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Analytical QCD ingredients for NNLL parton showers

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Frascati, 27th July 2024

Based mainly on work in JHEP 12 (2021) with Basem El-Menoufi , and JHEP 05 (2024) with Melissa van Beekveld, Basem El-Menoufi, Jack Helliwell, Pier Monni.

Application in arXiv:2406.02661 (2024), van Beekveld, M.D., El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Monni, Salam, Scyboz, Soto-Ontoso, Soyez,submitted to PRL.

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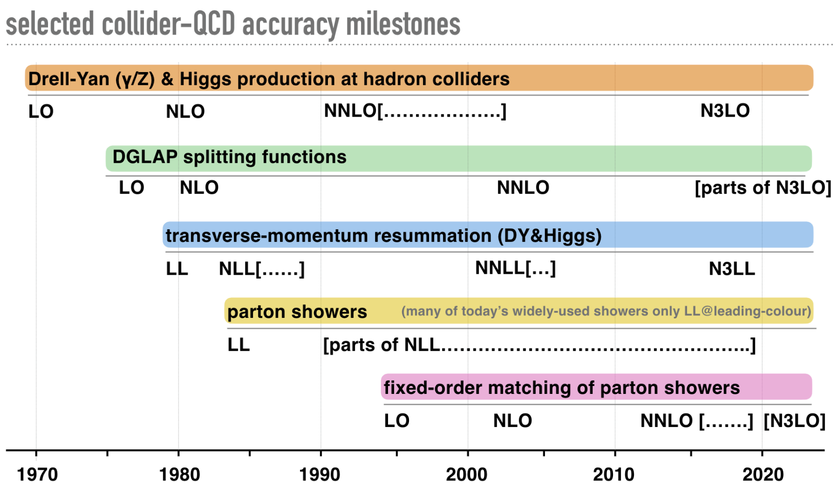
Outline

- Motivation for these studies : parton shower resummation accuracy
- Brief comments on changing state of the art
- Resummation coefficients for NNLL accuracy
- Beyond NLL in showers : Extending the K_{CMW} concept and derivation of $B_2(z)$, concept of effective emission probability
- Application in new NNLL showers
- Conclusions

Motivation

Parton shower accuracy



Taken from talk at Moriond QCD 2023 by G.Salam

- Log accuracy of showers under much scrutiny : a field that had stood relatively still for decades

- Over the same period substantial progress in understanding the structure of QCD in soft and collinear limits and in analytic resummation

Logarithmic accuracy definition

$$\Sigma(Q) = \sum_n c_n \alpha_s^n$$

Single scale observable. **Accuracy specified by maximum n.**

$$\Sigma(Q, vQ) = \sum_{n,m \leq 2n} c_{nm} \ \alpha_s^n L^m \qquad v \ll 1 \ \ L = \ln \frac{1}{v}$$

Multiscale observable. **Accuracy specified by n and m.**

$$\Sigma(Q, vQ) \sim \exp[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \cdots]$$

Multiscale observable **with exponentiation. Accuracy depends on g_n**

- g_1 is leading log (LL). Controls all double log (m= 2n) terms in expansion.
- Including g_2 gives NLL and g_3 is NNLL.
- NLL is a minimum for sensible pheno. studies and strong arguments for NNLL in context of current state-of the-art

Catani, Trentadue, Turnock and Webber 1992

Evolution of shower accuracy

- Shown in since 2007 **angular ordered showers do not reach NLL** for common class of (non-global) observables. Banfi, Corcella, MD 2007

- 2018 finding that widely used **dipole showers (e.g. Pythia 8) break even LL accuracy** beyond leading Nc.

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MD, Dreyer, Hamilton, Monni and Salam 2018

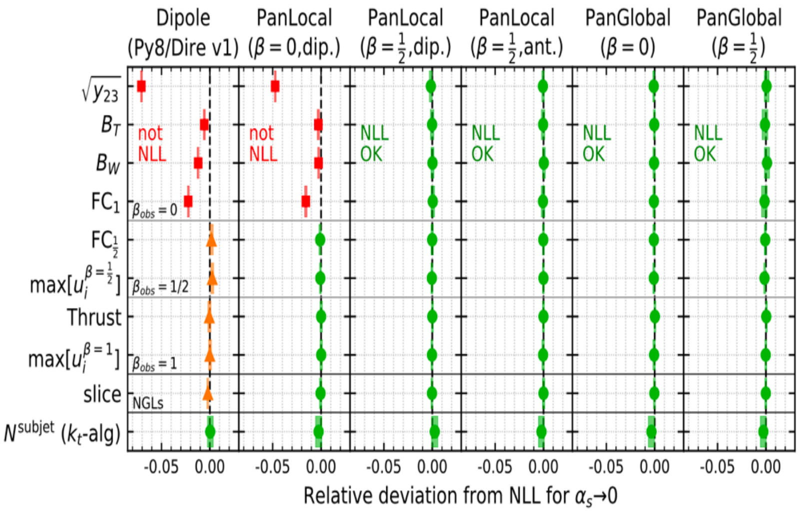
- 2020 - 2022 : **NLL dipole showers arrive** starting with first PanScales set of showers in 2020. Key later extensions and NLL showers by other groups.

MD et al 2020, Hamilton et al 2020, van Beekveld et al 2022, van Beekveld & Ferrario Ravasio 2023 , Nagy & Soper 2011, Forshaw et al. 2020, Herren et al. 2022

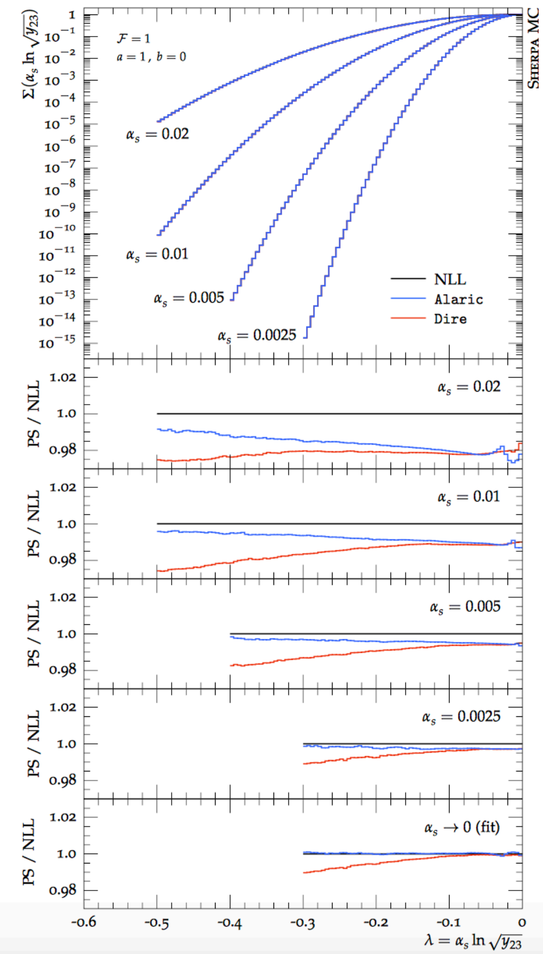
- 2023 & 2024 : **Advent of NNLL final state PanScales showers**

Ferrario Ravasio ett. al 2023, arXiv:2406.02661 (2024), van Beekveld, M.D., El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Monni, Salam, Scyboz, Soto-Ontoso, Soyez

NLL accurate showers



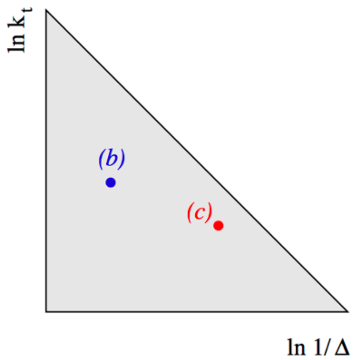
Plot from MD, Dreyer, Monni, Hamilton, Salam & Soyez 2020.



Herren, Hoeche, Schoenherr, Krauss 2022

- Principles identified for NLL in showers MD et al. 2020
- Demonstrably NLL dipole showers constructed
- A pathway to yet higher accuracy?

NLL criteria



$$\frac{d\mathcal{P}_{n\to n+1}}{d\ln v} = \sum_{\text{dipoles }\{\tilde{\imath},\tilde{\jmath}\}} \int d\bar{\eta}\frac{d\phi}{2\pi}\frac{\alpha_s(k_t)+K\alpha_s^2(k_t)}{\pi} \times \left[g(\bar{\eta})a_k P_{\tilde{\imath}\to ik}(a_k) + g(-\bar{\eta})b_k P_{\tilde{\jmath}\to jk}(b_k)\right],$$

K also referred to as K_1 or K_{CMW}

Can we build on accuracy principles identified for NLL?

For NLL: MD, Dreyer, Hamilton, Monni, Salam, Soyez 2020

- Need to reproduce QCD matrix elements in limit where all emissions strongly ordered in at least one of 2 possible logarithmic variables

- Correct inclusion of virtual corrections. Here showers simply exploit unitarity. Only degree of freedom left is coupling scheme.

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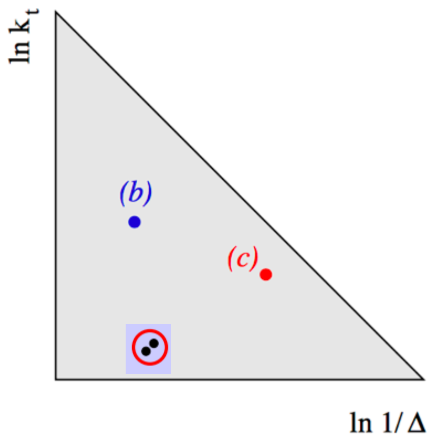
Towards NNLL



This suggests need for

- Getting real emission matrix-elements right in limit where pair of emissions are close in Lund plane → higher-order splitting kernels
- Known for over 2 decades. Campbell and Glover 1997, Catani & Grazzini 1998
- **A recent significant step : inclusion of fully differential double soft kernels in PanScales showers. Already gives NNLL for observables sensitive to soft emissions** Ferrario Ravasio et al 2023
- Including suitable analytical ingredients to take care of virtuals. At NLL done via K_{CMW} factor Catani, Marchesini Webber 1991 . But beyond NLL we need more.

Focus of This talk

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NNLL analytic ingredients

$$S_c(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\ \ln\frac{Q^2}{q^2} + B_c(\alpha_S(q^2))\right]\right\}$$

Collins Soper Sterman 1981

$$A_c(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_c^{(n)}$$

$$B_c(\alpha_S) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n B_c^{(n)}$$

$$A_q^{(1)} = C_F \ ,$$

$$A_q^{(2)} = \frac{1}{2}C_F K$$

$$B_q^{(1)} = -\frac{3}{2}C_F$$

Correctly taken care of in NLL

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_R n_f$$

- Use NNLL resummation to guide us. Typified by form factor in CSS approach. But basic idea more general
- Resummation accuracy controlled by “A” series of coefficients for the soft limit and “B” series for hard-collinear limit
- To go to NNLL we **need to account in the collinear series for B_2** (and in soft series for A_3.)

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B_2 and collinear emissions

- A_2 (or K_{CMW}) governs intensity of soft radiation from a hard parton. Related to a physical coupling definition in soft limit

- Similarly B_2 relates to intensity of collinear radiation off a given parton

- *Observable dependent* but always takes the form

$$B_2^f = -\gamma_f^{(2)} + b_0 X_v^f \qquad b_0 = \frac{11}{6} C_A - \frac{2}{3} T_R N_f$$

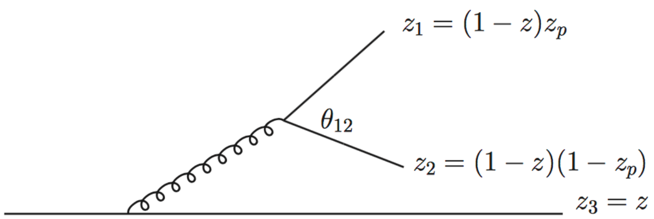
Davies and Stirling 1984
Catani, De Florian and Grazzini 2001
Banfi et al. 2019

$$\gamma_q^{(2)} = C_F^2 \left(\frac{3}{8} - \frac{\pi^2}{2} + 6\zeta(3) \right) + C_F C_A \left(\frac{17}{24} + \frac{11\pi^2}{18} - 3\zeta(3) \right) - C_F T_R n_f \left(\frac{1}{6} + \frac{2\pi^2}{9} \right)$$

$$-\gamma_g^{(2)} = \frac{4}{3} C_A T_R n_f + C_F T_R n_f - C_A^2 \left(\frac{8}{3} + 3\zeta_3 \right)$$

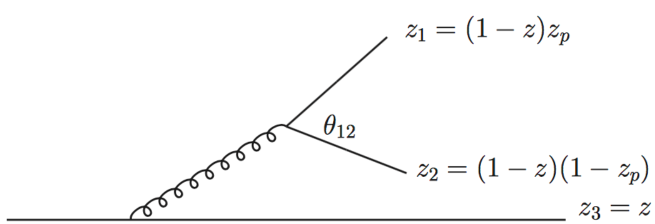
⇨ Endpoints of NLO DGLAP kernels

Computing a differential $B_2(z)$

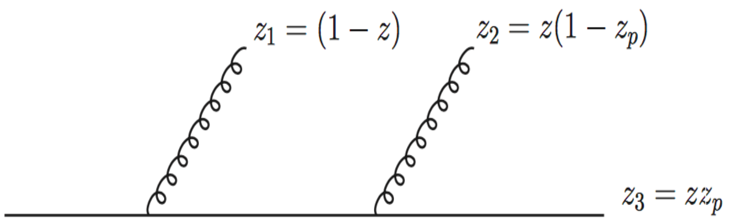


- In a shower approach we could encode info. on B_2 as function of emission kinematics
- Conceptually related to extension of K_{CMW} into collinear limit i.e. derive a function $B_2(z)$
- K_{CMW} computed from double-soft splitting kernels
- B_2 related to triple-collinear splittings

Splitting kernels : quarks

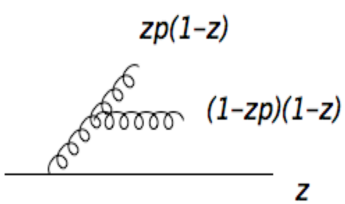


$C_F\ T_R\ n_f$ and $C_F\ (C_F\text{-}C_A/2)$ pieces



Pure C_F^2 piece

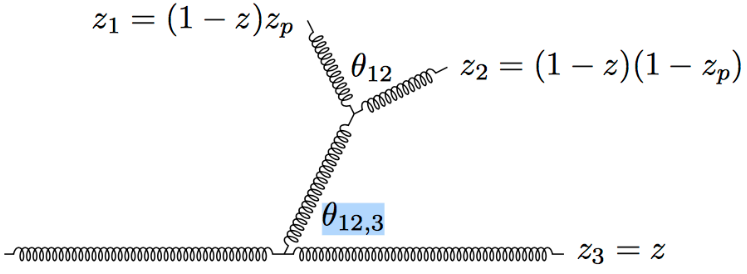
$$\left\langle \hat{P}_{\bar{q}'_1 q'_2 q_3} \right\rangle = \frac{1}{2} C_F T_R \frac{s_{123}}{s_{12}} \left[-\frac{t^2_{12,3}}{s_{12}s_{123}} + \frac{4z_3 + (z_1 - z_2)^2}{z_1 + z_2} + (1-2\varepsilon)\left(z_1 + z_2 - \frac{s_{12}}{s_{123}} \right) \right]$$



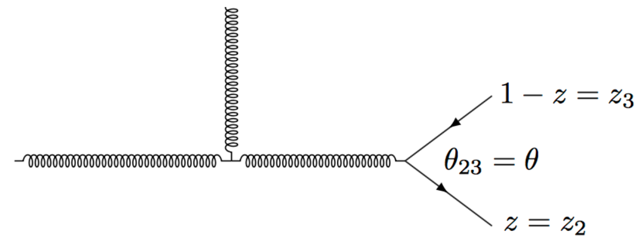
Pure $C_F\ C_A$ piece

Quark jets have four distinct pieces from 3 branching processes.

Splitting kernels : gluons

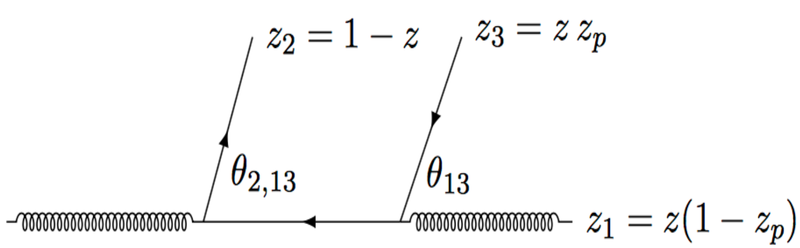


$C_A{}^2$ term



$C_A\ T_R\ n_f$ term

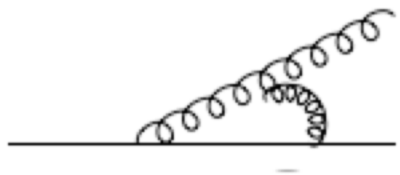
$$\langle \hat{P}^{(\mathrm{ab})}_{g_1 q_2 \bar{q}_3} \rangle = -2 - (1-\epsilon)s_{23}\left(\frac{1}{s_{12}} + \frac{1}{s_{13}}\right) + 2\frac{s_{123}^2}{s_{12}s_{13}}\left(1 + z_1^2 - \frac{z_1 + 2z_2z_3}{1-\epsilon}\right) - \frac{s_{123}}{s_{12}}\left(1 + 2z_1 + \epsilon - 2\frac{z_1+z_2}{1-\epsilon}\right) - \frac{s_{123}}{s_{13}}\left(1 + 2z_1 + \epsilon - 2\frac{z_1+z_3}{1-\epsilon}\right)$$



$C_F\ T_R\ n_F$ term

- 3 real emission kernels to consider
- Additionally a pure $T_R{}^2$ term from virtual corrections

Virtual corrections



$$
\begin{aligned}
P^{(1)}_{q\to gq} = \frac{c_\Gamma g_s^2}{\epsilon^2}\left(\frac{-s_{12}-\imath 0}{\mu^2}\right)^{-\epsilon}\Bigg[P^{(0)}_{q\to gq}\Bigg(&\frac{(C_F-C_A)\left(\epsilon(\delta\epsilon^2+\epsilon-3)+1\right)}{(\epsilon-1)(2\epsilon-1)} \\
&+ (C_A-2C_F)\,{}_2F_1\left(1,-\epsilon;1-\epsilon;\frac{z_1}{z_1-1}\right) - C_A\,{}_2F_1\left(1,-\epsilon;1-\epsilon;\frac{z_1-1}{z_1}\right) + C_F\Bigg) \\
&+ \frac{g_s^2 C_F}{z_1}\,\frac{(z_1-2)(z_1-1)\epsilon^2(\delta\epsilon-1)\,(C_A-C_F)}{(\epsilon-1)(2\epsilon-1)}\Bigg] + \text{c.c.}\,,
\end{aligned}
$$

- Also need the one-loop corrections to a collinear 1 to 2 splitting
- Taken from De Florian, Rodrigo, Sborlini (2013)
- Perform an integral over real emission phase space at fixed kinematics for a suitably defined first splitting. Do this in dim. reg. and combine with virtual piece.

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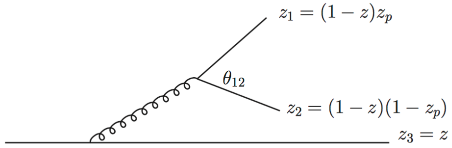
Calculations and Results

Calculations : set up

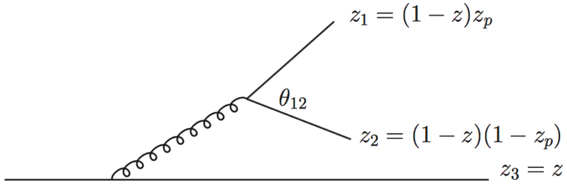


- We compute a collinear limit NLO correction to a given 1 to 2 splitting

$$\frac{\theta^2}{\sigma_0}\frac{d^2\sigma}{dz\, d\theta^2} = \int \mathrm{d}\Phi_3(z_i,\theta_{ij})\frac{\left(8\pi\alpha_s\mu^{2\epsilon}\right)^2}{s_{123}^2}\langle\hat{P}\rangle\,\theta^2\,\delta\left(\theta^2-\theta^2\left(z_i,\theta_{ij}\right)\right)\delta(z-z(z_i))\,\Theta_{\mathrm{cut}}(\theta_{ij})$$

- Fix energy and angle of the initial collinear splitting
- Our definition of energy fraction and angle are based on triple collinear configurations.
- Multiple definitions possible but in soft and collinear limits always point back to a unique 1 to 2 splitting.

Results : $C_F\, T_R\, n_f$ piece



Consider fixing z and parent angle $\theta_g^2 = z_p\theta_{13}^2 + (1-z_p)\theta_{23}^2 - z_p(1-z_p)\theta_{12}^2$

or other related quantity e.g. jet mass $\rho = z_1 z_3\theta_{13}^2 + z_2 z_3\theta_{23}^2 + z_1 z_2\theta_{12}^2$

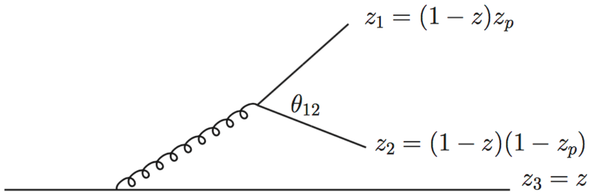
$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(\rho(1-z)\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\, dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(z(1-z)^2\theta_g^2\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

Also recovers soft limit exp. for scale and scheme

- NLO Results related by LO substitution
- Effect of gluon virtuality incorporated in K factor and z dependence
- Results contain info. on scale and scheme of coupling beyond soft limit

Results : $C_F(C_F\text{-}C_A/2)$ piece



- Identical particle interference term is purely finite.. Cannot identify parent uniquely but this ambiguity is irrelevant. Fixing any angle gives same result

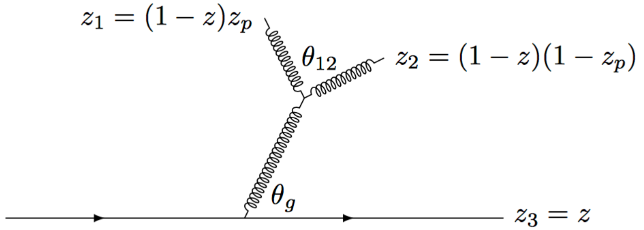
- Can look at either quark or antiquark distribution. Fixing z of either quark gives :

$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{(\mathrm{id.})} = \left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\, dz}\right)^{(\mathrm{id.})} = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}^{(\mathrm{id.})}(z),$$

$$\mathcal{P}^{(\mathrm{id.})}(z) = \left(4z - \frac{7}{2}\right) + \frac{5z^2-2}{2(1-z)}\ln z + \frac{1+z^2}{1-z}\left(\frac{\pi^2}{6} - \ln z \ln(1-z) - \mathrm{Li}_2(z)\right).$$

- Fixing z of anti-quark gives directly the non-singlet MSbar fragmentaton function $P_{q\bar{q}}^{V,(1)}$ (Eq. 4.108 of Ellis, Stirling, Webber text)

Results : pure C_F C_A piece



More involved calc. due to soft divergences.
Final answer similar to nf piece.

$$\left(\frac{\rho}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\rho\, dz}\right)^{\text{nab.}} = C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{P}^{(\text{nab.})}(z;\rho)$$

$$\mathcal{P}^{(\text{nab.})}(z;\rho) = \left(\frac{1+z^2}{1-z}\right)\left(-\frac{11}{6}\ln\left(\rho(1-z)\right) + \frac{67}{18} - \frac{\pi^2}{6} + \ln^2 z + \text{Li}_2\left(\frac{z-1}{z}\right) + 2\,\text{Li}_2(1-z)\right) + \\ + \frac{3}{2}\frac{z^2 \ln z}{1-z} + \frac{1}{6}(8-5z)$$

Note again appearance of KCMW coeff. and $b_0 \ln k_t$ term with rest giving hard collinear extension

Extracting $B_2(z)$

- Involves removing higher log order ingredients from our results.
- Illustrate on n_f term for quark jets

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\,dz}\right)^{C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(\frac{2}{3}\ln\left(z(1-z)^2\theta_g^2\right) - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

First remove soft limit terms

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma^{(2)}}{d\theta_g^2\,dz}\right)^{\text{soft},C_F T_R n_f} = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{2}{1-z}\left(\frac{2}{3}\ln\left((1-z)^2\theta_g^2\right) - \frac{10}{9}\right)$$
$$= C_F \frac{2}{1-z}\left(\frac{\alpha_s}{2\pi}\right)^2 \left(-b_0^{(n_f)} \ln\frac{k_t^2}{E^2} + K^{(n_f)}\right) \quad ,$$

Remove also remaining NLL hard collinear term $\propto -(1+z)\ln\theta_g^2$

$$\mathcal{B}_2^{q,n_f}(z;\theta_g^2) = C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\,\frac{2}{3}\ln z - (1+z)\left(\frac{2}{3}\ln(1-z)^2 - \frac{10}{9}\right) - \frac{2}{3}(1-z)\right)$$

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Extracting $B_2(z)$: gluon splitting channels

Similar exercise gives pure C_F CA term :

$$\mathcal{B}_2^{q,(\mathrm{nab.})}(z;\theta_g^2) = C_F C_A \left(\frac{\alpha_s}{2\pi}\right)^2 \left((1+z)\left(\frac{11}{6}\ln(1-z)^2 - \frac{67}{18} + \frac{\pi^2}{6}\right) + \frac{3}{2}\frac{z^2\ln z}{1-z} + \frac{8-5z}{6} + \frac{1+z^2}{1-z}\left(-\frac{11}{6}\ln z + \ln^2 z + \mathrm{Li}_2\left(\frac{z-1}{z}\right) + 2\mathrm{Li}_2(1-z)\right)\right)$$

Results for other variables follow from single emission kinematic relationship e.g.

$$\mathcal{B}_2^{q,n_f}(z;\rho) = \mathcal{B}_2^{q,n_f}(z;\theta_g^2) - C_F T_R n_f \left(\frac{\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\frac{2}{3}\ln z - (1+z)\frac{2}{3}\ln(1-z)\right)$$

For analogous C_F C_A relation replace 2/3 by -11/6.

Identical fermion term universal (no NLL piece to remove):

$$\mathcal{B}_2^{q,(\mathrm{id.})}(z) = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{\alpha_s}{2\pi}\right)^2\left(4z - \frac{7}{2}\right) + \frac{5z^2-2}{2(1-z)}\ln z + \frac{1+z^2}{1-z}\left(\frac{\pi^2}{6} - \ln z\ln(1-z) - \mathrm{Li}_2(z)\right)$$

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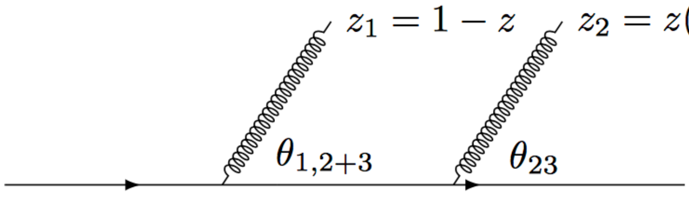
$B_2(z)$ for $C_F{}^2$ channel

- Here definition of "first emission" may be given by some ordering
- We take ordering in angle as simple choice with $\theta_{13} > \theta_{23}$

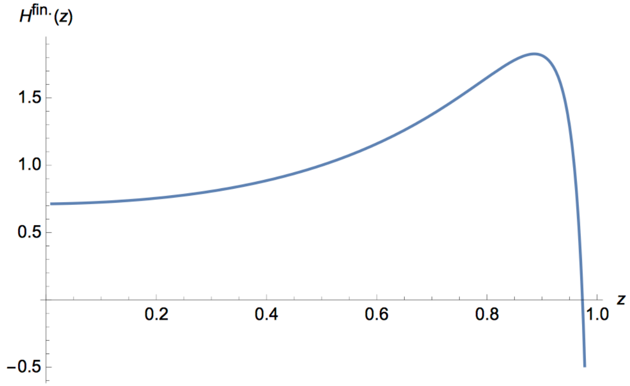


- Obtain B_2 as difference between triple-collinear and iterated 1 to 2 splittings and phase-space + virtual corr. to 1 to 2 splitting.

- Schematically amounts to computing

$$B_2(z) = \int d\Phi_3\, P_{1\to 3}\, \delta(z - (1 - z_1))\, \theta(\theta_{13} > \theta_{23}) - \int d\Phi_2^2\, P_{1\to 2}^2\, \delta(z - (1 - z_1))\, \theta(\theta_{13} > \theta_{23}) + V_1(z, \epsilon)$$

$B_2(z)$ for $C_F{}^2$ channel



Here due to ordering part of the result is numerical :

$$\mathcal{B}_2^{q,(\text{ab.})}(z) \quad = \left(\frac{C_F\alpha_s}{2\pi}\right)^2 \left(\frac{1+z^2}{1-z}\left(-3\ln z + 2\text{Li}_2\left(\frac{z-1}{z}\right) - 2\ln z\ln(1-z)\right) - 1 + H^{\text{fin.}}(z)\right)$$

$$\int_0^1 H^{\text{fin}}(z)\,dz = 4\zeta(3) - \frac{31}{8}$$

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Integrals over z

Integrals over z produce the expected form

$$B_2^{q,(\mathrm{ab.})} = \left(\frac{2\pi}{\alpha_s}\right)^2 \int_0^1 \mathcal{B}_2^{q,\mathrm{ab.}}(z)\, dz = \pi^2 - 8\zeta(3) - \frac{29}{8}$$

$$B_2^{q,(\mathrm{ab.})} + B_2^{q,(\mathrm{id.}),C_F^2} = -\gamma_q^{(2,C_F^2)} = C_F^2\left(\frac{\pi^2}{2} - 6\zeta(3) - \frac{3}{8}\right)$$

Combining also other channels we recover full $-\gamma_q^{(2} + C_F b_0 X_v$

with

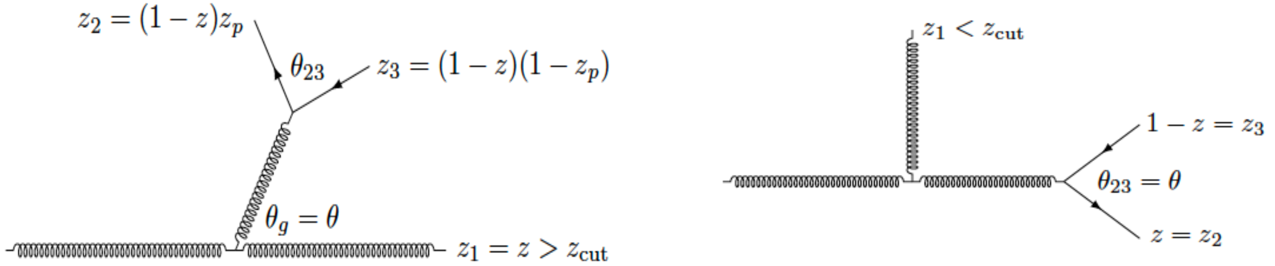
$$X_{\theta_g^2} = \frac{2\pi^2}{3} - \frac{13}{2}$$

$$X_\rho = \frac{\pi^2}{3} - \frac{7}{2}$$

→ Agrees with hard-collinear NNLL piece in resummation literature

Becher & Schwartz 2008. Banfi et. al. 2014, 2019

Gluon jet subtleties



- Gluon jets can be handled similarly modulo a few mainly technical differences/subtleties.
- Two histories involved at Born level
- Independent and correlated emission pictures mixed within same colour channel
- IR divergences associated to both emissions in a g to gg branching. Makes definition of basic 1 to 2 splitting more subtle.

IRC safe procedure for defining z and θ based on SoftDrop declustering

van Beekveld et. al. 2023

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Gluon jets results summary

- We obtain results for each channel with a fully analytic result in the $C_A\, T_R\, n_f$

$$\mathcal{B}_2^{g,C_A T_R}(z) = -p_{qg}(z)\left(\ln^2 z + \ln^2(1-z)\right) + \frac{1}{9}(28 - 41z + 41z^2)$$
$$+ \ln z\left(\frac{4}{3(1-z)} - \frac{26}{3}z^2 + 8z - 7\right) + \ln(1-z)\left(\frac{4}{3z} - \frac{26}{3}(1-z)^2 + 8(1-z) - 7\right)$$

- In other channels we have a semi-analytic result c.f. the C_F^2 channel of quark jets The results are consistent with

$$\int_0^1 dz\, B_2(z) = -\gamma_g^{(2)} + b_0 X_\theta^2 \quad X_\theta^2 = \left(-\frac{67}{9} + \frac{2\pi^2}{3}\right) C_A + \frac{23}{9} T_R n_f$$

N.B. In the C_A^2 channel a clustering correction appears specific to C/A clustering. Identical to that of SoftDrop jet mass.

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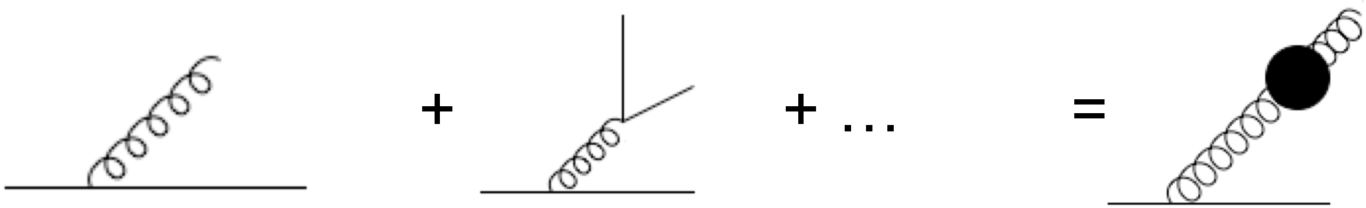
Applications

Inclusive emission probability



- Look to define an effective emission prob. relevant for NNLL Sudakov in parton showers or jet calculus.

- Consider combination with LO . Can for example be written as

$$\left(\frac{\theta_g^2}{\sigma_0}\frac{d^2\sigma}{d\theta_g^2 dz}\right)^{\text{tot.}} = C_F\left(\frac{1+z^2}{1-z}\right)\left[\frac{\alpha_s\left(E^2\right)}{2\pi} + \left(\frac{\alpha_s}{2\pi}\right)^2\left(-b_0\ln\left((1-z)^2\theta_g^2\right)+K\right) \quad -\left(\frac{\alpha_s}{2\pi}\right)^2 b_0 \ln z\right] \quad +\mathcal{R}$$

Suggests modification of argument of running coupling in h.c. limit

$$\equiv \frac{C_F}{2\pi}\left(\frac{1+z^2}{1-z}\right)\alpha_s\left(E_j^2 z(1-z)^2\theta_g^2\right)\left(1+\frac{\alpha_s}{2\pi}\mathcal{K}(z)\right)$$

Higher order effects included in scale and scheme of coupling

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Applications

Can use incl. emission probability concept in a couple of ways :

- Directly to derive new resummed results for pure collinear sensitive observables e.g. groomed jet substructure observables.

- More generally can also relate to definition of NNLL Sudakov for final state parton showers.

$$S = \exp\left[-4\int \frac{dk_t}{k_t}\int_{k_t}^{1} dz \frac{\alpha_{\mathrm{eff}}}{2\pi} M(k)P(z)\Theta(V(k) > v)\right]$$

The effective coupling contains NLO ingredients from branching of soft and hard-collinear emissions

arXiv:2406.02661 (2024), van Beekveld, M.D., El-Menoufi, Ferrario Ravasio, Hamilton, Helliwell, Karlberg, Monni, Salam, Scyboz, Soto-Ontoso, Soyez

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NNLL shower appplication

$$\alpha_{\rm eff} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} \left(K_1 + \Delta K_1(y) + B_2(z) \right) + \frac{\alpha_s^2}{4\pi^2} K_2 \right],$$

van Beekveld et. al. 2024

Contains terms related to:

- Inclusive branching of soft emission to order α_s^3

Included via $K_1, \Delta K_1(y), K_2.$

- Inclusive branching of hard-collinear emission to α_s^2

Included via $B_2(z)$

- K_1 or K_{CMW} was already needed for NLL
- Remaining ingredients are NNLL
- The shower quantities above are **NOT the same as those known from analytic resummation.**
- $B_2(z)$ **also not the same as our calculation**

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NNLL shower application

To achieve NNLL shower required **several non-trivial developments**

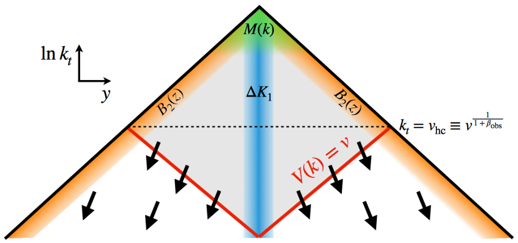
- Inclusion of higher-order splitting kernels. **Double-soft kernels suffice** for very wide class of “global event shape” observables. Ferrario Ravasio et. al. 2023

- **Crucial highly subtle element of relating the shower K coefficients and $B_2(z)$** to known results used in resummation and our calculations.

- Requires deep insight into shower dynamics and role of analytic ingredients.

- Key observation is that shower does not conserve same kinematic quantities as analytic calcs. Difference can be viewed as **kinematic drifts** of shower emissions. van Beekveld et. al. 2024

NNLL showers



$$\langle \Delta_x \rangle = \lim_{\tilde{\imath} \to \mathrm{SC}} \frac{1}{\mathcal{B}_{\tilde{\imath}}} \int d\Phi^{\mathrm{PS}}_{ij|\tilde{\imath}} \mathcal{R}_{ij} \times (x_{i+j} - x_{\tilde{\imath}}) \, .$$

- Average drifts are computable allowing relation of shower to NLO/resummation ingredients.

In particular we have $B_2^{\mathrm{int,PS}} = B_2^{\mathrm{int,NLO}} - \langle \Delta_{\ln z} \rangle,$

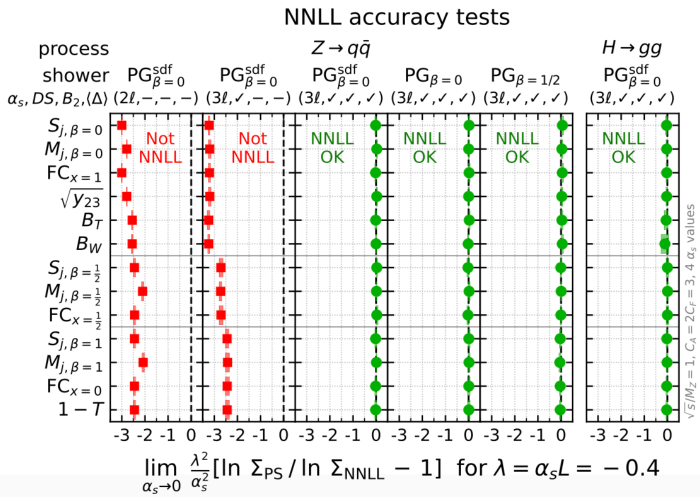
van Beekveld et. al. 2024

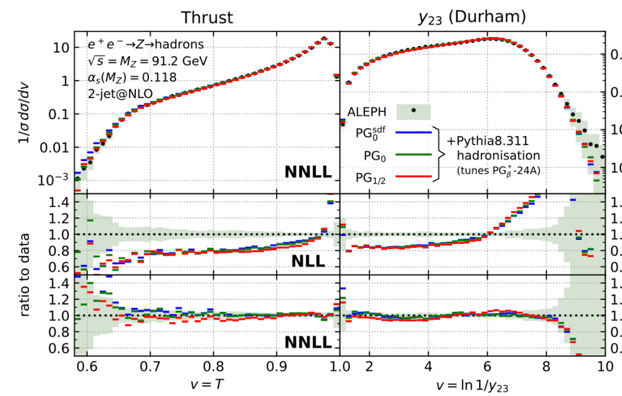
where $B_2^{\mathrm{int,NLO}}$ derives from the integral of the $B_2(z)$ presented before.

- Similar considerations for other terms. In particular $\int \Delta K_1(y) dy$ derives from rapidity drift.
- K_2 receives contribution from k_t drift relative to resummation version.

Banfi et al 2018.
Catani et al 2019

NNLL showers





Uses $\alpha_s(M_z) = 0.118$

van Beekveld et. al. 2024

After accounting for this we show :

- analytic proof of resummation accuracy.
- Numerical comparisons to resummation for several observables in two different processes involving quark and gluon jets. Show agreement!
- Sizeable NNLL terms.
- Comparisons to data showing improvement relative to NLL and signalling need for NNLL accuracy in pheno.

Summary and Conclusions

- First ever NNLL accurate showers recently achieved by PanScales collaboration.

- Needs higher-order kernels + specific analytic ingredients. Inclusion of double-soft kernels in PanScales a very significant step.

- Discussed in detail one key analytical ingredient $B_2(z)$. This relates to NNLL hard collinear Sudakov form factor

- Needed to be combined with analogous soft limit ingredients and crucially with understanding of relationship to corresponding shower quantities.

- This allowed for NNLL final state showers in the context of wide class of observables.

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- Next step will involve inclusion of triple-collinear splittings and differential $B_2(z)$ relevant e.g. for fragmentation functions. Will achieve almost complete NNLL final state picture.