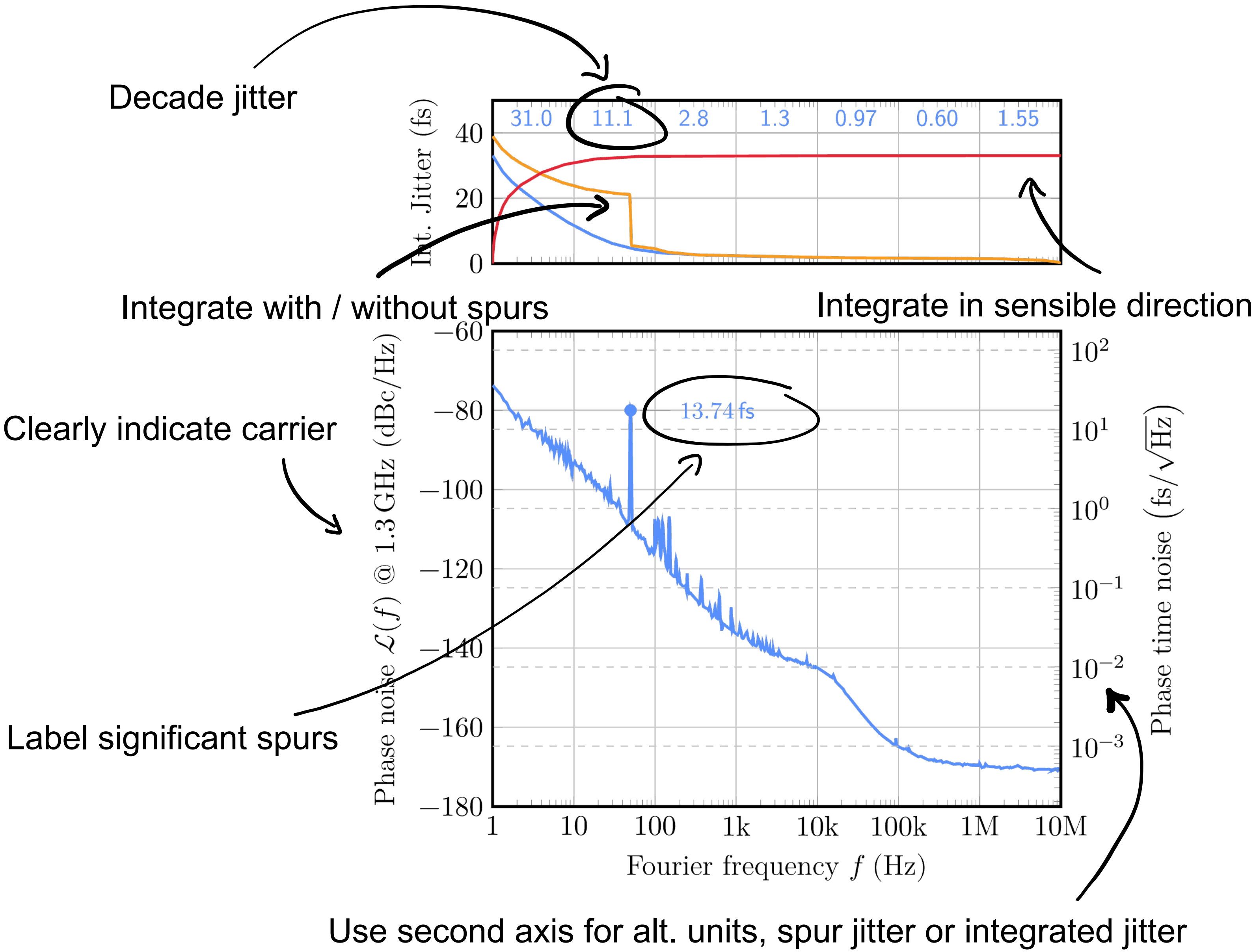
An example from the FSWP analyser



Noise Plots

Do’s & Don’ts

- Clearly declare omitted spurs.
- Avoid non-standard units.
- Use caption to clarify measurement conditions.



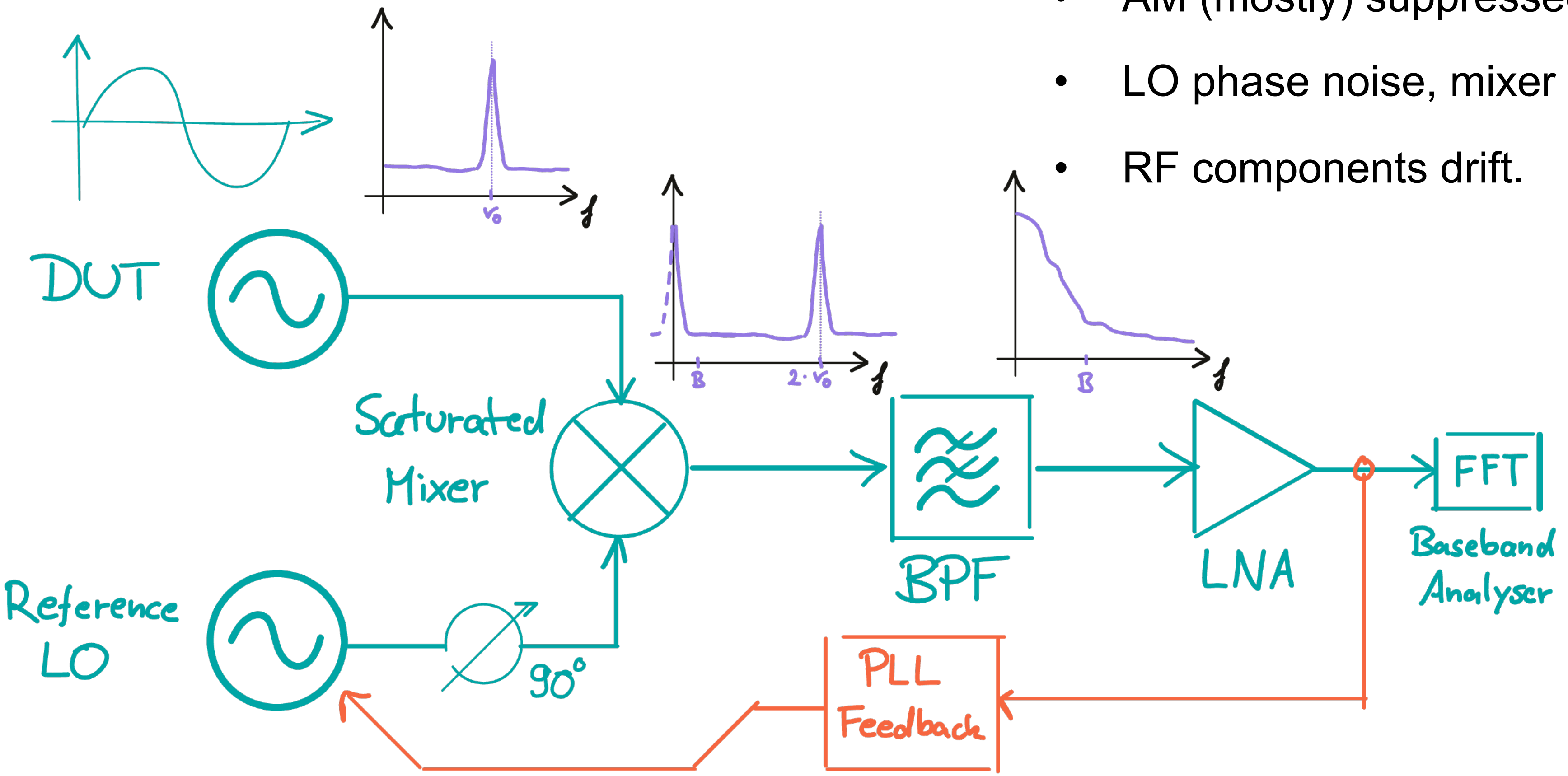
- There is no one-fits-all solution.

Phase & Amplitude Noise Measurement Techniques

Phase Noise Measurements

Saturated Mixer

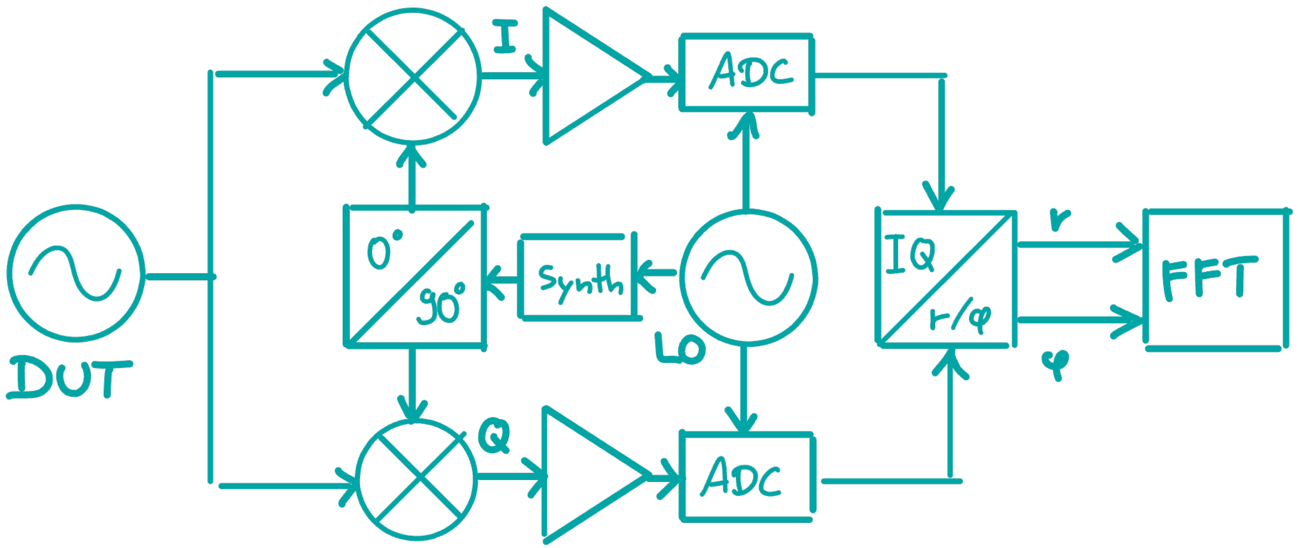
- Mixing in quadrature with LO @ ν_0 yields zero-IF.
- Only linear in small angle approximation
  → only for small PM.
- Low bandwidth PPL to LO ensures zero-IF.
  → PLL response needs to be taken into account!
- AM (mostly) suppressed.
- LO phase noise, mixer & LNA flicker added to DUT noise.
- RF components drift.



Phase Noise Measurements

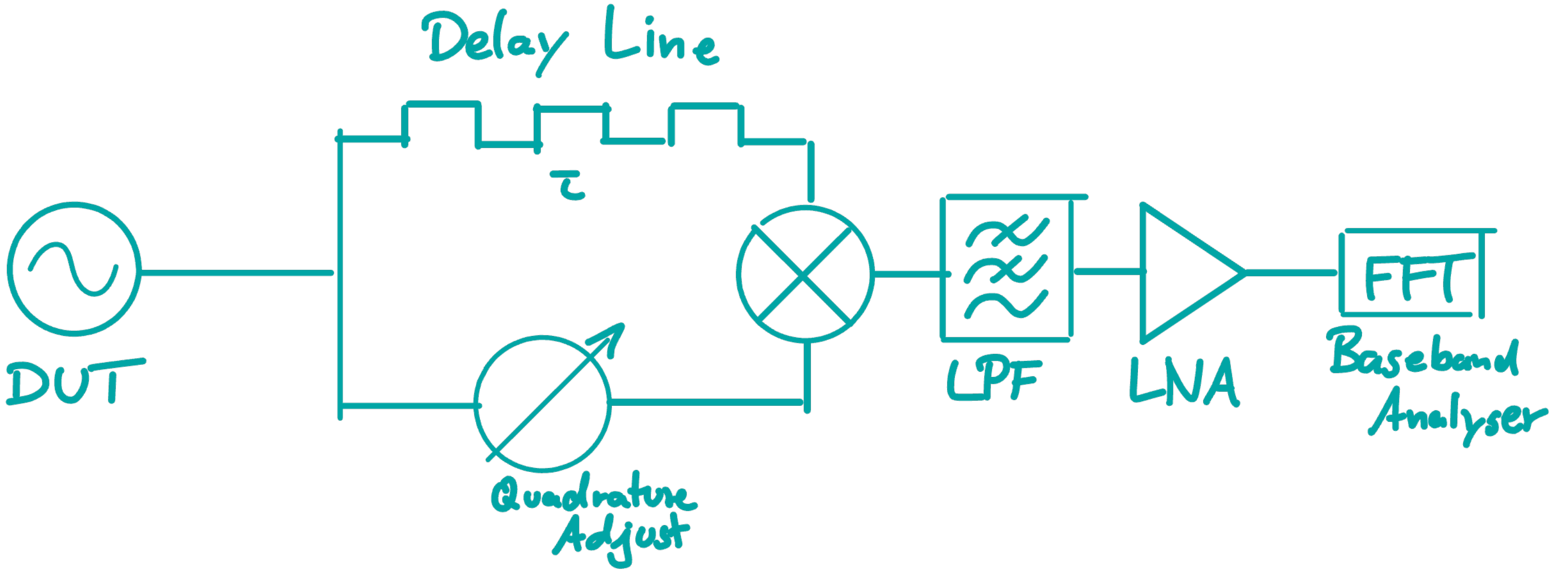
I/Q Detection

- Dual saturated mixer, in-phase + quadrature.
- Demodulation: $\varphi = \tan^{-1}\frac{Q}{I},\ a = \frac{\sqrt{I^2+Q^2}}{A} - 1.$
- Not reliant on small angle approximation.
- I/Q calibration necessary.
- LO derived ADC clock allows for phase-coherent IF sampling
  → Better harmonic disturbance rejection.
- Closer to a 360° phase detection system needed by LLRF controls.

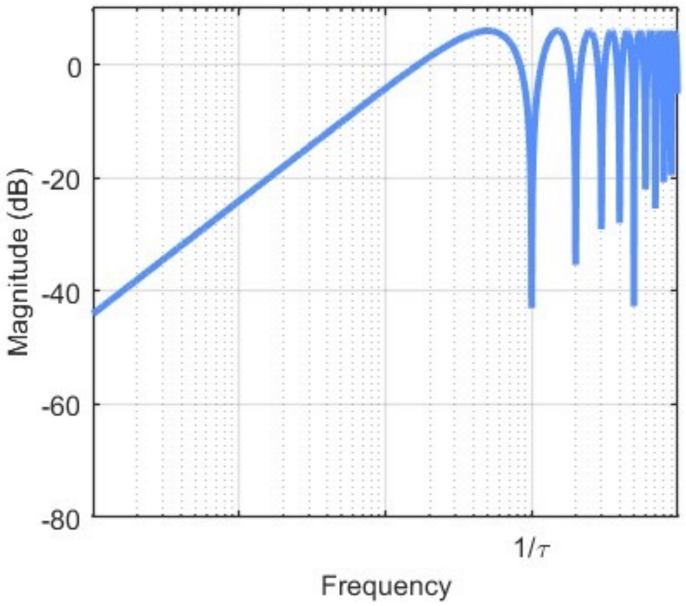


Phase Noise Measurements

Frequency Discriminator Method



$$\varphi(t) - \varphi(t-\tau) \rightarrow \left|1 - e^{-i2\pi f\tau}\right|^2 S_\varphi(f)$$



- Self-referenced, no LO.
- Delay line loss → better for optical.
- Delay line drifts.
- Low sensitivity / SNR for slow DUT phase drifts.
- Elimination at $f = N/\tau$ → limited frequency range.

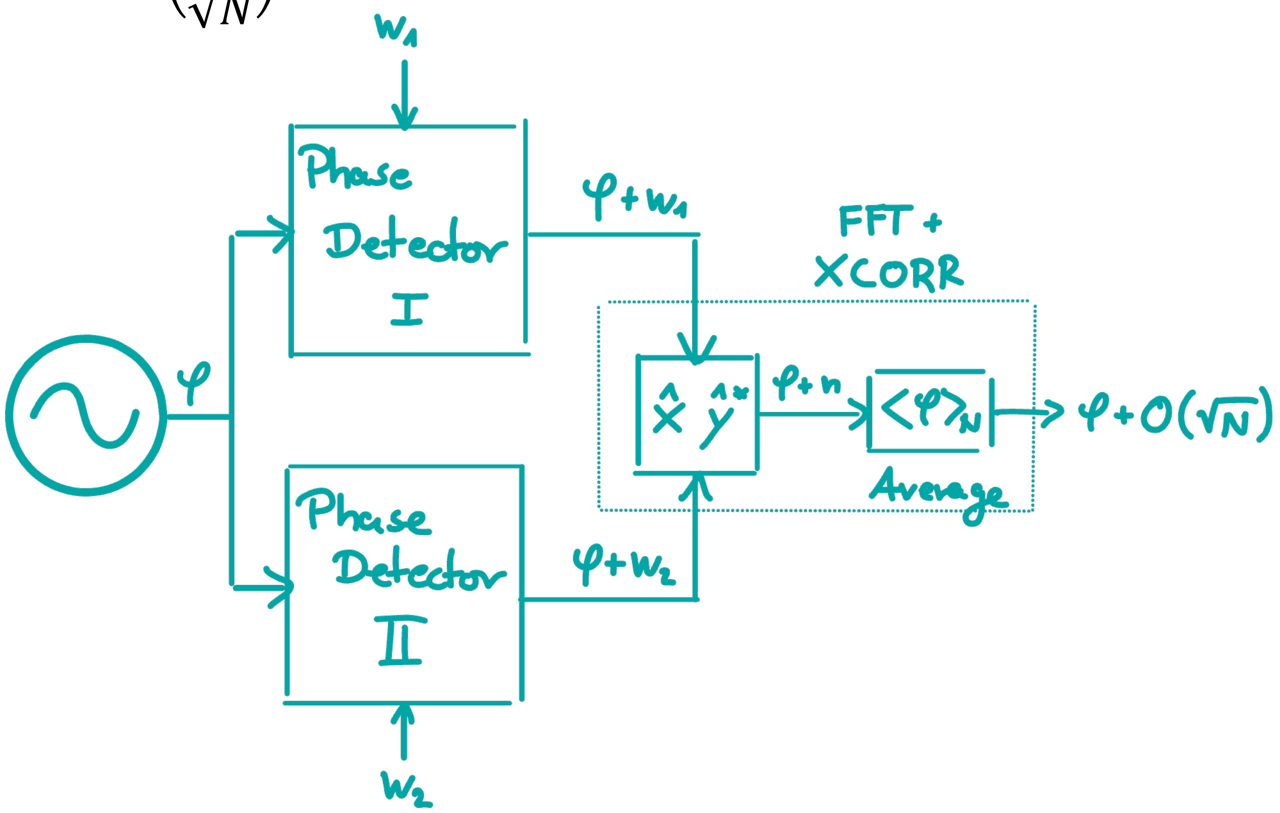
Improving Phase Noise Measurements

Cross-Correlation Techniques

- Cross-correlation is convolution in TD, i.e. multiplication in FD:

$$\mathrm{XCORR}(\varphi + w_1, \varphi + w_2\,) = (\varphi + w_1) \star (\varphi + w_2) = \bar{\hat{\varphi}} \cdot \hat{\varphi} + O\left(\frac{1}{\sqrt{N}}\right)$$

- Phase detector noise scales with $\frac{1}{\sqrt{N}}$
  - -3 dB @ N = 4
  - -15 dB @ N = 1024.
- Sample variance shrinks at the same time.
- Common noise not supressed (e.g. AM).
- Cross-spectral breakdown for phase inverted common noise!



Absolute, Residual and Additive Measurements

Absolute

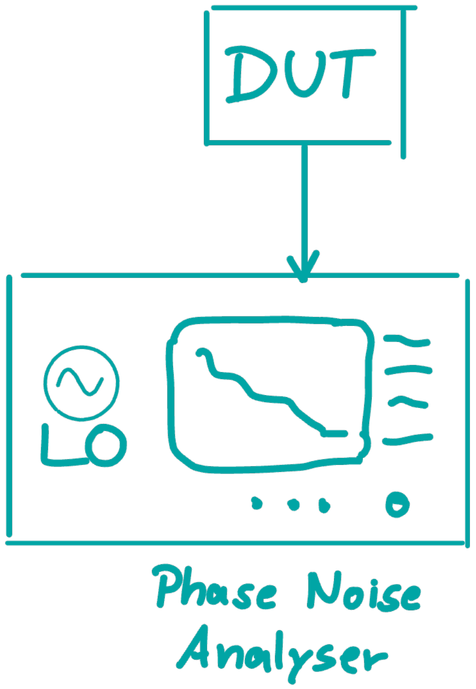
- Stability of an oscillator
- Analyser LO better than DUT (or XCORR)

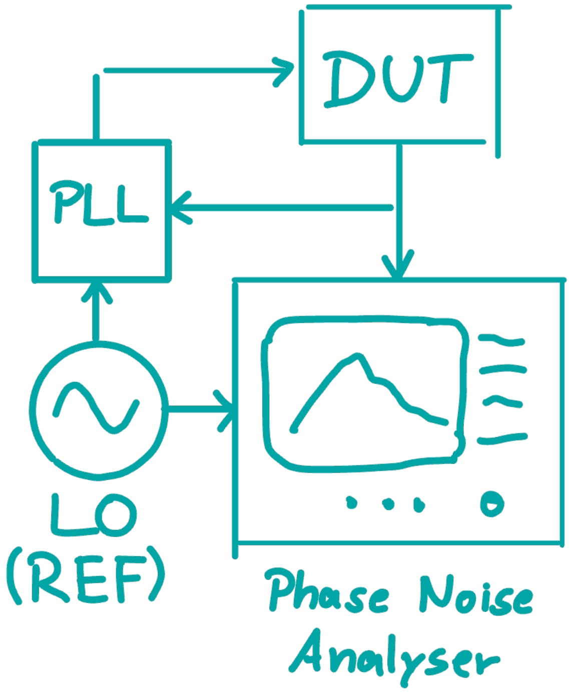
Residual

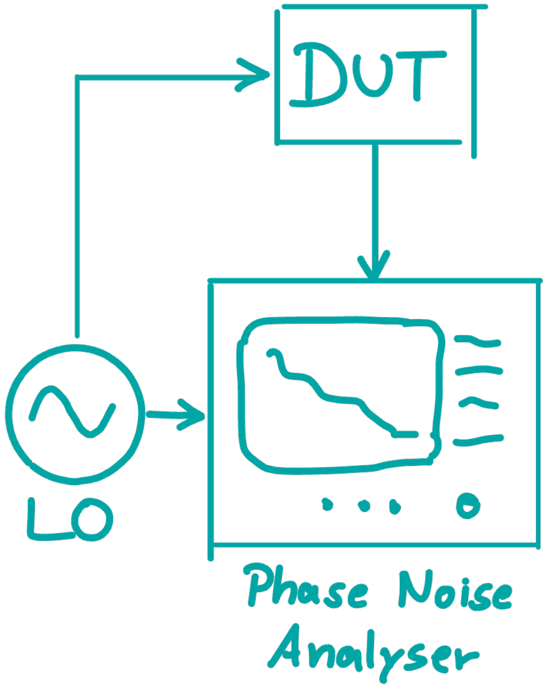
- Locking performance of PLL
- Reference as ext. analyser LO

Additive

- Excess noise of components
- Int. or ext. analyser LO





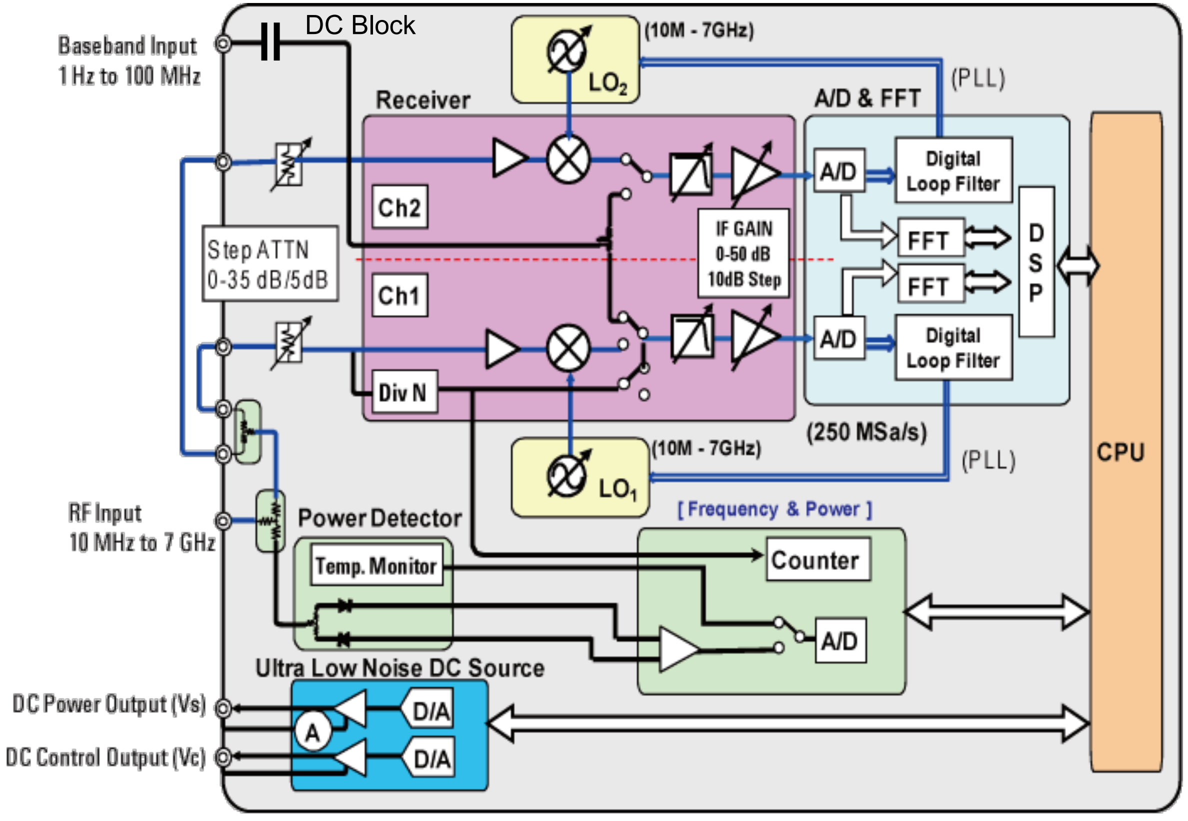


Phase Noise Analysers

Phase Noise Analysers

Agilent / Keysight E5052B

- Double saturated mixer scheme
- Found in many labs, weighs a ton.
- Important settings:
  - Attenuation (ideally $0\text{-}5\ \mathrm{dBm}$ after att.)
  - IF Gain (as high as possible without saturation error)
  - XCORR (double until PN doesn't improve anymore)
  - LO optimization, Capture range "normal"
- ±35V / 50 Ohm baseband input (DC block capacitor)
- Careful: Baseband shows V/Hz, but really is $\mathrm{V}/\sqrt{\mathrm{Hz}}$!
- (now superseded by SSA-X series)



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Phase Noise Analysers

Rohde & Schwarz FSWP / FSPN

- Double I/Q-detection scheme
- Non-zero IF leads to less distortion, no PLL
- Important settings:
  - Resolution Bandwidth (5%, 2%, 1% are common)
  - XCORR (aim for 10dB headroom to device sensitivity)
- Useful: Auto-Setup, fast, sensitivity indicator
- ±2V / 50 Ohm baseband input
- Pulsed measurements (bandwidth limited to $f_{\text{pulse}}/2$)

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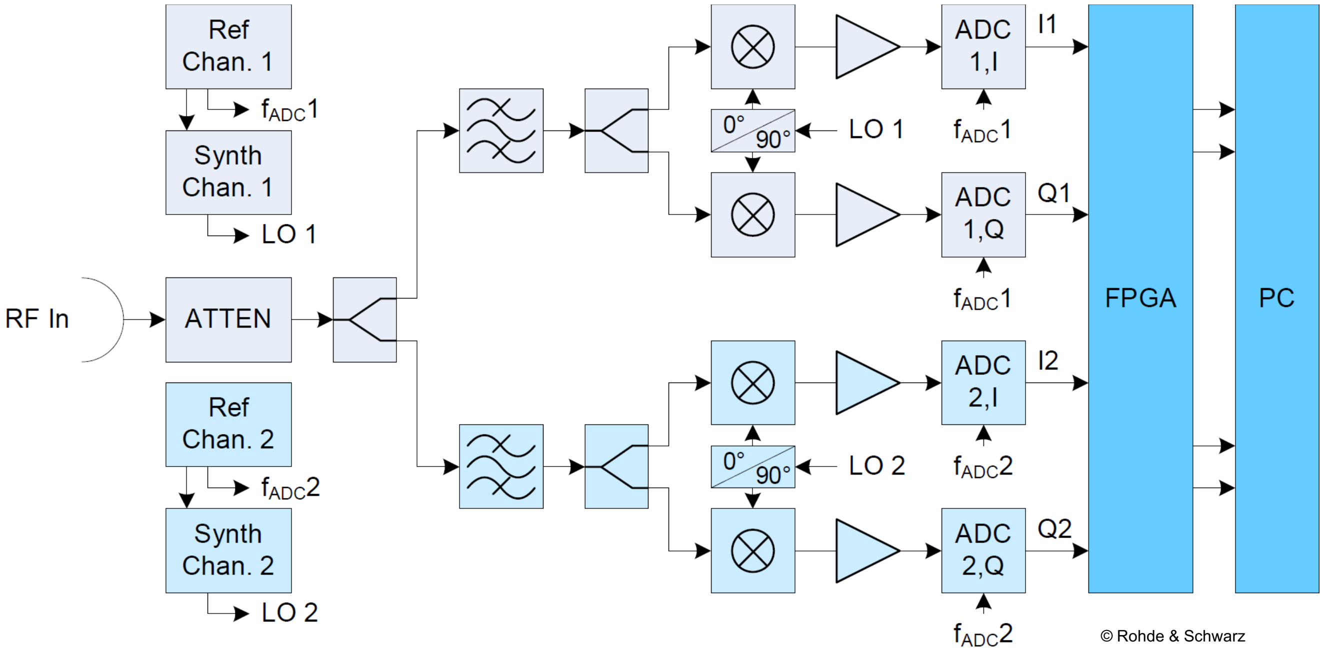

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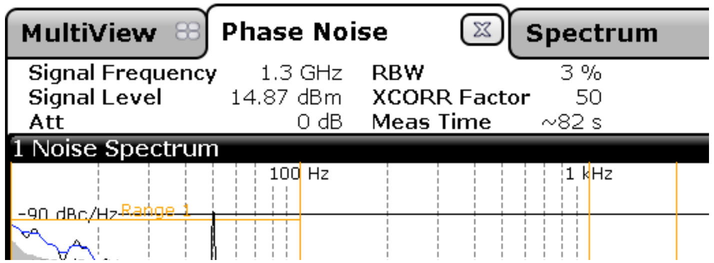
Practical Measurement Considerations

Proper Documentation is Key

A few minutes can save hours of work



- Measurements are worthless without measurement conditions – which you **will** forget.
- Keep screenshot, showing device settings.
- Store as many info as possible in trace file.
- A photo takes just two seconds.
- Use presets for consistent measurements.
- Note down external factors (PLL settings, optical power, LNA supply voltage, …).

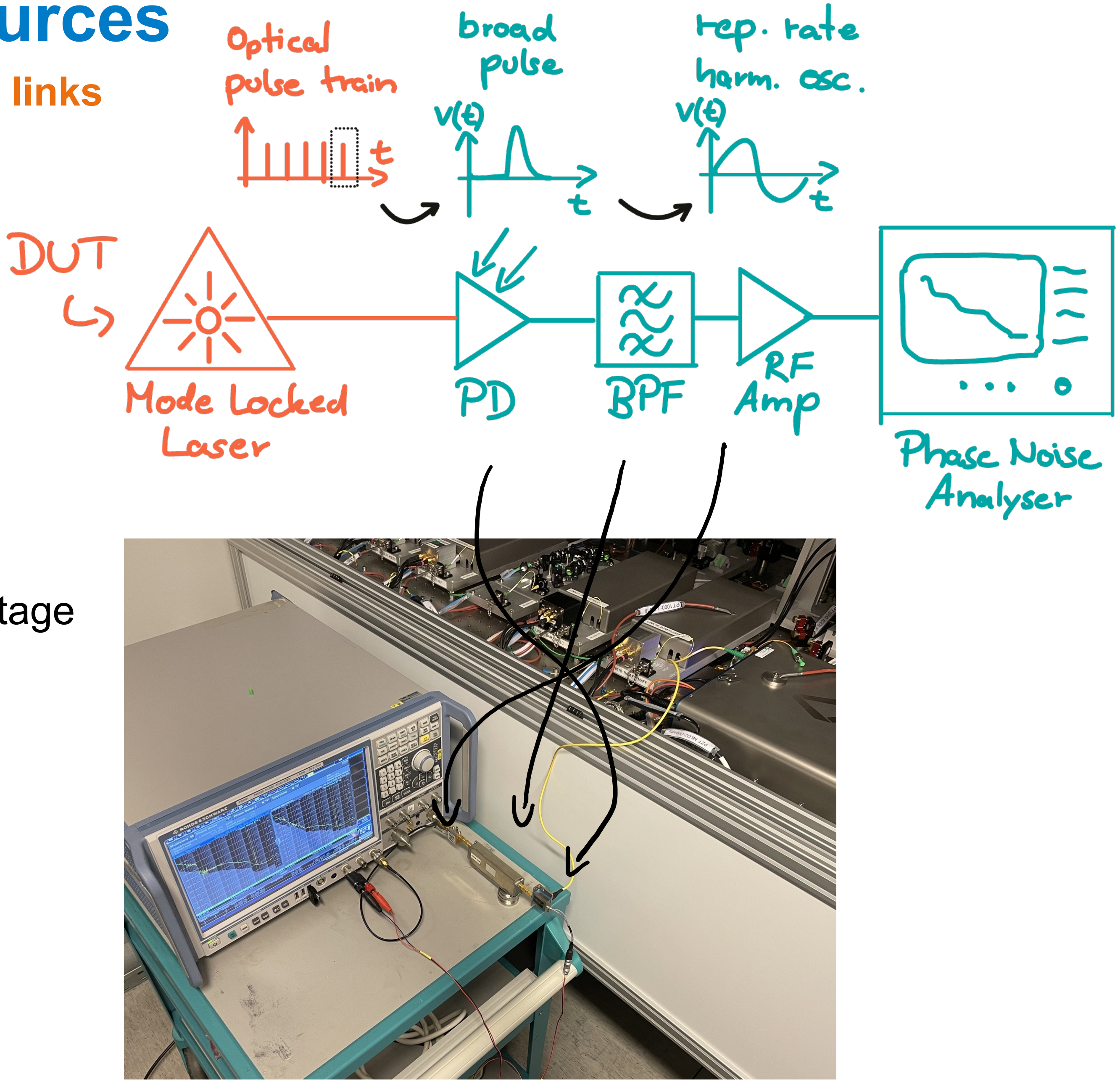
Beware:

- Ground-loops, unshielded cables, bad soldering, broken / unreliable components, sufficient signal power, power grid disturbances.

Measurements on Optical Sources

Mode-locked lasers & Pulsed optical fibre timing links

- Free-running MLL are “drifting sources”.
  → Generally unreliable below 10 Hz offset.
- Understand your detection chain:
  - Photodiode AM/PM dominates after ~ 1-10 kHz.
  - Amplifier excess noise
- 50 Ohm system, PD properly terminated?
- Photodiode bias voltage and amplifier supply voltage ideally from analyser (ground loops).
- Analyser itself can be the disturbance source!
  - Heat
  - EMI (Spurious)

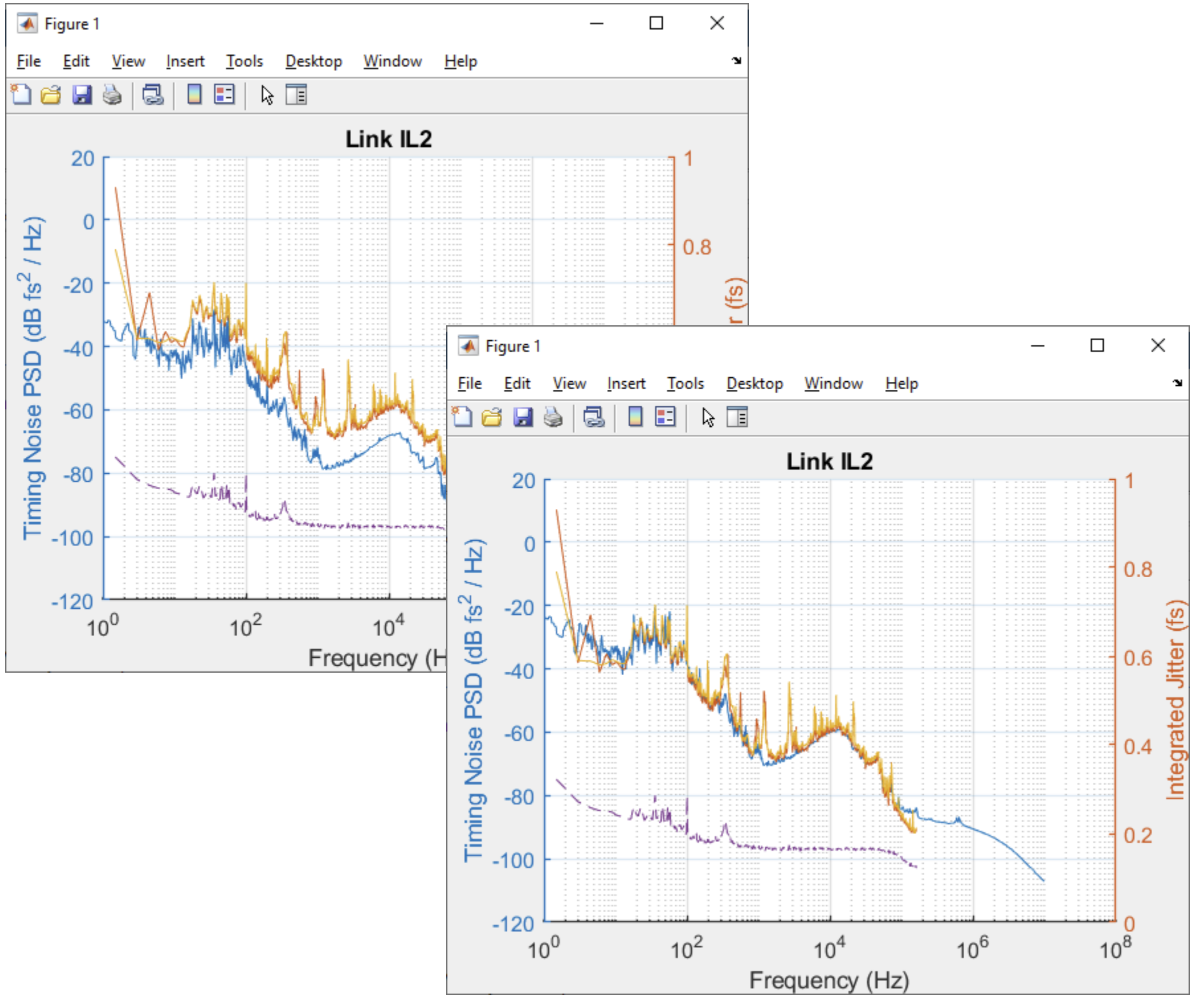


Baseband Measurements / External Phase Detectors

- Useful for:
  - Power supply noise (careful HV!)
  - External phase detectors
  - Relative intensity noise
- Standard units $\frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}}$ displayed in dB. But:

$$20\log_{10}\left[\frac{\mathrm{V}}{\sqrt{\mathrm{Hz}}}\right] = 10\log_{10}\left[\frac{\mathrm{V}^2}{\mathrm{Hz}}\right]$$

- Watch out for impedance mismatches!
- Correlation measurements only possible with certain devices.
- Ground-loops are critical! Use isolation transformer.
- Very sensitive, even to “swinging” cables.



Measuring at the Limit

Problems & Physical Boundaries

- Mechanical Stability

$$-180\ \mathrm{dB\ rad} \rightarrow 4 \cdot 10^{-12}\ \mathrm{m}$$

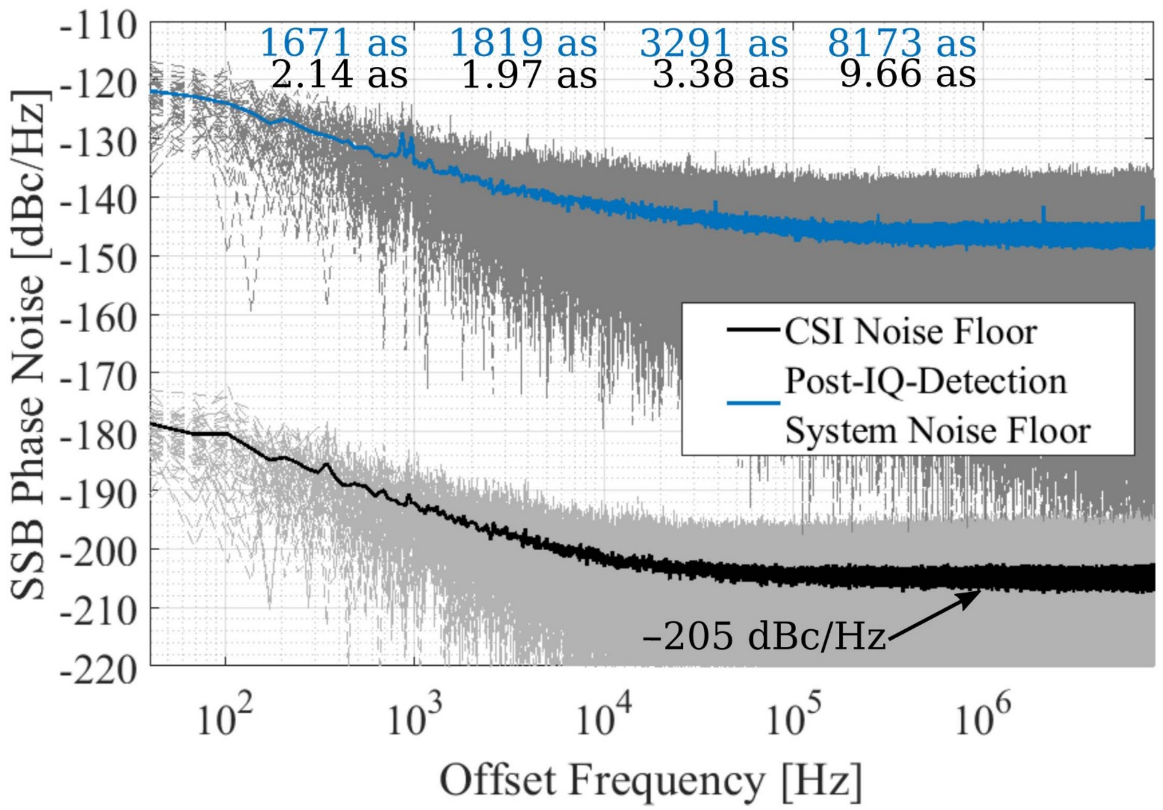
- Theoretical DUT thermal noise floor at (300K, > 1kHz offset from carrier)

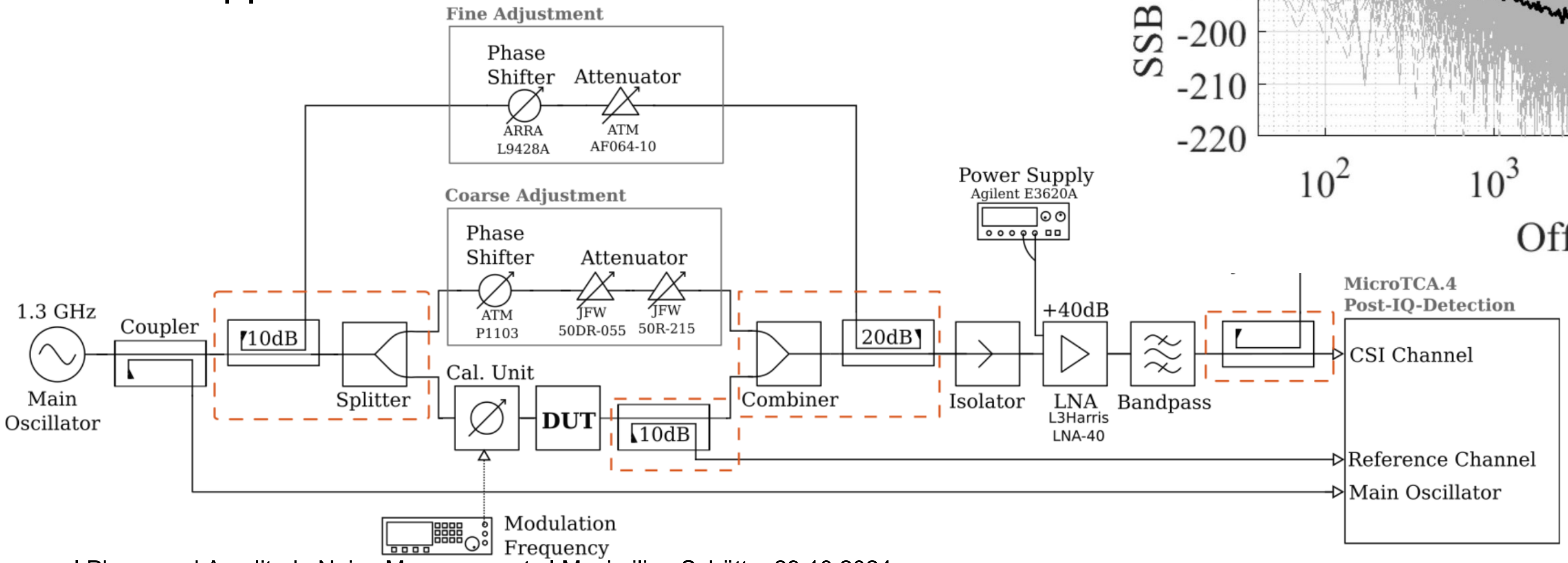
$$-177\,\frac{\mathrm{dBc}}{\mathrm{Hz}} - P_{\mathrm{carrier}}$$

- Cross-correlations do not help indefinitely
  (exponential measurement time, numerical problems, cross-spectrum collapse).
- “Close to the carrier”, analyser sensitivity is limited by RF component flicker in the receiver.
- Harmonic distortion during down-conversion & ADC sampling.
- Baseband is noisy!

Measuring at the Limit via Carrier Suppression Interferometry

- Mixing moves carrier power to DC
- Carrier suppression interferometry deletes carrier power
- No additive 1/f flicker when amplifying!
- Springer et al. - Phase Noise Measurements for L-Band Applications at Attosecond Resolution





References & Further Reading

For the serious phase noise enthusiast

Theory

- Demir – "Phase Noise in Oscillators A Unifying Theory and Numerical Methods for Characterisation" (1998)
- Rubiola – "Phase Noise and Frequency Stability in Oscillators", Cambridge University Press (2009) (and other works)
- Da Dalt – "Understanding Jitter and Phase Noise", Cambridge University Press (2018)
- Gardner – "Phaselock Techniques", Wiley, 3rd ed. (2005)

Practice

- Rubiola – Slides from rubiola.org
- Rohde & Schwarz – "Understanding Phase Noise Measuring Techniques"
- NIST – various resources and papers

Thank you!

Questions?

Contact

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- MSK LbSync Team: Sebastian Schulz, Thorsten Lamb, Matthias Felber, et al.