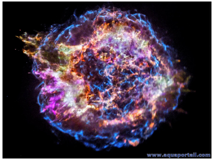


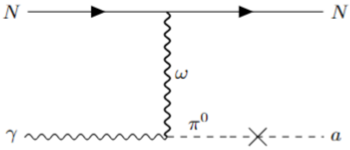
COST ACTION MEETING

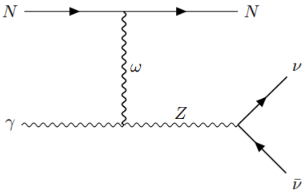
Photo-production in supernovae and neutron stars

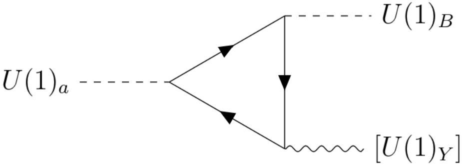
Miguel Vanvlasselaer
miguel.vanvlasselaer@vub.be

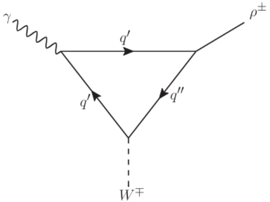
VUB and IIHE

What are the Wess-Zumino-Witten (WZW) interactions ?









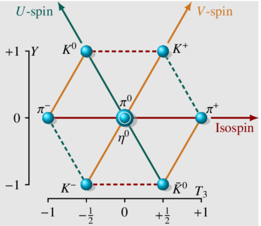
Computing the vertex

$$L \subset \omega \cdot Z \cdot F + \omega \cdot d\pi_0 \cdot F$$

Pion chiral lagrangian and WZW term

- Pion chiral lagrangian for the pseudo scalar mesons

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4} \mathrm{Tr}\left(D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U\right) , \qquad U = e^{2i\vec{\sigma}\cdot\vec{\pi}/f_\pi}$$



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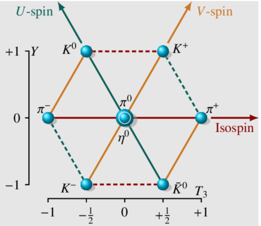

- Gauging the chiral lagrangian:

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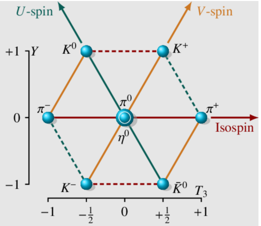

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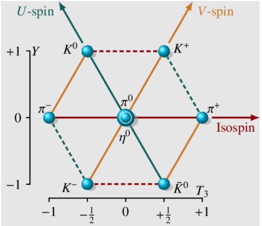

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- Solution: Wess-Zumino-Witten Phys. Lett. B 37 (1971) 95, Nucl. Phys. B 223 (1983) 422.

$$S_{\mathrm{WZW}} = \kappa \int_D d^5x\, \omega \;, \qquad \omega = -\frac{i}{240\pi^2}\, \epsilon^{\mu\nu\rho\sigma\tau}\, \mathrm{Tr}\left(\mathcal{U}_\mu\, \mathcal{U}_\nu\, \mathcal{U}_\rho\, \mathcal{U}_\sigma\, \mathcal{U}_\tau\right) \qquad \mathcal{U}_\mu = U^\dagger \partial_\mu U$$

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- Induces a conserved current

$$\hat{J}^\mu = \frac{1}{48\pi^2}\,\epsilon^{\mu\nu\rho\sigma\tau}\,\partial_\nu\ \text{Tr}\left[\{Q, U^\dagger\}\,\partial_\rho U\ U^\dagger \partial_\sigma U\ U^\dagger \partial_\tau U\right] \ .$$

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- Full expression for $S_{\text{WZW}}(U, A_\mu)$

$$\kappa\int_D d^5x\,\omega - \kappa e\int d^4x\,A_\mu J^\mu + \underbrace{\frac{ie^2}{24\pi^2}\int d^4x\,\epsilon^{\mu\rho\sigma\lambda}\,A_\rho\left(\partial_\mu A_\nu\right)\,\left[\text{Tr}\left(\{Q^2,U^\dagger\}\partial_\sigma U\right) - QUQ\partial_\sigma U^\dagger\right]}_{=\frac{ie^2}{48\pi^2}\frac{\pi_0}{f_\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}.$$

We found the anomaly !

Repeat the procedure with non-abelian chiral subgroups

Nucl. Phys. B 223 (1983) 422: Witten, PhysRevD.30.594: Kaymakcalan, Rajeev and Schechter

$\alpha \equiv dUU^\dagger \qquad \beta \equiv U^\dagger dU.$

$$\Gamma_{WZW}(U, \mathcal{A}_L, \mathcal{A}_R) = \Gamma_0(U) + \mathcal{C}\int \mathrm{Tr}\Big\{(\mathcal{A}_L\alpha^3 + \mathcal{A}_R\beta^3) - \frac{i}{2}[(\mathcal{A}_L\alpha)^2 - (\mathcal{A}_R\beta)^2]$$
$$+ i(\mathcal{A}_L U\mathcal{A}_R U^\dagger\alpha^2 - \mathcal{A}_R U^\dagger\mathcal{A}_L U\beta^2) + i(d\mathcal{A}_R dU^\dagger\mathcal{A}_L U - d\mathcal{A}_L dU\mathcal{A}_R U^\dagger)$$
$$+ i\big[(d\mathcal{A}_L\mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)\alpha + (d\mathcal{A}_R\mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)\beta\big] + (\mathcal{A}_L^3\alpha + \mathcal{A}_R^3\beta)$$
$$- (d\mathcal{A}_L\mathcal{A}_L + \mathcal{A}_L d\mathcal{A}_L)U\mathcal{A}_R U^\dagger + (d\mathcal{A}_R\mathcal{A}_R + \mathcal{A}_R d\mathcal{A}_R)U^\dagger\mathcal{A}_L U$$
$$+ (\mathcal{A}_L U\mathcal{A}_R U^\dagger\mathcal{A}_L\alpha + \mathcal{A}_R U^\dagger\mathcal{A}_L U\mathcal{A}_R\beta) + i\left[\mathcal{A}_L^3 U\mathcal{A}_R U^\dagger - \mathcal{A}_R^3 U^\dagger\mathcal{A}_L U - \frac{1}{2}(U\mathcal{A}_R U^\dagger\mathcal{A}_L)^2\right]\Big\}.$$

What is the pheno in all those terms ? HHH:0705.0697, 0708.1281, Gardner, He: 1101.1128

$$\mathcal{L}^{\pi}_{\mathrm{WZW}} \supset \frac{N_C}{48\pi^2} g_2^2 \tan\theta_W\ \epsilon^{\mu\nu\rho\sigma}\ F_{\mu\nu}\mathcal{A}_\rho Z_\sigma - 2\frac{N_C}{48\pi^2}\frac{\partial_\alpha\pi_0}{f_\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}\mathcal{A}_\beta$$

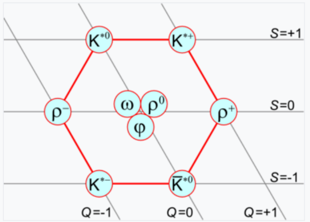
How to introduce vector mesons ?

$$\mathcal{L} \subset \omega \cdot F \cdot ... \;??$$

Vector mesons not part of the U matrix ! How to introduce them ?

$\Downarrow$ HHH: Phys. Rev. D 77 (2008) 085017

Introduce a background field B with QN of $\omega, \rho ...$ $\quad \delta B \to j^\mu$



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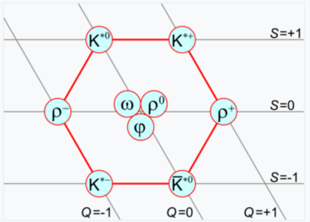

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$$\Gamma_c = -S^{\text{Bardeen}}_{\text{WZW}}(U = 1, \mathcal{A}_\mu + B_\mu) \quad \text{(Bardeen counterterms)}$$

$$\underbrace{\epsilon \cdot BA\partial A}_{\text{from WZW terms}} - \underbrace{\epsilon \cdot BA\partial A}_{\text{from counter terms}} = 0 \quad \Rightarrow \quad \text{three-legs GB term drop out}$$

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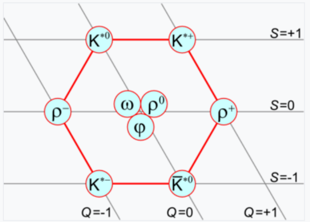

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- Non-vectorlike: Recipe is

$$S_{\rm full} = S_{WZW}(U, A_L + B_L, A_R + B_R) - \Gamma_c$$

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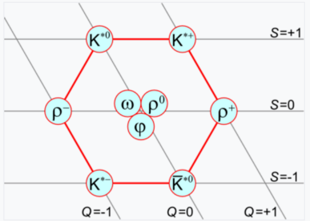

- vertex $\gamma - \omega - Z$

$$\frac{N_C}{48\pi^2} g_2^2 g_\omega \tan\theta_W \; \epsilon^{\mu\nu\rho\sigma} \; F_{\mu\nu}\omega_\rho Z_\sigma$$

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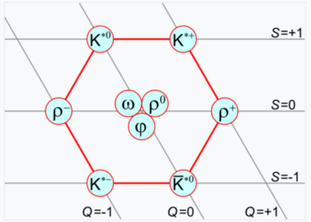

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$$\frac{\pi_0}{f_\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} = -2\frac{\partial_\alpha \pi_0}{f_\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}\mathcal{A}_\beta \to -2g_\omega\frac{\partial_\alpha \pi_0}{f_\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}\omega_\beta$$

Pion-axion mixing: $a - \pi_0$

Introduce the axion

- The KSVZ or KSVZ-like axion

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + \frac{\alpha_s}{8\pi}\frac{a}{f_a}G\widetilde{G} + \frac{1}{2}\left(\partial_\mu a\right)^2 - \frac{1}{2}m_{a_0}^2 a^2$$

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- axion and pion lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu a \quad \partial_\mu \pi_0\right)\mathcal{K}\begin{pmatrix}\partial_\mu a \\ \partial_\mu \pi_0\end{pmatrix} - \left(a \quad \pi_0\right)\mathcal{M}_{LO}^2\begin{pmatrix}a \\ \pi_0\end{pmatrix}$$

The mixing angle for the QCD axion Krauss and Wise 86', A. Notari, F. Rompineve, and G. Villadoro 22'

$$\theta_{\pi^0-a}^{\text{QCD}} \simeq \underbrace{\frac{1}{2}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}}_{\epsilon} \equiv \frac{C_A^{\text{QCDa}} f_\pi}{f_a}, \qquad \pi = \pi_{\text{phys}} + a_{\text{phys}}\epsilon$$

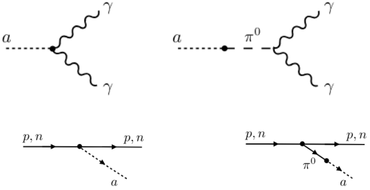
Prescription

$$\mathcal{M}_{i\to fa} = \theta_{\pi^0-a}^{\text{QCD}}\mathcal{M}_{i\to f\pi_0}$$

The mixing angle for heavy ALP

Bauer, Neubert, Renner, Schnubel, Thamm: gluon coupling

$$\hat{q}(a) = e^{-2i\vec{\kappa}_q c_{GG} a/f_a}\vec{q} \qquad \hat{m}_q(a) = e^{-2i\vec{\kappa}_q c_{GG} a/f_a}\vec{m}, \quad \text{Tr}[\vec{\kappa}] = 1$$



$$C_\gamma = -1.92 c_\gamma - \frac{m_a^2}{m_\pi^2 - m_a^2}\left(c_g \delta + c_{uu} - c_{dd}\right)$$

$$g_{ap} \to g_0(c_{uu} + c_{dd} + 2c_{GG})$$

- The mixing angle for heavy ALP Di Luzio, Giannotti, Nardi,Visinelli 20', Grilli di Cortona, Hardy, Pardo Vega, Villadoro 15'

$$\theta_{\pi^0 - a} \simeq \frac{1}{2}\frac{f_\pi}{f_a}\left(2c_{GG}\frac{(\kappa_u - \kappa_d)m_a^2}{m_\pi^2 - m_a^2}\delta - \frac{m_\pi^2}{m_\pi^2 - m_a^2}\Delta_\kappa\right) \equiv \frac{C_A^{\text{ALP}} f_\pi}{f_a}, \qquad \Delta_\kappa = 0$$

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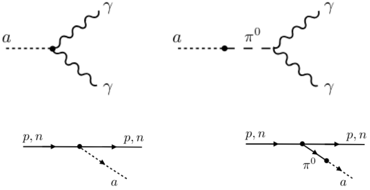

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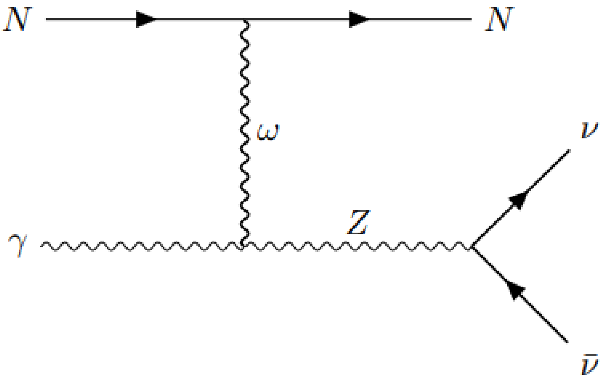
-

$$\pi = \pi_{\mathrm{phys}} - a_{\mathrm{phys}}\epsilon\frac{m_a^2}{m_a^2 - m_\pi^2} \qquad \Rightarrow \qquad \mathcal{M}_{i\to fa} = \theta^{\mathrm{ALP}}_{\pi^0-a}\mathcal{M}_{i\to f\pi_0}$$

Computing the full interaction



Road to our Lagrangian

- $$\mathcal{L}_{\text{WZW}} \supset \frac{N_C}{48\pi^2} g_2^2 g_\omega \; \tan\theta_W \; \epsilon^{\mu\nu\rho\sigma} \; F_{\mu\nu}\omega_\rho Z_\sigma \; + \dots$$

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- vertex γNNa

$$\mathcal{L}_{\text{int}} = C_A \frac{eN_c}{24\pi^2} \frac{g_\omega^2}{m_\omega^2} \epsilon_{\mu\nu\alpha\beta} \frac{\partial_\mu a}{f_a} F^{\nu\alpha} \bar{N}\gamma^\beta N$$

Summary of the results

- Photo-production of a neutrino pair: $\gamma N \to N\nu\bar{\nu}$

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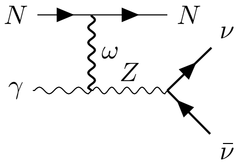

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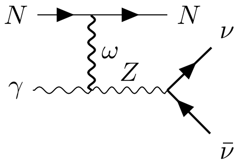
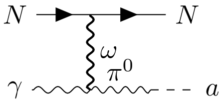


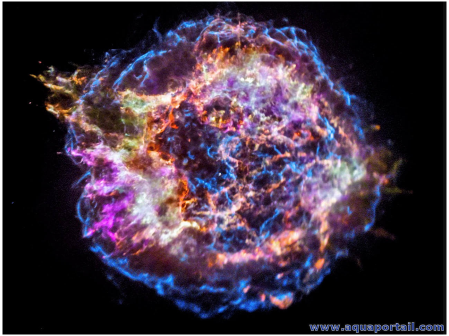
- Photo-production of axions: $\gamma N \to Na$

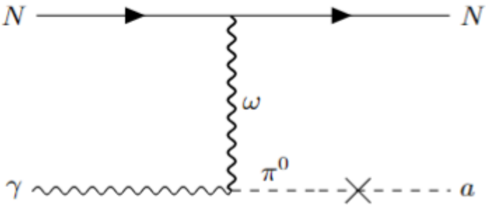
$$\mathcal{L}_{\rm int} = C_A\frac{eN_c}{24\pi^2}\frac{g_\omega^2}{m_\omega^2}\epsilon_{\mu\nu\alpha\beta}\frac{\partial_\mu a}{f_a}F^{\nu\alpha}\bar{N}\gamma^\beta N$$



Emission of axions from SN

Photo-production of axions in Supernovae





With Sabyasachi Chakraborty (IIT-Kanpur) and Aritra Gupta (IFIC Valencia): 2403.12169

The cooling argument in supernovae and SN1987A

- Neutrino received during SN1987A:

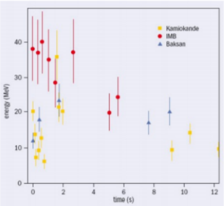


Figure: Credit:NirCam JWST

The cooling argument in supernovae and SN1987A

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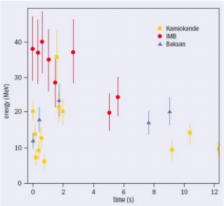

- Raffelt bound:

$$\frac{Q_{\text{additional}}}{\rho} \lesssim 10^{19} \text{ erg } s^{-1} g^{-1}$$

Figure: Credit:NirCam JWST

The cooling argument in supernovae and SN1987A

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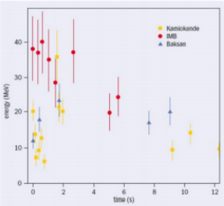

- Raffelt bound:

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- This is a coupling constraint !

Figure: Credit:NirCam JWST

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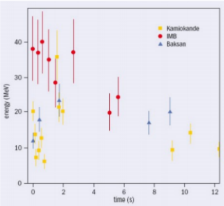

- Raffelt bound:

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- This is a coupling constraint !
- We need: Emissivity $Q =$, energy per unit of time and cc emitted

Figure: Credit:NirCam JWST

How to carry energy away from the SN?

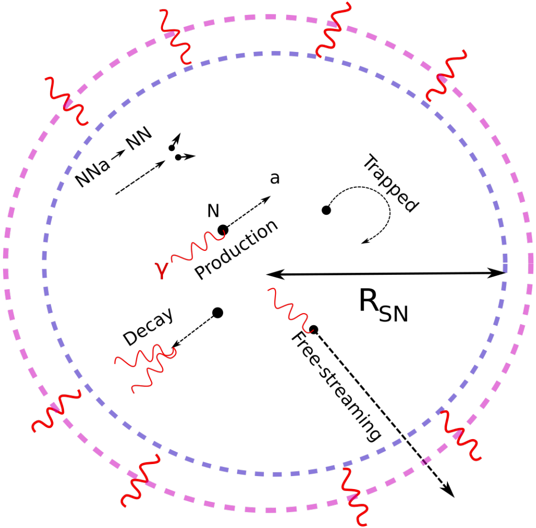
Supernovae extreme medium:
$\rho \sim \rho_0, \qquad T \sim 30 - 50$ MeV

bremsstrahlung and pion conversion

$$NN \to NNa \qquad N\pi \to Na$$

What about the photo-production ?

$$\boxed{N\gamma \to Na}$$

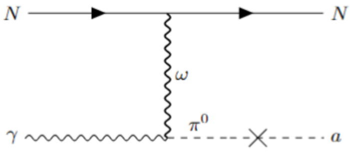


Emissivities of the $\gamma N \to Na$: main result

$$Q_{N\gamma\to Na} \approx \int \frac{dE_\gamma}{\pi^2} E_\gamma^2 f_\gamma(p_\gamma) \underbrace{n_B}_{\text{targets}} \underbrace{\sigma_{\gamma N\to Na}(E_\gamma)}_{\text{x-section}} \times \underbrace{E_\gamma}_{\text{energy escaping}}$$

- WZW Non-degenerate regime

$$\frac{Q^{\mathrm{WZW,ND}}_{N\gamma\to Na}}{10^{34}\ \text{erg/s/cm}^3} \approx 5.6\ C_A^2\ g_{40}^4\ T_{40}^8\ \rho_{15} \left(\frac{10^9\ \text{GeV}}{f_a}\right)^2$$



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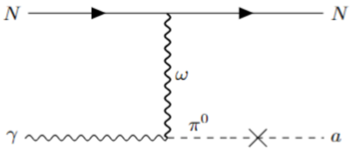

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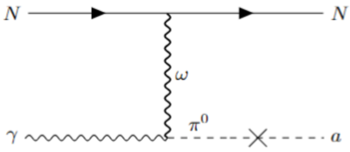
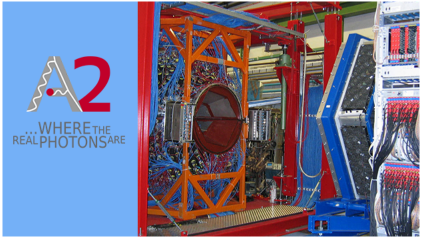
- ND computation holds for $T \gtrsim 30$ MeV.

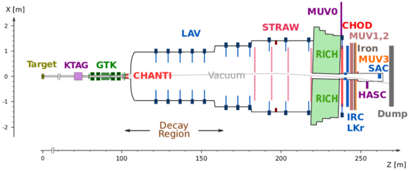


What is the value of g_ω ?

- Large range of theoretical prediction: $g_\omega \in 8 - 60$!

MAMI and NA60 collabs





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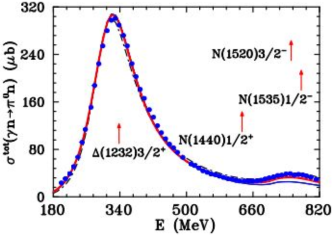
- Large range of theoretical prediction: $g_\omega \in 8 - 60!$
- $\sigma_{\gamma N \to N\pi_0}$ (MAMI) and $\Gamma_{\omega \to \pi_0 \gamma}$ (MAMI and NA60) measured



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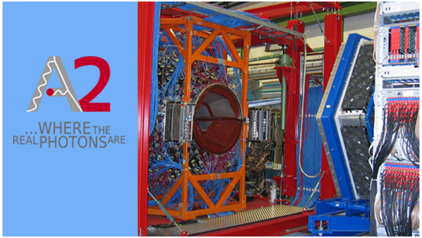

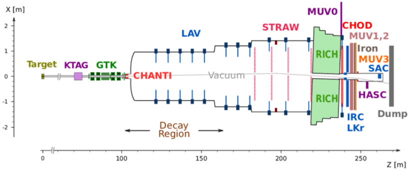

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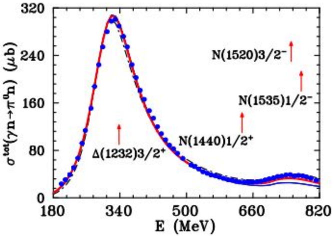

- Consistent with $g_\omega \sim 8 - 10$

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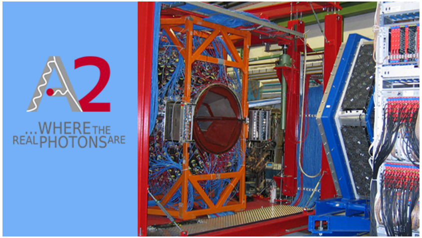

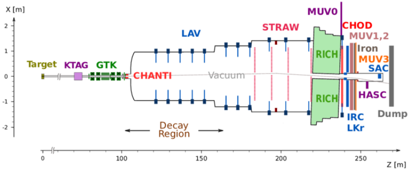

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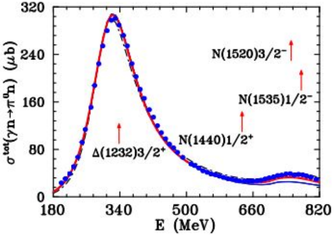

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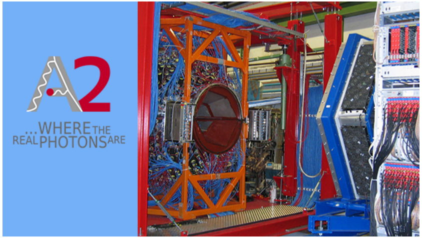
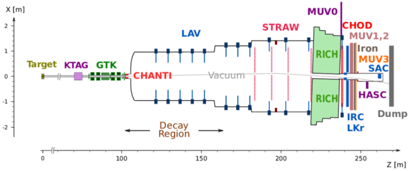
- Data driven method available: GlueX Aloni, Fanelli, Soreq, Williams: PRL 123, 071801

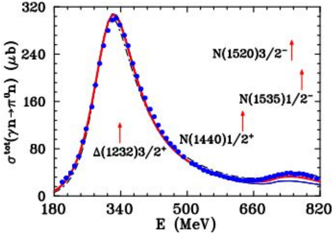
MAMI and NA60 collabs

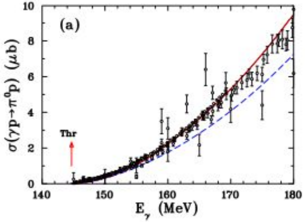




Data-driven photo production

- $\sigma_{\gamma N\to Na} \approx \frac{f_\pi C_A}{f_a}\sigma_{\gamma N\to N\pi_0}, \quad E > 145$ MeV





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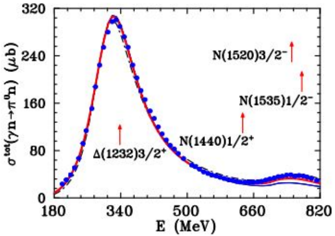

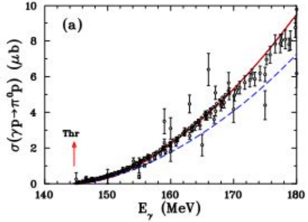

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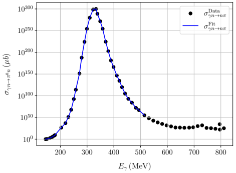
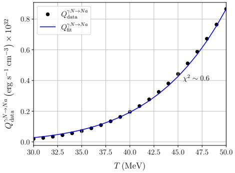
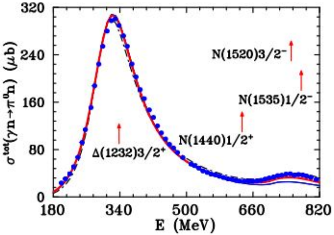
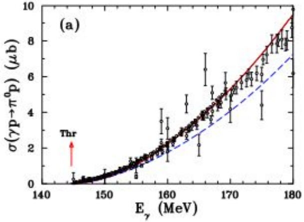
- Axion emissivities:

$$\frac{Q_{\rm data}^{\gamma N \to N a}}{1.6 \times 10^{33}\ \text{cm}^{-3} s^{-1} \text{erg}} \approx \left(\frac{C_A 10^9}{f_a/\text{GeV}}\right)^2 \times \rho_{15} T_{40}^{6.73}$$









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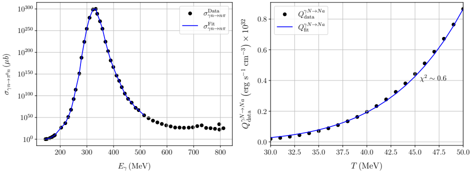
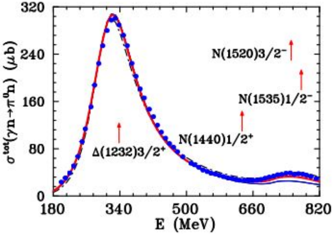
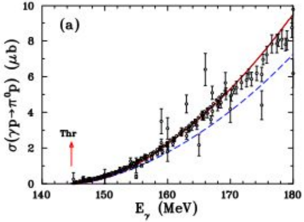


- The data-driven piece dominates if $g_\omega < 20$:

$$Q^{\rm data,ND}_{N\gamma\to Na} \gg Q^{\rm WZW}_{N\gamma\to Na}$$





In medium effects

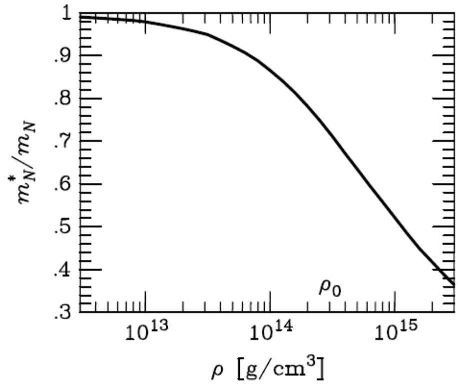


Figure: Credit: Raffelt

$\rho_{\mathrm{SN}} \sim$ few ρ_0

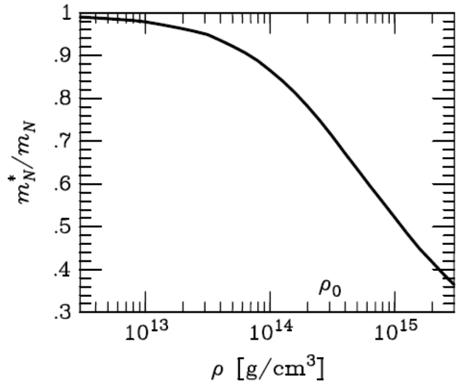
- The Brown-Rho scaling Brown, Rho 91'

$$\frac{f_\pi^\star}{f_\pi} \approx \frac{m_\omega^\star}{m_\omega} \approx \frac{m_N^\star}{m_N} \Rightarrow \frac{g_\omega^\star}{g_\omega} \approx \frac{g_\pi^\star}{g_\pi} \approx \frac{g_A^\star}{g_A}$$

g_A dependence: Voskresensky 01', Raffelt 01'

$$g_A^\star/g_A \approx 1/(1+(1/3)(m_N^\star/m_N)(\rho/\rho_0)^{1/3})$$

In medium effects



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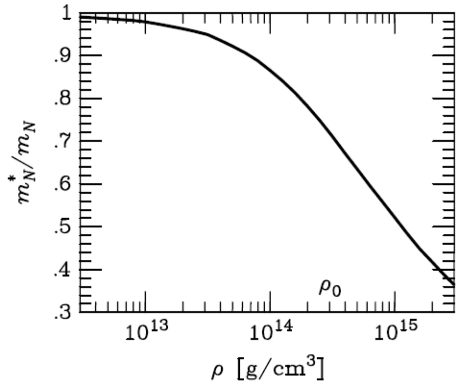
- For $NN \to NNa$: very mild

In medium effects



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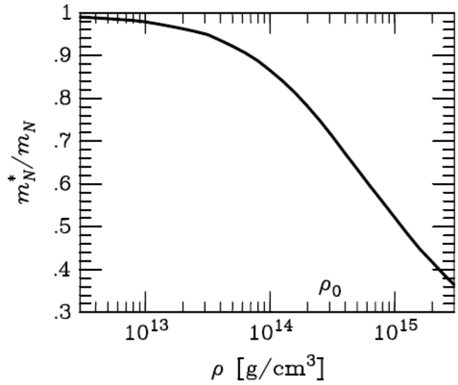
$$\frac{Q^\star_{N\gamma\to Na}}{Q_{N\gamma\to Na}} \approx \frac{m_\omega^4}{g_\omega^4}\frac{g_{\omega\star}^4}{m_{\omega\star}^4} \in [0.8, 1.5] \quad \rho \in [0.5, 2]\rho_0$$

In medium effects



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- For data-driven ??

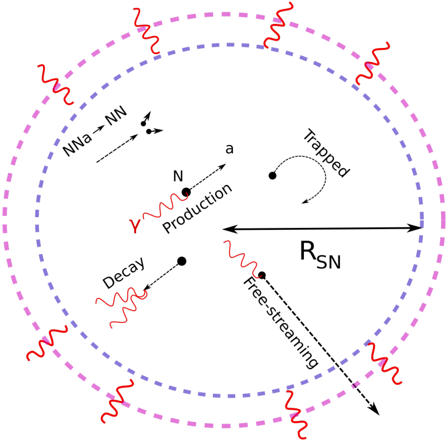
Emissivities of the $\gamma N \to Na$: further subtleties

- Absorption via $aN \to \gamma N$ or $NNa \to NN$:

$$L_a \approx \frac{1}{\Gamma_{aN\to\gamma N}} + \frac{1}{\Gamma_{NNa\to NN}}$$

$$\frac{L_a^{apn\to pn}}{10\ \text{Km}} \approx \frac{10^2}{X_p C_{ap}^2}\left(\frac{f_a}{10^9\,\text{GeV}}\right)^2 \frac{E_a}{1\ \text{GeV}}$$

We cut if $L_a < R_{SM} \sim 10$ km



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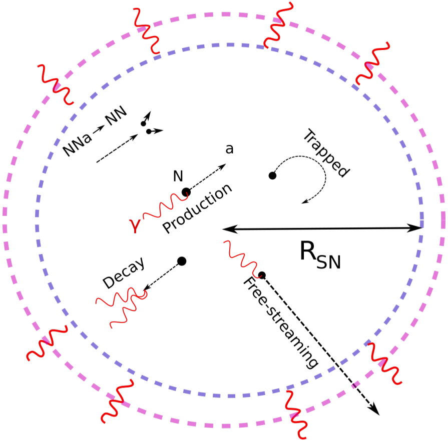
We cut if $L_a < R_{SM} \sim 10$ km

- Lapse effects: Caputo,Janka, Raffelt, Vitagliano: 2201.09890

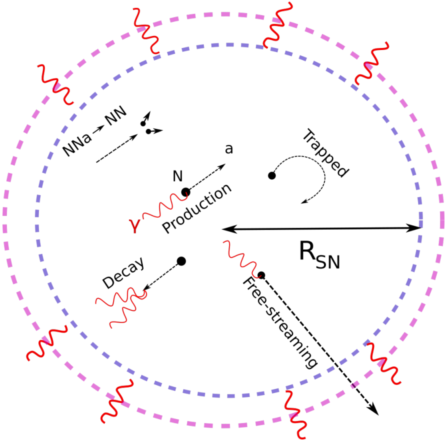
$$E_\infty = E_{\text{emission}} \times \alpha \qquad n_a^\infty = n_{\text{emission}} \times \alpha$$

$$\implies Q_\infty = \alpha^2 Q_{\text{emission}}$$

$$\alpha(M, r) \approx \sqrt{1 - 2M/r}$$



Emissivities of the $\gamma N \to Na$: further subtleties when axions are massive

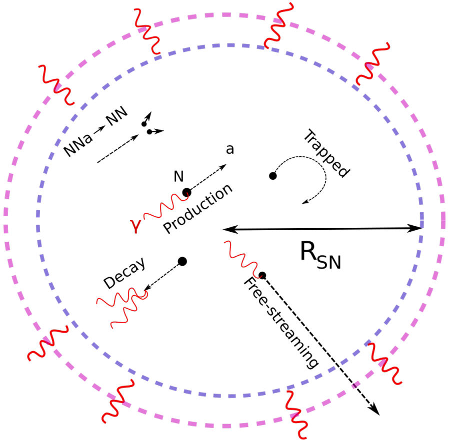


Massive axions regime

- Redshift effect: Caputo,Janka, Raffelt, Vitagliano: 2201.09890

$$Q \approx \int_{m_a/\alpha} dE \ldots$$

Emissivities of the $\gamma N \to N a$: further subtleties when axions are massive



Massive axions regime

- Redshift effect: Caputo,Janka, Raffelt, Vitagliano: 2201.09890

$$Q \approx \int_{m_a/\alpha} dE \ldots$$

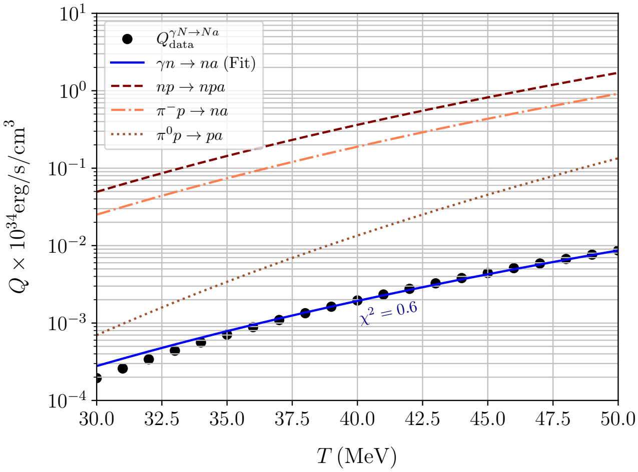
- Decay of the heavy axions:

$$L_{a\to\gamma\gamma} \approx \frac{4\times 10^4 \text{ km}}{(G_{a\gamma\gamma}/10^{-9}\text{GeV}^{-1})^2}\frac{E_a/100 \text{ MeV}}{(m_a/100 \text{ MeV})^4}$$

Cooling if $\quad L_{a\to\gamma\gamma} > R_{\rm SN}$

Impact of the photo-production on axion constraints for KSVZ model

Contribution from photo-production to the emissivity for QCD axion



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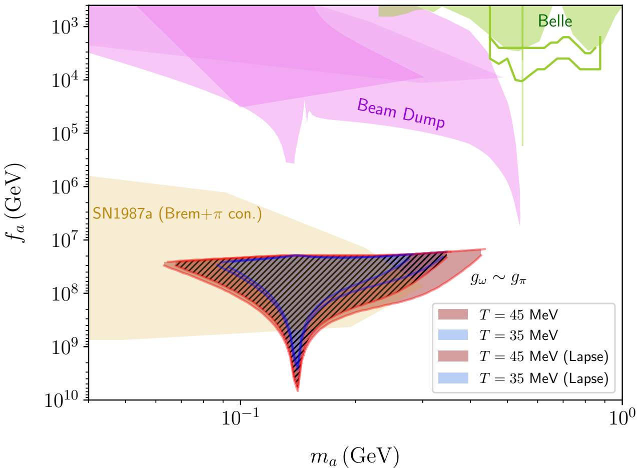
Contribution from photo-production to the emissivity for massive axion

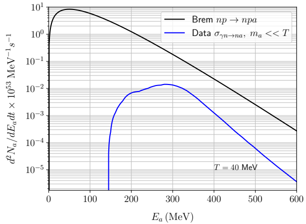


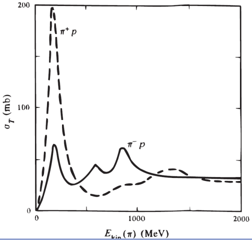
Can we observe the axions emitted from the supernovae

Bremsstrahlung peak: $E_a \sim 1.25T \sim 50 - 60$ MeV
Photo-production peak: $E_a \sim 6T \sim 250 - 300$ MeV

$$\frac{d^2N_a}{dE_a dt} \approx C_f \rho_{15} \left(\frac{C_A 10^9}{f_a/\text{GeV}}\right)^2 g_{40}^4 \left(\frac{E_a}{\text{MeV}}\right)^6 e^{-E_a/T} \ ,$$

$C_f = 4.6 \times 10^{42}$ MeV^{-1}s^{-1}





$$\sigma_{ap^+ \to N\pi^+} \approx 10^{-25} C_A^2 (f_\pi/f_a)^2 cm^{-2}$$

$$\frac{dN_\pi}{dt} \approx 6\rho_{15} C_A^4 \left(\frac{10^9}{f_a/\text{GeV}}\right)^4 g_{40}^4 T_{40}^7$$

$$\times \left(\frac{\text{Kpc}}{d}\right)^2 \left(\frac{M_{\text{detector}}}{\text{kton}}\right) \left(\frac{\text{g/mol}}{m_{H_2O}}\right)$$

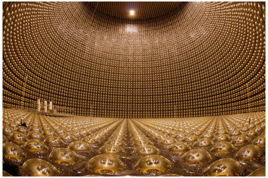
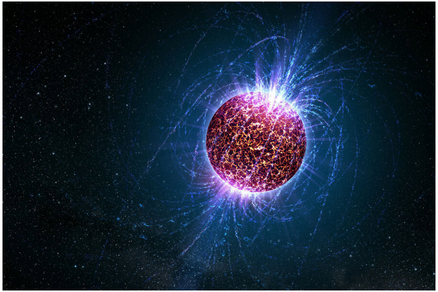
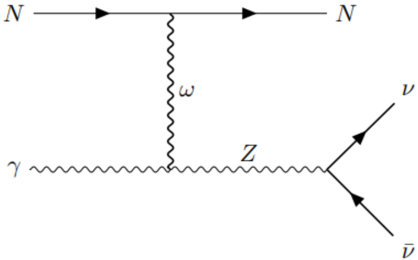


Photo-production of neutrino in NS

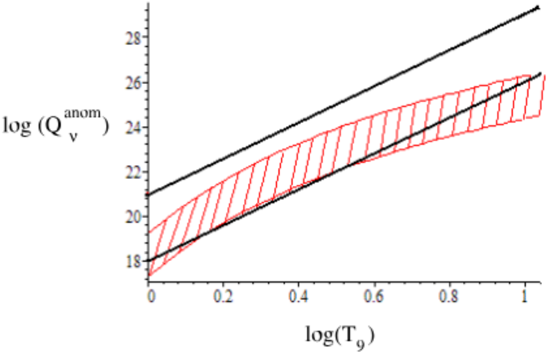




With Sabyasachi Chakraborty and Aritra Gupta: 2306.15872

WZW could contribution to NS cooling !

First computation of $N\gamma \to N\nu\bar{\nu}$ in [Harvey, Hill and Hill, arXiv:0708.1281]



But neglect the degeneracy effect ...

How do we compute the emissivity from a star ?

- Emissivity computation for $\gamma N \to NX$

$$Q_{\text{cooling}}^{N\gamma\to NX} \equiv \int \frac{d^3p_\gamma g_\gamma f_\gamma(p_\gamma)}{(2\pi)^3 2E_\gamma} \int \frac{g_N d^3p_{N_1} d^3p_{N_2}}{(2\pi)^3 2E_{N_1} (2\pi)^3 2E_{N_2}} \prod_{X_i} \int \frac{(2\pi)^4 d^3p_{X_i}}{(2\pi)^3 2E_{X_i}} \langle |\mathcal{M}|^2 \rangle \left(\sum_i E_{X_i} \right)$$
$$f_N(E_{N_1})\,(1 - f_N(E_{N_2}))\delta(E_1 - E_2 - Q_0)\,\delta^3(\vec{p}_1 - \vec{p}_2 - \vec{q})\,. \quad (1)$$

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$$f_N(E_{N_1}) \, (1 - f_N(E_{N_2})) \delta(E_1 - E_2 - Q_0) \, \delta^3(\vec{p}_1 - \vec{p}_2 - \vec{q}) \, . \tag{1}$$

- How to compute:

$$\boxed{\int \frac{g_N d^3p_{N_1} d^3p_{N_2}}{(2\pi)^3 2E_{N_1} (2\pi)^3 2E_{N_2}} f_N(E_{N_1}) \, (1 - f_N(E_{N_2}))}$$

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- Degenerate case: $(1 - f_N(E_{N_2})) \ll 1$: $\qquad \xi \equiv \frac{p_F^2}{2\pi m_N T} \gg 1$

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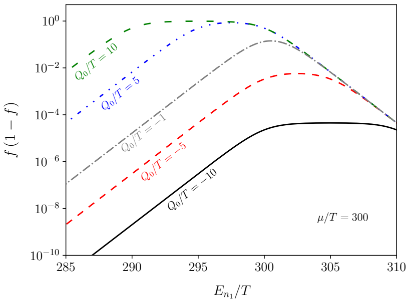
- Non-degenerate case: $(1 - f_N(E_{N_2})) \to 1$: $\quad \xi \ll 1$

Degenerate computation



$$Q^{2\to 3} = \frac{64\,n_F}{4}\,\frac{g_\gamma}{3}\kappa^2 \int \frac{d^3p_\gamma}{(2\pi)^3}\,\frac{f_\gamma}{2E_\gamma}|\vec{p}_\gamma|^2 \int \frac{d^3p_1\,d^3p_2}{(2\pi)^6 2E_1\,2E_2} E_1\,E_2\,(E_1+E_2)\,S(q^\mu)\ ,$$

$$S(Q_0, q) = \frac{M_N^2\,T}{\pi q}\,\frac{z}{1-e^{-z}}\Theta(\mu - E_-)\ .$$

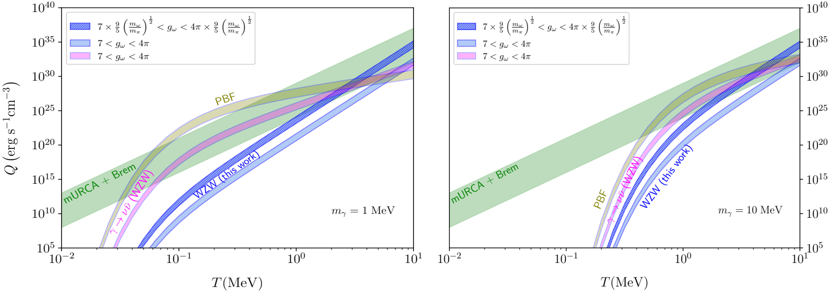
Standard cooling NS paradigm

- mURCA: $n\,n \longrightarrow n\,p\,e\,\bar{\nu}_e\,, \quad n\,p\,e \longrightarrow n\,n\,\nu_e$.:

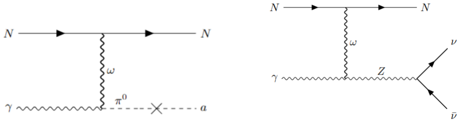
$$Q^{\mathrm{mURCA}} \simeq 10^{26-29}\left(\frac{T}{1\,\mathrm{MeV}}\right)^8 \mathrm{erg\,s^{-1}\,cm^{-3}}$$

- Bremsstrahlung

$$Q^{\,\nu-\mathrm{Brem}} \simeq 10^{24-28}\left(\frac{T}{1\,\mathrm{MeV}}\right)^8 \mathrm{erg\,s^{-1}\,cm^{-3}}$$

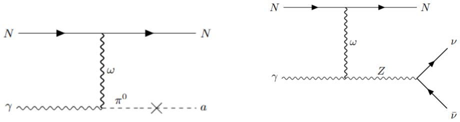


Conclusions and outlook



- Several new photo-production from anomaly: $\gamma N \to N\nu\nu$, $\gamma N \to Na$, ...ect.

Conclusions and outlook



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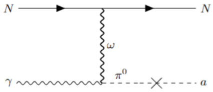

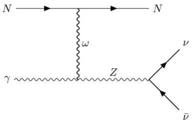

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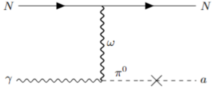
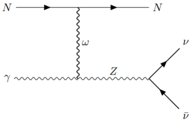
Conclusions and outlook





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- In SN, $N\gamma \to \gamma\nu\nu$, likely always subdominant wtr to traditional channels.
- pheno of WZW: "Circulez, Y'a rien à voir"?

Better keep exploring: Harvey Hill Hill arxiv:0712.1230

$$\begin{aligned}
\Gamma_{AAB} &= \mathcal{C}\int dZZ\left[\frac{s_W^2}{c_W^2}\rho^0 + \left(\frac{3}{2c_W^2}-3\right)\omega - \frac{1}{2c_W^2}f\right] + dAZ\left[-\frac{s_W}{c_W}\rho^0 - \frac{3s_W}{c_W}\omega\right] + dZ\left[W^-\rho^+ + W^+\rho^-\right]\frac{s_W^2}{c_W} \\
&\quad - s_W dA\left[W^-\rho^+ + W^+\rho^-\right] + (DW^+W^- + DW^-W^+)\left[-\frac{3}{2}\omega - \frac{1}{2}f\right], \\
\Gamma_{ABB} &= \mathcal{C}\int Z\Big\{d\rho^0\Big[-\frac{3}{2c_W}\omega - \frac{s_W^2}{c_W}a^0 + \left(-\frac{3}{2c_W}+3c_W\right)f\Big] + d\omega\Big[-\frac{3}{2c_W}\rho^0 + \left(-\frac{3}{2c_W}+3c_W\right)a^0 - \frac{s_W^2}{c_W}f\Big] \\
&\quad + da^0\left[\frac{s_W^2}{c_W}\rho^0 + \left(\frac{3}{2c_W}-3c_W\right)\omega - \frac{1}{2c_W}f\right] + df\Big[\left(\frac{3}{2c_W}-3c_W\right)\rho^0 + \frac{s_W^2}{c_W}\omega - \frac{1}{2c_W}a^0\Big]\Big\} \\
&\quad + s_W dA\left(\rho^0a^0 + 3\rho^0 f + 3\omega a^0 + \omega f + \rho^+a^- + \rho^-a^+\right) - \frac{s_W^2}{c_W}dZ\left(\rho^+a^- + \rho^-a^+\right) \\
&\quad + \frac{3}{2}\left[W^+D\rho^- + W^-D\rho^+\right](-\omega + f) + \frac{3}{2}\left[W^+(-\rho^- + a^-) + W^-(-\rho^+ + a^+)\right]d\omega \\
&\quad + \frac{1}{2}\left[W^+Da^- + W^-Da^+\right](-3\omega - f) + \frac{1}{2}\left[W^+(-3\rho^- - a^-) + W^-(-3\rho^+ - a^+)\right]df\,, \\
\Gamma_{BBB} &= \mathcal{C}\int 2\Big[\left(\rho^-f + \omega a^-\right)D\rho^+ + \left(\omega a^+ + \rho^+ f\right)D\rho^- + \left(\omega a^0 + \rho^0 f\right)d\rho^0 + \left(\rho^+a^- + \rho^-a^+ + \omega f + \rho^0a^0\right)d\omega\Big], \\
\Gamma_{AAAB} &= \mathcal{C}\int iW^+W^-Z\left[3c_W\omega + \left(c_W + \frac{1}{2c_W}\right)f\right], \\
\Gamma_{AABB} &= \mathcal{C}\int i\Big\{W^+W^-\left[\frac{3}{2}(\rho^0+a^0)\omega - \frac{1}{2}(\rho^0-a^0)f\right] \\
&\quad + W^+Z\Big[\left(\frac{3c_W}{2}-\frac{1}{c_W}\right)\rho^-f - \frac{3c_W}{2}\rho^-\omega - \frac{c_W}{2}a^-f + \frac{3c_W}{2}\omega a^-\Big]
\end{aligned}$$