Constraining Theories of Gravity by GINGER Experiment

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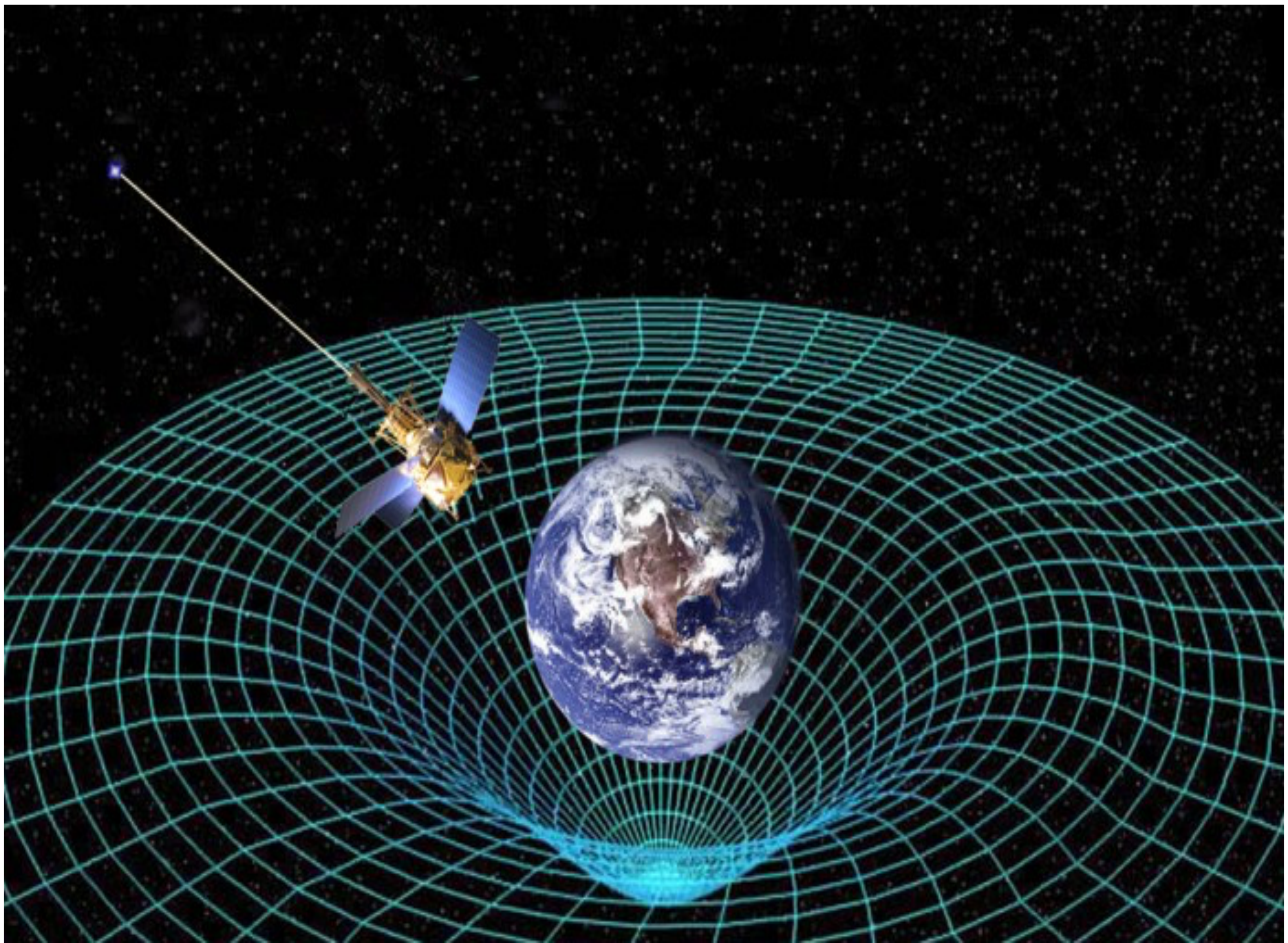

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Plan of the presentation

- *Shortcomings of General Relativity*
- *Modified Theories of Gravity*
- *Spherical Symmetry and Weak-Field Limit*

- **Modified gravity models**

  - Scalar-tensor model
    - Outcomes
    - Future perspectives
  - Horava-Lifshitz model
    - Outcomes
    - Future perspectives

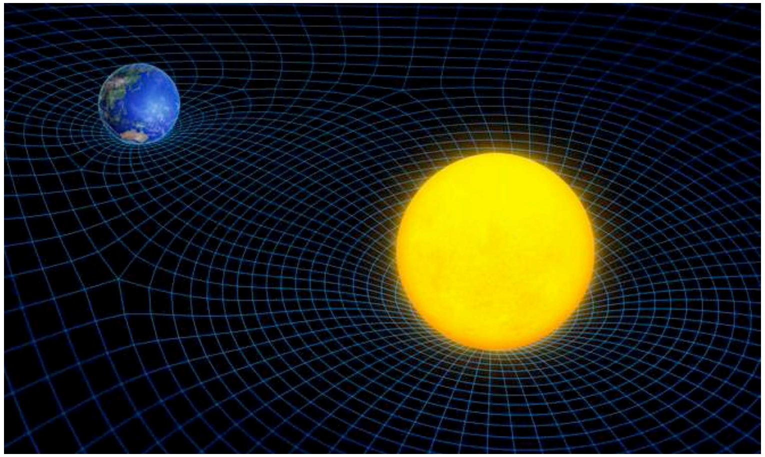
- *Future Perspectives*
- *Conclusions*

General Relativity: foundations and predictions

General Relativity

Describe the gravitational interaction through the spacetime curvature

First theory to successfully pass the Solar System Tests



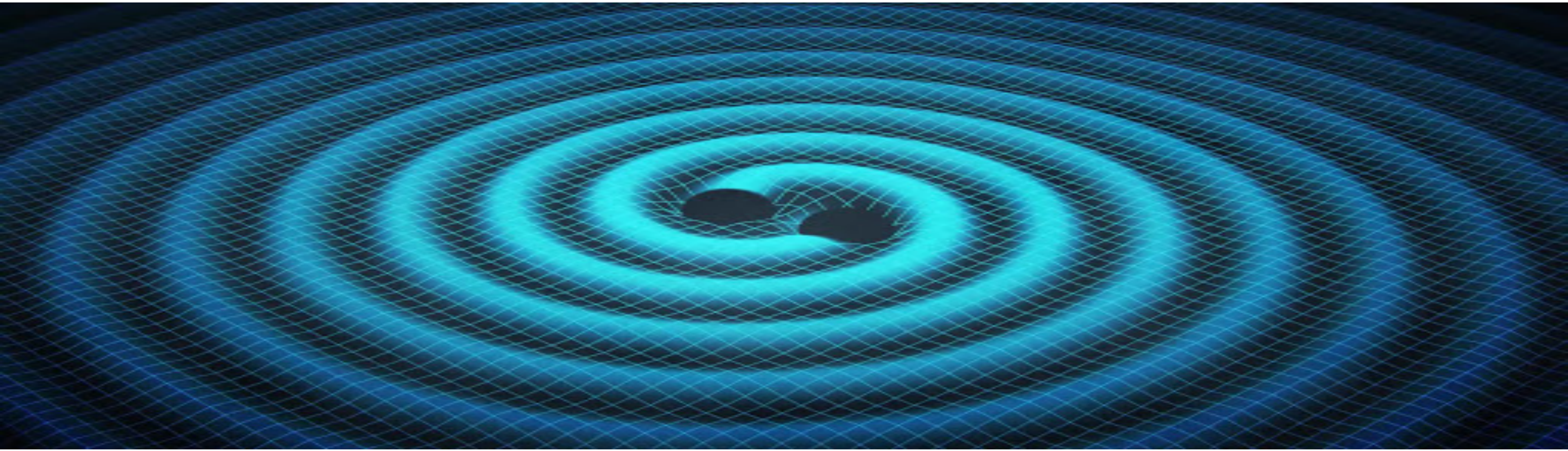
In a static and spherically Symmetric background

Schwarzschild Solution

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{sen}^2\theta d\phi^2$$

- **Black Holes**

- **Gravitational Waves**



• *Lense Thirring Effect*

This effect predicted by GR can be obtained starting from a Kerr-like metric

$$ds^2 = \mathcal{A}(t,r,\theta)dt^2 + \mathcal{B}(t,r,\theta)dr^2 + \mathcal{C}(t,r,\theta)d\theta^2 + \mathcal{D}(t,r,\theta)\sin^2\theta d\phi^2 + \mathcal{E}(t,r,\theta)dt\,d\phi$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$ds^2 = \left(1 - \frac{r_S}{r}\right)dt^2 - \frac{1}{1 - \frac{r_S}{r}\frac{J^2}{M^2 r^2}}dr^2 - r^2 d\theta^2 - \left(r^2 + \frac{J^2}{M^2} + \frac{r_S J^2}{M^2 r}\right)d\phi^2 - \frac{2 r_S J}{M r}dt\,d\phi$$

<ins>*Angular Momentum*</ins>

Correction to the precession of a gyroscope near a large rotating mass, due to the dragging of the spacetime!

$$\Omega_{LT}^{(GR)} = \frac{r_S}{4 M r^3} J$$

General Relativity: shortcomings

Shortcomings of GR

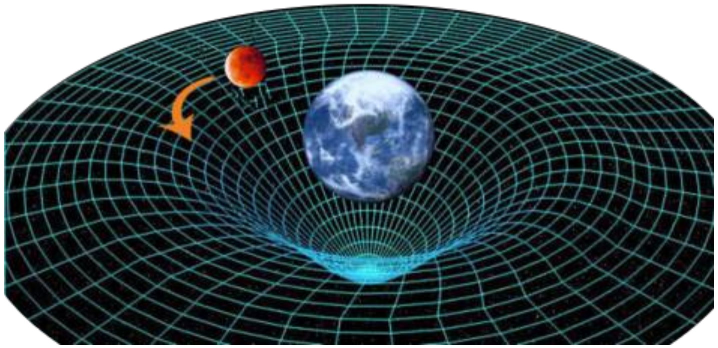
Large Scales

- Universe accelerated expansion
- Inflation
- Galaxy Rotation Curve
- Mass-Radius Diagram of some Neuton Stars
- Fine-tuning cosmological parameters

Small Scales

- Renormalizability
- GR cannot be quantized
- GR cannot be treated under the same standard of other interactions
- Discrepancy between theoretical and experimental value of Λ
- Classical spacetime singularities

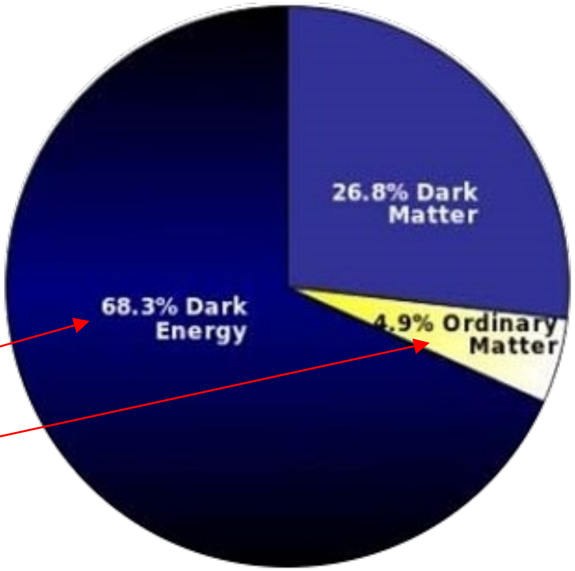
No theory is capable of solving these problems at once so far



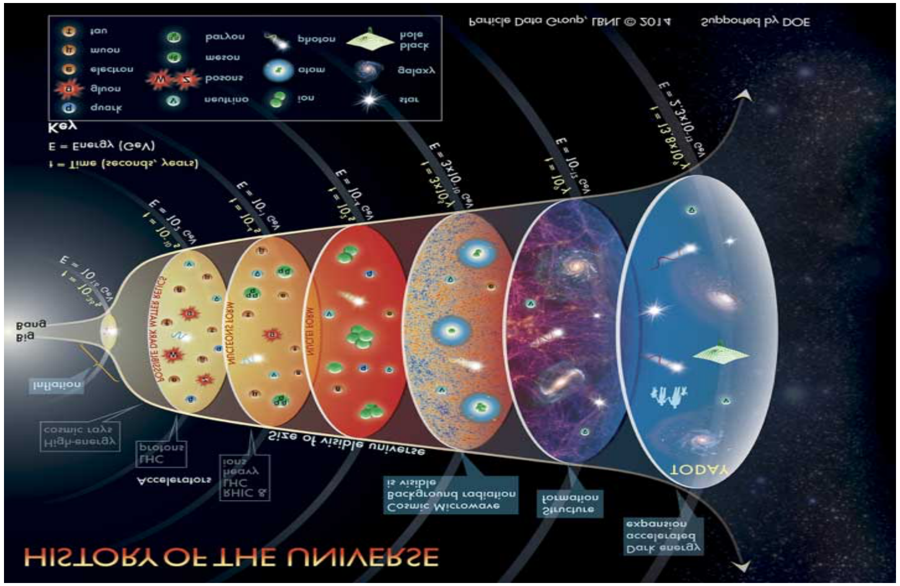
Cosmological Level

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Needs extra scalar fields to predict inflation



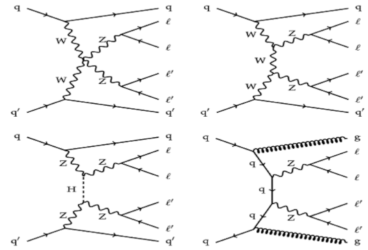
Modified Theories of Gravity

Classification

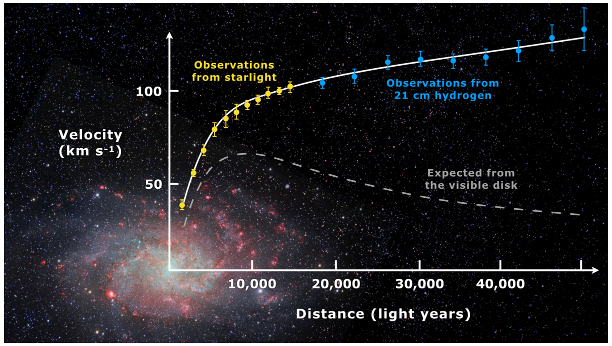
- Extended action → $f(R)$
- Coupling To Scalar fields → $\varphi \cdot R$
- Modified Action → $f(T)$

Motivations:

1. Could account for UV and IR quantum corrections

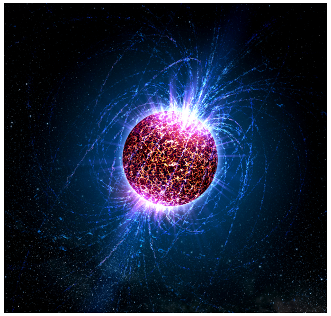


2. Can fit the galaxy rotation curve



3. Contains GR as a particular limit
4. Reproduce both late and early time cosmic evolutions

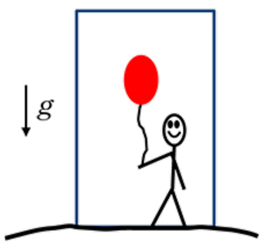
5. Predict the right mass-radius relation of some neutron star without invoking exotic EOS

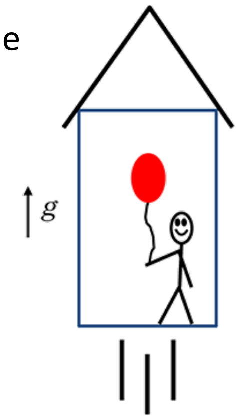


Modified theories of gravity

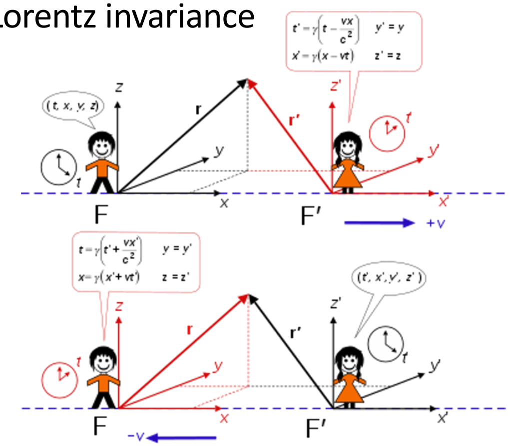
- Relax some assumptions of GR

➢ Equivalence principle



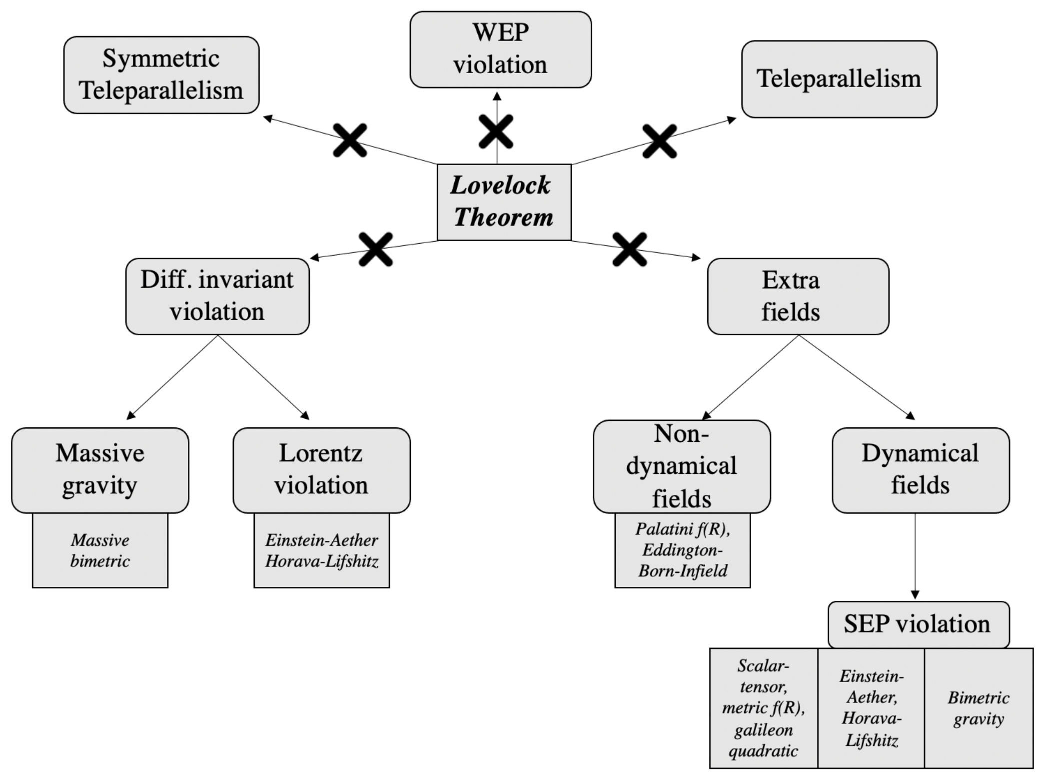


➢ Lorentz invariance



➢ Second-order field equations

$$S = \int \sqrt{-g} F(\phi, R, \Box^z R, R^{\mu\nu} R_{\mu\nu}, R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}) \qquad z \in \mathbb{Z}$$

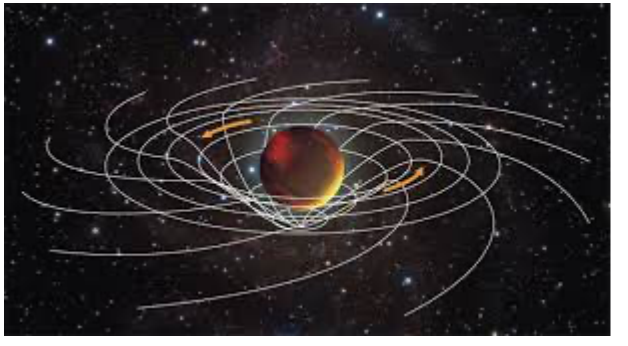


Examples of Modified Gravity Potentials

Modified Gravity Model	Corrected potential	Yukawa parameters
$f(R)$	$\Phi(r) = -\frac{G_N M}{r}\left[1 + \alpha\, e^{-m_R r}\right]$ $\mathbf{A} = \frac{G_N}{r^3}\,\mathbf{r}\times\mathbf{J}$	${m_R}^2 = -\frac{f_R(0)}{6 f_{RR}(0)}$
$f(R, \Box R) = R + a_0 R^2 + a_1 R \Box R$	$\Phi(r) = -\frac{G_N M}{r}\left(1 + c_0 e^{(-r/l_0)} + c_1 e^{(-r/l_1)}\right)$ $\mathbf{A} = \frac{G_N}{r^3}\,\mathbf{r}\times\mathbf{J}$	$c_{0,1} = \frac{1}{6} \mp \frac{a_0}{2\sqrt{9a_0^2 + 6a_1}}$ $l_{0,1} = \sqrt{-3a_0 \pm \sqrt{9a_0^2 + 6a_1}}$
$f(R,\, R_{\alpha\beta}R^{\alpha\beta}, \phi) + \omega(\phi)\phi_{;\alpha}\phi^{;\alpha}$	$\Phi(r) = -\frac{G_N M}{r}\Big[1 + g(\xi,\eta)\, e^{-m_R \tilde{k}_R\, r} +$ $+[1/3 - g(\xi,\eta)]\, e^{-m_R \tilde{k}_\phi\, r} - \frac{4}{3}\, e^{-m_Y r}\Big]$ $\mathbf{A} = \frac{G_N}{r^3}\left[1 - (1 + m_Y r)\, e^{-m_Y r}\right]\mathbf{r}\times\mathbf{J}$	${m_R}^2 = -\frac{1}{3 f_{RR}(0,0,\phi^{(0)}) + 2 f_Y(0,0,\phi^{(0)})}$ ${m_Y}^2 = \frac{1}{f_Y(0,0,\phi^{(0)})}$ ${m_\phi}^2 = -\frac{f_{\phi\phi}(0,0,\phi^{(0)})}{2\omega(\phi^{(0)})}$ $\xi = \frac{3 f_{R\phi}(0,0,\phi^{(0)})^2}{2\omega(\phi^{(0)})}$ $\eta = \frac{m_\phi}{m_R}$ $g(\xi,\,\eta) = \frac{1 - \eta^2 + \xi + \sqrt{\eta^4 + (\xi-1)^2 - 2\eta^2(\xi+1)}}{6\sqrt{\eta^4 + (\xi-1)^2 - 2\eta^2(\xi+1)}}$ $\tilde{k}^2_{R,\phi} = \frac{1 - \xi + \eta^2 \pm \sqrt{(1 - \xi + \eta^2)^2 - 4\eta^2}}{2}$

Is it possible to find out probes and test-beds for ETGs?

- *Geodesic motions around compact objects e.g- SgrA**
- **Lense-Thirring effect** →



- *Exact torsion-balance experiments*
- *Microgravity experiments from atomic physics*
- *Violation of Equivalence Principle (effective masses related to further gravitational degrees of freedom)*

Gravity models constrained by GINGER

Horava-Lifshits Gravity

$$S = \int d^3x\, dt\, \sqrt{-g} \left\{ \frac{2}{\kappa^2} \left(K_{ij}K^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2w^4} \left(\nabla_i R_{jk} \nabla^i R^{jk} - \nabla_i R_{jk} \nabla^j R^{ik} - \frac{1}{8} \nabla_i R \nabla^i R \right) \right\}$$

$$ds^2 = N^2 dt^2 - g_{ij} \left(dx^i + N^i dt \right) \left(dx^j + N^j dt \right)$$

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) \qquad K^2 = g_{ij} K^{ij}$$

General Scalar-Tensor Gravity

$$S = \int \sqrt{-g} \left[f\left(R, R_{\alpha\beta} R^{\alpha\beta}, \phi \right) + \omega(\phi) \nabla_\alpha \phi \nabla^\alpha \phi \right] d^4x,$$

$$Y \equiv R^{\mu\nu} R_{\mu\nu}$$

$\omega(\phi) \longrightarrow$ Kinetic term: general function of ϕ

Both provide Schwarzschild solution as a particular limit

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 - \left(1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{sen}^2 \theta d\phi^2$$

Why using GINGER to constrain ETGs?

1. **GINGER is an Earth-based apparatus**
2. **GINGER provides a measurement: ‘repeateble local here and now’**
3. **The conditions of the experiment are controllable**
4. **First direct measurement of a Post Newtonian effects on the Earth surface**
5. **Experiment based at different latitudes can be compared to improve the precision of measurements and further constrain the models**

Moreover…

- *De Sitter and Lense Thirring precessions are the two main effects of GR (and extensions) on Earth*
- *Due to the non linearity of GR, measurements at different curvature are required*

General Scalar-Tensor Theory

$$S = \int \sqrt{-g}\left[f\left(R, R_{\alpha\beta}R^{\alpha\beta}, \phi\right) + \omega(\phi)\nabla_\alpha\phi\nabla^\alpha\phi\right] d^4x$$

Field equations

$$f_R R_{\mu\nu} - \frac{f + \omega(\phi)\nabla^\alpha\phi\nabla_\alpha\phi}{2} g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R + g_{\mu\nu}\Box f_R + 2 f_Y R^\alpha_\mu R_{\alpha\nu}$$

$$-2 f_Y\left(\nabla_\alpha\nabla_\nu R^\alpha_\mu + \nabla_\alpha\nabla_\mu R^\alpha_\nu\right) + \Box\left(f_Y R_{\mu\nu}\right) + g_{\mu\nu}\nabla_\beta\nabla_\alpha\left(f_Y R^{\alpha\beta}\right) + \omega(\phi)\nabla_\mu\phi\nabla_\nu\phi = 0$$

Klein-Gordon equation

$$2\omega(\phi)\Box\phi + \omega_\phi(\phi)\nabla_\alpha\phi\nabla^\alpha\phi - f_\phi = 0$$

Properties:

- **Explain late and early time evolution without DM and DE**
- **Fit the experimental observations at the astrophysical level**

Horava-Lifshitz Theory

Properties:

- **Theory of quantum gravity capable of solving the small-scale shortcomings of GR**
- **Lorentz-Invariance emerges at large distances**
- **Successfully passes the Solar System Tests**

***One possible* spherically symmetric solution:**

$$g_{00} = (g_{11})^{-1} = 1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}$$

$\omega \longrightarrow$ Constant

Schwarzschild solution: $4M/\omega r^3 \ll 1$

***However*.....**

Exact spherically symmetric solutions in ETGs are very rare

↕

Weak Field Limit

General description of Weak-Field limit

Motivations:

Often exact solutions in ETGs cannot be found analytically

The typical values of the Newtonian gravitational potential Φ are larger than 10^{-5} in the Solar System (in geometrized units, Φ is dimensionless).

Scheme:

Linearization of the metric tensor

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g^{(2)}_{00} + g^{(4)}_{00} + \dots & g^{(3)}_{0i} + \dots \\ g^{(3)}_{0i} + \dots & -\delta_{ij} + g^{(2)}_{ij} + \dots \end{pmatrix}$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}(\partial_\sigma\partial_\mu h^\sigma_\nu + \partial_\sigma\partial_\nu h^\sigma_\mu - \partial_\mu\partial_\nu h - \Box h_{\mu\nu} - \eta_{\mu\nu}\partial_\rho\partial_\lambda h^{\rho\lambda} + \eta_{\mu\nu}\Box h)$$

First case: $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ *gravity*

Application of the PN limit to $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

Linearization of the metric tensor

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g_{00}^{(2)} + g_{00}^{(4)} + \dots & g_{0i}^{(3)} + \dots \\ g_{0i}^{(3)} + \dots & -\delta_{ij} + g_{ij}^{(2)} + \dots \end{pmatrix} = \begin{pmatrix} 1 + 2\phi + 2\Xi & 2A_i \\ 2A_i & -\delta_{ij} + 2\Psi\delta_{ij} \end{pmatrix}$$

- *Three potentials arise: two scalar potentials and one vector potential*
- *Φ, Ψ are proportional to the power c^{-2} (Newtonian limit) while A_i is proportional to c^{-3} and Ξ to c^{-4} (post-Newtonian limit)*

$$ds^2 = \mathcal{A}(t,r,\theta)dt^2 + \mathcal{B}(t,r,\theta)dr^2 + \mathcal{C}(t,r,\theta)d\theta^2 + \mathcal{D}(t,r,\theta)\sin^2\theta d\phi^2 + \mathcal{E}(t,r,\theta)dt\,d\phi$$

$$g_{00} \equiv \mathcal{A}(t,r,\theta)$$
$$g_{0i} = \mathcal{E}(t,r,\theta)$$
$$g_{ij}\delta^{ij} = \mathcal{B}(t,r,\theta) + \mathcal{C}(t,r,\theta) + \mathcal{D}(t,r,\theta)$$

Kerr spacetime

S. Capozziello, G. Lambiase, M. Sakellariadou and A. Stabile, ``Constraining models of extended gravity using Gravity Probe B and LARES experiments,'' Phys. Rev. D **91** (2015) no.4, 044012

Application of the PN limit to $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

By means of the decomposition of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

$$\begin{aligned} h_{00} &\sim \mathcal{O}(2) \\ h_{0i} &\sim \mathcal{O}(3) \\ h_{ij} &\sim \mathcal{O}(2), \end{aligned}$$

The function f, up to the c^{-4} order, can be developed as:

$$f(R, R_{\alpha\beta}R^{\alpha\beta}, \phi) = f_R(0,0,\phi^{(0)})R + \frac{f_{RR}(0,0,\phi^{(0)})}{2}R^2 + \frac{f_{\phi\phi}(0,0,\phi^{(0)})}{2}(\phi - \phi^{(0)})^2 + f_{R\phi}(0,0,\phi^{(0)})R\phi + f_Y(0,0,\phi^{(0)})R_{\alpha\beta}R^{\alpha\beta},$$

S. Capozziello, G. Lambiase, M. Sakellariadou and A. Stabile, ``Constraining models of extended gravity using Gravity Probe B and LARES experiments,'' Phys. Rev. D **91** (2015) no.4, 044012

Application of the PN limit to $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

***Result**:*

- *Form of the vector potential* →

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1-(1+m_Y|\mathbf{x}|)\,e^{-m_Y|\mathbf{x}|}\right]\hat{\mathbf{x}}\times\mathbf{J}$$

- *Form of the scalar potential* →

$$\phi(r) = -\frac{GM}{r}\left[1+g(\xi,\eta)e^{-m_R\tilde{k}_R r} + [1/3-g(\xi,\eta)]e^{-m_R\tilde{k}_\phi r} - \frac{4}{3}e^{-m_Y r}\right]$$

with the definitions:

$$m_R^2 = -\frac{1}{3f_{RR}\left(0,0,\phi^{(0)}\right)+2f_Y\left(0,0,\phi^{(0)}\right)}$$

$$m_Y^2 = \frac{1}{f_Y\left(0,0,\phi^{(0)}\right)} \qquad \eta = \frac{m_\phi}{m_R}$$

$$m_\phi^2 = -\frac{f_{\phi\phi}\left(0,0,\phi^{(0)}\right)}{2\omega\left(\phi^{(0)}\right)} \qquad g(\xi,\eta) = \frac{1-\eta^2+\xi+\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}{6\sqrt{\eta^4+(\xi-1)^2-2\eta^2(\xi+1)}}$$

$$\xi = \frac{3f_{R\phi}\left(0,0,\phi^{(0)}\right)^2}{2\omega\left(\phi^{(0)}\right)} \qquad \tilde{k}_{R,\phi}^2 = \frac{1-\xi+\eta^2\pm\sqrt{\left(1-\xi+\eta^2\right)^2-4\eta^2}}{2}$$

Lense-Thirring precession in $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity

$$\Omega_{\rm LT}^{\rm (EG)} = \frac{1}{2}(\epsilon^{ijk}\partial_i A_k)(\epsilon_{\ell nk}\partial^\ell A^k) = \frac{G}{r^3}\sqrt{(\epsilon_{\ell km}\partial^m \epsilon^{ijk} J_i x_j)^2} = -e^{-m_Y r}\left(1 + m_Y r + m_Y^2 r^2\right)\Omega_{\rm LT}^{\rm (GR)}$$

where

$$\mathbf{A}(\mathbf{x}) = \frac{G}{|\mathbf{x}|^2}\left[1 - (1 + m_Y|\mathbf{x}|)\, e^{-m_Y|\mathbf{x}|}\right]\hat{\mathbf{x}} \times \mathbf{J} \quad Y \equiv R^{\mu\nu}R_{\mu\nu}$$

$$\Omega_{LT}^{(GR)} = \frac{r_S}{4Mr^3} J \quad m_Y^2 = \frac{1}{f_Y\left(0, 0, \phi^{(0)}\right)}$$

For $f_Y \to 0$ i.e. $m_Y \to \infty$, we obtain the same outcome for the gravitational potential of $f(R, \phi)$-theory

Experimental constraints

Gyroscopic and Lense-Thirring effects…

$$g_{\mu\nu} \sim \begin{pmatrix} 1 + g_{00}^{(2)} + g_{00}^{(4)} + \dots & g_{0i}^{(3)} + \dots \\ g_{0i}^{(3)} + \dots & -\delta_{ij} + g_{ij}^{(2)} + \dots \end{pmatrix} = \begin{pmatrix} 1 + 2\Phi + 2\Xi & 2A_i \\ 2A_i & -\delta_{ij} + 2\Psi\delta_{ij} \end{pmatrix}.$$

For the model $f(R,\, R_{\alpha\beta}R^{\alpha\beta}, \phi) + \omega(\phi)\phi_{;\alpha}\phi^{;\alpha}$

we have

$$\left\{ \begin{aligned} \mathbf{A} &= \frac{G_N}{r^3}\left[1 - (1 + m_Y r)\, e^{-m_Y r}\right] \mathbf{r} \times \mathbf{J}. \\ \Omega^{\mathrm{LT}}_{(\mathrm{EG})} &= -e^{-m_Y r}\left(1 + m_Y r + m_Y^2 r^2\right) \Omega^{\mathrm{LT}}_{(\mathrm{GR})} \end{aligned} \right\} \rightarrow$$

$${m_Y}^2 = \frac{1}{f_Y(0,0,\phi^{(0)})}$$

$$Y = R_{\alpha\beta}R^{\alpha\beta}$$

The analysis provides

Gravity Probe B

$$m_Y > 7.1 \cdot 10^{-5} m^{-1}$$

GINGER

$$m_Y \geq 1.88 \cdot 10^{-6} m^{-1}$$

but this precision can be further improved

LARES

$$m_Y > 1.2 \cdot 10^{-6} m^{-1}$$

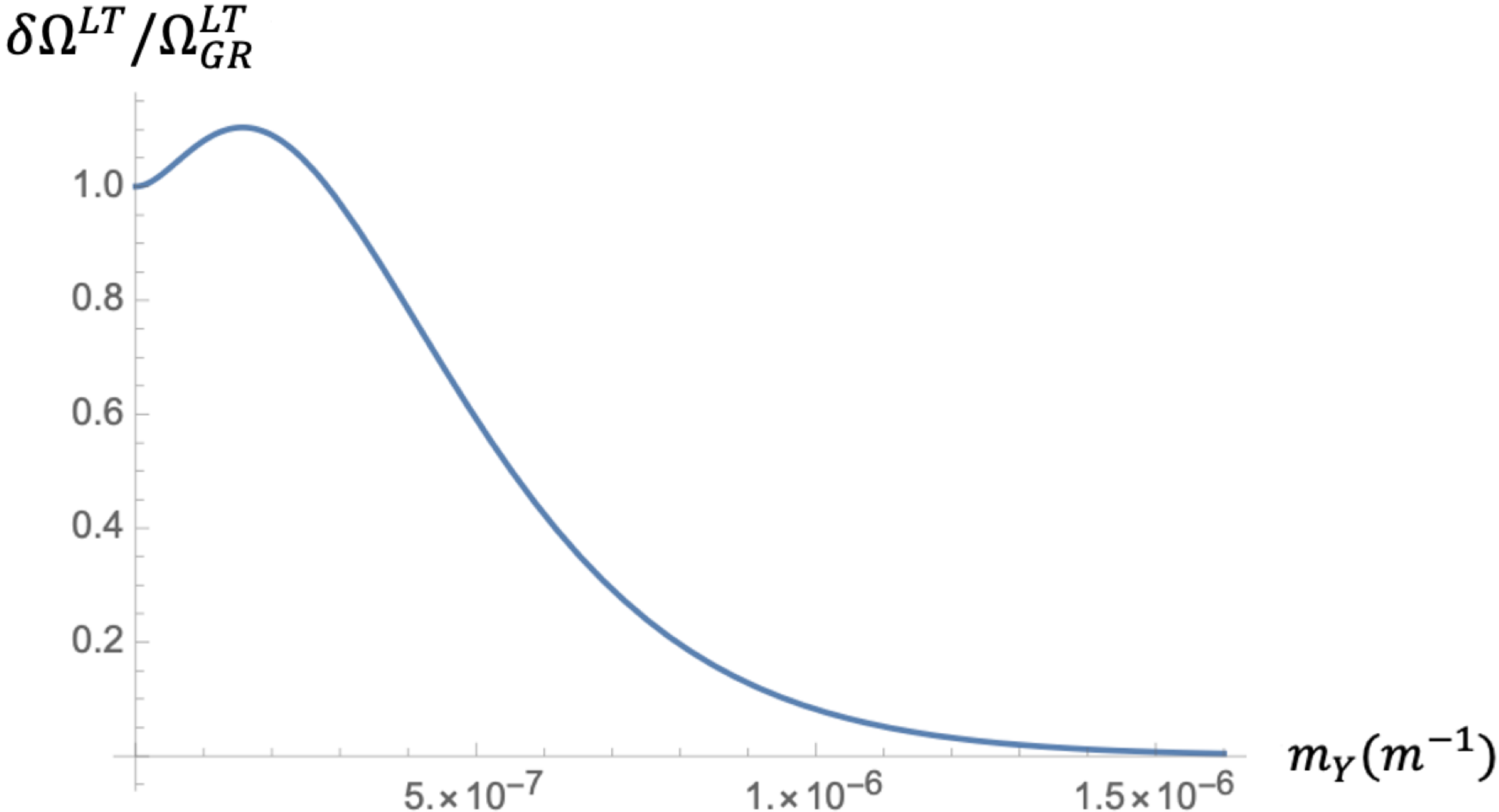


Fig. 1 $\delta\Omega^{LT}/\Omega^{LT}_{GR}$ as a function of m_Y. Note that GR is recovered in the limit $m_Y \to \infty$

S. Capozziello, et al. ``Constraining Theories of Gravity by GINGER experiment,'' Eur. Phys. J. Plus **136** (2021) no.4, 394

Future perpsectives in the context of the scalar-tensor model

Considering the definition of the circular velocity $v_c(r) = \sqrt{r\,\frac{\partial\Phi(r)}{\partial r}}$

GINGER can be used to constrain the effective potentials of modified theories of gravity and, consequently, the corresponding action

Modified Gravity Model	Corrected potential	Yukawa parameters
$f(R)$	$\Phi(r) = -\frac{G_N M}{r}\left[1+\alpha\, e^{-m_R r}\right]$ $\mathbf{A} = \frac{G_N}{r^3}\,\mathbf{r}\times\mathbf{J}$	${m_R}^2 = -\frac{f_R(0)}{6 f_{RR}(0)}$
$f(R,\Box R) = R + a_0 R^2 + a_1 R\Box R$	$\Phi(r) = -\frac{G_N M}{r}\left(1 + c_0 e^{(-r/l_0)} + c_1 e^{(-r/l_1)}\right)$ $\mathbf{A} = \frac{G_N}{r^3}\,\mathbf{r}\times\mathbf{J}$	$c_{0,1} = \frac{1}{6} \mp \frac{a_0}{2\sqrt{9a_0^2+6a_1}}$ $l_{0,1} = \sqrt{-3a_0 \pm \sqrt{9a_0^2+6a_1}}$

with $f_R(0)$ being the first derivative of $f(R)$ with respect to the scalar curvature R, evaluated in $R = 0$

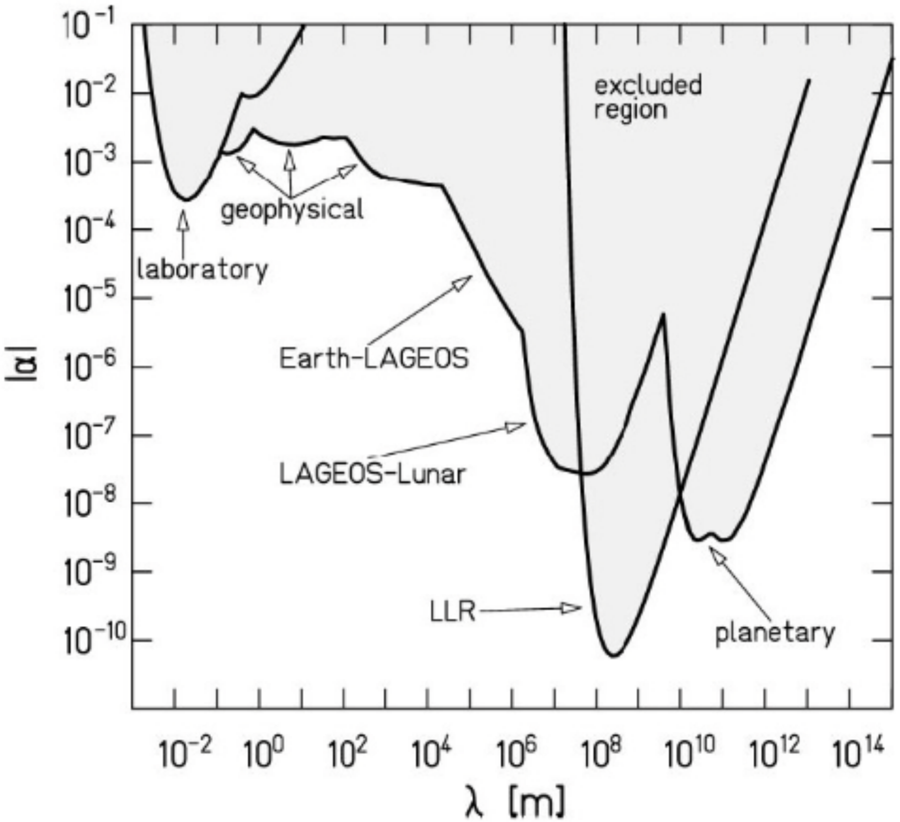
- The analysis can yield a validity range for α and λ. Once the effective potential is constrained, the gravitational model is selected
- Free parameter values can be compared with those selected by other experiments

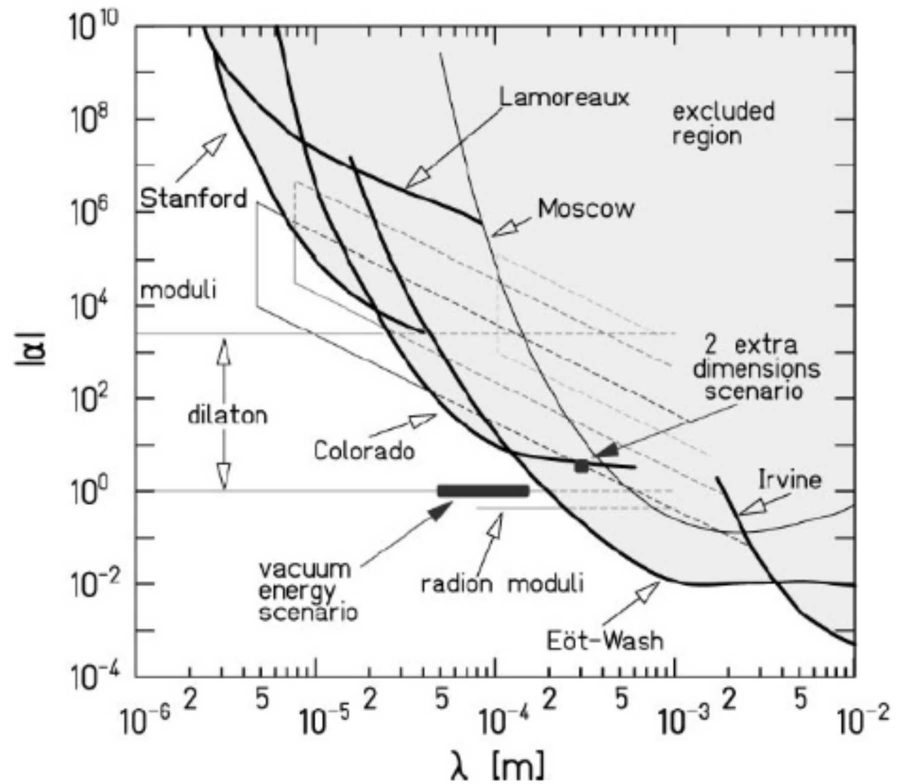
Constraints on f(R) gravity potential provided by other experiments

- α: Dimensionless strength parameter
- λ: Length scale or range

F(R) Gravity Potential:

$$\Phi(r) = \frac{GM}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$





…to be compared with future GINGER outcomes

GINGER results: the case of Horava-Lifshitz Gravity

Horava-Lifshitz Theory

Properties:

- **Theory of quantum gravity capable of solving the small-scale shortcomings of GR**
- **Lorentz-Invariance emerges at large distances**
- **Successfully passes the Solar System Tests**

One possible **spherically symmetric solution:**

$$g_{00} = (g_{11})^{-1} = 1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}$$

$\omega \longrightarrow$ Constant

Schwarzschild solution: $4M/\omega r^3 \ll 1$

Application of PN limit to *Horava-Lifshitz Gravity*

$$S=\int d^3x\,dt\,\sqrt{-g}\left\{\frac{2}{\kappa^2}\left(K_{ij}K^{ij}-\lambda K^2\right)-\frac{\kappa^2}{2w^4}\left(\nabla_i R_{jk}\nabla^i R^{jk}-\nabla_i R_{jk}\nabla^j R^{ik}-\frac{1}{8}\nabla_i R\nabla^i R\right)\right\}$$

$$K_{ij}=\frac{1}{2N}\left(\dot{g}_{ij}-\nabla_i N_j-\nabla_j N_i\right)\qquad K^2=g_{ij}K^{ij}$$

Linearization of the metric tensor

$$g_{\mu\nu}\sim\begin{pmatrix}1+g_{00}^{(2)}+g_{00}^{(4)}+\dots & g_{0i}^{(3)}+\dots\\ g_{0i}^{(3)}+\dots & -\delta_{ij}+g_{ij}^{(2)}+\dots\end{pmatrix}$$

With similar computations as the previous case, the ratio between the Horava-Lifshitz and General Relativity Gyroscopic precession is

$$\frac{\Omega^G_{HL}}{\Omega^G_{GR}}=\frac{1}{3}\left(1+2\frac{G}{G_N}a_1-2\frac{a_2}{a_1}\right)$$

with

a_1, a_2 constants *to be constrained*

Ω^G_{HL} → *Gyroscopic precession*

G → effective gravitational constant

$$\frac{\Omega^G_{HL}}{\Omega^G_{GR}} = \frac{1}{3}\left(1 + 2\frac{G}{G_N}a_1 - 2\frac{a_2}{a_1}\right)$$

Importance of constraining a_1, a_2

It has been shown that, in order for the matter coupling to be consistent with solar system tests, the gauge field and the Newtonian potential must be coupled to matter in a specific way, but there are no indication on how to obtain the precise prescription from the action principle. Recently such a prescription has been generalised and a scalar-tensor extension of the theory has been developed to allow the needed coupling to emerge in the IR without spoiling the power-counting renormalizability of the theory.

Matter action

$$S_M = \int dtd^3x\tilde{N}\sqrt{\tilde{g}}\ \mathcal{L}_M\ (\tilde{N}, \tilde{N}_i, \tilde{g}_{ij}; \psi_n)$$

Lapse function

Scalar Potential

$$\begin{aligned}\tilde{N} &= (1 - a_1\sigma)N, \\ \tilde{N}^i &= N^i + Ng^{ij}\nabla_j\phi, \\ \tilde{g}_{ij} &= (1 - a_2\sigma)^2 g_{ij},\end{aligned}$$

Vector

$$\sigma = \frac{A - \mathcal{A}}{N}, \quad \text{with} \quad \mathcal{A} = -\dot{\phi} + N^i\nabla_i\phi + \frac{1}{2}N\nabla^i\phi\nabla_i\phi.$$

a_1, a_2 are then related to the potentials and can be constrained by GINGER measure as….

Terrestial experiment: GINGER

GINGER measures the difference in frequence of light of two beams circulating in a laser cavity in opposite directions. This translates into a time difference between the right-handed beam propagation time and the left-handed one

$$\delta\tau = -2\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}}\, ds^i$$

The difference in time can be linked to the Sagnac frequence $\boldsymbol{\Omega}_S$*, measured by GINGER*

Perimeter

Laser wavelength

$$c\delta\tau = N(\lambda_+ - \lambda_-) = Nc\left(\frac{f_- - f_+}{f^2}\right) = \frac{P\lambda}{c}\,\delta f \equiv \frac{P\lambda}{c}\,\Omega_S$$

Wavelength difference

Splitting in terms of frequence between the two beams

GIGNER in Horava-Lifshitz Gravity

$$\delta\tau = -2\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}} ds^i \Longleftrightarrow \Omega_S = -\frac{2c^2\sqrt{g_{00}}}{P\lambda} \oint \frac{g_{0i}}{g_{00}} ds^i$$

In Horava-Lifshitz gravity, it is

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2 R}\sin\alpha\sin\theta - \frac{GI_E}{c^2R^3}(2\cos\theta\cos\alpha + \sin\theta\sin\alpha)\right]$$

Sagnac term

Lense-Thirring term

- A → Area encircled by the light beams
- α → Angle between the local radial direction and the normal to the plane of the array-laser ring
- θ → Colatitude of the laboratory
- Ω_E → Rotation rate of the Earth as measured in the local reference frame
- I_E → Momentum of Inertia
- P → Perimeter
- λ → Laser wavelength

Horava-Lifshitz vs General Relativity

General Relativity

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \quad \left[\cos(\theta+\alpha) - \frac{G_N M}{c^2 R}\sin\alpha\sin\theta - \frac{G I_E}{c^2 R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

$G = G_N$

$G = G_N$

$G = G_N$

Horava-Lifshitz Gravity

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E \quad \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2 R}\sin\alpha\sin\theta - \frac{G I_E}{c^2 R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

Advantages to use GINGER

- The actual precision of GINGERINO is 1/1000 in the geodesic term, 1/100 in the LT term
- GINGER experiment should overcome such uncertainty providing a precision of 1/1000 in the LT term
- The presence of two rings yields a dynamic measure of the angle **α**

$$\Omega_S = \frac{4A}{P\lambda}\Omega_E\left[\cos(\theta+\alpha) - \underbrace{\left(1+\frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\alpha\sin\theta}_{\text{Geodesic Term}} - \underbrace{\frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)}_{\text{LT Term}}\right]$$

Notice that:

- *While the measure of the LT term can constrain the value of G, from the measure of the geodesic term we can get the value of a_1 and a_2*
- *The precision of GINGERINO is 10^{-15} rad/s, which corresponds to a precision of 1.4 $\cdot\, 10^{-11}$ with respect to the dominant term.*

Outcomes

It is possible to find relations among a_1, a_2 and the gyroscopic precession in Horava gravity:

Using

$$\left|\frac{\Omega^G_{TOT}-\Omega^G_{GR}}{\Omega^G_{GR}}\right| = \left|\frac{2}{3}\left(\frac{G_{HL}}{G_N}a_1 - \frac{a_2}{a_1} - 1\right)\right|$$

and

$$\left|\frac{\Omega^{LT}_{TOT}-\Omega^{LT}_{GR}}{\Omega^{LT}_{GR}}\right| = \left|\frac{G_{HL}}{G_N} - 1\right|,$$

we find

$$0.999\,G_N < G_{HL} < 1.001\,G_N.$$

For instance, setting

$$\frac{G_{HL}}{G_N} = 0.999$$

$$a_1(0.999a_1 - 0.99985) < a_2 < a_1(0.999a_1 - 1.00015)$$
if $a_1 < 0$,

$$a_1(0.999a_1 - 1.00015) < a_2 < a_1(0.999a_1 - 0.99985)$$
if $a_1 > 0$.

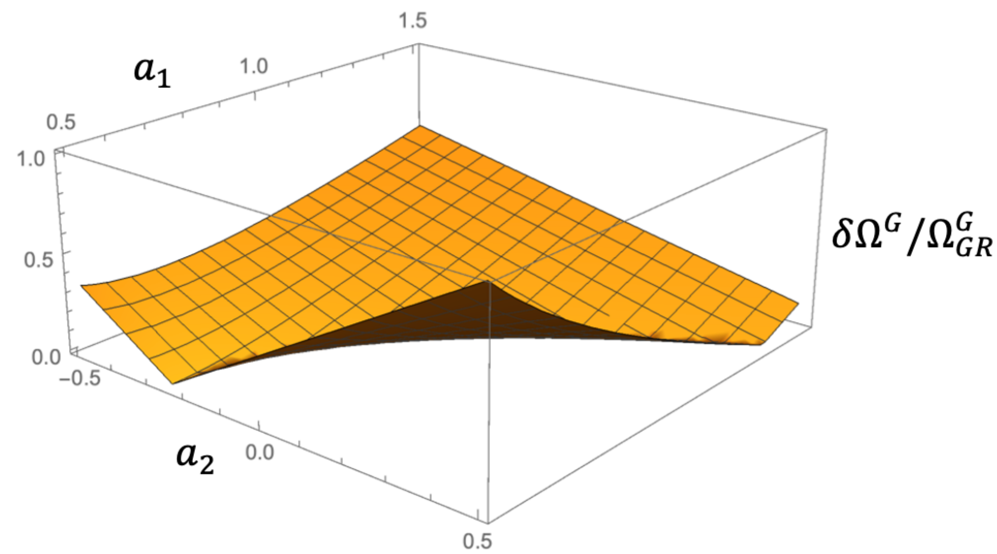
$$\frac{G_{HL}}{G_N} = 1.001$$

$$a_1(1.001a_1 - 0.99985) < a_2 < a_1(1.001a_1 - 1.00015)$$
if $a_1 < 0$

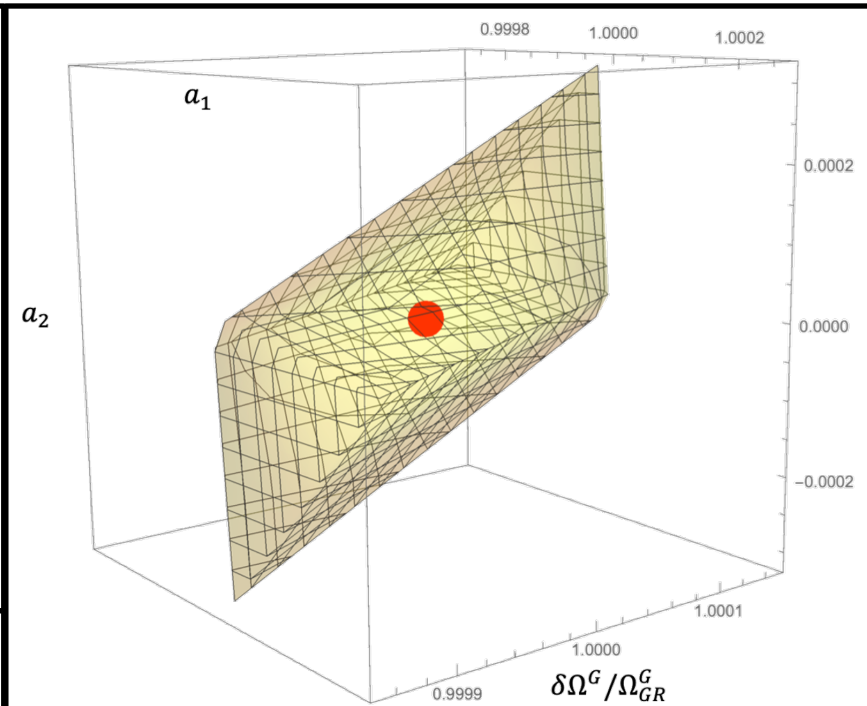
$$a_1(1.001a_1 - 1.00015) < a_2 < a_1(1.001a_1 - 0.99985)$$
if $a_1 > 0$.

S. Capozziello, et al. ``Constraining Theories of Gravity by GINGER experiment,'' Eur. Phys. J. Plus **136** (2021) no.4, 394

General relation among a_1, a_2 and the gyroscopic precession in Horava gravity:



$\delta\Omega^G/\Omega^G_{GR}$ as a function of a_1 and a_2, with G_{HL}/G_N fixed to 1.001



Graphical representation of the equation

$$\left|\frac{\Omega^G_{TOT}-\Omega^G_{GR}}{\Omega^G_{GR}}\right| = \left|\frac{2}{3}\left(\frac{G_{HL}}{G_N}a_1 - \frac{a_2}{a_1} - 1\right)\right| < 10^{-4}$$

used to constrain the free parameters. Red dot denotes the values of free parameters for which GR is recovered.

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Future perpsectives in the context of the Horava-Lifshitz model

The parameters can be further constrained by investigating the Lense-Thirring contribution

So that →

The corresponding model can be selected

$$\Omega_S = \frac{4A}{P\lambda}\,\Omega_E \left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}\,a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2 R}\sin\alpha\sin\theta - \frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right]$$

GINGER measure

Conclusions

Conclusions

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1.$$

- ***In the context of ETGs, we have studied the linearized field equations in the limit of weak gravitational fields and small velocities generated by rotating gravitational sources, aimed at constraining the free parameters, which can be seen as effective masses (or lengths).***

- ***The precession of spin of a gyroscope orbiting around a rotating gravitational source can be studied.***

- ***Gravitational field gives rise, according to GR predictions, to geodesic and Lense-Thirring processions, the latter being strictly related to the off-diagonal terms of the metric tensor generated by the rotation of the source (Kerr metric)***

- ***The gravitational field generated by the Earth can be tested by Gravity Probe B and LARES satellites. These experiments tested the geodesic and Lense-Thirring spin precessions with high precision.***

- ***The corrections on the precession induced by scalar, tensor and curvature corrections can be measured and confronted with data.***

- ***GINGER can put constraints on modified gravity models***

- ***Being an earth-based experiment, it exhibits several advantages***

- ***So far, GINGER data has been used to constrain scalar-tensor models and Horava-Lifshitz model***

- ***In future works, we aim to:***

  - *Consider other modified/extended theories of gravity*

  - *Use measurements on the gyroscopic effect to constrain potentials coming from GR alternatives*

Conclusions

In $f(R, R^{\mu\nu}R_{\mu\nu}, \varphi)$ gravity, GP-B and LARES satellites provide

$$m_Y \geq 7.3 \times 10^{-7} m^{-1}$$

$$m_Y > 1.2 \times 10^{-6} \mathrm{m}^{-1}$$

Perspective: constraint on m_y by GINGER

Perspective: constraints on a_1, a_2 by GINGER

In Horava-Lifshitz gravity, the weak-field limit provide

$$\begin{aligned} c\,\delta\tau &= \frac{4A\Omega_E}{c}\left[\cos(\theta+\alpha) - \left(1 + \frac{G}{G_N}a_1 - \frac{a_2}{a_1}\right)\frac{GM}{c^2R}\sin\theta\sin\alpha \right. \\ &\quad \left. - \frac{GI_E}{c^2R^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)\right] \end{aligned}$$