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# A theoretical framework for isotope production at the SPES ISOL facility

## $^{28}\text{Mg}$ as a case-study

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ISOTOPIC TIME MACHINE  
FROM CULTURAL HERITAGE  
TO SUSTAINABLE FUTURE

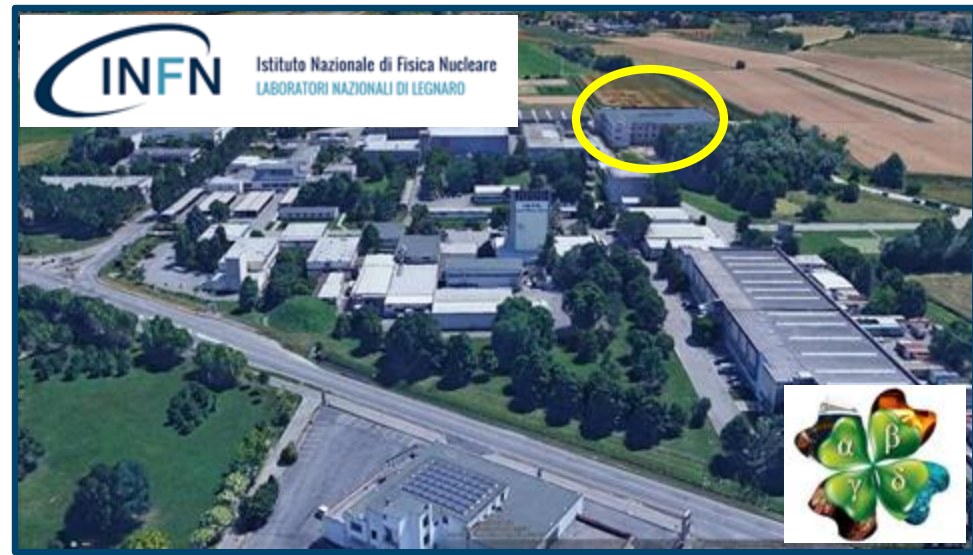




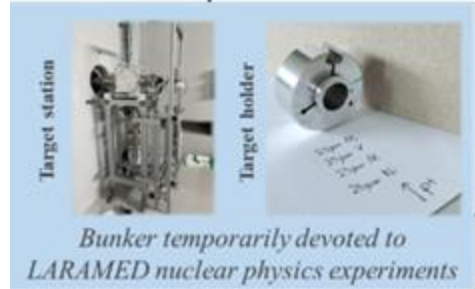
# SPES at INFN-LNL

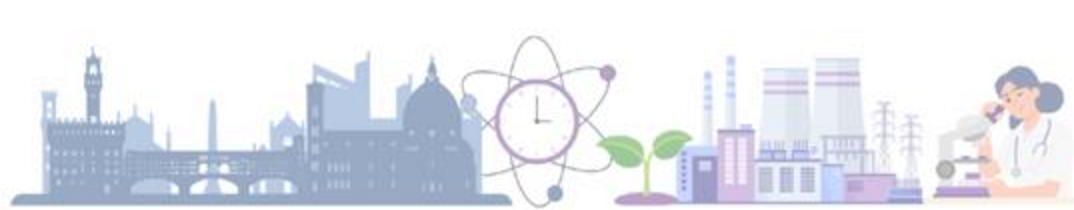


Two beamlines for long-living isotope production



**SPES:** B70 cyclotron (BEST Cyclotron Systems) with energy 30-70 MeV and maximum intensity 750  $\mu$ A. First Isotope Separation On-Line (**ISOL**) beam in November '24





# The SPES\_MED project

SPES  MED

INFN-CSN3

**General aim:** study the production of medical radionuclides at SPES

## WP1

Cross-section measurements  
for direct production

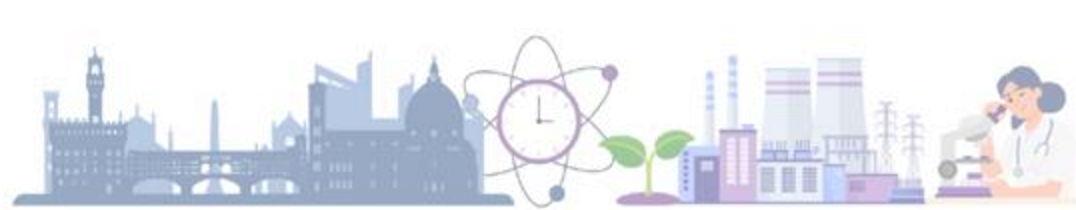
## WP2

Yield measurements  
for ISOL production

## WP3

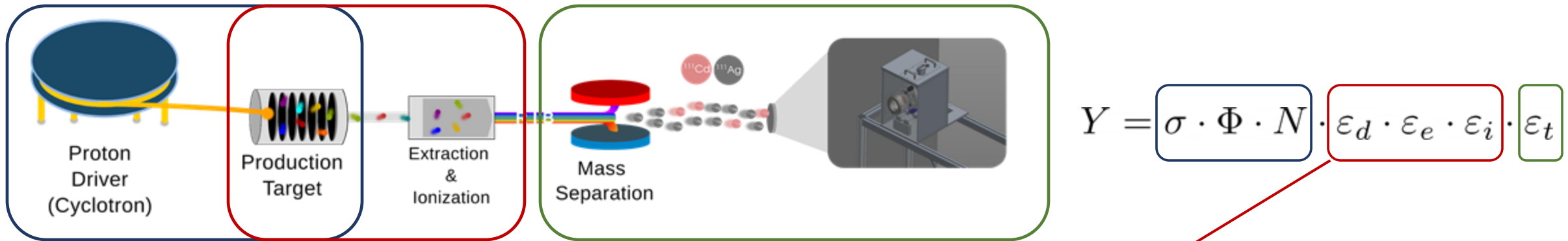
Models and simulations for nuclear  
reactions and ISOL processes





- SPES ion sources:
- surface
  - plasma
  - laser (LIS) ←

# A theoretical framework for ISOL



Atom population in the target box:

Atom population in the ion source:

$$\dot{N}_t = R - \kappa N_t - \lambda N_t$$

$$\dot{N}_i = \kappa N_t - \epsilon r N_i - \zeta N_i - \lambda N_i$$

Labels for the equations:

- Production** (points to  $R$ )
- Extraction** (points to  $\kappa N_t$ )
- Decay** (points to  $\lambda N_t$ )
- Ionization (pulse efficiency × pulse rate)** (points to  $\epsilon r N_i$ )
- Loss** (points to  $\zeta N_i$ )

This term represents the ion current coming out of the LIS ←

**Time-resolved model:**

- ✓ sensitive to more parameters
- ✓ allows for continuous optimization



# Case study: $^{28}\text{Mg}$ in SiC targets

- $^{28}\text{Mg}$ :  $\beta^-$  and  $\gamma$ -emitting **radiotracer** with biological and biomedical applications
- $t_{1/2} = 20.9$  h
- Mg isotopes can be produced by ISOL using **SiC targets**

Typical parameters of SiC targets

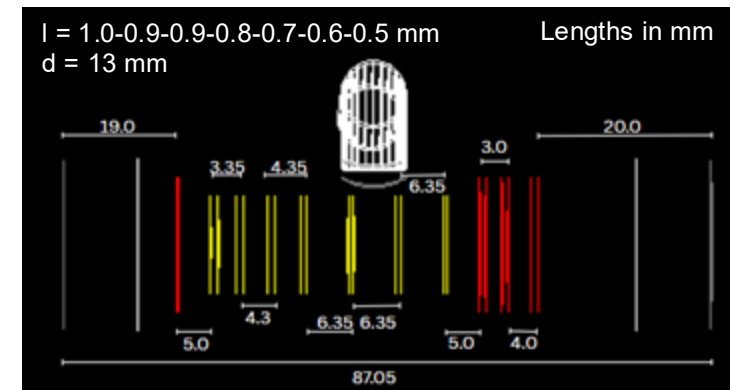
|        | $\rho$ ( $\text{g}/\text{cm}^3$ ) | $r$ ( $\mu\text{m}$ ) | $r_{\text{pori}}$ ( $\mu\text{m}$ ) |
|--------|-----------------------------------|-----------------------|-------------------------------------|
| SiC SA | 3.10                              | [4 – 10]              | no pori                             |
| SiC SP | 3.04                              | [4 – 10]              | 50                                  |

Good resistance up to  $T \sim 1800$  °C

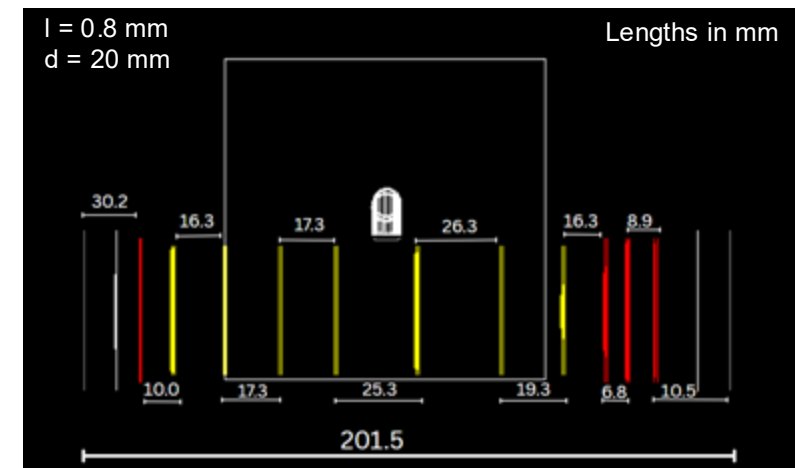
[T Ogura et al 2018](#)  
[WS Watson et al 1979](#)  
[M Manziolaro et al 2021](#)

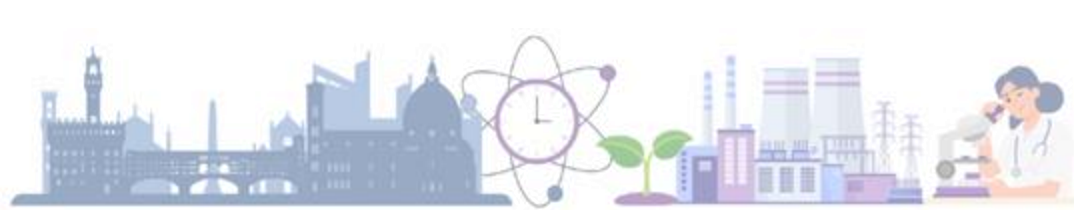
Two SPES target geometries

“small”



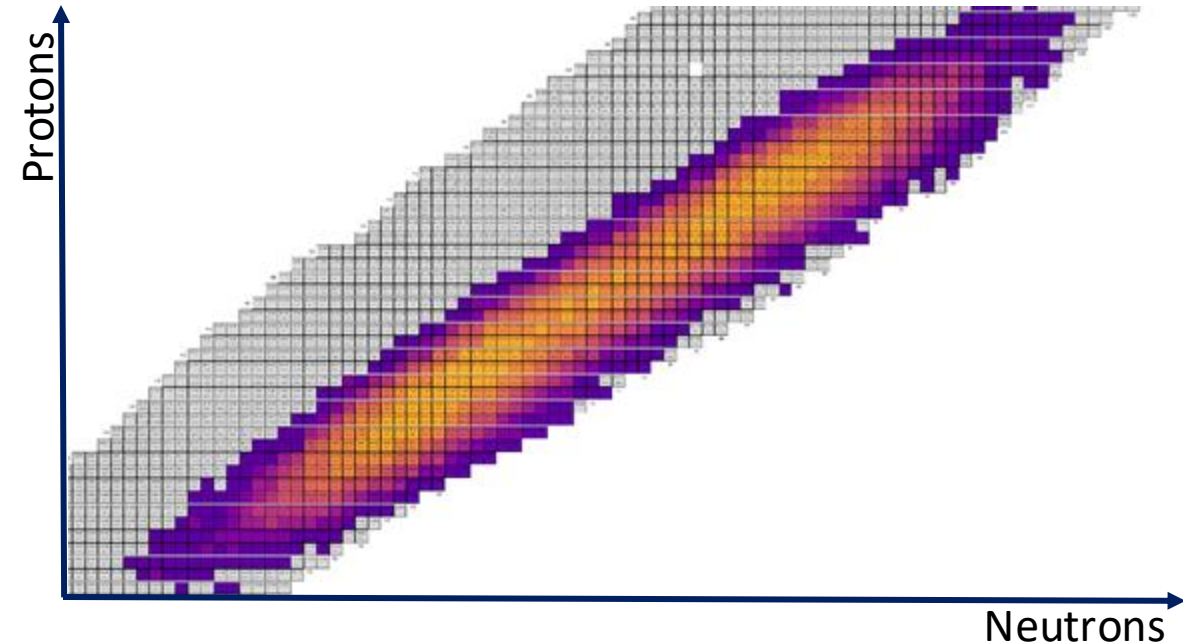
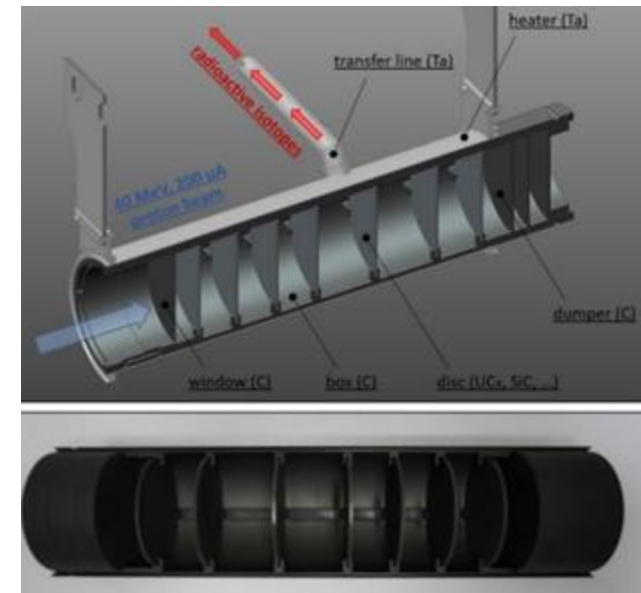
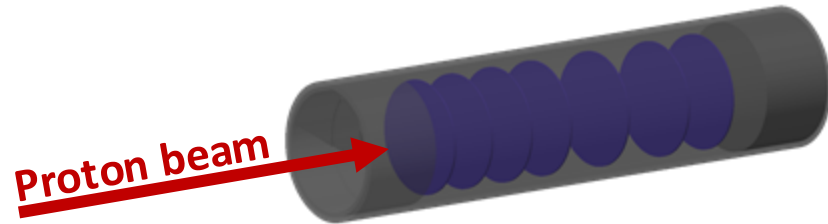
“big”





# Generation of isotopes

- **Standard procedure** using Monte Carlo codes
- The peculiar 7-disk geometry of the SPES target can be built





Simulations performed in the  
*CloudVeneto* infrastructure  
( $10^{10}$  events, uncertainty < 3%)

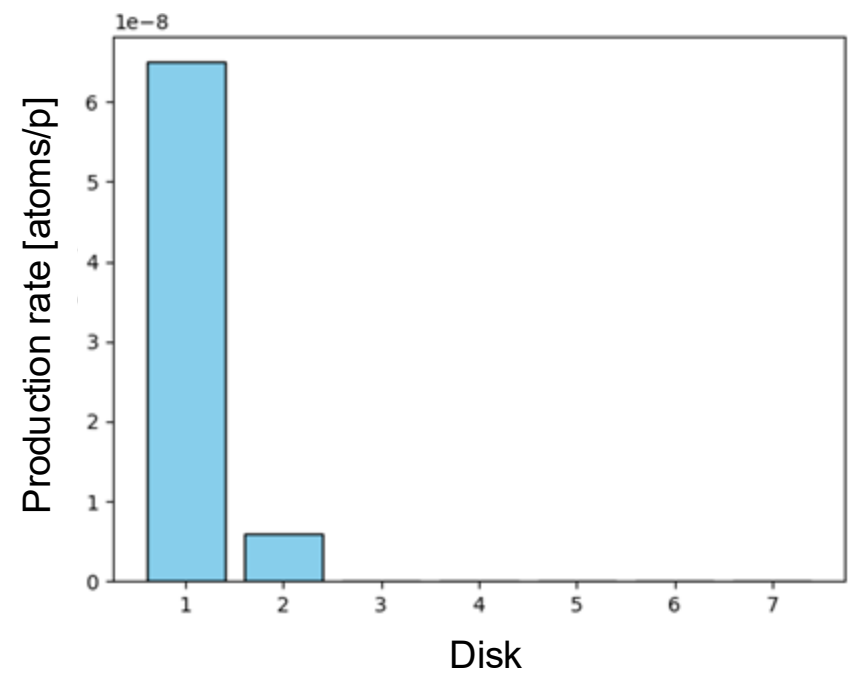
# $^{28}\text{Mg}$ production

With a 40 MeV, 200  $\mu\text{A}$  proton beam, the main production channel is likely to be  $^{30}\text{Si}(p,3p)^{28}\text{Mg}$  (compound nucleus and proton evaporation).

- Production rates predicted by Geant4:
- “Small” target:  $8.2 \times 10^{-8}$  atoms/p
  - “Big” target:  $6.4 \times 10^{-8}$  atoms/p

Disk distribution is also useful information.

$^{28}\text{Mg}$  distribution in small SiC target discs





# Diffusion

GEANT4 application G4\_ISOL\_RELEASE

[M Ballan et al 2020](#)

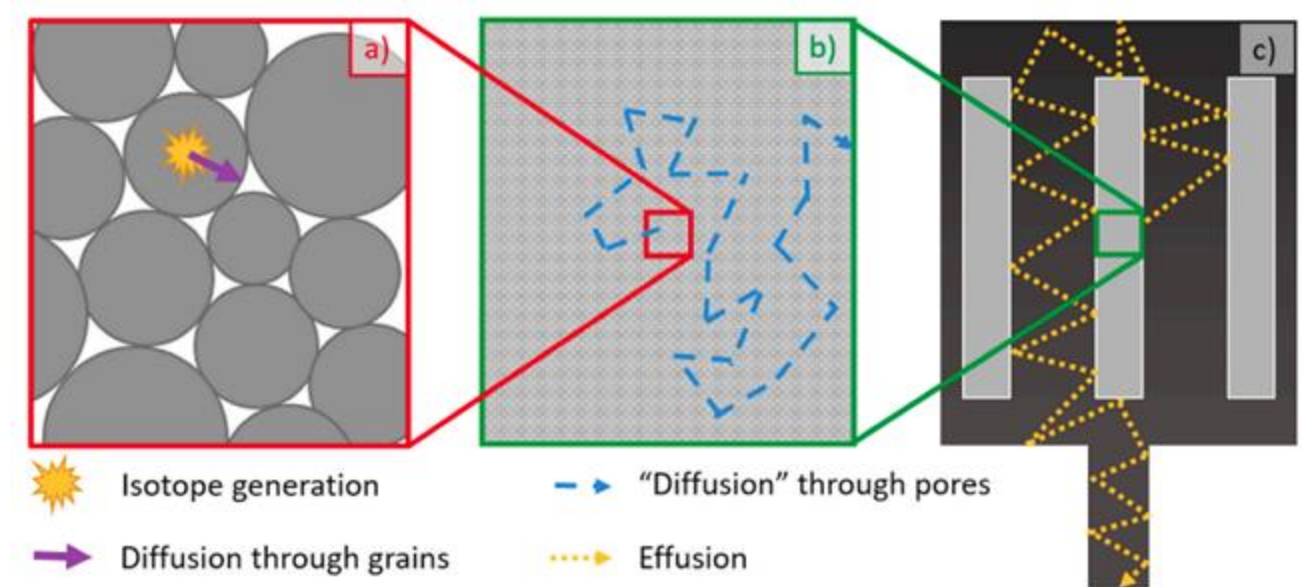
From the solution of Fick's laws in a sphere, the mean escape time of an **atom diffusing in a target grain** with radius  $a$  is

$$\langle t \rangle_{\text{diff}} = \frac{a^2}{15D}$$

with (Arrhenius equation)

$$D(T) = D_0 \exp\left(-\frac{E_a}{kT}\right)$$

$\langle t \rangle_{\text{diff}}$  computed for each event using a normal distribution for  $a$ .



**Generalized diffusion through pores** can be modeled with a different  $D_0$ .

[M Fujioka, Y Arai 1981](#)

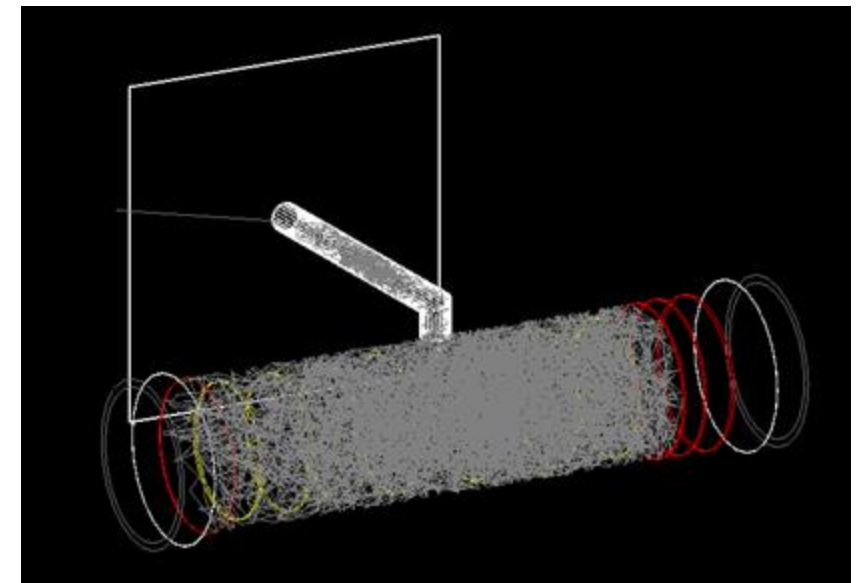
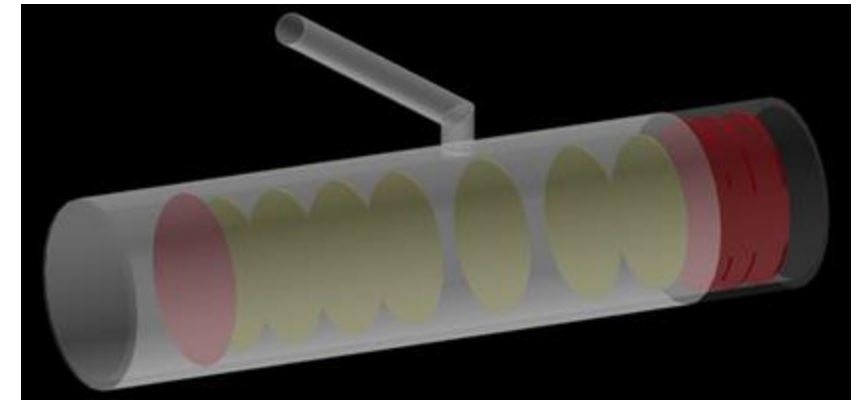
# Effusion

The **porous diffusion** and **effusion** time is simulated by letting the particles bounce until they reach the transfer line. Mean free path for porous diffusion:  $MFP = 2D_p/v$ .

**Sticking** time in the Ta box walls and **reabsorption** probability in the disks can also be set.

$$\langle t \rangle = \langle t \rangle_{\text{diff}} + \langle t \rangle_{\text{diff,p}} + \langle t \rangle_{\text{eff}} = \kappa^{-1}$$

*Free path of a single atom before exiting the target box*





# Diffusion and effusion of $^{28}\text{Mg}$

$10^6$  events: uncertainty < 0.1%

| $T$    | $\langle a \rangle$ | $\sigma_a$      | $D_0$                           | $E_a$                     |
|--------|---------------------|-----------------|---------------------------------|---------------------------|
| 1873 K | 7 $\mu\text{m}$     | 1 $\mu\text{m}$ | 0,024871 $\text{cm}^2/\text{s}$ | 92,2436 $\text{kcal/mol}$ |

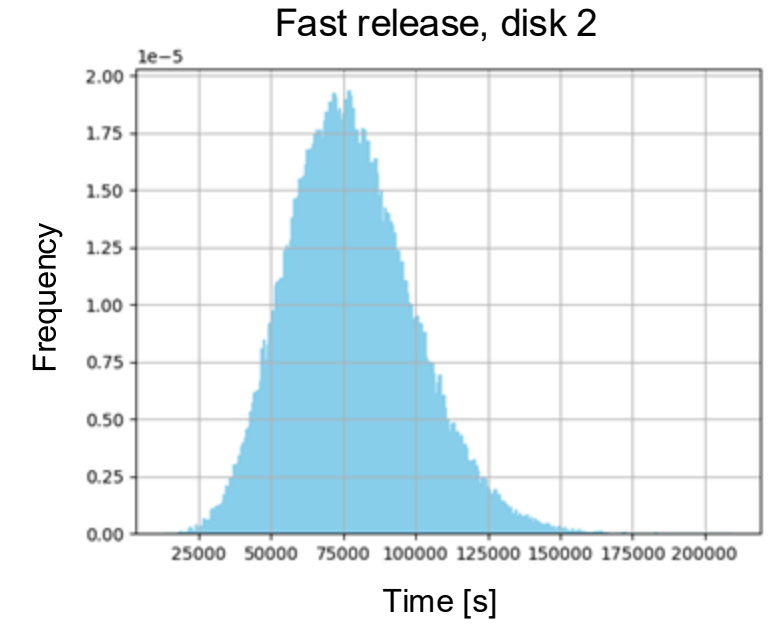
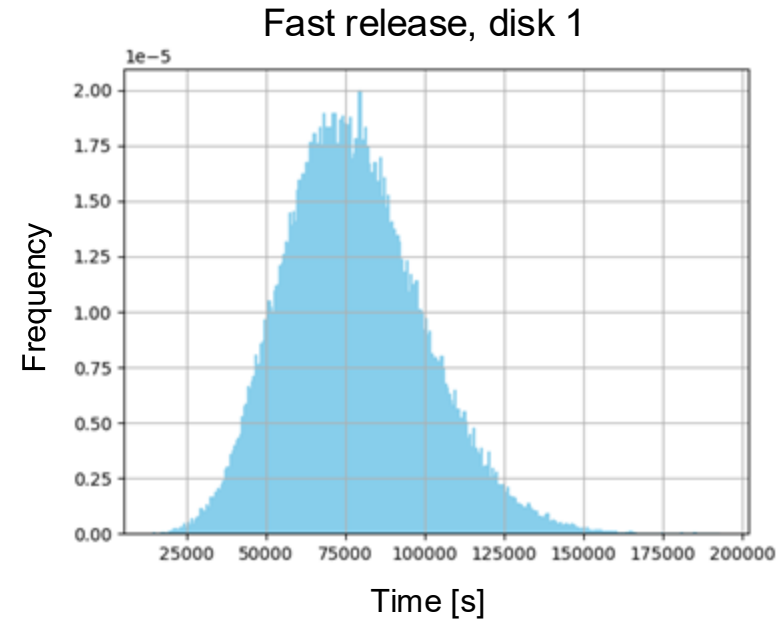
$D_{0,p}$  between  $10^{10} \text{ cm}^2/\text{s}$  ("slow") and  $10^{20} \text{ cm}^2/\text{s}$  ("fast").

Sticking times < 1 ms: neglected.

➤ No significant difference between discs and target configuration

➤  $\langle t \rangle_{\text{eff}} < 1 \text{ s}$ ,  $\langle t \rangle_{\text{diff,p}} \ll 1 \text{ s}$

➤  $\langle t \rangle_{\text{diff}} = 21.6 \text{ h}$  dramatically high even in porous target and comparable with  $t_{1/2}$





# Diffusion and effusion of $^{28}\text{Mg}$ improved

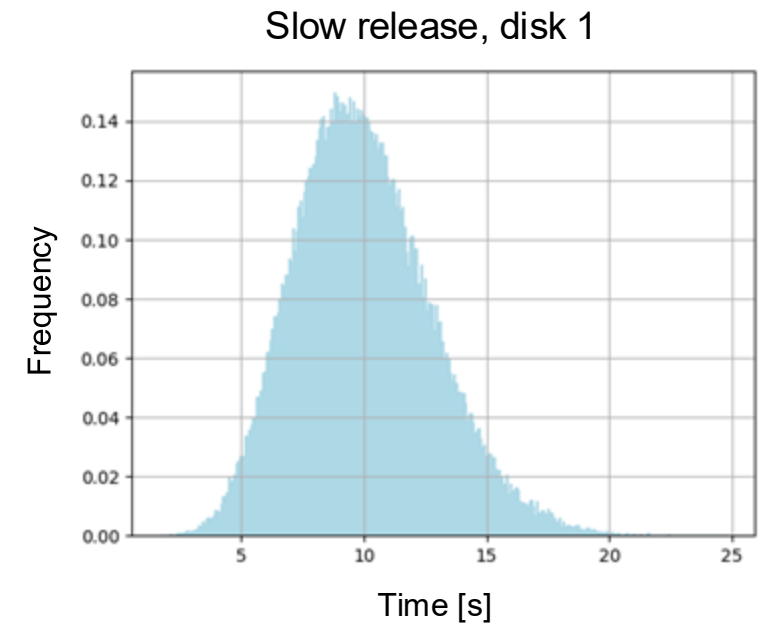
A suitable release could be obtained by slightly improving

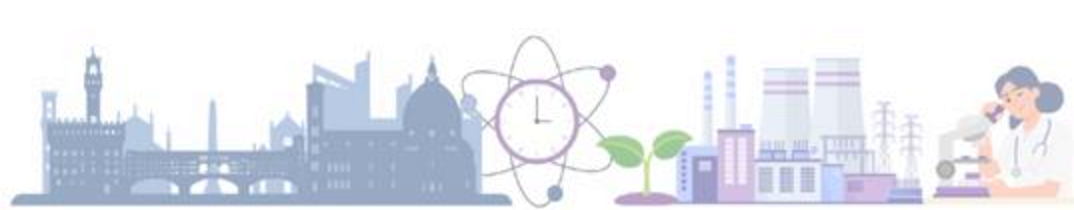
- **grain size** (reduction by 1 order of magnitude)
- **thermal resistance** (up to 2000 °C)

| $T$    | $\langle a \rangle$ | $\sigma_a$        | $D_0$                           | $E_a$                     |
|--------|---------------------|-------------------|---------------------------------|---------------------------|
| 2273 K | 0.7 $\mu\text{m}$   | 0.1 $\mu\text{m}$ | 0,024871 $\text{cm}^2/\text{s}$ | 92,2436 $\text{kcal/mol}$ |

In this way:

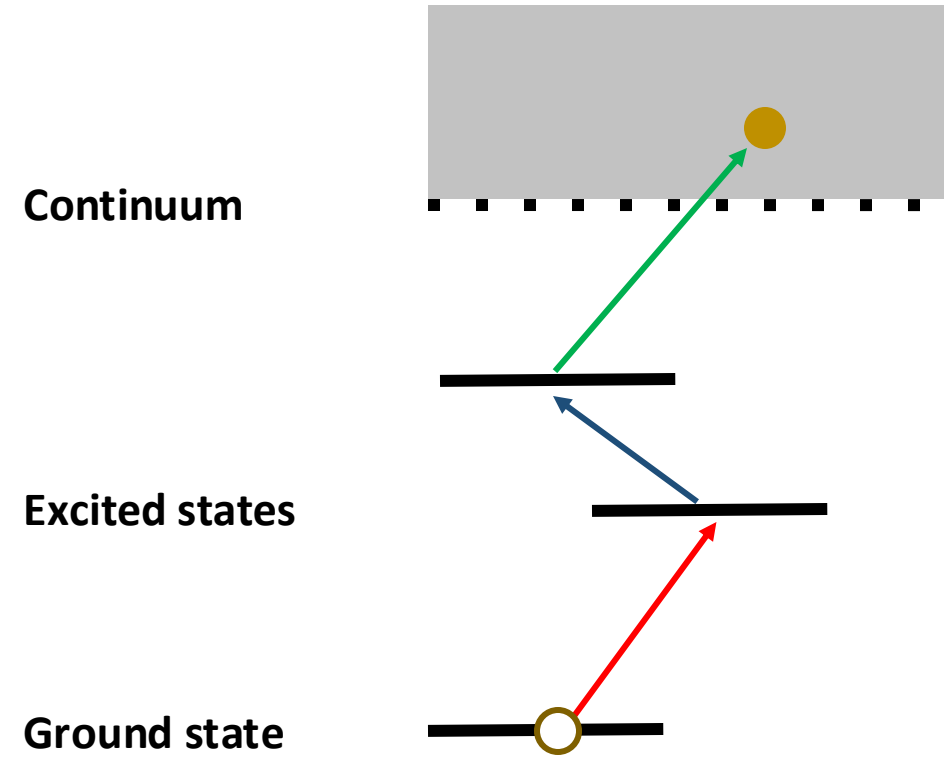
- $\langle t \rangle = 9.961(3) \text{ s}$  in the small target
- $\langle t \rangle = 10.271(3) \text{ s}$  in the big target





# Photo-ionization in the SPES LIS

- **Level population dynamics** of a laser photo-ionization scheme studied with the quantum **density matrix**
- This, combined with a **volume factor**  $\eta$ , gives the **single-pulse ionization efficiency** of the photon-atom interaction,  $\varepsilon$
- Global **photo-ionization rate** obtained multiplying  $\varepsilon$  times the laser **pulse rate**  $r$





# Photo-ionization scheme

$$\dot{\rho}_{11}(t) = -\frac{i}{2} \Omega_{21}(t) \rho_{12}(t) + \frac{i}{2} \Omega_{12}(t) \rho_{21}(t) + 2(A_{21} \rho_{22}(t) + A_{31} \rho_{33}(t)),$$

$$\dot{\rho}_{22}(t) = -\frac{i}{2} \Omega_{12}(t) \rho_{21}(t) + \frac{i}{2} \Omega_{21}(t) \rho_{12}(t) - \frac{i}{2} \Omega_{32}(t) \rho_{23}(t) + \frac{i}{2} \Omega_{23}(t) \rho_{32}(t) + 2A_{32} \rho_{33}(t) - 2A_{21} \rho_{22}(t),$$

$$\dot{\rho}_{33}(t) = -\frac{i}{2} \Omega_{23}(t) \rho_{32}(t) + \frac{i}{2} \Omega_{32}(t) \rho_{23}(t) - 2(A_{32} + A_{31} + \gamma_i(t)) \rho_{33}(t),$$

$$\dot{\rho}_{\text{ion}}(t) = 2\gamma_i(t) \rho_{33}(t).$$

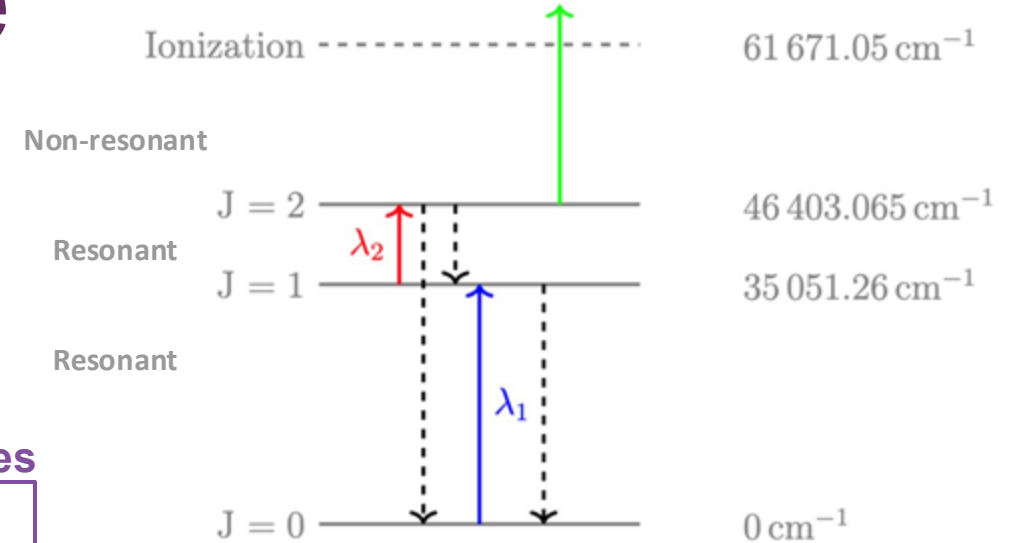
## Populations

$$\dot{\rho}_{12}(t) = \frac{i}{2} \Omega_{12}(t) (\rho_{22}(t) - \rho_{11}(t)) - \frac{i}{2} \Omega_{32}(t) \rho_{13}(t) - (i\Delta_1 + A_{21} + A_{32} + A_{31} + 2\gamma_{L1}) \rho_{12}(t),$$

$$\dot{\rho}_{23}(t) = \frac{i}{2} \Omega_{23}(t) (\rho_{33}(t) - \rho_{22}(t)) - \frac{i}{2} \Omega_{21}(t) \rho_{13}(t) - (i\Delta_2 + A_{21} + A_{32} + A_{31} + \gamma_i(t) + 2\gamma_{L2}) \rho_{23}(t),$$

$$\dot{\rho}_{13}(t) = \frac{i}{2} \Omega_{12}(t) \rho_{23}(t) - \frac{i}{2} \Omega_{23}(t) \rho_{12}(t) - (i(\Delta_1 + \Delta_2) + A_{21} + A_{32} + A_{31} + \gamma_i(t) + 2\gamma_{L1} + 2\gamma_{L2}) \rho_{13}(t).$$

## Coherences



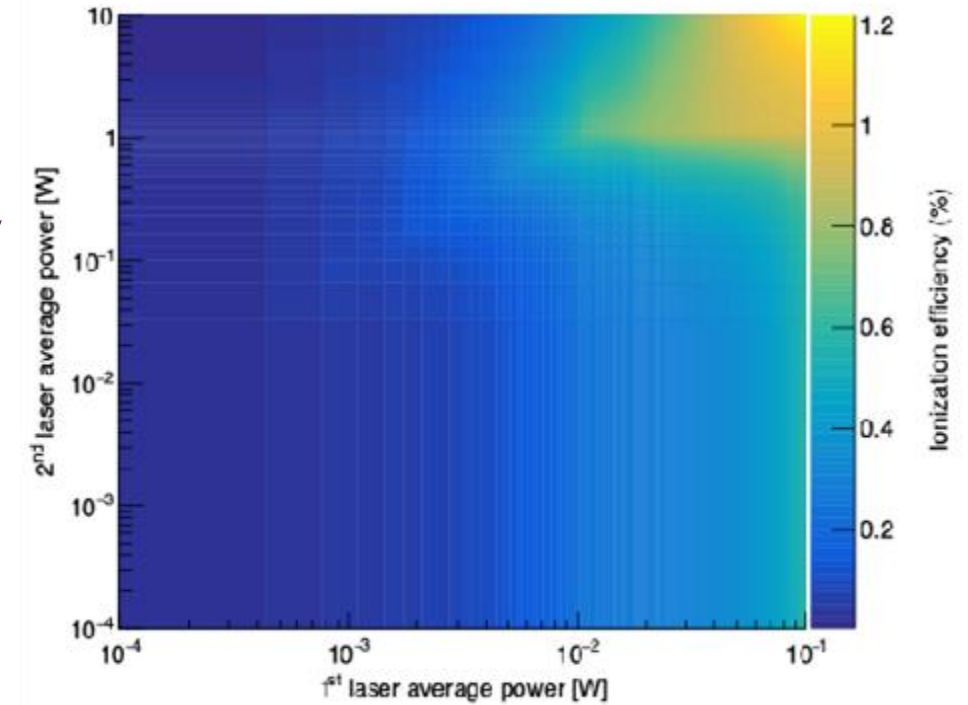
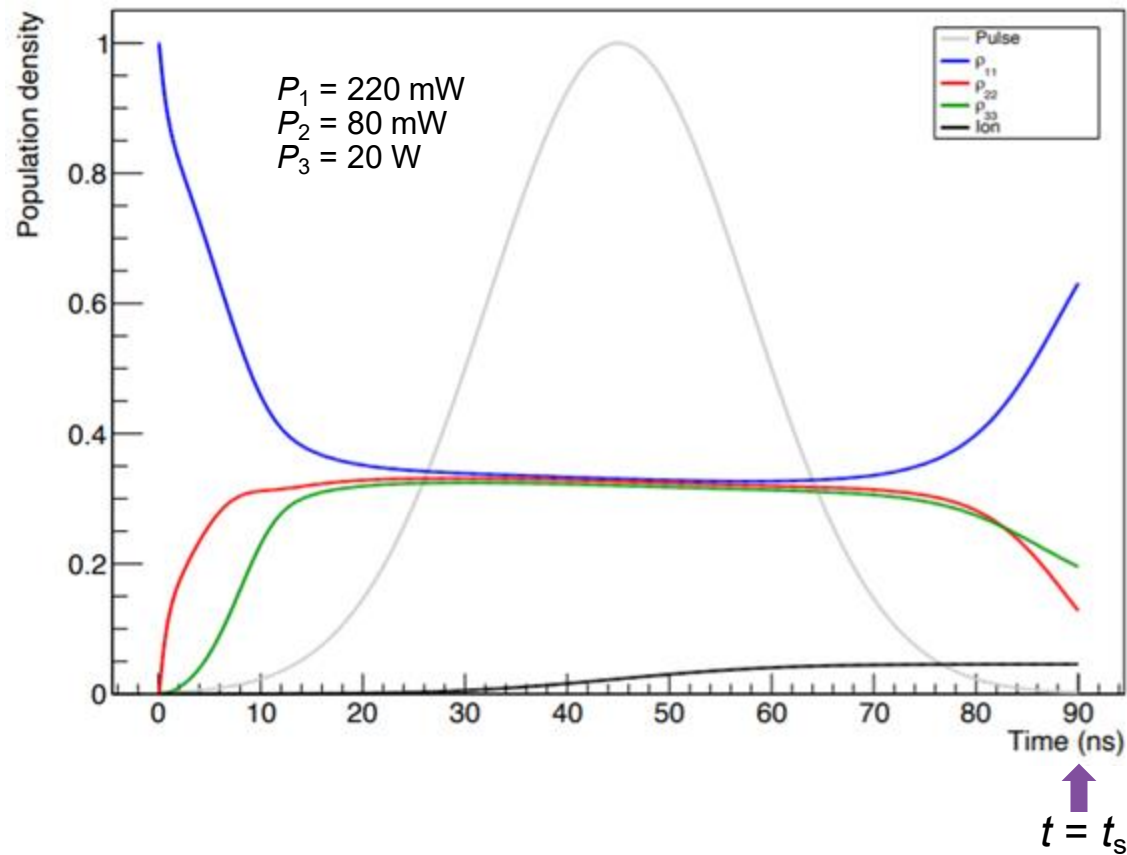
$A_{ji}$ : Einstein coefficients (decay rates)

$\Omega_{ij}$ : Rabi frequencies (oscillation frequencies between levels,  $\propto P^{1/2}$ )

$\gamma_i$ : non-resonant ionization rate ( $\propto P$ )



# Photo-ionization efficiency



With powers reachable in the SPES Online Laser Lab:

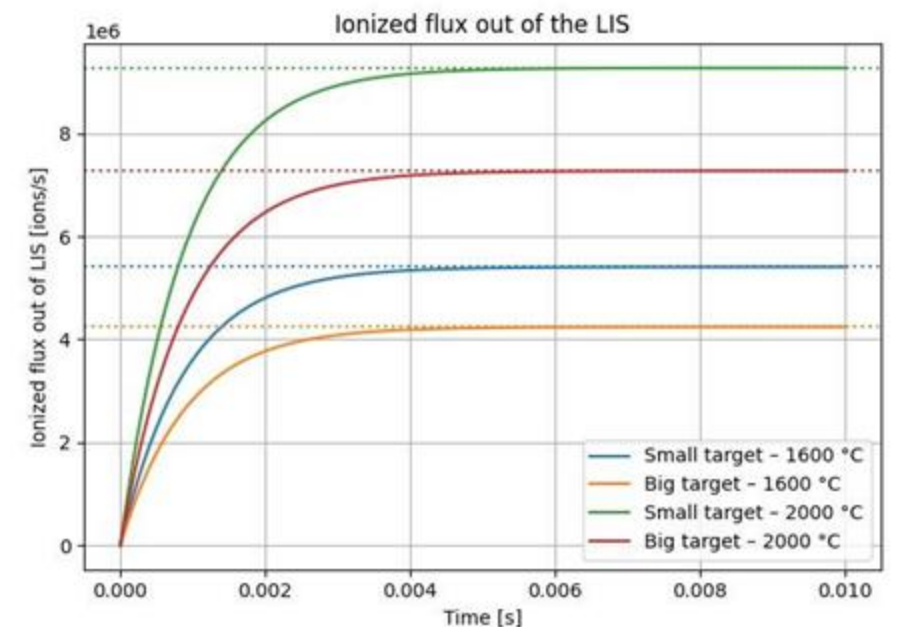
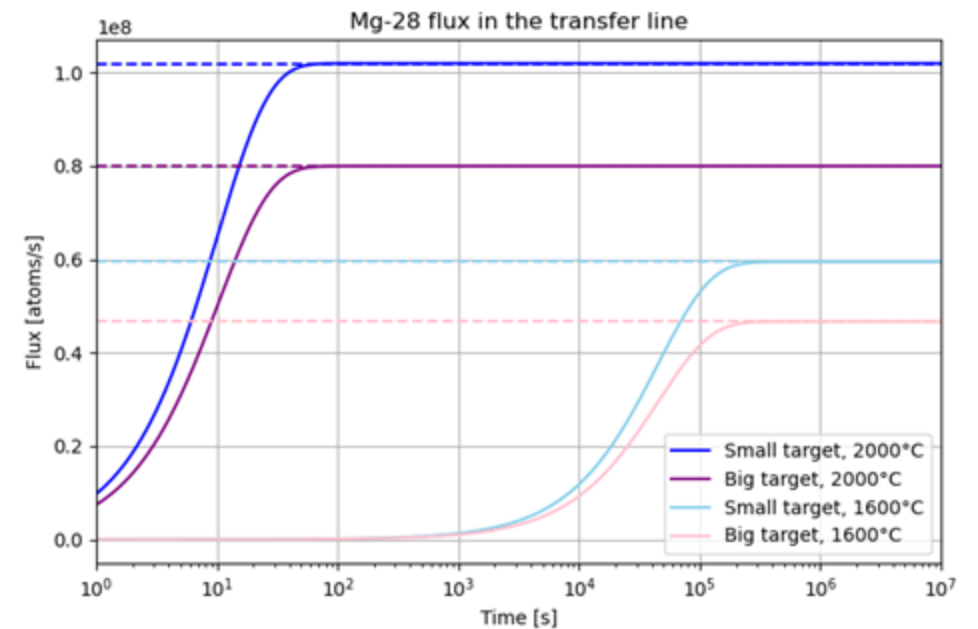
$$\varepsilon = \rho_{\text{ion}}(t_{\text{sat}}) \cdot \eta \sim 1\%$$

$\eta \sim 0.2$  is the ratio between the **laser interaction volume** and the SPES LIS. This is a good result, as  $r = 10^4 \text{ s}^{-1}$ .

[A Arzenton et al 2025](#)

# Overall ion flux

- $^{28}\text{Mg}$  flux in the transfer line becomes steady in an acceptable time only at high  $T$
- Once the steady state in the transfer-line flux is achieved, the **ion flux** from the LIS becomes steady in a negligible time
- The steady state of the ion flux is strongly **influenced by the loss rate** (here  $10^3 \text{ s}^{-1}$ ), which is tunable with experimental data
- Under this assumption,  **$\sim 0.3 \text{ MBq}$**  would be produced in **1h**





# Conclusions

- ❖ **Theoretical workflow** developed, needs experimental validation
- ❖ New **Mg laser photo-ionization scheme** proposed
- ❖ The production of  **$^{28}\text{Mg}$**  is critically **hindered by diffusion in grains**
- ❖ According to calculations, SiC targets with slightly smaller grain size and higher thermal resistance could produce  $^{28}\text{Mg}$



# Thank you!