



UNIVERSITY
OF TRENTO



Trento Institute for
Fundamental Physics
and Applications

CASTLE 2024

17th–20th September 2024

Tagliolo di Monferrato

FORECASTING THE SENSITIVITY OF PULSAR TIMING ARRAYS TO SCALAR-INDUCED GW

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September, 19th 2024

OUTLINE

1. PULSAR TIMING ARRAYS

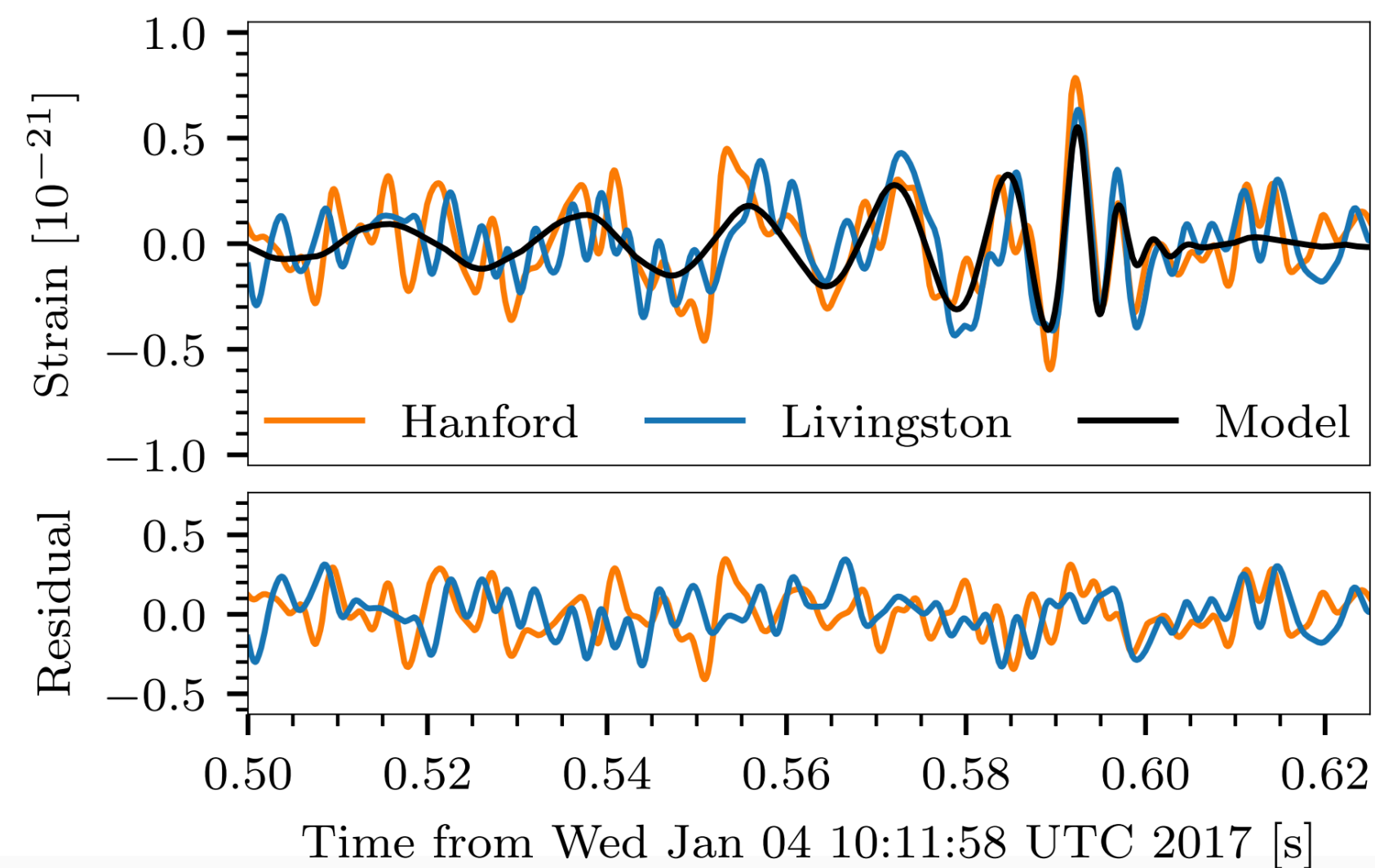
2. FORECASTS: INTRODUCING A NEW METHOD

3. SCALAR-INDUCED GW: CONSTRAINING A POTENTIAL SIGNAL

MORPHOLOGY OF A GW SIGNAL

DETERMINISTIC SIGNAL

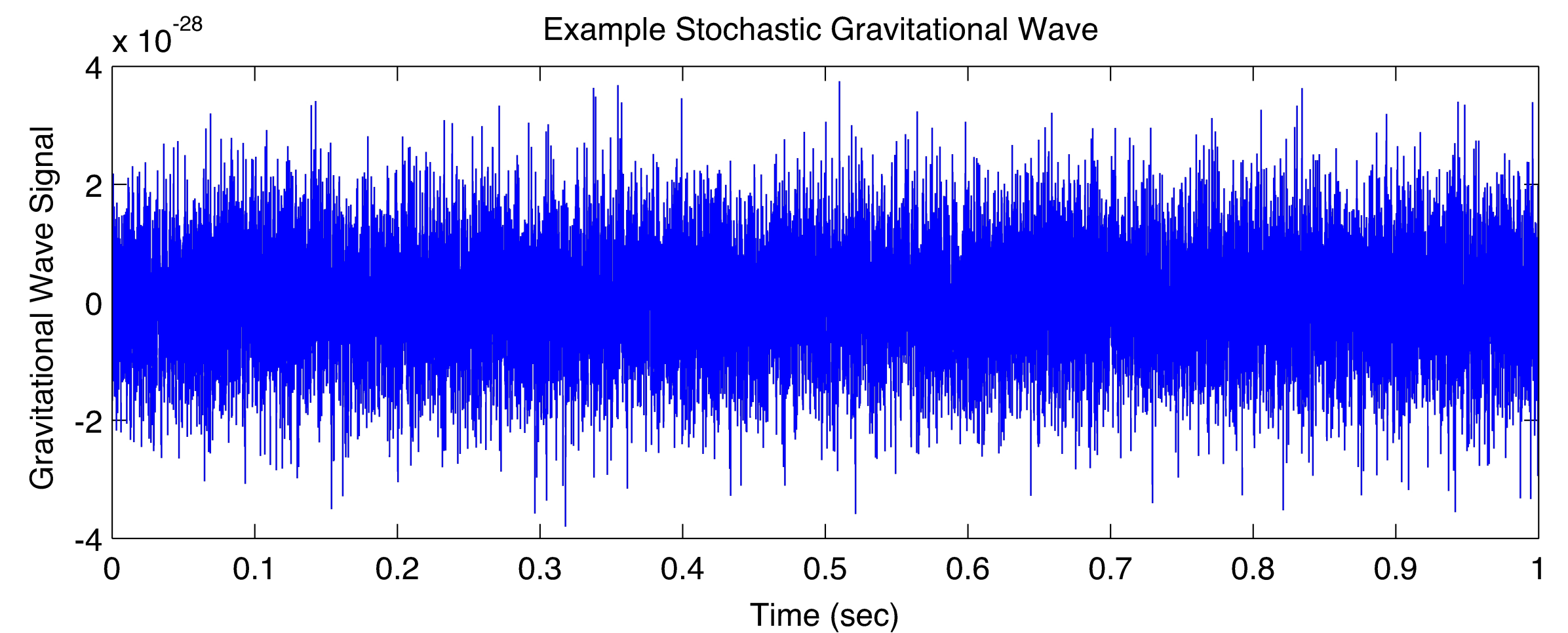
- Generated by binaries
- Time coherency
- Well localized in the sky



Strain data from Hanford and Livingston detectors at the time of GW170104. Credit: [1]

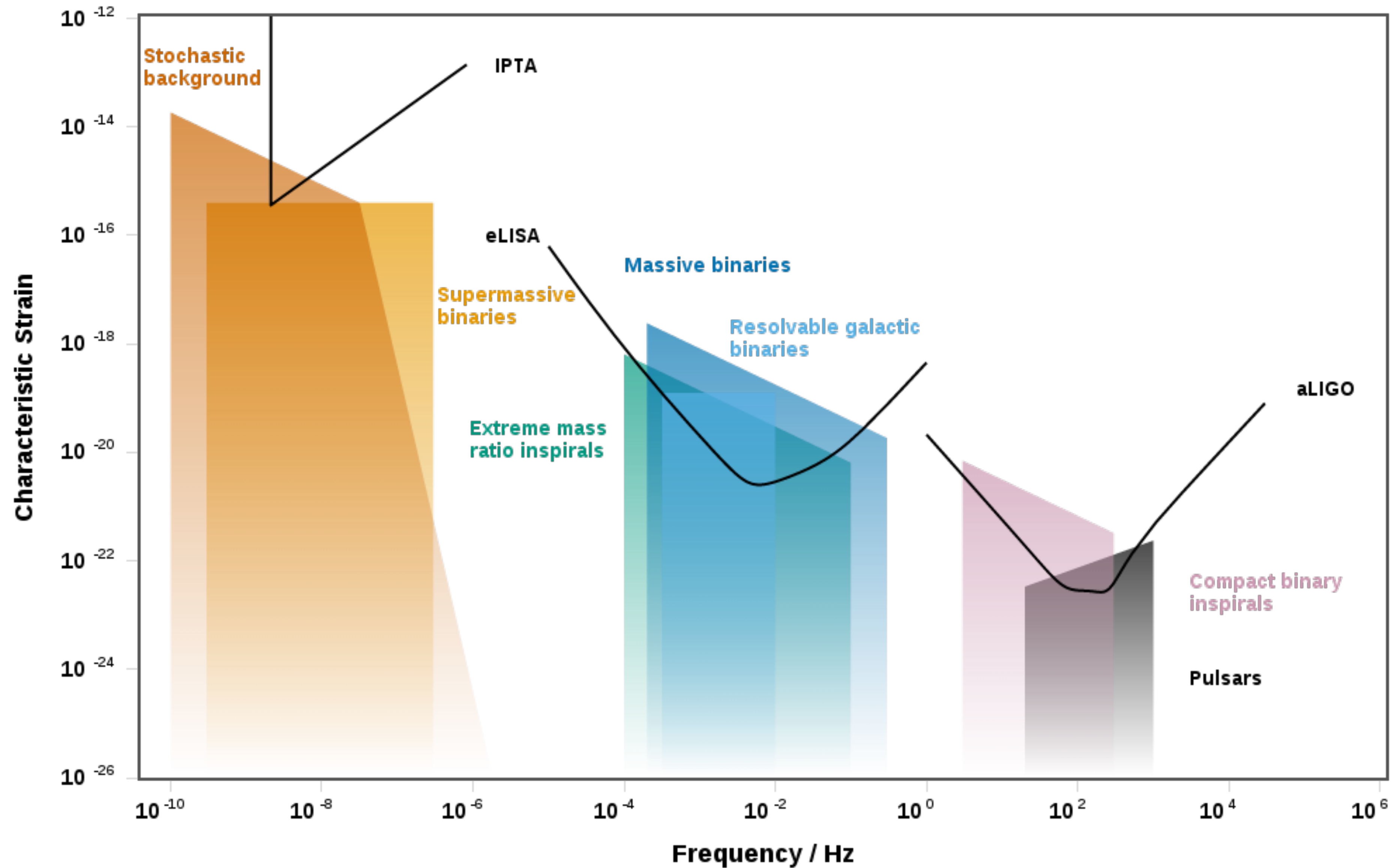
STOCHASTIC BACKGROUND

- Superposition of many signals
- No time coherency
- Diffuse in the sky



Example of a signal from a stochastic GW source. Credit: [2]

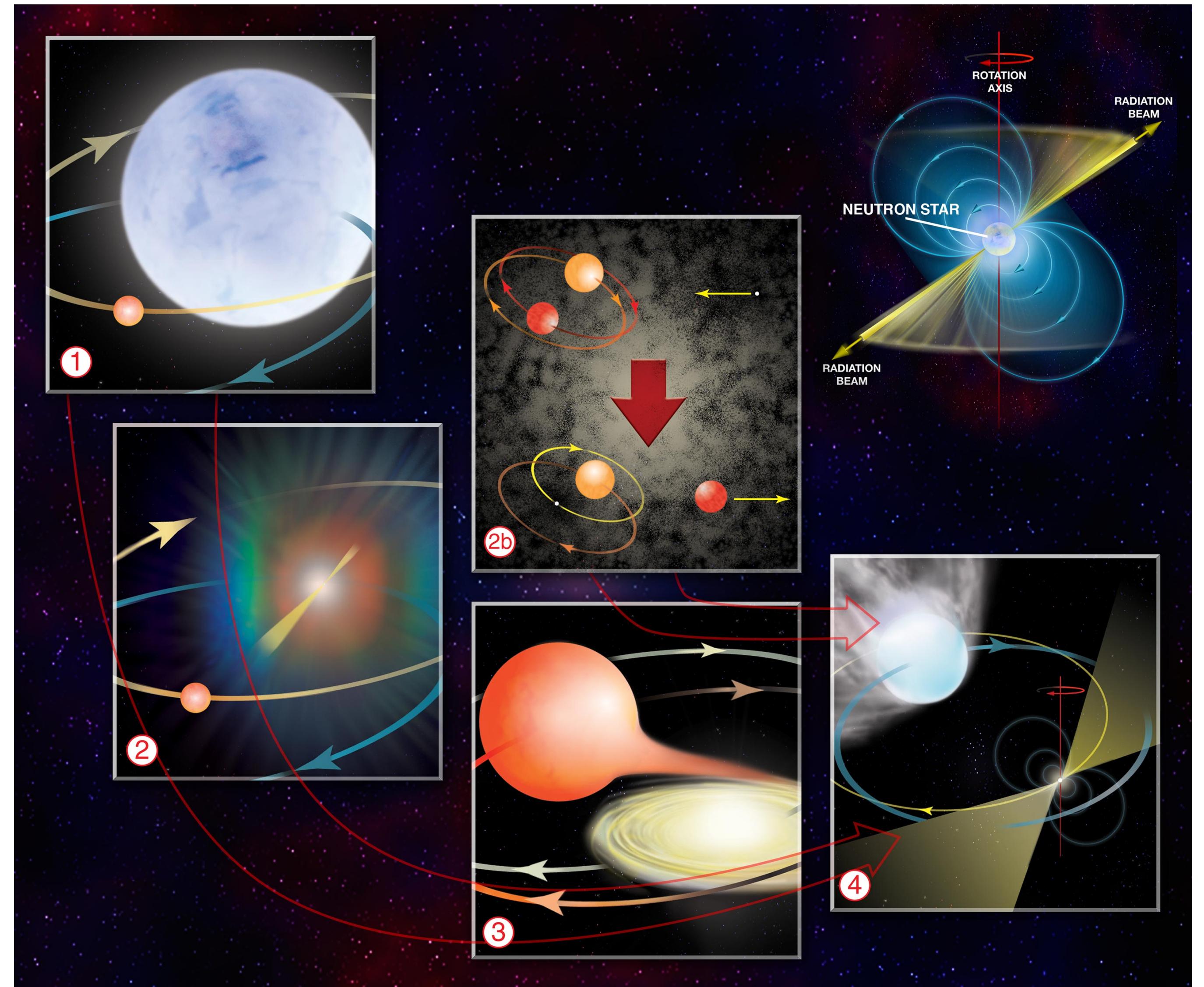
PRESENT AND FUTURE DETECTORS



Gravitational-wave sensitivity-curve plot using characteristic strain. Credit: [3]

MILLISECOND PULSARS

- Pulsars with a rotational period less than 10 ms
- Spun up through accretion of matter from a **companion star** in a close binary system
- **Lighthouse** effect: we observe the beam radiation
- ~ 400 known pulsars
- Observed in the **radio** frequencies

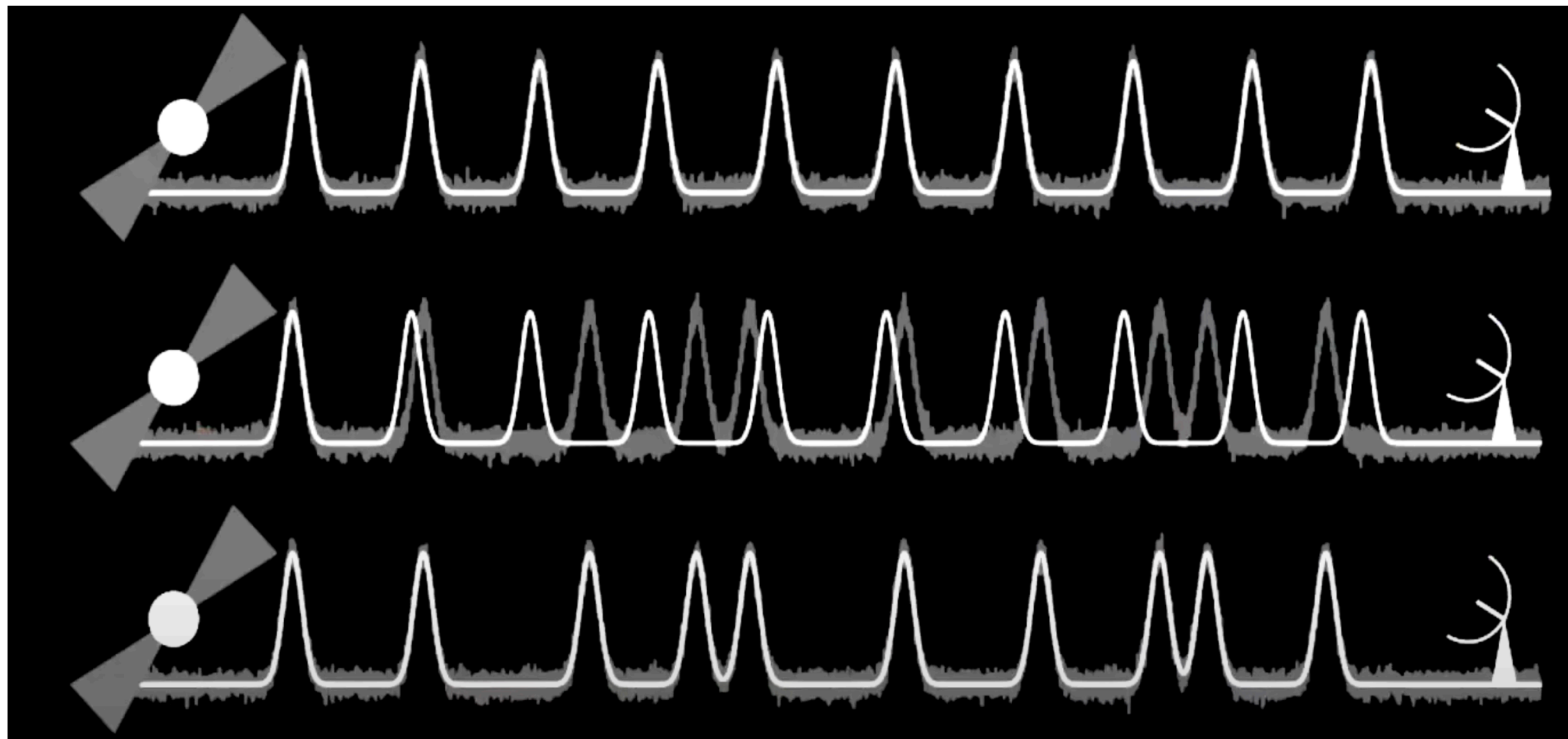


Steps astronomers say are needed to create a pulsar with a superfast spin

Credit: [4]

PULSAR TIMING

- Accounting for each pulse



Many factors dictating the pulse arrival times:

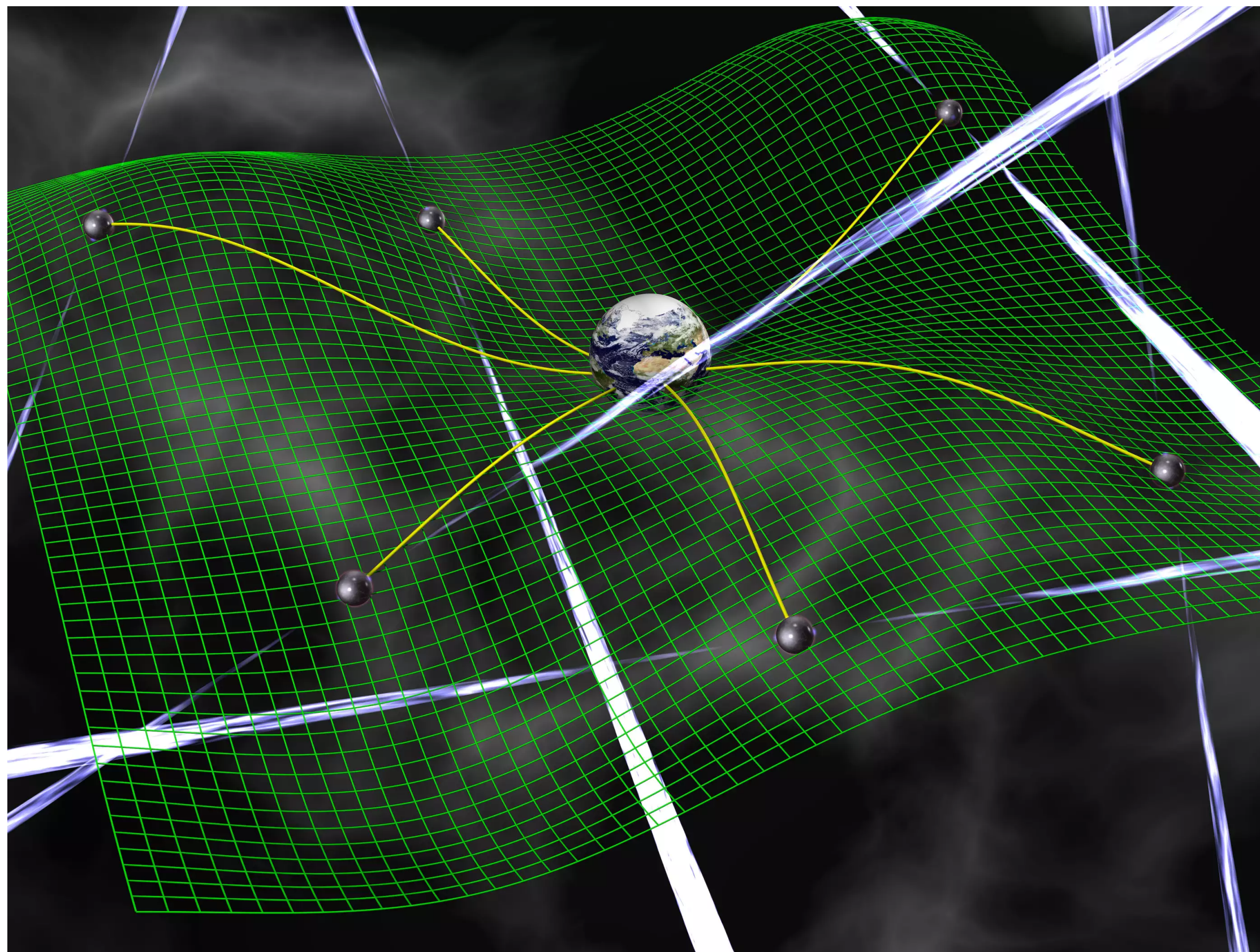
- Pulse period, period derivatives, location on the sky
- GR effects, dispersion measure

Pulsar timing process. Credit: [5]

- All these factors must be taken into account to build a pulsar timing model
- **Time residuals:** difference between the measured TOA and the model

PULSAR TIMING ARRAYS

- Taking many pulsars reduces the noise contribution



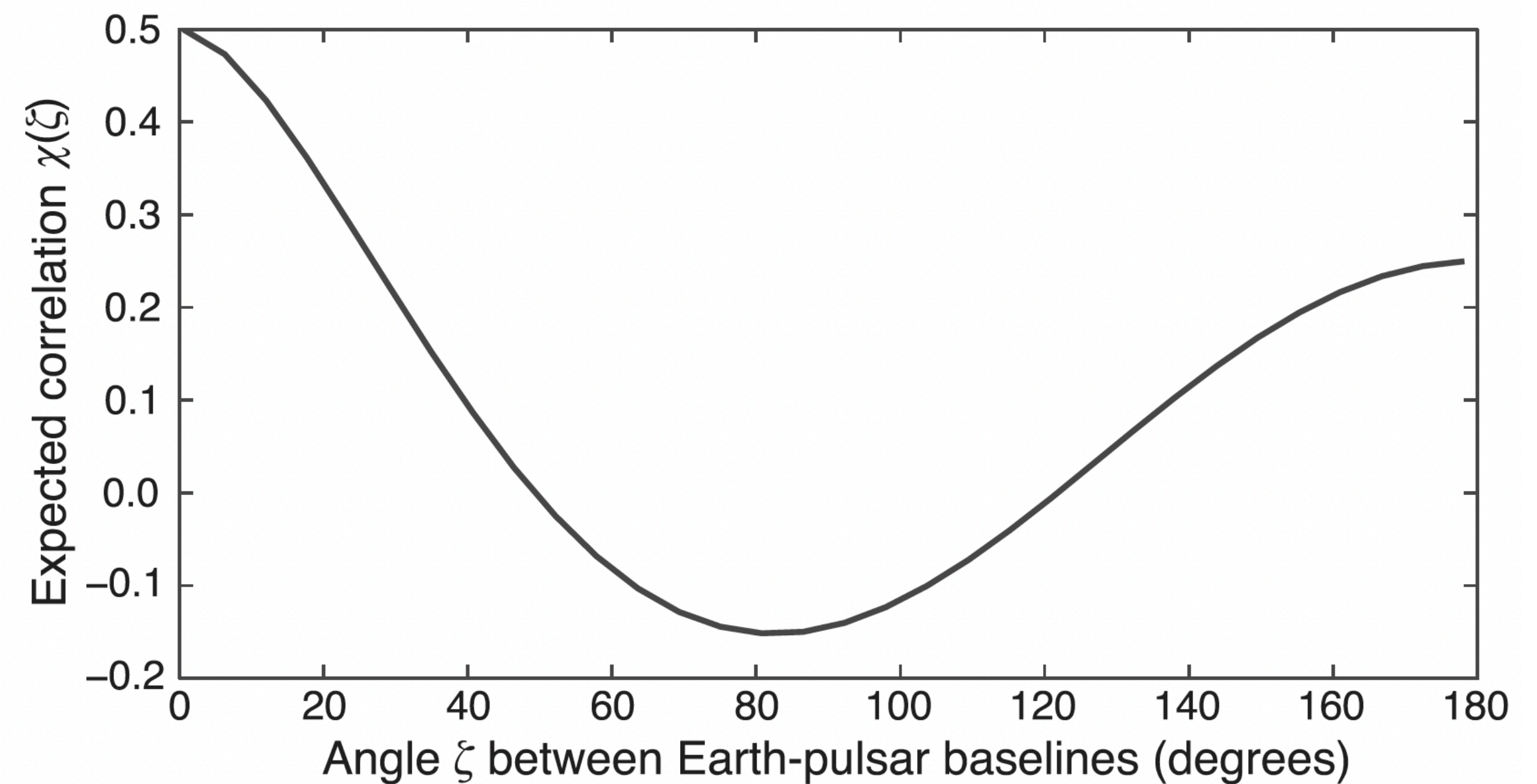
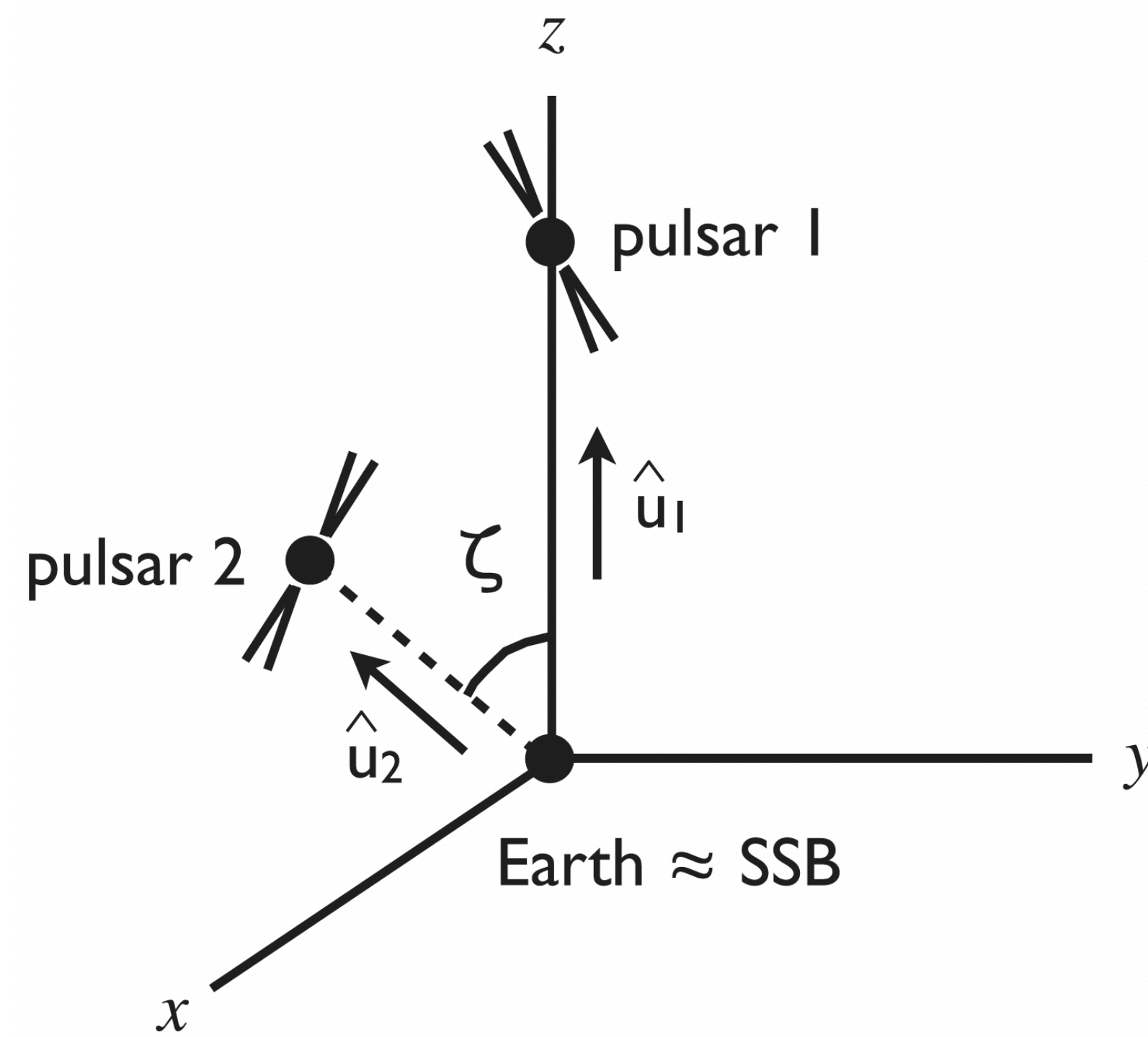
- Each pulsar is monitored by one or more telescopes
- Several frequencies are monitored
- Long acquisition time ($>$ yrs) to go to very low frequencies

$$f_{yr} \sim 3.17 \times 10^{-8} \text{ Hz}$$

Pulsar timing arrays. Credit: [6]

THE MODEL: HELTINGS & DOWNS CURVE

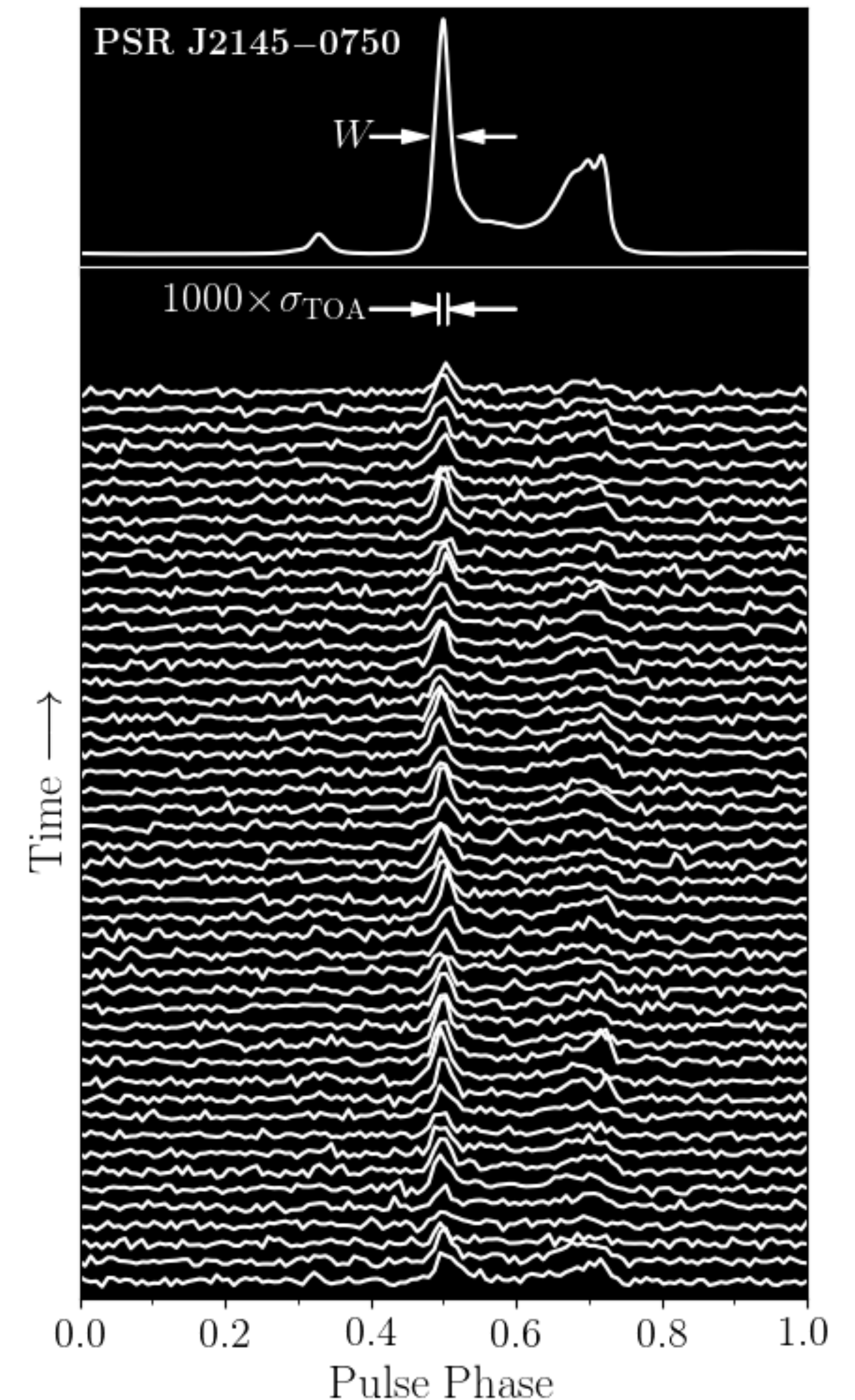
- For an isotropic, unpolarized stochastic background of quadrupolar gravitational radiation with $+$ and \times polarisation modes, the expected correlated response of a pair of Earth-pulsar baseline follows the Hellings and Downs curve



Credit: [7]

NOISE SOURCES

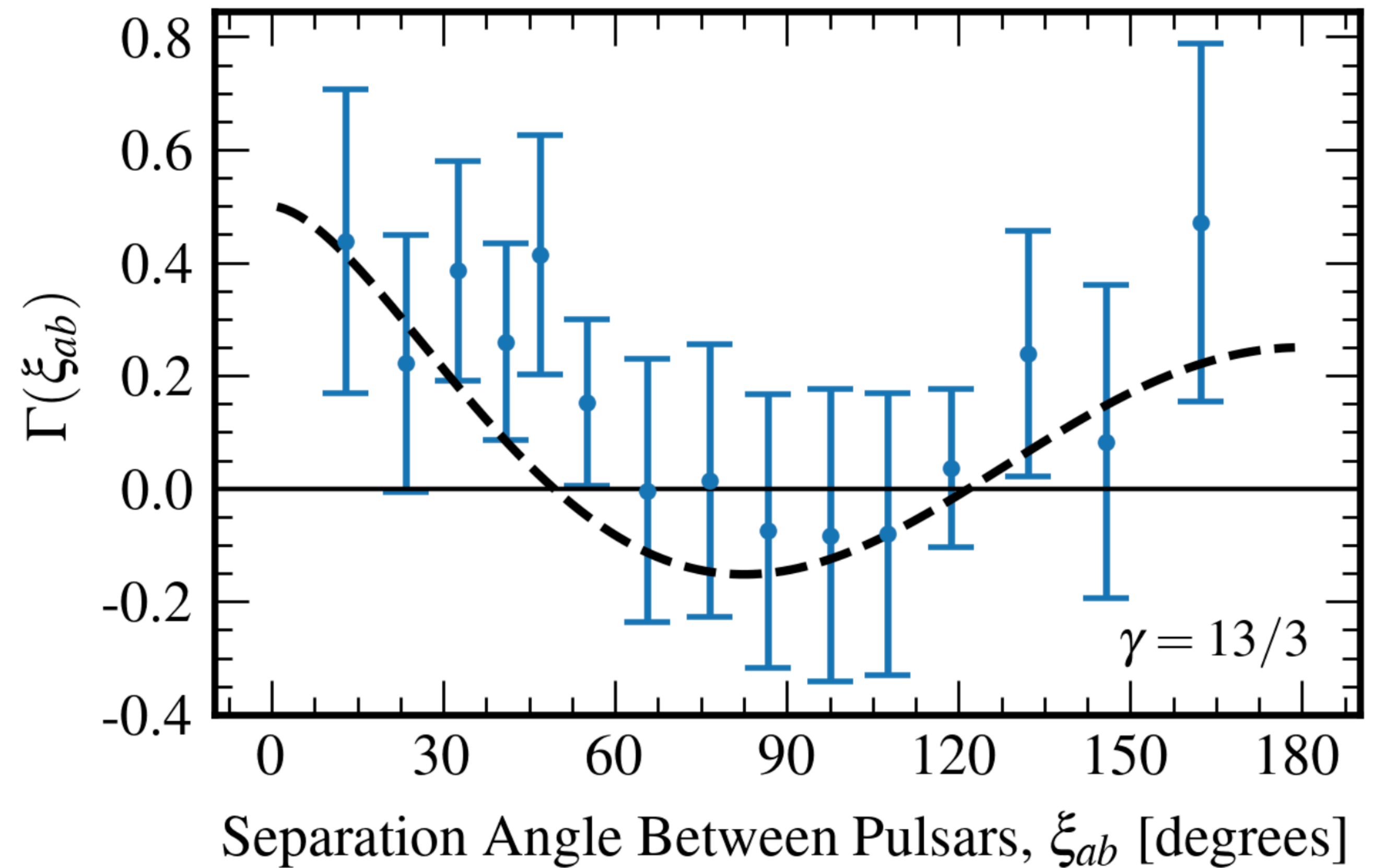
- **White noise:** due to instrumental noise and change of shape of different pulses
- **Red noise:** due to the stochasticity in the rotation of pulsars
- **Chromatic noise components:** temporal variations in dispersion measure and scattering variations due to the time-varying electron column density along the line of sight
- Any of these is expected to behave as a quadrupole!



Jitter in the NANOGrav pulsar J2145-0750. Credit: [8]

EVIDENCE FOR A STOCHASTIC GW BACKGROUND

- 67 pulsars
- Observation time: 16.03 yrs
- Not a 5σ evidence, but observation by independent collaborations



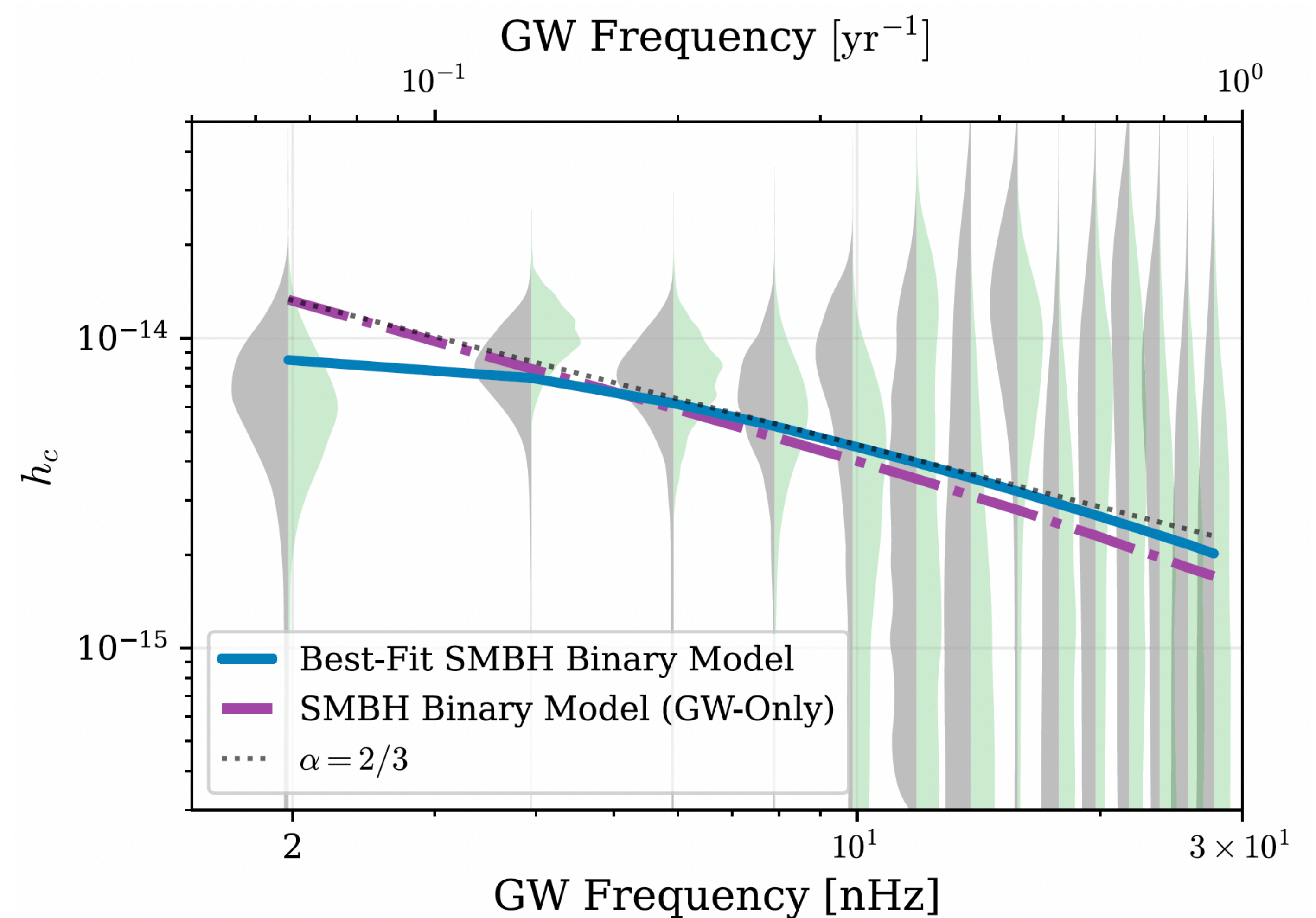
Credit: [9]

ASTROPHYSICAL OR COSMOLOGICAL ORIGIN?

➤ The observed signal is consistent with SMBBHs

➤ GW strain is modelled with a power law

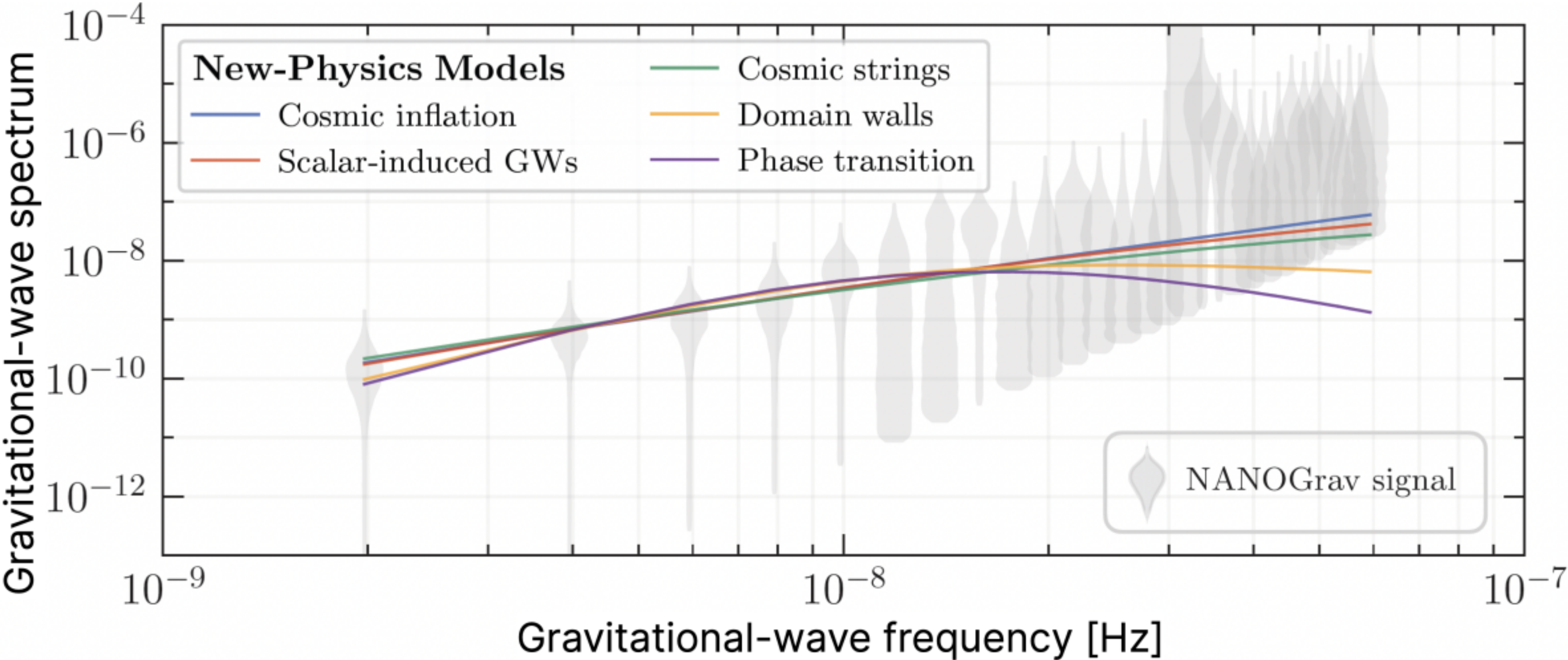
$$h_c(f) = A_{GWB} \left(\frac{f}{f_{yr}} \right)^\alpha$$



Credit: [10]

ASTROPHYSICAL OR COSMOLOGICAL ORIGIN?

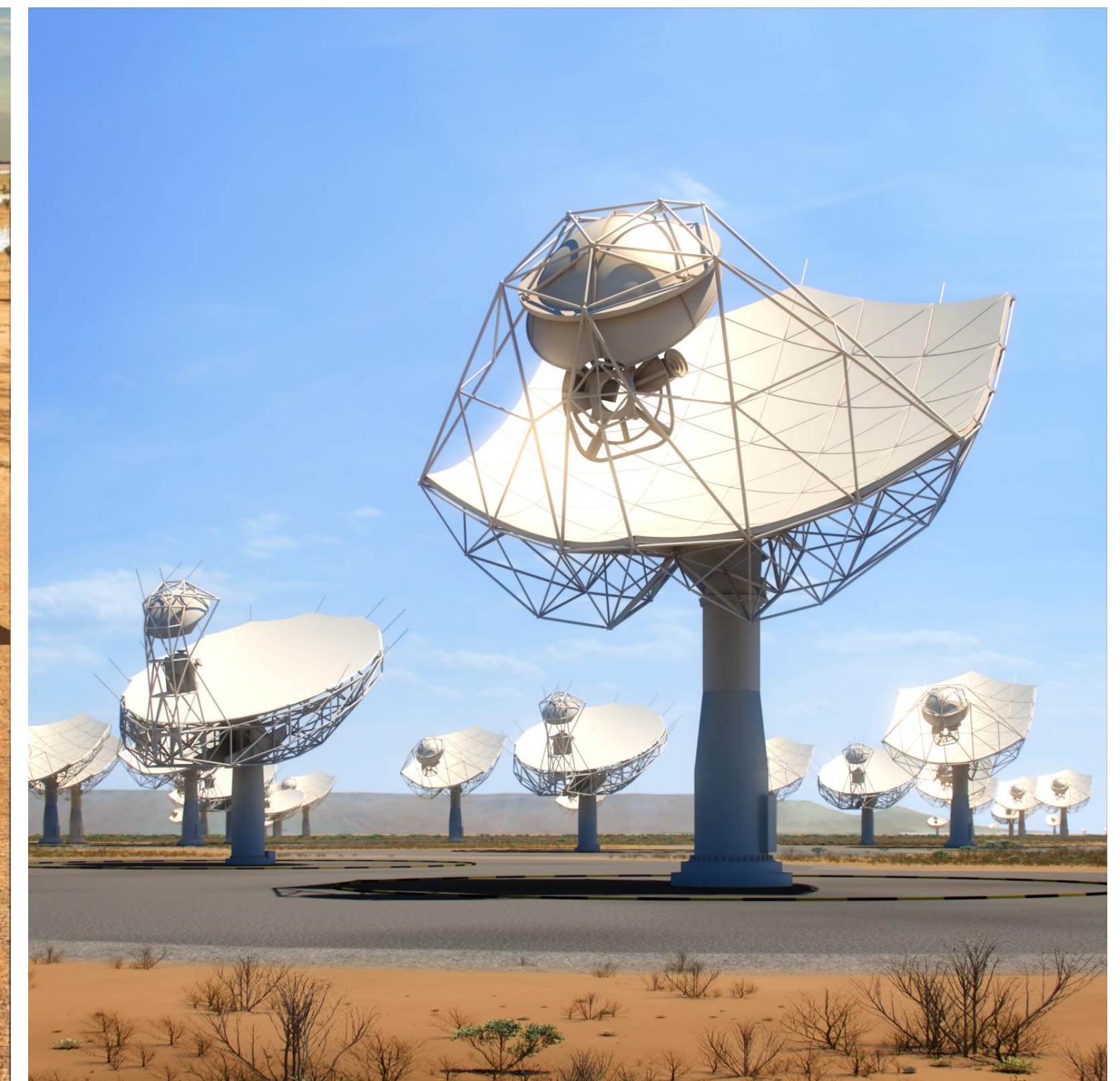
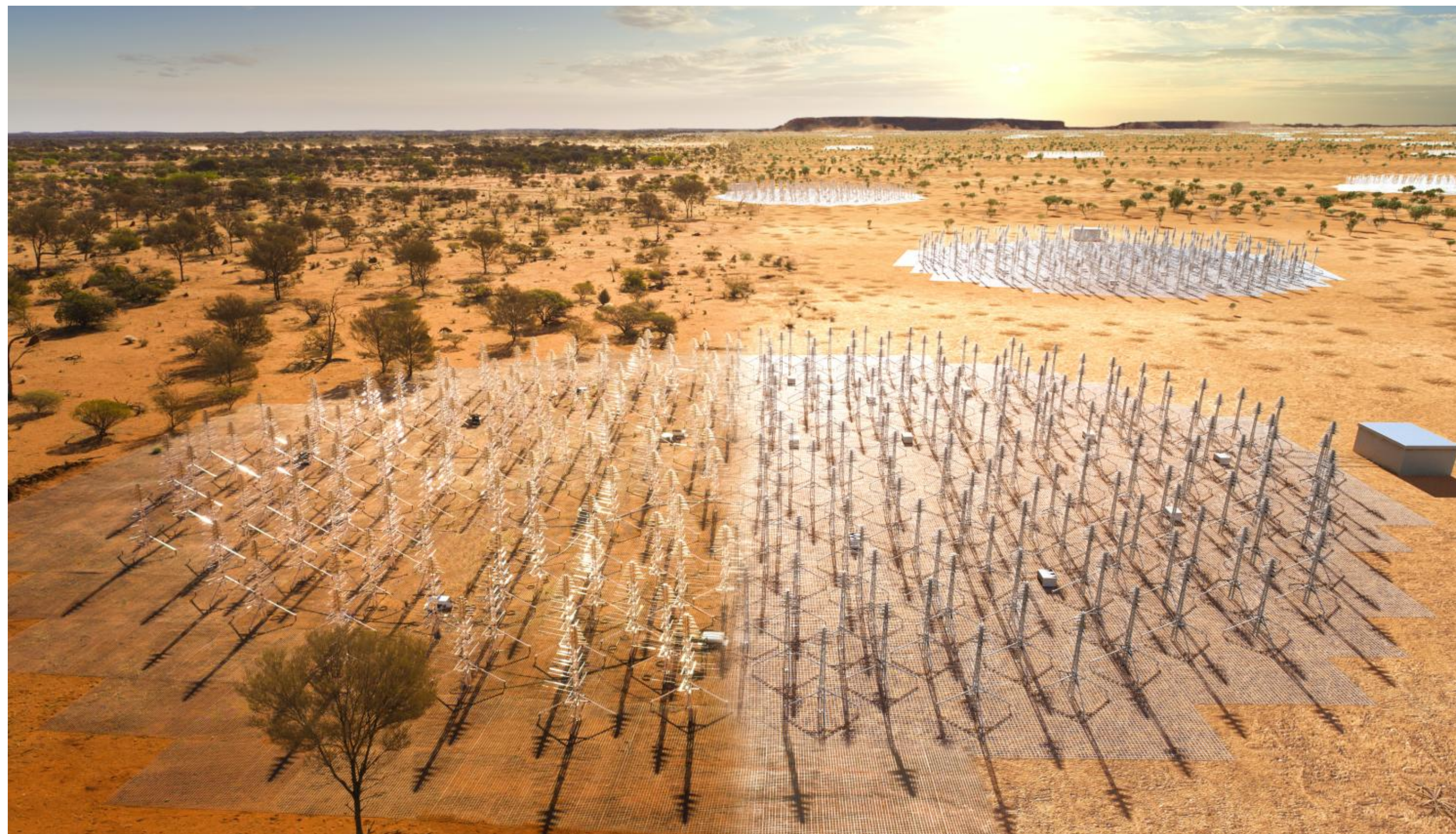
► The observed signal is also consistent with many other sources



Credit: [11]

WHAT'S NEXT?

- **More data:** longer observation time, more pulsars and frequency points
- New generation telescopes: the **Square Kilometer Array** (SKA) will provide high-precision pulsar timing measurement with uncertainties below ~ 100 ns



SPEED UP THE PRESENT ANALYSIS

S. Babak, M. Flaxa, G. Franciolini, & M. Pieroni *PR D* **110**, 063022 (2024)

P. F. Depta, V. Domcke, G. Franciolini, & M. Pieroni arXiv:2407.14460

- Data in the frequency domain:

$$\langle \tilde{s} \rangle = 0 \quad \text{for SGWB}$$

$$\langle \tilde{n} \rangle = 0 \quad \text{for noise}$$

- Information is in the variance: $\langle \tilde{d}^2 \rangle = \langle \tilde{s}^2 \rangle + \langle \tilde{n}^2 \rangle = RP_h + P_n$
- Simplifying assumptions: **Gaussian**, **(isotropic)** and **stationary** data for each pulsar

$$-\ln \mathcal{L}(\tilde{d} | \theta) \propto \sum_{k,IJ} \ln[C_{IJ}(f_k, \theta)] + \tilde{d}_I^k C_{IJ}^{-1}(f_k, \theta) \tilde{d}_J^{k*}$$

$$C_{IJ}(f) = P_{n,IJ}(f) + \chi_{IJ} R(f) P_h(f)$$

* I, J are pulsar indices

SPEED UP THE PRESENT ANALYSIS

S. Babak, M. Flaxa, G. Franciolini, & M. Pieroni *PR D* 110, 063022 (2024)

- Fisher matrix estimates on uncertainties

$$F_{\alpha\beta} \equiv -\frac{\partial^2 \log \mathcal{L}}{\partial \theta^\alpha \partial \theta^\beta} = T_{obs} \sum_{f_k} \frac{\partial \log C}{\partial \theta^\alpha} \frac{\partial \log C}{\partial \theta^\beta}$$

with $C(f_k, \theta_0) = \tilde{d}^k \tilde{d}^{k*}$ and θ_0 is the maximum likelihood estimator of model parameters

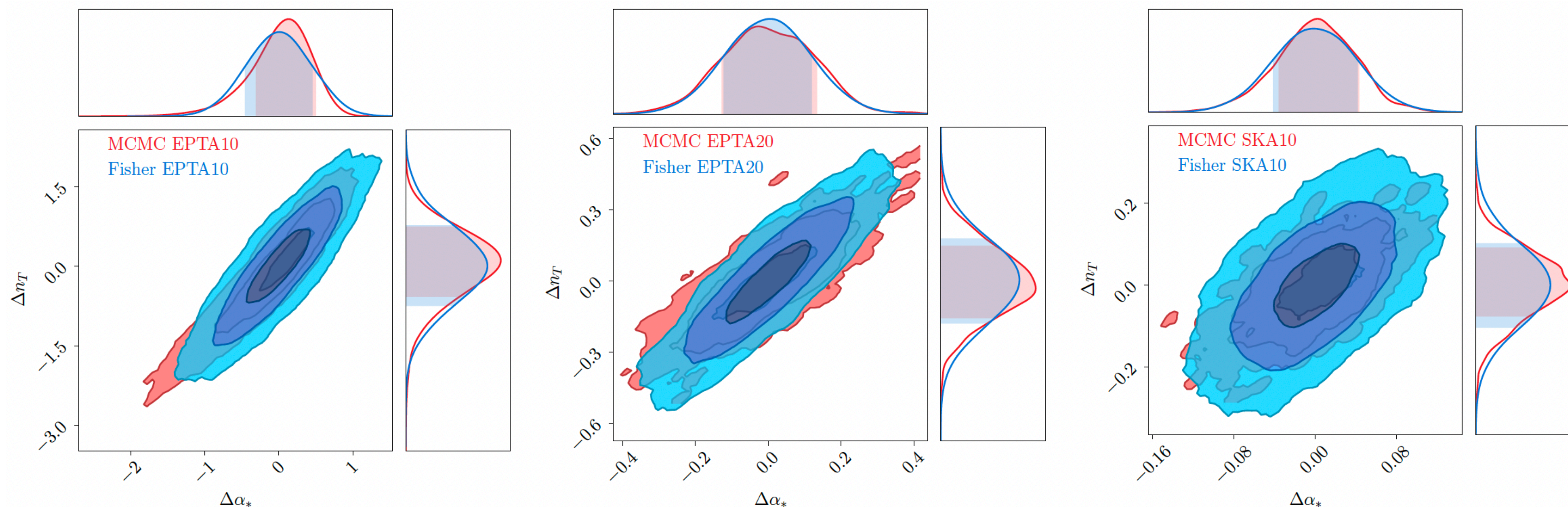
- By inverting the FIM one can estimate the uncertainties as $\sigma_\alpha \equiv \sqrt{F_{\alpha\alpha}^{-1}}$
- The python code `fastPTA` is available: <https://github.com/Mauropieroni/fastPTA/>

COMPARISON WITH FULL TIME DOMAIN BAYESIAN ANALYSIS

S. Babak, M. Flaxa, G. Franciolini, & M. Pieroni *PR D* 110, 063022 (2024)

- Is this method consistent with the standard approach?
- Let's consider a power law signal and test against current data + simulated mock data for future configurations

$$h^2\Omega_{GW} = 10^\alpha \left(\frac{f}{f_{yr}} \right)^{n_T}$$



Fully consistent!

Computational
time of a day vs
few seconds

SCALAR INDUCED GW

V. Dandoy, V. Domcke, & F. Rompineve *SciPost Phys. Core* **6**, 060 (2023)

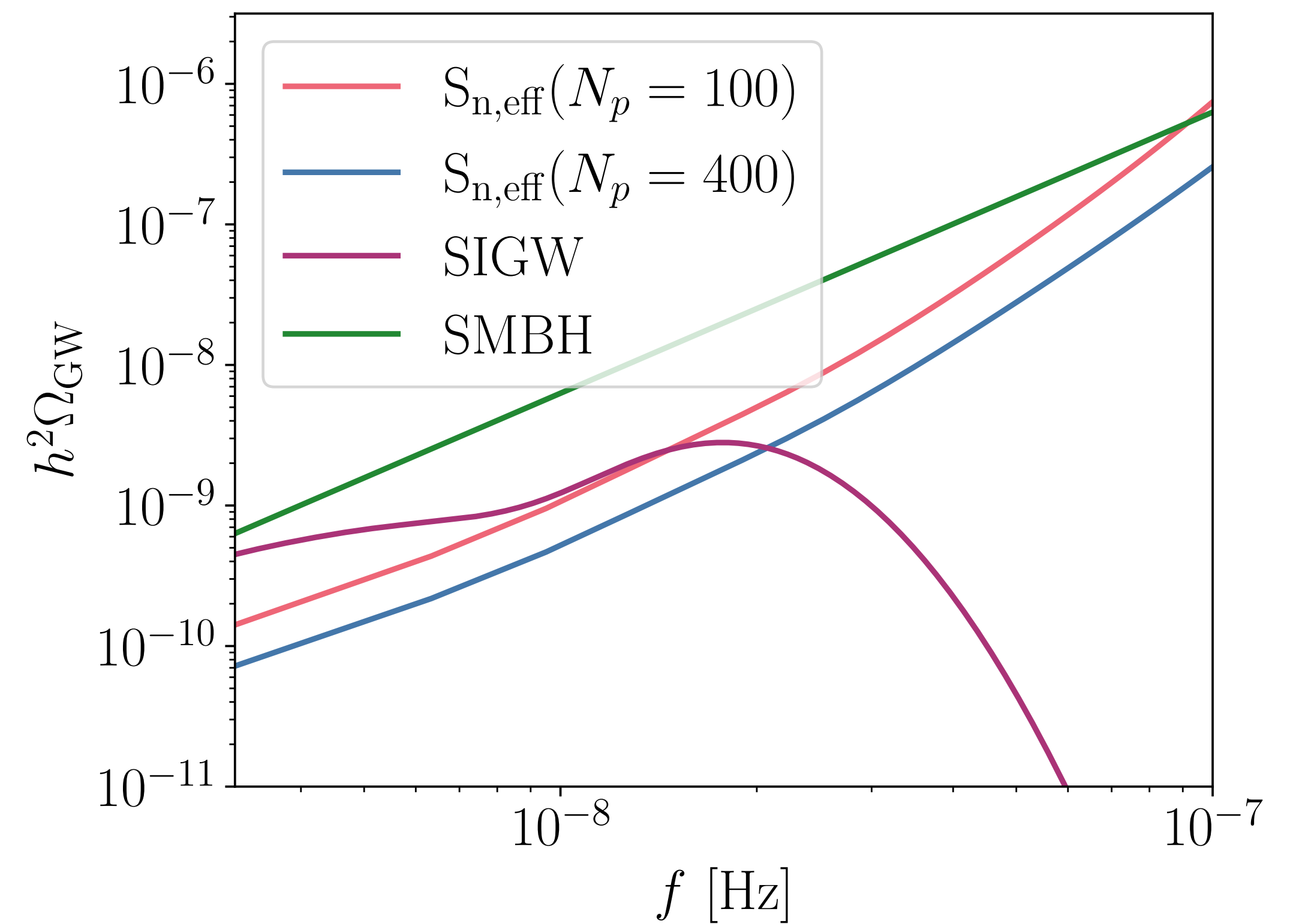
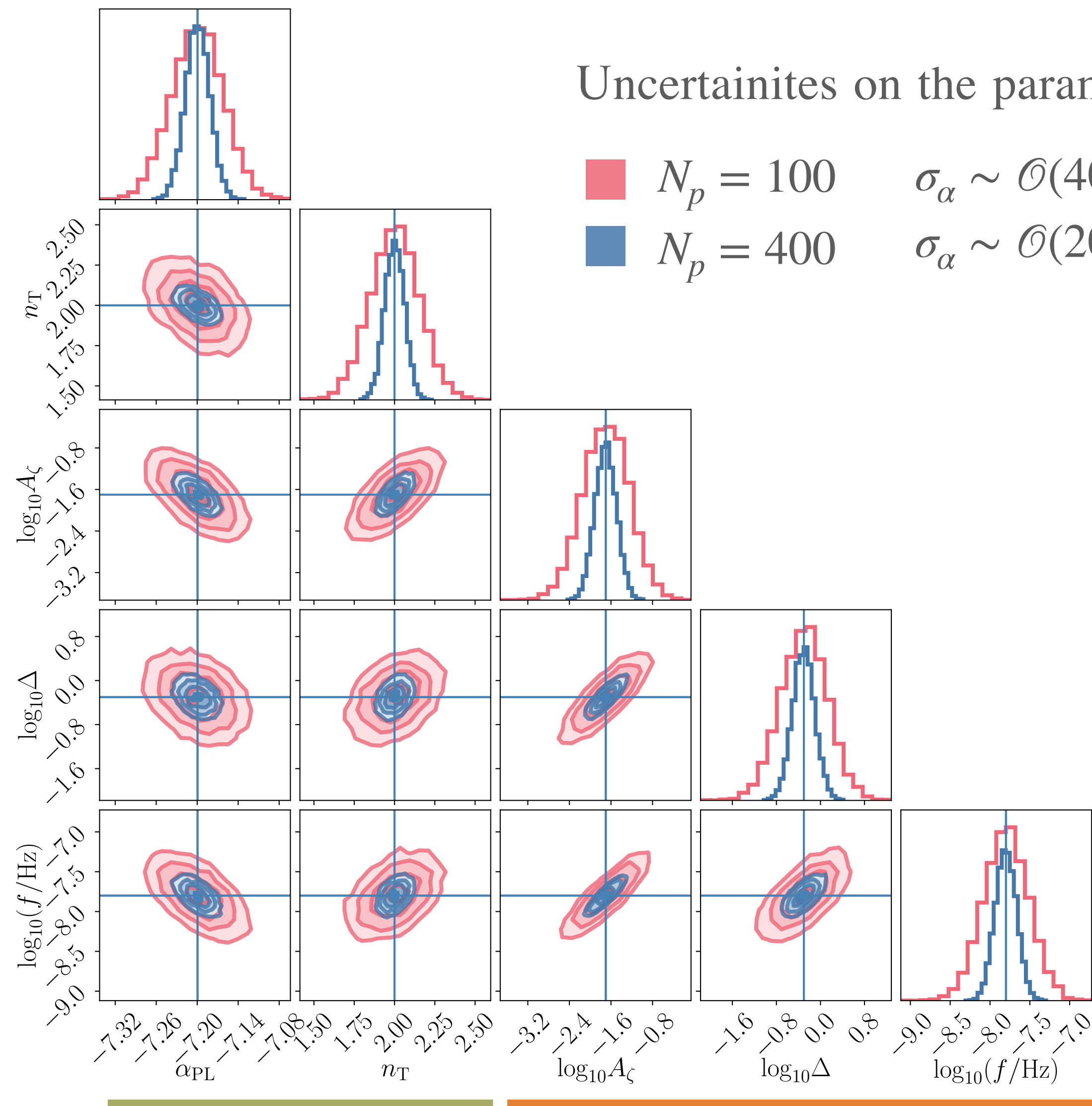
- **Aim:** forecasting the sensitivity of PTAs to a primordial signal: scalar-induced gravitational waves (SIGW)
- Spectrum of GW induced at second order by scalar perturbations. We stay agnostic on the specific inflationary models and we adopt a **log-normal parametrization**

$$P_{\zeta}(k) = \frac{A_{\zeta}}{\sqrt{2\pi}\Delta} \exp \left[-\frac{\log^2(k/k_*)}{2\Delta^2} \right]$$

- It captures the possibility of a peak in the power spectrum at small scales

SCALAR INDUCED GW AND SMBBH

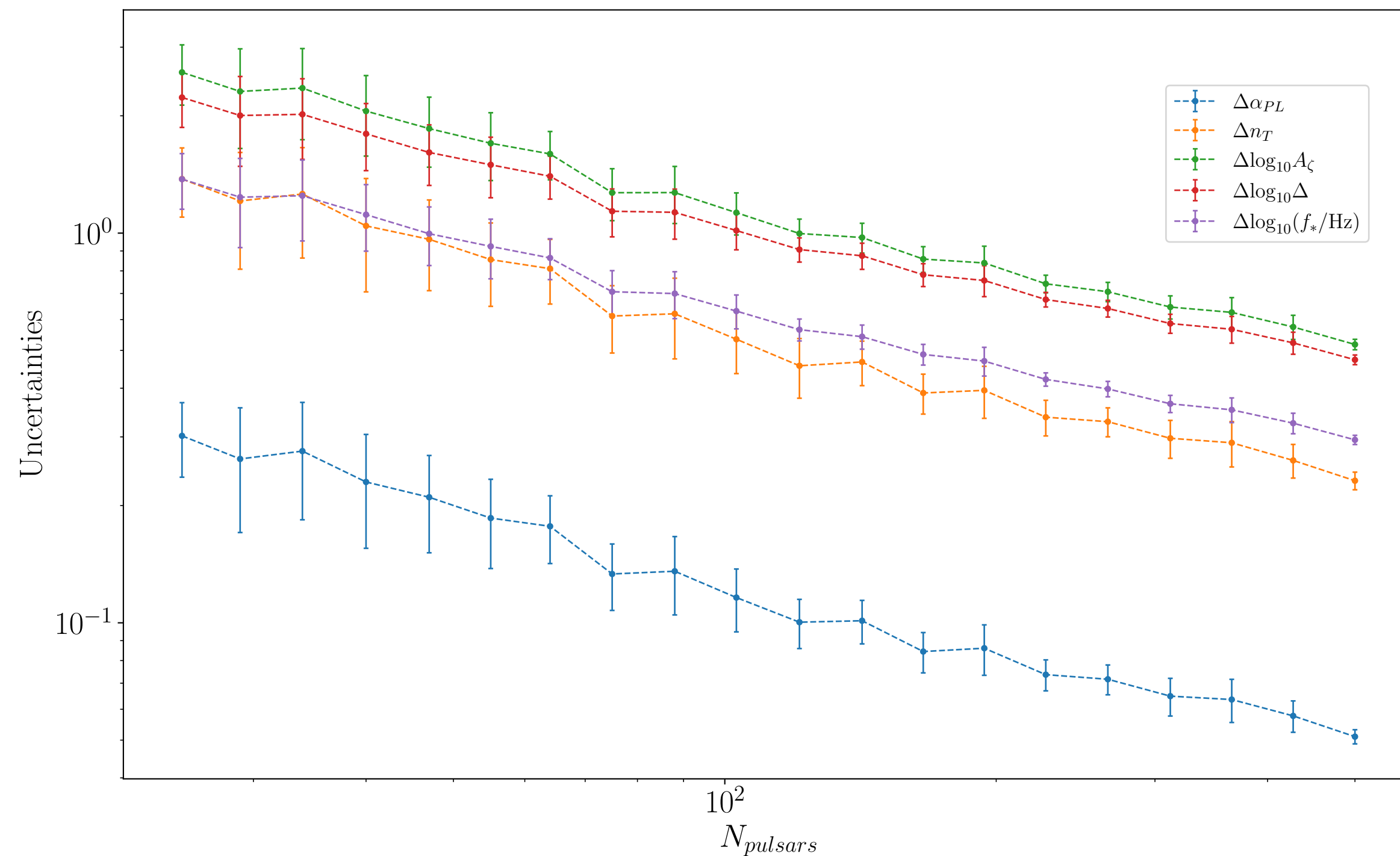
POWER-LAW MODEL (SMBBHs) + SIGWs (with maximal amplitude)



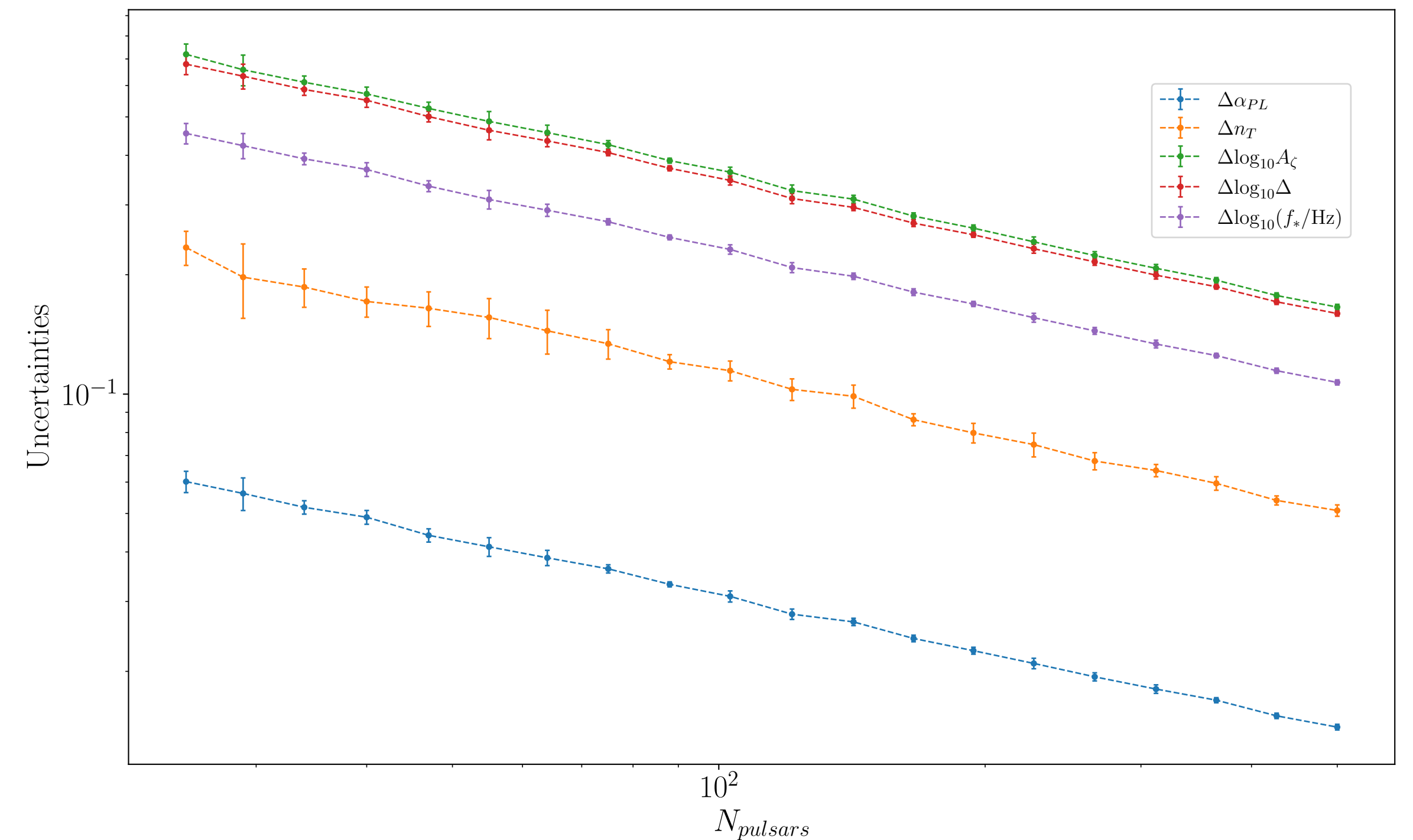
Future SKA-like: 10 yrs, N_p pulsars

SCALING OF THE UNCERTAINTIES WITH N

EPTA

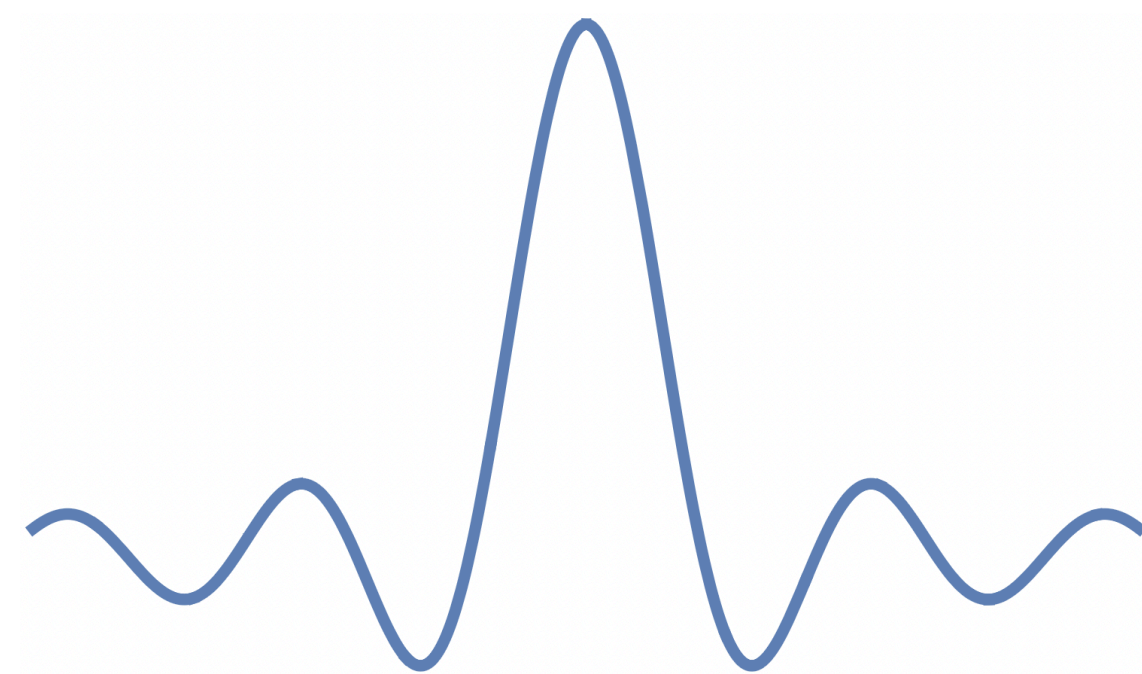


SKA



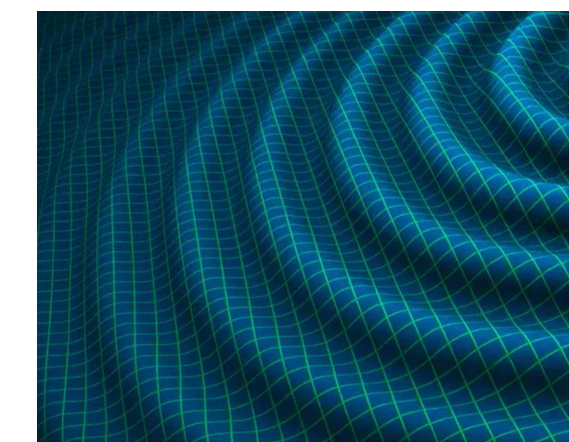
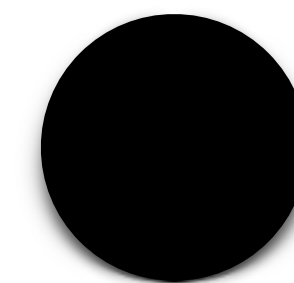
- Expected scaling $\propto 1/\sqrt{N}$
- Much better results for SKA

PRIMORDIAL BLACK HOLES



PBH COLLAPSE

EMISSION OF II ORDER GWs



- Fraction $f_{\text{PBH}} = \Omega_{\text{PBH}}/\Omega_{\text{DM}}$ of the total dark matter today
- **Avoid PBH overproduction!**

$$f_{\text{PBH}} < 1$$

Large signal amplitude
is needed to better
constrain SIGW

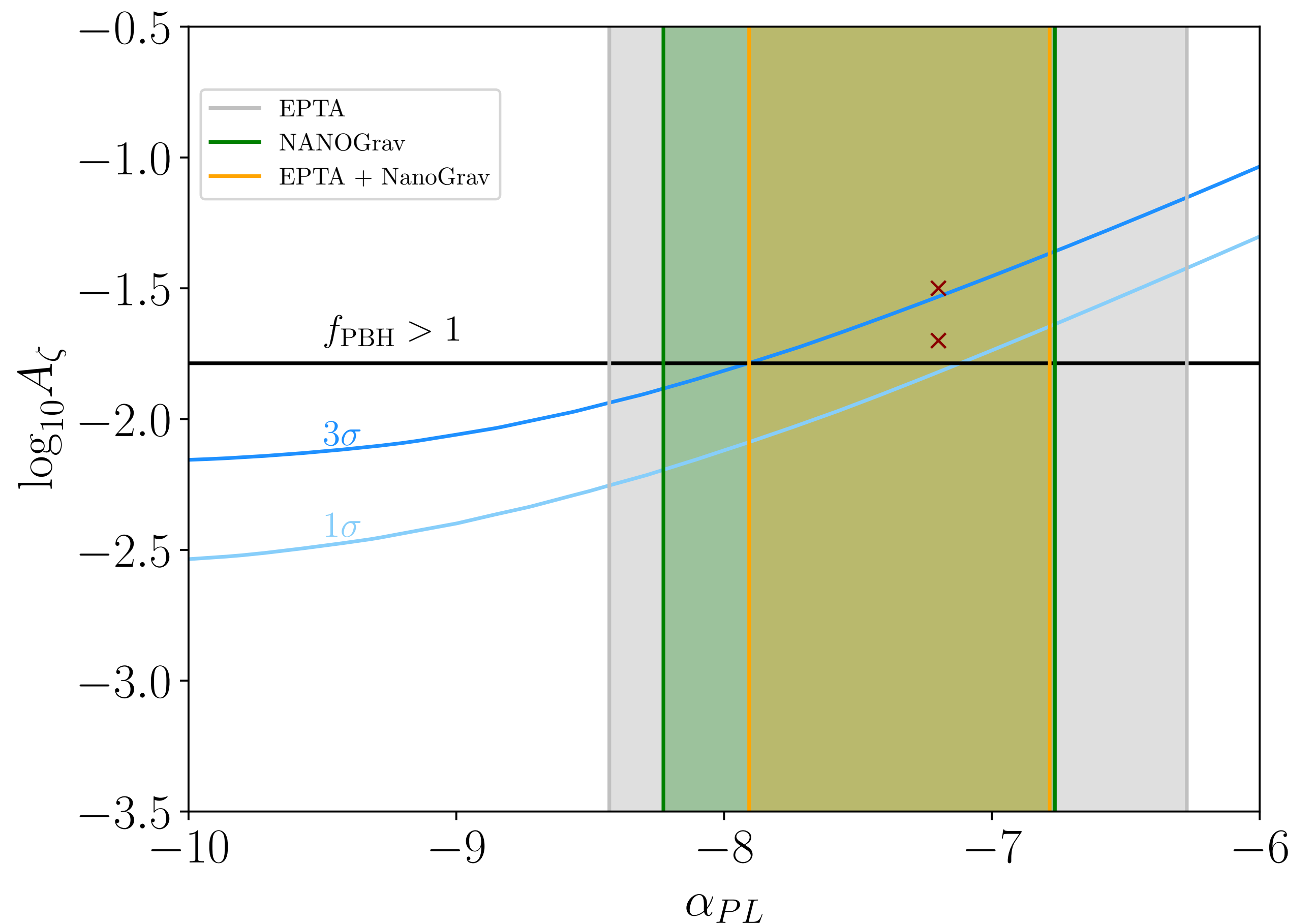


A too large amplitude
overproduces PBH

CONSTRAINING THE PARAMETER SPACE

► Look for a region in the parameter space of signal amplitudes where we can:

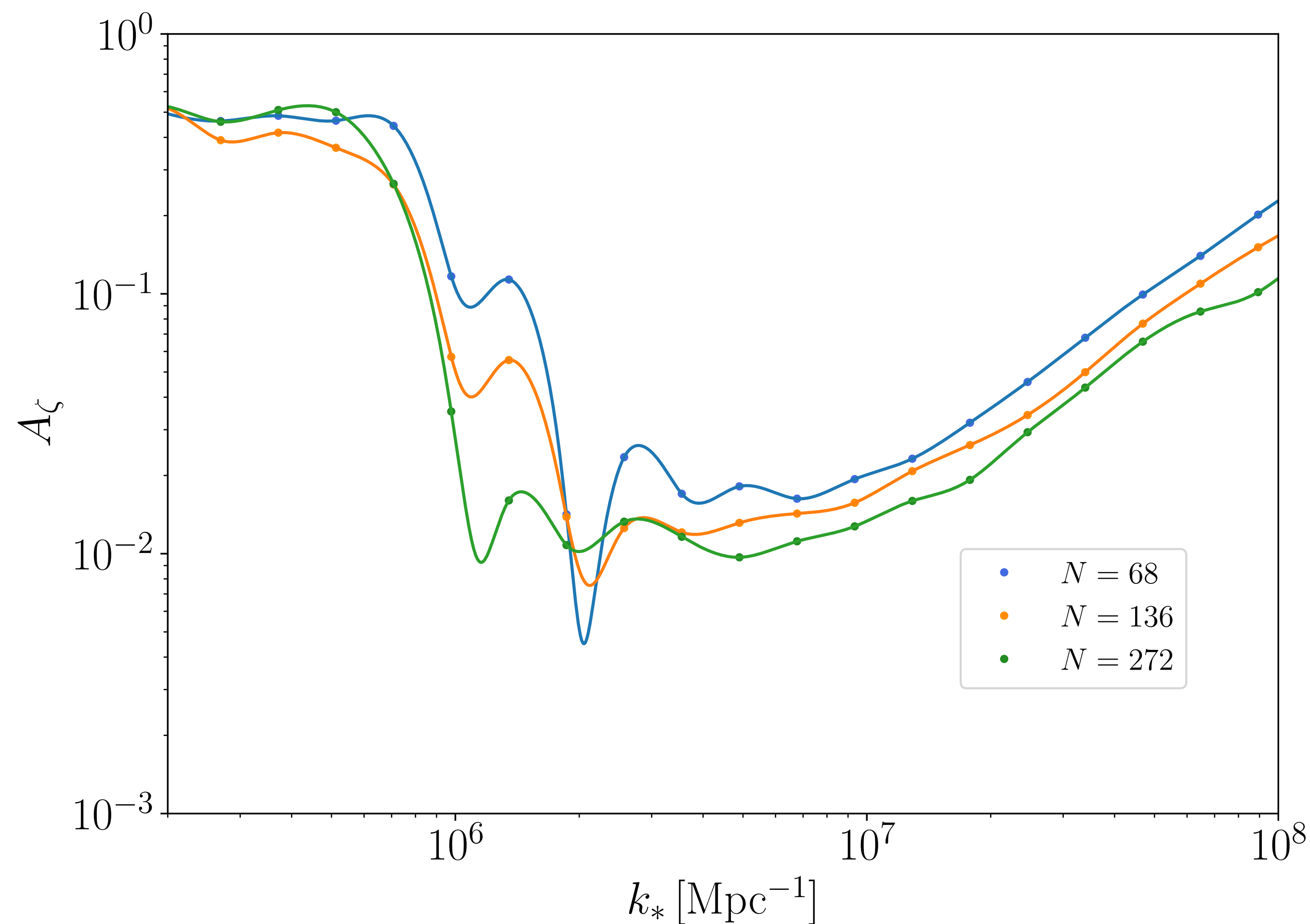
- Keep $f_{\text{PBH}} < 1$
- Measure $\log_{10} A_\zeta$ with $< 3\sigma$ uncertainties
- Have a power-law amplitude within the inferred posteriors



UPPER BOUND ON A_ζ

See [A. J. Iovino, G. Prerna, A. Riotto, & H. Veermae](#)
arXiv: 2406.20089 for comparison

- Upper bound from NANOGrav15 ($N_p = 68$) and scaling with increasing number of pulsars



SUMMARY

- Pulsar timing arrays (PTAs) provide an unprecedented opportunity to search for GWs in the nHz frequency range
- We expect to gain in sensitivity with next generation experiments
- Forecasting can be made faster
- Concrete possibility of measuring a scalar-induced GW signal and constraining primordial black holes (PBHs)

IMAGE CREDITS

[1] B. P. Abbott et al, *PRL* **118**, 221101 (2017)

[2] A. Stuver/LIGO

[3] Christopher Berry blog

[4] B. Saxton, NRAO/AUI/NSF

[5] H. Thankful Cremartie, *Carnegie Colloquium, October 17, 2023*

[6] David Champion/Max Planck Institute for Radio Astronomy

[7] Fredrick A. Jenet, Joseph D. Romano, *Am. J. Phys.*, **83**, 635-645 (2015)

[8] NANOGrav Collaboration, [arXiv:1810.06594](https://arxiv.org/abs/1810.06594)

[9] G. Agazie et al 2023 *ApJL* 951 L8

[10] G. Agazie et al 2023 *ApJL* 952 L37

[11] <https://nanograv.org/15yr/Summary/NewPhysics>

[12] <https://www.skao.int/en/explore/telescopes>

[13] V. Dandoy, V. Domcke, & F. Rompineve *SciPost Phys. Core* **6**, 060 (2023)

[14] B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *Rept. Prog. Phys.* **84**, no.11, 116902 (2021)

BACKUP SLIDES

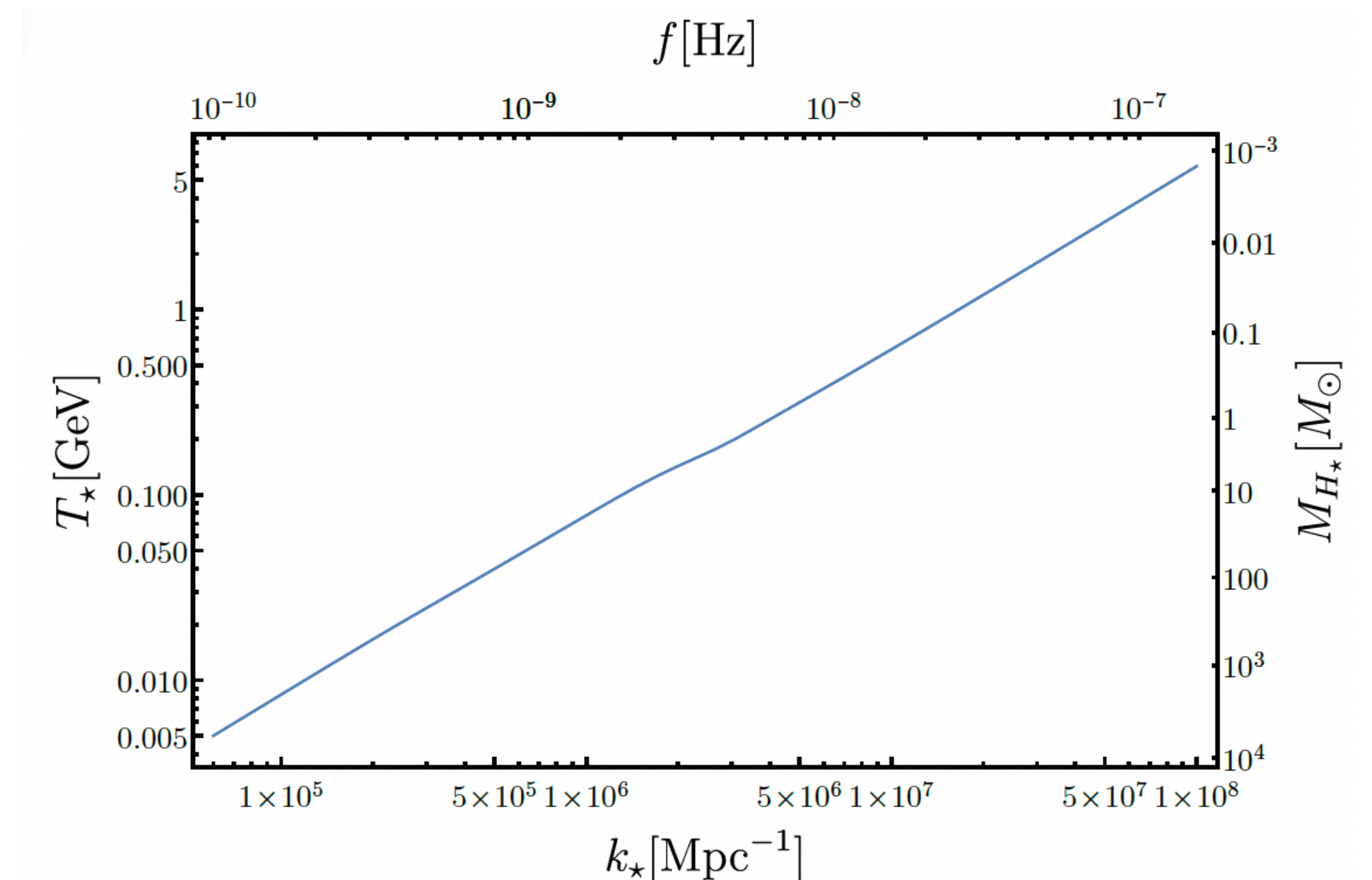
SCALAR INDUCED GW

- The spectrum of GWs can be computed analytically

$$\Omega_{GW} \left(\frac{f}{f_*}, \Delta, A_\zeta \right) = \dots$$

- The spectrum today can be easily evaluated with a temperature-dependent modulation (GWs are decoupled from the thermal bath and they always evolve as radiation)

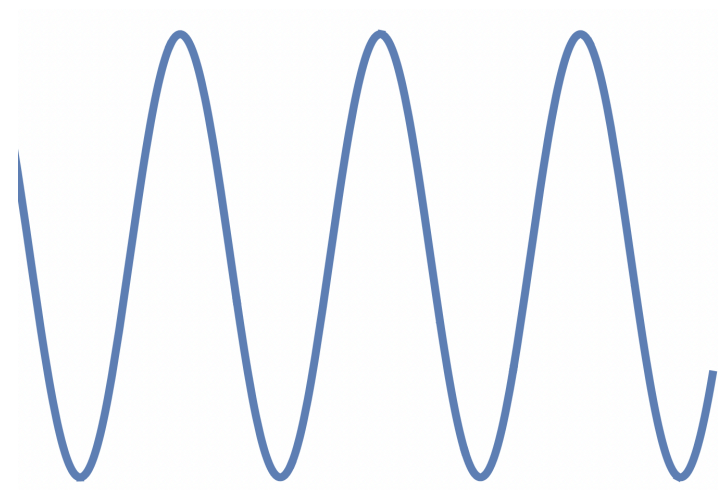
$$\begin{aligned} \Omega_{GW} h^2 &= \left[\frac{a(T)}{a_0} \right]^4 \left[\frac{H(T)}{H_0/h} \right]^2 \Omega_{GW}(T) \\ &\approx 1.6 \times 10^{-5} \left[\frac{g_*(T)}{100} \right] \left[\frac{g_{*s}(T)}{100} \right]^{-\frac{4}{3}} \Omega_{GW}(T) \end{aligned}$$



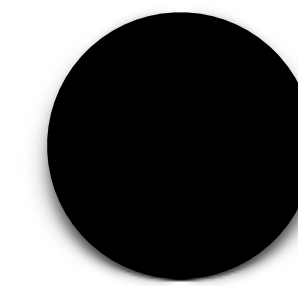
Credit: [13]

PRIMORDIAL BLACK HOLES

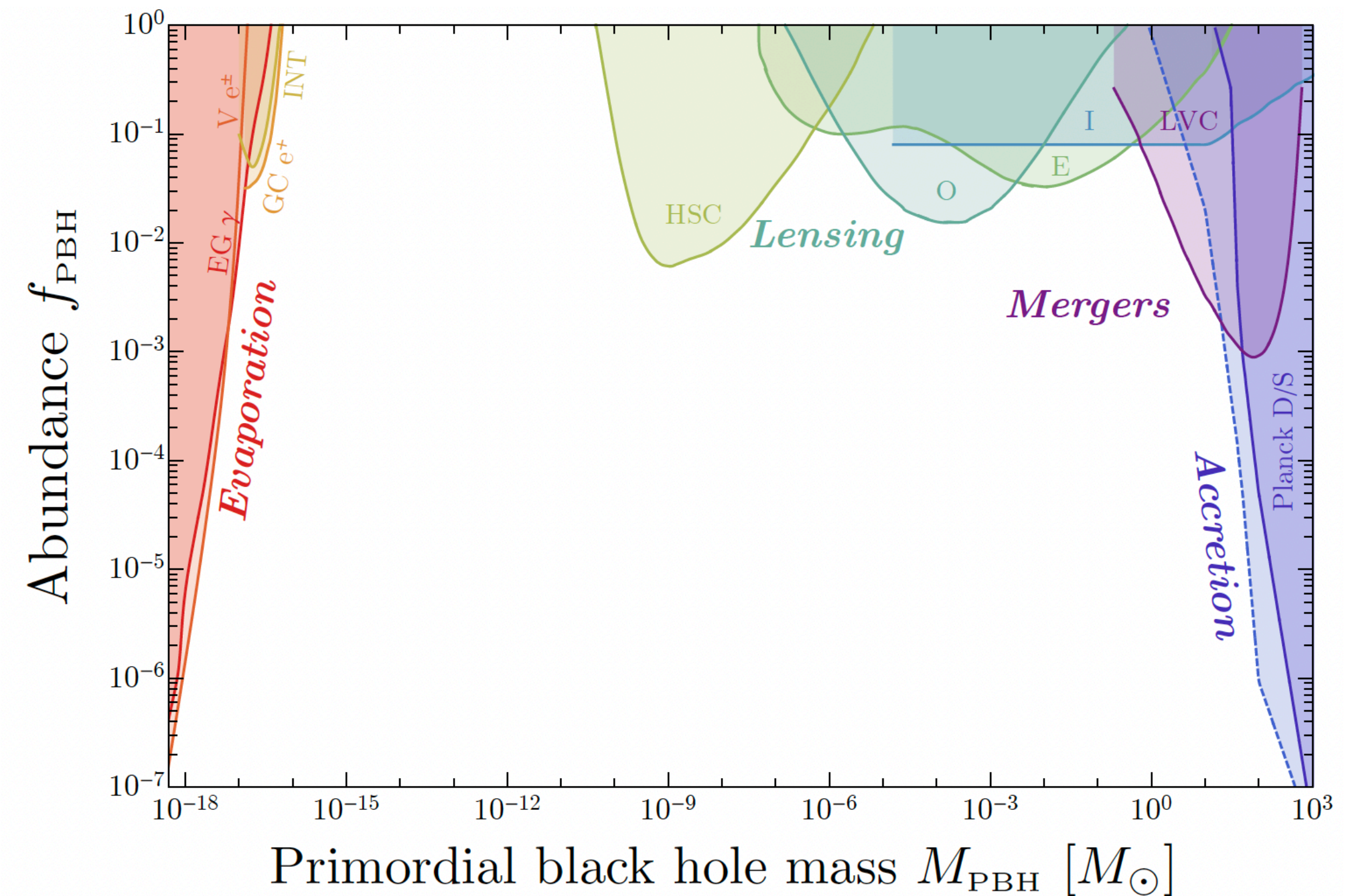
- Mass of the PBH and frequency of the GW are related by the Hubble scale at formation



$$f_{GW} \approx 3 \cdot 10^{-9} \text{ Hz} \left(\frac{m_{PBH}}{M_{\odot}} \right)^{-1/2}$$

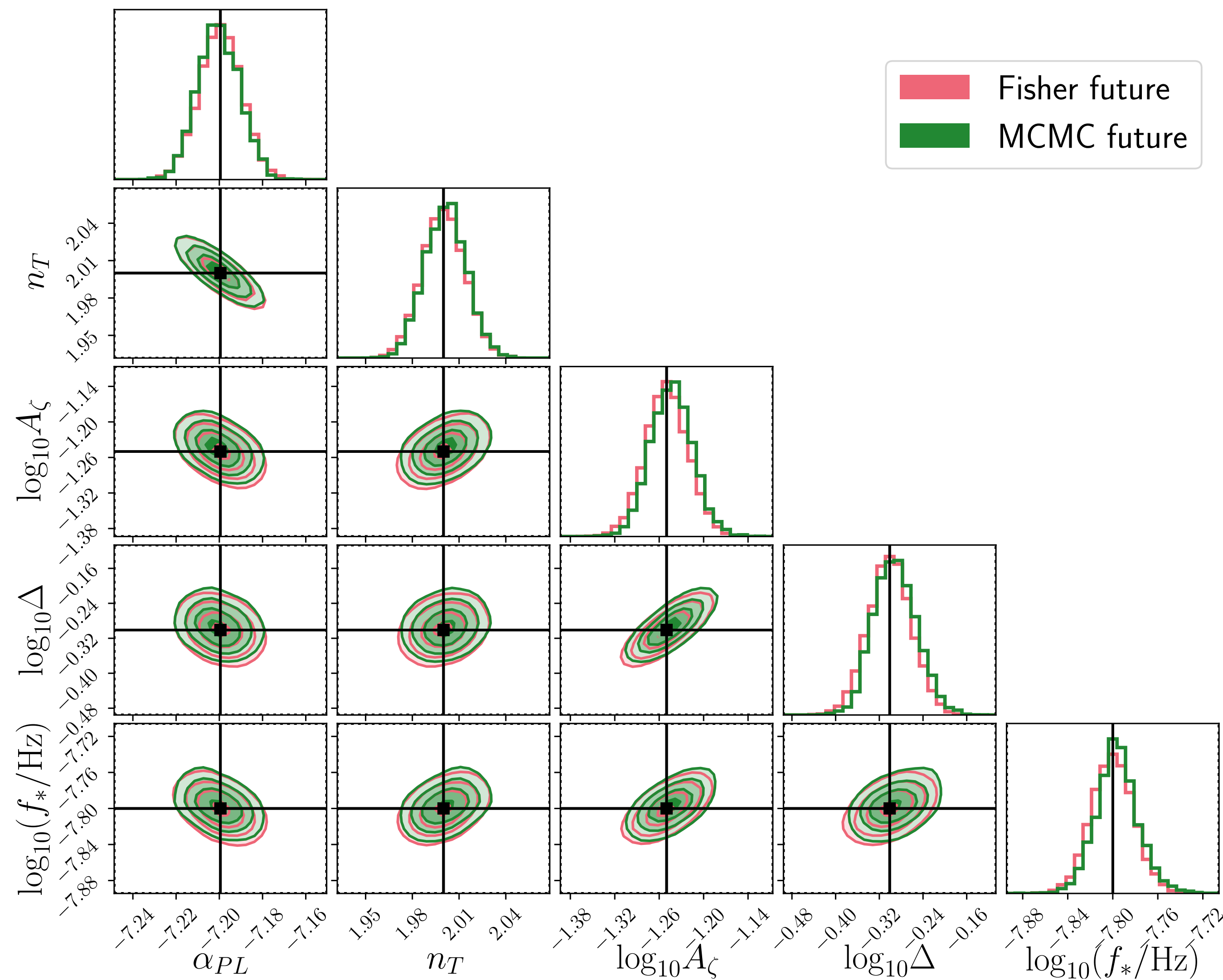


- Complementary constraints from PBH overproduction



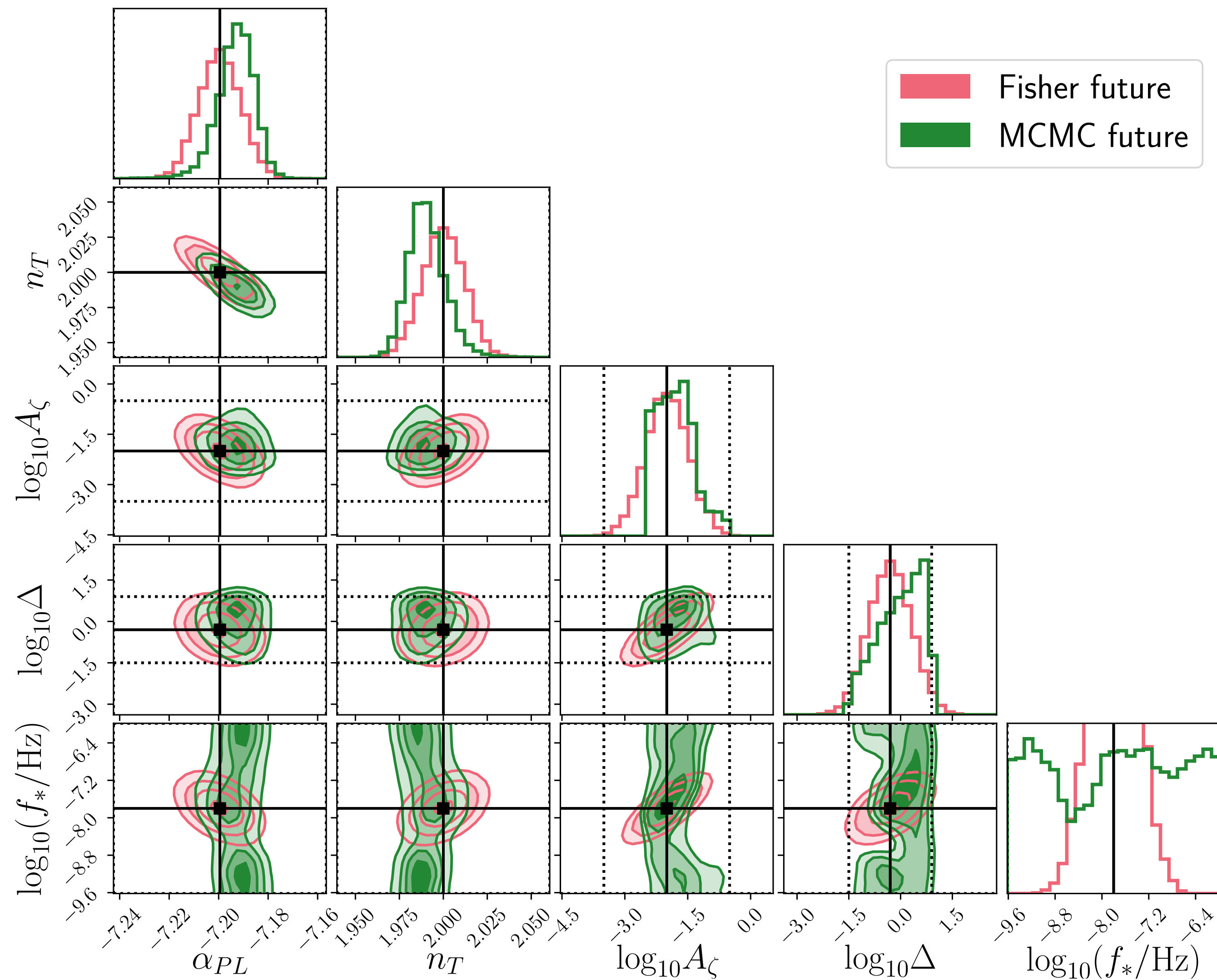
Credit: [14]

FIM VS MCMC FOR NOISELESS DATA



- EPTA-like **noiseless** data
- Large SNR to see if the two methods agree
($\log_{10} A_\zeta = -1.25$)

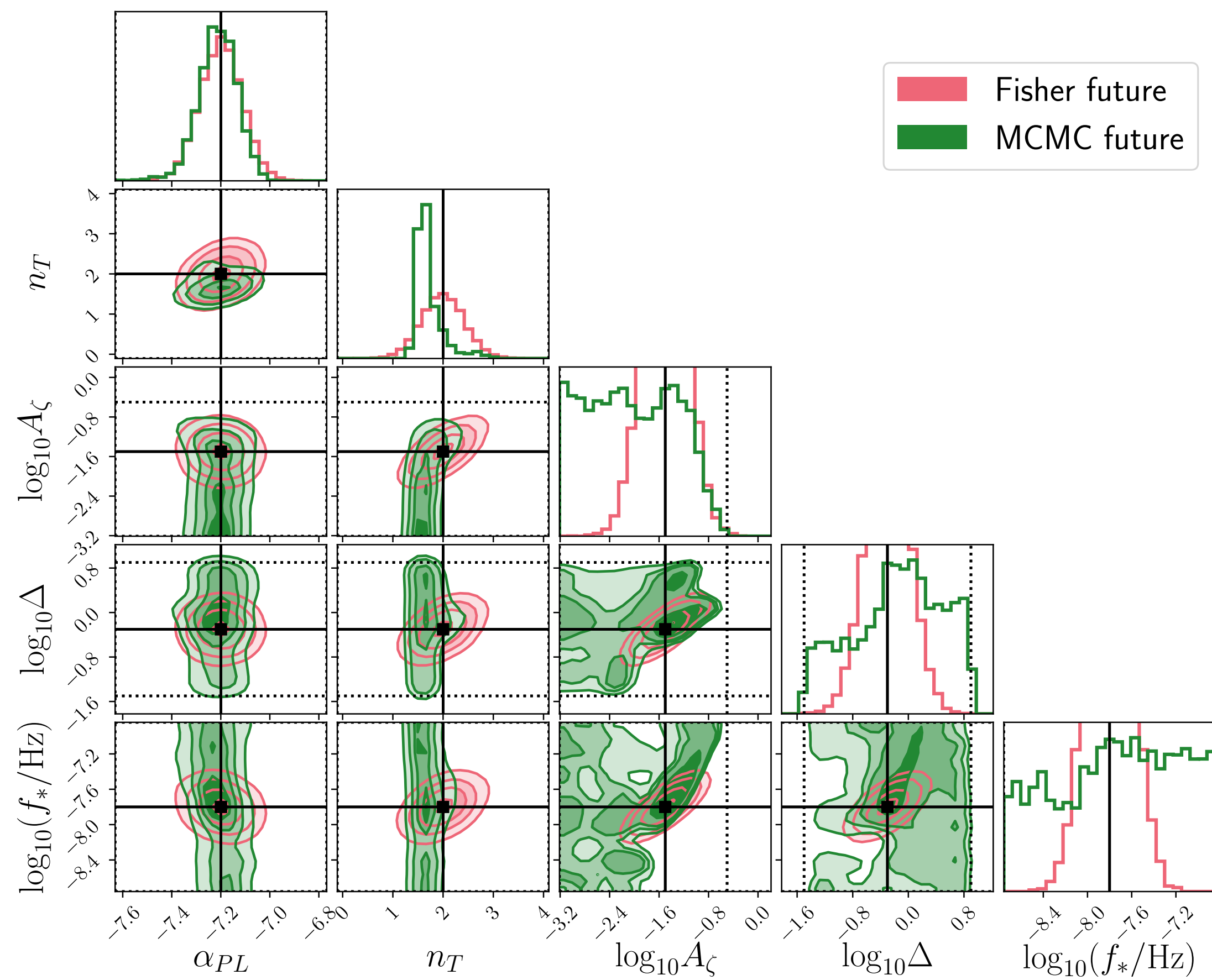
FIM VS MCMC FOR NOISELESS DATA



- EPTA-like **noiseless** data
- Lowering the SNR
($\log_{10} A_\zeta = -2$)
- Degeneracy on f_*
- Visible effect of the priors

COMPARISON BETWEEN EPTA AND SKA DATA

EPTA



SKA

