Primordial Black Holes: formation and cosmological impact in the current Universe

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Introduction: a very brief overview

- **Primordial Black Holes (PBHs)** [**Zeldovich & Novikov** (1967), **Hawking** (1971)] could form from the collapse of cosmological perturbation during the radiation dominated era.

$$p = \frac{\rho}{3}$$

- **PBHs could span a large wide range of masses and if not evaporated** [BH evaporation **Hawking** (1974)]: PBHs with ***$M > 10^{15}\, g$*** are interesting candidates for dark matter, intermediate mass black holes and the seeds of supermassive black holes.

- **Numerical hydrodynamical simulations in spherical symmetry** of a cosmological perturbation, characterized by an amplitude δ, have shown:

    - $\delta > \delta_c \;\Rightarrow\;$ PBH formation
    - $\delta < \delta_c \;\Rightarrow\;$ perturbation bounce
    - $\delta_c \sim c_s^2 \equiv \dfrac{\partial p}{\partial \rho}$ **(Carr 1975)**

Equation of State of the Early Universe

The radiation dominated Universe goes through 3 main transitions:

- Electroweak phase-transition
- **QCD phase-transition** (crossover)

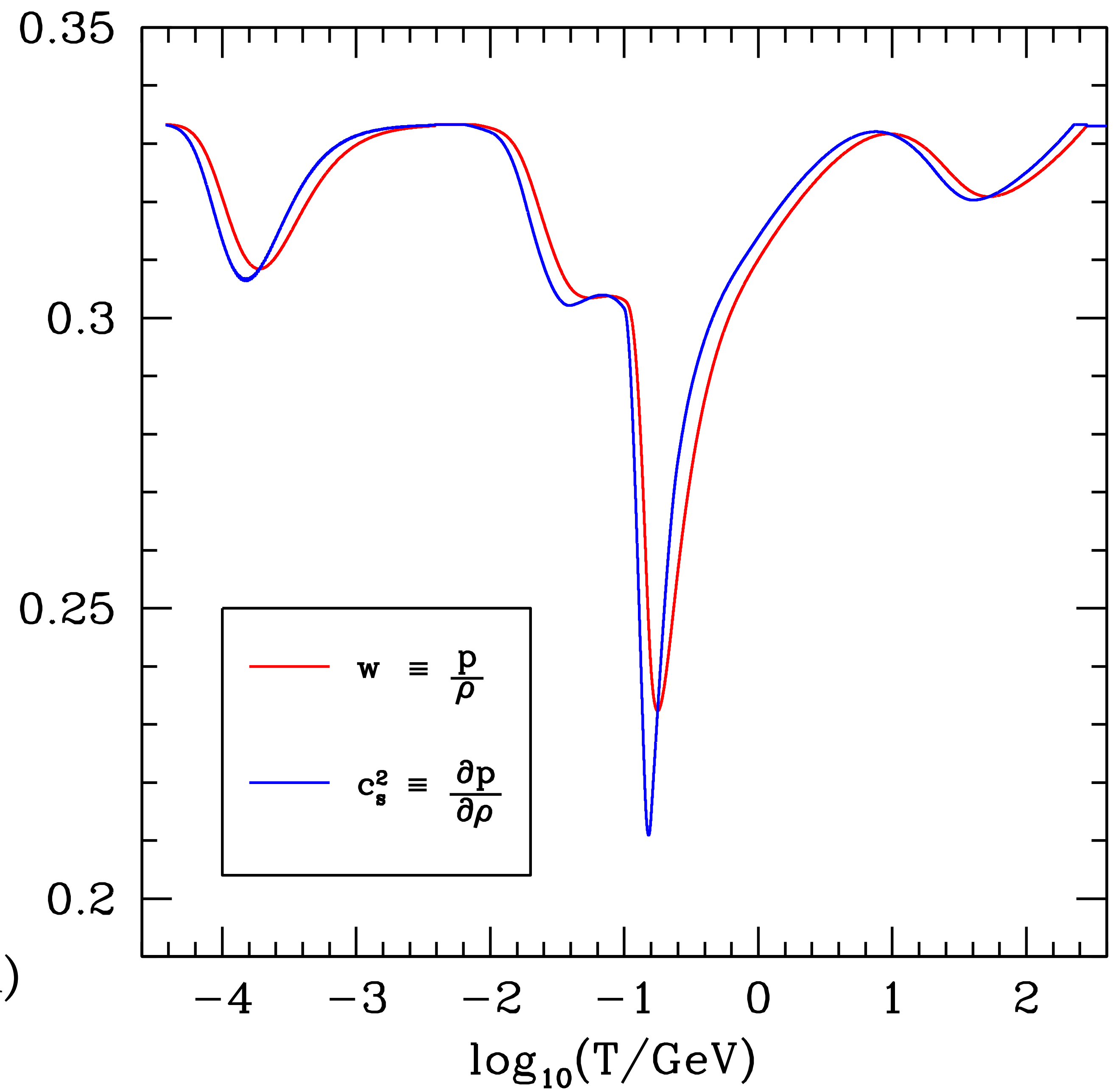
$$\rho = \frac{\pi^2}{30} g_{\rm eff}(T) T^4 \qquad s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3$$

$$p = sT - \rho = w(T)\rho$$

$$w(T) = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1$$

$$c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\rm eff} + T h'_{\rm eff})}{3(4g_{\rm eff} + T g'_{\rm eff})} - 1$$

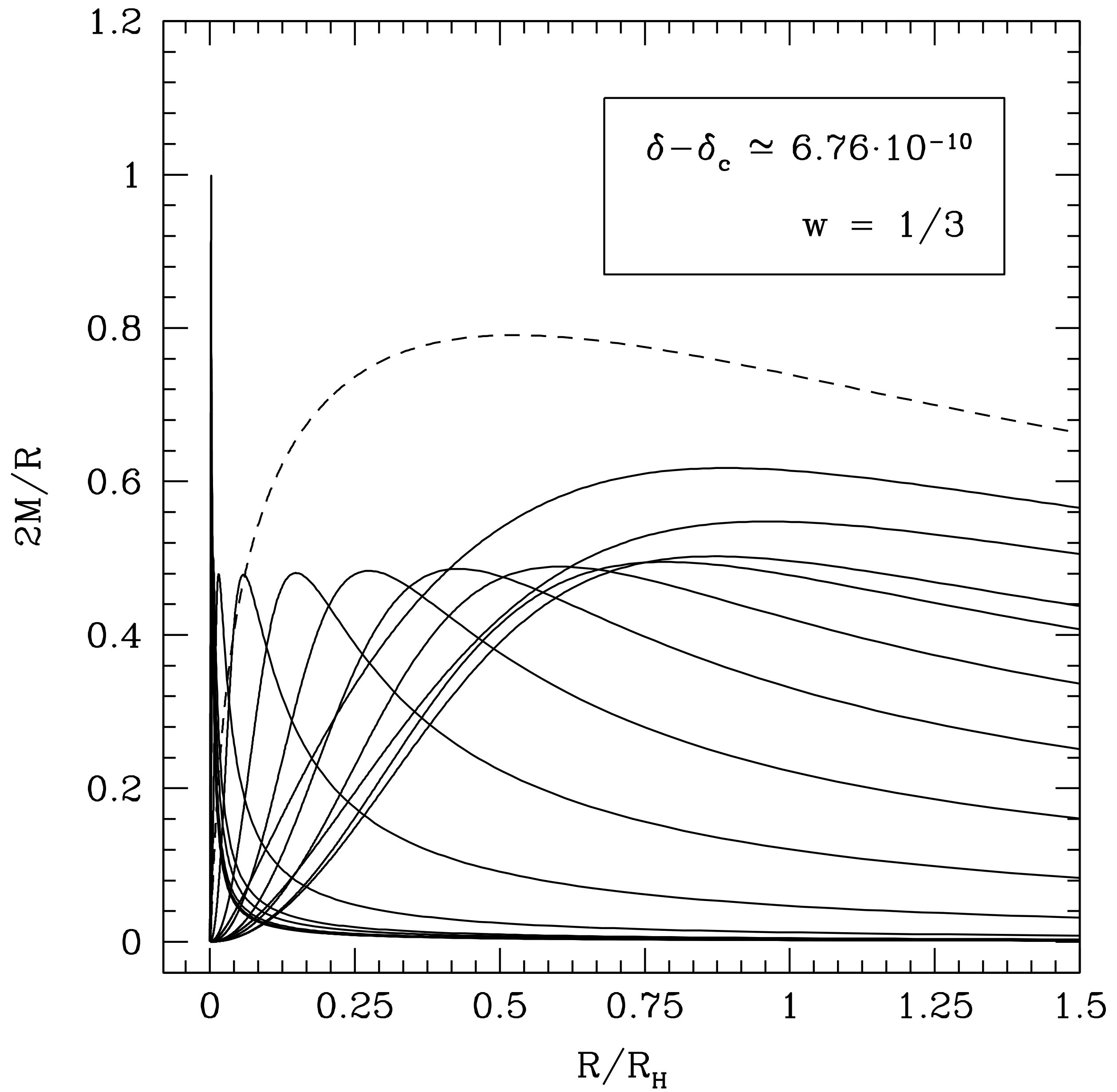
- Nucelosynthesis (e+e- annihilation)

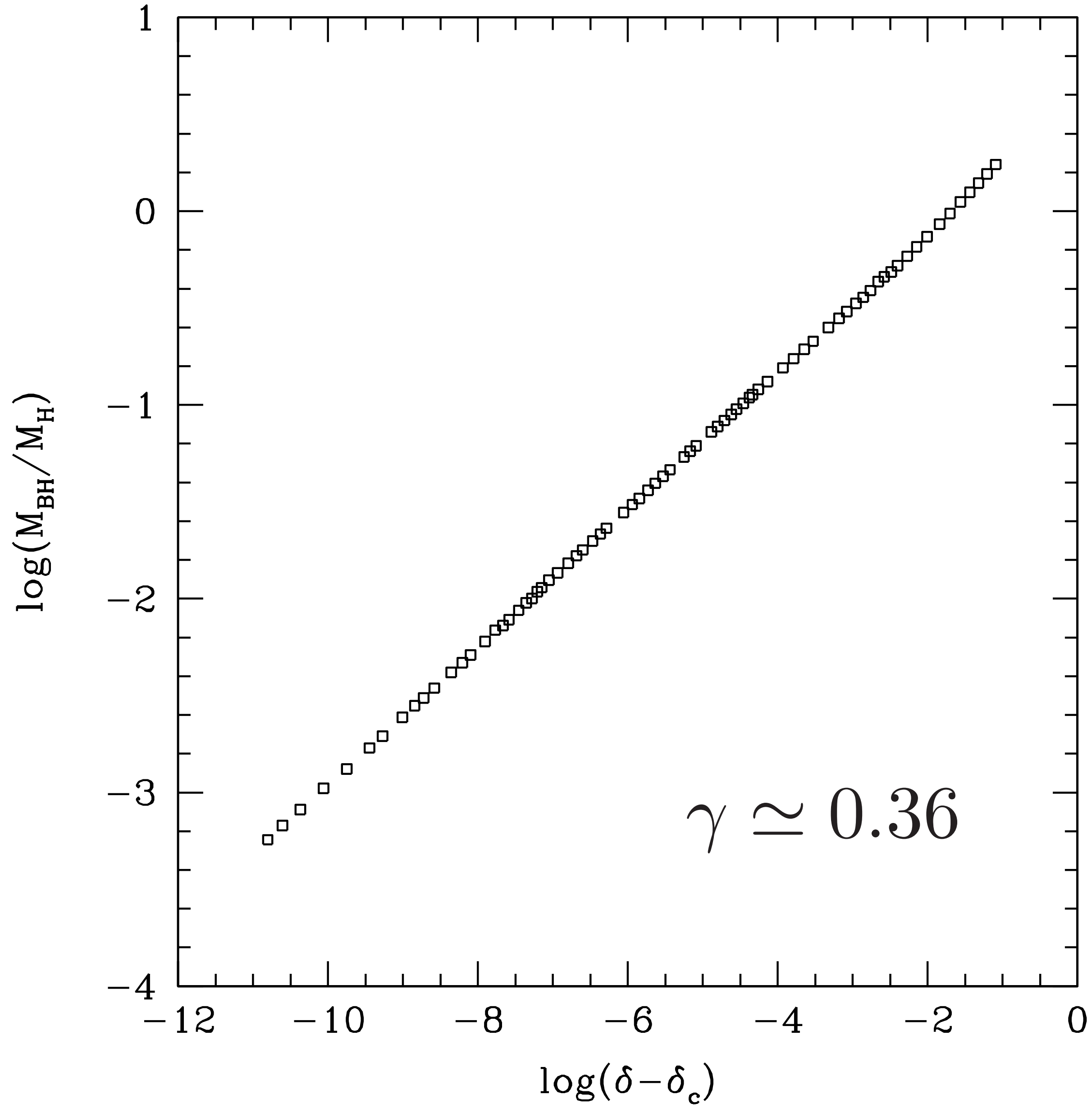


PBH formation in radiation dominated Universe

$$R(r,t) = 2M(r,t)$$

$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H$$





IM, J. Miller, Rezzolla, Polnarev - CQG (2005 - 2013)

$\mathcal{K}, \delta_c$ — shape dependent

Initial conditions: curvature profile

- The asymptotic metric ($t \rightarrow 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{4}{9}\left[\boxed{\nabla^2\zeta(r)} + \boxed{\frac{1}{2}\left(\nabla\zeta(r)\right)^2}\right] e^{-2\zeta(r)}$$

- The perturbation amplitude δ is measured by the peak of the compaction function, corresponding to the excess of mass of the over density.

$$\mathcal{C}(r) := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = \boxed{-\frac{4}{3}\tilde{r}\zeta'(r)}\left[1 + \boxed{\frac{1}{2}\tilde{r}\zeta'(r)}\right] \quad \Rightarrow \quad \delta = \boxed{\delta_G}\left[1 - \boxed{\frac{3}{8}\delta_G}\right]$$

PBH Abundance (Peak Theory)

C.Germani, IM - PRL (2019)

C. Yoo, T. Harada, J. Garriga, K. Kohri (2019)

- **PDF of δ follows a Gaussian distribution:** $P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$

$$\sigma^2 = \langle\delta^2\rangle = \int_0^\infty \frac{dk}{k}\mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k}(kr_m)^4 T^2(k, r_m) W^2(k, r_m)\mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}}\mathcal{K}\left(\frac{k_*}{a_m H_m}\right)^3 \sigma^\gamma \nu_c^{1-\gamma}\gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \qquad \nu_c \equiv \frac{\delta_c}{\sigma}$$

- **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8}\sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

- **Narrow peak:** $\frac{k_*}{\sigma} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22\sqrt{\frac{k_*}{\sigma\mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7\times 10^{-4}\frac{k_*}{\sigma} \gg 10^{-3}$

- **Broad peak:** $\frac{k_*}{\sigma} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46(\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3\times 10^{-3}$

- **Non linear effects:** $\delta = \delta_G\left[1 - \frac{3}{8}\delta_G\right] \;\Rightarrow\; 1.5 \lesssim \frac{\mathcal{P}_{0_{NL}}}{\mathcal{P}_{0_L}} = \frac{16\left(1-\sqrt{1-\frac{3}{2}\delta_c}\right)^2}{9\delta_c^2} \lesssim 4$

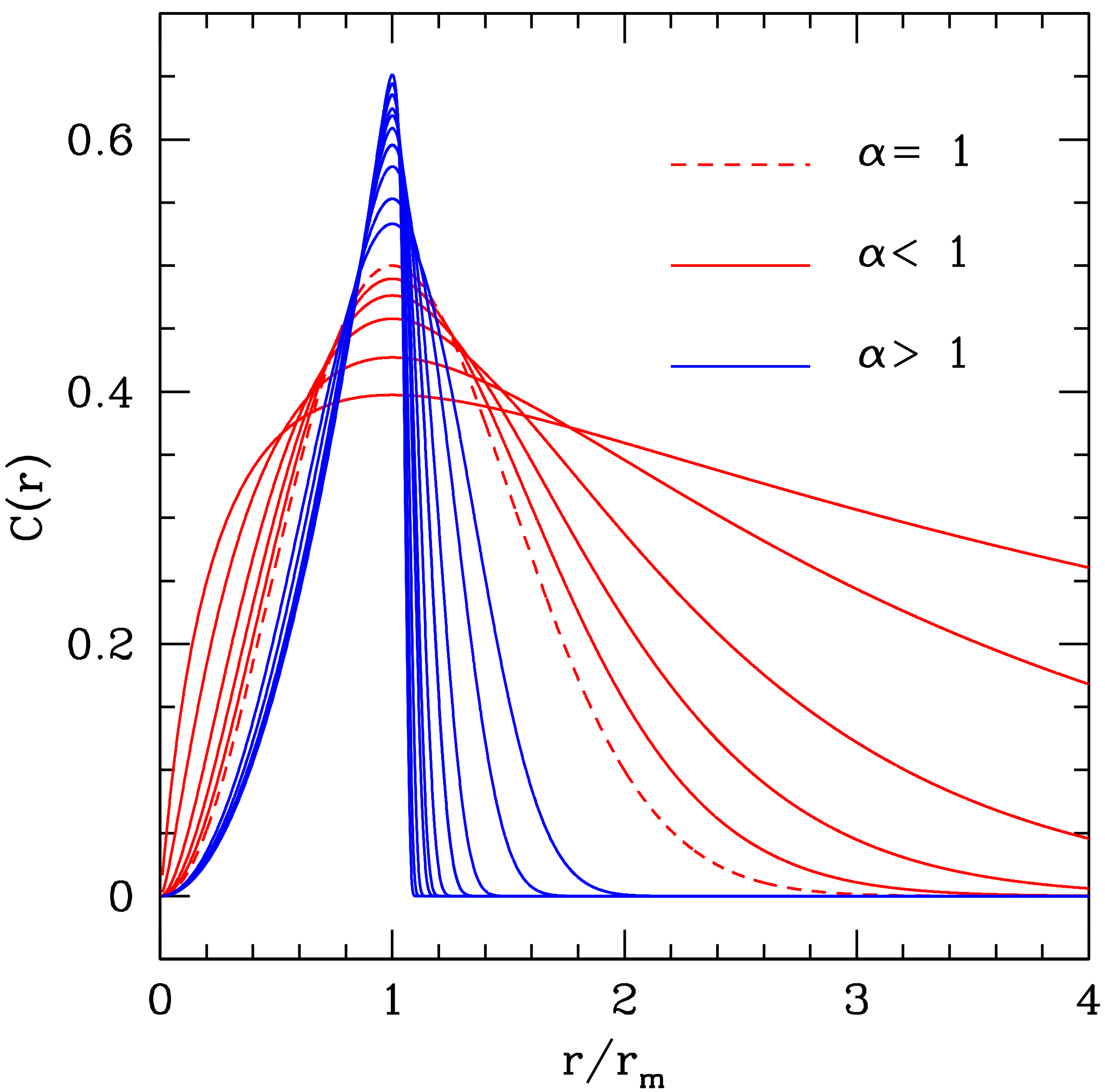
S.Young, IM, C.Byrnes JCAP (2019)

De Luca, Franciolini, Kehagias, Peloso, Riotto and Unal (2019)

PBH threshold

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

$$\alpha \equiv -\frac{\mathcal{C}''(\tilde{r}_m)\tilde{r}_m^2}{4\mathcal{C}(\tilde{r}_m)} = \frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$



$$\tilde{r} = r e^{\zeta(r)}$$

$$\boxed{0.4 \leq \delta_c(\alpha) \leq \frac{2}{3}}$$

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

I. Musco - PRD (2019)

PBH threshold

$$\mathcal{C}'(r_m) = 0, \quad \Phi_m \equiv -r_m \zeta'(r_m)$$

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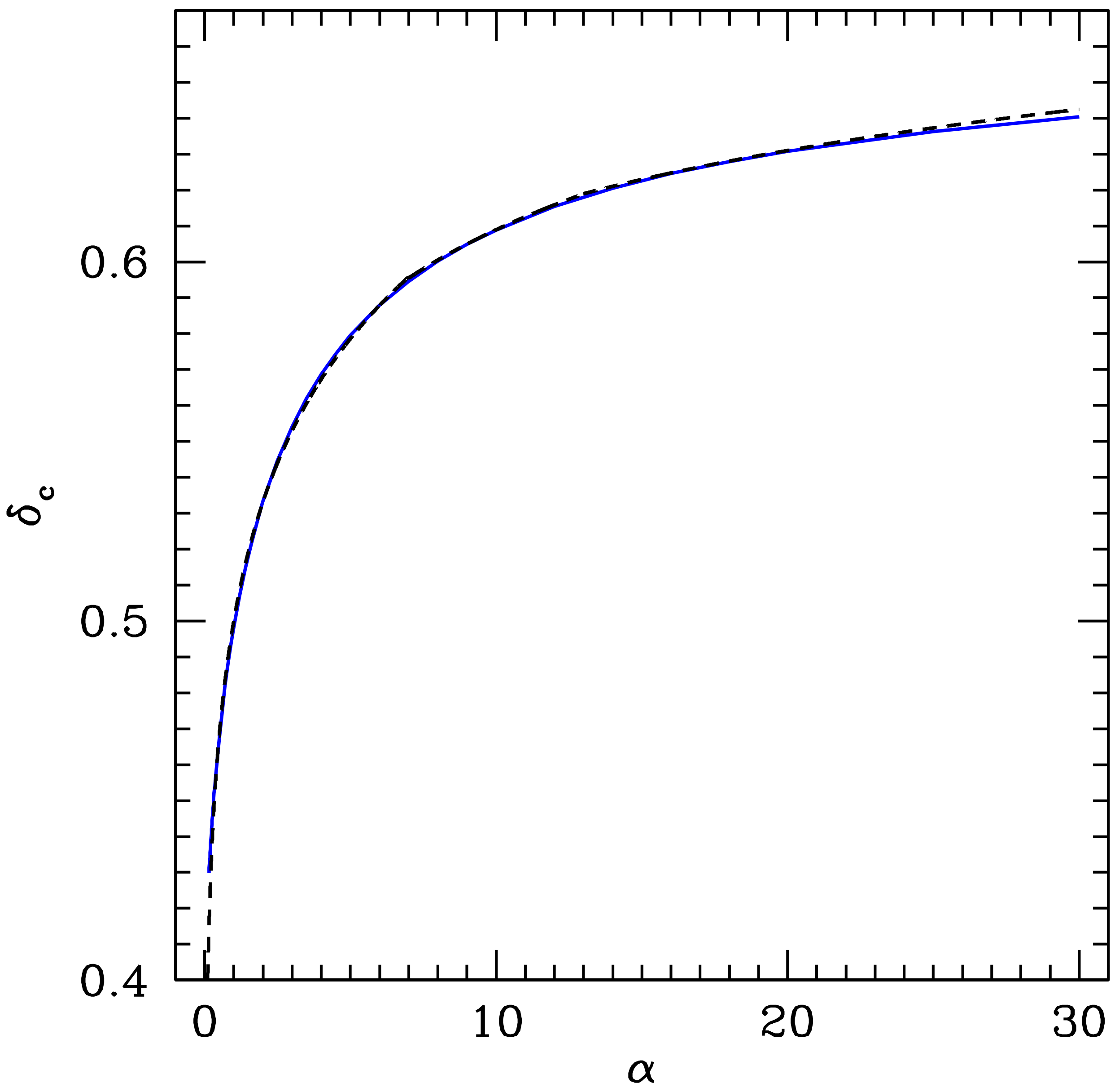

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I. Musco - PRD (2019)

PBH threshold prescription

Curvature power spectrum $\mathcal{P}_\zeta$

$\Downarrow$

Characteristic overdensity scale $k_* \hat{r}_m$

$\Downarrow$

Characteristic shape parameter α

$\Downarrow$

Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation**: take the primordial power spectrum $\mathcal{P}_\zeta$ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k,\eta)$

$$P_\zeta(k,\eta) = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k) T^2(k,\eta).$$

2. **The comoving length scale** $\hat{r}_m$ of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ. Compute the value of $k_*\hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2\hat{r}_m^2 - 1)\frac{\sin(k\hat{r}_m)}{k\hat{r}_m} + \cos\left(k\hat{r}_m\right)\right] P_\zeta(k,\eta) = 0\,.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha)\left[1+F(\alpha)\right]\alpha = -\frac{1}{2}\left[1+\hat{r}_m\frac{\int dk k^4 \cos\left(k\hat{r}_m\right) P_\zeta(k,\eta)}{\int dk k^3 \sin\left(k\hat{r}_m\right) P_\zeta(k,\eta)}\right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5}e^{-1/\alpha}\frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha},\frac{1}{\alpha}\right)}}\,.$$

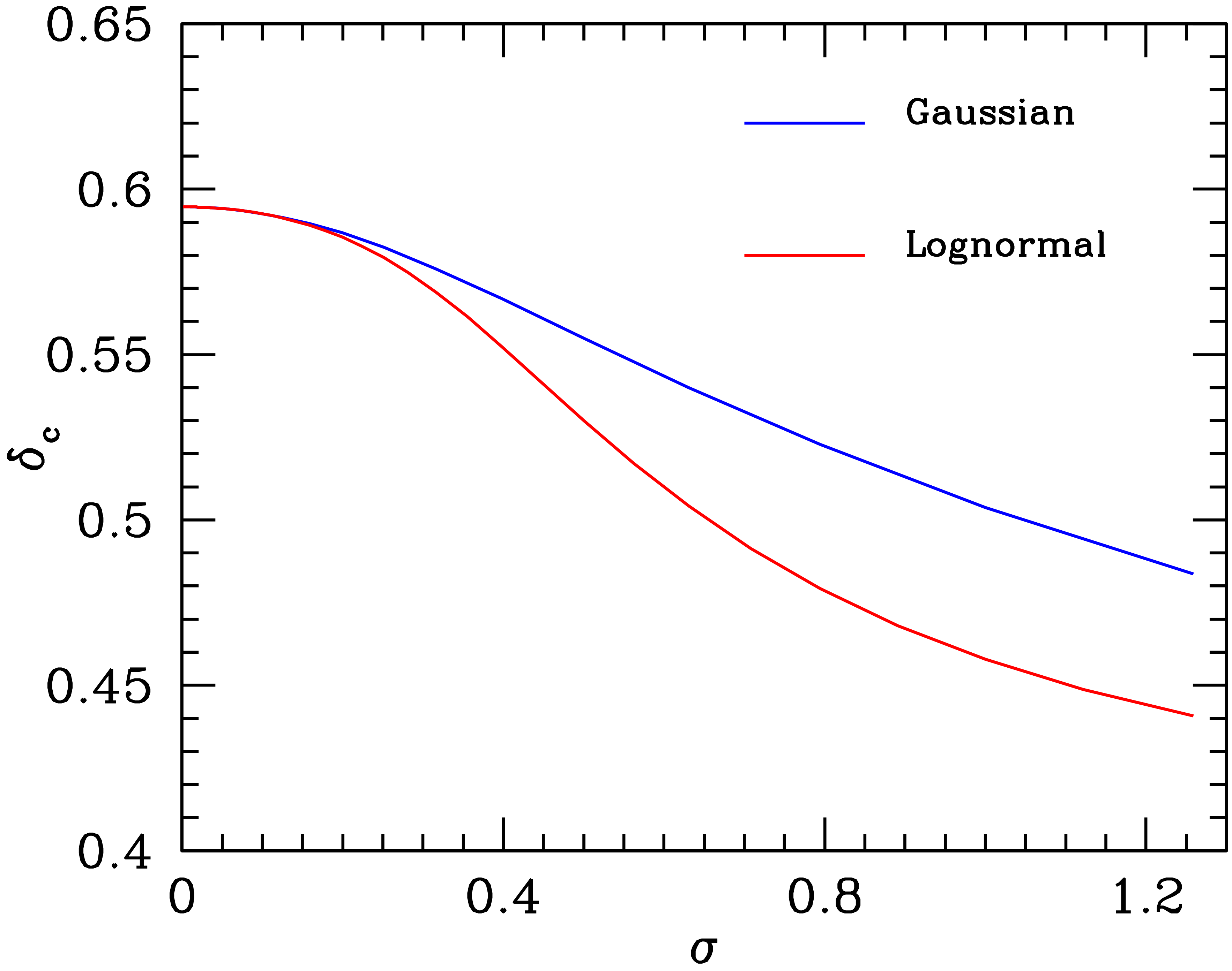
4. **The threshold δ_c:** compute the threshold as function of α, fitting the numerical simulations, at *superhorizon scales*, making a linear extrapolation at horizon crossing ($aHr_m = 1$).

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

Power Spectrum:

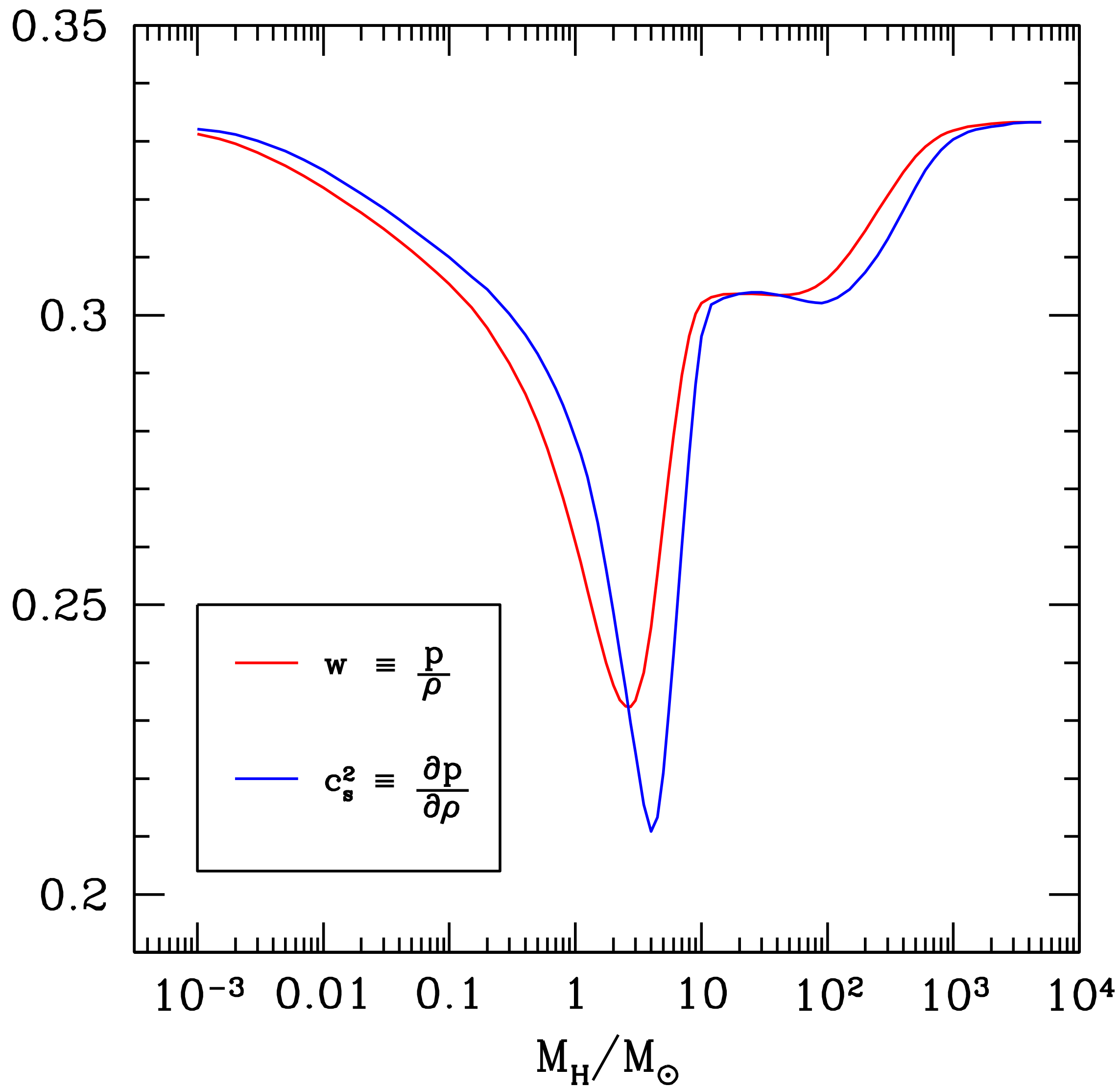
Gaussian: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-(k-k_*)^2/2\sigma^2\right]$

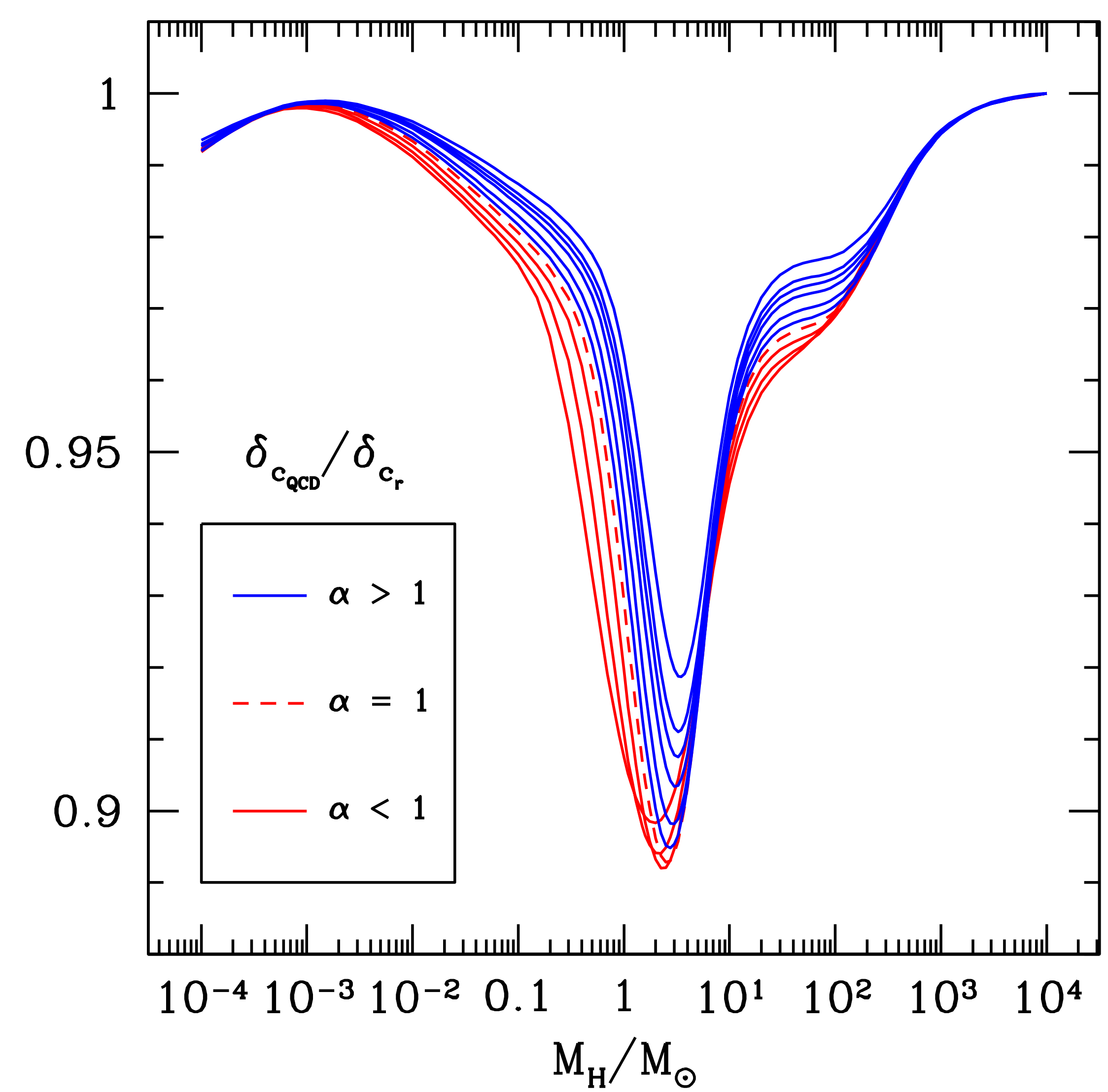
Lognormal: $\mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-\ln^2(k/k_*)/2\sigma^2\right]$



PBH Threshold during the QCD

IM, K. Jedamzik, Sam Young - PRD (2024)





Depending on the shape, the threshold for PBH formation during the QCD phase transition is reduced about 10% around the minimum of $w(T)$.

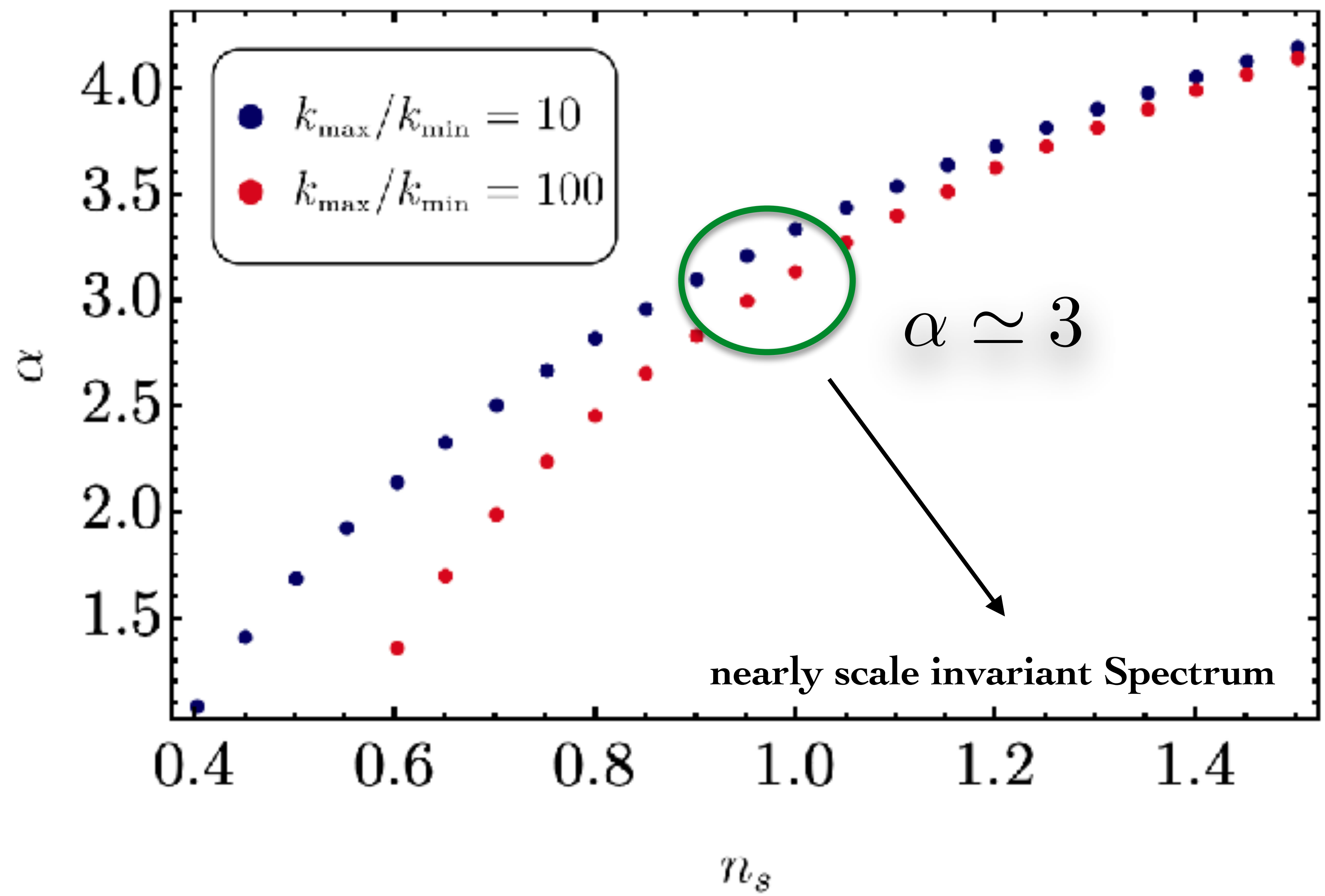
Significant enhancement of PBH formation around the solar mass scale: abundance increased of about O(3) with respect radiation!

Scale invariant Power Spectrum

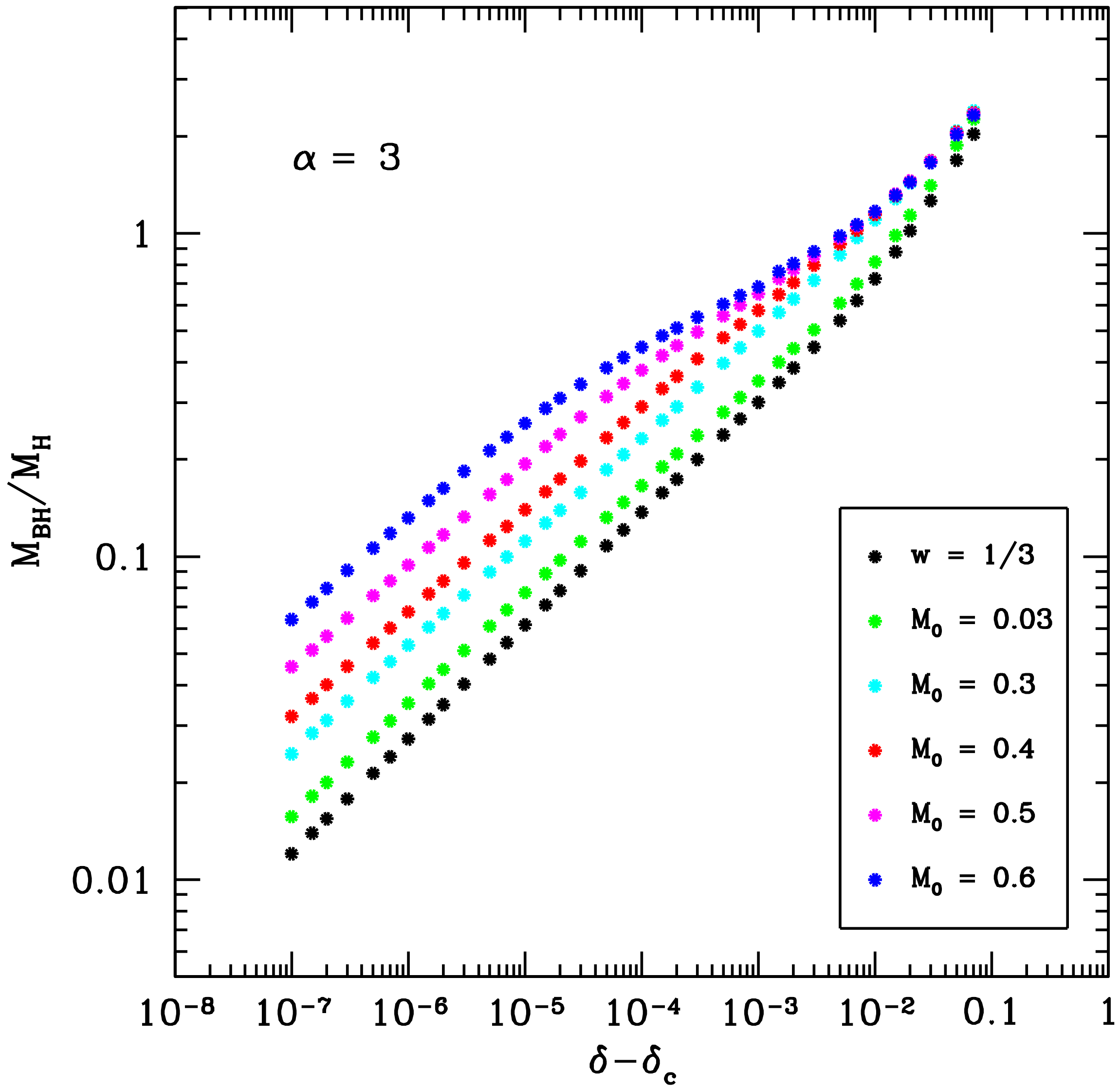
$$P_\zeta(k) = A\,(k/k_{\rm min})^{n_s-1}\,\Theta(k-k_{\rm min})\Theta(k_{\rm max}-k)$$

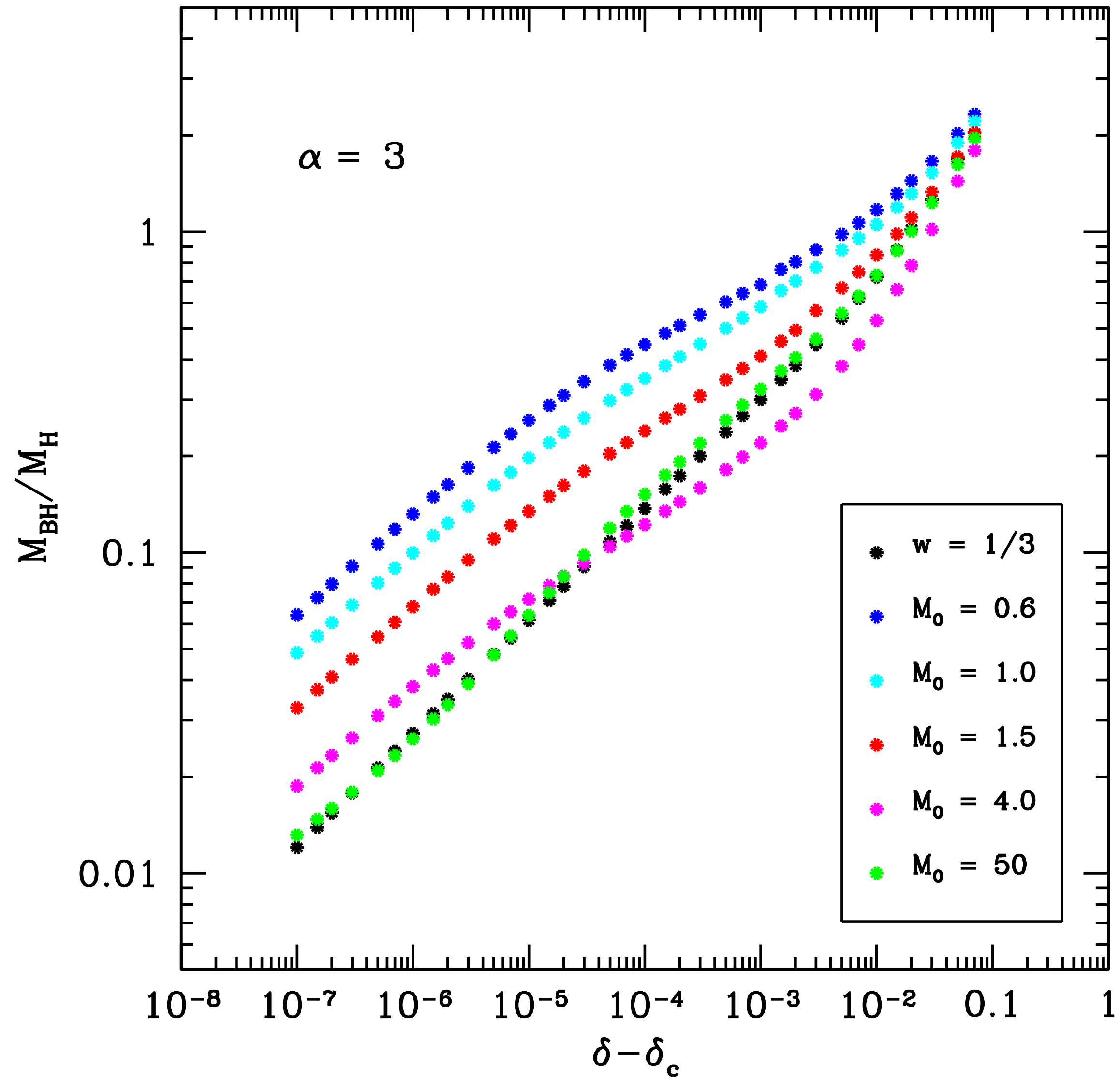
n_s — spectrum tilt

$k_{\rm max}/k_{\rm min}$ — cut-off scale



PBH scaling law during the QCD





$$M_{PBH} = \mathcal{K}(\delta - \delta_c)^{\gamma} M_H$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

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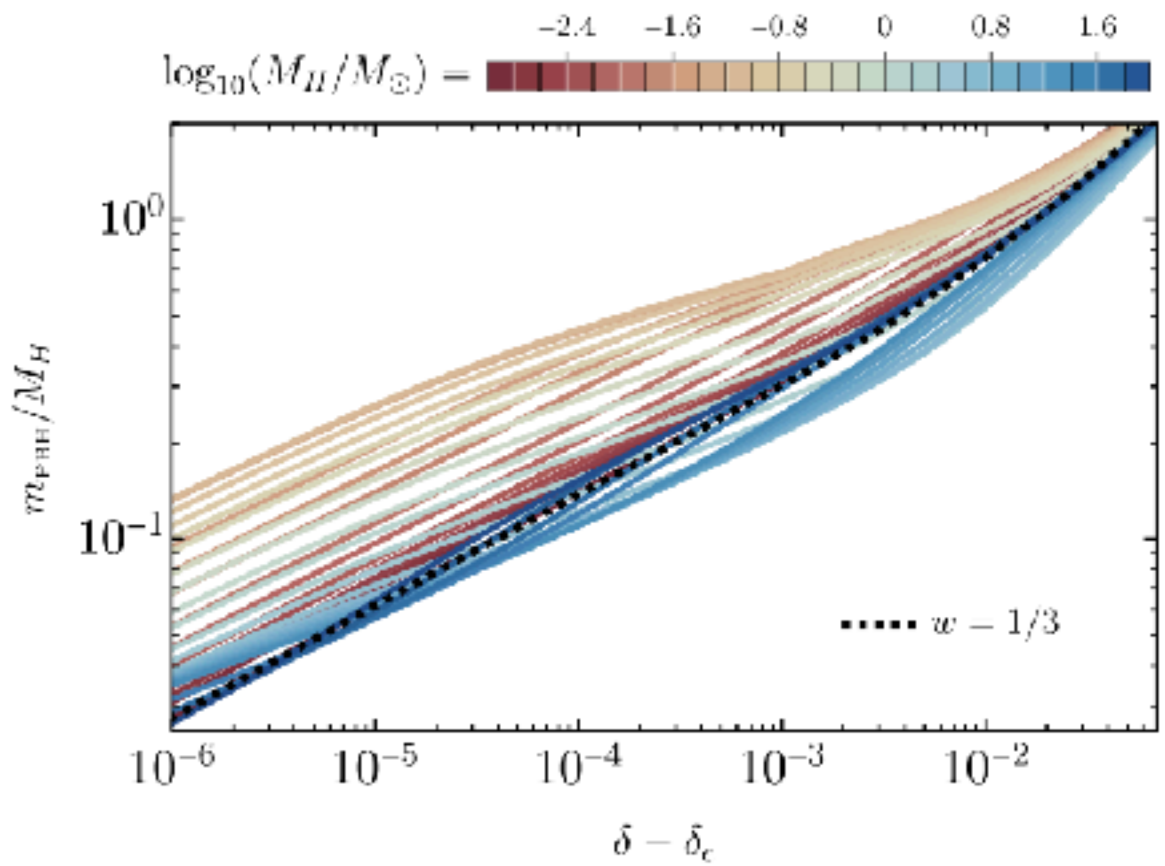

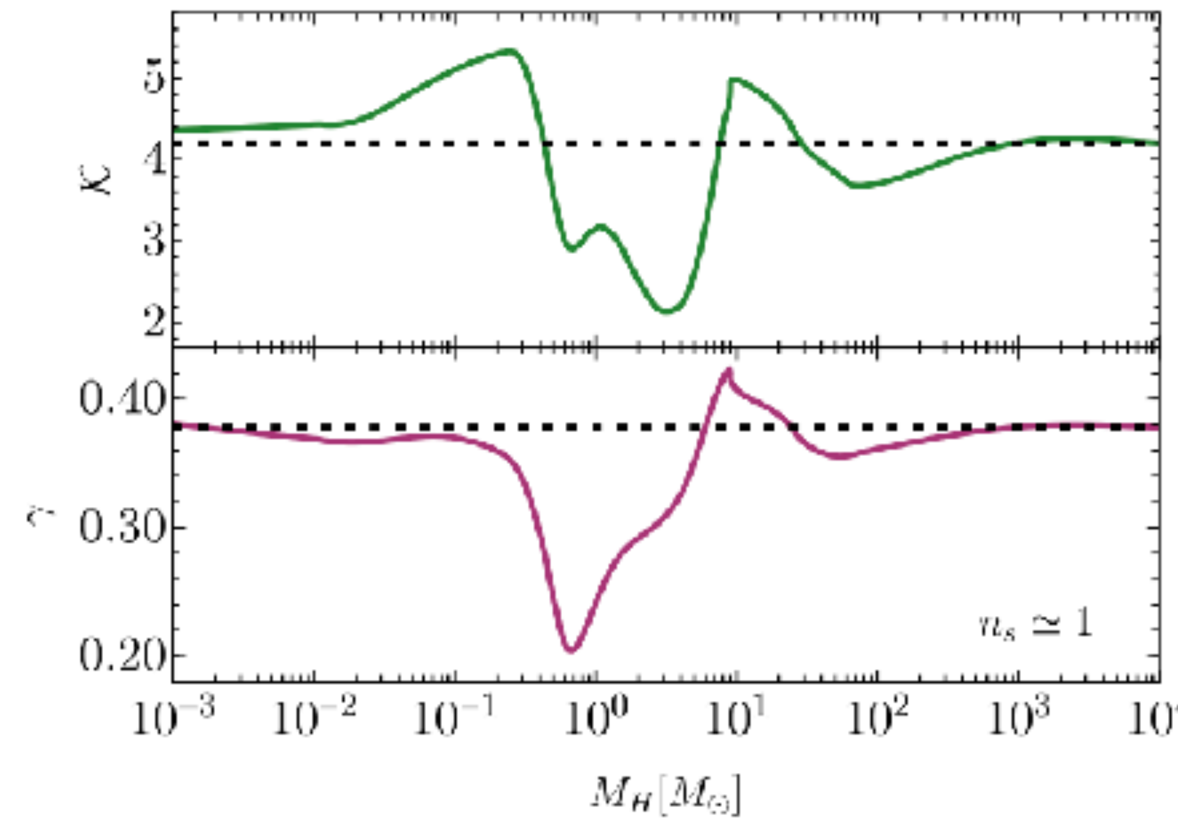

$$\boxed{M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H}$$

$$\delta_c(M_H), \gamma(M_H), \mathcal{K}(M_H)$$

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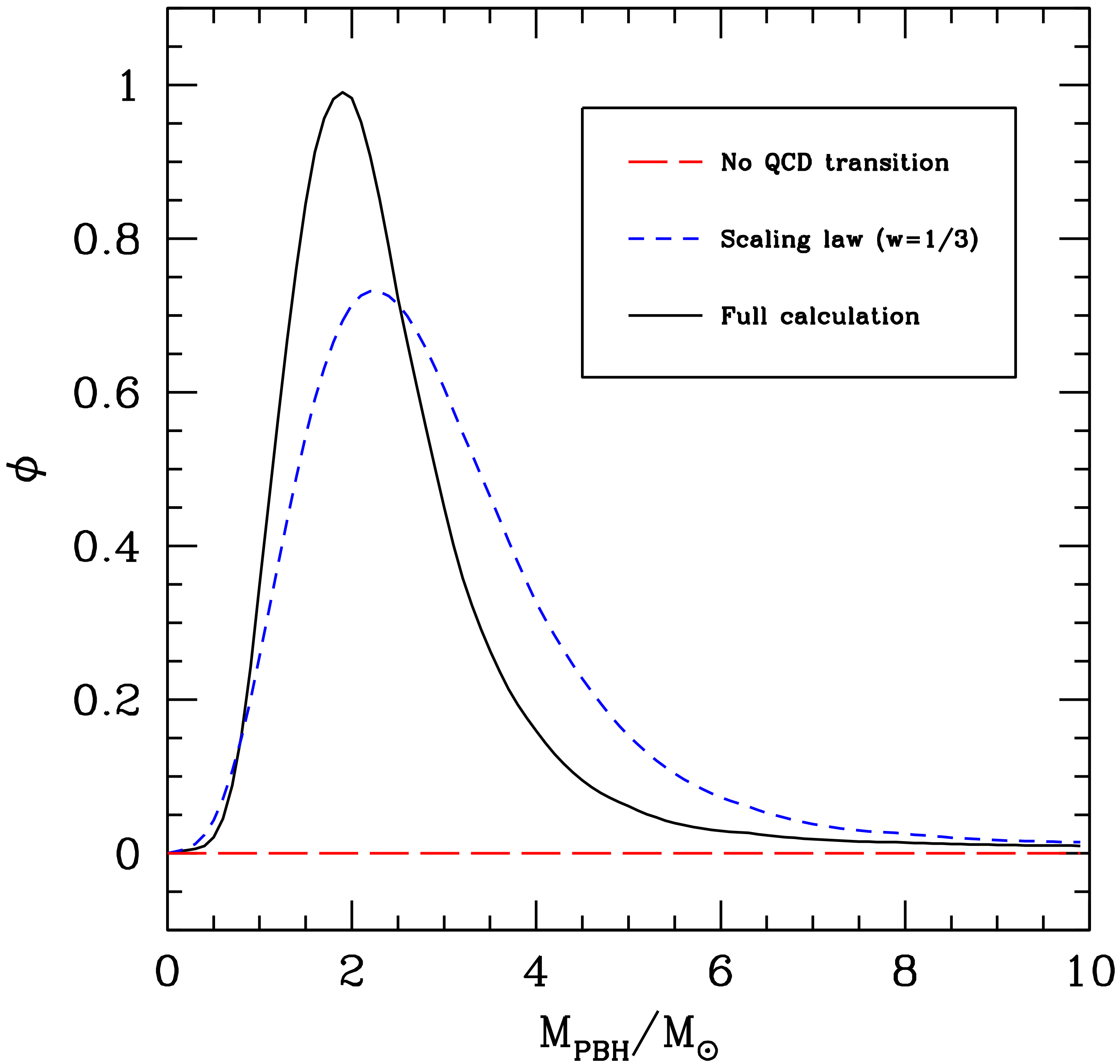
PBH mass function during the QCD

Mass Function ψ(mPBH): fraction of PBHs with mass in the infinitesimal interval of M_{PBH}

$$\phi(M_{\rm PBH}) = \frac{1}{\Omega_{\rm PBH}} \frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}M_{\rm PBH}}$$

$$\int {\rm d}m_{\rm PBH}\ \phi(m_{\rm PBH}) = 1$$

- The main effect is given by the modification of the threshold.
- The modified scaling law gives a pile up of PBHs on smaller masses.



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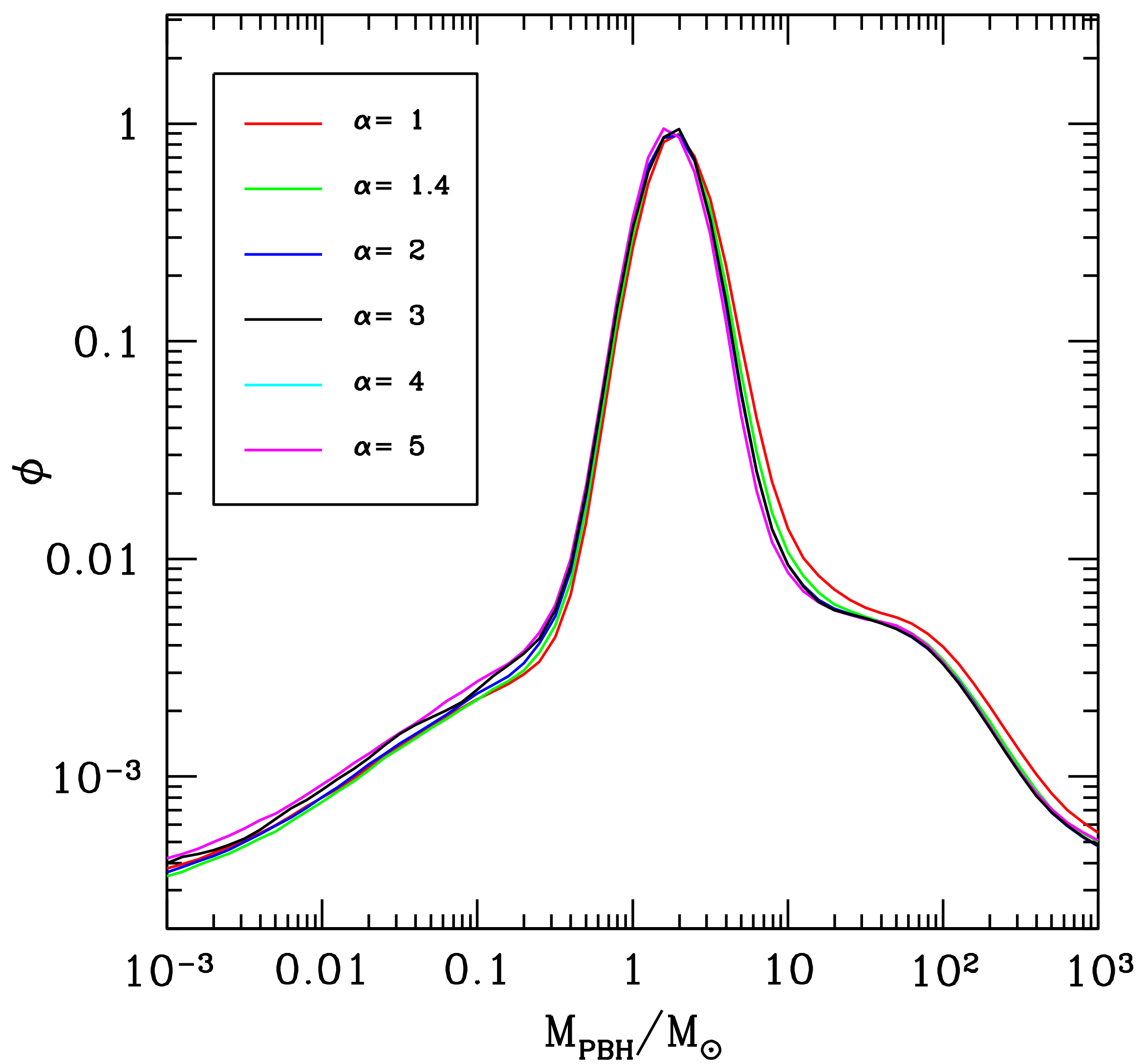
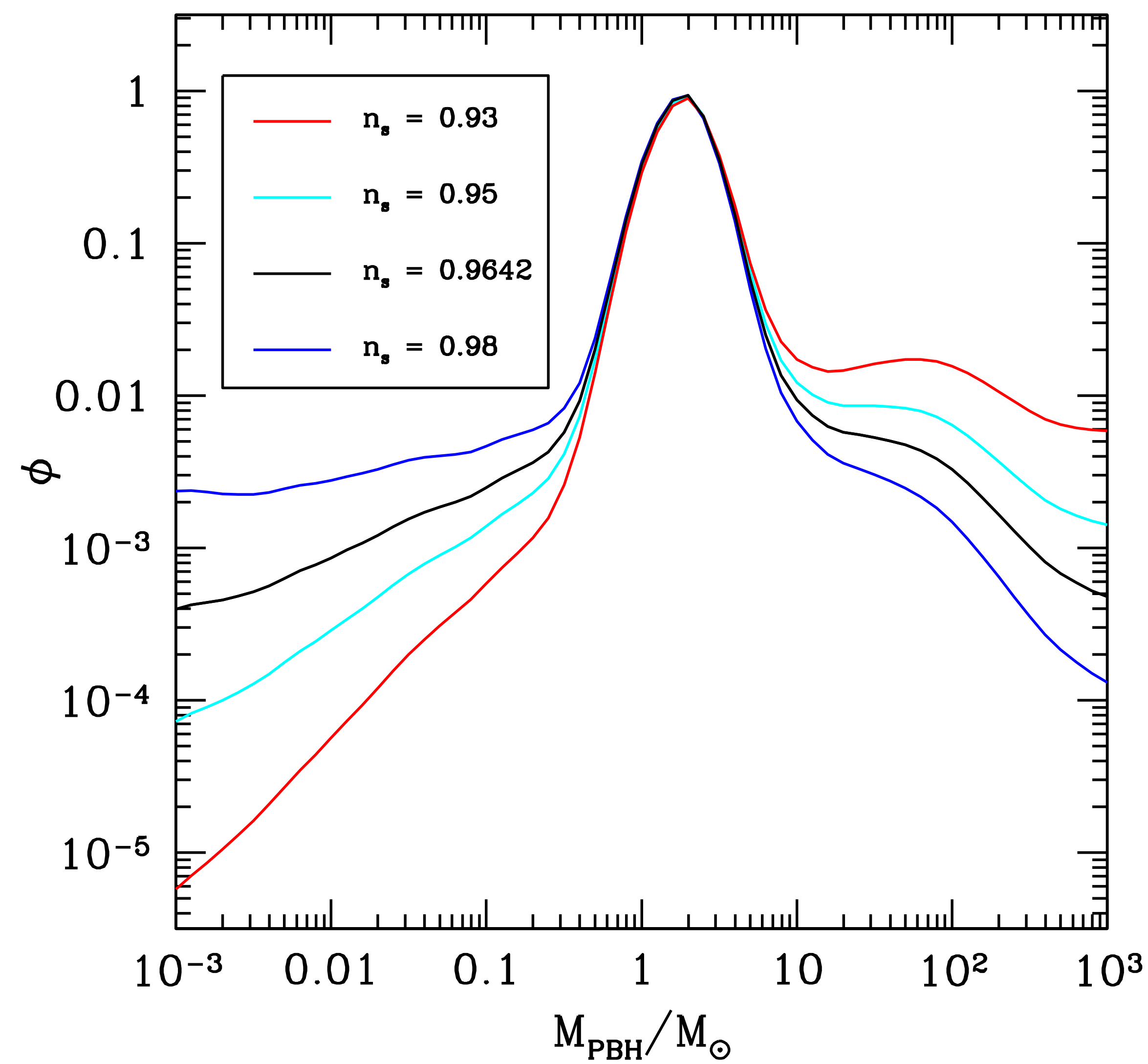
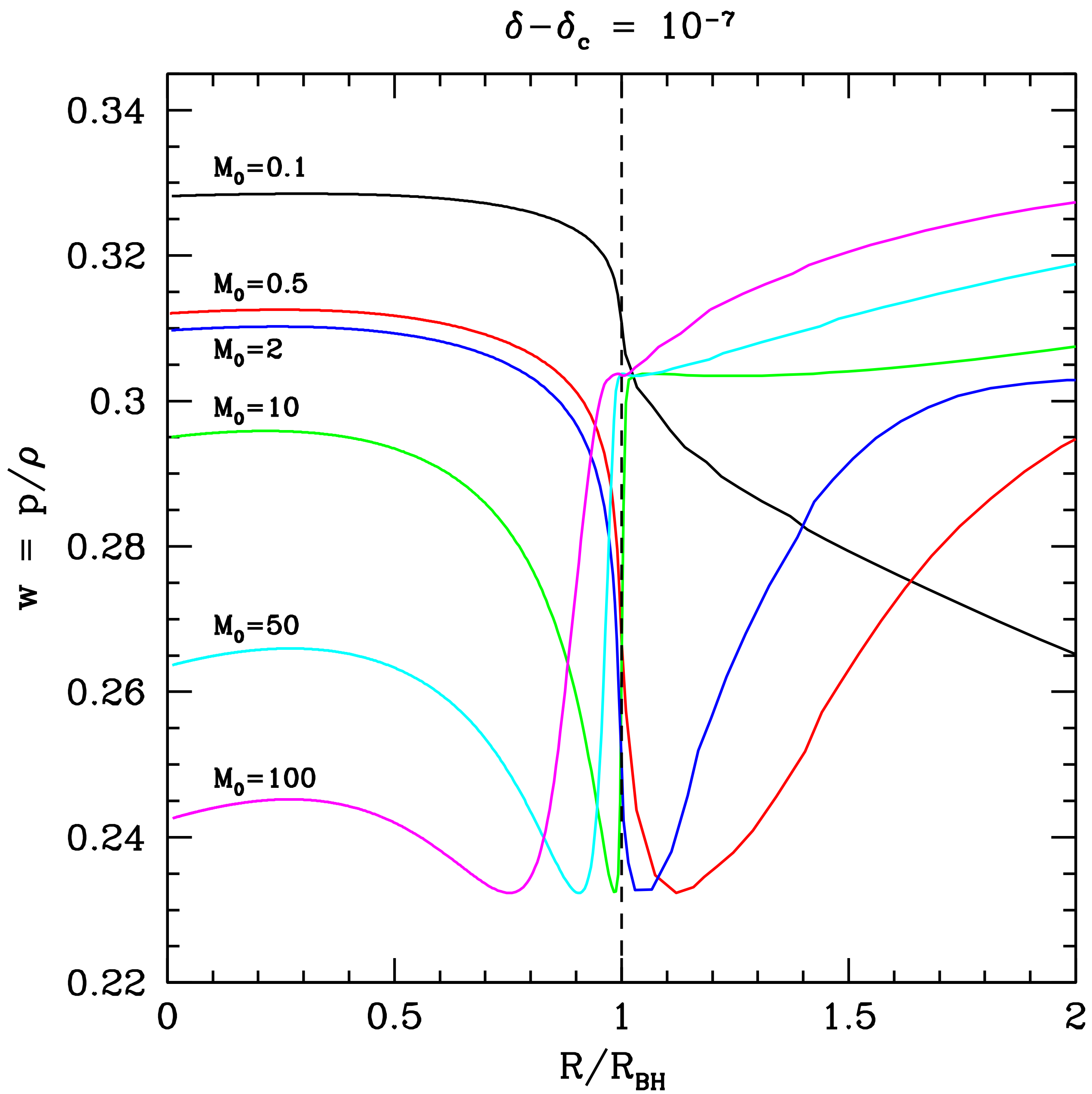
PBH mass function during the QCD: shape/tilt dependence

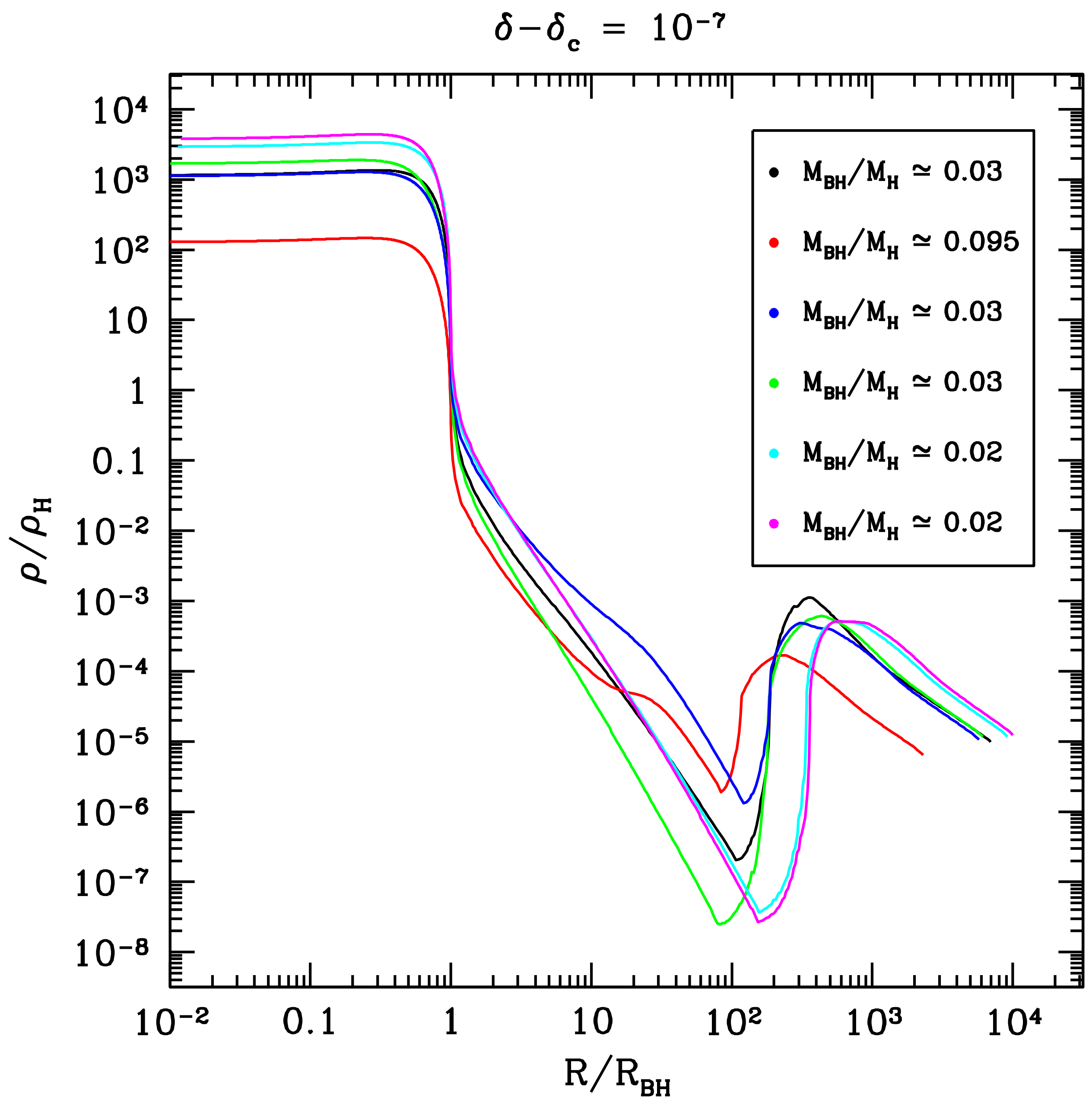




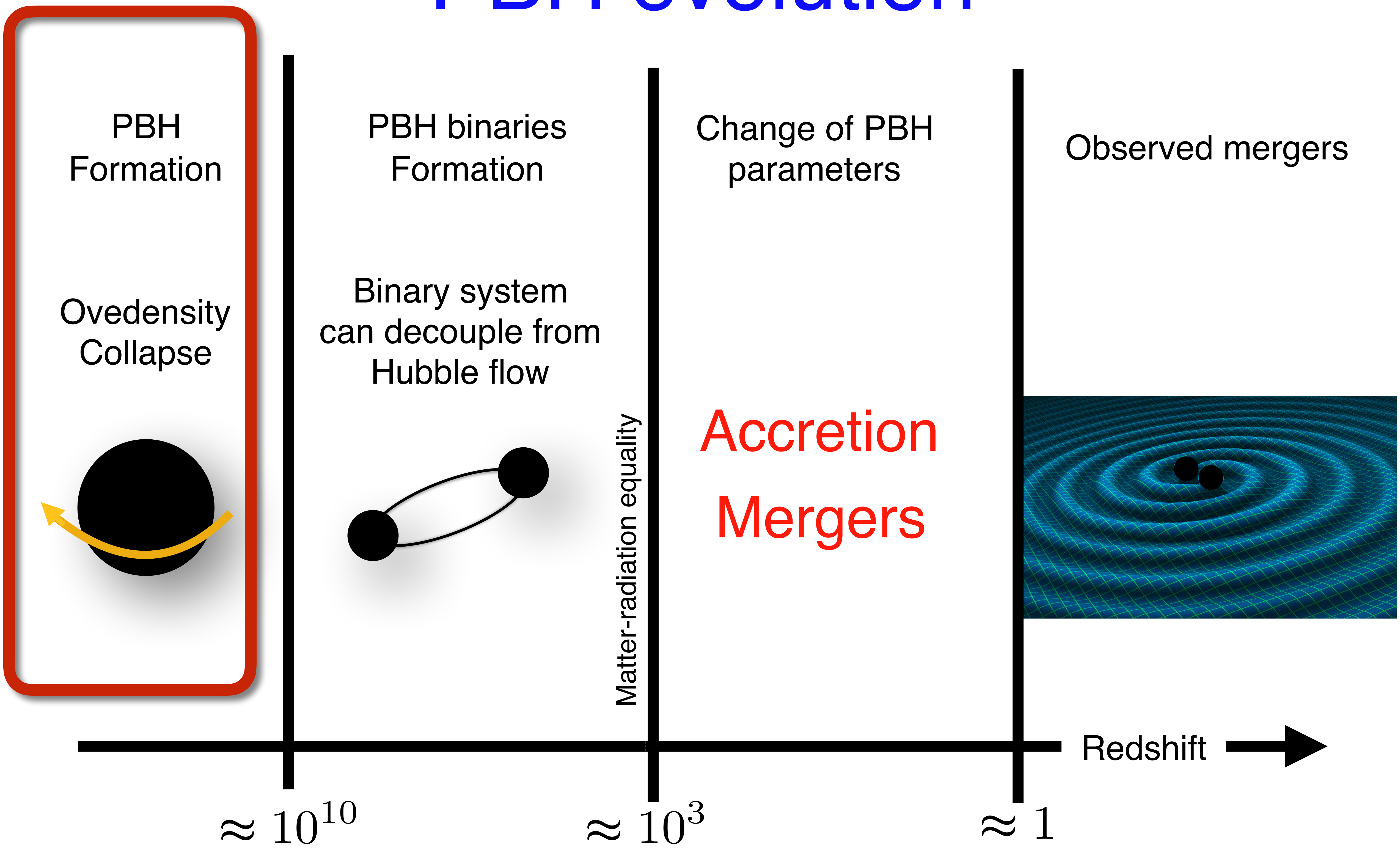
- Given the PBH abundance, the shape does not play a significant role on the mass function (attractor solution)!
- The tilt of the power spectrum does not affect the peak of the mass function.

PBH formation during the QCD





PBH evolution

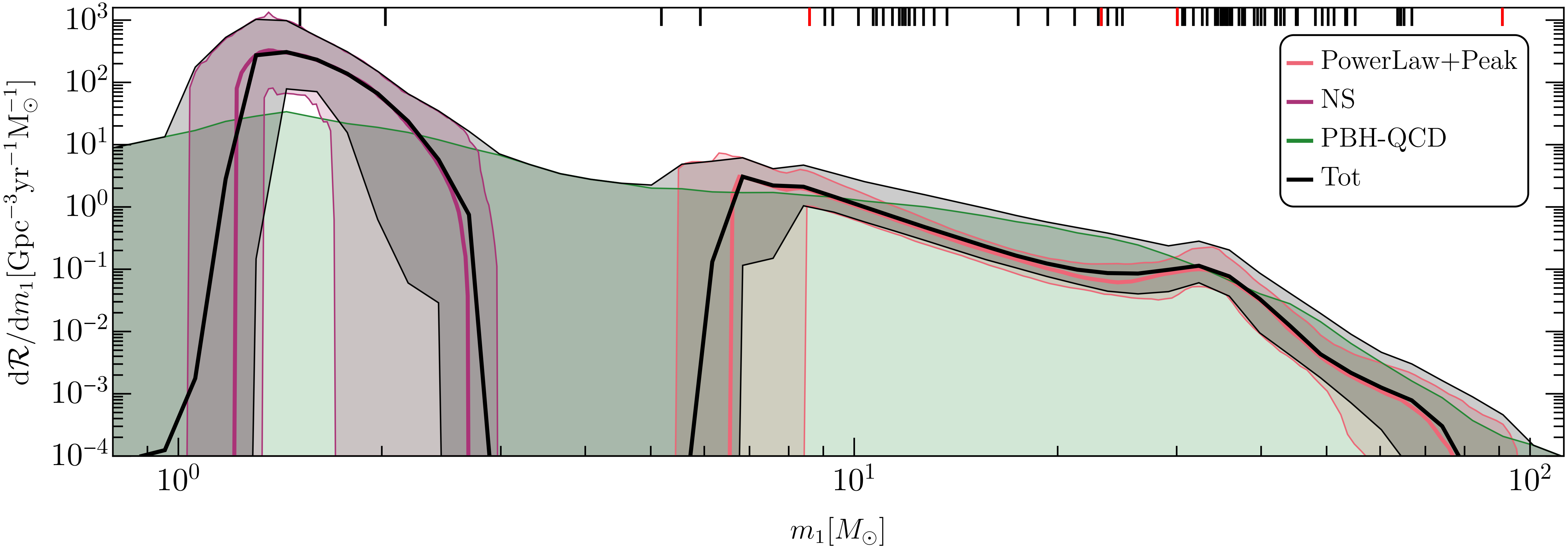


GWs from PBH mergers

G. Franciolini, IM, P.Pani, A urbano - PRD (2022)

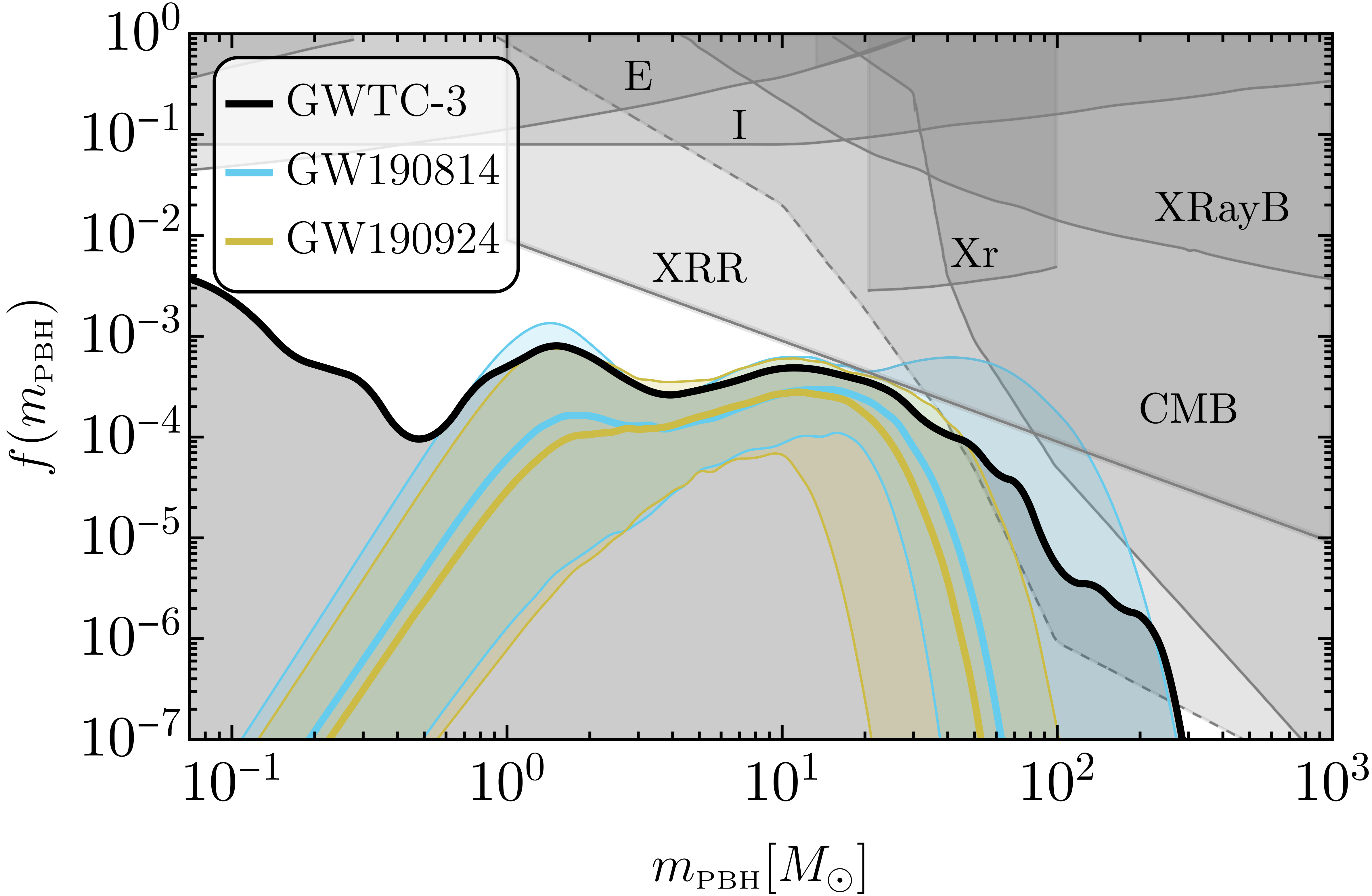
- Making **Bayesian inference analysis** we found that a sub-population of PBHs is compatible with the LVK catalog.

- PBHs give a natural explanation for the events in with BH mass gap: in particular GW190814 falling within the lower mass gap (predictions for O4 and O5).

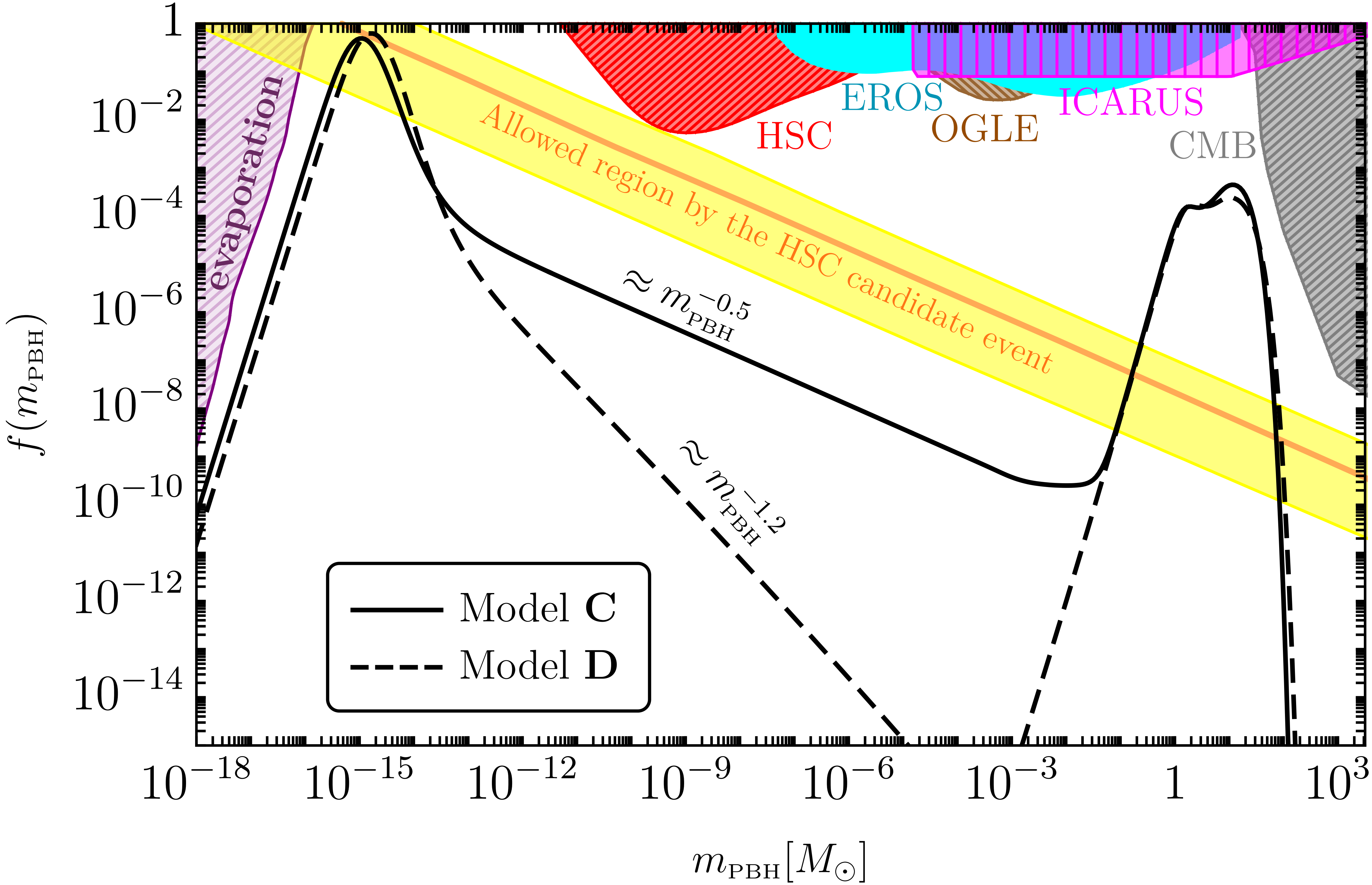


GW event	PBH prob. [%]	$m_1[M_\odot]$	$m_2[M_\odot]$
GW151012	1.2	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$
GW190412	25.4	$30.1^{+4.7}_{-5.1}$	$8.3^{+1.6}_{-0.9}$
GW190512_180714	1.6	$23.3^{+5.3}_{-5.8}$	$12.6^{+3.6}_{-2.5}$
GW190519_153544	1.5	$66.0^{+10.7}_{-12.0}$	$40.5^{+11.0}_{-11.1}$
GW190521	7.2	$95.3^{+28.7}_{-18.9}$	$69.0^{+22.7}_{-23.1}$
GW190602_175927	2.7	$69.1^{+15.7}_{-13.0}$	$47.8^{+14.3}_{-17.4}$
GW190701_203306	1.4	$53.9^{+11.8}_{-8.0}$	$40.8^{+8.7}_{-12.0}$
GW190706_222641	1.3	$67.0^{+14.6}_{-16.2}$	$38.2^{+14.6}_{-13.3}$
GW190828_065509	2.8	$24.1^{+7.0}_{-7.2}$	$10.2^{+3.6}_{-2.1}$
GW190924_021846	40.3	$8.9^{+7.0}_{-2.0}$	$5.0^{+1.4}_{-1.9}$
GW191109_010717	2.9	65^{+11}_{-11}	47^{+15}_{-13}
GW191129_134029	1.2	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$
GW190425	2.8	$2.0^{+0.6}_{-0.3}$	$1.4^{+0.3}_{-0.3}$
GW190426_152155	1.2	$5.7^{+3.9}_{-2.3}$	$1.5^{+0.8}_{-0.5}$
GW190814	29.1	$23.2^{+1.1}_{-1.0}$	$2.59^{+0.08}_{-0.09}$
GW190917_114630	3.0	$9.3^{+3.4}_{-4.4}$	$2.1^{+1.5}_{-0.5}$
GW200105_162426	3.6	$8.9^{+1.2}_{-1.5}$	$1.9^{+0.3}_{-0.2}$
GW200115_042309	1.2	$5.9^{+2.0}_{-2.5}$	$1.44^{+0.85}_{-0.29}$

PBH - DM constraints



PBHs and Dark Matter (asteroidal mass)



Conclusions

- The **non linear threshold for PBH** and **the mass function** could be fully computed from the **shape of the power spectrum of cosmological perturbations**, making relativistic numerical simulations.

- A softening of the equation of state **(QCD) significantly enhances the formation of PBHs**, with a **mass distribution peaked between 1 and 2 solar masses (**the range of heavy NSs and light BHs).

- This could gives **a sub-population of BH mergers compatible with the LVK catalog**, explaining mass gap events as **GW190814**.

- Our analysis predicts a **constraint on the abundance of DM in PBHs formed during the QCD (up to 0.1%)**, compatible with the current observational constraints.

- **A large enough feature of the power spectrum could account for all dark matter in PBHs in the asteroidal mass range** (USR inflation models).