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DI TORINO



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Detecting Relativistic Doppler in Galaxy Clustering by Multi-tracing a Single Galaxy Population

[F. Montano & S. Camera, PDU 46 (2024) 101570, [arXiv:2309.12400](https://arxiv.org/abs/2309.12400)]

[F. Montano & S. Camera, PDU 46 (2024) 101634, [arXiv:2407.06284](https://arxiv.org/abs/2407.06284)]

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18th September 2024
CASTLE 2024 – Tagliolo Monferrato

Relativistic galaxy number counts

The leading local contributions to the number density contrast of galaxies are [Yoo (2010); Bonvin & Durrer (2011); Challinor & Lewis (2011)]:

$$\Delta(\vec{x}) = b\delta(\vec{x}) - \frac{1}{\mathcal{H}} \partial_r v_r(\vec{x}) - \alpha v_r(\vec{x}),$$

with:

- $\alpha = -\varepsilon + 2Q - 2 \frac{Q-1}{r\mathcal{H}} + \frac{\mathcal{H}'}{\mathcal{H}^2}$,
- r = comoving radial distance,
- b = linear galaxy bias,
- $\delta = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$ = matter density contrast,
- \mathcal{H} = conformal Hubble factor,
- v = velocity field,
- Q = magnification bias,
- ε = evolution bias.

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with:

Sample-dependent quantities

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- \mathcal{H} = conformal Hubble factor,
- v = velocity field,
- Q = magnification bias,
- \mathcal{E} = evolution bias.

Auto- and cross-correlation measurements

- $\langle \delta_X(\vec{k}) \delta_Y(\vec{k}') \rangle \propto \delta^D(\vec{k} + \vec{k}') P_{XY}(k)$

$$P_{XY}(z, k, \mu) = \left[(b_X + f\mu^2)(b_Y + f\mu^2) + \left(\frac{\mathcal{H}f\mu}{k}\right)^2 \alpha_X \alpha_Y + i \frac{\mathcal{H}f\mu}{k} (\alpha_X(b_Y + f\mu^2) - \alpha_Y(b_X + f\mu^2)) \right] P_m(k)$$

- $X = Y \rightarrow$ auto-correlation
- $X \neq Y \rightarrow$ cross-correlation

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$$= \left[(b_X + f\mu^2)(b_Y + f\mu^2) + \left(\frac{\mathcal{H}f\mu}{k} \right)^2 \alpha_X \alpha_Y \right.$$

$$\left. + i \frac{\mathcal{H}f\mu}{k} (\alpha_X (b_Y + f\mu^2) - \alpha_Y (b_X + f\mu^2)) \right] P_m(k)$$

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- $P_{XY}(z, k, \mu) = P_{YX}^*(z, k, \mu) \rightarrow P_{XY}(z, k, \mu) = P_{YX}(z, k, -\mu)$
- The Doppler contribution is **proportional to k^{-1} in the imaginary part** of the cross-power spectrum [McDonald (2009)].

Multi-tracer power spectrum

We can put together information given by auto- and cross-power spectra to obtain tighter constrains [Percival et al. (2004); Fonseca *et al.* (2015)].

- We have now:

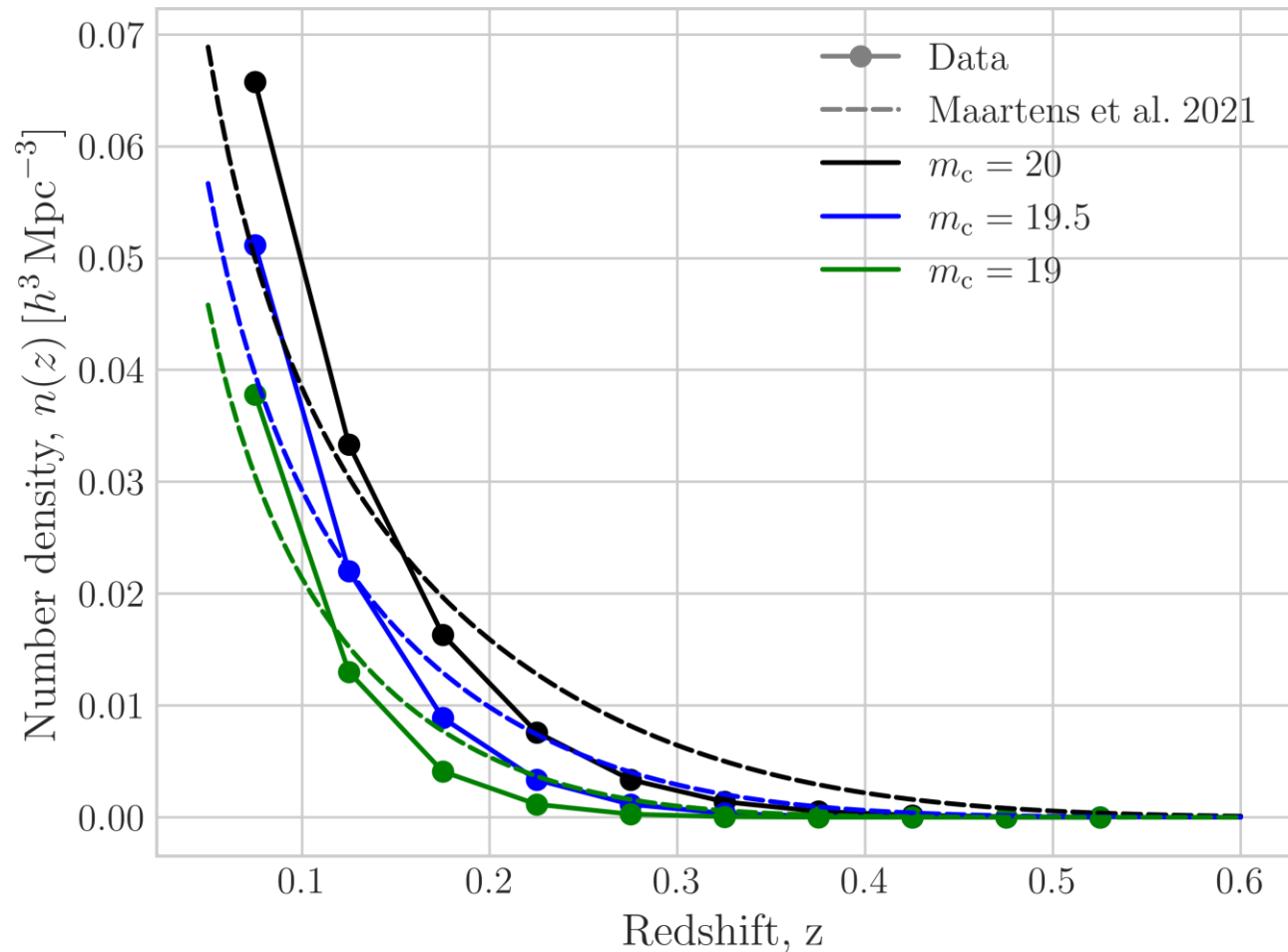
$$P = \begin{pmatrix} P_{XX} \\ P_{XY} \\ P_{YY} \end{pmatrix}, \quad \Gamma = \frac{2}{N_{modes}} \begin{pmatrix} \tilde{P}_{XX}^2 & \tilde{P}_{XX}\tilde{P}_{XY} & \tilde{P}_{XY}^2 \\ \tilde{P}_{XX}\tilde{P}_{YX} & \frac{\tilde{P}_{XX}\tilde{P}_{YY} + \tilde{P}_{XY}\tilde{P}_{YX}}{2} & \tilde{P}_{XY}\tilde{P}_{YY} \\ \tilde{P}_{YX}^2 & \tilde{P}_{YX}\tilde{P}_{YY} & \tilde{P}_{YY}^2 \end{pmatrix}$$

- Multi-tracer power spectrum with P_{FF}, P_{FB}, P_{BB} [Montano & Camera (2024)].

The Doppler contribution is sample-dependent

We use:

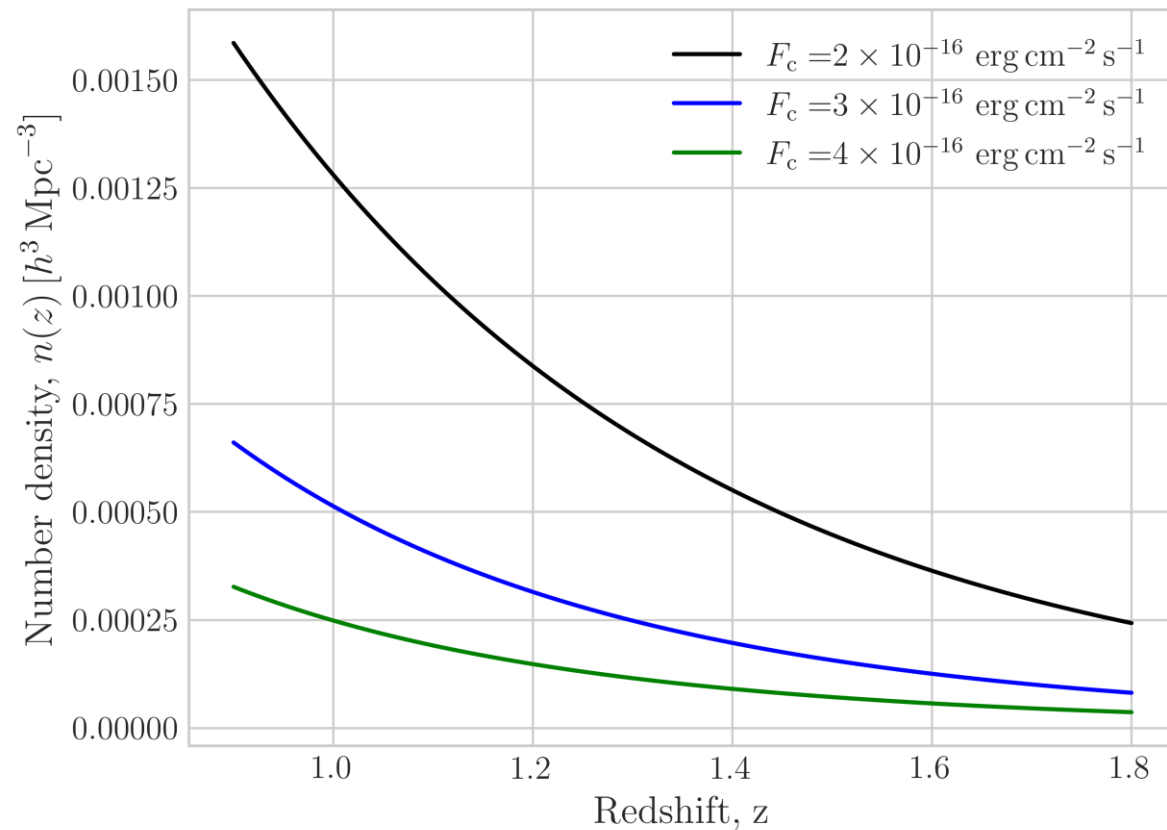
- A low-redshift DESI-like Bright Galaxy Sample (BGS) [Smith et al. (2023)];



The Doppler contribution is sample-dependent

We use:

- A low-redshift DESI-like Bright Galaxy Sample (BGS) [Smith et al. (2023)];
- A population of H α galaxies observed by a *Euclid*-like survey [Maartens et al. (2021)].



Luminosity cut technique

[Bonvin et al. (2014, 2016, 2023); Gaztanaga et al. (2017)]

- **Complete sample (T):** all the galaxies that are observed with a flux density F higher than a fixed minimum flux

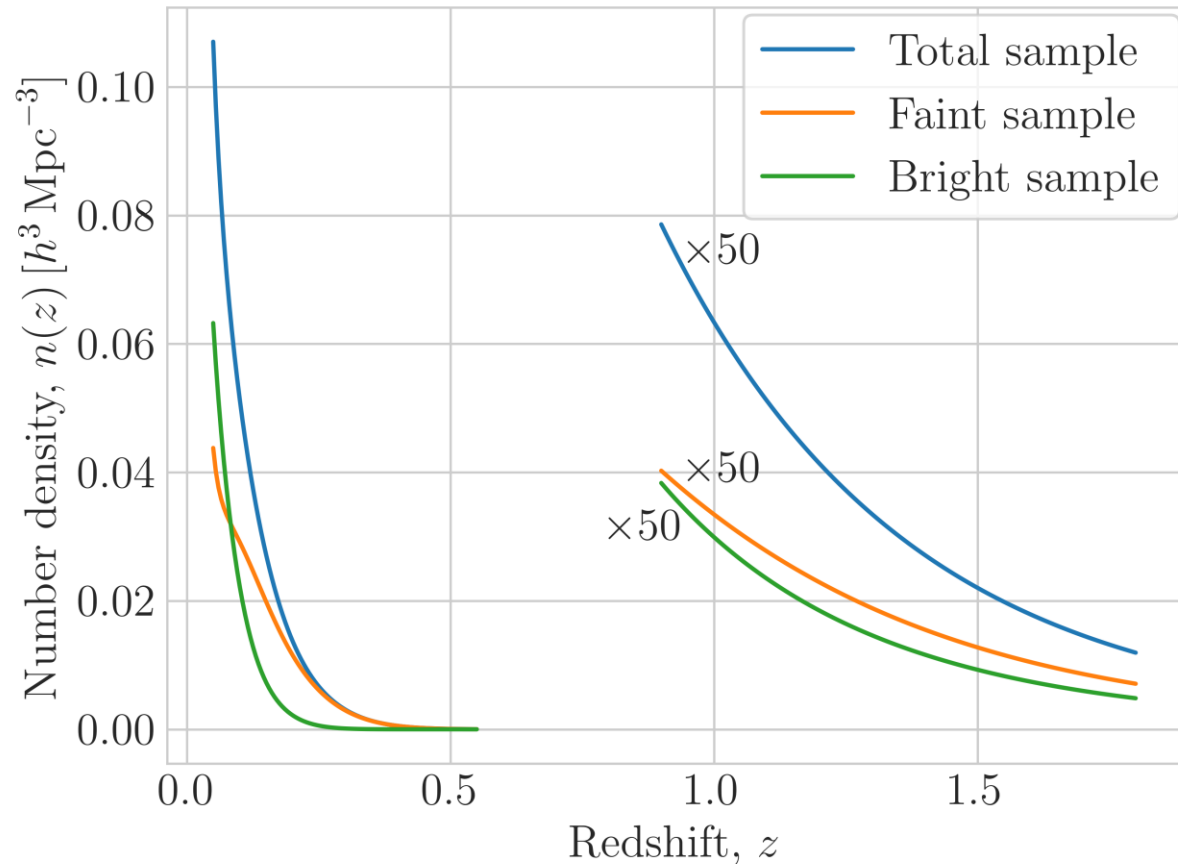
$$F > F_c$$

- **Faint sample (F):** all the galaxies with

$$F_c < F < F_s$$

- **Bright sample (B):** all the galaxies with

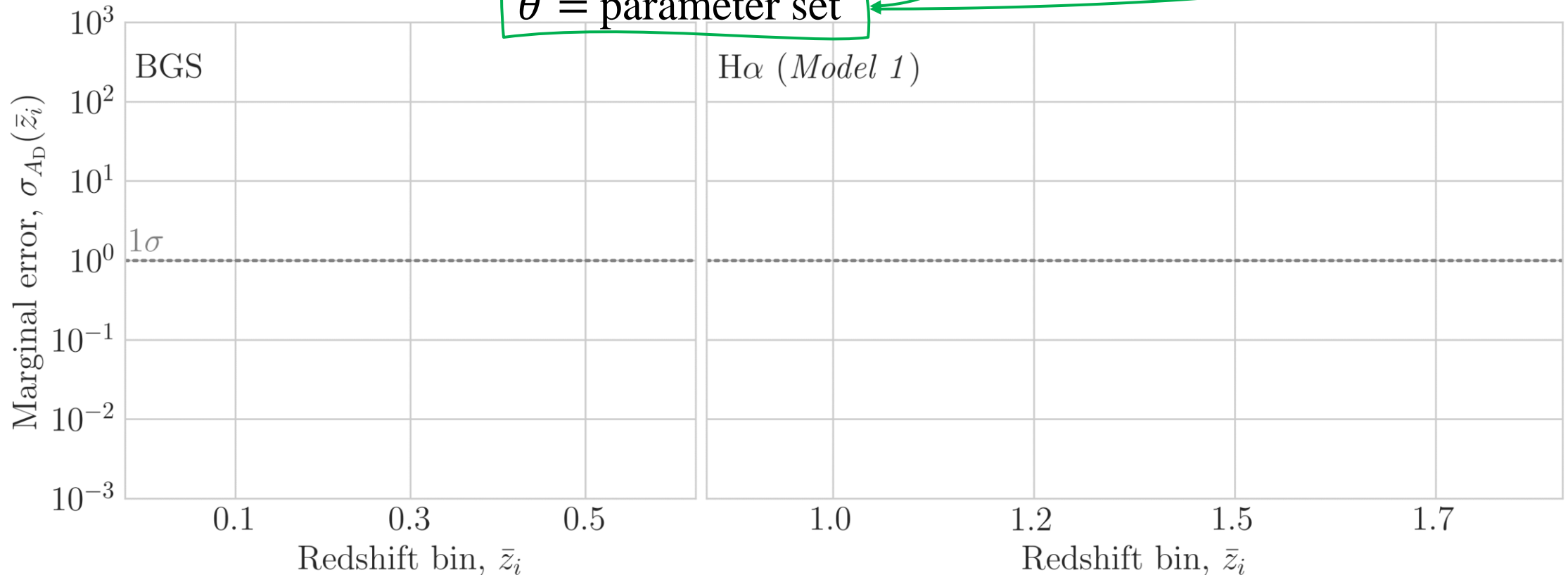
$$F > F_s$$



Information matrix analysis

$$P = \begin{pmatrix} P_{FF} \\ P_{FB} \\ P_{BB} \end{pmatrix}, \quad I_{\alpha\beta}(z_i) = \sum_{m,n} \frac{\partial P(z_i, \mu_m, k_n)^H}{\partial \theta_{(\alpha)}} \Gamma^{-1} \frac{\partial P(z_i, \mu_n, k_m)}{\partial \theta_{(\beta)}}$$

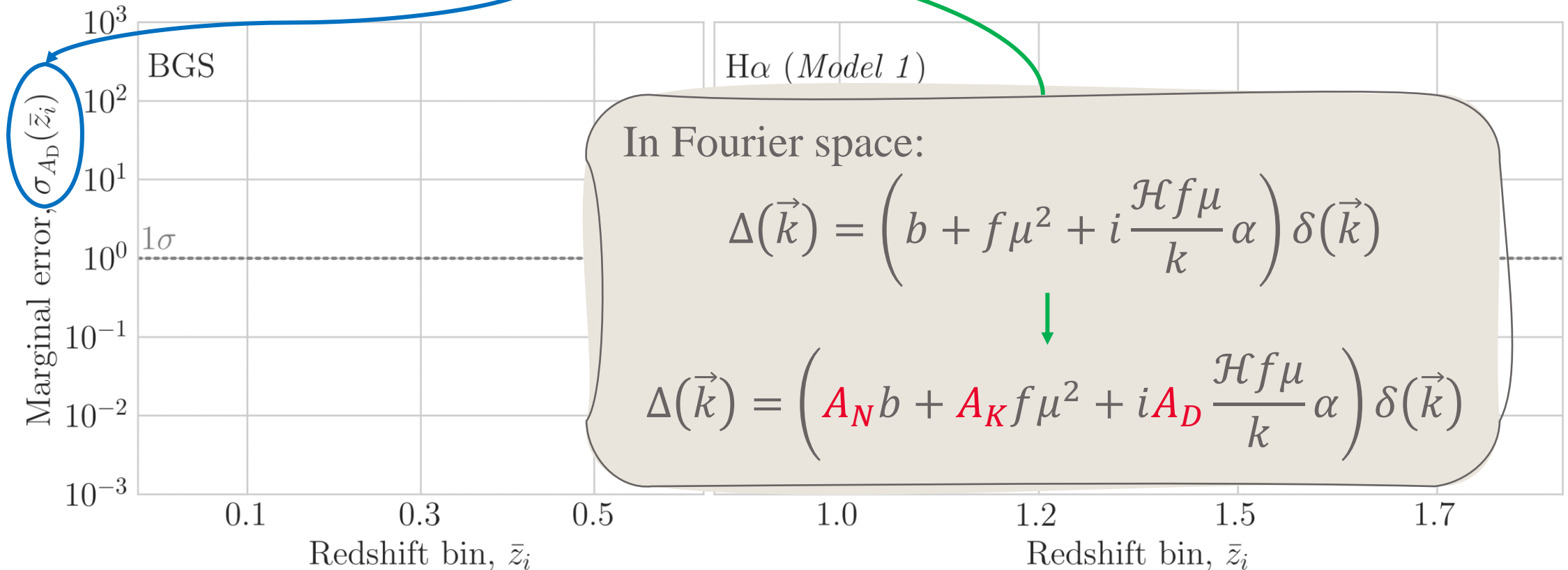
$\theta =$ parameter set



- H α emitters, *Model 3* luminosity function [Pozzetti et al. (2016)]:
 - $F_c = 2 \times 10^{-16} \text{ erg cm}^{-2} \text{ s}^{-1}$
 - $\Delta z \sim 0.23$

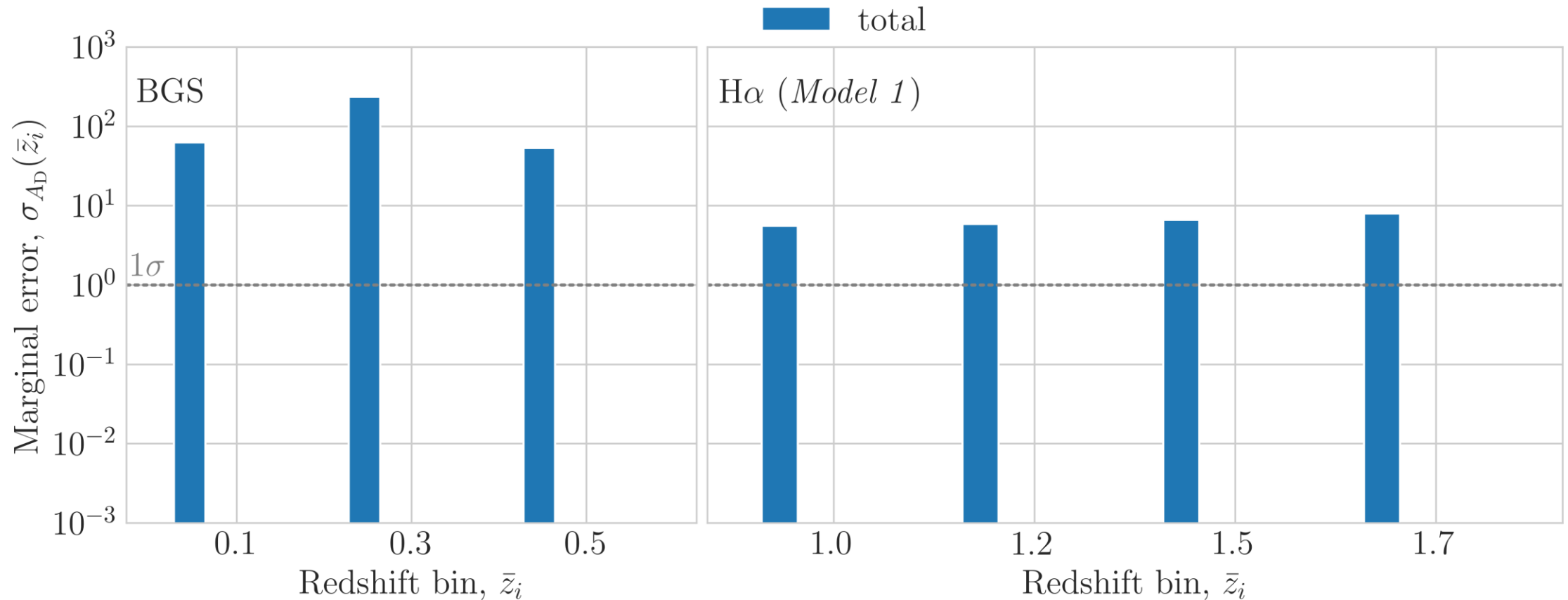
- BGS:
 - $m_c = 20.175$
 - $\Delta z \sim 0.17$
 - $f_{sky} = 0.36$

$$\theta_\alpha = \{A_N, A_K, A_D\}$$



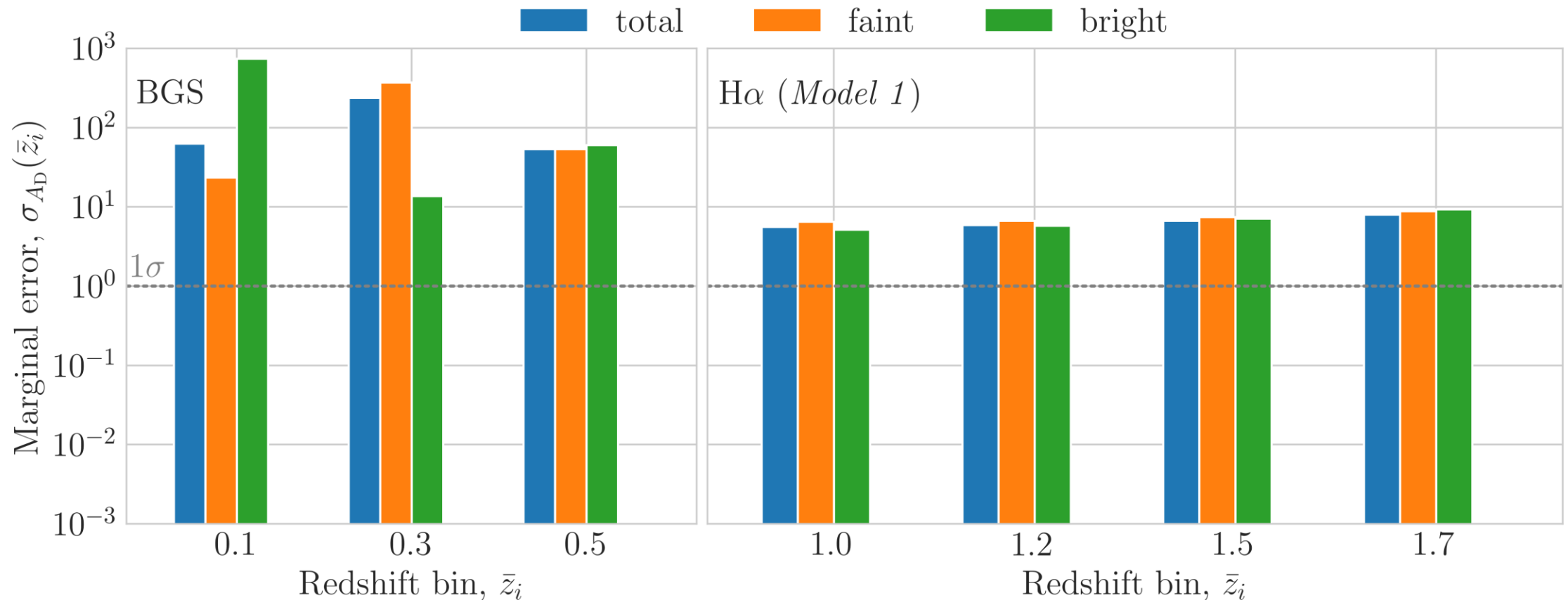
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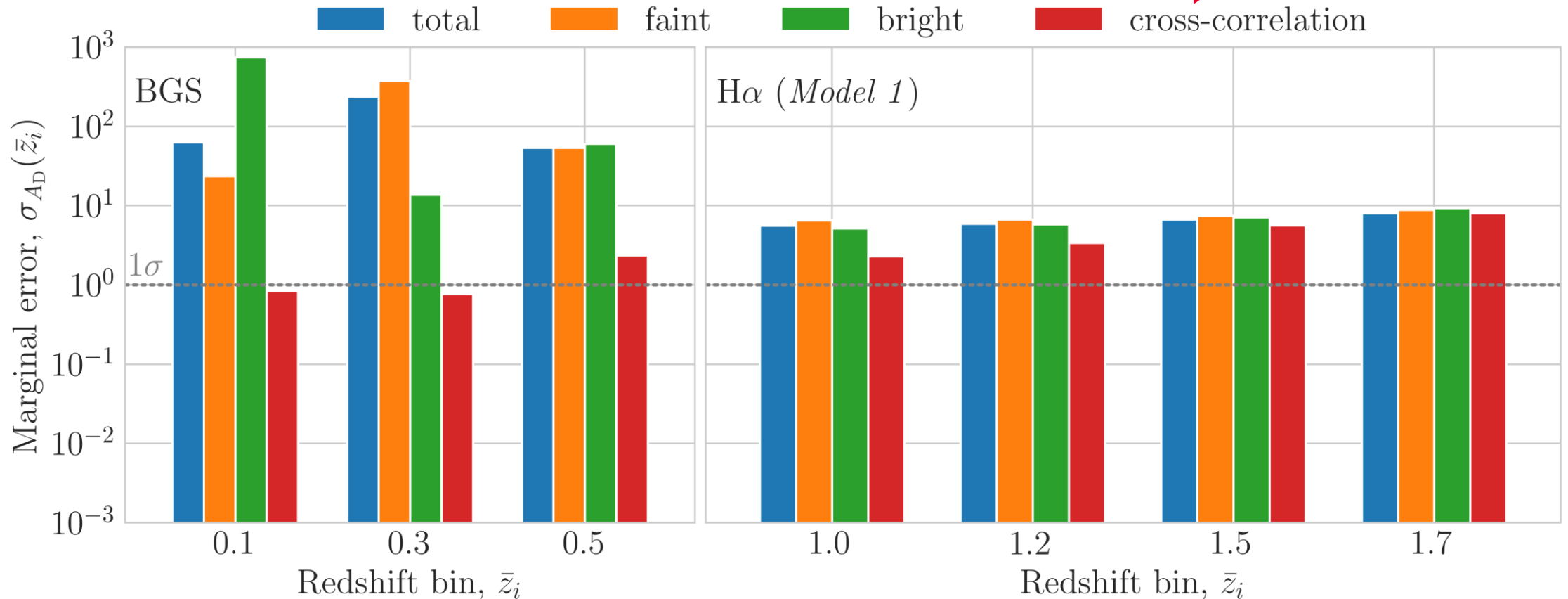
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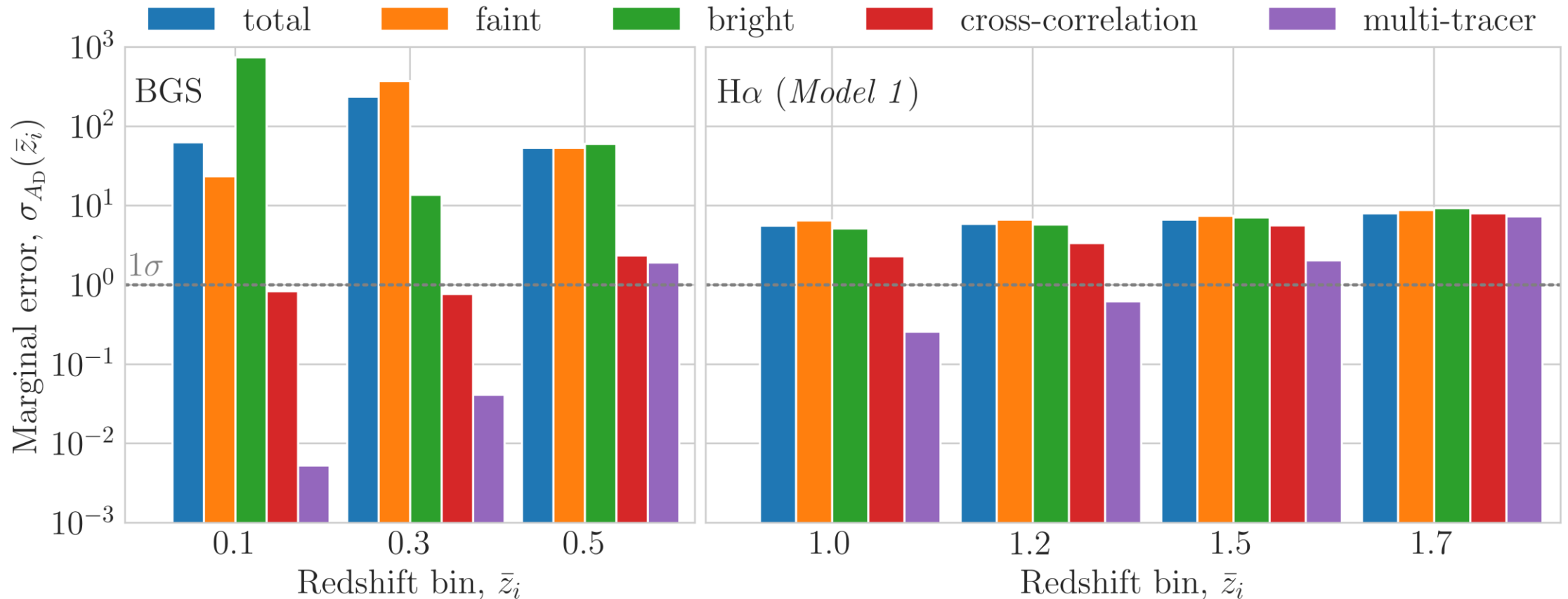
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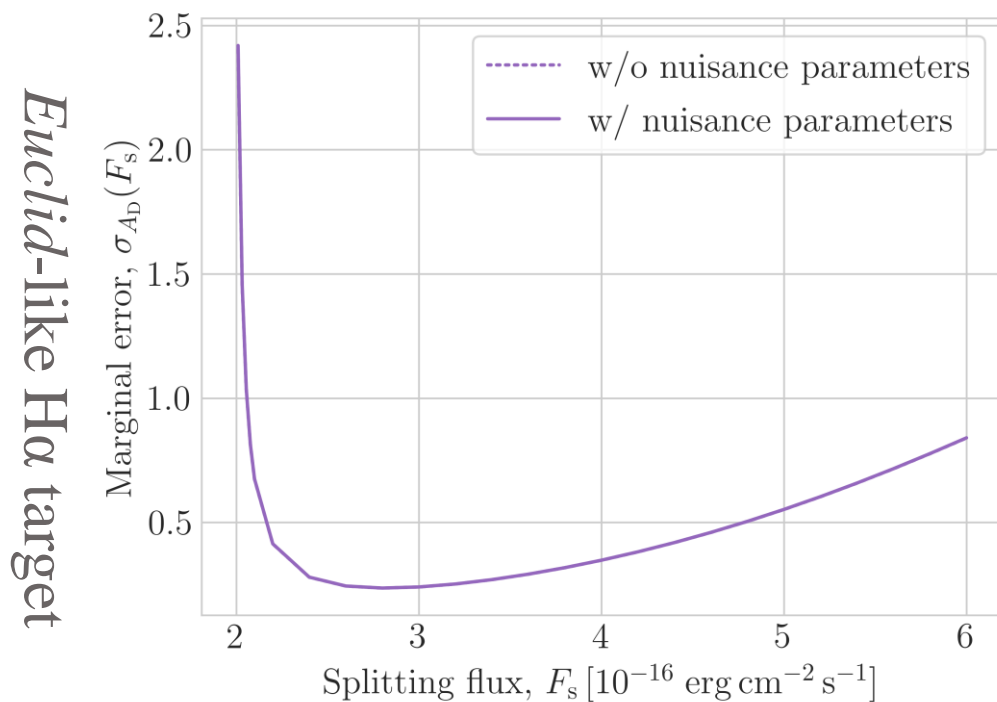
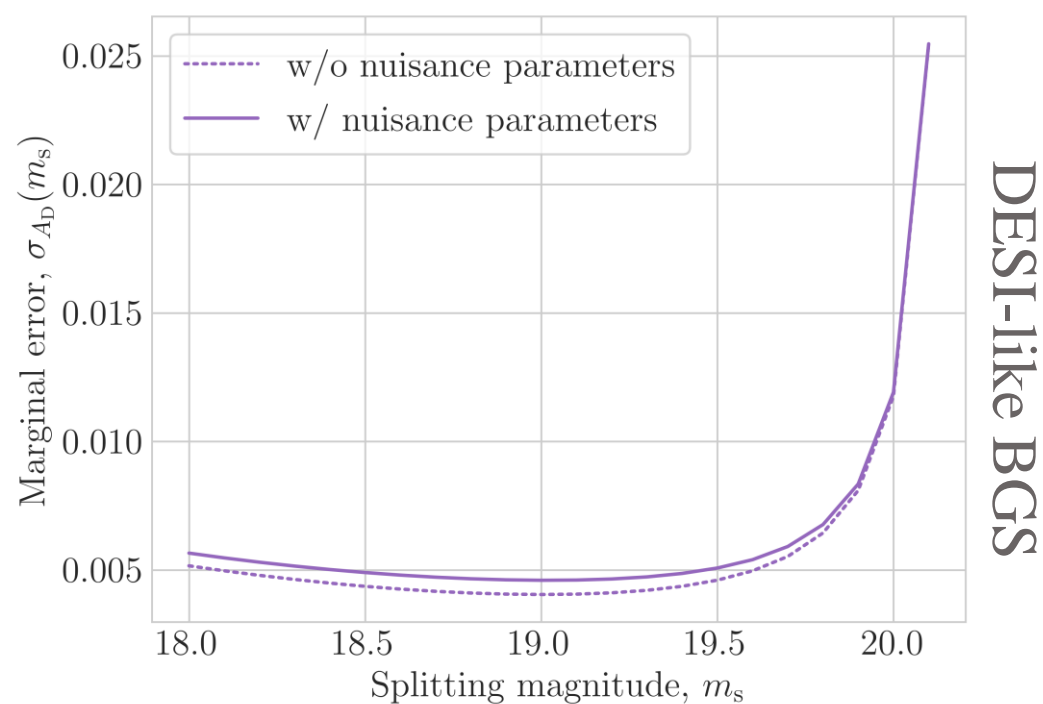


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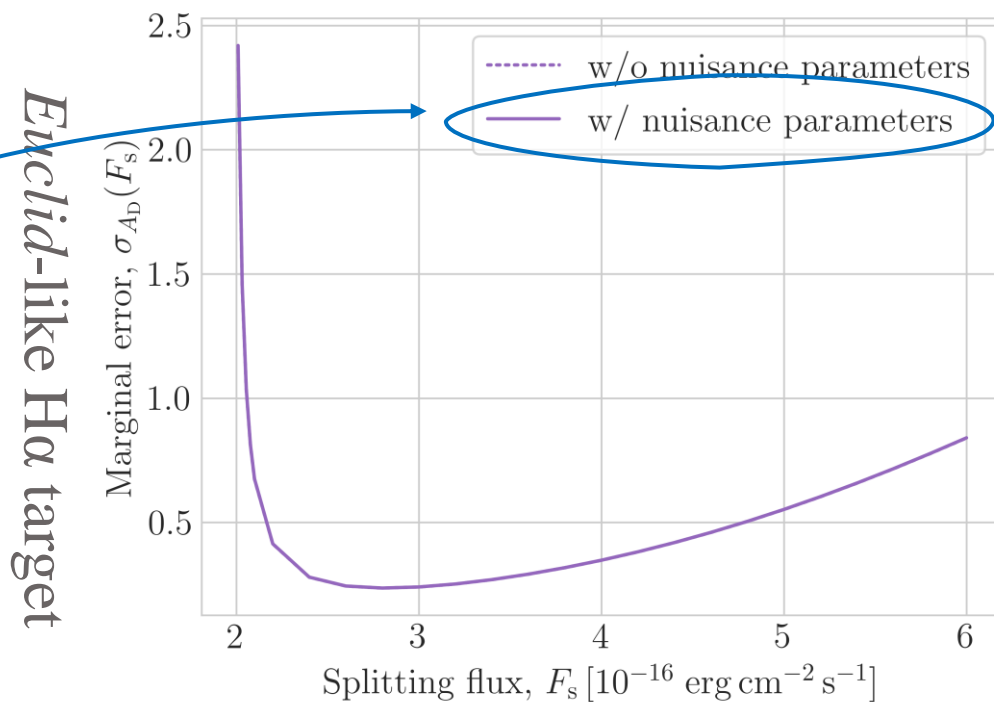
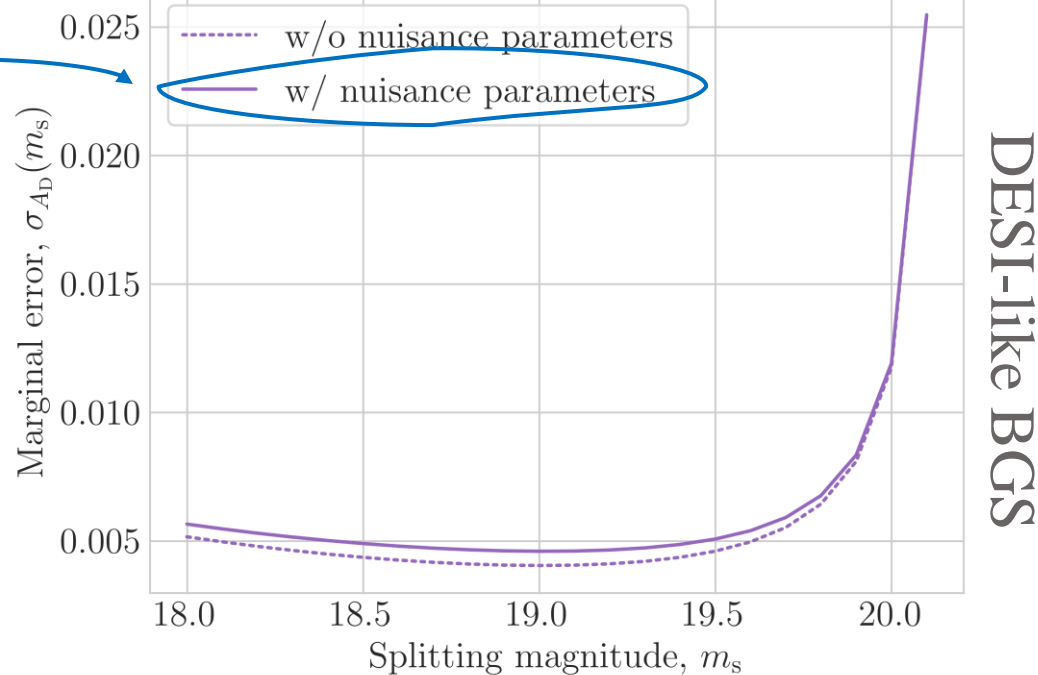


We can study how the probability of detecting a relativistic contribution depends upon the splitting flux adopted.



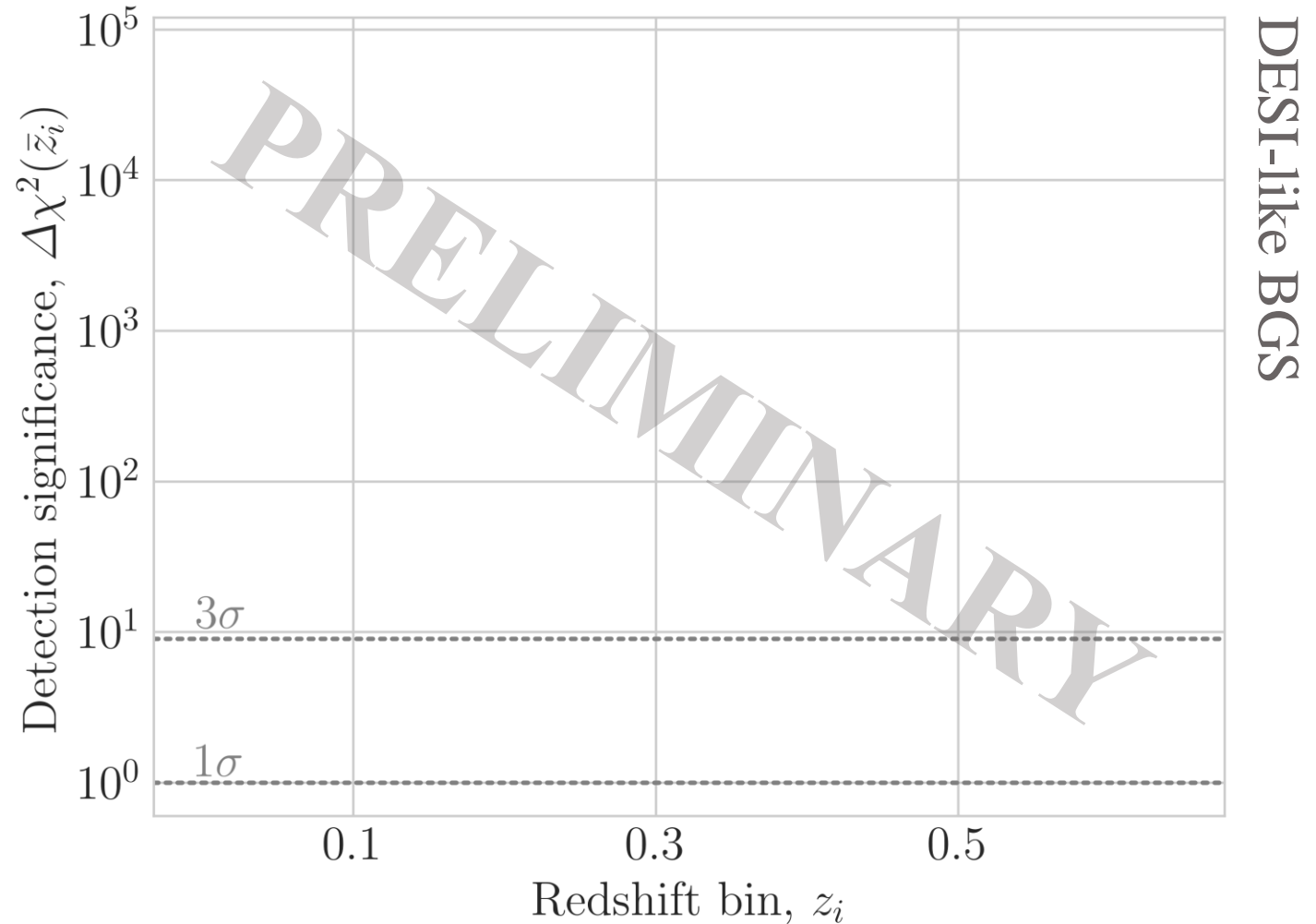
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$$\theta_\alpha = \left\{ A_N, A_K, A_D, \left\{ N_{FF}^{(i)} \right\}, \left\{ N_{FB}^{(i)} \right\}, \left\{ N_{BB}^{(i)} \right\} \right\}$$



What about multiple splits?

Can we further increase the signal by considering more than 2 sub-samples?



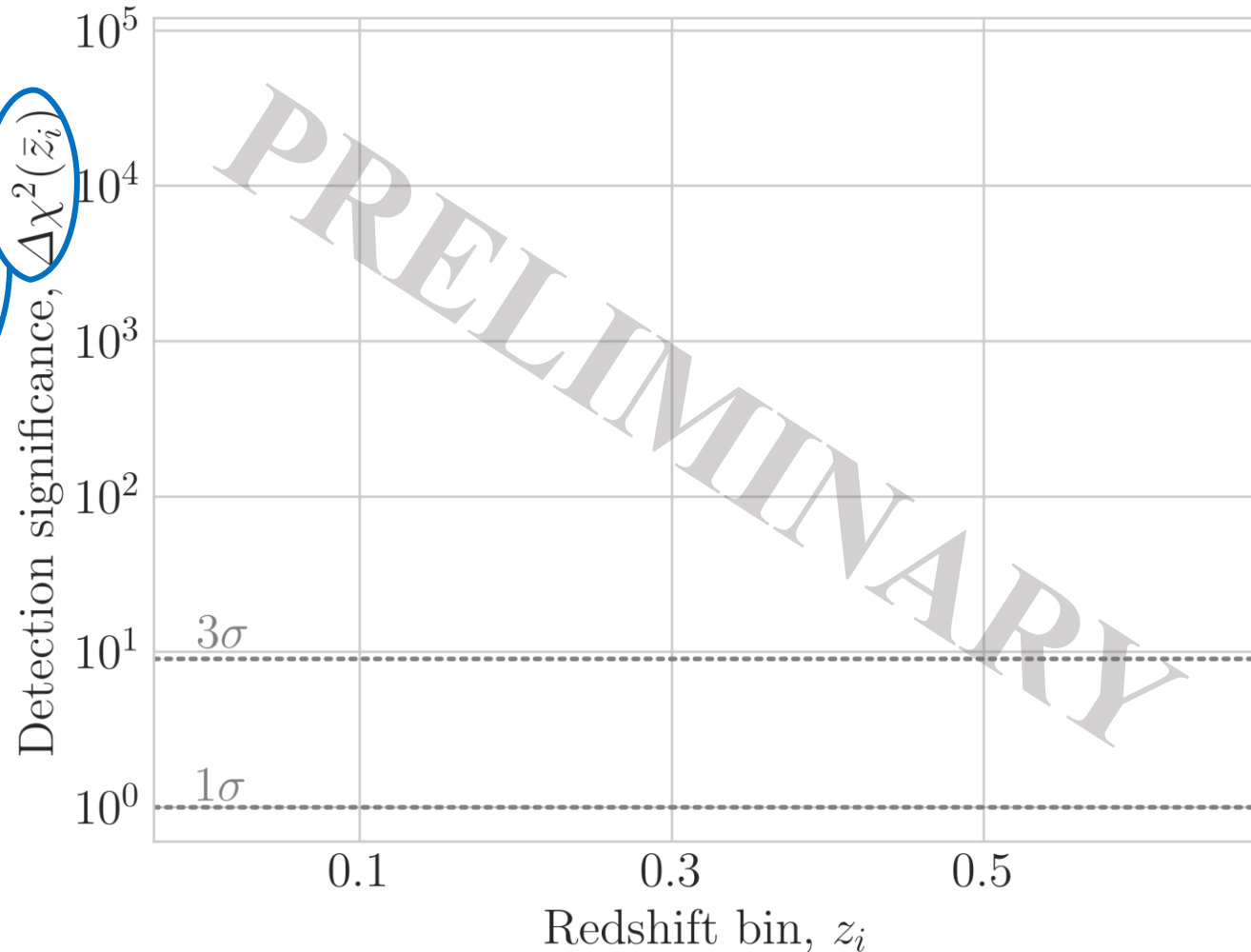
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Detection significance analysis

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$$\Delta\chi^2(\bar{z}_i) = \sum_{k,\mu} \Delta\mathbf{P}^H \Gamma^{-1} \Delta\mathbf{P}$$

$\Delta\mathbf{P}$ computed with a null-hypothesis of no Doppler



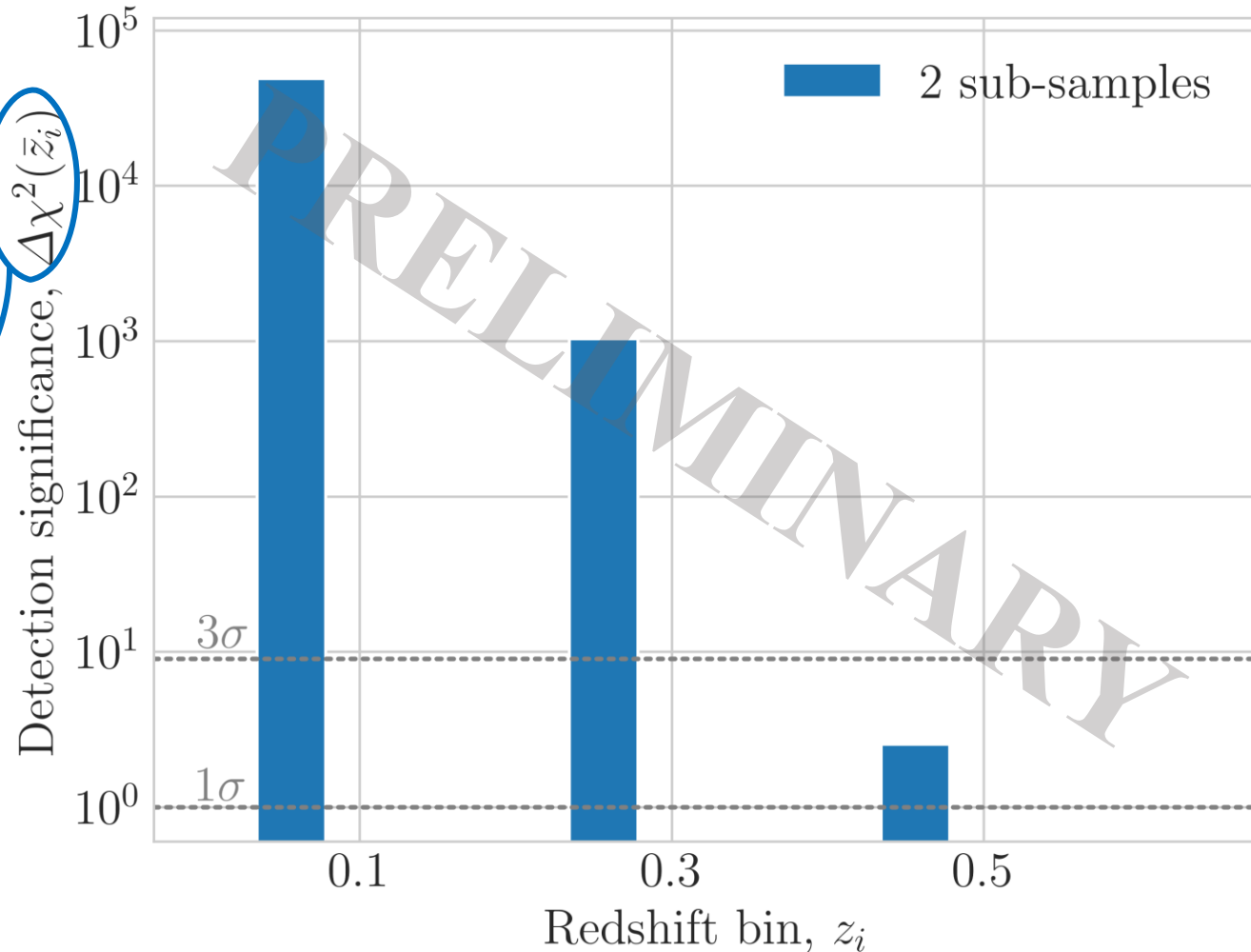
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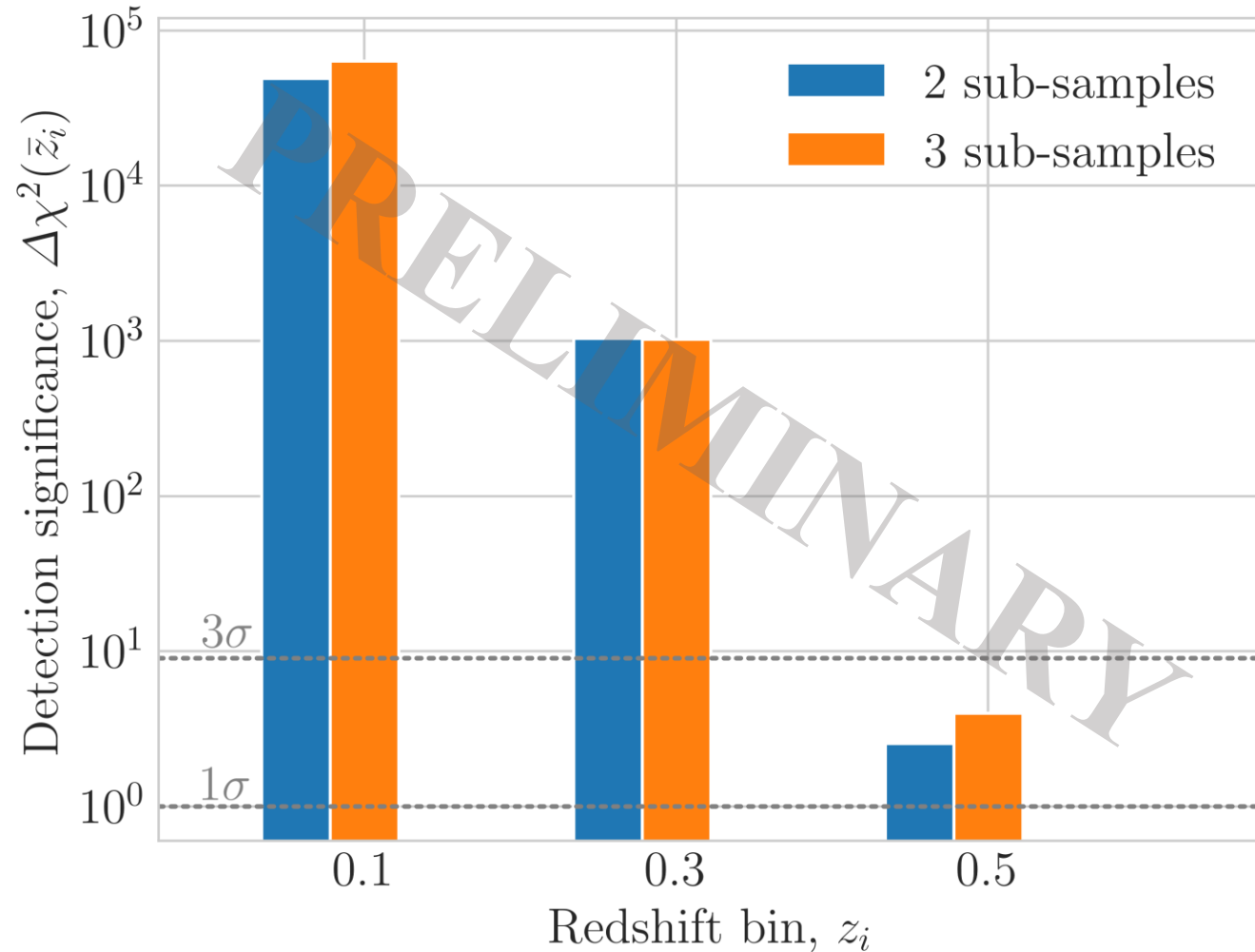


DESI-like BGS

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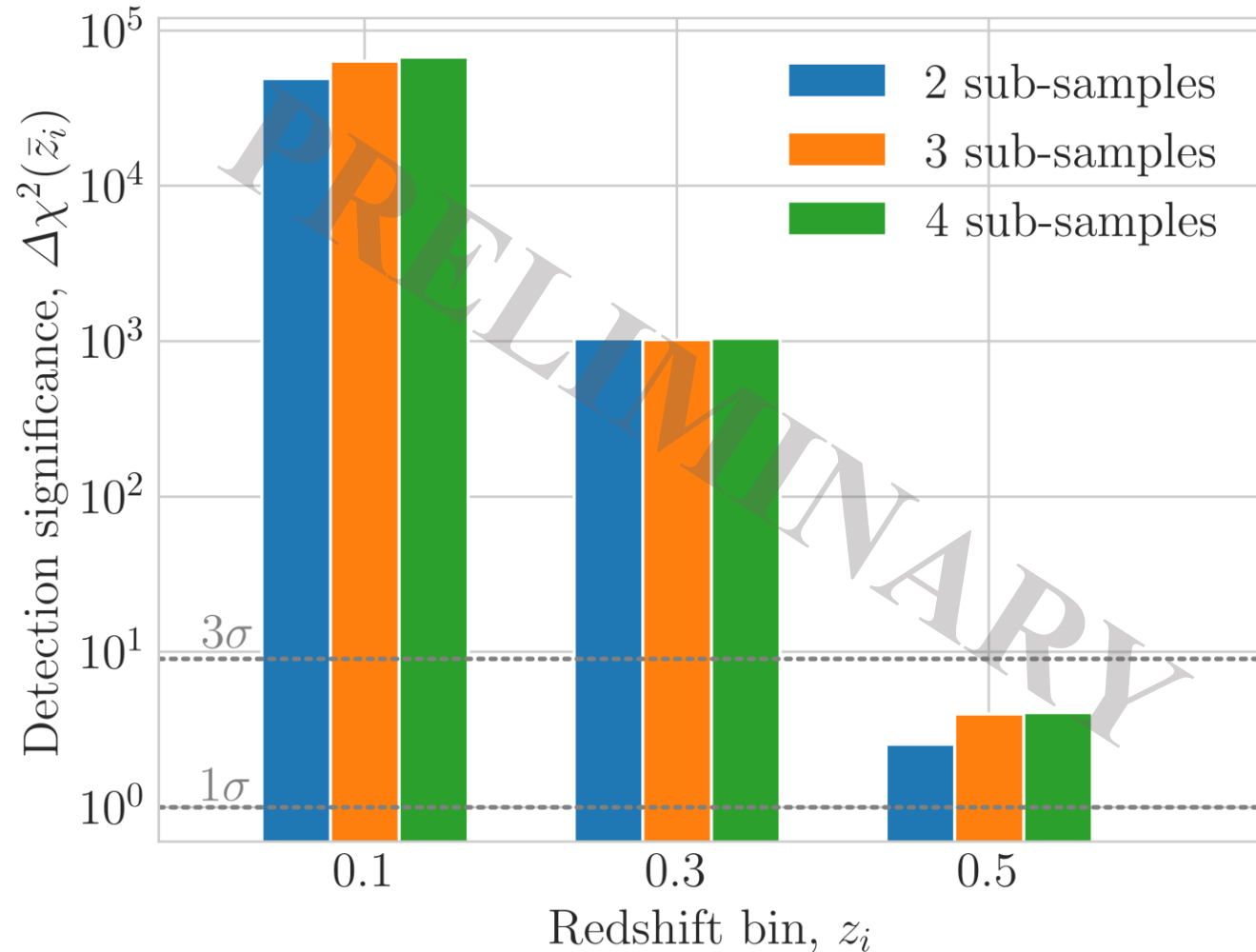


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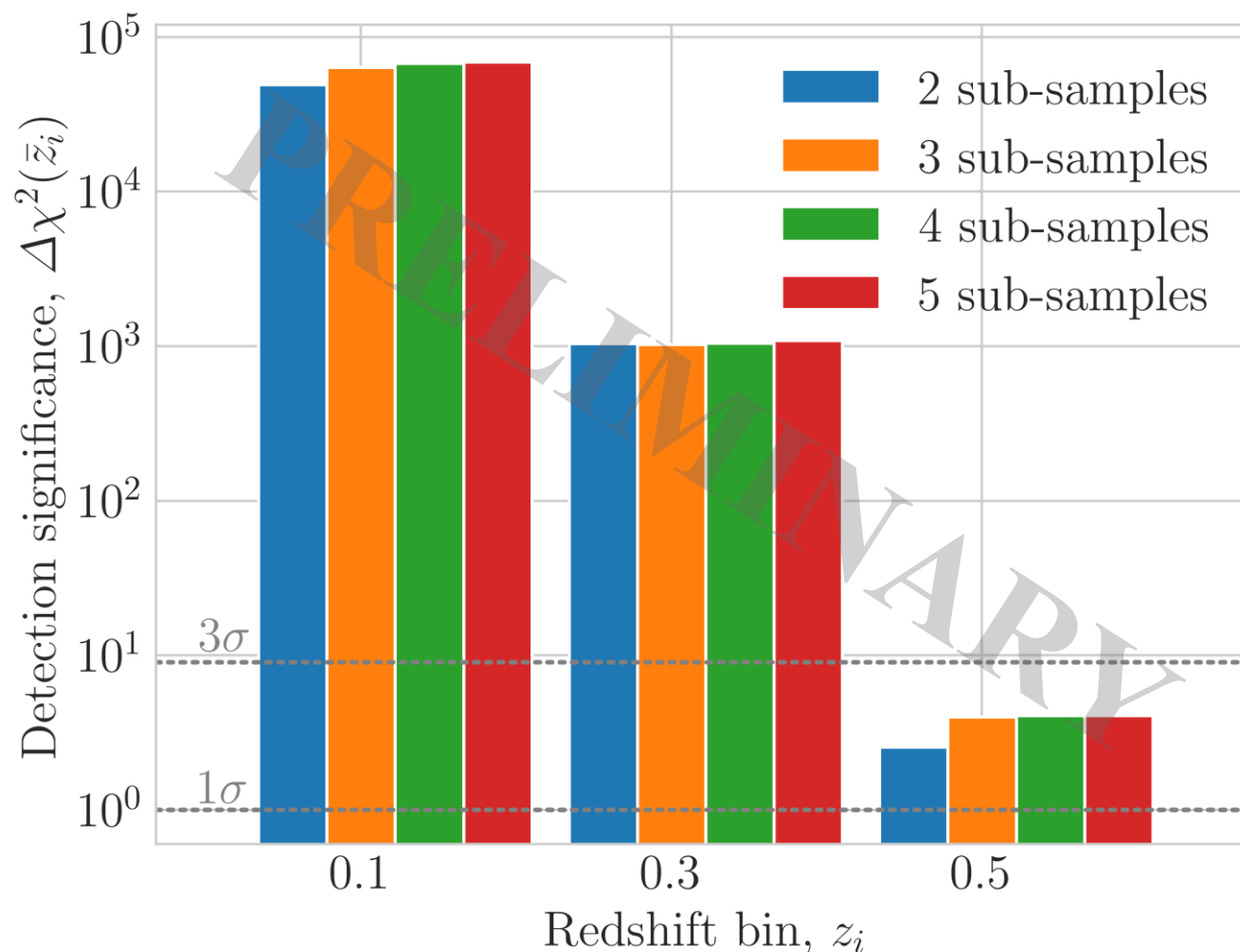
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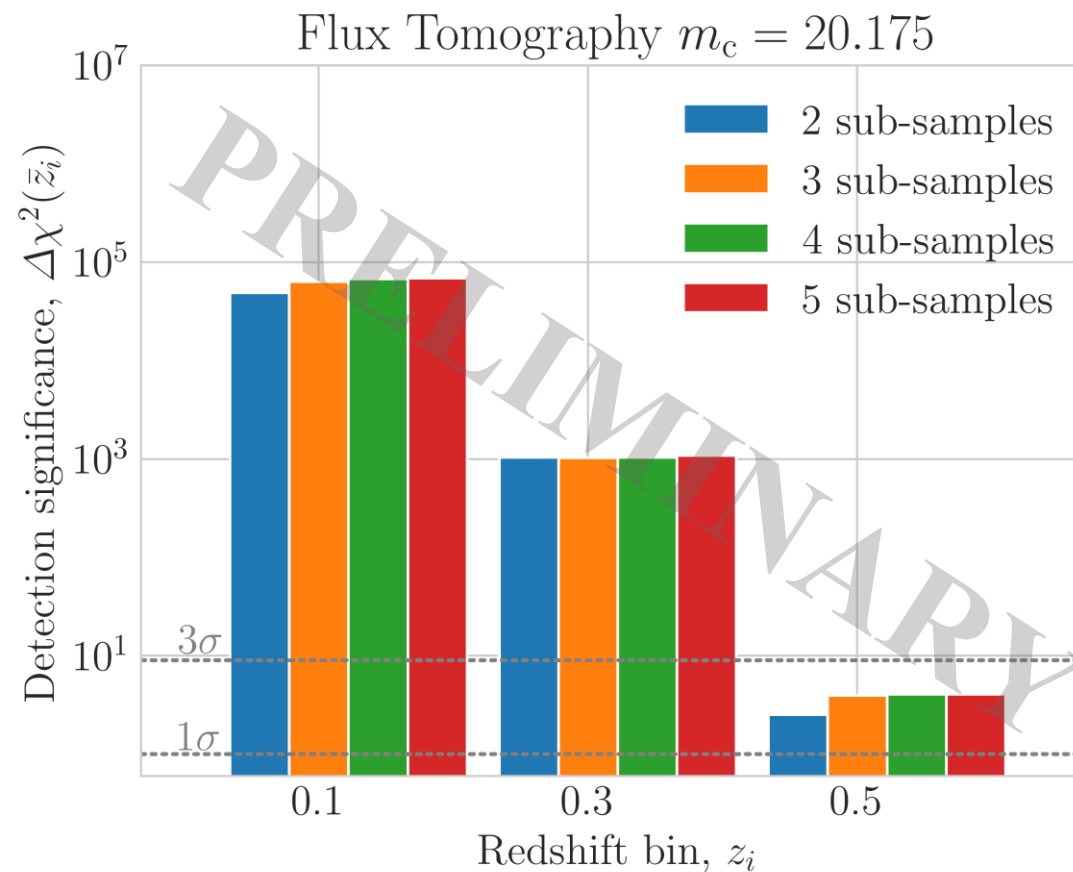
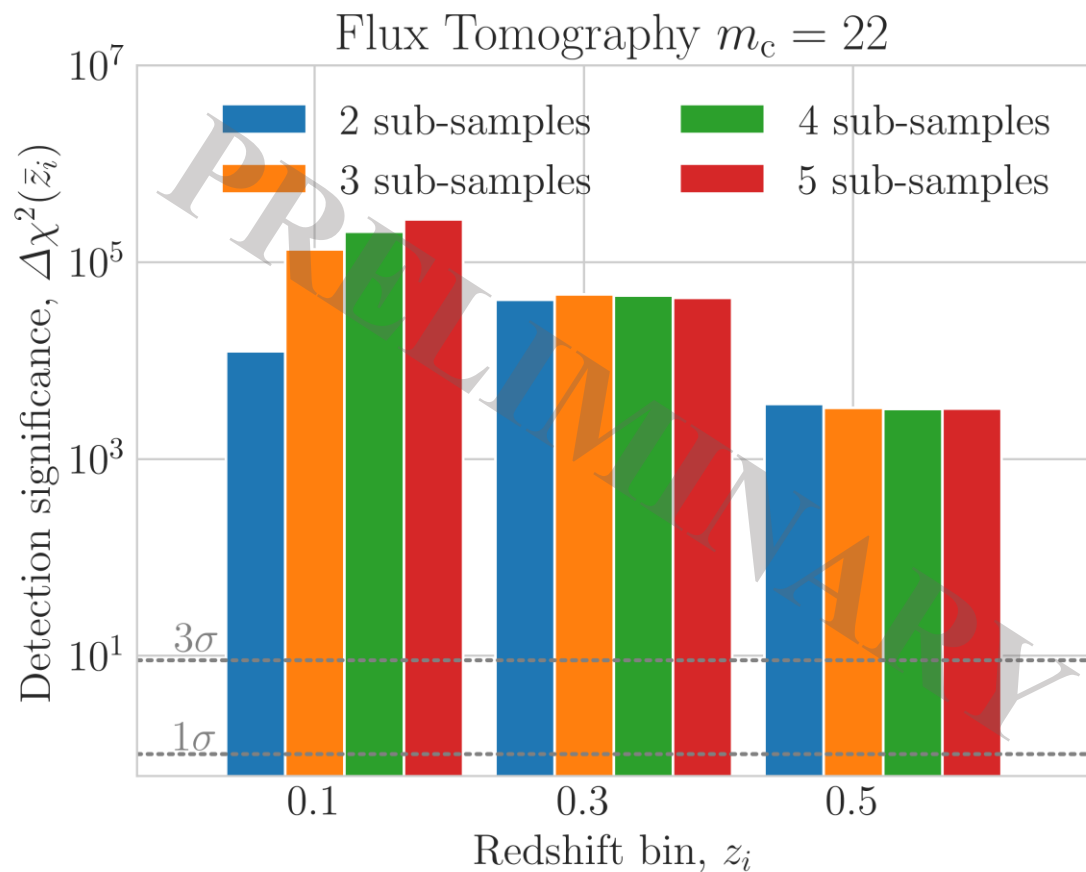
We seem to be going towards a **saturation** of the information we can extract from a single galaxy population



DESI-like BGS

What about multiple splits?

Detection significance analysis



What about integrated effects?

[Marco Novara, FM & S. Camera, (2024 TBS)]

Effects included in the angular power spectrum analysis

$$\Delta_l = \Delta_l^N + \Delta_l^{Doppler} + \Delta_l^{lensing} + \Delta_l^{GR}$$

[Castorina & Di Dio (2022)]

$$\begin{aligned} \Delta(\mathbf{n}, z) = & b_1 D_m + \mathcal{H}^{-1} \partial_r v_{\parallel} \\ & + \frac{5s_b - 2}{2} \int_0^r dr' \frac{r - r'}{rr'} \Delta_{\Omega} (\Psi + \Phi) \\ & + \mathcal{R} (v_{\parallel} - v_{\parallel o}) - (2 - 5s_b) v_{\parallel o} \\ & + \left\{ \left(\mathcal{R} - \frac{2 - 5s_b}{\mathcal{H}_0 r} \right) \mathcal{H}_0 V_o + (\mathcal{R} + 1) \Psi - \mathcal{R} \Psi_o + (5s_b - 2) \Phi + \dot{\Phi} \mathcal{H}^{-1} \right. \\ & \quad \left. + (f_{\text{evo}} - 3) \mathcal{H} V \right\} \\ & + \frac{2 - 5s_b}{r} \int_{\tau}^{\tau_o} (\Psi + \Phi) d\tau' + \mathcal{R} \int_{\tau}^{\tau_o} (\dot{\Psi} + \dot{\Phi}) d\tau', \end{aligned}$$

where we have introduced the redshift dependent parameter

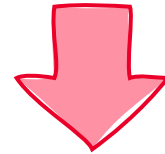
$$\mathcal{R} = 5s_b + \frac{2 - 5s_b}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f_{\text{evo}}.$$

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$$\Delta_l = \Delta_l^N + \Delta_l^{\text{Doppler}} + \Delta_l^{\text{lensing}} + \Delta_l^{\text{GR}}$$



Study of the relevance of the Doppler, local and integrated potential terms in a faint-bright multi-tracer angular power spectrum.

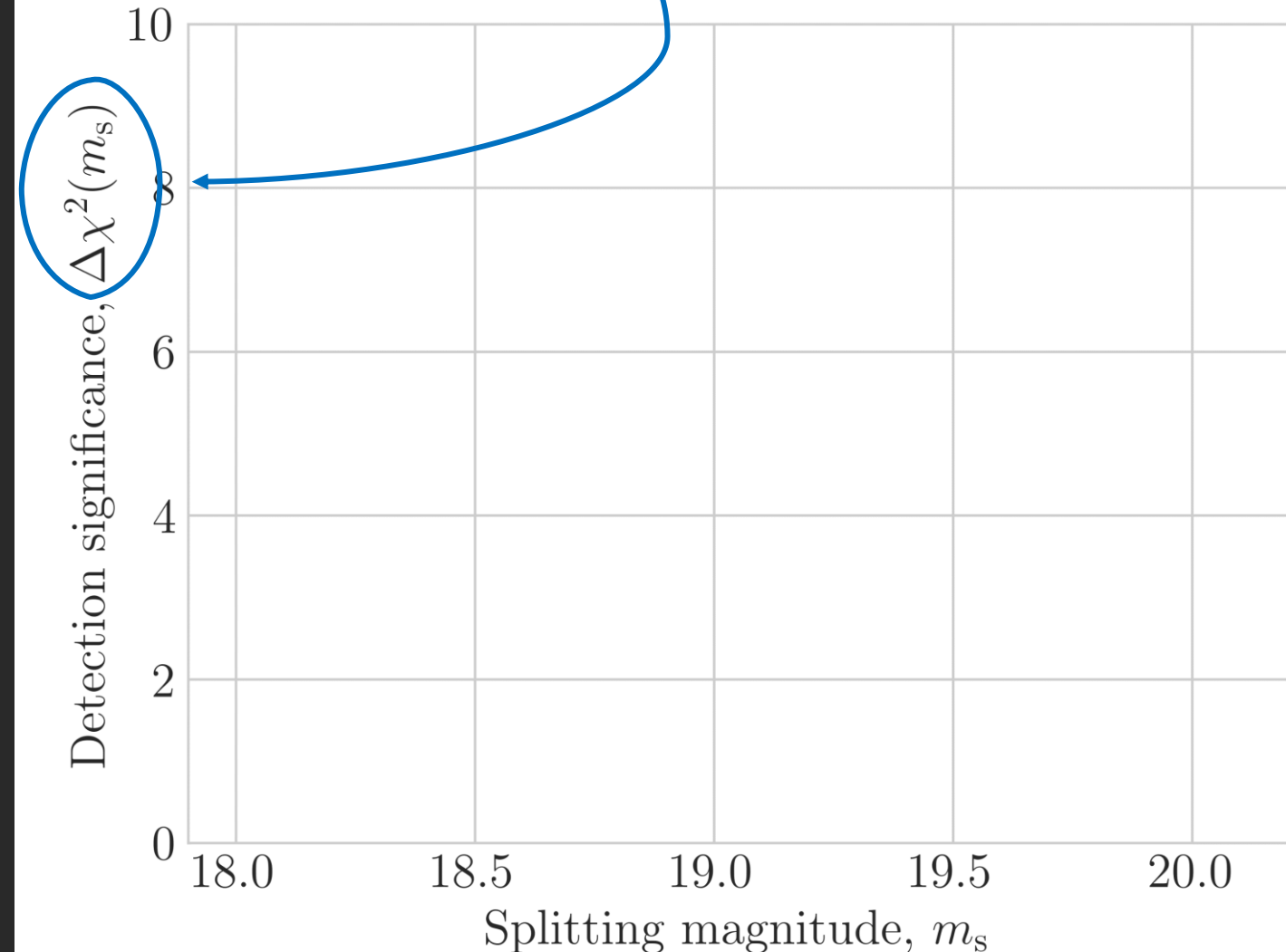
$$\Delta\chi^2 = \sum_l \text{tr}[S \Gamma^{-1} S \Gamma^{-1}]$$

$$S = (d - m)$$

$\Gamma = \text{Covariance}$

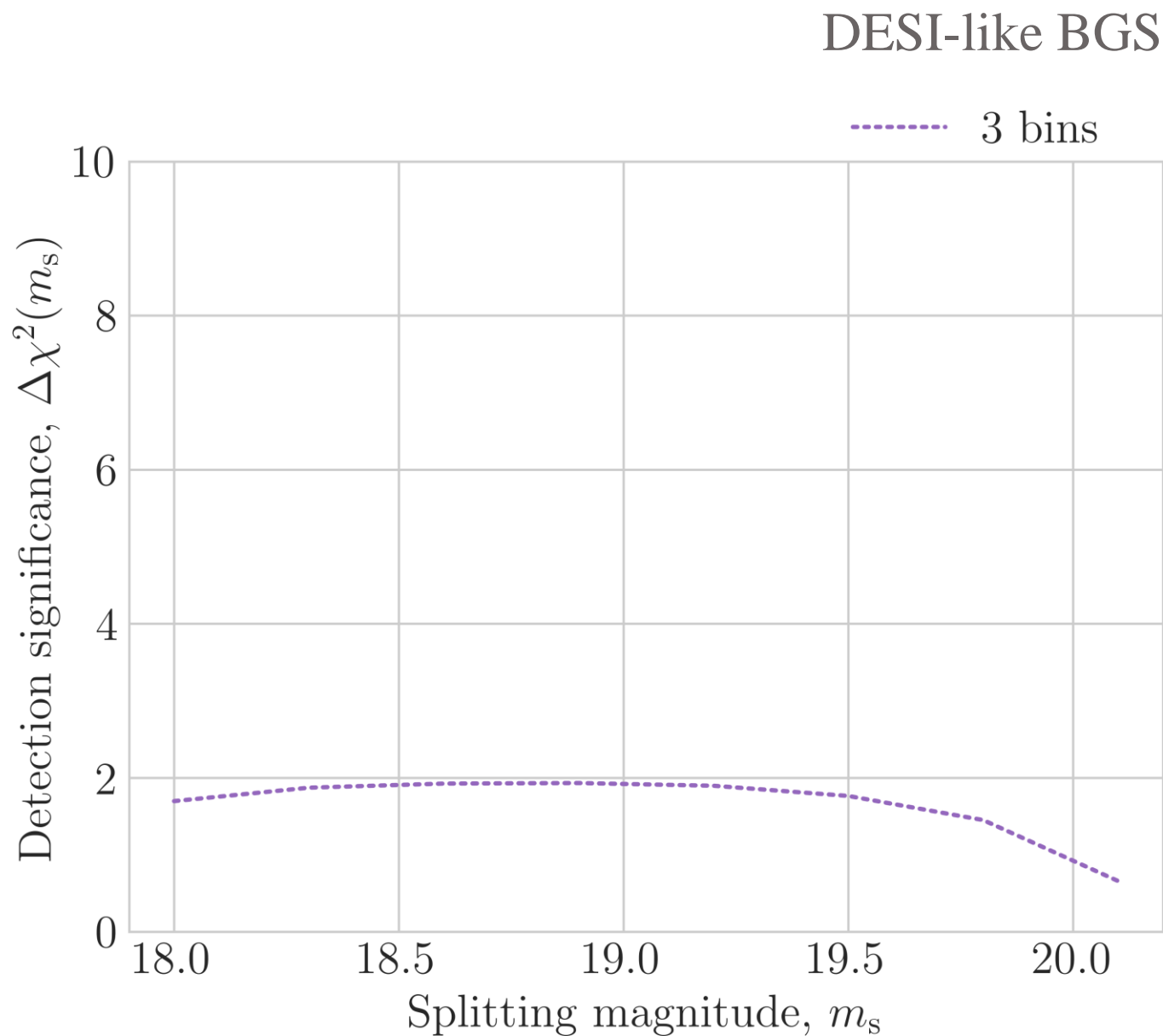
Null-hypothesis: $\Delta_l^{Doppler} = \Delta_l^{GR} = 0$

Also in harmonic space we can study how the statistical significance of the relativistic contribution depends upon the splitting flux adopted.



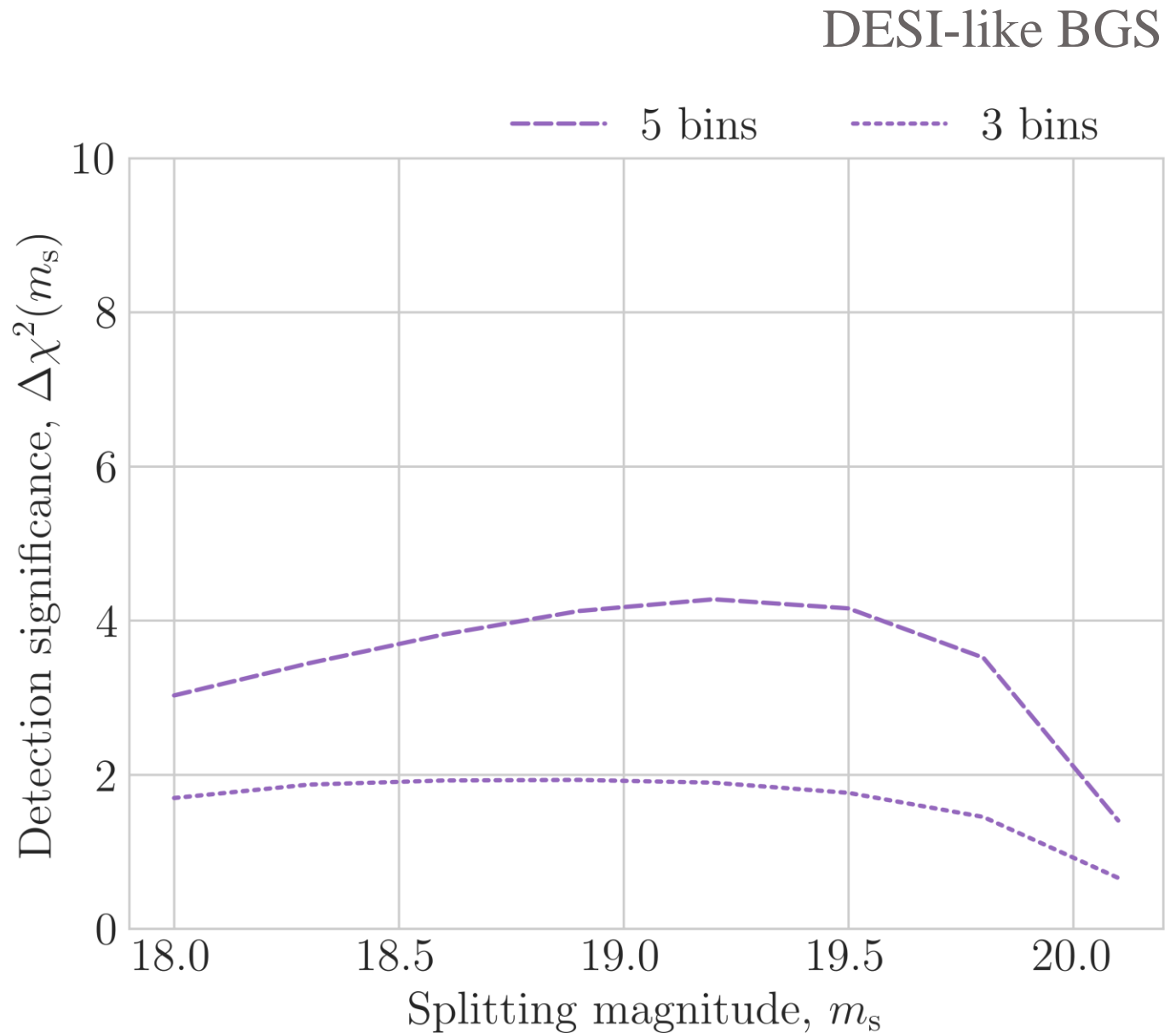
[Novara *et al.* (2024 TBS)]

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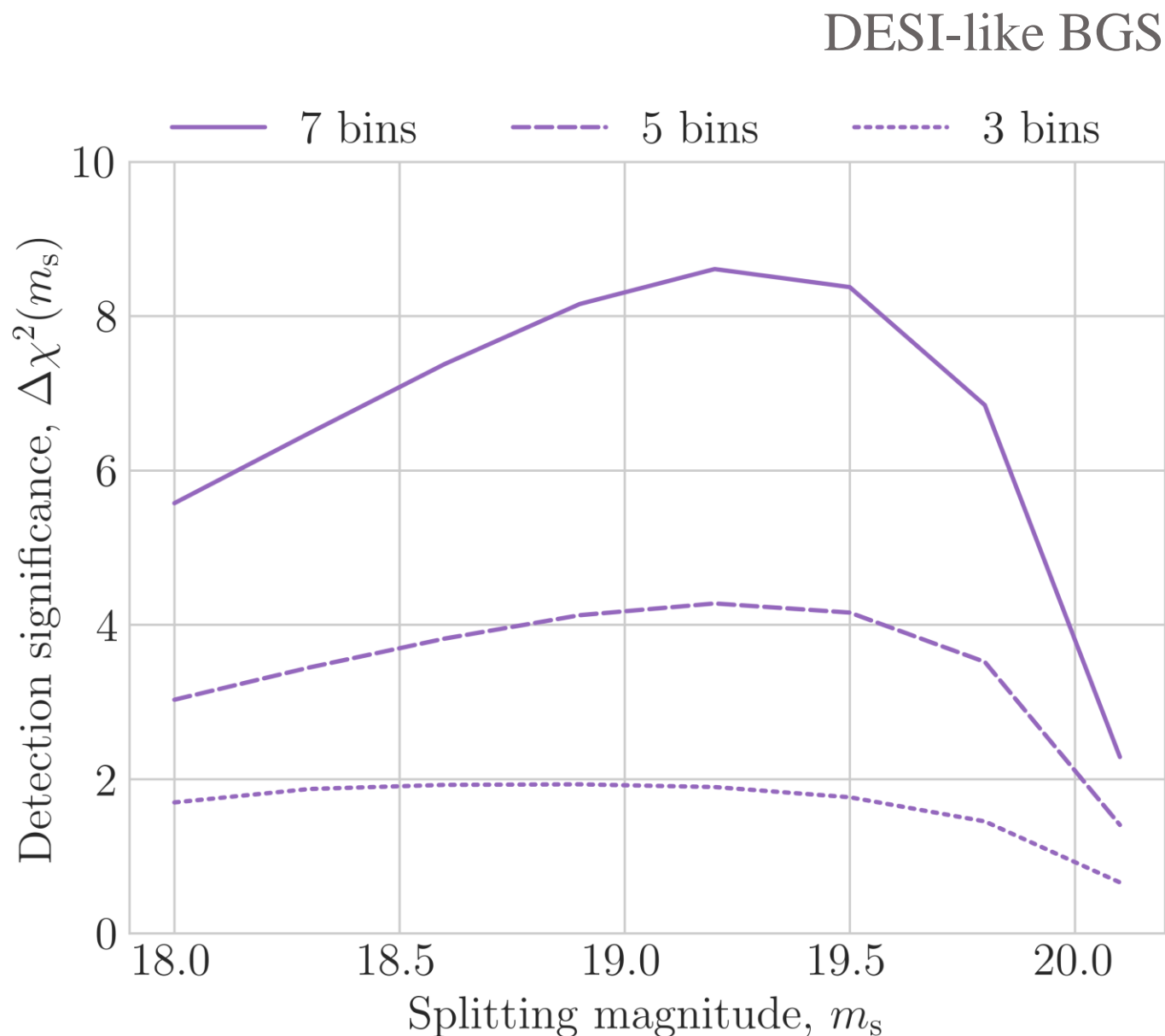
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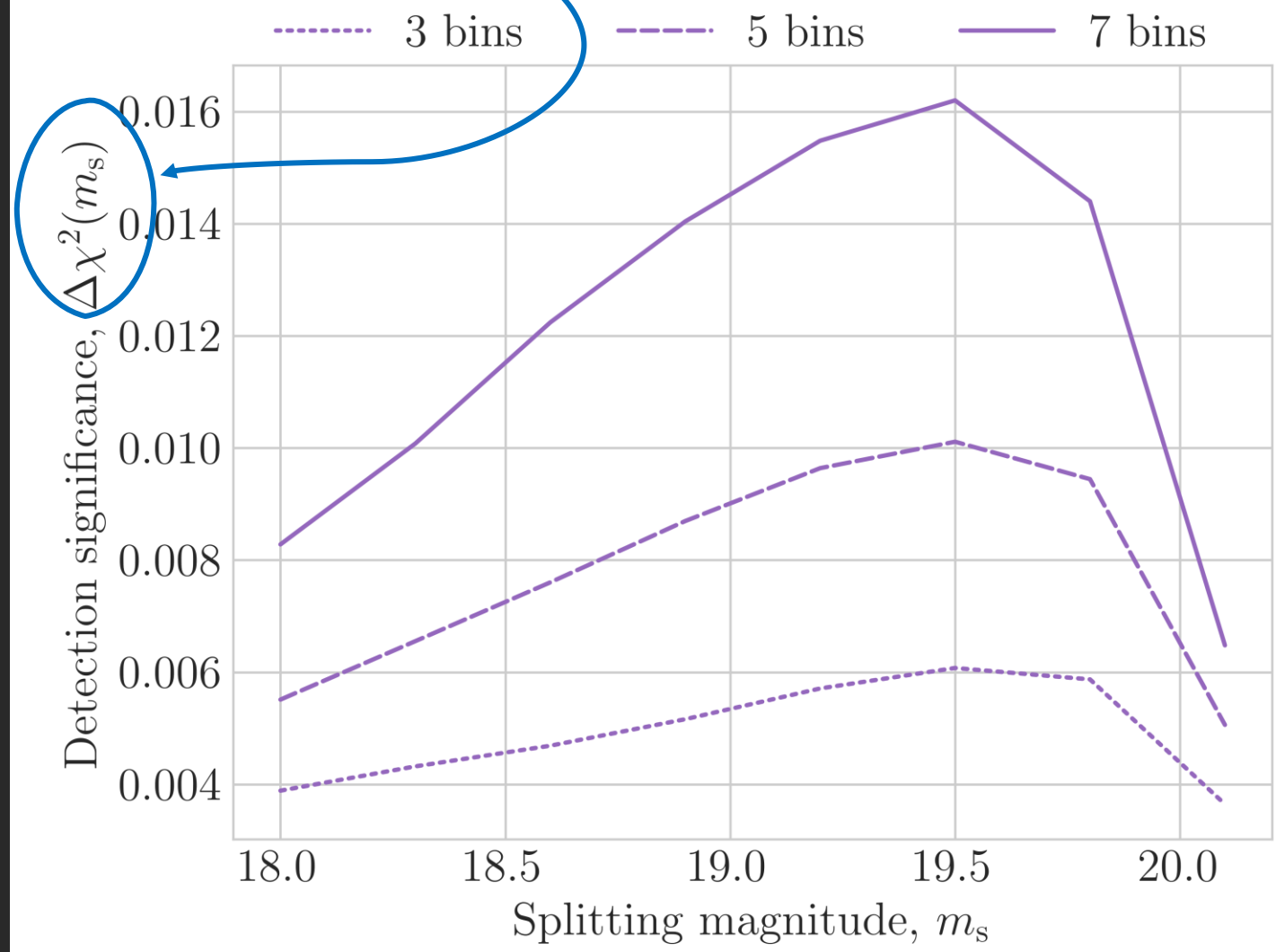


[Novara *et al.* (2024 TBS)]

DESI-like BGS

Null-hypothesis: $\Delta_l^{GR} = 0$

**Without considering
the Doppler term the
GR contribution
seems to be
undetectable**



[Novara *et al.* (2024 TBS)]

Future work

- An analysis of the performance of the luminosity cut technique using simulated data will demonstrate its reliability.
- Including wide-angle effects.

Take-home messages

- A multi-tracer approach is able to beat cosmic variance, even within a single dataset.
- Thanks to the increased sensitivity and the enhanced volume the upcoming galaxy surveys will shed light on the largest scales of the universe.



**Thanks for your
attention!**



Backup slides

Relativistic galaxy number counts

In Fourier space, our assumptions give us:

$$\Delta(\vec{k}) = \mathcal{Z}^{(1)}(\vec{k})\delta(\vec{k})$$

$$\mathcal{Z}_N^{(1)}(k, \mu) = b + f\mu^2$$

$$\mathcal{Z}_{GR}^{(1)}(k, \mu) = i \frac{\mathcal{H}}{k} \alpha f \mu$$

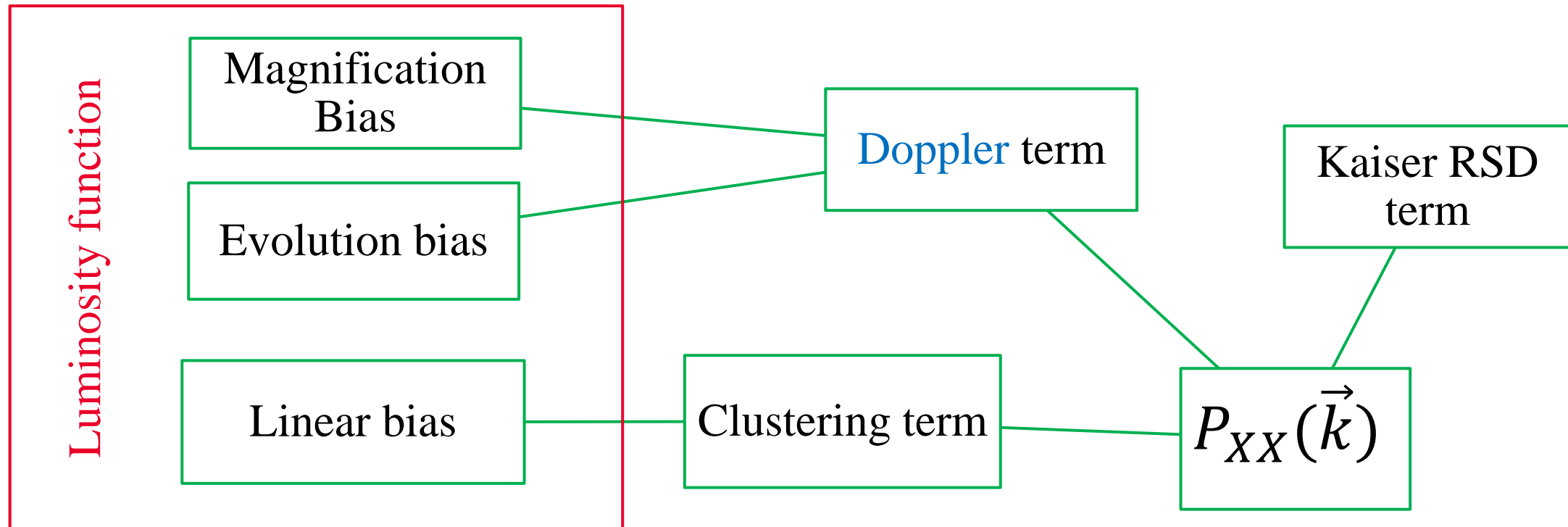
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$$\mathcal{R} = 5s_b + \frac{2 - 5s_b}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f_{\text{evo}}.$$

Relativistic Doppler in galaxy power spectra

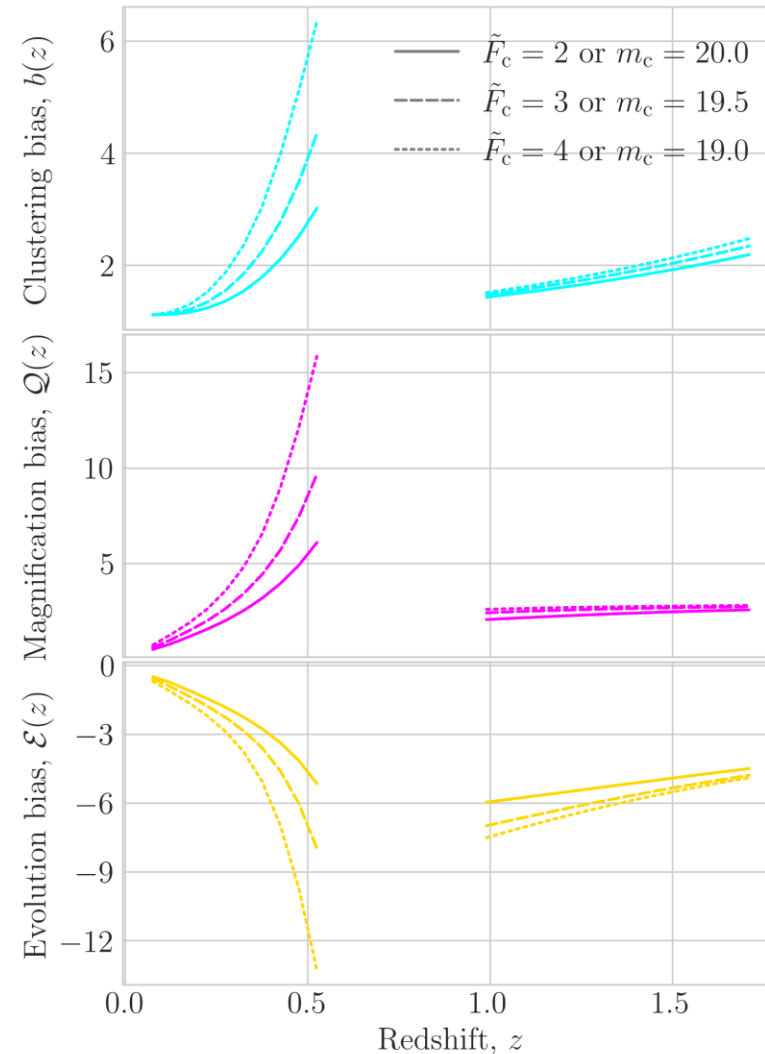


A sample optimisation work is required.

Linear bias in the case of multiple targets

The clustering bias for the faint population can be written as [Ferracho *et al.* (2014)]:

$$b_F = \frac{n_T b_T - n_B b_B}{n_F}$$



Magnification evolution biases for the sub-samples

Biases for the bright sample can be easily obtained from those of the total sample by substituting $F_c \rightarrow F_s$.

$$Q_B = - \frac{\partial \ln(n_B)}{\partial \ln(L_S)}$$
$$\mathcal{E}_B = - \frac{\partial \ln(n_B)}{\partial \ln(1+z)}$$

In the case of the faint sample, we have instead to consider the upper cut [Bonvin et al. (2023)].

$$Q_F = - \frac{\partial \ln(n_F)}{\partial \ln(L_C)} + \frac{\partial \ln(n_F)}{\partial \ln(L_S)}$$
$$\mathcal{E}_F = - \frac{\partial \ln(n_F)}{\partial \ln(1+z)}$$

Information matrix analysis

$$I_{\alpha\beta}(z_i) = \sum_{m,n} \frac{\partial P(z_i, \mu_m, k_n)^H}{\partial \theta_{(\alpha)}} \Gamma^{-1} \frac{\partial P(z_i, \mu_n, k_m)}{\partial \theta_{(\beta)}}$$

- Covariance:

$$\Gamma(z, \mu, k) = \frac{\widetilde{P}_{XX}(z, \mu, k)\widetilde{P}_{YY}(z, \mu, k) + \widetilde{P}_{XY}(z, \mu, k)\widetilde{P}_{YX}(z, \mu, k)}{N_{modes}(z, k, \mu)}$$

$$\widetilde{P}_{XY} = P_{XY} + \frac{\delta_X^Y}{n_X}$$

$$N_{modes}(z, k, \mu) = \frac{V(z, \Delta z)}{(2\pi)^3} 2\pi k^2 \Delta k \Delta \mu$$

$$V(z, \Delta z) = \frac{4\pi f_{sky}}{3} \left[r^3 \left(z + \frac{\Delta z}{2} \right) - r^3 \left(z - \frac{\Delta z}{2} \right) \right]$$

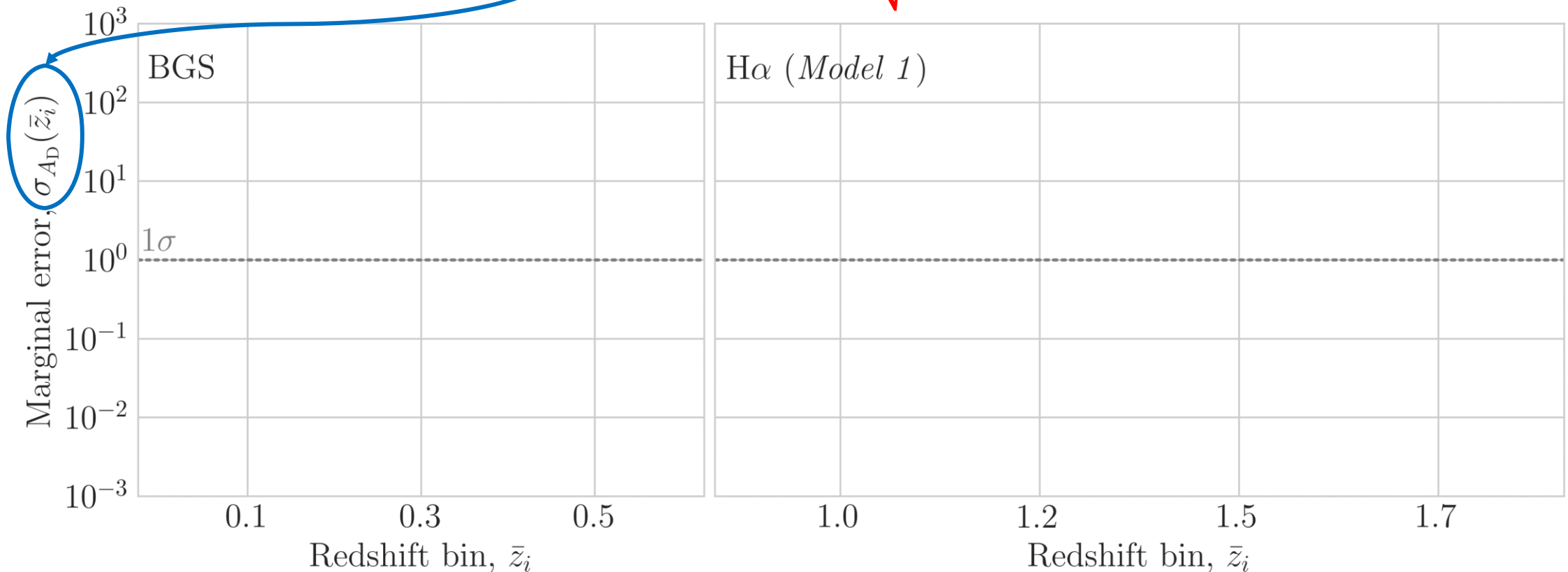
- Lowest and highest scale:

$$k_{min} = \frac{2\pi}{\sqrt[3]{V(z, \Delta z)}}, \quad k_{max} = 0.2 h \text{ Mpc}^{-1}$$

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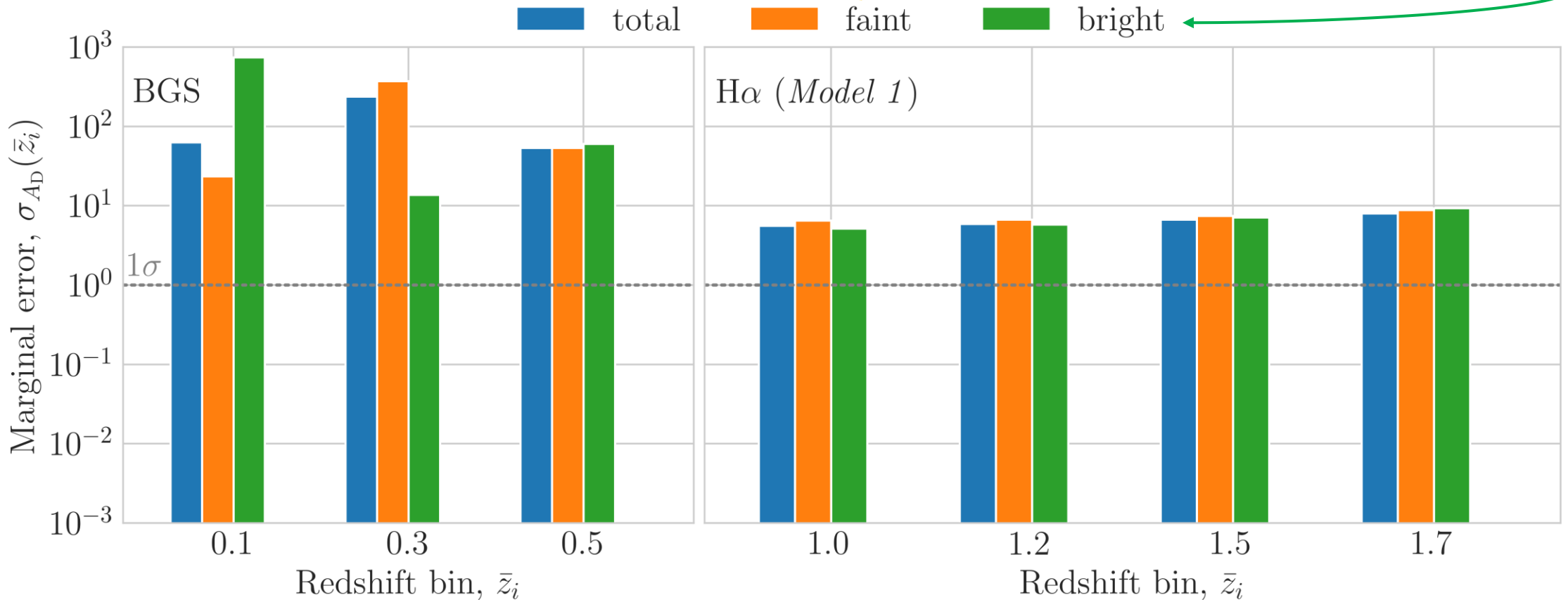
$$\theta_\alpha = \{A_N, A_K, A_D\} \rightarrow \sigma_{\theta_\alpha} = \sqrt{(I_{\alpha\alpha})^{-1}}$$



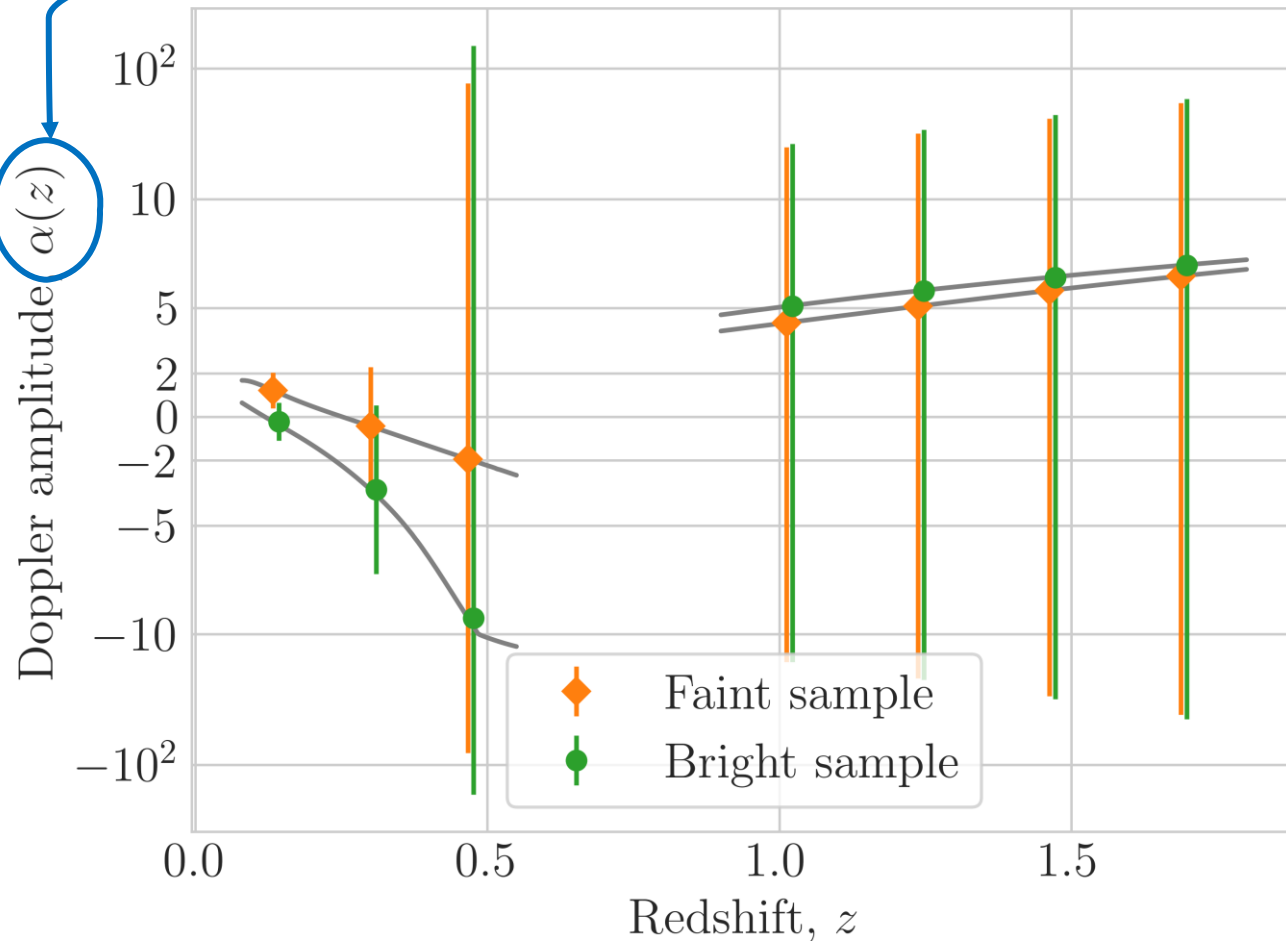
- H α emitters, *Model 3* luminosity function [Pozzetti et al. (2016)]:
 - $F_c = 2 \times 10^{-16} \text{ erg cm}^{-2} \text{ s}^{-1}$
 - $F_s = 2.8 \times 10^{-16} \text{ erg cm}^{-2} \text{ s}^{-1}$
 - $\Delta z \sim 0.23$

- BGS:
 - $m_c = 20.175$
 - m_s
 - Δz
 - $f_{sky} =$

$$\Gamma = \frac{2}{N_{\text{modes}}} \begin{pmatrix} \tilde{P}_{XX}^2 & \tilde{P}_{XX}\tilde{P}_{XY} & \tilde{P}_{XY}^2 \\ \tilde{P}_{XX}\tilde{P}_{YX} & \frac{\tilde{P}_{XX}\tilde{P}_{YY} + \tilde{P}_{XY}\tilde{P}_{YX}}{2} & \tilde{P}_{XY}\tilde{P}_{YY} \\ \tilde{P}_{YX}^2 & \tilde{P}_{YX}\tilde{P}_{YY} & \tilde{P}_{YY}^2 \end{pmatrix}$$



$$\theta_\alpha = \{b_F \sigma_8, b_B \sigma_8, f \sigma_8, \alpha_F, \alpha_B\}$$



- H α emitters, *Model 3*
luminosity function [Pozzetti et al. (2016)]:
 - $F_c = 2 \times 10^{-16} \text{ erg cm}^{-2} \text{ s}^{-1}$
 - $F_s = 2.8 \times 10^{-16} \text{ erg cm}^{-2} \text{ s}^{-1}$
 - $\Delta z \sim 0.23$
- BGS:
 - $m_c = 20.175$
 - $m_s = 19$
 - $\Delta z \sim 0.17$
- $f_{\text{sky}} = 0.36$