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Laboratori Nazionali di Frascati

Trinification from a "realistic" E_6 GUT model

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In collaboration with K. S. Babu, Borut Bajc
Based on 2305.16398 and 2403.20278.

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Outline

(1) Introduction and motivation:

Build a "realistic" E_6 *GUT model that has* $\mathrm{SU}(3)^3$ *(trinification) or* $\mathrm{SU}(6) \times \mathrm{SU}(2)$ *symmetry at an intermediate breaking stage.*

(2) Model building considerations

(2.1) Introducing the group E_6

(2.2) How to get trinification as an intermediate symmetry

(3) Spontaneous symmetry breaking via **650** of E_6

(4) A realistic model: $\mathbf{650} \oplus \mathbf{27} \oplus \mathbf{351}'$

(4.1) Symmetry breaking patterns

(4.2) Yukawa sector

(4.3) Unification analysis

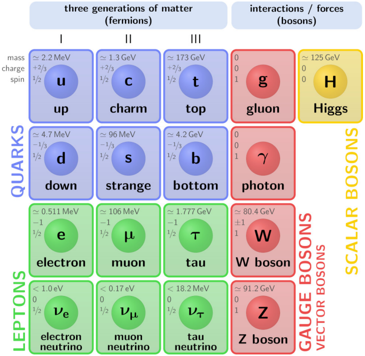
(4.4) Proton decay

(5) Conclusions

(1) Introduction and motivation

Introduction — Standard Model

- Standard Model (SM) is well established at accessible energies



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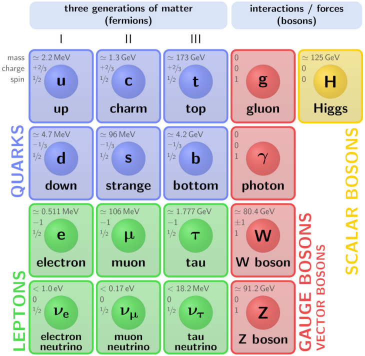

- Gauge group:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$$

- Field content:

fermions: $3 \times (Q \oplus u^c \oplus d^c \oplus L \oplus e^c)$

scalars: H

$$Q \sim (\mathbf{3}, \mathbf{2}, +\tfrac{1}{6}), \qquad L \sim (\mathbf{1}, \mathbf{2}, -\tfrac{1}{2}),$$
$$u^c \sim (\bar{\mathbf{3}}, \mathbf{1}, -\tfrac{2}{3}), \qquad e^c \sim (\mathbf{1}, \mathbf{1}, +1),$$
$$d^c \sim (\bar{\mathbf{3}}, \mathbf{1}, +\tfrac{1}{3}), \qquad H \sim (\mathbf{1}, \mathbf{2}, +\tfrac{1}{2}).$$

- Parameters: (19)

$$g,\ \{y, \theta, \delta\},\ \{\mu^2, \lambda\},\ \theta_{QCD}$$

Introduction — unification of gauge couplings $\rightarrow$ GUT?

- Going beyond SM: puzzles
  (a) ν masses and mixings
  (b) dark matter (DM)
  (c) baryon asymmetry in universe
  (d) $\theta_{QCD} \approx 0$
  (e) ...

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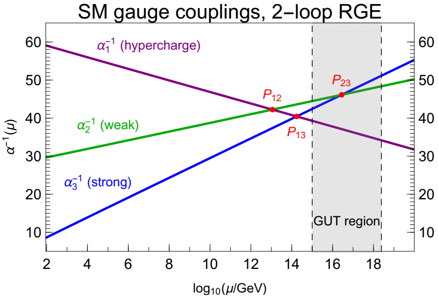
- Curious observation: unification of SM gauge couplings at high E?

  → **Grand Unified Theories**

  (above puzzles can be addressed also in **GUT** framework)



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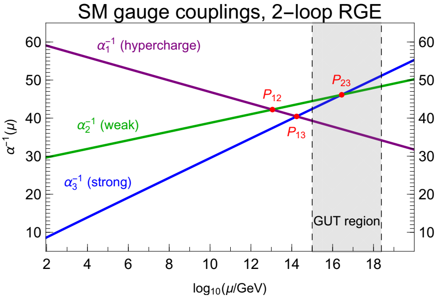

- Unification window for M_U:

  $> 10^{15}\,\mathrm{GeV}$ (proton decay)

  $< 2.4 \cdot 10^{18}\,\mathrm{GeV}$ (Planck scale)

Non-trivial feature:
M_U compatible with window!

Introduction — GUT possibilities

- **GUT**: Yang-Mills theory with a unified group G:
  (1) G is simple
  (2) $G \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
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C_n	$\mathrm{Sp}(2n)$	rotations in $\mathbb{H}^n$	/
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Minimal choices for G:

$$G_{\mathrm{SM}} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$$

Motivation — novel intermediate symmetries from E_6

- Breaking can occur in multiple stages (phase transitions):

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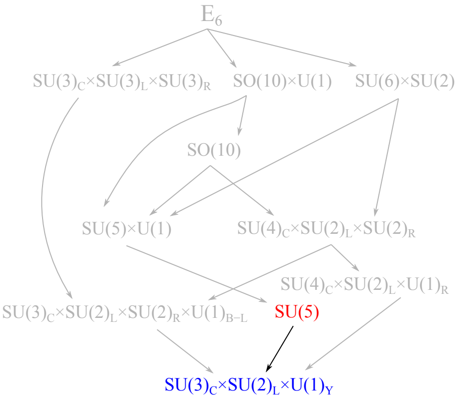

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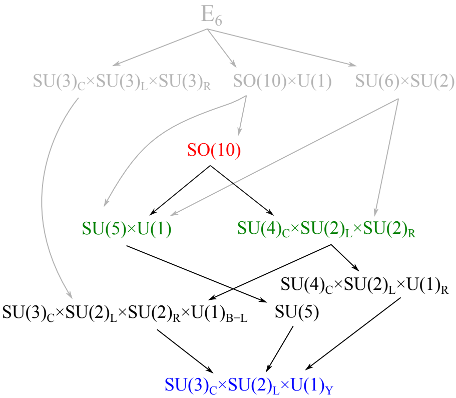

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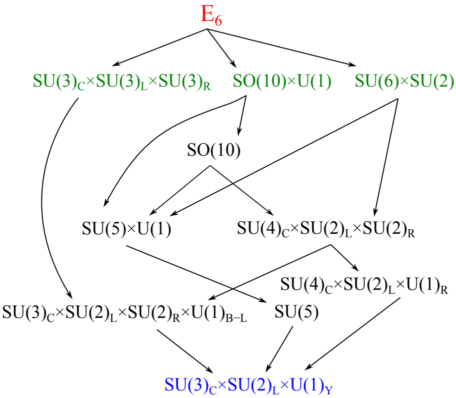

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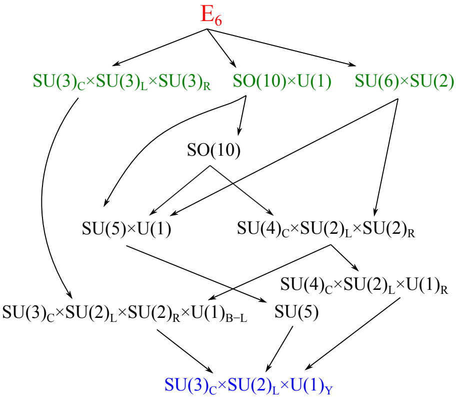
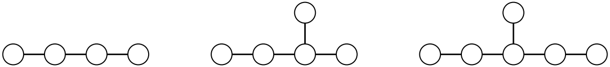


Motivation for this talk:

Build a realistic E_6 GUT model that can break through trinification

(2) Model building considerations

Model building — E_6 group



$$\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$$

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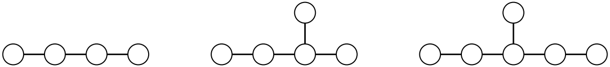

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rank	4		5		6
dimension	24		45		78
fund. irrep	**5**		**10**		**27**

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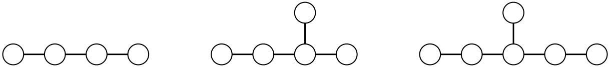

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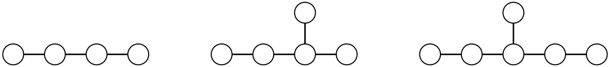

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- Irreducible representations of E_6 (some complex):

$$\mathbf{1},\quad \mathbf{27}^{i},\quad \mathbf{78}^{i}{}_{j},\quad \mathbf{351}^{[ij]},\quad \mathbf{351}'^{(ij)},\quad \mathbf{650}^{i}{}_{j},\quad \mathbf{1728}^{ij}{}_{k},\quad \mathbf{2430}^{ij}{}_{kl},\quad \ldots$$

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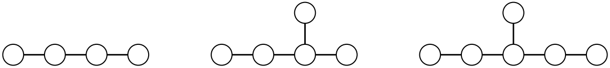

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- Invariant tensors: d_{ijk}, d^{ijk} (completely symmetric), δ^i_j.
(Used also in irrep constraints, e.g. $\mathbf{351}'^{ij} d_{ijk} = 0$.)

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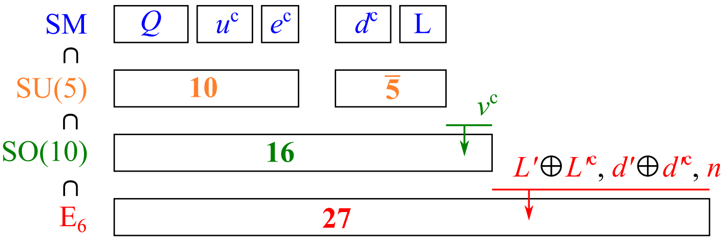
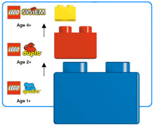
Model building — decomposing the fundamental of E_6

- Matter unification in a fermionic **27**:





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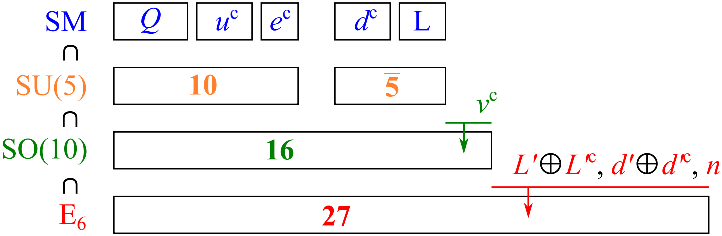

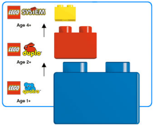

- Fermion exotics in E_6: (heavy)

(a) right-handed neutrinos: $\nu^c, n \sim (1,1,0)$

(b) vector-like d-quarks: $d' \oplus d'^c \sim (3,1,-\tfrac{1}{3}) \oplus (\bar{3},1,+\tfrac{1}{3})$

(c) vector-like L-leptons: $L' \oplus L'^c \sim (1,2,-\tfrac{1}{2}) \oplus (1,2,+\tfrac{1}{2})$

Model building — how to get trinification from E_6?

- Trinification is a maximal subgroup of E_6.
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(1) **R** must have a trinification singlet (framed):

$$\mathbf{27},\quad \mathbf{78},\quad \mathbf{351},\quad \mathbf{351'},\quad \boxed{\mathbf{650}},\quad \mathbf{1728},\quad \boxed{\mathbf{2430}}.$$

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(1) **R** must have a trinification singlet (framed):

27, **78**, **351**, **351**$'$, **650** (framed), **1728**, **2430** (framed).

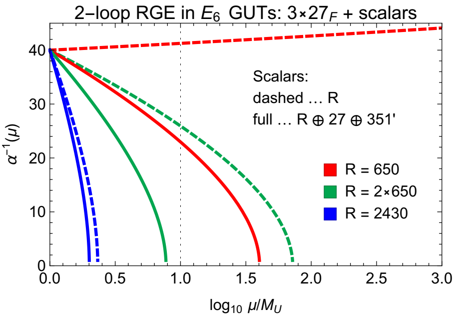
(2) **Perturbativity constraint**:

→ Landau pole at scale Λ

→ non-renormalizable operators suppressed by $(M_U/\Lambda)^{D-4}$

→ under control when $M_U/\Lambda \lesssim 10^{-1}$

→ Only viable choice: **650**



(3) Symmetry breaking via **650** of E_6

Breaking E_6 via **650** — preliminary considerations

- We study breaking via a real scalar **650** [2305.16398]:

$$\mathrm{E}_6 \xrightarrow{\langle \mathbf{650} \rangle} H \tag{1}$$

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→ Maximal groups of E_6: (not the same as in Michel, but helps)

$\mathbf{1}_H \in \mathbf{650}$, $G_{\mathrm{SM}} \subset H$	$\mathbf{1}_H \in \mathbf{650}$	$\mathbf{1}_H \notin \mathbf{650}$
$\mathrm{SU}(3) \times \mathrm{SU}(3) \times \mathrm{SU}(3)$	$\mathrm{G}_2 \times \mathrm{SU}(3)$	$\mathrm{Sp}(8)$
$\mathrm{SU}(6) \times \mathrm{SU}(2)$	F_4	G_2
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→ **Considered candidates**: 1st and 2nd column, **650** has H-singlet(s)

→ **Limitation**: we don't look one level deeper for little groups of 3rd column

Breaking E_6 via **650** — scalar potential and finding solutions

- Scalar potential with **650** $\equiv \mathbf{X}$: $\qquad (D^{ij}{}_{kl} := d^{ijm} d_{klm})$

$$
\begin{aligned}
V(\mathbf{X}) = &-M^2 \cdot \mathrm{Tr}(\mathbf{X}^2) \\
&+ m_1 \cdot \mathrm{Tr}(\mathbf{X}^3) + m_2 \cdot X^i{}_l\, X^j{}_m\, X^k{}_n\, d^{lmn} d_{ijk} \\
&+ \lambda_1 \cdot (\mathrm{Tr}(\mathbf{X}^2))^2 + \lambda_2 \cdot \mathrm{Tr}(\mathbf{X}^4) + \lambda_3 \cdot (\mathbf{X}^2)^k{}_i\, (\mathbf{X}^2)^l{}_j\, D^{ij}{}_{kl} \\
&+ \lambda_4 \cdot X^i{}_{i'} X^j{}_{j'} X^k{}_{k'} X^l{}_{l'}\, D^{i'j'}{}_{kl}\, D^{k'l'}{}_{ij} + \lambda_5 \cdot X^i{}_l\, X^j{}_m\, (\mathbf{X}^2)^k{}_n\, d^{lmn} d_{ijk}.
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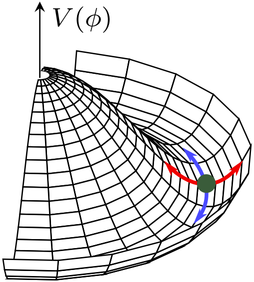
$$\begin{aligned} V(\mathbf{X}) = &-M^2 \cdot \mathrm{Tr}(\mathbf{X}^2) \\ &+ m_1 \cdot \mathrm{Tr}(\mathbf{X}^3) + m_2 \cdot X^i{}_l\, X^j{}_m\, X^k{}_n\, d^{lmn} d_{ijk} \\ &+ \lambda_1 \cdot (\mathrm{Tr}(\mathbf{X}^2))^2 + \lambda_2 \cdot \mathrm{Tr}(\mathbf{X}^4) + \lambda_3 \cdot (\mathbf{X}^2)^k{}_i\, (\mathbf{X}^2)^l{}_j\, D^{ij}{}_{kl} \\ &+ \lambda_4 \cdot X^i{}_{i'} X^j{}_{j'} X^k{}_{k'} X^l{}_{l'}\, D^{i'j'}{}_{kl}\, D^{k'l'}{}_{ij} + \lambda_5 \cdot X^i{}_l\, X^j{}_m\, (\mathbf{X}^2)^k{}_n\, d^{lmn} d_{ijk}. \end{aligned} \tag{2}$$

- *Singlet ansatz:* take only $\mathbf{1}_H$-directions in $\mathbb{R}^{650}$
  - → self-consistent ansatz
  - → need to solve only $\partial_{\mathbf{1_H}} V = 0$, other directions automatic
  - → compute **all** masses with $\partial_i \partial_j V$
  - → For local minimum: masses positive or would-be Goldstones



Breaking E_6 via **650** — good solutions for intermediate groups H

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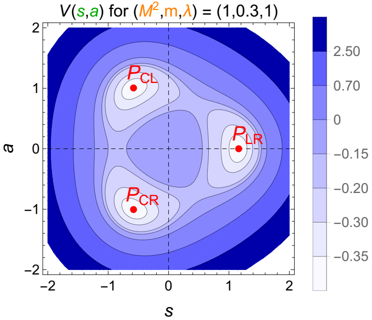

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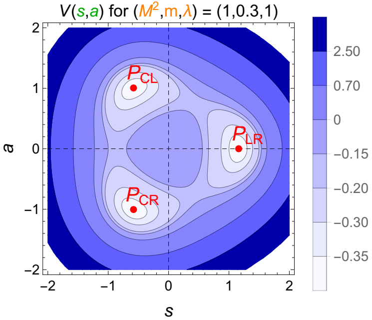

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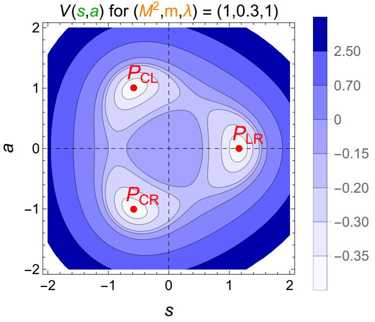

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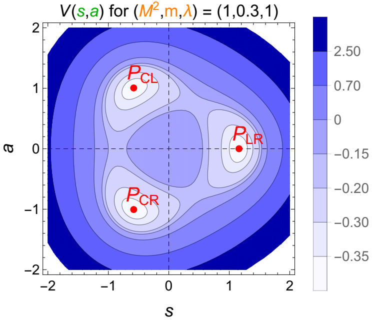

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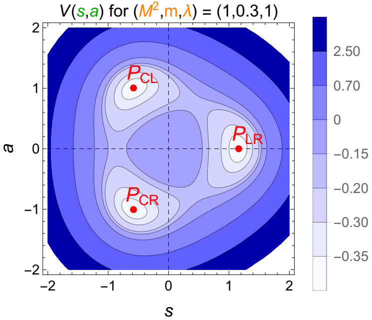

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  - → $m_H = \sum \alpha_i m_i$, $\lambda_H = \sum \beta_i \lambda_i$ are H-dependent
  - → Minimize $V(x)$ analytically (easy): we get solutions for

$$\mathrm{SU}(6)\times\mathrm{SU}(2), \qquad \mathrm{SO}(10)\times\mathrm{U}(1), \qquad \mathrm{F}_4, \qquad \mathrm{G}_2\times\mathrm{SU}(3). \qquad (3)$$

(4) Realistic E_6 model: $\mathbf{650} \oplus \mathbf{27} \oplus \mathbf{351}'$

Realistic E_6 model — setup

- Field content: [2403.20278]

fermions:	$3 \times \mathbf{27}_F$
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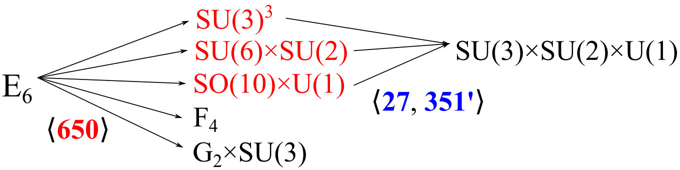

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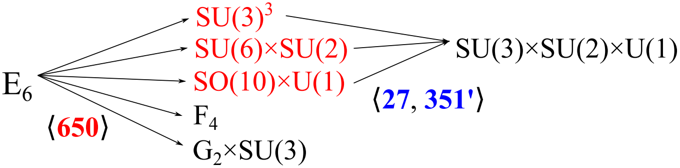

- Renormalizable Yukawa sector: 2 symmetric matrices

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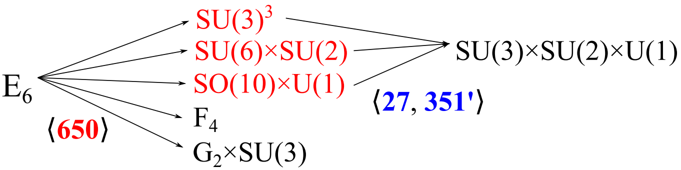

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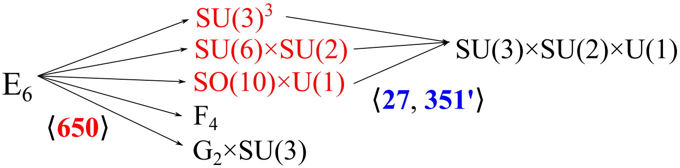

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  - Realistic, analog of the $\mathrm{SO}(10)$ case: $\mathbf{16}_F\,\mathbf{16}_F\,\mathbf{10} + \mathbf{16}_F\,\mathbf{16}_F\,\overline{\mathbf{126}}$

Realistic E_6 model — symmetry breaking and SM embeddings

- Breaking pattern to SM: $\mathrm{E}_6 \xrightarrow{\langle \mathbf{650} \rangle} H \xrightarrow{\langle \mathbf{27}, \mathbf{351}' \rangle} G_{\mathrm{SM}}$

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- Viable H-vacua leading to G_{SM}:

name	intermediate symmetry	viable?	
trinification	$\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$	✓✓✓	(LR,CL,CR)
standard	$\mathrm{SU}(6)_{CL} \times \mathrm{SU}(2)_R$	✓	
flipped	$\mathrm{SU}(6)_{CL} \times \mathrm{SU}(2)_{R'}$	—	unifies in $\mathrm{SU}(6)$
LR-flipped	$\mathrm{SU}(6)_{CR} \times \mathrm{SU}(2)_L$	✓	
standard	$\mathrm{SO}(10) \times \mathrm{U}(1)$	—	unifies in $\mathrm{SO}(10)$
flipped	$\mathrm{SO}(10)' \times \mathrm{U}(1)'$	✓	

Realistic E_6 model — Yukawa sector

- Yukawa sector Lagrangian:

$$\mathcal{L} \supset \mathbf{Y}_{27}\, \mathbf{27}_F{}^i\, \mathbf{27}_F{}^j\, \mathbf{27}^k\, d_{ijk} + \mathbf{Y}_{351'}\, \mathbf{27}_F{}^i\, \mathbf{27}_F{}^j\, \mathbf{351}'^*{}_{ij} + h.c. \tag{5}$$

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$$\begin{aligned}\mathcal{L}_{ude} = &\quad (u)^T \left(-\mathbf{Y}_{27} v_7 + \tfrac{1}{\sqrt{15}} \mathbf{Y}_{351'} v_8\right) (u^c)\,, \\ &+ \begin{pmatrix} d^c \\ d'^c \end{pmatrix}^T \begin{pmatrix} \mathbf{Y}_{27}\, v_1^* - \tfrac{1}{\sqrt{15}} \mathbf{Y}_{351'} v_2^* & \mathbf{Y}_{27} V_1 + \tfrac{1}{\sqrt{15}} \mathbf{Y}_{351'}\, W_1{}^* \\ -\mathbf{Y}_{27} v_4^* + \tfrac{1}{\sqrt{15}} \mathbf{Y}_{351'} v_5^* & -\mathbf{Y}_{27} V_2 + \tfrac{1}{\sqrt{15}} \mathbf{Y}_{351'}\, W_2{}^* \end{pmatrix} \begin{pmatrix} d \\ d' \end{pmatrix} \\ &+ \begin{pmatrix} e \\ e' \end{pmatrix}^T \begin{pmatrix} -\mathbf{Y}_{27} v_1^* - \tfrac{1}{2}\sqrt{\tfrac{3}{5}} \mathbf{Y}_{351'} v_2^* + \tfrac{1}{2} \mathbf{Y}_{351'} v_3^* & \mathbf{Y}_{27} V_1 - \tfrac{1}{2}\sqrt{\tfrac{3}{5}} \mathbf{Y}_{351'}\, W_1{}^* \\ \mathbf{Y}_{27} v_4^* + \tfrac{1}{2}\sqrt{\tfrac{3}{5}} \mathbf{Y}_{351'} v_5^* + \tfrac{1}{2} \mathbf{Y}_{351'} v_6^* & -\mathbf{Y}_{27} V_2 - \tfrac{1}{2}\sqrt{\tfrac{3}{5}} \mathbf{Y}_{351'}\, W_2{}^* \end{pmatrix} \begin{pmatrix} e^c \\ e'^c \end{pmatrix}\end{aligned} \tag{6}$$

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- Exotics at M_I, SM fermions at EW. **Mixing**: (d^c, d'^c) and (e, e').

Realistic E_6 model — neutrinos

- Explicit fermion mass matrices for neutrinos

$$\mathcal{L}_\nu = \tag{7}$$

$$\begin{pmatrix}\nu \\ \nu'\end{pmatrix}^T \begin{pmatrix} \mathbf{Y}_{27}\, v_7 + \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, v_8 - \tfrac{1}{2}\mathbf{Y}_{351'}\, v_9 & -\tfrac{1}{\sqrt{2}}\mathbf{Y}_{351'}\, v_{11} & \mathbf{Y}_{27}\, V_1 - \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, W_1{}^* \\ -\tfrac{1}{\sqrt{2}}\mathbf{Y}_{351'}\, v_{10} & -\mathbf{Y}_{27}\, v_7 - \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, v_8 - \tfrac{1}{2}\mathbf{Y}_{351'}\, v_9 & -\mathbf{Y}_{27}\, V_2 - \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, W_2{}^* \end{pmatrix} \begin{pmatrix}\nu^c \\ n \\ \nu'^c\end{pmatrix}$$

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- Neutrino spectrum:

Realistic E_6 model — neutrinos

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- Neutrino spectrum:

$\rightarrow$ Heavy vector-like neutrino pair at M_I: $(\alpha\nu + \beta\nu') \oplus \nu'^c$

Realistic E_6 model — neutrinos

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$$
\mathcal{L}_\nu = \tag{7}
$$

$$
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$$

$$
+ \begin{pmatrix} \nu^c \\ n \\ \nu'^c \end{pmatrix}^T \begin{pmatrix} \mathbf{Y}_{351'}\, {W_3}^* & \tfrac{1}{\sqrt{2}}\mathbf{Y}_{351'}\, {W_4}^* & -\mathbf{Y}_{27}\, v_4^* - \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, v_5^* + \tfrac{1}{2}\mathbf{Y}_{351'}\, v_6^* \\ \tfrac{1}{\sqrt{2}}\mathbf{Y}_{351'}\, {W_4}^* & \mathbf{Y}_{351'}\, {W_5}^* & \mathbf{Y}_{27}\, v_1^* + \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, v_2^* + \tfrac{1}{2}\mathbf{Y}_{351'}\, v_3^* \\ -\mathbf{Y}_{27}\, v_4^* - \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, v_5^* + \tfrac{1}{2}\mathbf{Y}_{351'}\, v_6^* & \mathbf{Y}_{27}\, v_1^* + \tfrac{1}{2}\sqrt{\tfrac{3}{5}}\mathbf{Y}_{351'}\, v_2^* + \tfrac{1}{2}\mathbf{Y}_{351'}\, v_3^* & 0 \end{pmatrix} \begin{pmatrix} \nu^c \\ n \\ \nu'^c \end{pmatrix}
$$

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→ Heavy vector-like neutrino pair at M_I: $(\alpha\nu + \beta\nu') \oplus \nu'^c$

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Realistic E_6 model — neutrinos

- Explicit fermion mass matrices for neutrinos

$$
\begin{aligned}
\mathcal{L}_\nu = & \\
& \begin{pmatrix} \nu \\ \nu' \end{pmatrix}^T \begin{pmatrix} \mathbf{Y}_{27} v_7 + \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} v_8 - \frac{1}{2}\mathbf{Y}_{351'} v_9 & -\frac{1}{\sqrt{2}}\mathbf{Y}_{351'} v_{11} & \mathbf{Y}_{27} V_1 - \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} W_1{}^* \\ -\frac{1}{\sqrt{2}}\mathbf{Y}_{351'} v_{10} & -\mathbf{Y}_{27} v_7 - \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} v_8 - \frac{1}{2}\mathbf{Y}_{351'} v_9 & -\mathbf{Y}_{27} V_2 - \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} W_2{}^* \end{pmatrix} \begin{pmatrix} \nu^c \\ n \\ \nu'^c \end{pmatrix} \\
& + \begin{pmatrix} \nu^c \\ n \\ \nu'^c \end{pmatrix}^T \begin{pmatrix} \mathbf{Y}_{351'} W_3{}^* & \frac{1}{\sqrt{2}}\mathbf{Y}_{351'} W_4{}^* & -\mathbf{Y}_{27} v_4^* - \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} v_5^* + \frac{1}{2}\mathbf{Y}_{351'} v_6^* \\ \frac{1}{\sqrt{2}}\mathbf{Y}_{351'} W_4{}^* & \mathbf{Y}_{351'} W_5{}^* & \mathbf{Y}_{27} v_1^* + \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} v_2^* + \frac{1}{2}\mathbf{Y}_{351'} v_3^* \\ -\mathbf{Y}_{27} v_4^* - \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} v_5^* + \frac{1}{2}\mathbf{Y}_{351'} v_6^* & \mathbf{Y}_{27} v_1^* + \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{Y}_{351'} v_2^* + \frac{1}{2}\mathbf{Y}_{351'} v_3^* & 0 \end{pmatrix} \begin{pmatrix} \nu^c \\ n \\ \nu'^c \end{pmatrix}
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\tag{7}
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    Majorana terms for $\{\nu^c, n\}$
  - → Seesaw type I: $(-\beta^*\nu + \alpha^*\nu')$ at EW^2/M_I
    $\{\nu^c, n\}$ at M_I

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- Simplification of mass matrices:
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$$\mathbf{m}_u = -\mathbf{Y}_{27} v_7 + \tfrac{1}{\sqrt{15}} \mathbf{Y}_{351'} v_8, \tag{9}$$
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- Works! Analogous to $\mathrm{SO}(10)$ fit with $\mathbf{10} \oplus \mathbf{126}$ [Ohlsson'19].

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- After careful considerations. . . the effective models are:

vacuum	scalars with ESH $+\ \mathbb{Z}_2^{\psi}$
$3_C 3_L 3_R \rtimes LR$	$2 \times (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$
$3_C 3_L 3_R \rtimes CL$	$2 \times (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ $+2 \times (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) + (\bar{\mathbf{6}}, \mathbf{1}, \bar{\mathbf{6}})$
$3_C 3_L 3_R \rtimes CR$	$2 \times (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ $+2 \times (\mathbf{3}, \mathbf{3}, \mathbf{1}) + (\mathbf{6}, \mathbf{6}, \mathbf{1})$

vacuum	scalars with ESH $+\ \mathbb{Z}_2^{\psi}$
$6_{CL} 2_R$	$(\mathbf{15}, \mathbf{1}) + (\overline{\mathbf{21}}, \mathbf{3}) + (\bar{\mathbf{6}}, \mathbf{2}) + (\mathbf{84}, \mathbf{2})$
$6_{CR} 2_L$	$(\mathbf{15}, \mathbf{1}) + (\overline{\mathbf{105'}}, \mathbf{1}) + (\bar{\mathbf{6}}, \mathbf{2}) + (\mathbf{84}, \mathbf{2})$
$10' \, 1'$	$(\mathbf{16}, +1) + (\mathbf{126}, +2) + (\mathbf{10}, -2)$

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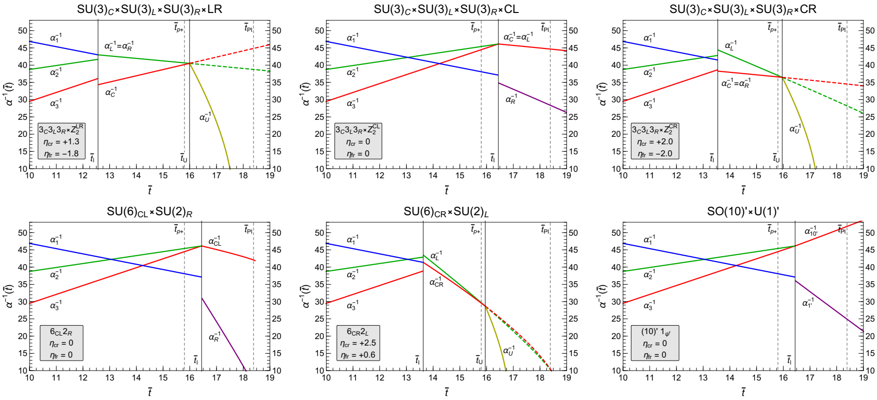

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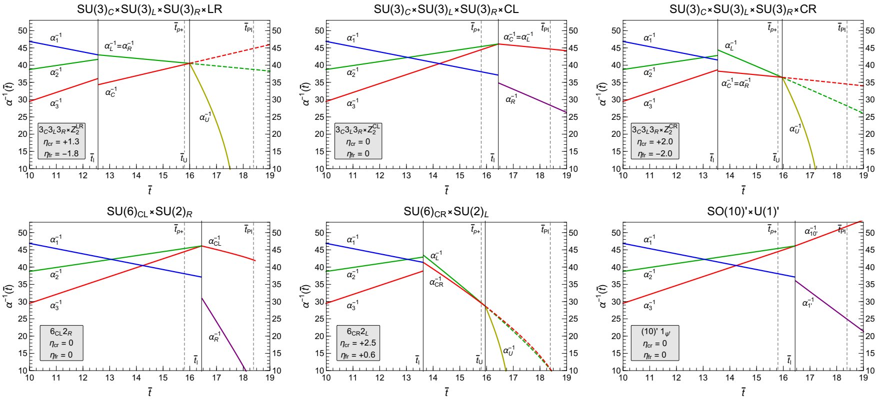

- Viable cases: $3_C 3_L 3_R \rtimes LR$, $3_C 3_L 3_R \rtimes CR$, $6_{CR} 2_L$

Proton decay in the E_6 model

- Gauge mediators of proton decay in E_6: X, X', X''

label	$3_C 2_L 1_Y$	SU(5)	SO(10)	E_6
X	$(3, 2, -5/6)$	24	45	78
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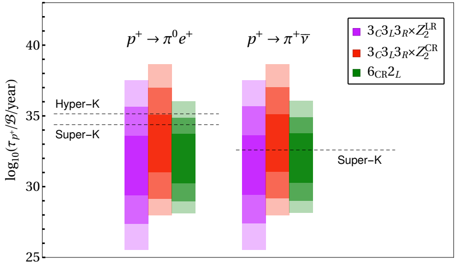

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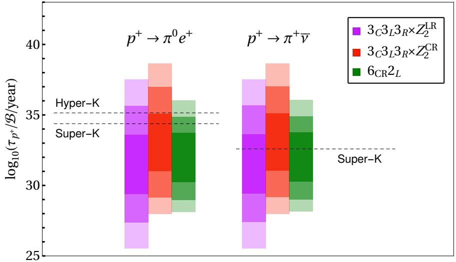

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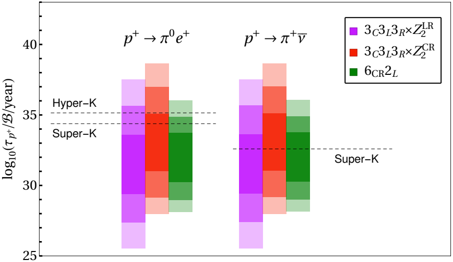

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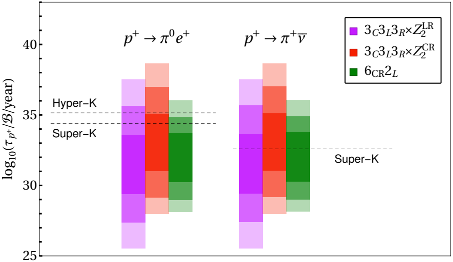
- Large uncertainties in τ_{p^+} (M_{GUT} varies with spectrum)



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Thank you for your attention!