Black hole microstates geometries and their holographic duals

Sami Rawash

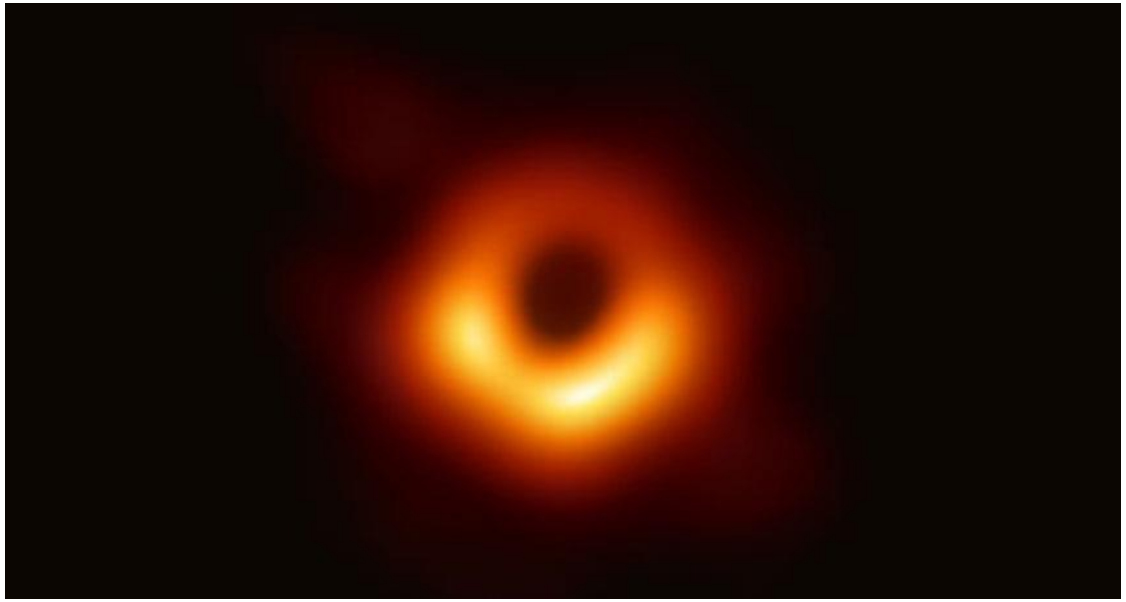
22 May 2024

Based on: 1904.12880, 2105.13046
2112.08378, 2212.08585

Outline of the talk:

- Motivations
- Black holes in string theory and the fuzzball proposal
- Tools and techniques to test the conjecture: holography, shockwaves and computational methods

Evidences for the existence of black holes:



Experimentally, they confirm GR predictions. Theoretically, we know that this is not the ultimate description of black hole physics.

GR description:

- Black holes are singular solutions of Einstein's equations.

- No Hair Theorem: uniqueness of black hole solution for fixed macroscopic charges (mass M, charge Q and angular momentum J).

- Black holes obey thermodynamic laws: thermal equilibrium (T=surface gravity), conservation of energy and

$$S_{BH} = \frac{A_{hor}}{4\, G_N} \qquad \Delta S_{BH} \geq 0$$

But at this level, this is just an analogy… Black holes are black: does this temperature have a physical meaning?

QFT in classical BH background:

- Black holes are not black: they emit thermal radiation, due to pair production at the horizon

Open questions in GR:

- Statistical mechanics interpretation of BH entropy: $S_{BH} \sim \log(\#microstates)$

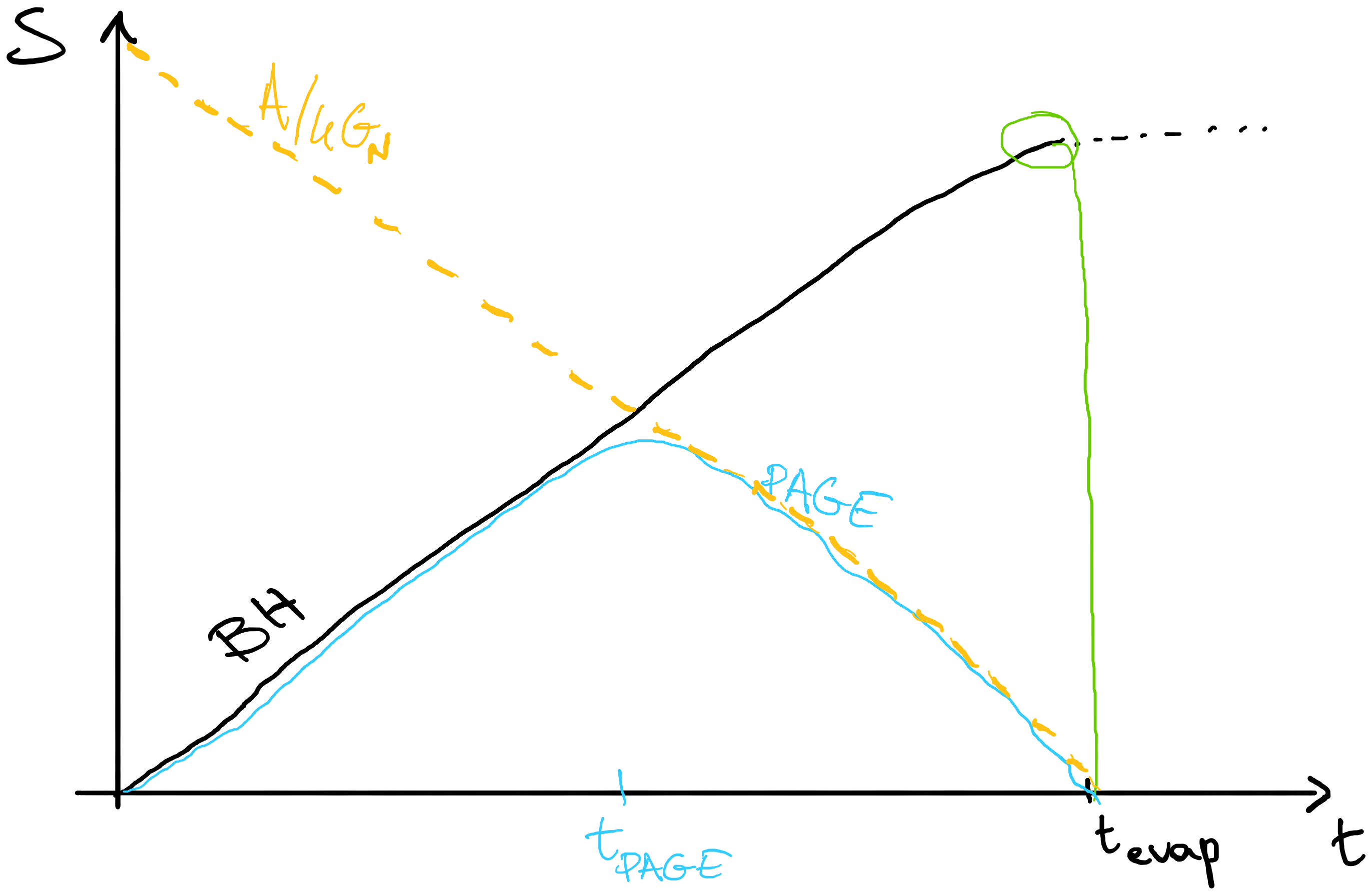
What/where are the BH microstates? How do they look like?

- Information paradox: Hawking evaporation can map a pure state into a mixed one

$\Longrightarrow$ The process in not unitary!

Why is this a difficult problem?

The Page curve describes the evolution of Entanglement Entropy between an evaporating body and the radiation, if the evaporation process is unitary



Evaporation process described by Hawking

Hawking process valid until BH is Planck size, all information released at the end.

Page curve: valid for a unitary process.

Message:
We expect Quantum Gravity to shed light upon this issues, but it is not clear how…

Black holes in string theory:

- Black holes are bound states of N strings and D-branes (N>>1).

- Can we count microscopic degeneracy?

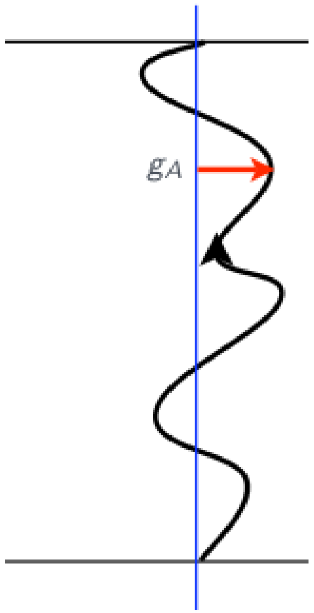
We consider BPS (supersymmetric) states: the degeneracy is protected by supersymmetry

$\Rightarrow$ We can extrapolate the degeneracy at strong coupling (BH regime) from the one at weak coupling (Strominger Vafa '96)

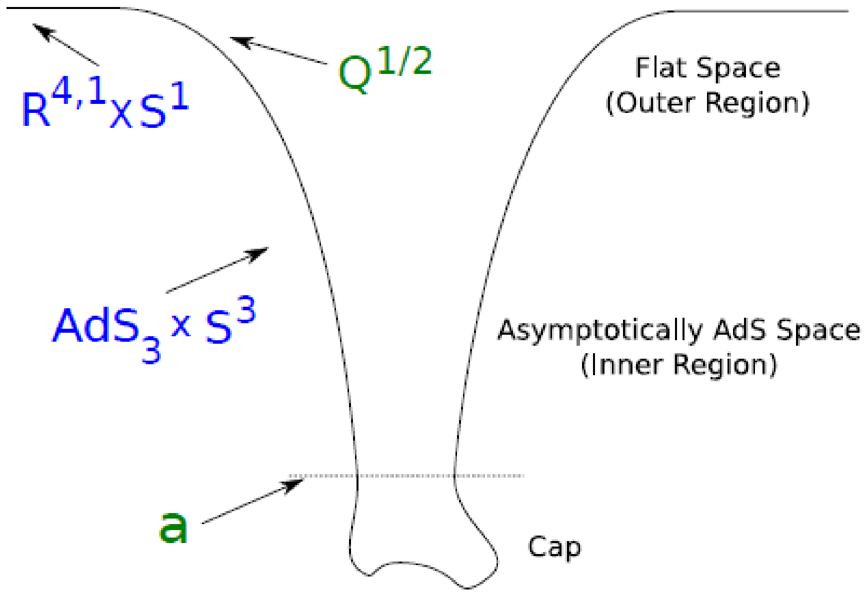
- String theory captures correctly the d.o.f. of black holes. This does not answer the question: How do individual microstates look like in the BH regime?

D1D5 system: (Type IIB on $R^{1,4} \times S^1 \times T^4$: D1 branes wrap S^1 and D5 branes wrap $S^1 \times T^4$)

Let's start in the F1-P frame (dual to D1-D5): a microstate is described by 8 functions g_A that define the profile of the oscillating string in the transverse directions



At strong coupling (in D1D5 frame)

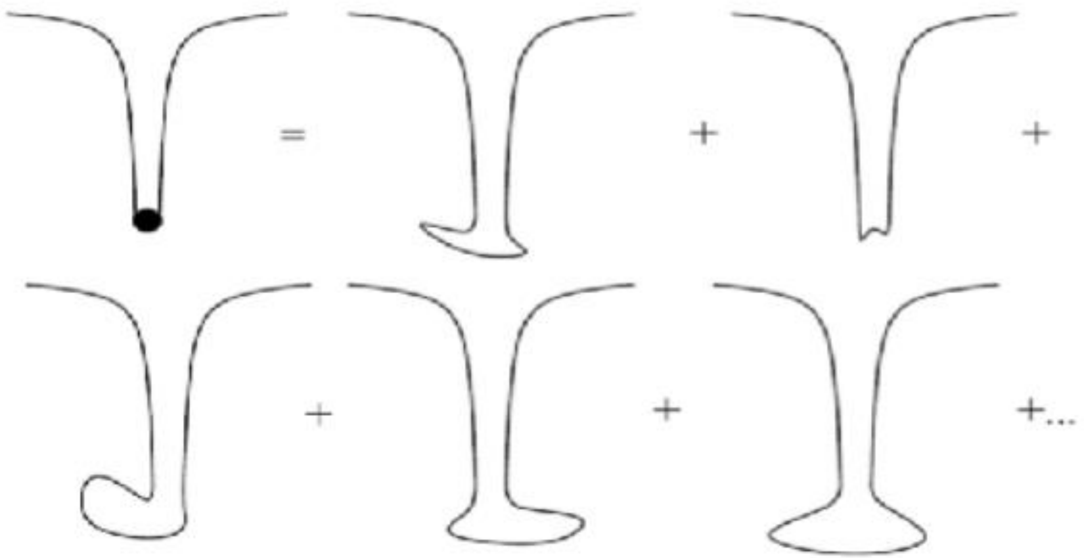


The backreaction on spacetime is non-singular and horizonless

(Lunin, Mathur, '01)

Fuzzball proposal:

For black hole microstates quantum gravity effects become important at the scale of the horizon, due to the size of the underlying bound state.



Typical microstates are expected to be highly quantum. A subset of them are coherent states, solution of the supergravity e.o.m., that are low curvature, non-singular and horizonless.

→ The classical black hole solution is a coarse-grained description of the system. It is accurate for some purposes, but not for "fine grained" questions.

State of the art:

- Supersymmetric D1-D5 BH: good understanding of all the microstates, but this black hole is microscopic…
- Supersymmetric D1-D5-P BH: two classes of solutions (superstrata and multicenter), but they are not enough to account for all the entropy
- Non-extremal BH: JMaRT solution, very atypical

$\Longrightarrow$ This proposal is a conjecture, it is important to develop tools and techniques to test and corroborate it

Holography:

- What are the holographic duals of the microstates we have constructed?
- Can we enforce the case of interpreting them as BH microstates?
- Can the CFT intuition guide the construction of new microstate?

Shockwaves:

- Can we provide a coarse-grained description of the microstates, that still capture the overall physics?
- Are the microstates instable? Do instabilities drive them toward BH geometries?
- How do particles inside the “would be horizon” backreact on space-time?

Computational methods:

- Given that constructing BH microstates is so difficult, can we exploit computer science tools to get physically relevant, numerical solutions?

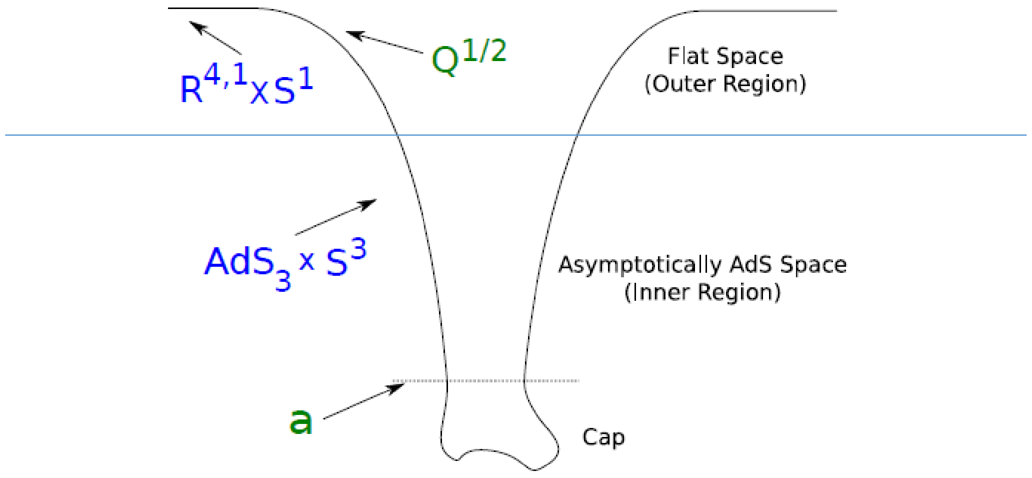
Holography

Duality between spectrum (and dynamics) of string theory in an asymptotically AdS space and that of gauge invariant operators in the dual CFT.

In our case:

(Type IIB on $R^{1,4} \times S^1 \times T^4$: D1 branes wrap S^1, D5 branes wrap $S^1 \times T^4$ and S^1possibly carries P)

In the near horizon limit, the geometry is $AdS_3 \times S^3$ ⟹ Holography is applicable



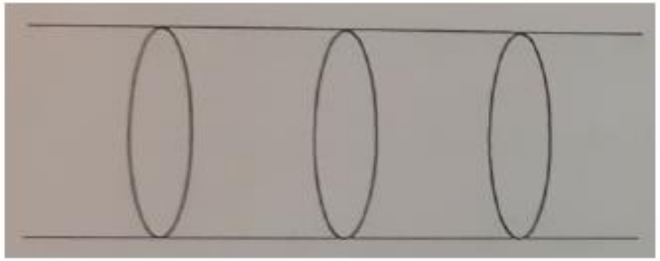
Duality ⟷ D1D5 CFT

D1D5 CFT:

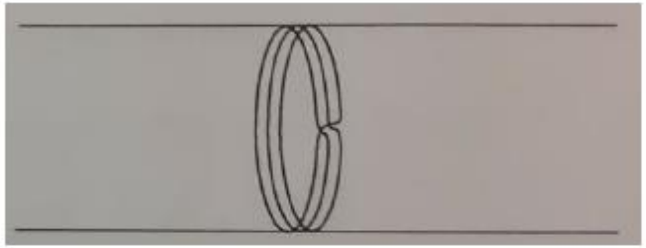
Symmetries: Virasoro, (4,4) SUSY, $SU(2)_L \times SU(2)_R$ with R-symmetry

Free orbifold point: CFT is a sigma model with target space $(T^4)^N/S_N$
Field content: N copies of 4 free bosons and 4 free fermions $(X_{(r)}, \psi_{(r)})$, r = 1,…,N

Orbifold theory → States are in correspondence with conjugacy classes of S_N through boundary conditions of the fields



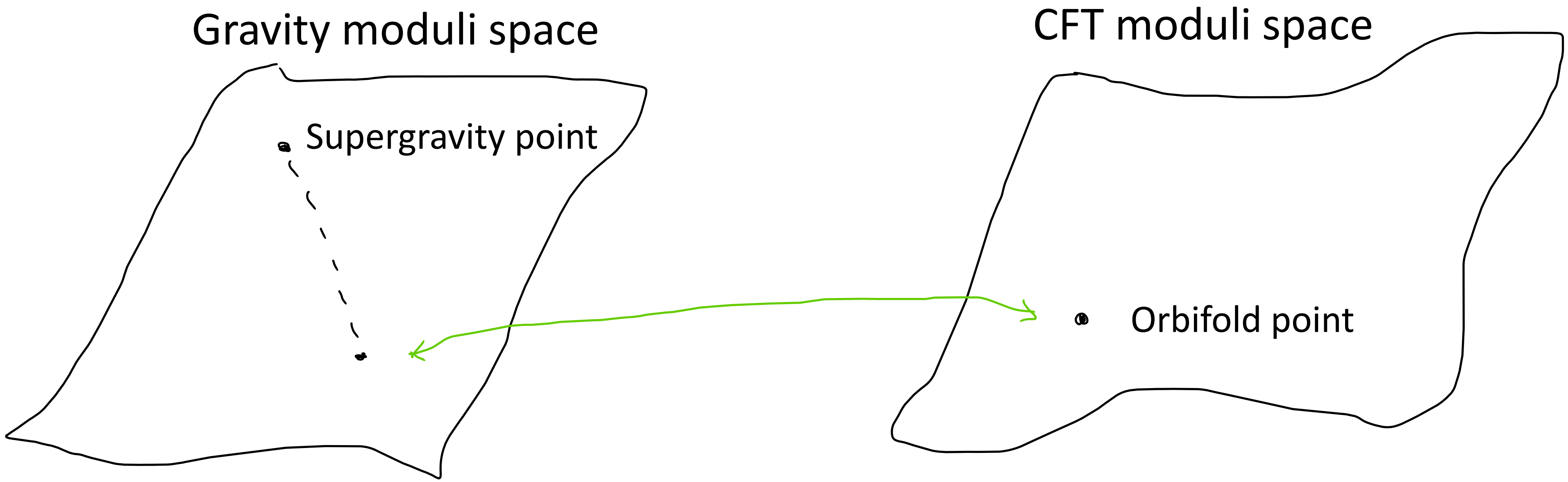
Boundary conditions:
$X_{(1)} \to X_{(1)}, \quad X_{(2)} \to X_{(2)}, \quad X_{(3)} \to X_{(3)}$



Boundary conditions:
$X_{(1)} \to X_{(2)} \to X_{(3)} \to X_{(1)}$

Twist operator: Σ_k changes boundary conditions.

Strategy: gain insights on the gravity theory by studying the D1D5 CFT, but …



Focus on protected quantities (moduli independent): expectation values of chiral primary operators in 1/4 (1/8)-BPS states

Limitations:

i) $|s\rangle$ is characterized by expectation values of all the operators in the theory, not only chiral primaries…

ii) Expectation values of simple operators on a microstate are indistinguishable from thermal state up to $O(e^{-N})$

Precision holography dictionary:

Expectation values of chiral primary operators (and descendants) of dimension d

Terms of order r^{-2-d} in the geometry expansion around the vacuum $AdS_3 \; x \; S^3$

$|s\rangle$ is a CFT state dual to a microstate geometry, $O_i^{(d)}$ is the operator dual to the field $\phi_i^{(d)}$

$$\phi_i^{(d)} = \frac{c_i}{r^{2+d}} + O(r^{-d-3}) \quad \longleftrightarrow \quad \left\langle s \middle| O_i^{(d)} \middle| s \right\rangle \propto c_i$$

The dictionary for operators of dimension 1 has been completely worked out (Skenderis, Taylor, ’06/’07)
(Giusto, Moscato, Russo, ‘15)

Dimension 2

Selection rules imply that CFT operators and supergravity fields that are related by the duality share the same quantum numbers

Degeneracies:
There are single and multi-trace operators with the same quantum numbers
There are different single-traces with the same quantum number

$$\phi_I^{(2)} \quad \longleftrightarrow \quad \tilde{O}_I^{(2)} = \alpha\, O_I^{(2)} + \beta\, O'^{(2)}_I + \gamma_{Iij} O_i^{(1)} O_j^{(1)} + \cdots$$

Identify the operator mixing:

- Brute force computation (assuming enough geometries where $\phi_I^{(2)}$ is turned on are known)
- Choose a convenient basis: single-particle basis (where all extremal 3-pt functions are zero)

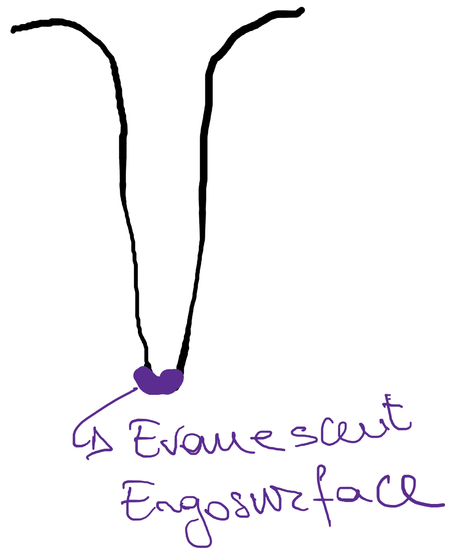
Shockwaves on microstate geometries

Shockwave: backreaction of high-energy, supersymmetry preserving massless quanta.

- Provide a coarse grained description of a system that requires d.o.f. beyond supergravity

(Lunin, Mathur '01)

- Enables studies of instabilities of microstates. Perturb the system with a massive probe



- Linear level: the particles moves on a geodesics of the background
- Non-linear level: If one includes backreaction, the particle radiates energy, and it will approach the evanescent ergosurface. But locally, the energy of a massive particle that approaches a null trajectory is enormous. This leads to an instability.

(Eperon, Reall, Santos '16)

Instability drives the microstates towards typical region of phase space (2 charge example)

(Marolf, Michel, Puhm '16)

Aim: construct first family of 3-charge microstate solutions with a shockwave in the core

Multicenter solutions: 3-charge supersymmetric microstates, # centers=2 (GLMT solution)

(Bena, Warner ‘05)

(Giusto, Mathur, Saxena ‘04)

$AdS_3 x\, S^3$
(with Z_k orbifold) $\xrightarrow{\text{Spectral flow}}$ Near horizon limit of GMLT solution $\xrightarrow{\text{Glue with flat space}}$ Asymptotically flat GLMT solution

Spectral Flow: large coordinate transformation

$$\phi \to \phi - \frac{s+1}{k}\, t + \frac{s}{k}\, y \qquad\qquad \psi \to \psi - \frac{s+1}{k} y + \frac{s}{k} \mathrm{t}$$

Global charges: increases the energy and the left angular momentum coordinate transformation

Starting point: $AdS_3 \times S^3$ (Z_k orbifold)

$$ds^2_{Ads_3\times S^3} = R^2_{Ads}\left(-(1+r^2)dt^2 + \frac{1}{1+r^2}dr^2 + r^2 dy^2 + d\theta^2 + \sin^2\theta\, d\phi^2 + \cos^2\theta\, d\psi^2\right)$$

Backreaction of massless particles at $r = 0, \theta = \frac{\pi}{2}$. We impose:

- Supersymmetry is preserved
- Good classical limit: Take many of these particles, uniformly distributed along ϕ.
- Each particle can have different energy, but such that $E \gg \frac{1}{R_{Ads}}$, so that the wavelength $\lambda \ll R_{Ads}$ (Pointlike)

$$ds^2 = ds^2_{Ads_3\times S^3} + q\, R^2_{Ads} \frac{\left((r^2+1)dt + \sin^2\theta\, d\phi\right)^2 - (r^2 dy \;- \cos^2\theta\, d\psi)^2}{r^2 + \cos^2\theta}$$

$0 \leq q < 1$ parametrises the strength of the shock wave.

(Lunin, Mathur '01)

Strategy to build the solutions:

- Spectral flow: GLMT solution with a backreacted shockwave in the decoupling limit
- Asymptotically flat extension: first family of 3-charge microstates with a backreacted shockwave in the core

- Proposed CFT dual states: the shockwave is represented by states in a highly twisted sector of the CFT (order $N^{>0}$ copies of the CFT are glued together)
- The shockwave is not located on the evanescent ergosurface of the solution (no direct connection with the instability)
- The shockwave drives the microstates toward more typical regions of the phase space

Computational methods

Aim: design a method to derive approximate multicenter solutions

Multicenter solutions: involve a Gibbons-Hawking metric $\Rightarrow$ Spacetime has non trivial topology

They are defined by a set of harmonic functions in 3-dim Euclidean space

$$H = \sum_{a=0}^{n-1} \frac{q_a}{r_a}\,, \qquad K^i = \sum_{a=0}^{n-1} \frac{k_a^i}{r_a}\,, \qquad L^i = l_0^i + \sum_{a=0}^{n-1} \frac{l_a^i}{r_a}\,, \qquad M = m_0 + \sum_{a=0}^{n-1} \frac{m_a}{r_a}\,,$$

a=0,1,…,n-1 labels the number of centers
i=1,2,3 labels the gauge fields
r_a is the distance from the a-th center

Mathematically, by fixing the positions of the centers and the coefficients of the harmonic functions you get a solution. Physically, there are further constrains (which make life harder)

Interested in solutions that:

i) are horizonless
ii) are smooth (up to possible orbifold singularities)
iii) are asymptotically flat in 5D
iv) respect charge quantization conditions
v) are macroscopic (have large asymptotic charges)

$\Rightarrow$ Constraints on the positions of the centers and the coefficients of the harmonic functions

Constraints:

iv) k^i_a: flux parameters must be integers in appropriate units

i), ii) imply the Bubble Equations

$$\sum_{b\neq a} \frac{q_a q_b}{r_{ab}} \Pi^0_{ab}\left(\Pi^1_{ab}\Pi^2_{ab} - \frac{1}{2g^2}\mathbb{T}_{ab}\right) = \sum_{b,i} q_a q_b l^i_0 \Pi^i_{ab}\,,$$

$$\Pi^i_{ab} = \frac{k^i_b}{q_b} - \frac{k^i_a}{q_a}\,, \qquad \mathbb{T}_{ab} = \frac{1}{q_a^2} + \frac{1}{q_b^2}\,.$$

Relation between k^i_a and r_{ab} :

- Solve for the k^i_a: obtain irrational fluxes
- Solve for r_{ab}: obtain unphysical distances (violations of triangular inequality)

Ways out:

i) Arrange special locations of the centers, so that distances are rational (solve for k_a^i)

(Avila, Ramirez, Ruiperez '17)

ii) Consider generic locations, and construct approximate solutions

We focus on the second approach, and developed an algorithm that comprises two parts:

i) Bayesian optimization: algorithm to find global maxima of black box functions

→ We generate good seed solutions by considering k_a^i as dependent variables. We optimize positions of the centers so that the k_a^i we obtain generate large asymptotic charges

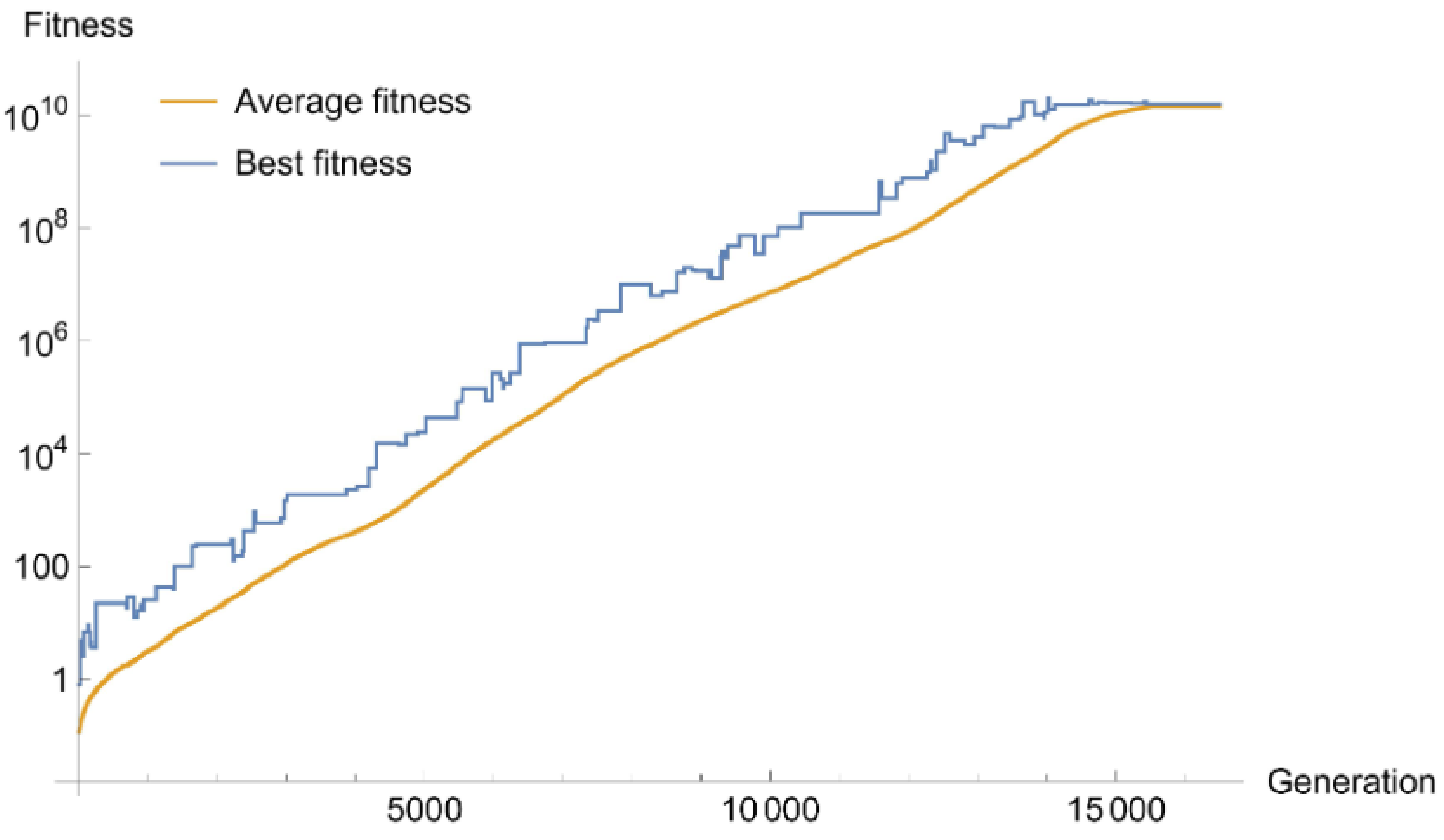
ii) Evolutionary algorithm: optimization method inspired by Darwin's theory

→ Round k_a^i of the seed solution and consider the positions as dependent variables.

- Population of solutions
- Fitness function, in our case the Bubble equations (defines the selection method)
- Reproduction mechanism (inheritance)

The algorithm can be used to derive solutions with an arbitrary number of centers (constraint: computational resources)

5 centers example:



Respects all the constraints imposed by physics, but for the bubble equations which are solved to machine precision

Conclusions and future directions

We have introduced the "fuzzball proposal", which at the current state of things is a conjecture. Various tools can be employed to test and corroborate it

- Holography:

The method to develop the dictionary is mature and can be used at higher orders, even though it computationally hard. So far, used to identify holographic duals to known microstate geometries; it could be worth to further explore it to guide new microstate geometries constructions.

- Shockwaves:

Provide a coarse-grained descriptions of microstates that cannot be fully described in supergravity. It would be interesting to study backreactions of shockwaves in ergoregions of non-extremal microstates

- Computational methods:

Used to derive approximate microstate geometries. Use these methods to generate a training set to be employed in ML setups.

THANK YOU!