

# Birefringence noise – static birefringence

May 31<sup>st</sup>, 2024

*G. Zavattini, University and INFN - Ferrara*

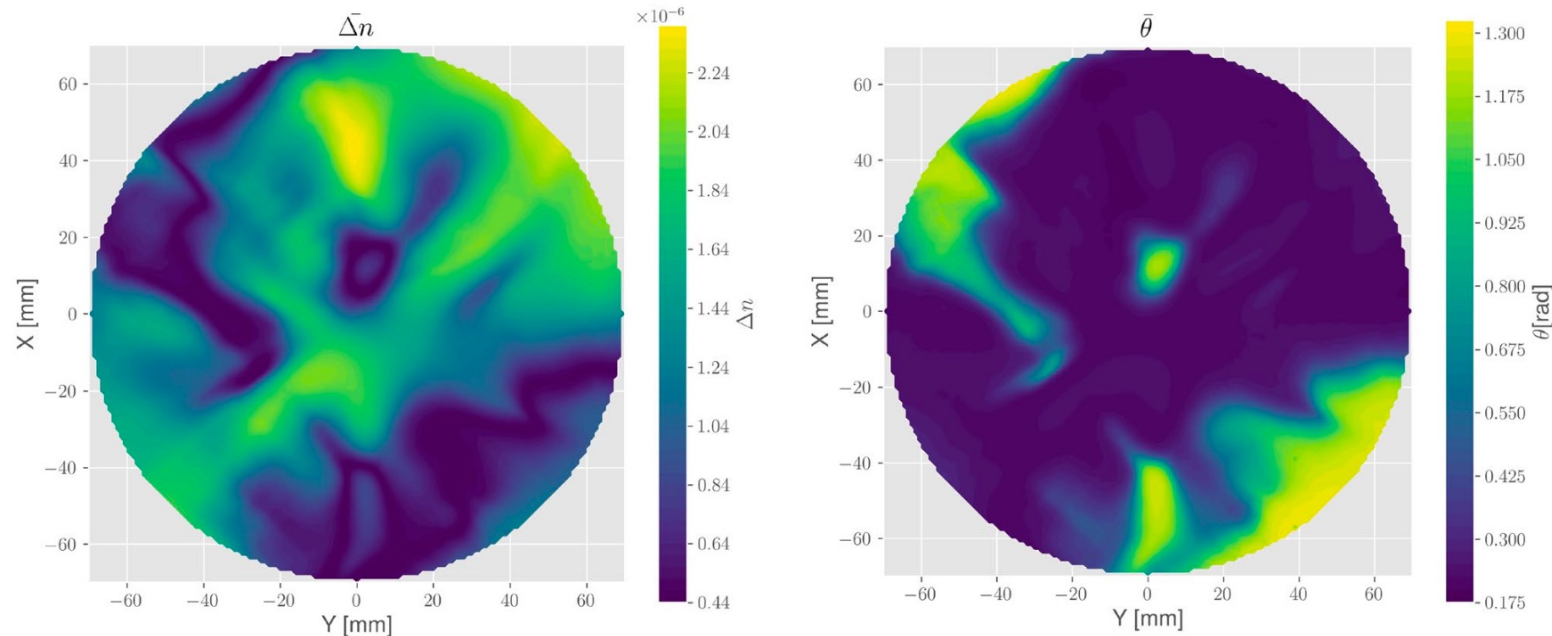
## Group composition:

Guido Zavattini	Staff Ferrara
Andrea Mazzolari	Staff Ferrara
Giovanni Di Domenico	Staff Ferrara
Aurelie Mailliet	Post-Doc Ferrara
Alina Soflau	Master's student Ferrara
Federico Della Valle	Staff Siena
Emilio Mariotti	Staff Siena

# Comments and questions on substrate

## KAGRA

- Measured birefringence of sapphire is  $\Delta n \approx 10^{-6}$  with 15 cm thick substrate.
- Non uniform birefringence map of substrate (amplitude and direction). Phase shifts of 4 rad.
- $\Delta n \approx 10^{-7}$  in silicon\*. Non uniform here too. For ET the desired thickness is 57 cm.
- ➔ Total phase shift  $\approx 1$  rad.
- Is  $\Delta n \approx 10^{-7}$  still too large? ET desires X10 better sensitivity than KAGRA.  $\Delta n < 10^{-7}$ ?
- If uniform, align polarization with axis of system birefringence (entire cavity). If non uniform...



**Figure 4.** Mean distribution of both birefringence  $\Delta n$  and  $\theta$ -angle, calculated from the six input-polarization combinations which led to no miscalculations. (<https://doi.org/10.1038/s41598-023-45928-0>)

\*see also C. Krüger et al. Class. Quantum Grav. 33 (2016)

# Induced birefringence from stress

- Residual stress will generate a (static) birefringence map inside the sample
- External stress will also generate a birefringence

$$\Delta n = C_{\text{SOC}} (\sigma_1 - \sigma_2)$$

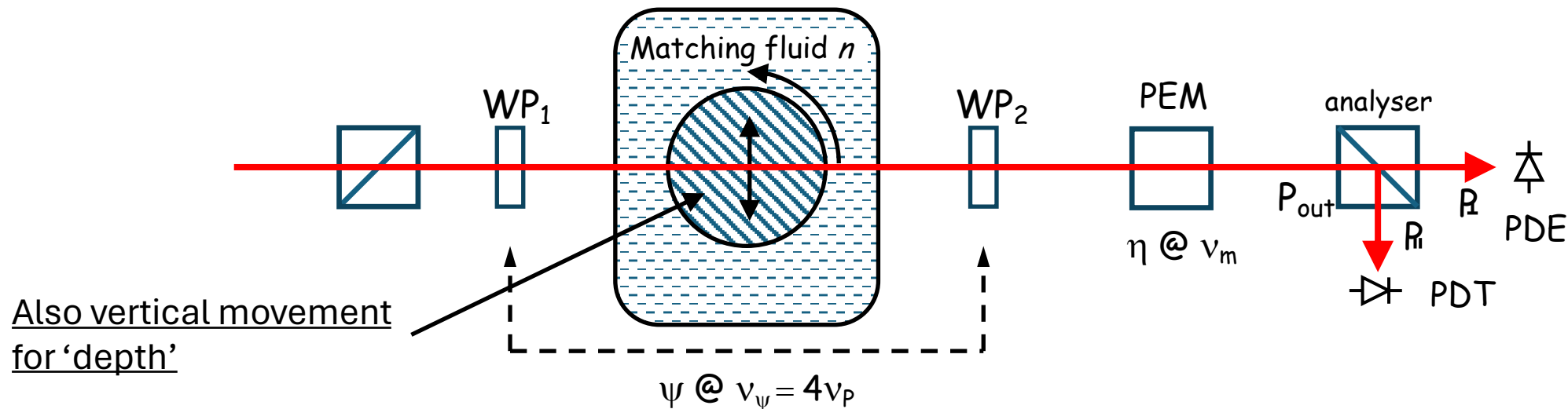
- $C_{\text{SOC}}$  = Stress optic coefficiente [ $\text{Pa}^{-1}$ ],  $\sigma_1$  and  $\sigma_2$  stress along perpendicular directions [ $\text{Pa}$ ]
- Typical values of stress optic coefficient:  $C_{\text{SOC}} \approx 10^{-12} \text{Pa}^{-1}$
- Fused silica:  $2.4 \times 10^{-12} \text{Pa}^{-1}$
- Crystalline Silicon (axes):  $(0.6 \div 1) \times 10^{-12} \text{Pa}^{-1}$
- Some initial work done for stress induced birefringence in Silicon as ET-LF substrate:  
C. Krüger et al. Class. Quantum Grav. 33 (2016) 015012
- Sapphire: could not find a value for  $C_{\text{SOC}}$ .

# Aim 1 – 2D map and involved groups

- Measure 2D birefringence maps (averaged along the beam path) as a function of the sample's preparation and material
- Start with relatively small samples
- With larger samples of several kilograms a dedicated XZ high load motorized system will be needed
- Any group growing substrates: IKZ for Silicon (already contacted), ILM for Sapphire (?), other materials like non doped YAG from IKZ or CRYTUR (already contacted with offer)

# To be implemented: birefringence tomography?

- Birefringence measurements as a function of depth?
- Is the birefringence near the surfaces?
- Is birefringence tomography possible? Birefringence is not a scalar.
- Need a matching fluid: Cargille for FS and maybe Sapphire
- No matching fluids with  $n = 3.5$  for Silicon



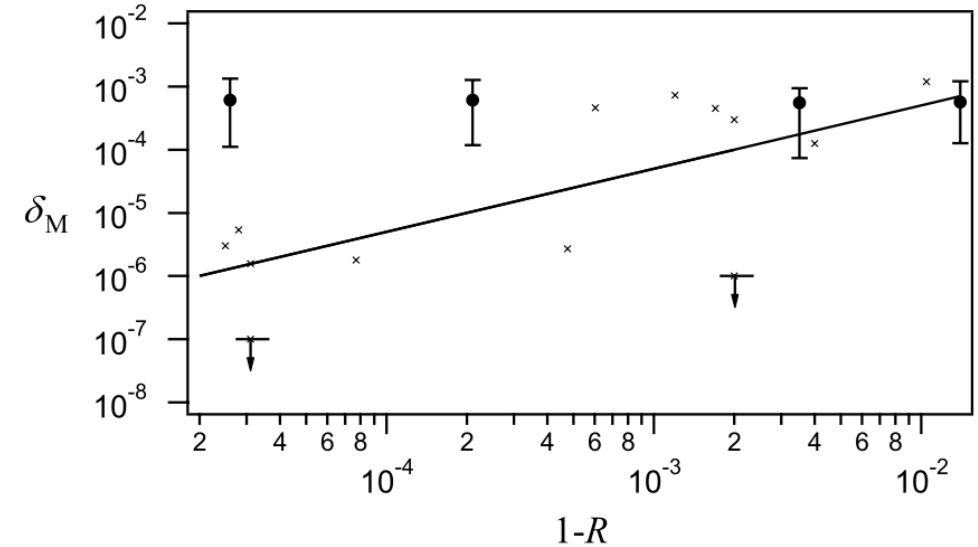
# Comments and questions on coatings

## MIRRORS

- Our experience and other's too (Toulouse BMV group) have found that the static birefringence of coatings:

$$\Delta n_{\text{high finesse}} < \Delta n_{\text{low finesse}}$$

- There seems to be a 'more' uniform map compared to substrates (over  $\approx$  few centimeters).
- The origin of this birefringence is not clear. C. Rizzo's group, Toulouse, attribute the birefringence to the stress in the first layer near to the substrate (F. Bielsa, Appl Phys B (2009) 97: 457–463). Is the cause of the stress the interface between the substrate and first layer of the coating?
- Experience from Si crystal bending using silicon nitride coatings for charged particle channeling (communication from A. Mazzolari) shows that with stoichiometry of silicon nitride coatings one can control the stress on silicon. Maybe the birefringence of mirror coatings with silicon nitride as the first layer could be reduced?



**Fig. 6** Two different numerical calculations for the induced phase retardation per reflection as a function of  $(1 - R)$ . *Solid curve*: birefringence only for the first layer just after the substrate. *Dots with error bars*: calculation with random birefringence per each layer. *Crosses*: measurements plotted in Fig. 3

# Comments and questions on coatings

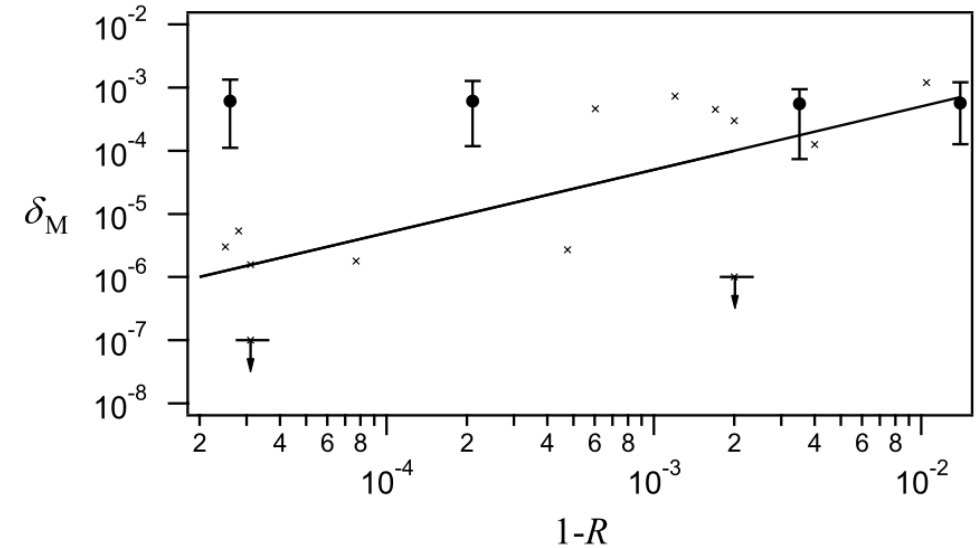
## MIRRORS

- Our experience and other's too (e.g. Toulouse BMV group) have found that the static birefringence of coatings:

$$\Delta n_{\text{high finesse}} < \Delta n_{\text{low finesse}}$$

- ET finesse  $\mathcal{F} = 900 \rightarrow 1-R = 3.5 \cdot 10^{-3} \rightarrow$  ellipticity/reflection  $\delta_M \approx 10^{-4}$
- Again, total ellipticity can be  $\approx 1$  rad (PR = 10)

- Study the mirror birefringence as a function of number of layers  $(1-R)$ ?
- Study the mirror birefringence comparing before and after annealing?
- What polarization state is the light used for locking? In our Fabry-Perot based polarimeter the static mirror birefringences were oriented to subtract each other and the polarisation aligned to the birefringence axis of the cavity as a whole. In this way the two eigenmodes of the cavity are almost superimposed.



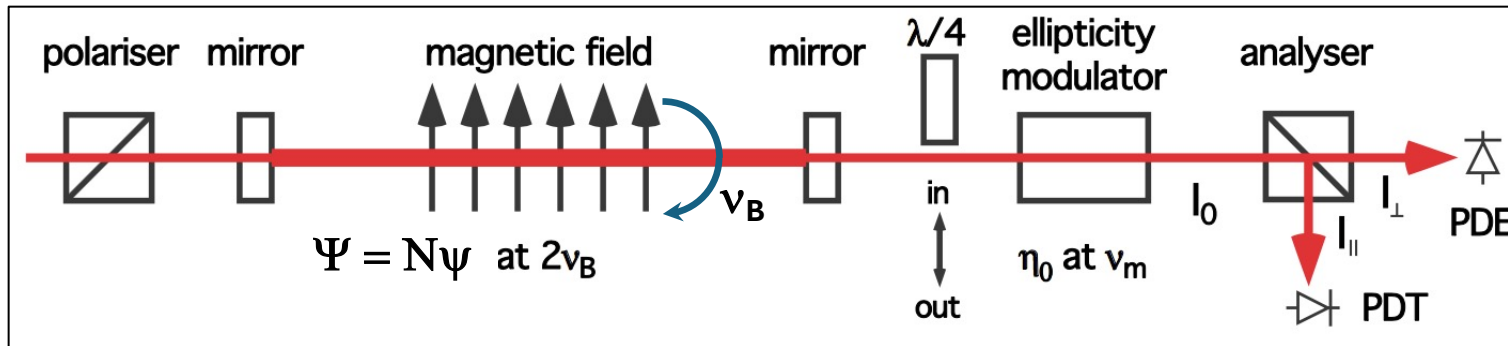
**Fig. 6** Two different numerical calculations for the induced phase retardation per reflection as a function of  $(1 - R)$ . *Solid curve*: birefringence only for the first layer just after the substrate. *Dots with error bars*: calculation with random birefringence per each layer. *Crosses*: measurements plotted in Fig. 3

# Aim 2 – 2D map of coatings and involved groups

- Understand the origin of the mirror coatings' birefringence
- Start with relatively small samples
- With larger samples of several kilograms a dedicated XZ high load motorized system will be needed
- Any group depositing coatings: Padova (already contacted and has provided samples), Sannio (already contacted and provided a sample), others?



# Birefringence noise – PVLAS general scheme



F. Della Valle et al. Eur. Phys. J. C (2016) 76:24

A. Ejlli et al. Physics Reports 871 (2020) 1–74

VMB:

@  $B_{\text{ext}} = 2.5 \text{ T}$

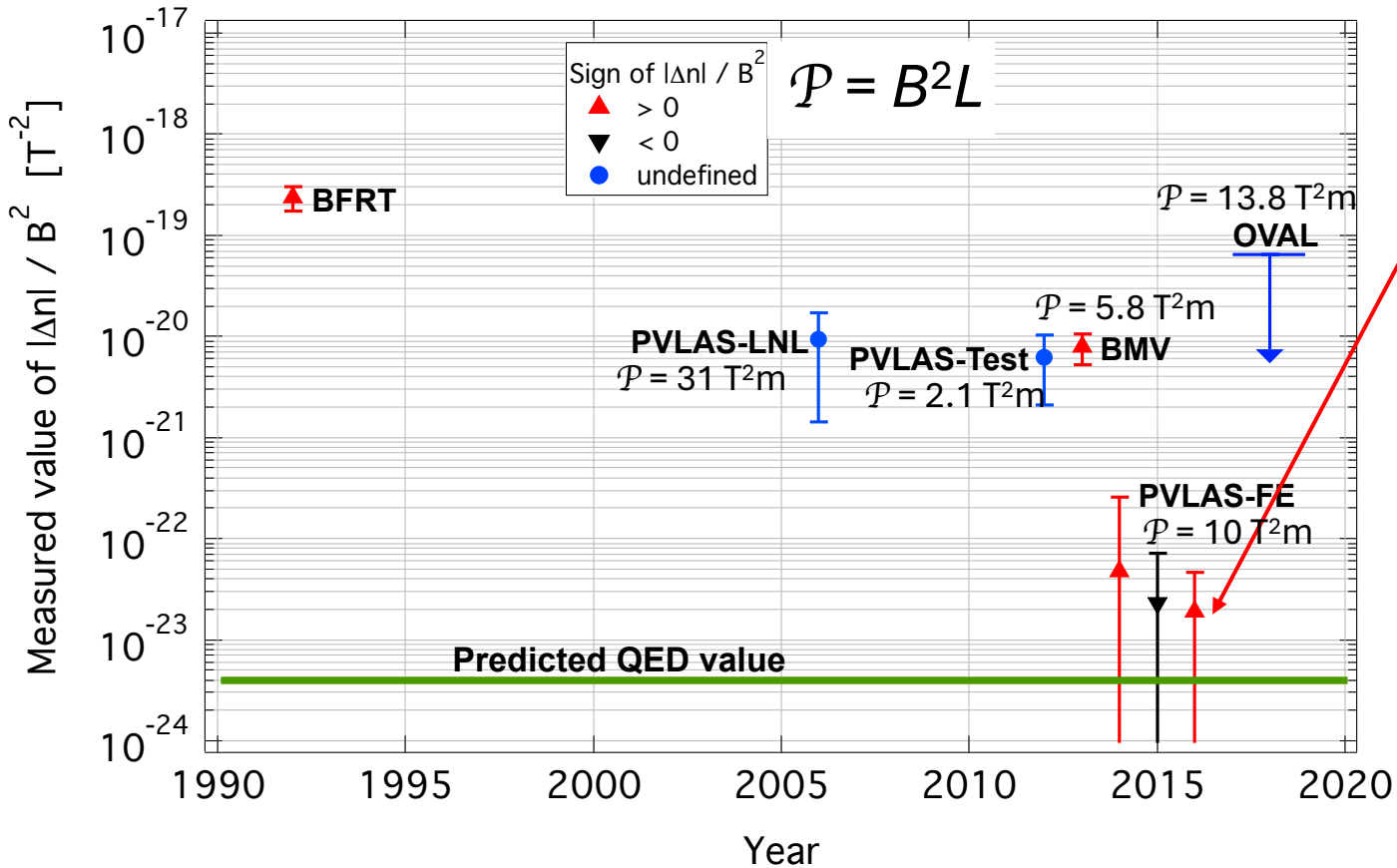
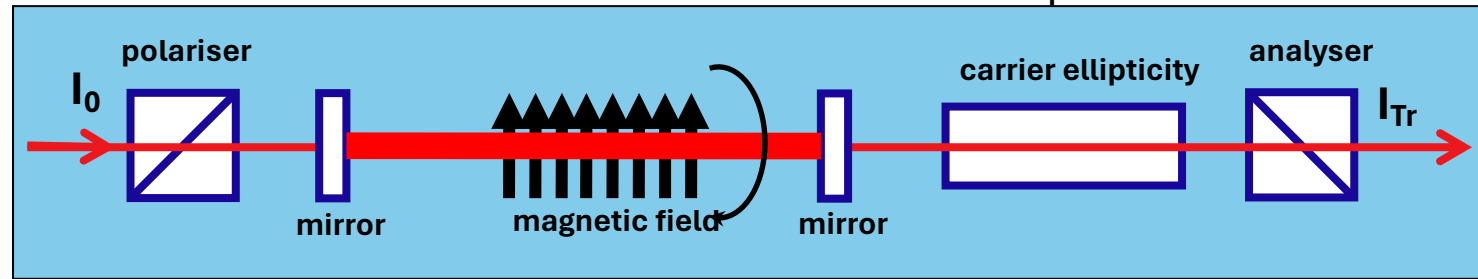
$\Delta n = 2.5 \cdot 10^{-23}$

- $L$  is the length of the birefringent medium (in the PVLAS experiment  $\Delta n_B \propto B^2$  )
- Single pass ellipticity:  $\psi = \frac{\pi \int \Delta n_B dL}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t)$  Here  $\vartheta(t)$  is the angle between the polarisation and the birefringence axis.
- The Fabry-Perot cavity amplifies  $\psi$  by a factor  $N = 2\mathcal{F}/\pi$ . We had  $\mathcal{F} = 7 \times 10^5$
- The ellipticity modulator allows heterodyne detection which linearizes the ellipticity  $\psi$  to be measured and allows the distinction between a rotation and an ellipticity. The insertion of the  $\lambda/4$  wave plate allows measuring rotations.
- The rotating magnetic field modulates the desired signal due to VMB.

$$\Rightarrow I_{\text{out}} \simeq I_0 \left\{ \eta^2(t) + 2\eta(t)N\psi(t) + 2\eta(t)\Gamma(t) + \dots \right\}$$

# State of the art

General scheme: modulated or pulsed field



- The PVLAS - FE result remains the most sensitive measurement yet performed:  
 $\Delta n / B^2 = (1.9 \pm 2.7) \cdot 10^{-23} \text{ T}^{-2}$  with 2.5 T

- Permanent magnets allowed careful debugging of systematics:  $B^2L = 10 \text{ T}^2\text{m}$

- Optical path difference sensitivity:  
 $S_{\text{OPD}} = 4 \cdot 10^{-19} \text{ m} / \sqrt{\text{Hz}} @ \approx 16 \text{ Hz}$

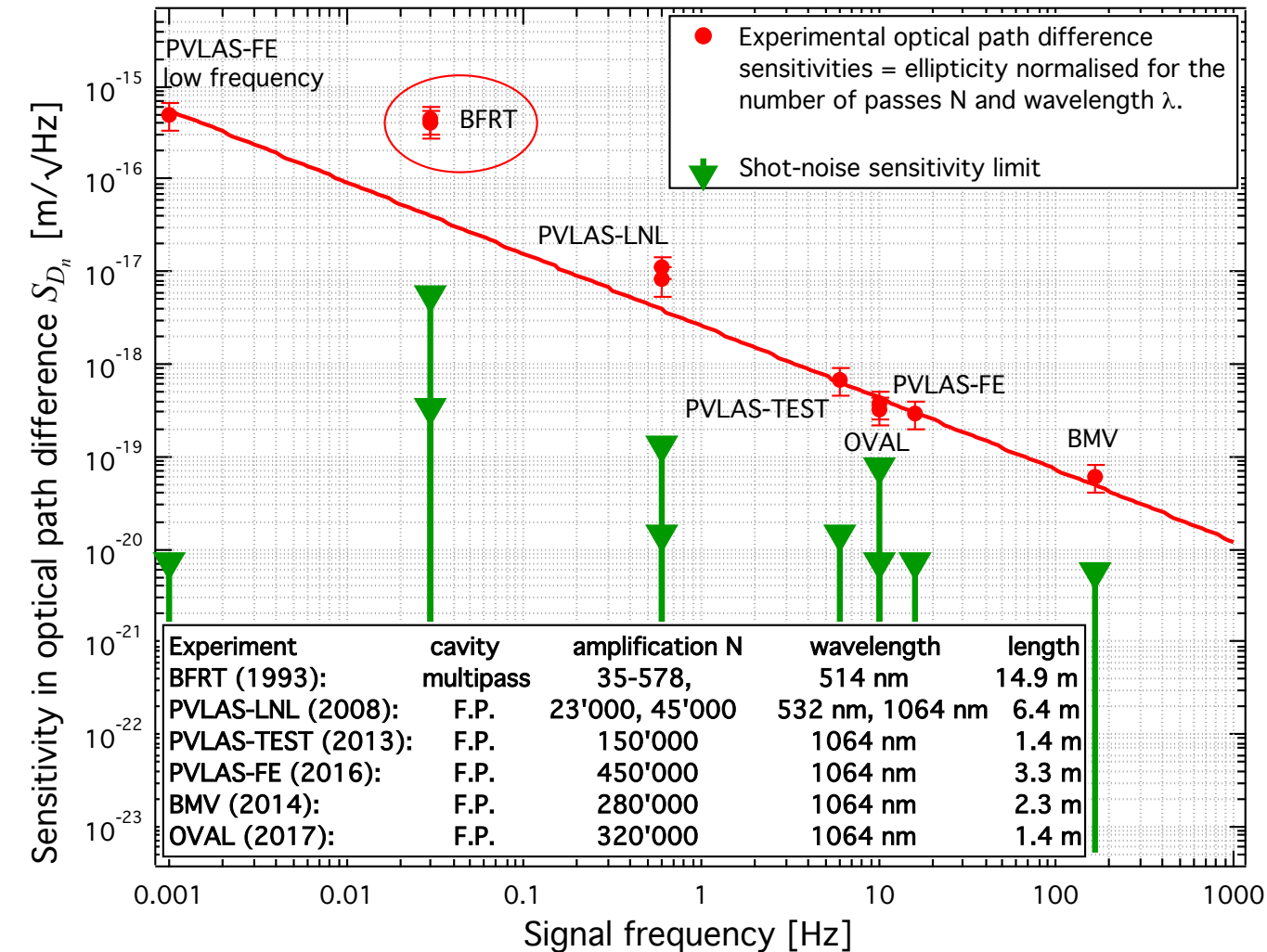
- Cavity amplification was  $N \approx 4.5 \cdot 10^5$

- Intrinsic noise from the mirrors limited the sensitivity and the SNR

- Measured noise: x30 shot-noise @ 16 Hz

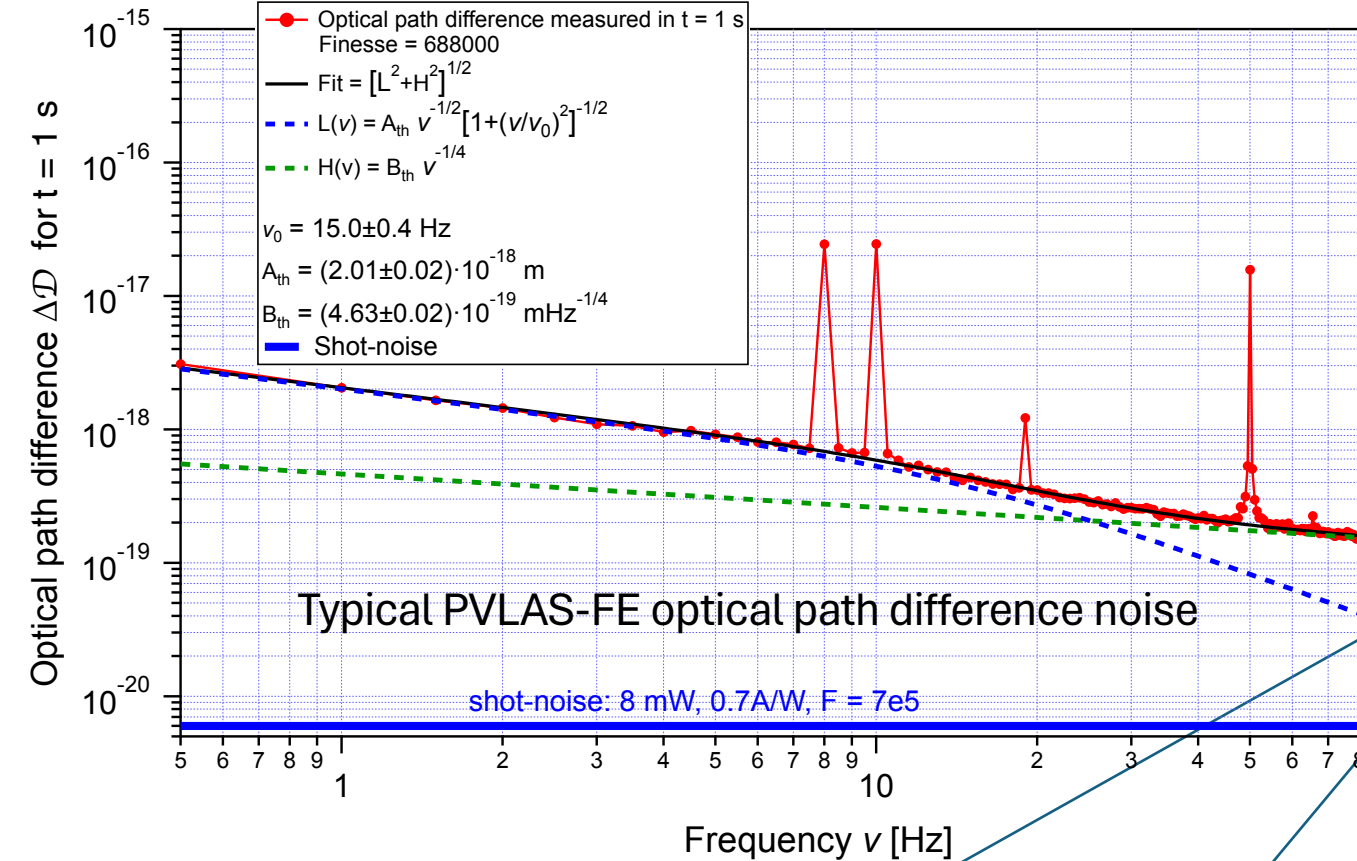
# Intrinsic mirror birefringence noise

## Limits in the sensitivity of a polarimeter



- No experimental effort has reached shot-noise sensitivity (green) with a high finesse F.P.
- There seems to be a common problem afflicting all experiments
- This noise seems to be an intrinsic property of the cavity mirrors
- With a low finesse cavity one does reach shot-noise. The limit is not the method.

# Intrinsic mirror birefringence noise

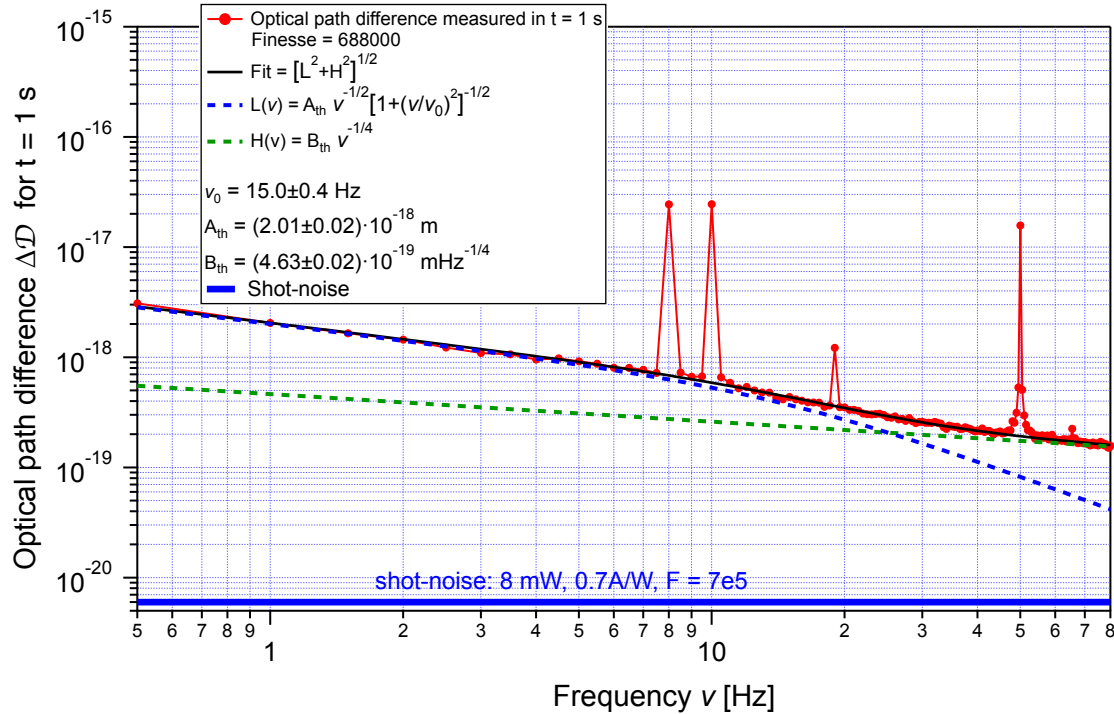


- Finesse  $\mathcal{F} = 6.88 \times 10^5$
- Peaks at 8 Hz and 10 Hz represent Cotton-Mouton calibration signals from 850  $\mu$ bar Argon gas.
- The peak at 19 Hz is generated by a Faraday rotation leakage due to the total cavity static birefringence from the mirrors. Eigenstate superposition measurement
- Brownian? Why the cut-off?
- Thermo-elastic model points to tantala.
- **For ET we can measure new coatings. Finesse must be  $F \geq 5e4$  ( $R \geq 99.995\%$ ): the amplified mirror noise must be greater than shot-noise.**
- **Will be testing crystalline GaAs/AlGaAs mirrors.**

$$S_{OPD}(\nu) = \sqrt{\left(\frac{A_{th} \nu^{-1/2}}{\sqrt{1 + (\nu/\nu_0)^2}}\right)^2 + (B_{th} \nu^{-1/4})^2}$$

$$A_{th} = (2.01 \pm 0.02) \times 10^{-18} \text{ m}, \quad \nu_0 = (15.0 \pm 0.4) \text{ Hz}, \quad B_{th} = (4.63 \pm 0.02) \times 10^{-19} \text{ m/Hz}^{1/4}$$

# Intrinsic mirror birefringence noise



- Estimated the thermoelastic birefringence noise in reflective coatings (ATFilm mirrors)\*
- $C_{SO}$  = stress optic coefficient
- $Y$  = Young's modulus
- $\alpha_T$  = thermal expansion coefficient
- $r_0$  = beam radius on mirror
- $C_T$  = specific heat capacity
- $\rho$  = density
- $\lambda_T$  = thermal conductivity

Temperature spectral density

$$S_T(\nu) = \sqrt{\frac{8k_B T^2}{\pi r_0^2 \sqrt{\pi \rho C_T \lambda_T \nu}}} \propto \nu^{-1/4}$$

Optical path difference spectrum

$$S_{\Delta D} = 2d_e \sqrt{2} C_{SO} Y \alpha_T S_T(\nu)$$

Fused silica

$$S_{\Delta D}^{(FS)} \sim 4 \times 10^{-21} \text{ m}/\sqrt{\text{Hz}} \quad @ \quad 1 \text{ Hz}$$

Tantala

$$S_{\Delta D}^{(Ta)} \sim (1 \div 5) \times 10^{-19} \text{ m}/\sqrt{\text{Hz}} \quad @ \quad 1 \text{ Hz}$$

Compatible with  
from the fit

$$B_{th} = (4.63 \pm 0.02) \times 10^{-19} \text{ m}/\text{Hz}^{1/4}$$

# Aim 3 – birefringence noise and involved groups

- New coatings. Finesse must be high:  $\mathcal{F} > 5 \cdot 10^4$
- Any group depositing coatings (?)

# Aim 4 – birefringence measurements at cryogenic temperatures

- What is the stress optic coefficient at cryogenic temperatures?  
Especially for Silicon and Sapphire.
- Does the birefringence of substrates and coatings improve/worsen?
- The Ferrara unit has cryogenic expertise and Milano (Marco Giammarchi) has expressed interest.
- This would need a significant financial investment