
Unfolding Systematics for GSI2021 analysis

FOOT physics meeting 11/06/2024

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Summary & next steps of the previous meeting (21/05/2024)

- Standard sanity check on the unfolding machinery has been presented
- No unexpected behavior of the machinery has been found
- Performing checks on the dependence of different figures of merit on the number of iterations, 3 iterations could be chosen as the optimal value for **iterative bayesian unfolding**

...About systematics of the unfolding method

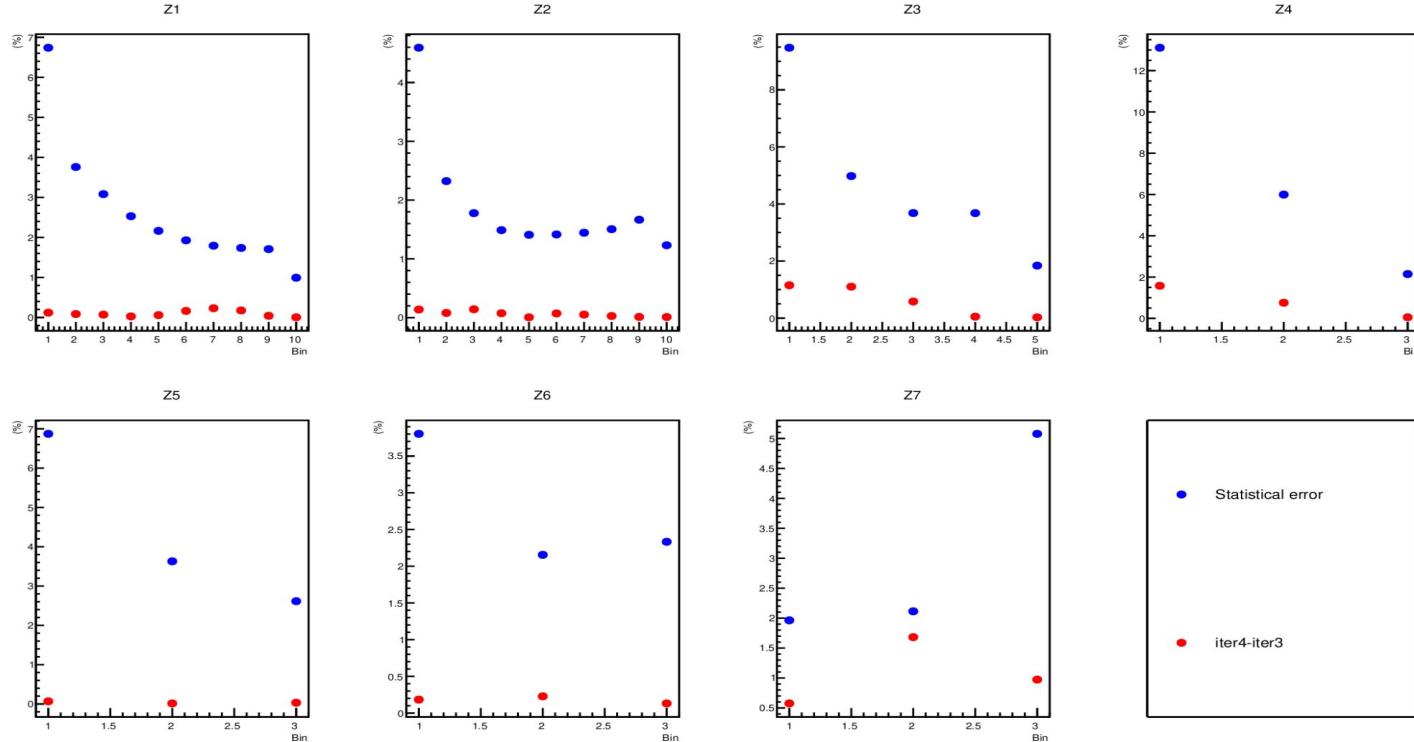
- “External”: difference compared to an alternative method
- “Internal”: difference between using 3 iteration and 4

expected to be at subpercent level

Table 1. Data tests performed on various unfolding methods.

| | Y_{unf} | $\chi^2_A / N_{\text{d.f.}}$ | $\text{Prob}(\chi^2_A, N_{\text{d.f.}})$ |
|--------------------------|------------------|-------------------------------------|--|
| expectation | 4584 | $N_{\text{d.f.}} / N_{\text{d.f.}}$ | 0.5 |
| bin-by-bin | 4521 | 34.7 / 0 | n.a. |
| matrix inversion | 4584 | 0 / 0 | n.a. |
| template fit | 4572 | 12.0 / 16 | 0.74 |
| constrained template fit | 4584 | 12.0 / 16 | 0.74 |
| Tikhonov $\tau = 0.0068$ | 4584 | 13.4 / 16 | 0.64 |
| Tikhonov $\tau = 0.012$ | 4584 | 15.0 / 16 | 0.52 |
| EM method $n = 0$ | 4537 | 5069 / 0 | n.a. |
| EM method $n = 20$ | 4585 | 5.9 / 0 | n.a. |
| EM method $n = 100$ | 4584 | 4.2 / 0 | n.a. |
| EM method $n = 1000$ | 4584 | 3.9 / 0 | n.a. |
| IDS $n = 1$ | 4584 | 76.8 / 0 | n.a. |
| IDS $n = 3$ | 4584 | 26.1 / 0 | n.a. |
| IDS $n = 10$ | 4584 | 8.0 / 0 | n.a. |
| IDS $n = 30$ | 4584 | 4.9 / 0 | n.a. |

Unfolding Method Systematic (internal)



Difference between 4 and 3 iterations one order of magnitude smaller than the statistical error → **the idea is to neglect it!**

Unfolding Method Systematic (external)

Singular Value Decomposition (SVD)

A. Hoecker and V. Kartvelishvili,
“SVD Approach to Data Unfolding,”
Nucl. Instrum. Meth. A 372
(1996) 469 [[arXiv:hep-ph/9509307](https://arxiv.org/abs/hep-ph/9509307)].

Iterative, Dynamically Stabilized (IDS)

Malaescu Bogdan
“An Iterative, Dynamically Stabilized
(IDS) Method of Data Unfolding”
(2011) [[arXiv:1106.3107](https://arxiv.org/abs/1106.3107)]

Unfolding Method Systematic (SVD kreg default)

```
SVD init 10 x 10 bins, kreg=5  
Preunfold integral = 127449  
Specunfold integral = 127419  
end loop for 2
```

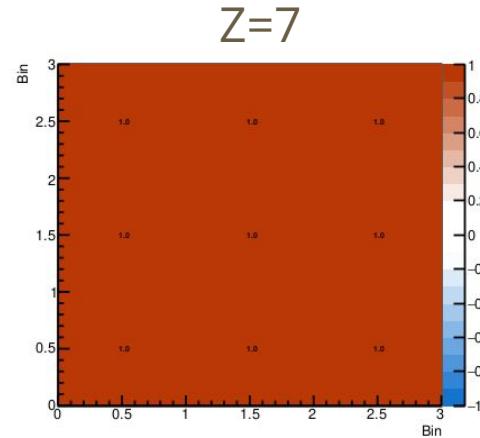
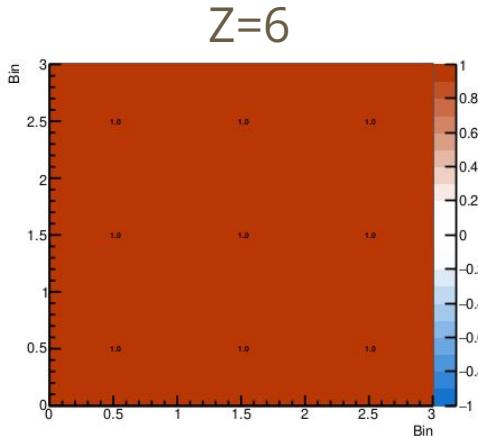
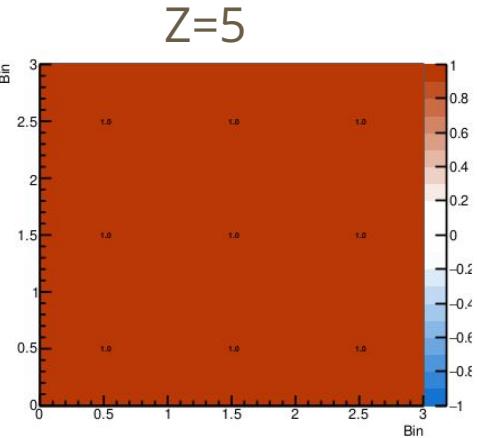
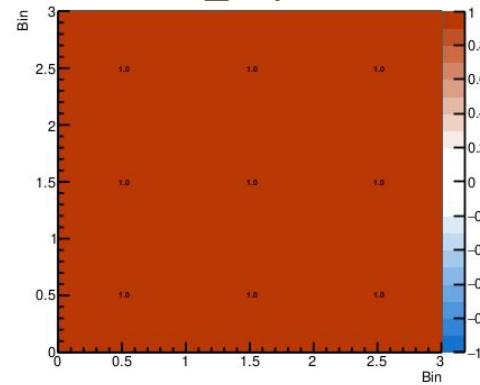
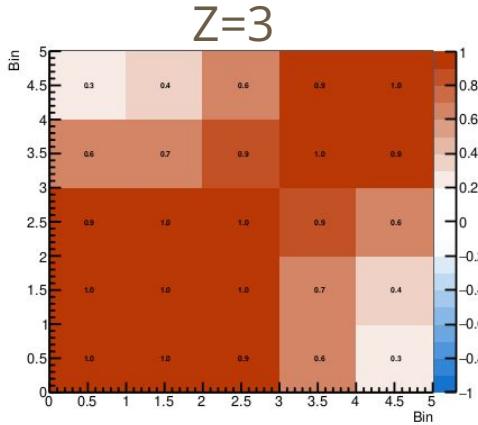
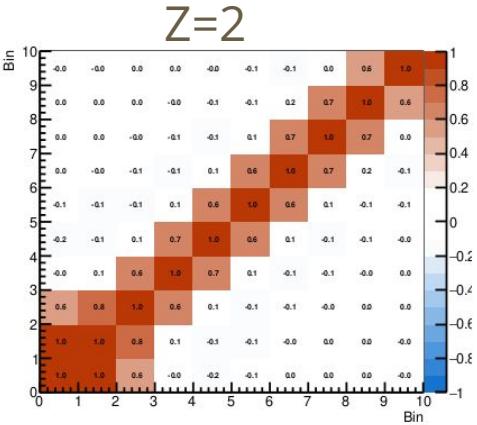
```
SVD init 5 x 5 bins, kreg=2
Preunfold integral = 13687.6
Specunfold integral = 13672.5
end loop for 3
```

```
SVD init 3 x 3 bins, kreg=1  
Preunfold integral = 7266.69  
Specunfold integral = 3647.41  
end loop for 4
```

```
SVD init 3 x 3 bins, kreg=1  
Preunfold integral = 7661.97  
Specunfold integral = 3817.45  
end loop for 5
```

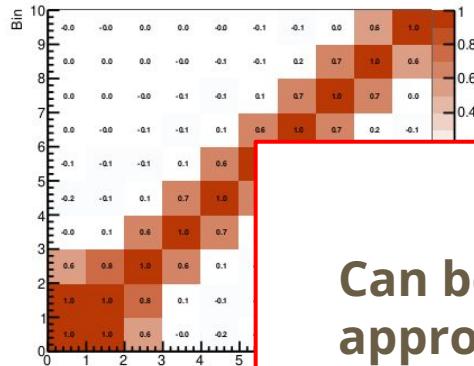
```
SVD init 3 x 3 bins, kreg= 1  
Preunfold integral = 17037.1  
Specunfold integral = 8513.49  
end loop for 6
```

SVD init 3 x 3 bins, kreg=1
Preunfold integral = 21141
Specunfold integral = 10771.6

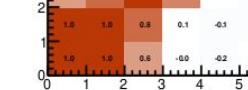


Unfolding Method Systematic (SVD kreg default)

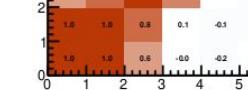
SVD init 10 x 10 bins, kreg=5
Preunfold integral = 127449
Specunfold integral = 127419
end loop for 2



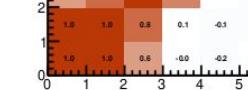
SVD init 5 x 5 bins, kreg=2
Preunfold integral = 13687.6
Specunfold integral = 13672.5
end loop for 3



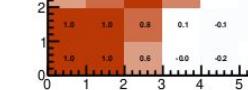
SVD init 3 x 3 bins, kreg=1
Preunfold integral = 7266.69
Specunfold integral = 3647.41
end loop for 4



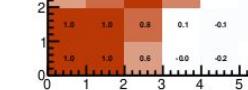
SVD init 3 x 3 bins, kreg=1
Preunfold integral = 7661.97
Specunfold integral = 3817.45
end loop for 5



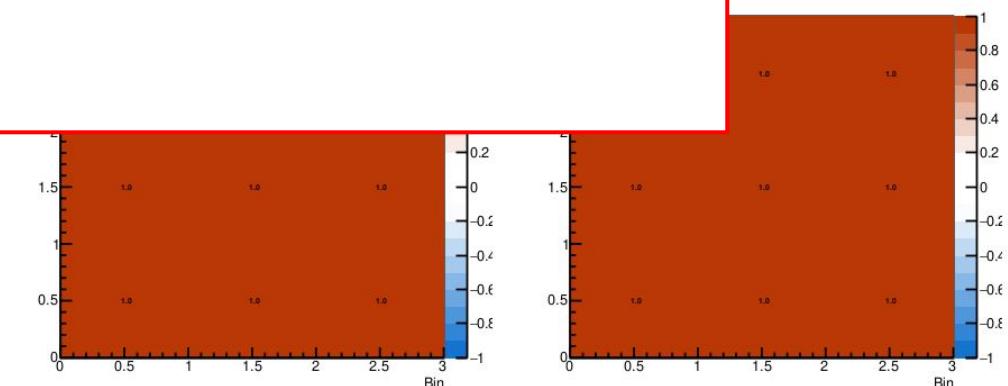
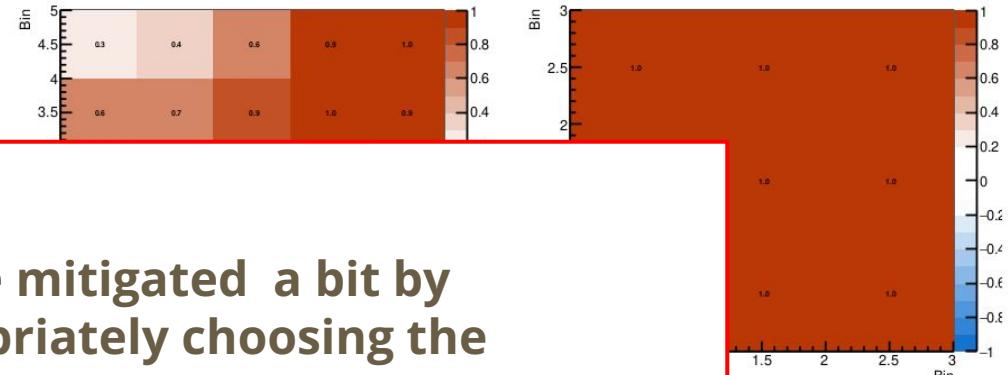
SVD init 3 x 3 bins, kreg= 1
Preunfold integral = 17037.1
Specunfold integral = 8513.49
end loop for 6



SVD init 3 x 3 bins, kreg=1
Preunfold integral = 21141
Specunfold integral = 10771.6



Can be mitigated a bit by
appropriately choosing the
regularization parameter Kreg



Unfolding Method Systematic (SVD kreg tuned)

```
SVD init 10 x 10 bins, kreg=8  
Preunfold integral = 127449  
Specunfold integral = 127444  
end loop for 2
```

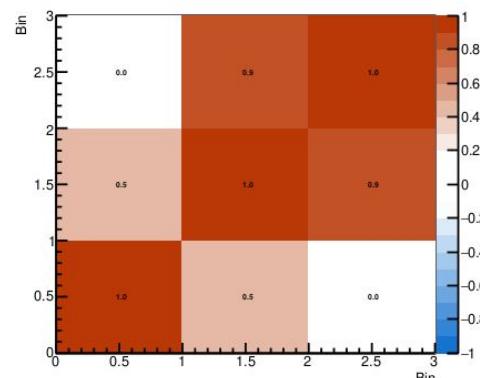
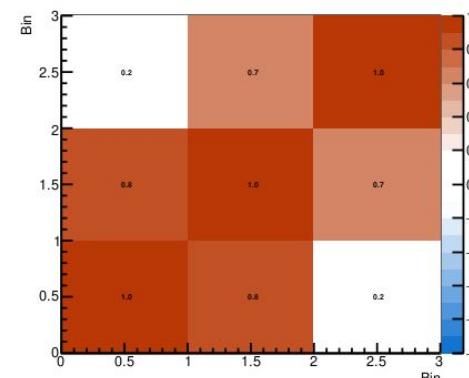
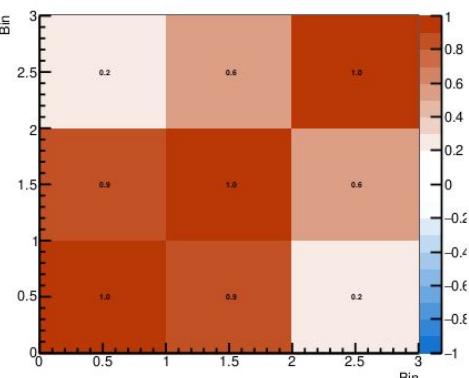
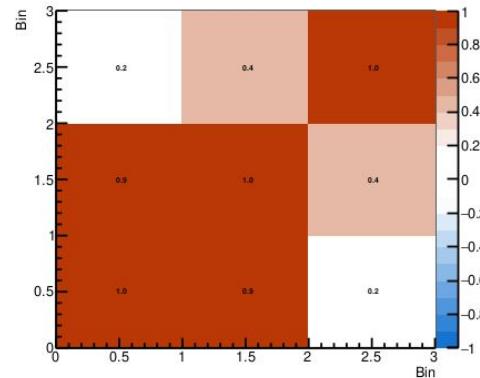
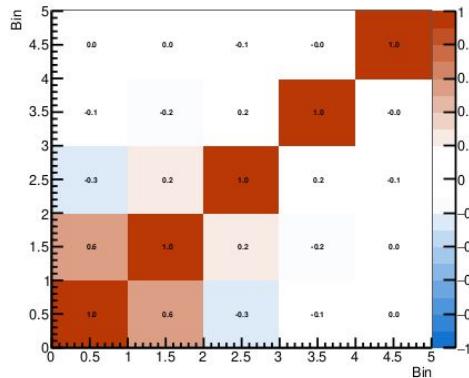
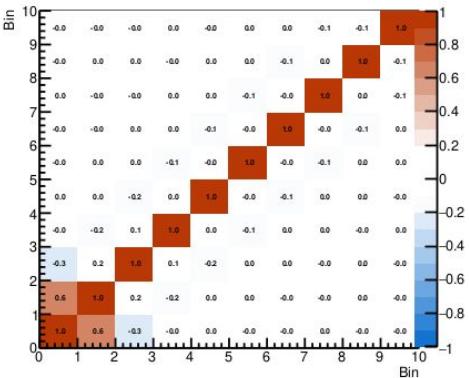
```
SVD init 5 x 5 bins, kreg=4
Preunfold integral = 13687.6
Specunfold integral = 13685.7
end loop for 3
```

```
SVD init 3 x 3 bins, kreg=2  
Preunfold integral = 7266.69  
Specunfold integral = 7282.13  
end loop for 4
```

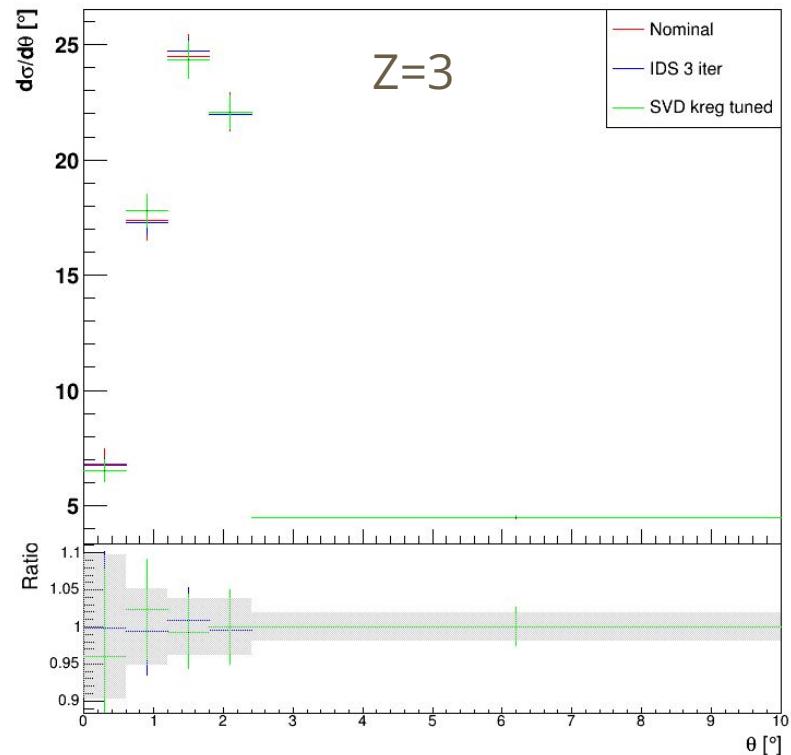
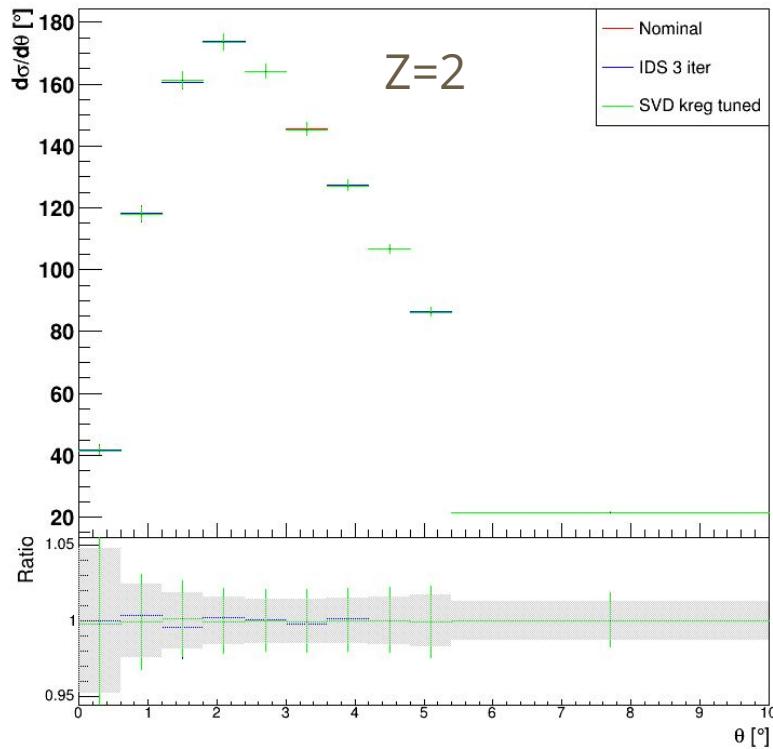
```
SVD init 3 x 3 bins, kreg=2
Preunfold integral = 7661.97
Specunfold integral = 7648.56
end loop for 5
```

```
SVD init 3 x 3 bins, kreg=2  
Preunfold integral = 17037.1  
Specunfold integral = 17046.6  
end loop for 6
```

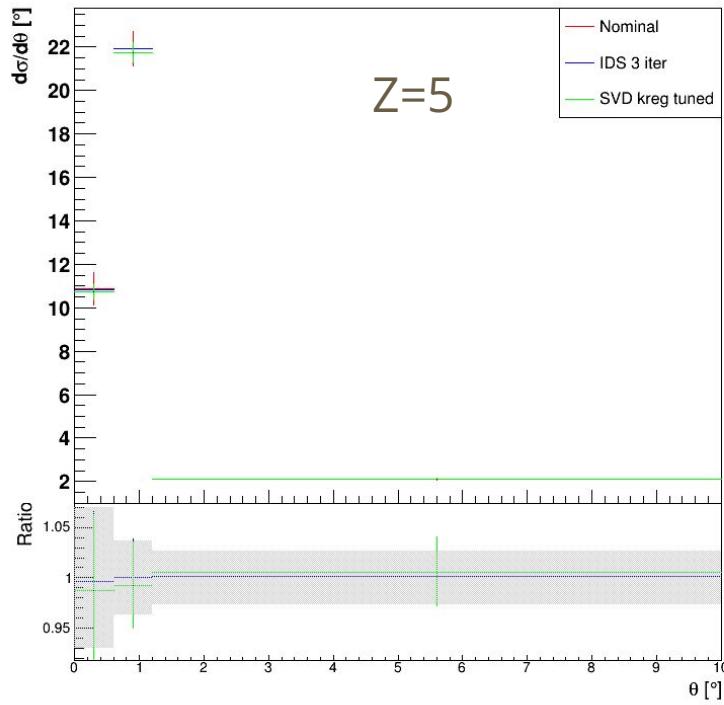
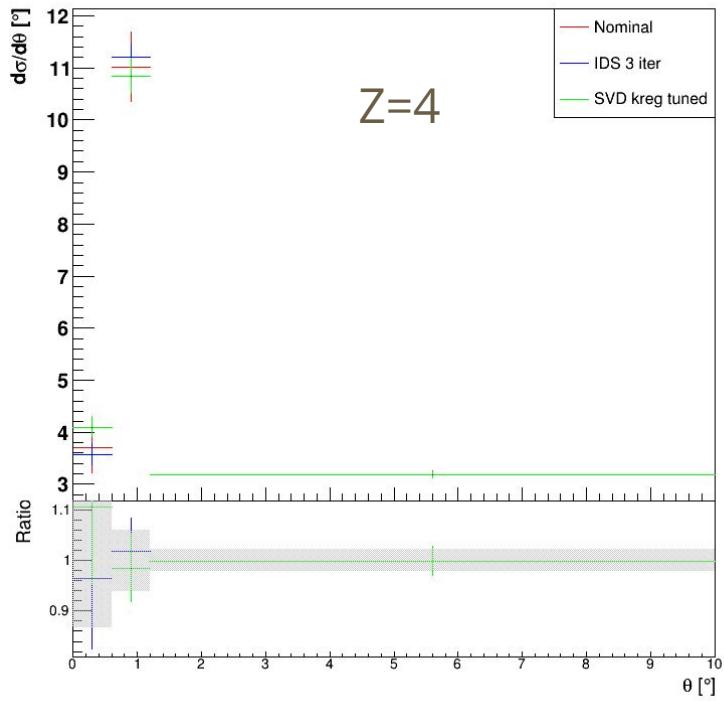
SVD init 3 x 3 bins, kreg=2
Preunfold integral = 21141
Specunfold integral = 21434.4



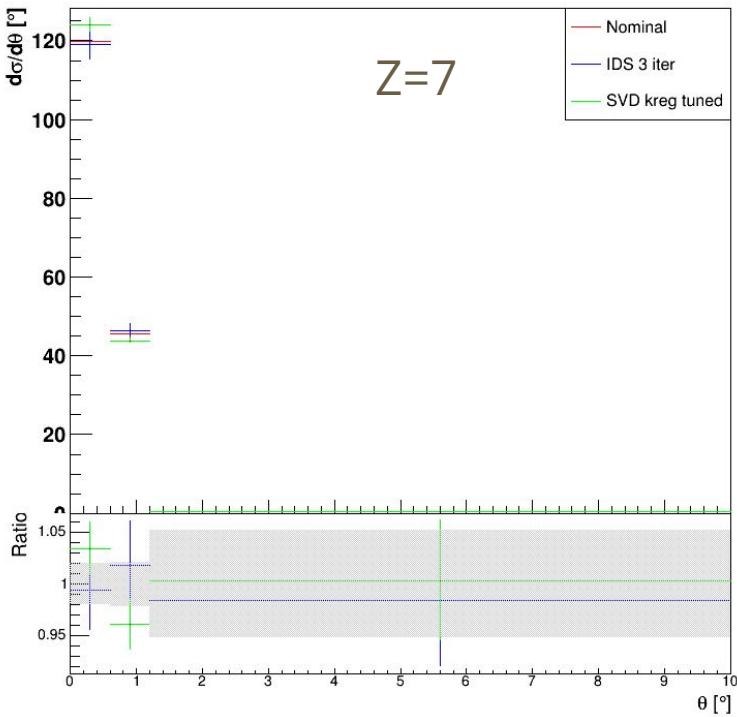
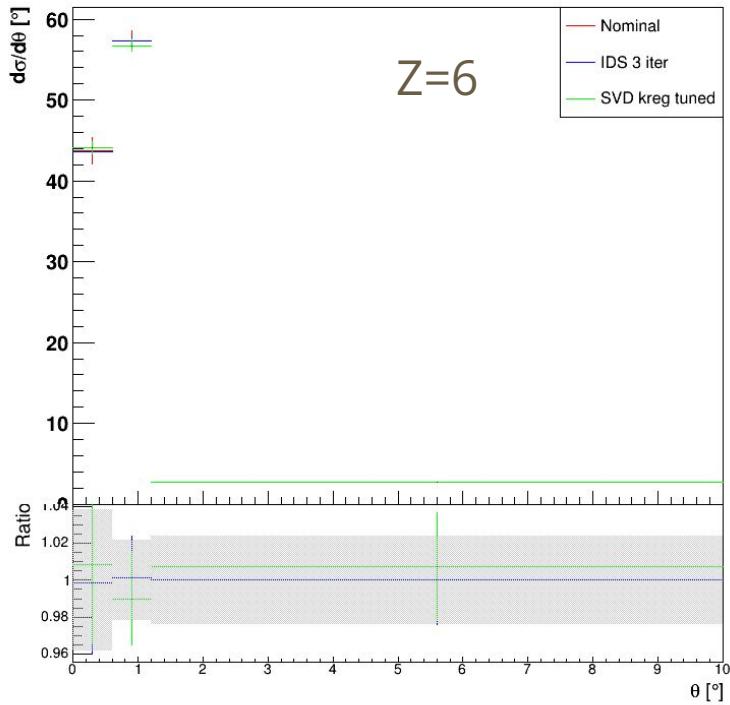
Systematics uncertainty: Unfolding method comparison



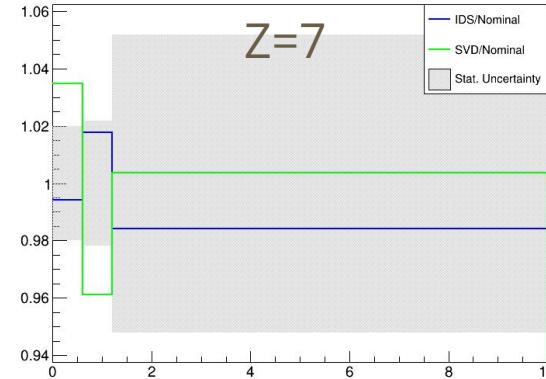
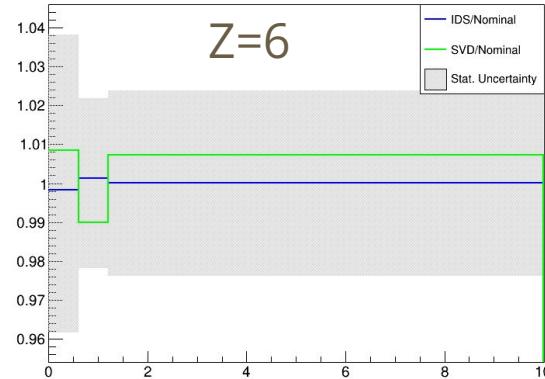
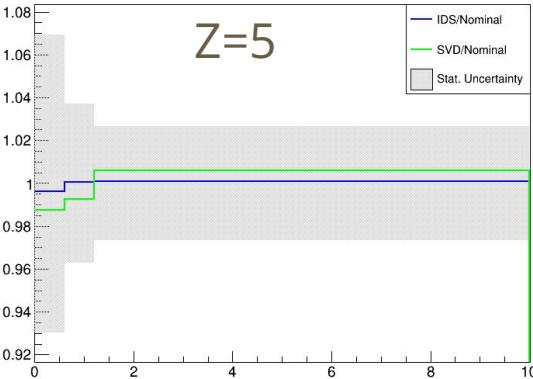
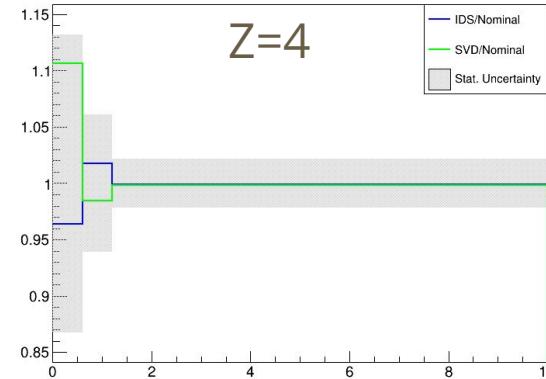
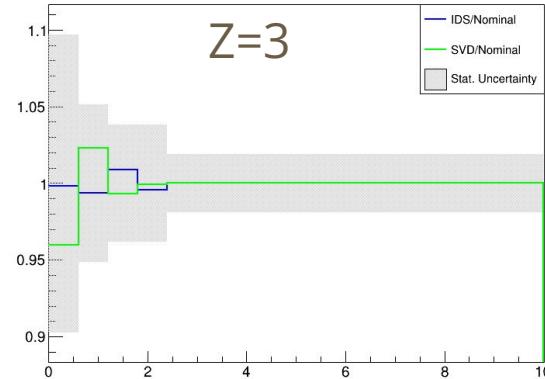
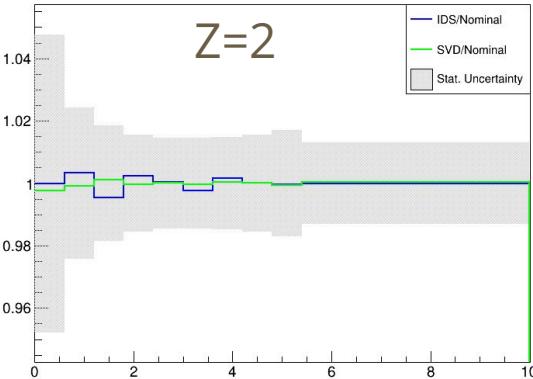
Systematics uncertainty: Unfolding method comparison



Systematics uncertainty: Unfolding method comparison



Systematics uncertainty: closer look to ratio plots



Systematics uncertainty: Values in a table

$Z=2$

| Bin | Center | Width | $\Delta(\text{IDS}/\text{Nominal}) [\%]$ | $\Delta(\text{SVD}/\text{Nominal}) [\%]$ |
|-----|--------|-------|--|--|
| 1 | 0.30 | 0.30 | -0.01 | -0.23 |
| 2 | 0.90 | 0.30 | 0.34 | -0.10 |
| 3 | 1.50 | 0.30 | -0.45 | 0.12 |
| 4 | 2.10 | 0.30 | 0.24 | -0.04 |
| 5 | 2.70 | 0.30 | 0.05 | 0.01 |
| 6 | 3.30 | 0.30 | -0.23 | -0.03 |
| 7 | 3.90 | 0.30 | 0.15 | 0.03 |
| 8 | 4.50 | 0.30 | 0.01 | 0.02 |
| 9 | 5.10 | 0.30 | -0.03 | -0.07 |
| 10 | 7.70 | 2.30 | -0.01 | 0.03 |

$Z=3$

| | | | | |
|---|------|------|-------|-------|
| 1 | 0.30 | 0.30 | -0.19 | -4.00 |
| 2 | 0.90 | 0.30 | -0.62 | 2.32 |
| 3 | 1.50 | 0.30 | 0.87 | -0.68 |
| 4 | 2.10 | 0.30 | -0.42 | -0.07 |
| 5 | 6.20 | 3.80 | 0.01 | 0.04 |

$Z=4$

| | | | | |
|---|------|------|-------|-------|
| 1 | 0.30 | 0.30 | -3.60 | 10.64 |
| 2 | 0.90 | 0.30 | 1.79 | -1.56 |
| 3 | 5.60 | 4.40 | -0.13 | -0.17 |
| 1 | 0.30 | 0.30 | -0.38 | -1.25 |

$Z=5$

| | | | | |
|---|------|------|-------|-------|
| 1 | 0.30 | 0.30 | -0.38 | -1.25 |
| 2 | 0.90 | 0.30 | 0.04 | -0.75 |
| 3 | 5.60 | 4.40 | 0.10 | 0.58 |
| 1 | 0.30 | 0.30 | -0.16 | 0.85 |

$Z=6$

| | | | | |
|---|------|------|-------|-------|
| 2 | 0.90 | 0.30 | 0.12 | -1.00 |
| 3 | 5.60 | 4.40 | 0.00 | 0.72 |
| 1 | 0.30 | 0.30 | -0.57 | 3.46 |
| 2 | 0.90 | 0.30 | 1.78 | -3.87 |
| 3 | 5.60 | 4.40 | -1.58 | 0.37 |

Table 1: Percentage differences for each bin compared to nominal.

**In general no big difference
is observed apart from Be
and N**

Summary and Conclusions

- Two different kind of the unfolding methods systematics has been investigated: (*internal, external*)
- The ***internal method systematics*** results at subpercent level almost everywhere in the unfolded distributions and one order of magnitude smaller wrt the statistical uncertainty —> **can be safely neglected**
- ***External method systematics*** by looking at two alternative unfolding methods:
 - Singular Value Decomposition (SVD)
 - Iterative Dynamically Stabilized (IDS)
- SVD suffers of no conservation of the integral and strong dependance on the regularization parameter choice, while IDS shows a more stable behavior.

Reasonable proposal: *Assess ONE unfolding systematics by taking the relative difference between our nominal method (Bayes) and the IDS one.*

backup