Solving Beautiful Puzzles

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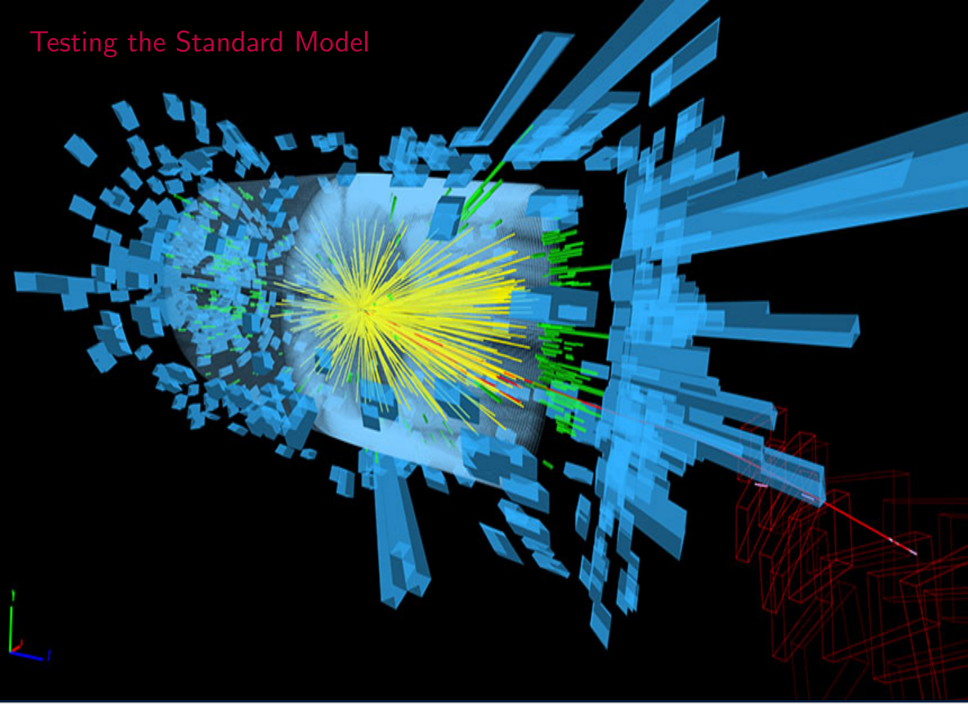
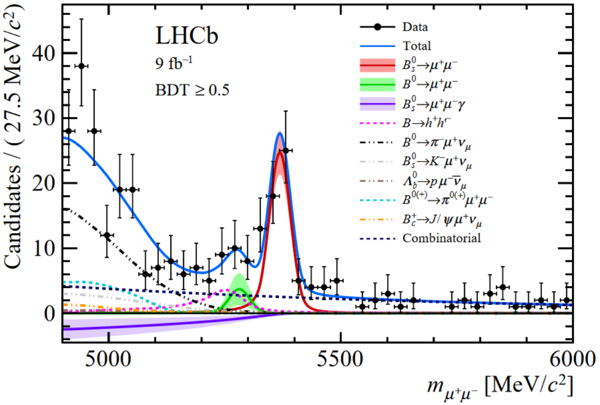
= Laboratori Nazionali di Frascati 2024 =

Testing the Standard Model



Testing the Standard Model: Indirect

LHCb Collaboration [Phys. Rev. Lett. 128, (2022) 041801]



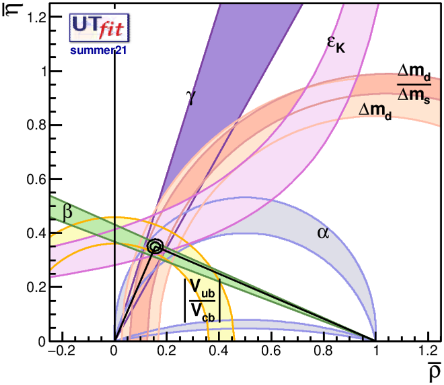
Precision frontier

Tiny deviations from SM predictions constrain effects of New Physics

The Flavour Puzzle

Thanks to Marcella Bona for providing the 2021 plots

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



Huge amounts of data + theory advances = Precision frontier

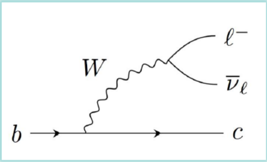
Tiny deviations from SM predictions constrain effects of New Physics

SM or beyond?

Challenge:

Disentangle SM long-distances effects from the effects of new interactions

Quark level process



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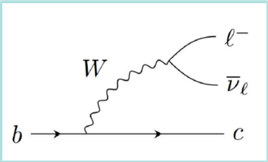
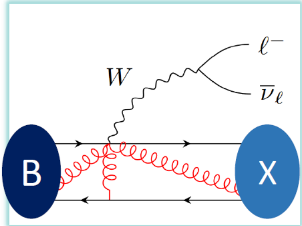


hadronization

Reality: Bound state



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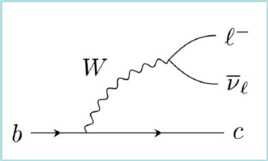

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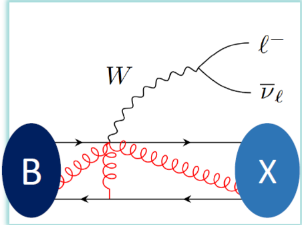

- Reliable theory uncertainties are essential!

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- Look for the cleanest observables/methods

SM or beyond?

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Disentangle SM long-distances effects from the effects of new interactions

- Reliable theory uncertainties are essential!
- Look for the cleanest observables/methods
- Some anomalies already spotted

Challenge:

Disentangle SM long-dist effects fr e effects ctions

Puzzles in semileptonic decays

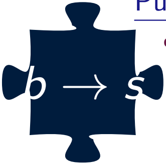
- Inclusive versus Exclusive
- V_{cb} and V_{ub}
- LFUV in R_D and R_{D*}

Puzzles in nonleptonic decays

- Missing CP violation
- $B \to \pi K$ puzzle
- $B \to D\pi$ puzzle

Puzzles in rare decays

- Anomalies in $b \to s\ell\ell$



Inclusive versus Exclusive Decays

The Beauty of Semileptonic Decays

Motivation:

- Theoretically relatively easy to describe: factorization of strong interaction effects

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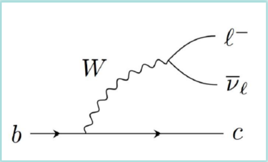

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Two options:

- Exclusive decays: pick one final state with the desired quarks ($V_{cb} \to D^{(*)}$ and $V_{ub} \to \pi$)
- Inclusive decays: everything you can think of! (denoted with X_c or X_u)

The Beauty of Semileptonic Decays

Motivation:

- Theoretically relatively easy to describe: factorization of strong interaction effects

Challenge:

- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
  - Calculable: Lattice or Light-cone sumrules = Exclusive Decays
  - Measurable: from data = Inclusive Decays

Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established framework
- Extract important CKM parameters $|V_{cb}|, |V_{ub}|$ (and $|V_{cs}$?)
- Extract power corrections from data
- Cross check of exclusive decays

The Heavy Quark Expansion

Inclusive Decays = Heavy Quark Expansion

- b quark mass is large compared to $\Lambda_{\rm QCD}$
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem → (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B\right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B| \ldots |B\rangle$ non-perturbative matrix elements → string of iD
- operators contain chains of covariant derivatives

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- Standard tool for inclusive $B \to X_c \ell \nu$ decays

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HQE elements: $\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\ldots(iD_{\mu_n})b_v|B\rangle$

- Extracted from kinematic moments of the data
- Ideas for the lattice Juetner et al. [202305.14092]

HQE parameters

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD,iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD,iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Powercounting in the HQE

I: $\boldsymbol{m_Q \sim m_q \gg \Lambda_{\rm QCD}}$ OPE for $b \to c\ell\bar{\nu}$

- q is treated as a heavy degree of freedom
- two-quarks operators: $\bar{Q}_v(iD^\alpha \cdots iD^\beta)Q_v$
- IR sensitivity to mass m_q

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8\log\rho + \ldots\right]\frac{\rho_D^3}{m_Q^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

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II: $m_Q \gg m_q \gg \Lambda_{\rm QCD}$ start with q dynamical

- four-quark operators $(\bar{Q}_v\Gamma q)(q\bar{\Gamma}Q_v)$

$\to$ removed when matching onto two-quark operators

- RGE running gives $\log(m_q/m_Q)$

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III: $\boldsymbol{m_Q \gg m_q \sim \Lambda_{\rm QCD}}$ OPE for $c \to s\ell\bar{\nu}$ Fael, Mannel, KKV [1910.05234]

- q dynamical degree of freedom
- four-quark operators remain in OPE
- no explicit $\log(m_q/m_Q)$: hidden inside new non-perturbative HQE parameters

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IV: $m_Q \gg \Lambda_{\rm QCD} \gg m_q$ for $b \to u$ and $c \to d$ transitions

Powercounting in the HQE

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III and IV have four-quark (weak annihilation) effects

Weak Annihiliation

Uraltsev, Bigi, Voloshin, Mannel, Turczyk; Ligeti, Luke, Manohar, Phys. Rev.D82 (2010) 033003
Gambino, Kamenik, Nucl.Phys.B840 (2010) 424

- IR sensitivity to light quark gives additional four-quark non-pert. parameters:

$$\langle B|(\bar{b}_v\gamma^\nu P_L q)(\bar{q}\gamma^\mu P_L b_v)|B\rangle = 2M_B\left[T_1(\mu)g^{\mu\nu} + T_2(\mu)v^\mu v^\nu\right]$$

- Starting at $\mathcal{O}(1/m_b^3)$ and mix with ρ_D^3 under renormalization
- Challenging to study non-perturbatively
- Very important to achieve precise $B \to X_{d,s}\ell\ell$ predictions
  Hurth, Huber, Lunghi, Jenkins, Qin, KKV [2007.04191,2404.03517]

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- Can be obtained from D and D_s semileptonics using HQE?

$$B_{WA}^{bq} = \frac{m_B f_B^2}{m_D f_D^2} B_{WA}^{cq}$$

- Effect is $(m_b/m_c)^3$ enhanced compared to B decays

Heavy quark expansion for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb ...
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin]

  $\rightarrow$ V_{ub}

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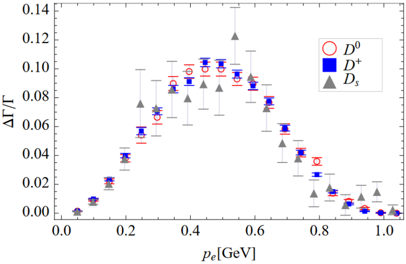
Challenges:

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

Extracting weak annihilation from data

CLEO data, Gambino, Kamenik [1004.0114]



- Lepton energy moments extracted from spectrum
- Kinetic mass for charm at $\mu = 0.5$ GeV threshold, HQE parameters as input
- Max 2% weak annihilation (WA) contribution to $B \to X_u \ell \nu$

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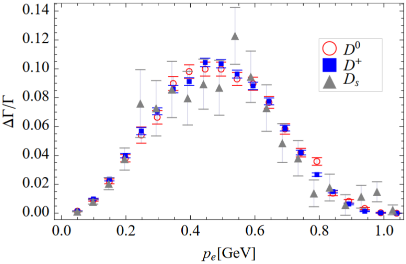

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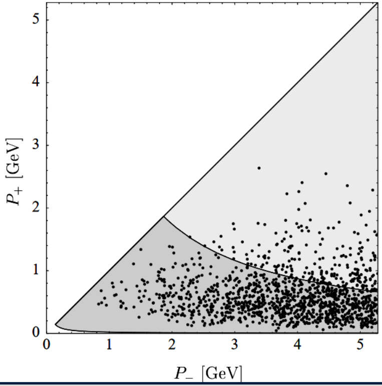
- In progress: Feasibility study to measure q^2 moments at BESIII Bernlochner, Gilman, Malde, Prim, KKV, Wilkinson

Inclusive $B \to X_u$ semileptonic decays

Modified Heavy Quark Expansion

- Cuts needed to suppress large charm background
- Pushes towards specific corner of the phase space
  - Local OPE as in $b \to c$ cannot work
  - Sensitivity to b-quark wave function properties (Fermi motion)
  - Deal with energetic light degrees of freedom
  - More than two scales involved!
- Expansion parameter $\Lambda_{\rm QCD}/(m_b - 2E_\ell)$
- Use light-cone OPE with light-cone directions n and $\bar{n}$



Factorization of scales

- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

  $\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
  $\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
  $\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions

Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v |B(v)\rangle$$

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$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v |B(v)\rangle$$

  - Universal
  - Similar to parton distribution in DIS

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- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n \cdot D)) b_v |B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega \theta(m_b(1-y) - \omega) f(\omega)$$

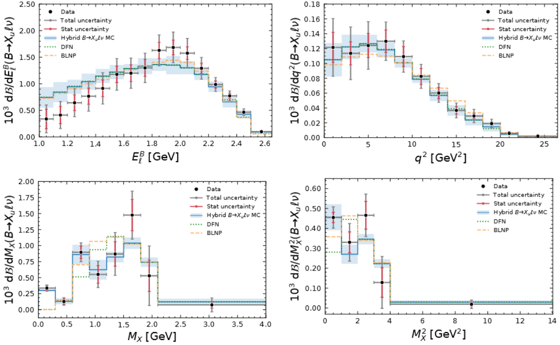
- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative and cannot be computed

Shape function parametrization

Differential spectra from BelleII [2107.13855]



- Often linked to $B \to X_s \gamma$
- Updated experimental measurements could constrain SFs further

Current status: inclusive V_{ub}

Belle [2102.00020]

Different frameworks for inclusive $B \to X_u$:

- BLNP: Bosch, Lange, Neubert, Paz uses Soft Collinear Effective Theory (SCET)
- GGOU Gambino, Giordano, Ossola, Uraltsev
  - OPE with hard-cutoff
  - No subleading SFs

Approaches to calculate the SF perturbatively:

- DGE: Dressed Gluon Exponentiation Andersen, Gardi
- ADFR Aglietti, Di Lodovico, Ferrerar, Ricciardi

Average of all four approaches:

$$|V_{ub}|_{\text{incl}} = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\tau_B \delta\Gamma(B \to X_u \ell\nu)}} = (4.10 \pm 0.28) \cdot 10^{-3}$$

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Exclusive world average: $|V_{ub}|_{\rm excl} = (3.44 \pm 0.12) \cdot 10^{-3}$

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Inclusive determinations need to be scrutinized

BLNP update - in progress -

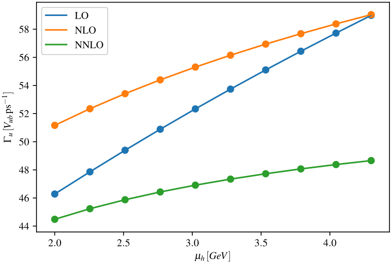
Bosch, Lange, Neubert, Paz [2005]
Greub, Neubert, Pecjak [0909.1609]; Beneke, Huber, Li [0810.1230]; Becher, Neubert [2005]

Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- In progress: include known α_s^2 corrections

Shape function parametrization

Preliminary! Olschewsky, Lange, Mannel, KKV [240x.xxxx]



- α_s^2 corrections give large corrections [see also Pezcjak 2019]
- Required to make precision predictions

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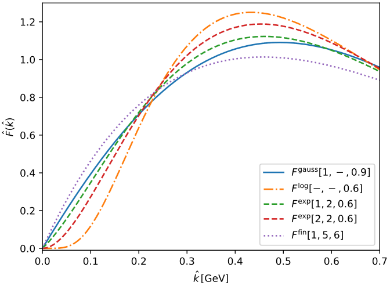
Update of BLNP approach

- Systematic framework: Soft Collinear Effective Theory (SCET)
- In progress: include known α_s^2 corrections
- Moments of shape functions can be linked to HQE parameters in $b \to c$
  - In progress: include higher-moments
  - kinetic mass scheme as in $b \to c$
- Shape function is non-perturbative and cannot be computed
  - In progress: new flexible parametrization

Shape function parametrization

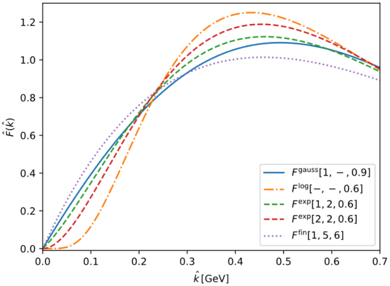
Olschewsky, Lange, Mannel, KKV [240x.xxxx]



- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

Shape function parametrization

Olschewsky, Lange, Mannel, KKV [240x.xxxx]



- All moments of shape functions are linked to HQE parameters
- Allows for a range of different shapes $\rightarrow$ systematic uncertainty

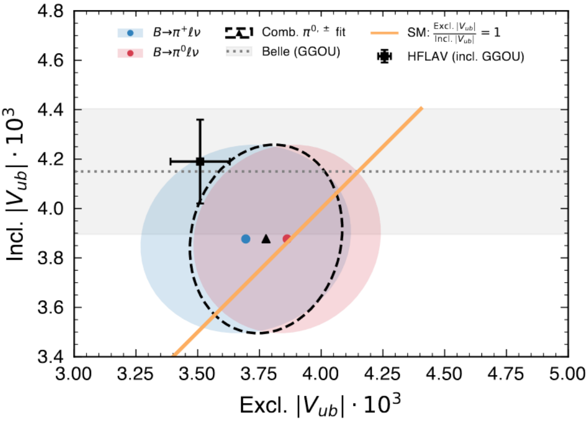
In progress:

Lange, Mannel, Olschewsky, KKV [in progress]

$$|V_{ub}|_{\text{incl}} = \quad \text{Stay Tuned!}$$

Inclusive versus Exclusive

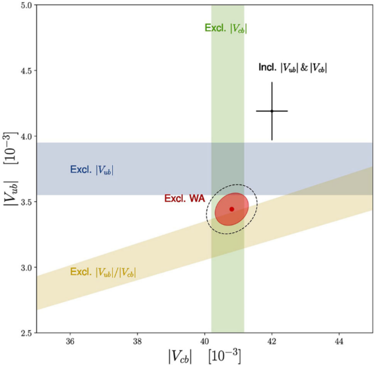
BelleII [2303.17309]



- First simultaneous measurement
- Experimental advantages due to common backgrounds and modeling

Current status $|V_{xb}|$ puzzles

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w



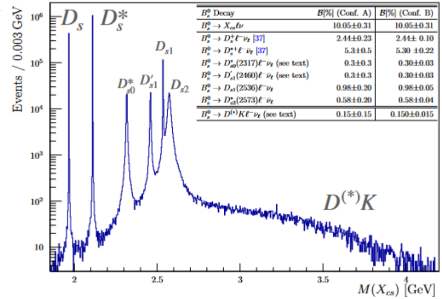
- Includes also ratio measurements of $|V_{ub}/V_{cb}|$ from LHCb (B_s and Λ)

Inclusive Measurements at LHCb?

Inclusive B_s decays?

De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael

First study of the possiblities using sum-over-exclusive technique



B_s^0 Decay	$\mathcal{B}$[%] (Conf. A)	$\mathcal{B}$[%] (Conf. B)
$B_s^0 \to X_{cs}\ell\nu$	10.05±0.31	10.05±0.31
$B_s^0 \to D_s^+\ell^-\nu_\ell$ [37]	2.44±0.23	2.44± 0.10
$B_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [37]	5.3±0.5	5.30 ±0.22
$B_s^0 \to D_{s0}^*(2317)\ell^-\nu_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D'_{s1}(2460)\ell^-\bar{\nu}_\ell$ (see text)	0.3±0.3	0.30±0.03
$B_s^0 \to D_{s1}(2536)\ell^-\nu_\ell$	0.98±0.20	0.98±0.05
$B_s^0 \to D_{s2}^*(2573)\ell^-\bar{\nu}_\ell$	0.58±0.20	0.58±0.04
$B_s^0 \to D^{(*)}K\ell^-\nu_\ell$ (see text)	0.15±0.15	0.150±0.015

- B_s spectrum well-separated
- Only M_X^2 moments available
- Study $SU(3)$ breaking of HQE

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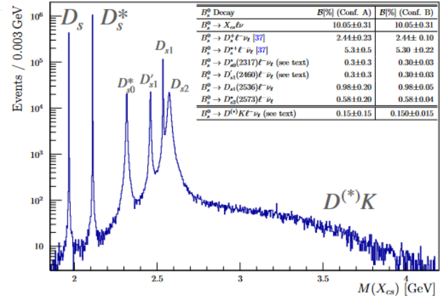

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- B_s spectrum well-separated
- Only M_X^2 moments available
- Study $SU(3)$ breaking of HQE

- Improve knowledge D_s^{**} states
- Understand non-resonant contribution
- $|V_{cb}|$ extraction requires Branching ratio from Belle II!

Inclusive $B \to X_s \ell\ell$ decays

Hurth, Huber, Lunghi, Jenkins, Qin, KKV [2007.04191, 2404.03517]

Set up an OPE as for $B \to X_u$

- Power-corrections give large uncertainties
- Normalizing to $B \to X_u$ may reduce uncertainty:

$$\mathcal{R}(q_0^2) = \int_{q_0^2}^{M_B^2} dq^2 \frac{d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2} \Bigg/ \int_{q_0^2}^{M_B^2} dq^2 \frac{d\Gamma(\bar{B} \to X_u \ell \bar{\nu})}{dq^2}$$

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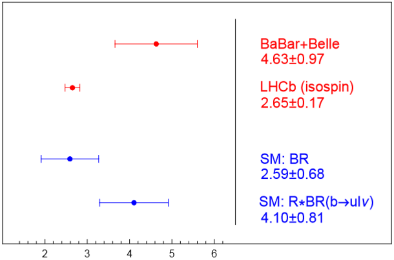

$\mathrm{BR}(B \to X_s \ell^+ \ell^-)[> 15] \times 10^7$

- At high-q^2 $X_s = K, K^*, K\pi, \ldots$
- Use sum-over-exclusives from LHCb measurements!

Inclusive $B \to X_s \ell\ell$ decays

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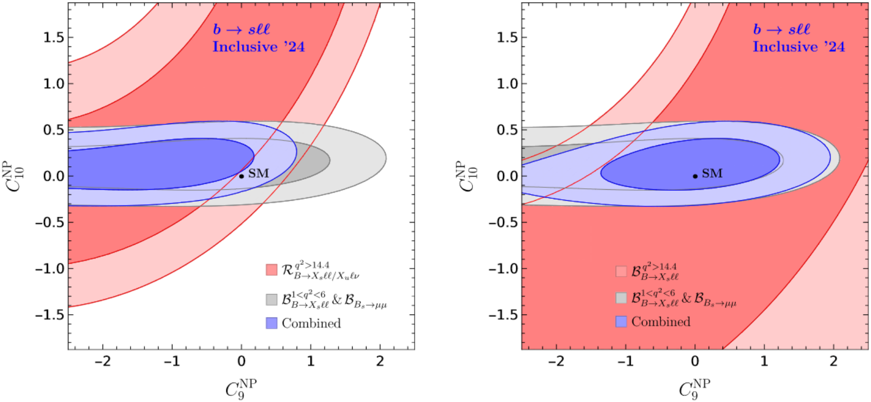
Important to cross-check the exclusive channels!



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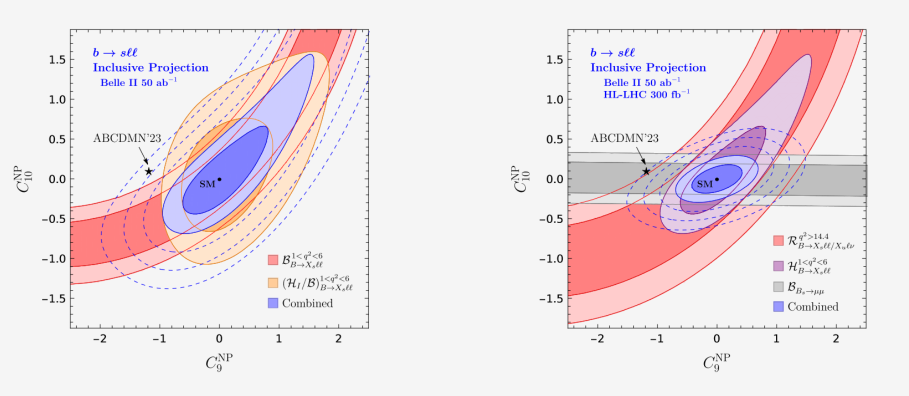

Outlook: Ratio of inclusive V_{ub}/V_{cb}

See Gambino , Giordano [0805.0271]

- New measurements by Belle [2311.00458]
- New! α_s^3 corrections for $b \to u$ Fael, Usovitsch [2310.03685]

- We can predict the $B \to X\ell\nu$ rate in local OPE

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- New measurements by Belle [2311.00458]
- New! α_s^3 corrections for $b \to u$ Fael, Usovitsch [2310.03685]
- In progress: Direct calculation of the ratio

$$C \equiv \left|\frac{V_{cb}}{V_{ub}}\right|^2 \frac{\mathcal{B}(B \to X_u \ell\nu)}{\mathcal{B}(B \to X_c \ell\nu)}$$

- Either in shapefunction region or in local OPE (see also Mannel, Rahimi, KKV [2105.02163])
- We can predict the $B \to X\ell\nu$ rate in local OPE

Solving beautiful puzzles

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- Need to revise previous assumptions and ensure reliable systematic uncertainties

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Close collaboration between theory and experiment necessary!

Backup

What mass to use?

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_Q(\mu)^{\rm kin} = m_Q^{\rm Pole} - \left[\overline{\Lambda}\right]_{\rm pert} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\rm pert} + \dots$$

$$[\overline{\Lambda}]_{\rm pert} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi} \mu \qquad [\mu_\pi^2]_{\rm pert} = C_F \frac{\alpha_s(m_c)}{\pi} \mu^2$$

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- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\Lambda_{\rm QCD} < \mu < m_Q$: expansion parameters μ/m_Q
  - Well established for m_B: $\mu/m_B \simeq 0.2$
  - Charm??
    - $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$
    - $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

Challenge: $\mu = 0.5$ GeV touches upon the non-perturbative regime?

Ratios of V_{cb} and V_{ub}: a B_s puzzle

Bolognani, van Dyk, KKV [2308.0437]
LHCb [2012.05143], Khodjamirian, Rusov [2017]

- Also $B_s \to K\mu\nu$ is sensitive to $|V_{ub}|$
- Only accessible at LHCb, but normalization needed
- Using $B \to D\mu\nu$ gives access to the ratio

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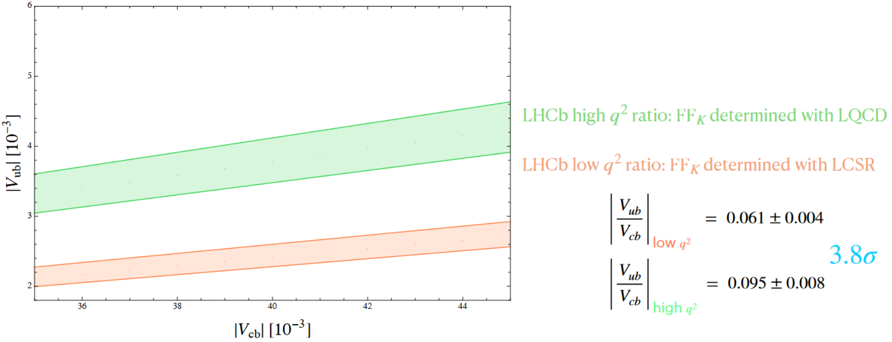

A puzzle in B_s decays?

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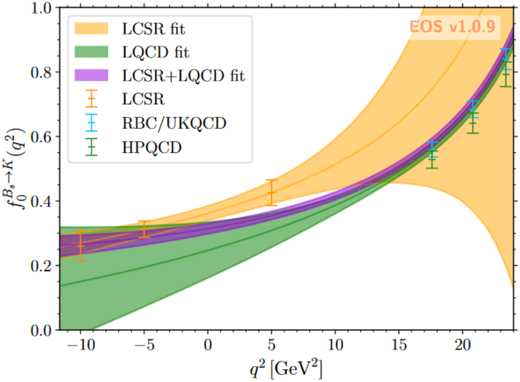
- Recent update: New form factor predictions combining lattice and light-cone sumrule information



A puzzle in B_s decays?

Bolognani, van Dyk, KKV [2308.0437]
LHCb [2012.05143], Khodjamirian, Rusov [2017]

- Recent update: New form factor predictions combining lattice and light-cone sumrule information
- Puzzle becomes less: 1.9σ difference

$$q^2 < 7\mathrm{GeV}^2 \rightarrow \frac{|V_{ub}|}{|V_{cb}|} = 0.0681 \pm 0.004 \qquad q^2 > 7\mathrm{GeV}^2 \rightarrow \frac{|V_{ub}|}{|V_{cb}|} = 0.0801 \pm 0.005$$

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344 Boushmelev, Mannel, KKV [2301.05607]

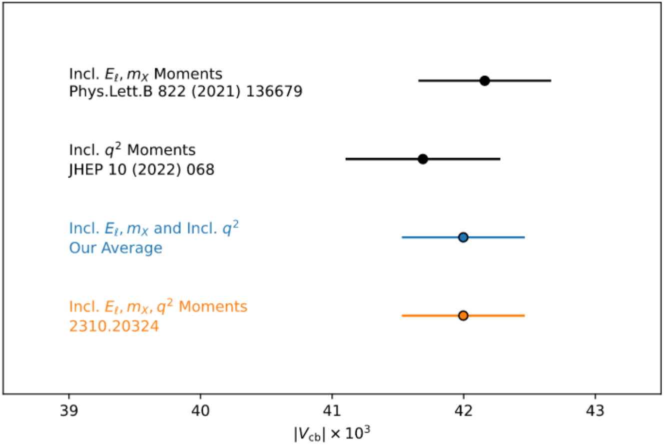
- m_c not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Replace m_c by moments of the spectral density!
- First study shows small improvement in pert. series
- In progress: Similar approach for the charm + power corrections

Summary of $|V_{cb}|$ inclusive

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w



- Need new branching ratio measurements!

Contamination of the $B \to X_c \ell \nu$ signal

Rahimi, Mannel, KKV [arXiv: 2105.02163]

Avoid background subtraction by calculating the full inclusive width:

$$d\Gamma(B \to X\ell) = d\Gamma(B \to X_c \ell \bar{\nu}) + d\Gamma(B \to X_u \ell \bar{\nu}) + d\Gamma(B \to X_c(\tau \to \ell \bar{\nu} \nu)\bar{\nu})$$

- $b \to u \ell \nu$ contribution: suppressed by V_{ub}/V_{cb}
- $b \to c(\tau \to \mu \nu \bar{\nu})\bar{\nu}$ contribution: phase space suppressed
- QED effects
- Quark-hadron duality violation?

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

Challenge:
estimate how much this description would improve V_{cb} determination

$b \to u\ell\nu$ contribution: Local OPE

Neubert (1994); Bosch, Paz, Lange, Neubert (2004,2005)

- Can be analyzed in local OPE as $B \to X_c\ell\nu$ by taking $m_c \to 0$ limit
- For V_{ub} determination
  - large charm background requires experimental cuts
  - reduces the inclusivity and local OPE no longer converges
  - spectrum described by non-local OPE
  - convolution of pert. coefficients with shape function

Goal:
provide theoretical description and compare with Monte-Carlo data used by Belle (II)

- NLO + $1/m_b^2$+$1/m_b^3$
- In agreement with partonic calc of DFN De Fazio, Neubert (1999); Gambino, Ossola, Uraltsev (2005)
- First study: no α_s for $1/m_b^2$, no additional uncert. due to missing higher orders
- Inputs HQE parameters from $B \to X_c\ell\nu$ study Gambino, Schwanda [2014]; Gambino, Healey, Turczy [2016]

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; De Fazio, Neubert 1999; Bosch, Lange, Neubert, Paz 2005

Compare local OPE with generator level Monte-Carlo data provided by Cao, Bernlochner

Monte Carlo:

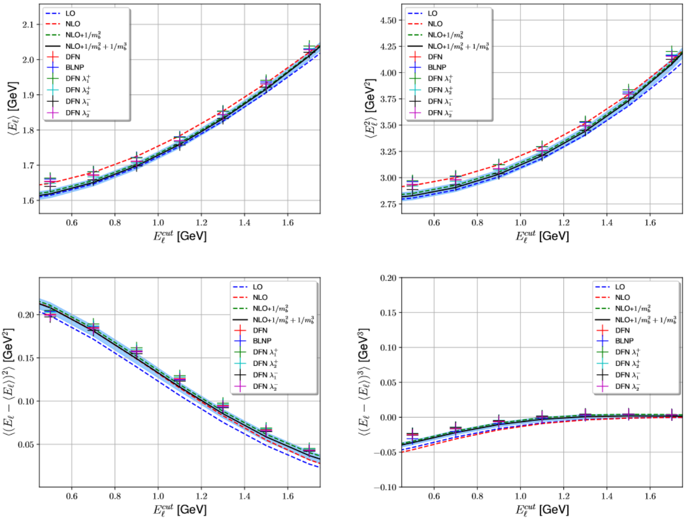
- BLNP: specific shape function input parameters shape function parameters $b = 3.95$ and $\Lambda = 0.72$
- DFN: α_s corrections convoluted with the exponential shape function model
  - Inputs from $B \to X_c \ell \nu$ and $B \to X_s \gamma$ data using KN-scheme Kagan, Neubert 1998
  - $(\lambda_1^+, \lambda_2^+, \lambda_1^-, \lambda_2^-)$ are obtained by varying $\bar{\Lambda}$ and μ_π^2 within 1σ Buchmuller, Flacher, 2006

Hadronic contributions: "hybrid Monte Carlo" Belle Collabroation [arXiv:2102.00020.]

- convolution with hadronization simulation based on PYTHIA
- plus explicit resonances: $\bar{B} \to \pi \ell \bar{\nu}$ and $\bar{B} \to \rho \ell \bar{\nu}$

Monte Carlo versus HQE

Rahimi, Mannel, KKV [arXiv: 2105.02163]; MC data by Lu Cao and Florian Bernlochner



MC-results are in good agreement with the HQE results

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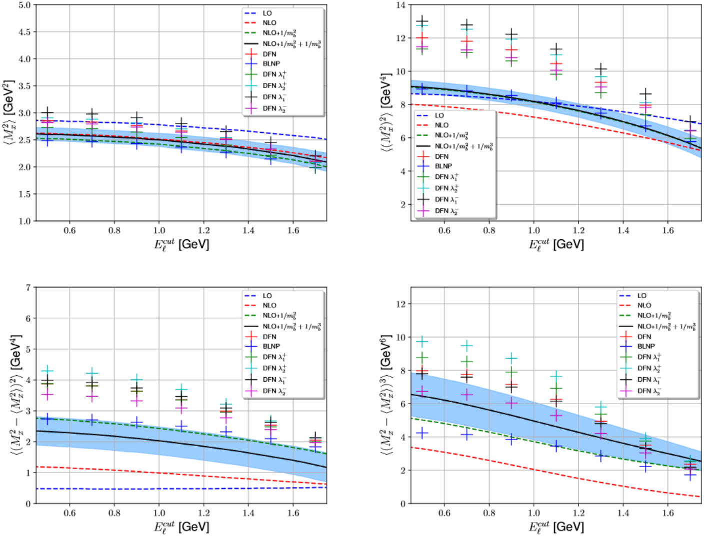

Wide spread between MC for higher moments

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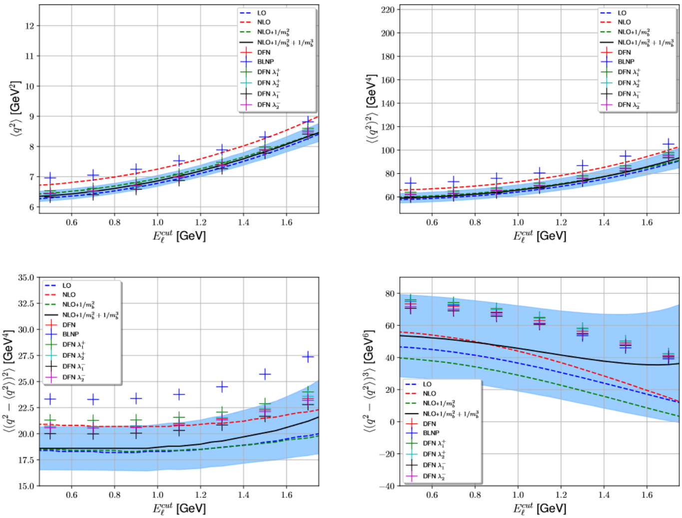

Monte Carlo versus local OPE: remarks

Rahimi, Mannel, KKV[arXiv: 2105.02163];

Remarks:

- DFN: Smearing corresponding to a shape function, mimicking some non-perturbative effects; may not capture all
- BLNP: should reproduce the HQE, with parameters adjusted to local HQE prediction
  - should include higher moments of the shape-function model?
  - include subleading shape functions?
- our HQE: interesting to include α_s to HQE parameters, α_s^2?

Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344

- m_c not observable $\rightarrow$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \rightarrow$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

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- Start from vacuum correlator

$$\int d^4x\, e^{-iqx} \langle 0| T[j_\mu(x) j_\nu(0)] |0\rangle = (g_{\mu\nu} q^2 - q_\mu q_\nu)\Pi(q^2)$$

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- Expand around $q^2 = 0$: ($\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s(\mu)}{\pi}\bar{C}_n^{(1)} + \ldots$)

$$\Pi(q^2) = \Pi(0) + \frac{4}{9}\frac{3}{16\pi^2}\sum_{n=1}^{\infty}\bar{C}_n\left(\frac{q^2}{4m_c^2}\right)$$

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- $\bar{C}_n$ known up to α_s^2 and related to moments

$$\bar{C}_n = (4m_c^2)^n M_n \quad \text{with} \quad M_n = \int\frac{ds}{s^{n+1}}R(s) \qquad (1)$$

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- Replace m_c:

$$m_c = \frac{1}{2}\left(\frac{\bar{C}_n}{M_n}\right)^{1/(2n)}$$

Interplay between electrons and muons

KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R(X_{e/\mu}) = 1.033 \pm 0.022$ with cut, see H. Junkerkalefeld [ICHEP]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ

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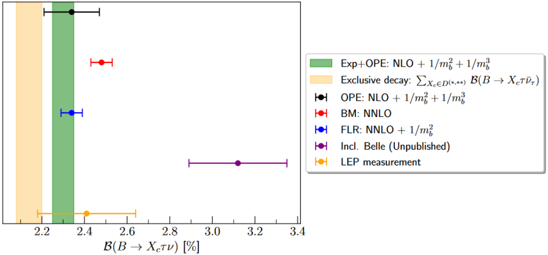
- Next step ratios with τ!

$$R_{\tau/\ell}(X) = 0.221 \pm 0.004$$



Interplay between electrons and muons

KKV, Rahimi [2207.03432]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e \bar{\nu}_e)}{\Gamma(B \to X_c \mu \bar{\nu}_\mu)}$$

- Belle II result: $R(X_{e/\mu}) = 1.033 \pm 0.022$ with cut, see H. Junkerkalefeld [ICHEP]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ
- Next step ratios with τ! Need new measurements!

$$R_{\tau/\ell}(X) = 0.221 \pm 0.004$$