# Beam-Based Voltage Calibration for Double-Harmonic RF Systems in the CERN Super Proton Synchrotron

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## ♦ Abstract

Accurate knowledge of the voltage and phase in an RF cavity gap is essential to preserve beam quality and to achieve efficient, precise real-time correction with LLRF feedback. Voltage calibration using longitudinal phase-space tomography is a well-established beam-based technique that has demonstrated remarkable precision in determining the RF voltage experienced by a particle bunch. In a double-harmonic RF system, beam-based voltage calibration involves minimizing a four-dimensional parameter space that depends on the phase-voltage parameters of both RF systems. This process can be computationally challenging, and it is often more practical to perform a sequence of two-parameter voltage measurements, referencing the higher-order cavity system with respect to the main one. The Super Proton Synchrotron (SPS) at CERN is equipped with 200 MHz and 800 MHz cavity systems, that operate in phase at the bunch position for a non-accelerating bucket. In this context, the latest beam-based voltage calibration campaign conducted in the SPS will be presented, comparing different approaches.

## 1. Longitudinal tomography

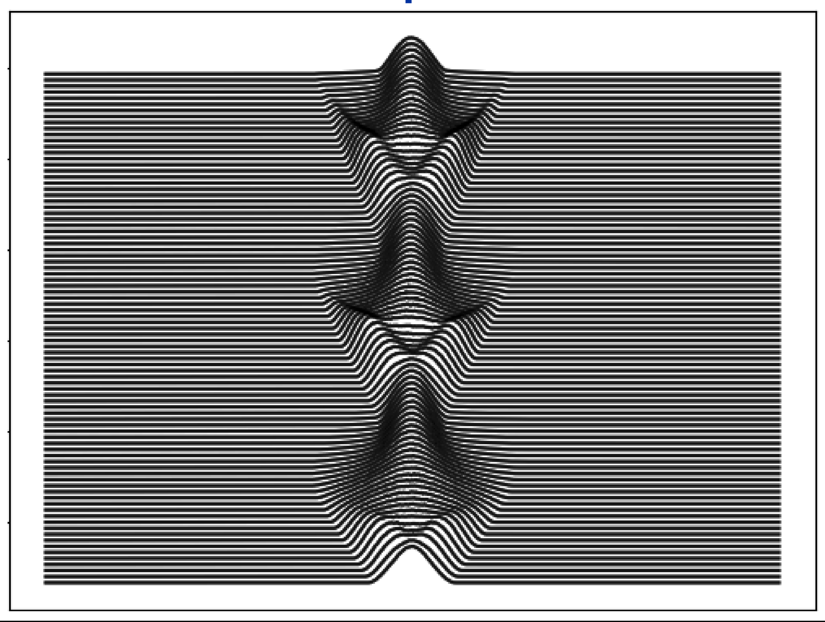
Voltage of the 200 MHz cavity system, $V_{200}$

Synchronous phase, $\phi_s$

Voltage of the 800 MHz cavity system, $V_{800}$

Relative phase between the two RF systems, $\Phi_2$

**Mountain range of longitudinal bunch profiles**



**Equation of the synchrotron motion in a double-harmonic RF system**

$$E_{n+1} = E_n + qV_{200}\left[\sin(\phi_n + \phi_s) + \frac{V_{800}}{V_{200}}\sin(r_h\phi_n + r_h\phi_s + \Phi_2)\right]$$

$$\phi_{n+1} = \phi_n + \frac{2\pi h\eta_0}{\beta_0^2 E_0}(E_{n+1} - E_0)$$

Energy of the particle

Ratio of harmonic numbers

Phase deviation of the particle

**Tomography algorithm [1]**

The tomography-based voltage calibration involves finding the optimal (effective) RF parameters $\hat{V}_{200}, \hat{V}_{800}, \hat{\phi}_s, \hat{\Phi}_2$, such that

$$\hat{V}_{200}, \hat{V}_{800}, \hat{\phi}_s, \hat{\Phi}_2 = \underset{V_{200}, V_{800}, \phi_s, \Phi_2}{\arg\min}\ D(V_{200}, V_{800}, \phi_s, \Phi_2)$$

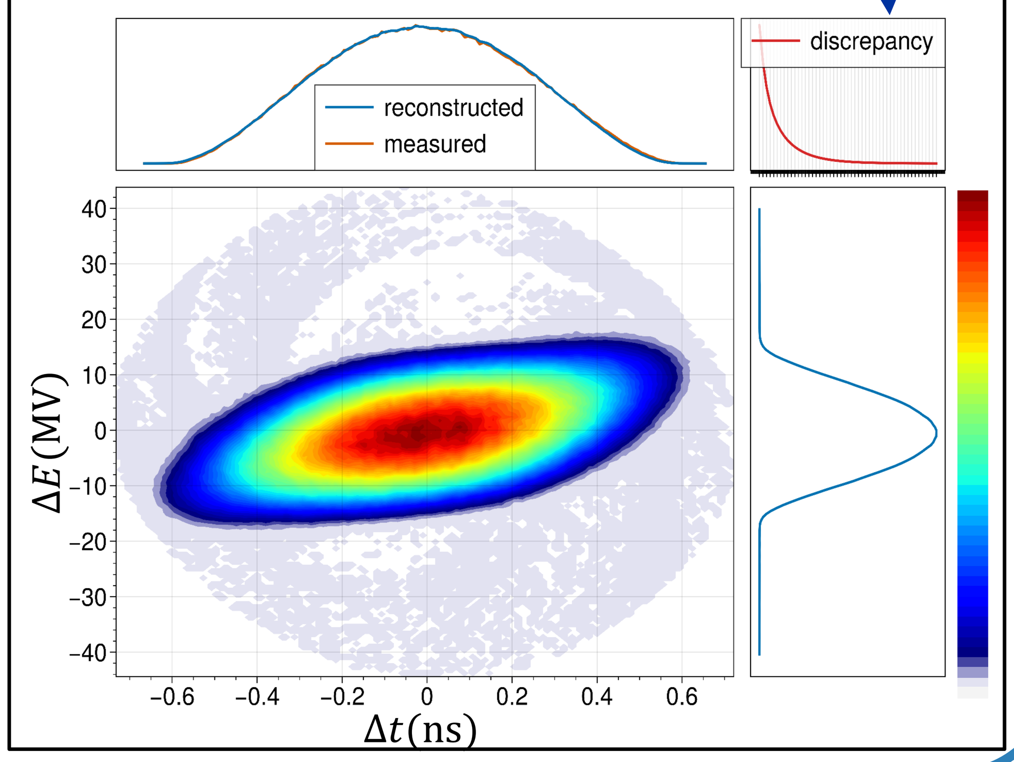
$$D = \sqrt{\frac{\sum_{i=1}^{f_r}\sum_{j=1}^{f_{\text{bin}}}|\Delta P_{ij}|}{N_{f_r} N_{\text{bin}}}}$$

Difference between the $i_{th}$ measured and reconstructed profiles at the $j_{th}$ bin

$N_{f_r}$: Number frames

$N_{\text{bin}}$: Number of bins

**Best reconstructed bunch profile**



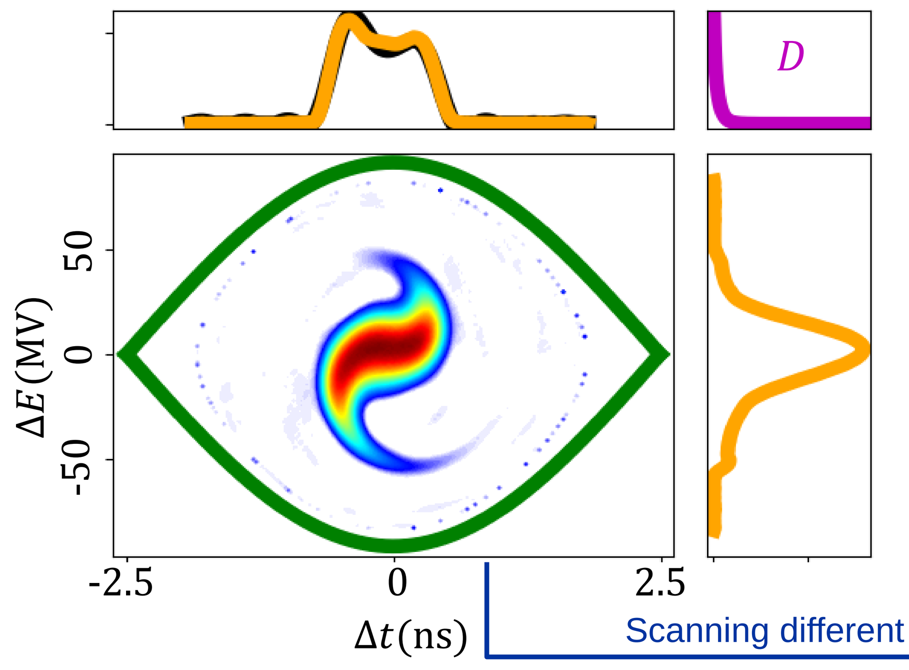

## 2. Voltage calibration of the 200 MHz cavity system

- $6 \times 200$ MHz traveling wave cavities (TWC) in total
- 800 MHz cavity system disabled → $V_{800} = 0$
→ Two independent variables

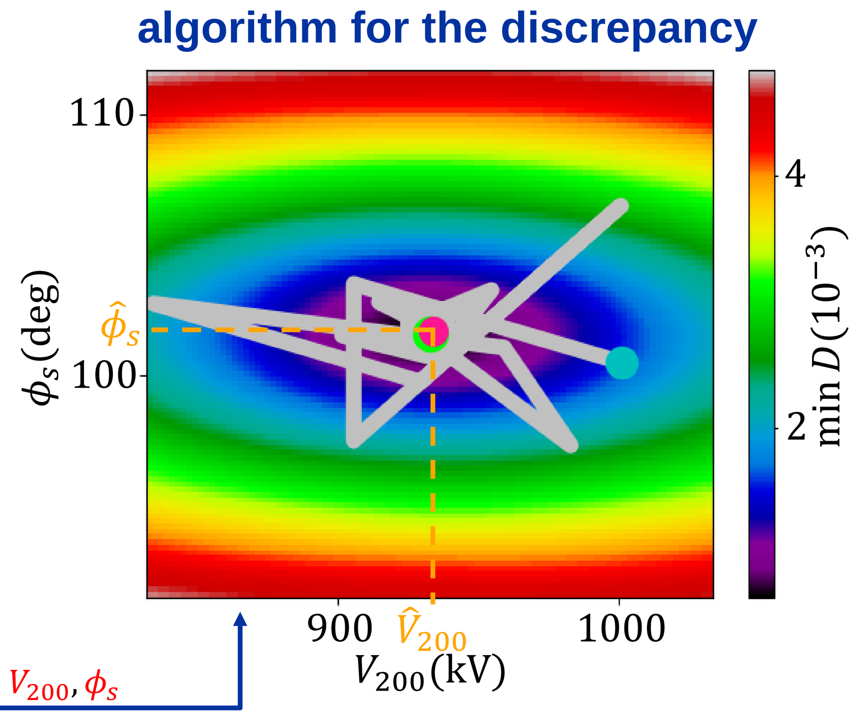
$$\hat{V}_{200}, \hat{\phi}_s = \underset{V_{200}, \phi_s}{\arg\min}\ D(V_{200}, \phi_s)$$

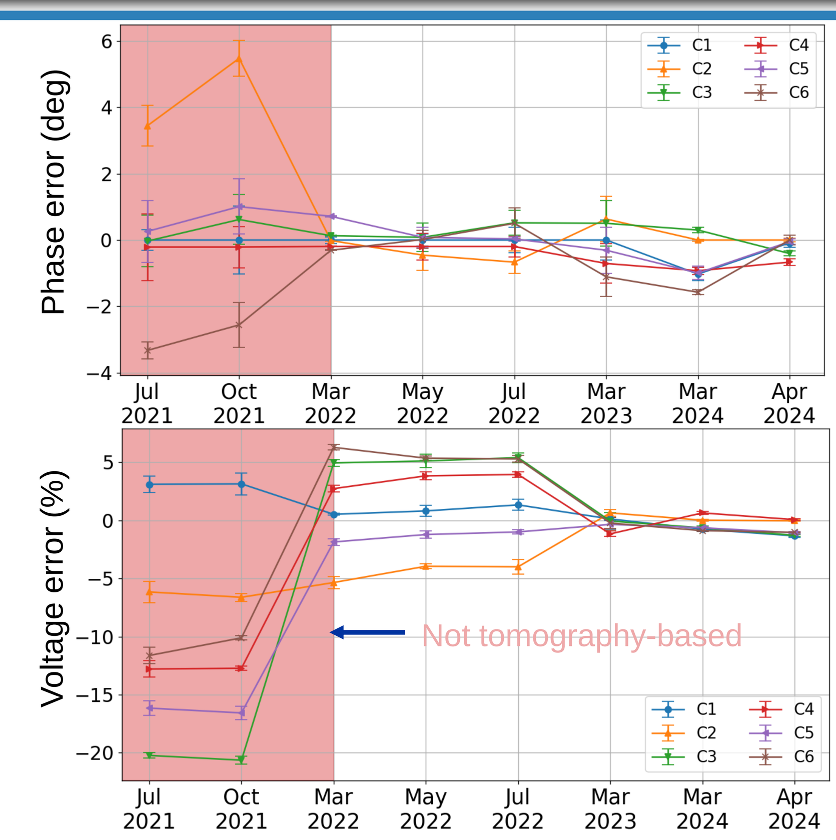
- Measurements of the longitudinal profiles were performed one cavity at a time to prevent the influence of imperfect phasing
- A mismatch between the bunch and the RF voltage is required (i.e., dipole or quadrupole oscillation)

**Example of tomography reconstruction in single-harmonic RF system**




**Calibration using a minimizer algorithm for the discrepancy**



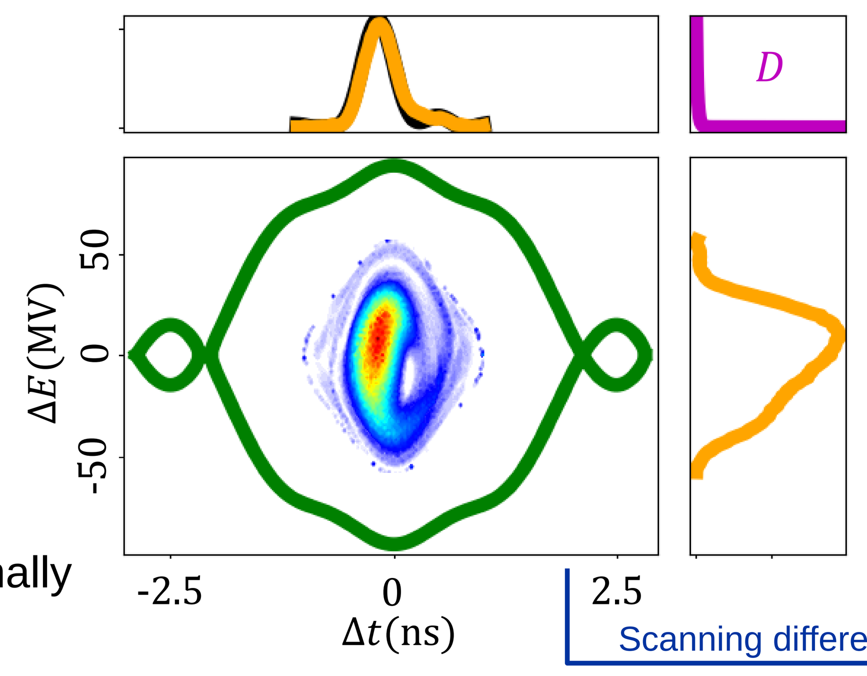

## 3. Voltage calibration in double-harmonic RF system

- 200 MHz and 800 MHz cavity system enabled
- 4th harmonic RF system consisting of 2 TWCs operating in counterphase with respect to the main RF system
→ Four independent variables

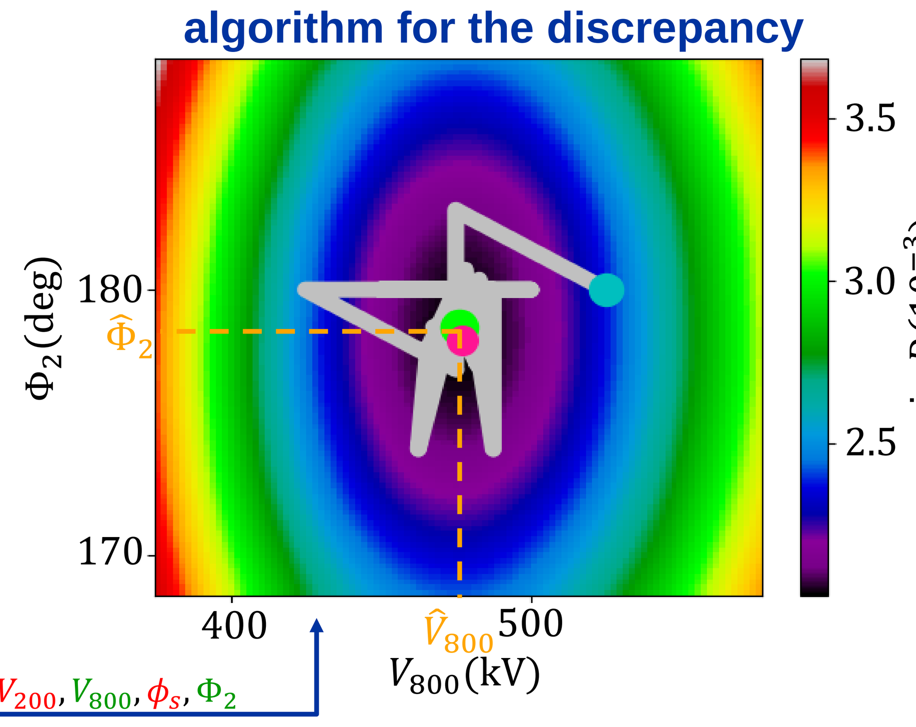
$$\hat{V}_{200}, \hat{V}_{800}, \hat{\phi}_s, \hat{\phi}_r = \underset{V_{200}, V_{800}, \phi_s, \Phi_2}{\arg\min}\ D(V_{200}, V_{800}, \phi_s, \Phi_2)$$

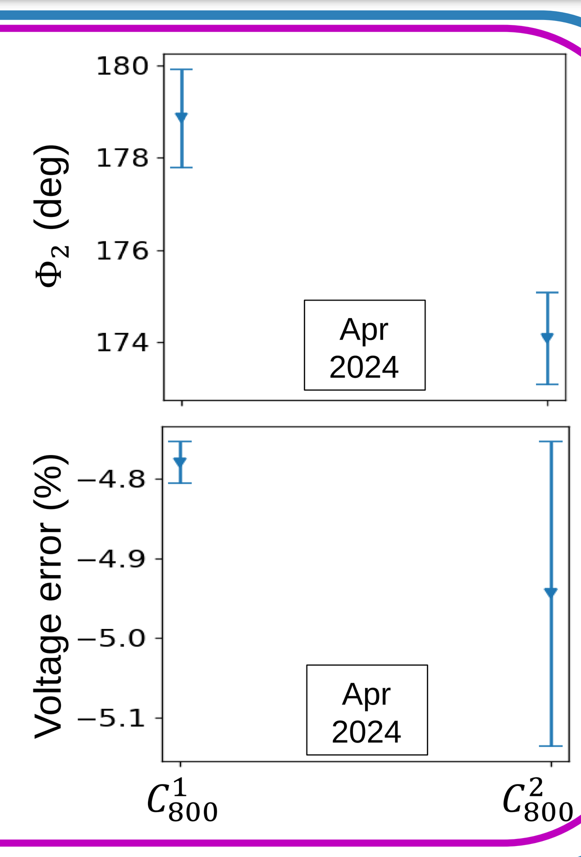
- Due to the nonlinearities of the 4th harmonic RF system, any longitudinal oscillations are rapidly suppressed (phase-mixing) → uncertainty on $V_{800}$ can be large
- Minimizing a 4D function can be computationally challenging

**Example of tomography reconstruction in double-harmonic RF system**




**Calibration using a 4-D minimizer algorithm for the discrepancy**

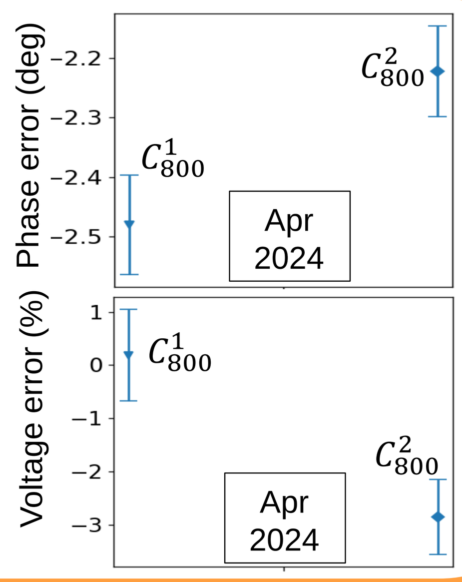



- 800 MHz cavity system as single harmonic RF system [2]
- Compute $\hat{V}_{200}$ and $\hat{\phi}_s$ a priori, and then perform the calibration for the 800 MHz cavities
→ Two independent variables

$$\hat{V}_{800}, \hat{\Phi}_2 = \underset{V_{800}, \Phi_2}{\arg\min}\ D(V_{800}, \Phi_2)$$

- The bucket of the 800 MHz can be relatively small leading to particle loss when the mismatch is introudced
→ Tomography reconstruction may not be accurate




## ♦ Conclusions

- Tomography-based calibration confirms remarkable precision and reproducibility
- For the 200 MHz cavity systems, the voltage and relative phase errors vary by less than 1% and 1 deg, respectively → not achievable with conventional RF measurements
- The 200 MHz cavity system can be treated independently if the uncertainty in $\hat{V}_{200}$ and $\hat{\phi}_s$ are relatively small → propagation of the error
- Alternatively, the double-harmonic RF system calibration can be performed by minimizing a 4-D discrepancy function
→ computationally demanding

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**References**

[1] C. H. Grindheim and S. Albright, "Longitudinal phase space tomography version 3," Tech. Rep. CERN-ACC-NOTE-2021-0004, Geneva Switzerland, 2021.

[2] D. Quartullo, S. Albright, H. Damerau, A. Lasheen, G. Papotti, and C. Zisou, "RF Voltage calibration using phase space tomography in the CERN SPS," 13th Int. Particle Acc. Conf. p. 844,TUPOST005, Bangkok, Thailand, 2022. doi:10.18429/JACoW-IPAC2022-TUPOST005.