UNIVERSITÀ DI TORINO McMULE

Seminars INFN Bari

The Muon Anomaly – An Intricate Landscape

Marco Rocco

Università di Torino

BARI, NOVEMBER 12$^{\text{TH}}$, 2024

a mysterious (?) particle

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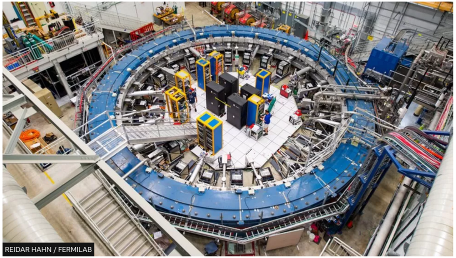
Science & Environment

Scientists at Fermilab close in on fifth force of nature

10 August



The findings come from the US muon g-2 experiment

The BBC, 10.08.2023

a mysterious (?) particle

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A new force of nature? Scientists close in on a fifth force as they discover a mysterious subatomic particle disobeying the laws of physics

- **Physicists observed the 'peculiar wobble' of subatomic particle called a muon**
- **Modern understanding of physics may be missing an 'unknown particle or force'**
- **The new findings replicate earlier results from 2021 but with four times the data**

By JONATHAN CHADWICK FOR MAILONLINE
UPDATED: 14:13, 11 August 2023

699
View comments

Scientists are closing in on identifying a new force of nature after observing the peculiar 'wobble' of a subatomic particle.

The Daily Mail, 11.08.2023

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- . . . and its (apparent) flaws

a (mostly) successful model

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- . . . and its (apparent) flaws
- how: the muon anomaly is a prime example

- this is *not* about a fifth force
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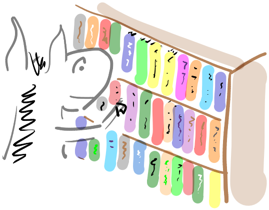
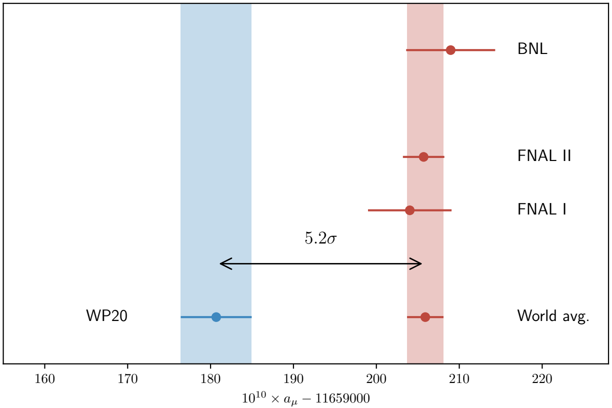
- it is about a very predictive model and its success
- . . . and its (apparent) flaws
- how: the muon anomaly is a prime example
- spoiler: hadronic physics at low energies is hard



it's messy, but let's start somewhere



- $a_\mu^{\text{exp;WA}} = 116\,592\,059(22) \cdot 10^{-11}$ [2402.15410, [World Average after BNL+FNAL1/2/3]]
- $a_\mu^{\text{WP}} = 116\,591\,810(43) \cdot 10^{-11}$ [2006.04822, [White Paper]]

- the muon has a spin S, coupling to a(n electro)magnetic field

$$\mathcal{H} = -\mu_\ell \cdot B - d_\ell \cdot E$$

$$\mu_\ell = -g_\ell \frac{e}{2m_\ell} S \quad \rightarrow \quad \boxed{a_\ell = \frac{g_\ell - 2}{2}}$$

$$d_\ell = -\eta_\ell \frac{e}{2m_\ell} S$$

- the muon has a spin S, coupling to a(n electro)magnetic field

– 1928: Dirac predicts $g_\ell = 2$

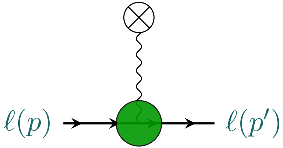
– 1934: Kinsler & Houston confirm the result with a large experimental error

– 1947: Kusch & Foley measure a 0.12% deviation from 2 (hence anomaly)

– 1948: Schwinger computes the first radiative correction to a_ℓ, i.e. $\frac{\alpha}{2\pi} \sim 0.116\%$

- radiative corrections to

$$\bar{u}(p')\,\Gamma^{\mu}\,u(p) = \bar{u}(p')\left(F_1(q^2)\gamma^{\mu} + F_2(q^2)\frac{i\sigma^{\mu\nu}}{2m_\ell}\,q_\nu\right)u(p) \qquad q = p' - p$$



- $F_1(0)$ is the (renormalised) electric charge
- $F_2(0)$ is the magnetic anomaly, and allows for a perturbative expansion in α

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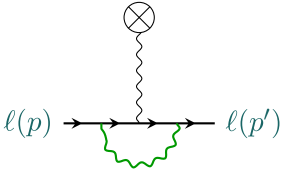

- $F_1(0)$ is the (renormalised) electric charge
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$\rightarrow$ the first non-trivial order is Schwinger's result

- QED contribution: loops with photons and leptons
- can be computed perturbatively

$$a_\ell^{\rm QED} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \dots$$

- a sum of mass-independent and mass-independent terms

$$C_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}\left(\frac{m_\mu}{m_e}\right) + A_2^{(2n)}\left(\frac{m_\mu}{m_\tau}\right) + A_2^{(2n)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

- higher orders?

  cp. $\Delta a_\mu^{\rm exp} \sim 22 \cdot 10^{-11}$ with $(\alpha/\pi)^4 \sim 2.91 \cdot 10^{-11}$ and $(\alpha/\pi)^5 \sim 6.76 \cdot 10^{-14}$

$C_\mu^{(2)}$	0.5
$C_\mu^{(4)}$	0.765 857 425(17)
$C_\mu^{(6)}$	24.050 509 96(32)
$C_\mu^{(8)}$	130.878 0(61)
$C_\mu^{(10)}$	750.72(93)

n	$C_\mu^{2n}(\alpha/\pi)^n \cdot 10^{11}$
1	116 140 973.321(23)
2	413 217.6258(70)
3	30 141.90233(33)
4	381.004(17)
5	5.0783(59)

QED numerics

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- $a_\mu^{\text{QED}}(\text{Cs18}) = 116\,584\,718.931(7)_{\text{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha} \cdot 10^{-11}$
- $a_\mu^{\text{exp;WA}} - a_\mu^{\text{QED}}(\text{Cs18}) = 7\,341(22) \cdot 10^{-11}$

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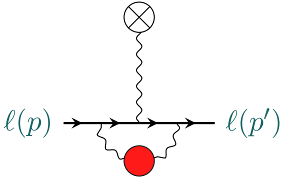
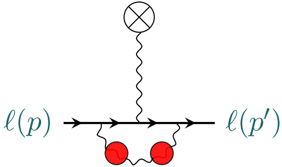
- $a_\mu^{\text{exp;WA}} - a_\mu^{\text{QED}}(\text{Cs18}) - a_\mu^{\text{EW;2L}} = 7\,187(22) \cdot 10^{-11}$

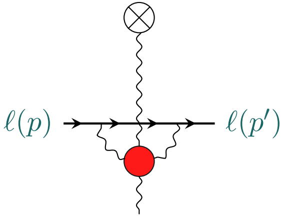
- LO hadronic vacuum polarisation $\sim \left(\frac{\alpha}{\pi}\right)^2$



- LO hadronic vacuum polarisation $\sim \left(\frac{\alpha}{\pi}\right)^2$
- NLO hadronic vacuum polarisation $\sim \left(\frac{\alpha}{\pi}\right)^3$



- LO hadronic vacuum polarisation $\sim \left(\frac{\alpha}{\pi}\right)^2$
- NLO hadronic vacuum polarisation $\sim \left(\frac{\alpha}{\pi}\right)^3$
- Hadronic light-by-light $\sim \left(\frac{\alpha}{\pi}\right)^3$



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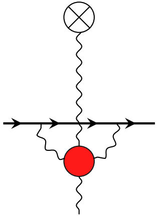
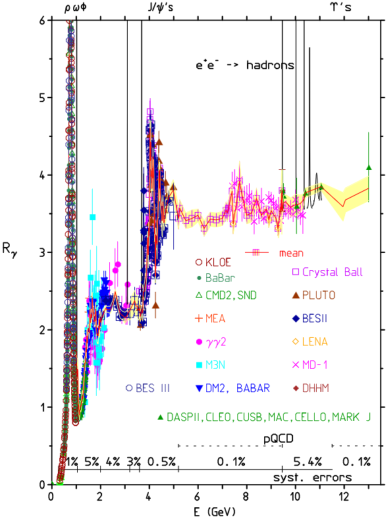


- ◇ dispersive approaches: $92(19) \cdot 10^{-11}$
- ◇ lattice QCD: $79(31)(18) \cdot 10^{-11}$ or $107(15) \cdot 10^{-11}$
- ◇ $a_\mu^{\text{exp;WA}} - a_\mu^{\text{QED}}(\text{Cs18}) - a_\mu^{\text{EW;2L}} - a_\mu^{\text{HLbL;WP}} = 7\,095(29) \cdot 10^{-11}$



[F. Jegerlehner]

- gauge invariance:

$$\Pi^{\mu\nu}(q^2) = -i(q^2 g^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)$$

- analyticity:

$$\Pi_{\text{ren}} = \Pi(q^2) - \Pi(0) = \frac{q^2}{\pi}\int_{4m_\pi^2}^\infty \mathrm{d}s \frac{\mathrm{Im}\Pi(s)}{s(s-q^2)}$$

- unitarity (optical theorem):

$$\mathrm{Im}\Pi(s) = -\frac{s}{4\pi\alpha}\,\sigma(e^+e^- \to \text{hadrons}) = -\frac{\alpha}{3}\,R_\gamma(s)$$

$$\Rightarrow a_\mu^{\text{HVP;LO}} = \frac{\alpha^2\, m_\mu^2}{3\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{d}s}{s^2}\,K(s)\,R_\gamma(s)$$

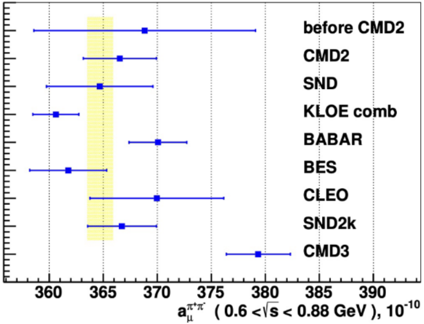
– measure $e^+e^- \to \text{hadrons}$ for $s > 0$ at 1% or better

– use pQCD for $s \to \infty$

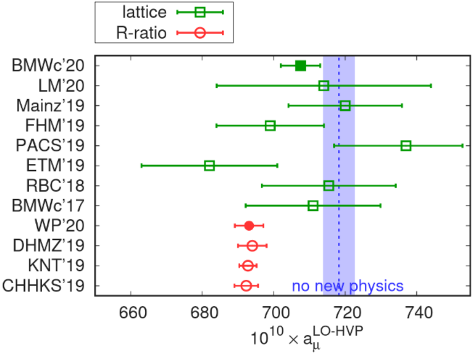
- use exclusive data from CMD, SND, BESIII, KLOE, BABAR, BELLEII . . .
  ↪ scan and ISR experiments
- different compilations resulting in different results

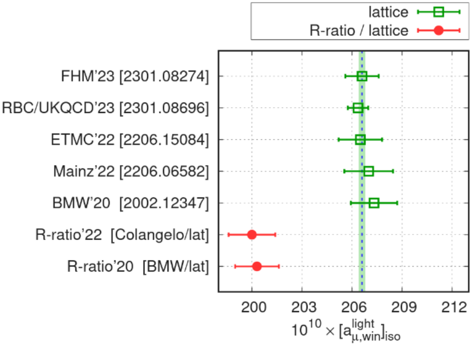
LO-HVP [J17]	6 881(41)
LO-HVP [DHMZ19]	6 939(40)
LO-HVP [KNT19]	6 927.8(24.2)
LO-HVP [WP20]	6 931(40)
NLO-HVP [Kurz+14]	–98.7(0.9)
NNLO-HVP [Kurz+14]	12.4(0.1)

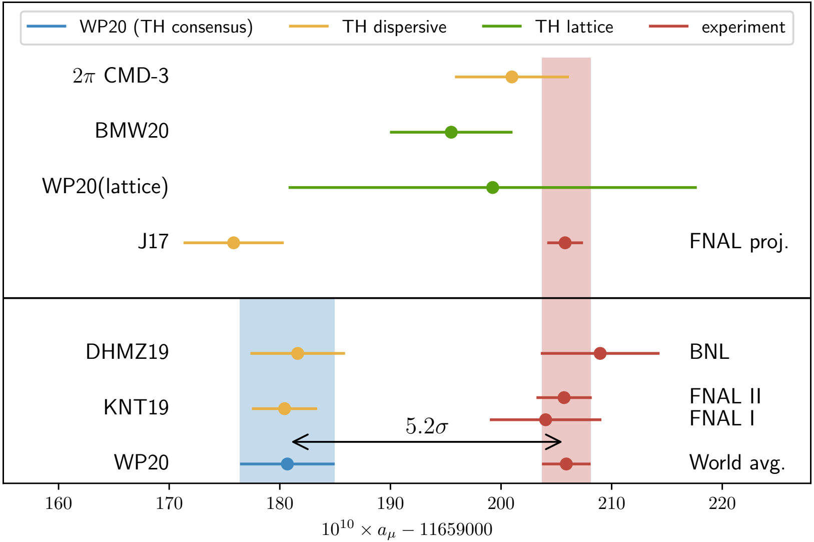
- use exclusive data from CMD, SND, BESIII, KLOE, BABAR, BELLEII . . .
  ↪ scan and ISR experiments
- different compilations resulting in different results
- tensions among datasets: e.g. KLOE vs BABAR, and $\pi\pi$ channel ($> 50\%$)

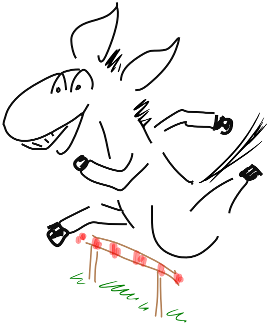


- non-perturbative, first-principle method based on a discretised space-time
- physical results: (1) chiral limit; (2) infinite-volume limit; (3) continuum limit
- $a_\mu^{\rm LO;HVP} = \alpha^2 \int_0^\infty {\rm d}t K(t) \langle J_\mu(t) J_\nu(0) \rangle \to$ BMWc: $7075(2.3)(5.0) \cdot 10^{-11}$






WP20 (TH consensus)
TH dispersive
TH lattice
experiment
2π CMD-3
BMW20
WP20(lattice)
J17
FNAL proj.
DHMZ19
KNT19
WP20
BNL
FNAL II
FNAL I
World avg.
5.2σ
160
170
180
190
200
210
220
$10^{10} \times a_\mu - 11659000$



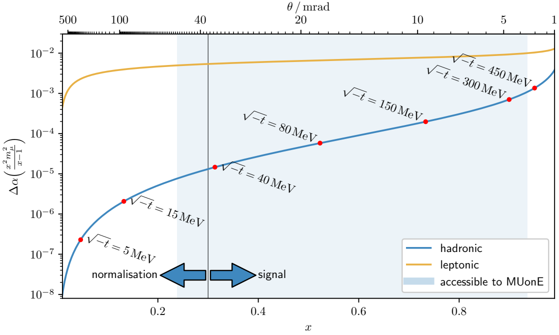
now what?

use space-like data at low energies

- collide muons against electrons
- measure scattering angles: θ_e and θ_μ
- reconstruct $\Delta\alpha^{\text{had}}(x<0)$
- apply the space-like dispersive formula ($x \propto t$)

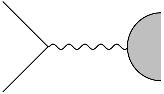
$$a_\mu^{\text{HVP;LO}} \propto \frac{\alpha}{\pi}\int_0^1 \mathrm{d}x\,(1-x)\,\Delta\alpha^{\text{had}}(x)$$

- realise the signal is $\mathcal{O}(10^{-3})$
- competitive extraction at 10^{-2}

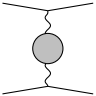


[Carloni Calame et al. 15; Abbiendi et al. 16]

time-like in $ee \to$ hadrons

$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

space-like in $e\mu \to e\mu$

$$\int \mathrm{d}t \Big(K'(t)$$  $$\Big)$$

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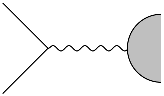
$\int \mathrm{d}s \Big(K(s)$  $\Big)$

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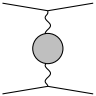
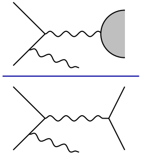
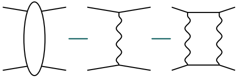
$\int \mathrm{d}t \Big(K'(t)$  $\Big)$

in practice . . .

radiative return measurement

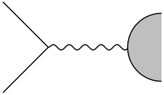
$\int \mathrm{d}s \Big(K(s)$  $\Big)$

loop-induced process

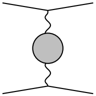
$\int \mathrm{d}t\, K'(t) \Big($  $\Big)$

(QED) radiative corrections are vital

time-like in $ee \to$ hadrons

$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

space-like in $e\mu \to e\mu$

$$\int \mathrm{d}t \Big(K'(t)$$  $$\Big)$$

in practice ...

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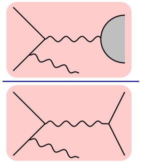
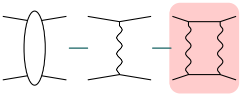
$$\int \mathrm{d}s \Big(K(s)$$  $$\Big)$$

loop-induced process

$$\int \mathrm{d}t \, K'(t) \Big($$  $$\Big)$$

(QED) radiative corrections are vital

bare marketing

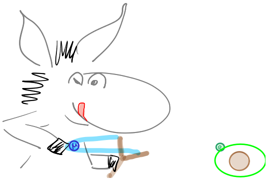
- higher-order predictions and comparison with precision experiments
- focus on $2 \to 2$ low-energy QED$+$ scattering processes
- **input**: matrix elements by us or others (at NNLO $+$ first visits at N3LO)
- **output**: physical cross section for any physical observable at fixed order
- at present an integrator, generator features under testing

McMule

Monte Carlo for MUons and other LEptons

code $\to$ `https://mule-tools.gitlab.io/`

docs $\to$ `https://mcmule.readthedocs.io/`



McMuone

[arXiv:2212.06481]

[Broggio, Engel, Ferroglia, Mandal, Mastrolia, Ronca, Rocco, Signer, Torres Bobadilla, Ulrich, Zoller]

basic setup

$$\mu\, e \to \mu\, e \quad @\ \mathrm{NNLO}$$

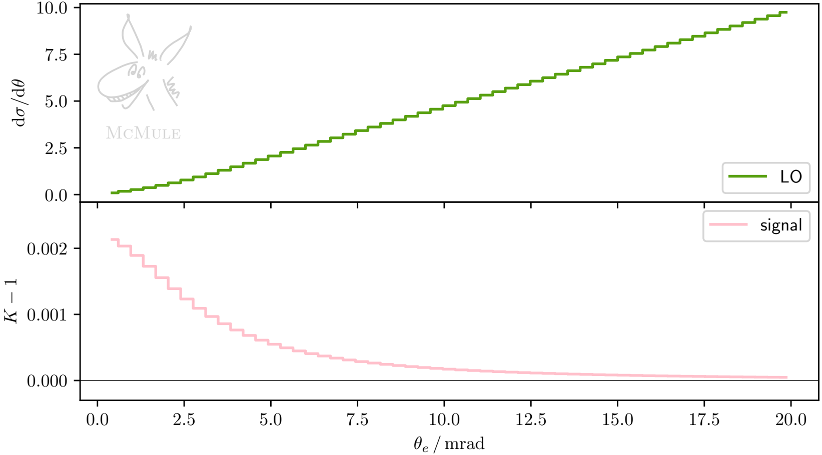
- kinematical setup mimics MUonE:

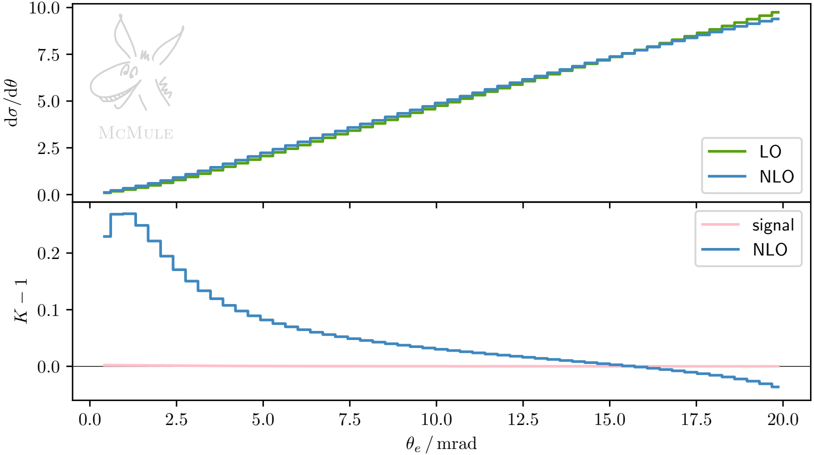
$$\texttt{S1} \quad :: \quad E_{\mu,i} = 160\,\mathrm{GeV} \qquad E_{e,f} > 1\,\mathrm{GeV} \qquad \theta_{\mu,f} > 0.3\,\mathrm{mrad}$$

- results for different kinematical scenarios and any IR safe observable
- first NNLO calculation with two different internal & external masses
- 11 authors, 12 institutes, 6 countries
- 5+ yr of work + run for 2.5 CPU yr
  (290 kWh energy / 1300 kettles / 60 kgCO2e / $\frac{1}{3}$ flight Milano–Bari)

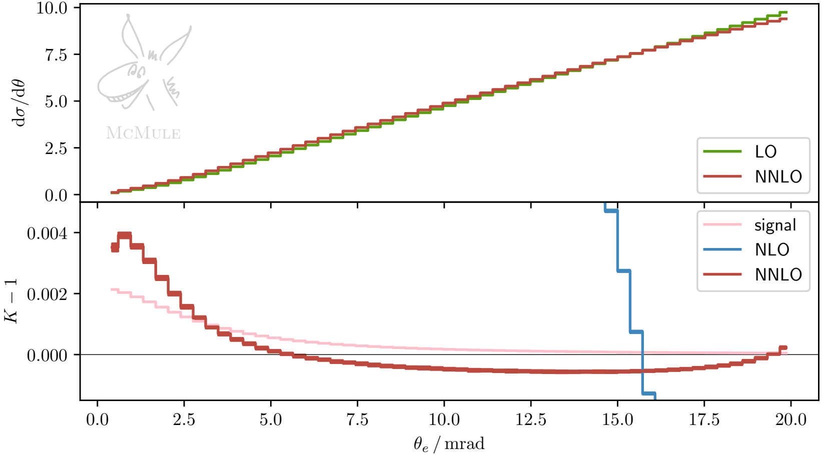
more at `mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy`

S1 θ_e spectrum

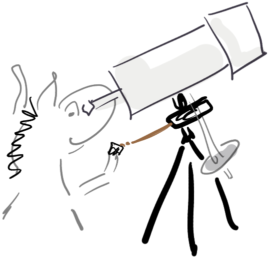




10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NLO
signal
NLO
0.2
0.1
0.0
K − 1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad



10.0
7.5
5.0
2.5
0.0
$\mathrm{d}\sigma/\mathrm{d}\theta$
LO
NNLO
0.004
0.002
0.000
$K-1$
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad



let's wrap it up

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- we are testing the sixth digit of a physical quantity, impressive!
- QED and EW contributions well under control
- Hadronic $\rightarrow$ Vacuum Polarisation
  - largest uncertainty in $\pi\pi$ channel
  - large range from KLOE to CMD-3, main source of prediction error

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future steps

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future steps

◇ data (re-)analysis at BABAR and KLOE

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Thank you!