Ferrara laboratory for birefringence measurements of substrates and reflective coatings

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Università degli Studi di Ferrara

Summary

- Birefringence measurements in transmission
- Birefringence measurements in reflection

Physics: birefringence and ellipticity

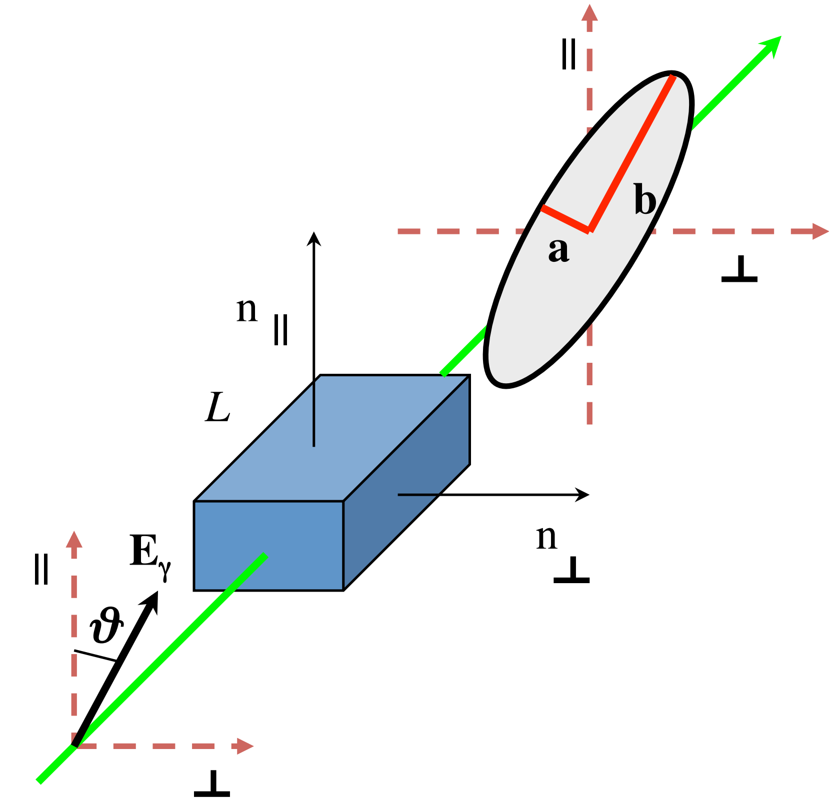
- In a birefringent medium $n_{\parallel} \neq n_{\perp}$
- Passing through a birifrangente medium, a linearly polarized beam

will acquire an ellipticity $\psi = \pm a/b$ (the sign determines the rotation direction of E_γ)

$$\mathbf{E}_\gamma = E_\gamma \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Delta\phi = \frac{2\pi(n_{\parallel} - n_{\perp})L}{\lambda}$$

$$\mathbf{E}'_\gamma = E_\gamma \begin{pmatrix} 1 + i\frac{\Delta\phi}{2}\cos 2\vartheta \\ i\frac{\Delta\phi}{2}\sin 2\vartheta \end{pmatrix}, \qquad \Delta\phi \ll 1$$

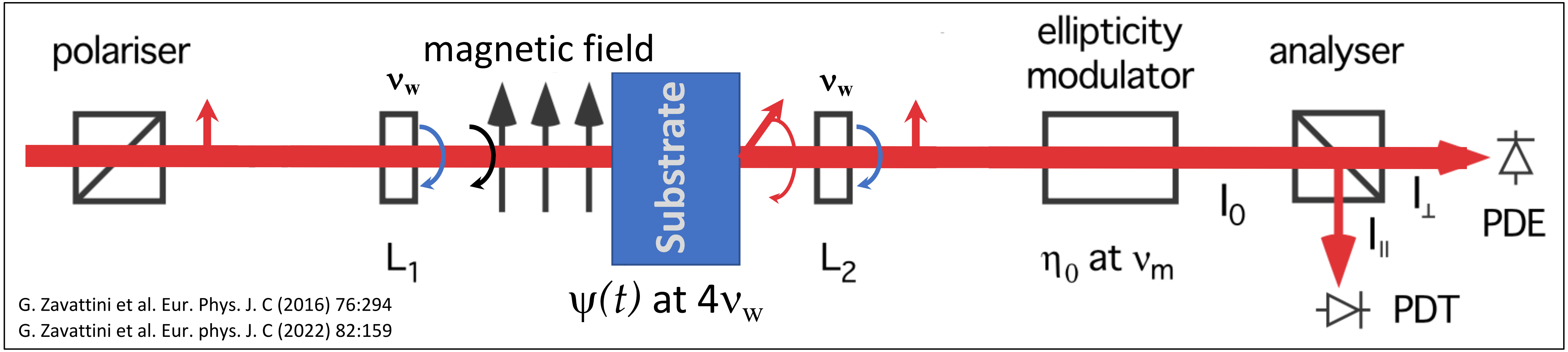
$$\psi = \pm\frac{a}{b} \approx \frac{\Delta\phi}{2}\sin 2\theta = \frac{\pi(n_{\parallel} - n_{\perp})L}{\lambda}\sin 2\vartheta$$



$$\Delta n \approx 10^{-7},\ L \approx 10\ \mathrm{cm},\ N \approx 10,\ \lambda = 1064\ \mathrm{nm} \ \longrightarrow\ \Delta\phi \approx 0.6\ \mathrm{rad} \approx 34^\circ$$

Substrate birefringence measurements

- Single pass ellipticity: $\psi(t) = \dfrac{\pi \int \Delta n \, dL}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t).$
- Here ϑ(t) is the angle between the polarisation and the birefringence axis. ϕ(t) is the HWP angle: $\vartheta(t) = 2\phi(t)$



$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2} \sin 2\phi(t) + \frac{\alpha_2(t)}{2} \sin[2\phi(t) + 2\Delta\phi(t)]$$

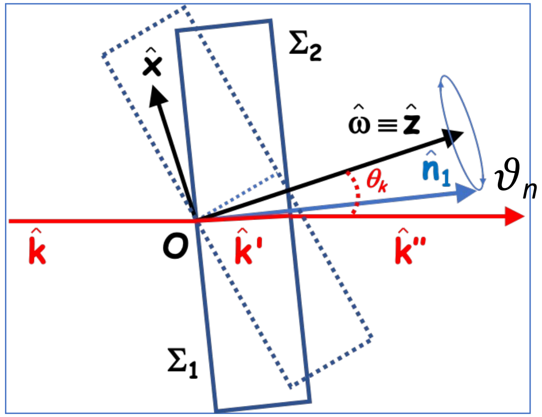
- $\alpha_{1,2}$ are the residual retardations from π of the HWPs. The modulator's frequency is ν_m = 50 kHz.
- The detected intensity is **demodulated** at the modulator's frequency ν_m to obtain the ellipticity spectrum.
- The ellipticity spectrum includes the desired signal, systematic effects and noise

$$I_{\text{out}} \simeq I_0 \left\{ \eta^2(t) + 2\eta(t)\psi(t) + 2\eta(t)\Gamma(t) + \ldots \right\}$$

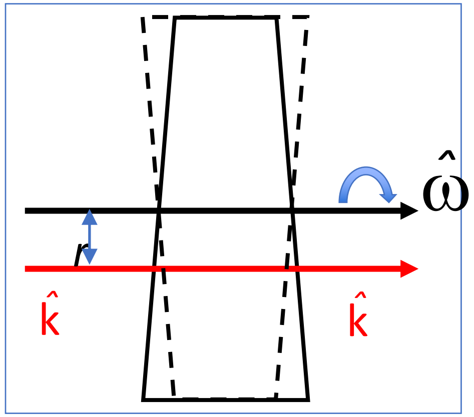
Generation of spurious harmonics from rotating HWPs

$$\alpha_{1,2}(\phi, \mathrm{T}, r) = \alpha_{1,2}^{(0)}(\mathrm{T}) + \alpha_{1,2}^{(1)} \cos\phi(\mathrm{t}) + \alpha_{1,2}^{(2)} \cos 2\phi(\mathrm{t}) + \ldots$$

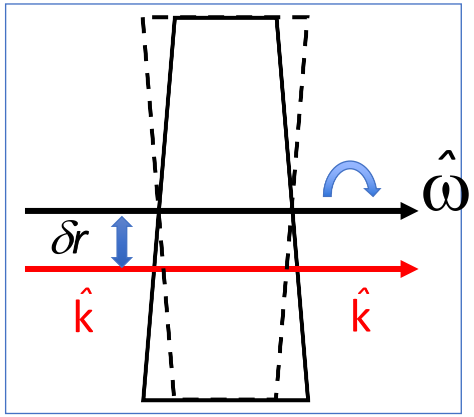
Temperature dependence of $\alpha_{1,2}^{(0)}(\mathrm{T}) = \frac{2\pi}{\lambda} \int \Delta \mathrm{n} \, \mathrm{dL}$



ALIGNMENT



WEDGE β



WEDGE + OSCILLATION @ ν_w

$$\alpha_{1,2}^{(1)} \approx \frac{2\pi}{\lambda} \Delta n \frac{D}{n^2} \vartheta_n \vartheta_k$$

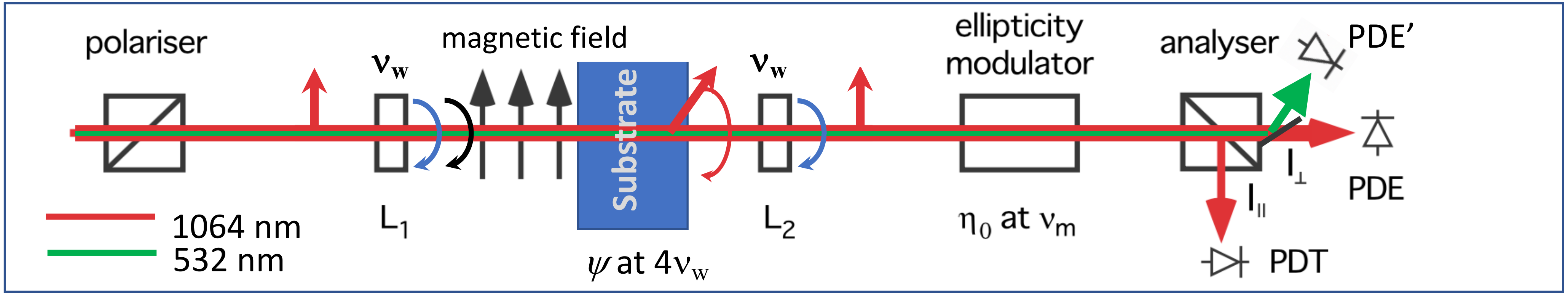
$$\alpha_{1,2}^{(2)} \approx \frac{2\pi}{\lambda} \Delta n \frac{D}{4n^2} \vartheta_n^2 \vartheta_k^2$$

$$\alpha_{1,2}^{(1)} \approx \frac{2\pi}{\lambda} \Delta n \, \Delta r_0 \, \beta \qquad \alpha_{1,2}^{(2)} \approx \frac{2\pi}{\lambda} \Delta n \, \delta r \, \beta$$

Generate 4th harmonic but can be controlled to < 10^{-5} level corresponding to an optical path difference $\int \Delta n \, dL \lesssim 10^{-12}$ m

✓ The HWPs can be aligned separately using a frequency doubled laser @ 532 nm

Baseline scheme for substrate birefringence measurements



$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2}\sin 2\phi(t) + \frac{\alpha_2(t)}{2}\sin[2\phi(t) + 2\Delta\phi(t)]$$

Signal @ $4\nu_w$

Spurious signals
Contain harmonics of ν_w

Relative rotation phase error
Degrades extinction

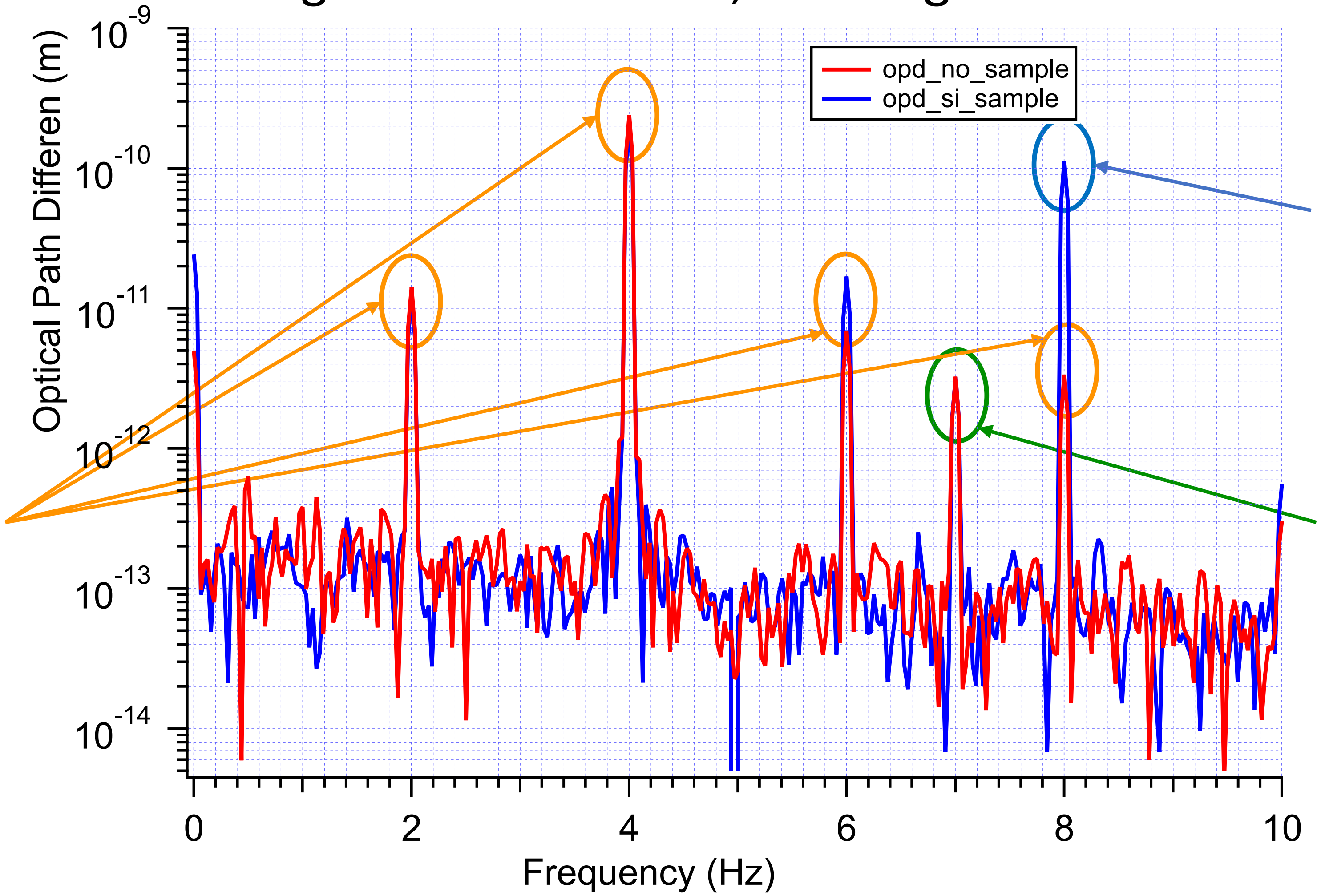
$\alpha_{1,2}$ are the phase errors from π of the two HWPs and $\phi(t)$ is their rotation angle

- ✓ 532 nm beam (HWP -> FWP) allows independent alignment of the rotating HWPs to reduce 1st, 3rd and 4th harm.
- ✓ At 1064 nm, control the temperature of the wave-plates to reduce the dominating 2nd harmonic
- ✓ Reduced systematic peaks such that $\alpha_{1,2}^{(1,2,3)} \lesssim 10^{-4}$ at all relevant harmonics and in particular, for the 4th harmonic, $\alpha_{1,2}^{(4)} \lesssim 10^{-5}$. Can be subtracted vectorially ➔ Ellipticity sensitivity $\psi_0 \approx 10^{-6}$
- ✓ Can produce X-Y ‘maps’ of the static average birefringence of a substrate: $\Delta n = \frac{\psi_0 \lambda}{\pi L}$
- ✓ Optical path difference sensitivity $S_{OPD} \lesssim 10^{-12}$ m
- ✓ Calibration with the Cotton-Mouton effect in air using a rotating 2.5 T permanent magnet

Example of a demodulated spectrum

$$OPD = \Delta n L = \frac{\psi_0 \lambda}{\pi}$$

Integration time = 32 s; Hanning window.



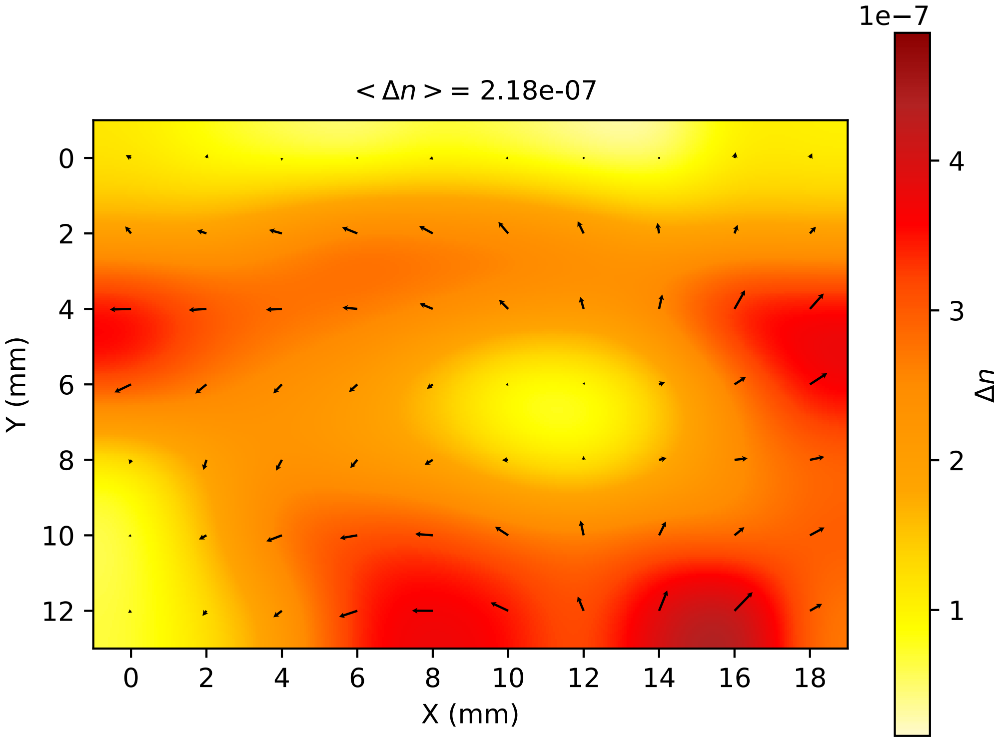
- Spurious harmonics from temperature and misalignment.

- Peak due to silicon birefringence: $\Delta n = 1.1 \times 10^{-7}$; L = 1 mm

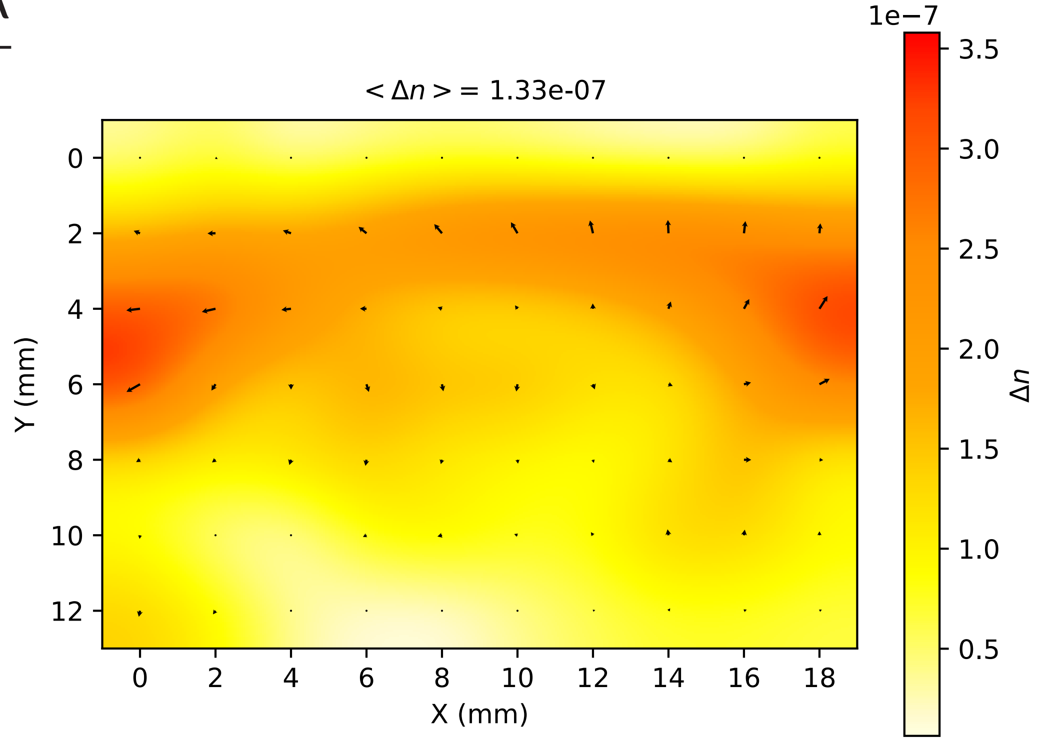
- Calibration Cotton-Mouton peak of air. $\Delta n = 3.9 \times 10^{-12}$; L = 0.84 m

Example of birefringent map: first samples

- Silicon crystal samples (100), L = 1 mm thick, 2.5 cm x 2.5 cm, cut in house from larger sample
- Measurements using 1064nm (significant absorption). Will be repeated with 1550nm
- Subtracted vectorially the waveplate contribution (small effect)
- Held with clamp from bottom edge (left): extra stress can be seen due to clamp.
- Held without clamp (right). Upper half maintains same birefringence.
- Non uniform birefringence.



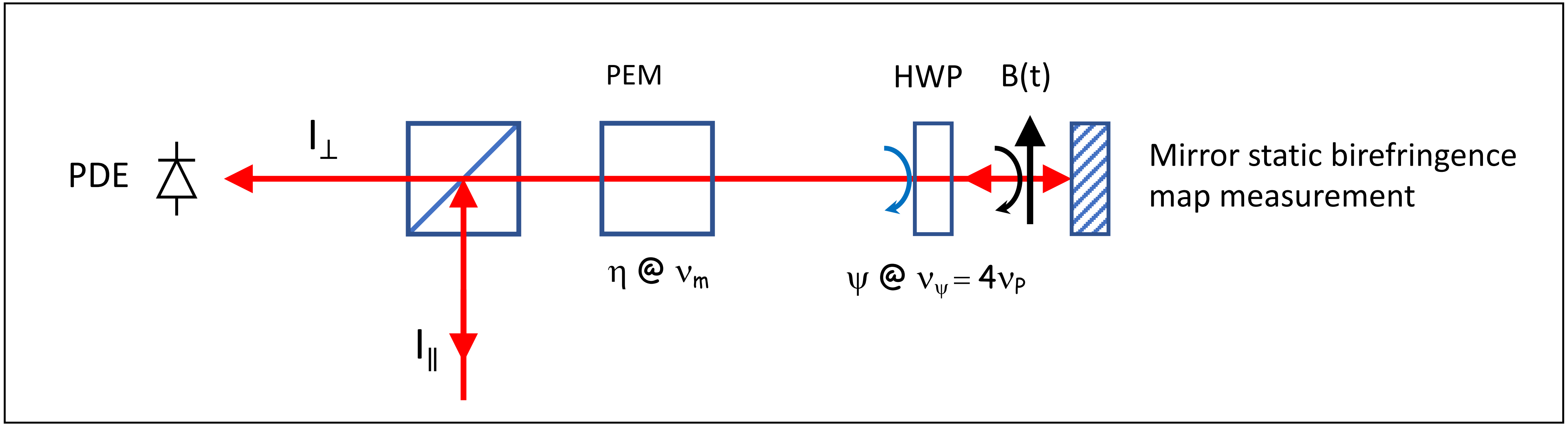
$$\Delta n = \frac{\psi_0 \lambda}{\pi L}$$



Reflective coating birefringence measurements

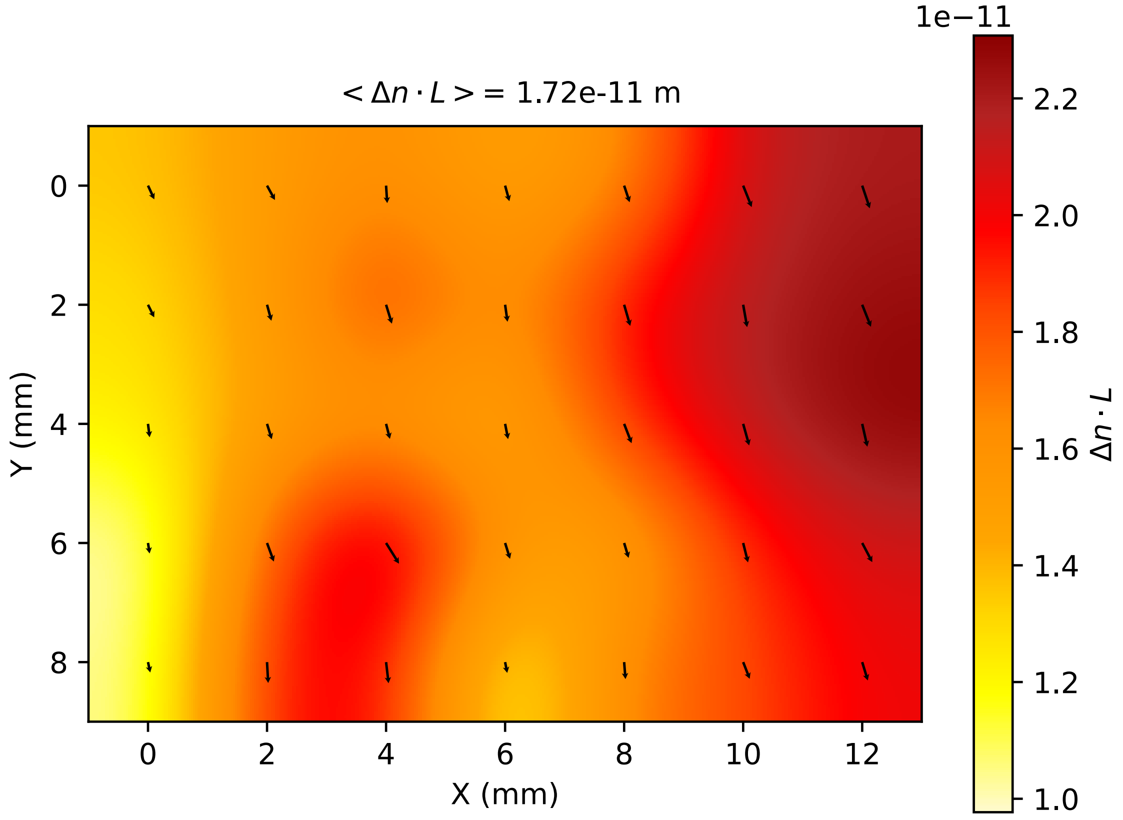
Reflection scheme for static birefringence maps of reflective coatings:

- At present we have a 1064 nm beam aligned.
- With a silver mirror the induced ellipticity is minimum and is, at present, associated to the rotating HWP .
- Will implement a 532 nm beam to distinguish the rotating HWP effect from the mirror effect.
- Also installed a rotating magnet for calibration.

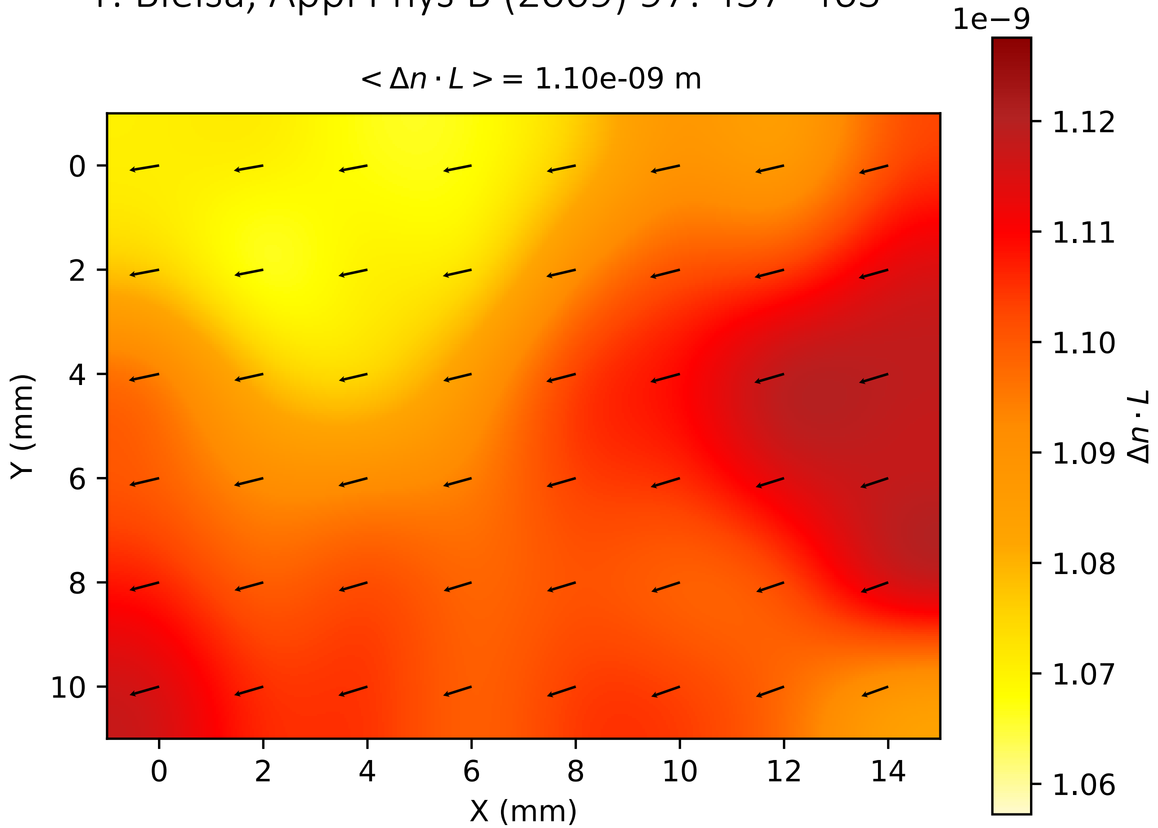


Example of birefringent map of coatings: first samples

- Silver mirror.
- Very low birefringence.
- Measured ellipticity is dominated by the rotating half-waveplate.



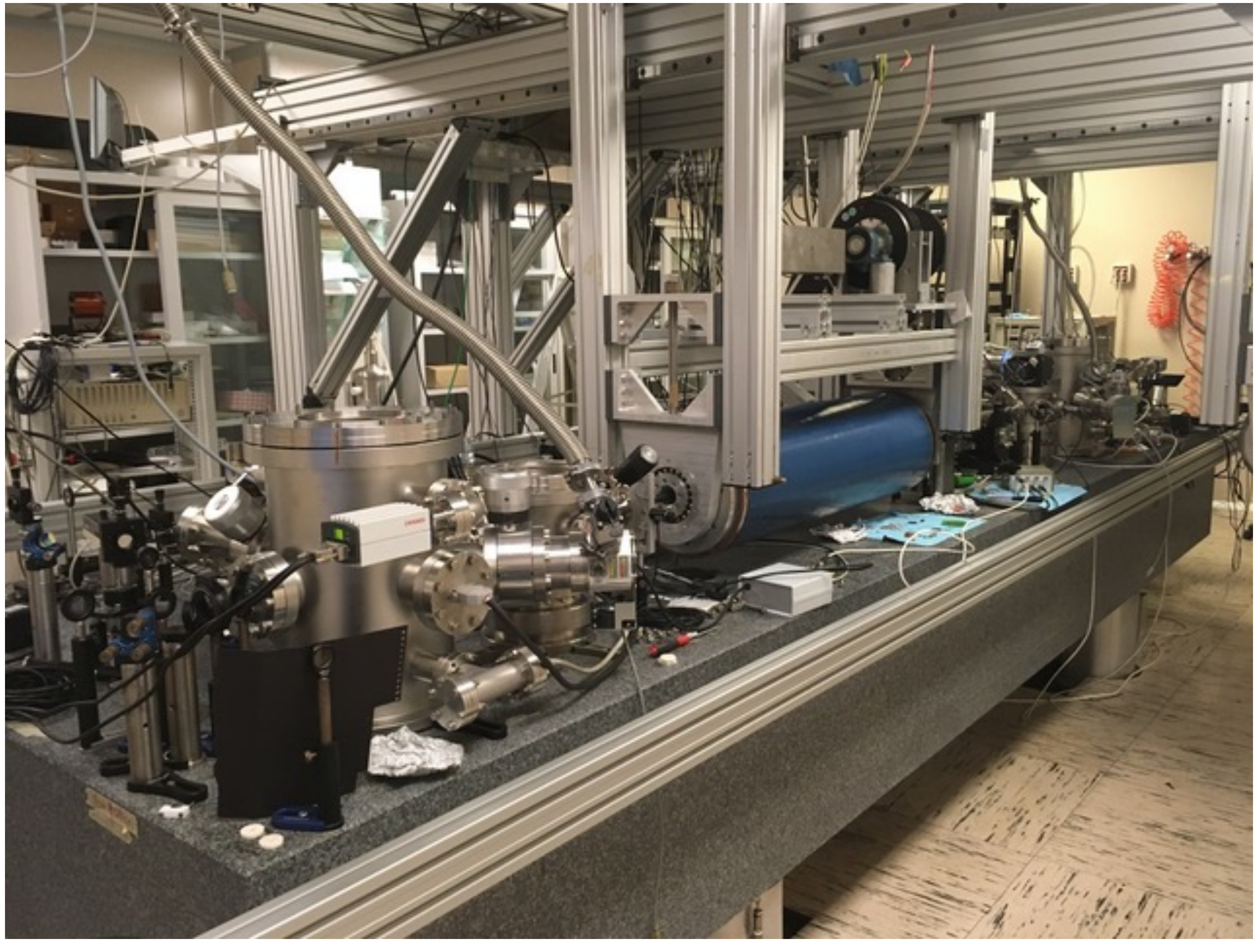
- Dielectric mirror with T ≈ 10^{-3}. 'Uniform'.
- Polarization can be aligned in cavities.
- Higher reflectivity, lower birefringence. For F ≈ 10^5, $\Delta n \cdot L \approx 3 \times 10^{-13}$ m.
- Brandi et al. Appl. Phys. B 65, 351–355 (1997); F. Bielsa, Appl Phys B (2009) 97: 457–463



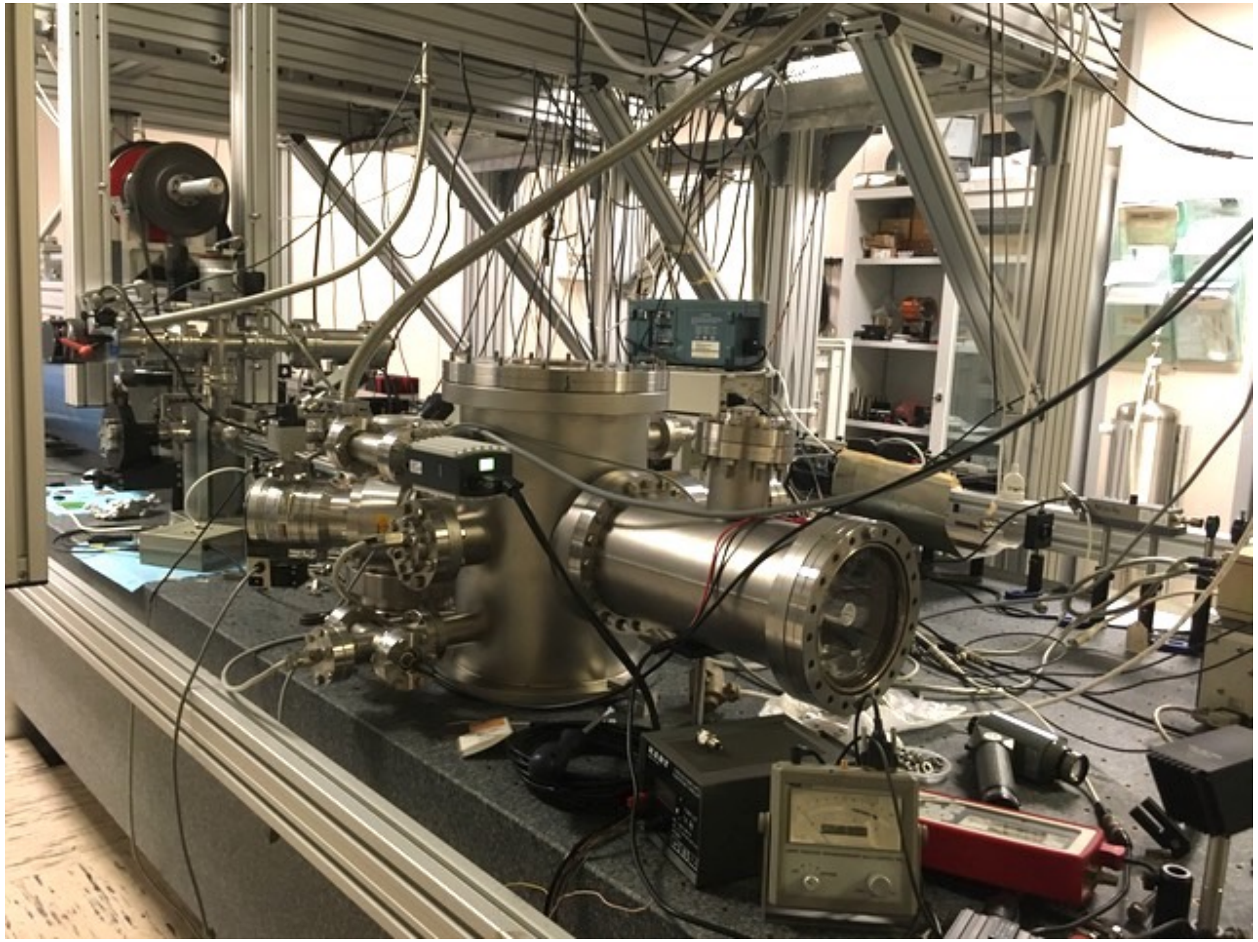
Pictures

At present being used with rotating HWPs.

General view from input side

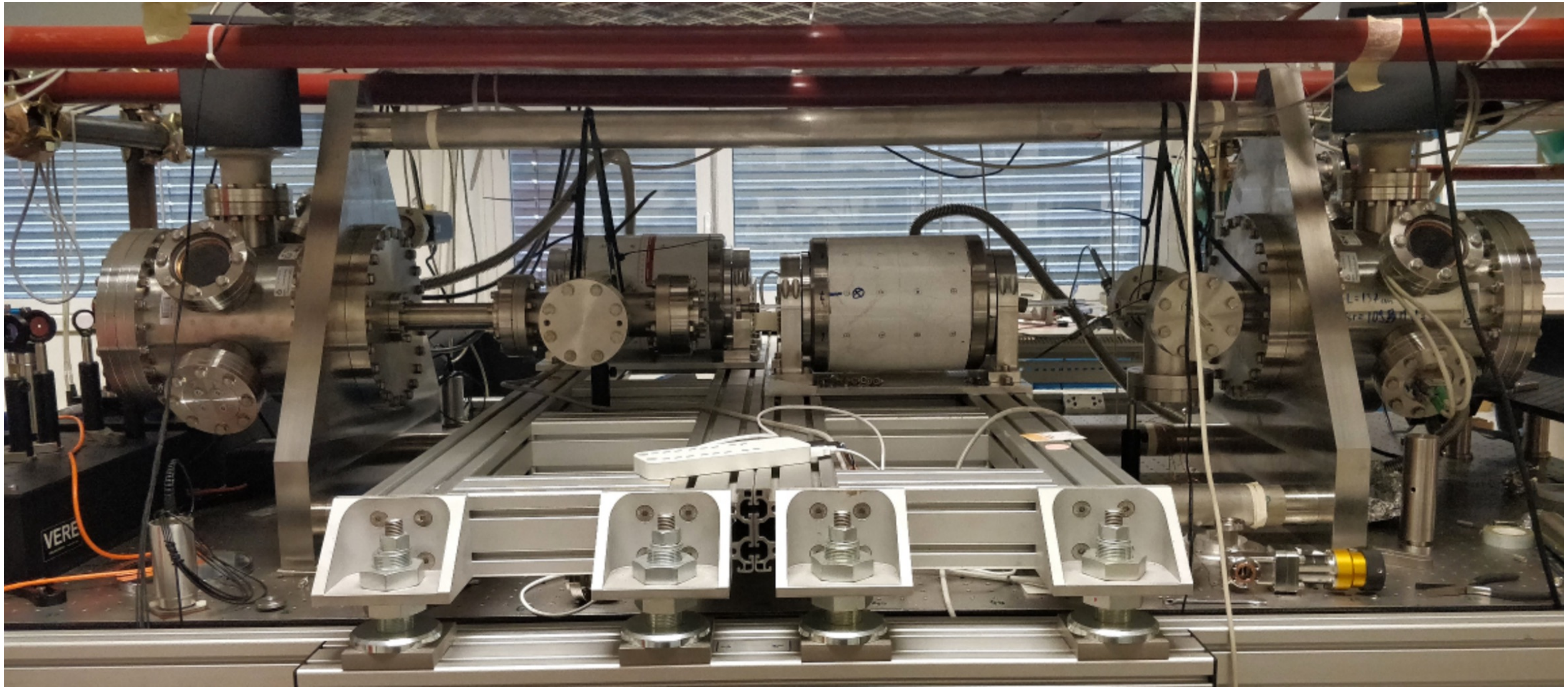


General view from output side



Pictures: lab2

Polarimeter at present being used with a low finesse cavity (F ≈ 3000).



Near future: will be dedicated to birefringence measurements with the rotating HWPs at 1064nm and 1550nm.

Comments and questions: 2

MIRRORS

- Our experience and other's too (Toulouse BMV group) have found that the static birefringence of coatings:

$$\Delta n_{\text{high finesse}} < \Delta n_{\text{low finesse}}$$

- There seems to be a 'more' uniform map compared to substrates (over ≈ few centimeters).
- The origin of this birefringence is not clear. C. Rizzo's group, Toulouse, attribute the birefringence to the first layer near to the substrate (F. Bielsa, Appl Phys B (2009) 97: 457–463). The cause is the stress between the substrate and first layer of the coating?
- Experience from Si crystal bending using silicon nitride coatings for charged particle channeling (communication from A. Mazzolari) shows that with stoichiometry of silicon nitride coatings one can control the stress on silicon. Maybe the birefringence of mirrors with silicon nitride as the first layer could be reduced?
- In our Fabry-Perot based polarimeter the static mirror birefringences were oriented to subtract each other and the polairsation aligned to the axis of the cavity as a whole. In this way the two eigenmodes of the cavity are almost superimposed.

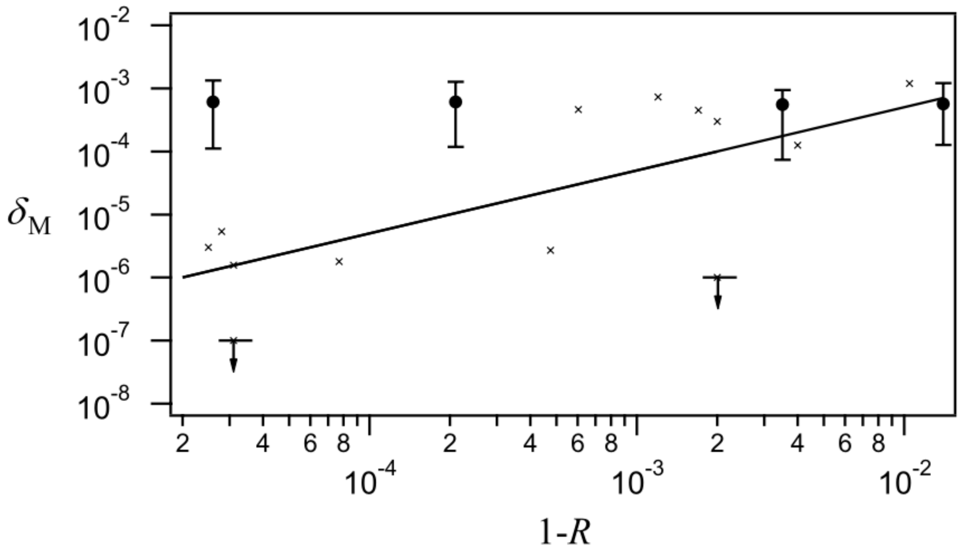


Fig. 6 Two different numerical calculations for the induced phase retardation per reflection as a function of $(1 - R)$. *Solid curve*: birefringence only for the first layer just after the substrate. *Dots with error bars*: calculation with random birefringence per each layer. *Crosses*: measurements plotted in Fig. 3

Comments and questions: 1

KAGRA

- Measured birefringence of sapphire is $\Delta n \approx 10^{-6}$ with 15 cm thick substrate.
- Non uniform birefringence map of substrate (amplitude and direction). Phase shifts of 4 rad.
- $\Delta n \approx 10^{-7}$ in silicon*. Non uniform here too. For ET the desired thickness is 67 cm.
- ➔ Total phase shift ≈ 1 rad.
- Is $\Delta n \approx 10^{-7}$ still too large? If uniform, align polarization with axis of system birefringence. If non uniform...

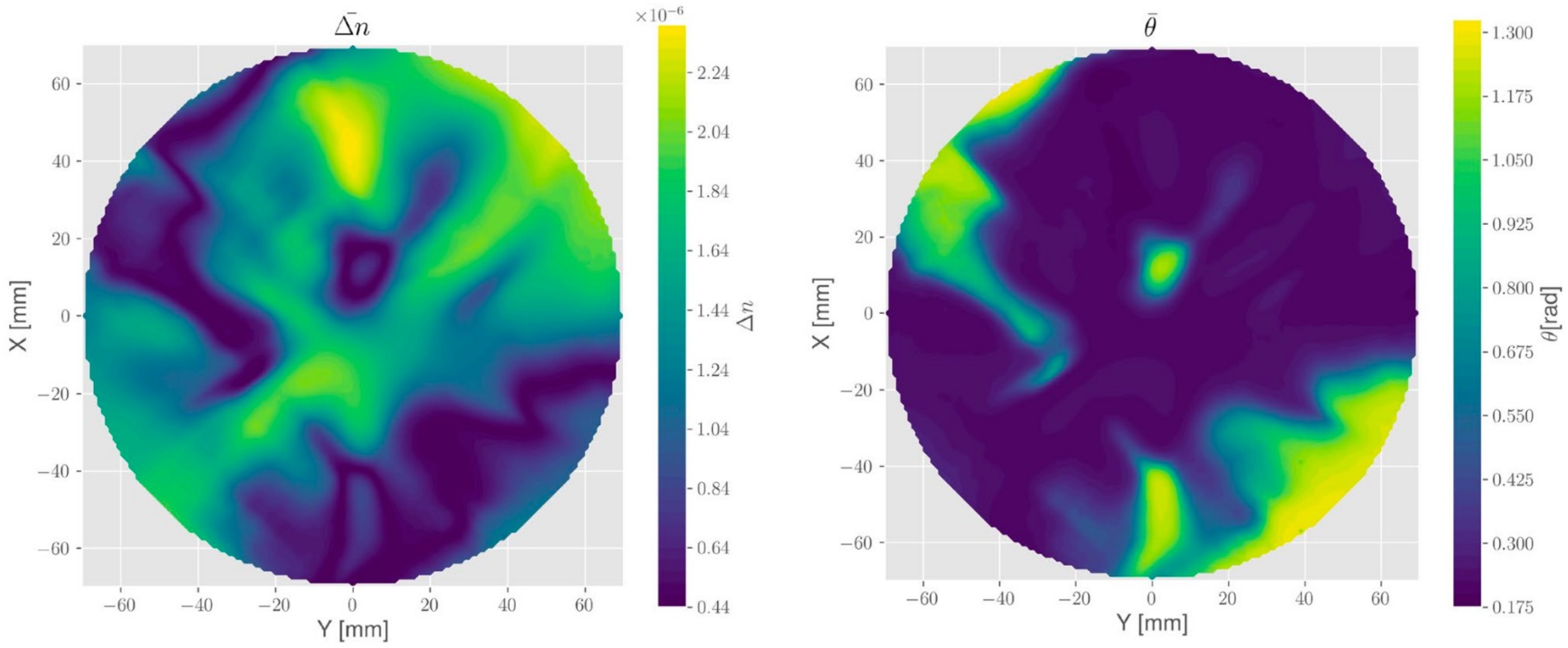


Figure 4. Mean distribution of both birefringence Δn and θ-angle, calculated from the six input-polarization combinations which led to no miscalculations. (https://doi.org/10.1038/s41598-023-45928-0)

*see also C. Krüger et al. Class. Quantum Grav. 33 (2016) 015012

Induced birefringence from stress

- Residual stress will generate a (static) birefringence map inside the sample
- External stress will also generate a birefringence

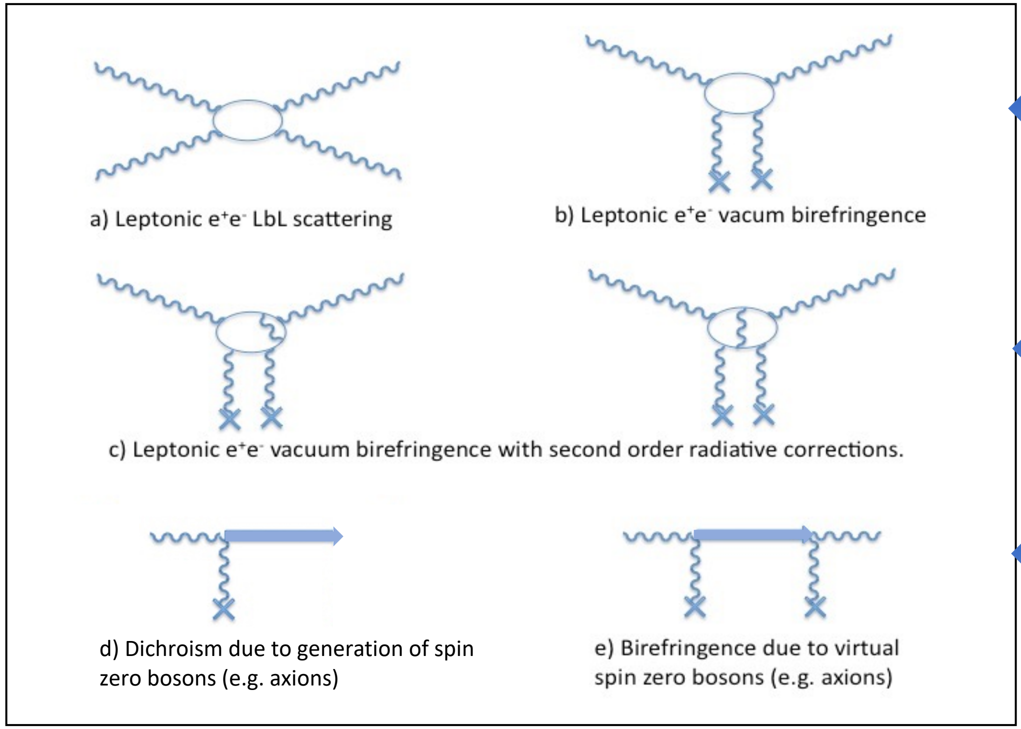
$$\Delta n = C_{\mathrm{SOC}}\left(\sigma_1 - \sigma_2\right)$$

- C_{SOC} = Stess optic coefficiente $[Pa^{-1}]$, σ_1 and σ_2 stress along perpendicular directions $[Pa]$
- Typical values of stress optic coefficient: $C_{\mathrm{SOC}} \approx 10^{-12}\ Pa^{-1}$
- Fused silica: 2.4 x $10^{-12}\ Pa^{-1}$
- Crystalline Silicon (axes): $(0.6 \div 1)$ x $10^{-12}\ Pa^{-1}$
- Some initial work done for stress induced birefringence in Silicon as ET-LF substrate: C. Krüger et al. Class. Quantum Grav. 33 (2016) 015012
- Sapphire: could not find a value for C_{SOC}.

Thank you for your attention

Background work in sensitive polarimetry

Experimental study of the induced birefringence by an external magnetic field in vacuum



Light-by-light interaction and vacuum magnetic birefringence.

Must be there: $\Delta n = 4X10^{-24}\ B^2$ with B in Tesla.

Includes MCPs

Radiative correction 1.45%

Contributions from hypothetical neutral light particles coupling to two photons: ALPs

Euler-Kockel-Heisenberg Lagrangian predicts VMB

$$\mathcal{L}_{\text{EK}} = \frac{1}{2\mu_0}\left(\frac{E^2}{c^2} - B^2\right) + \frac{A_e}{\mu_0}\left[1\left(\frac{E^2}{c^2} - B^2\right)^2 + 7\left(\frac{\vec{E}}{c}\cdot\vec{B}\right)^2\right] + \ldots$$

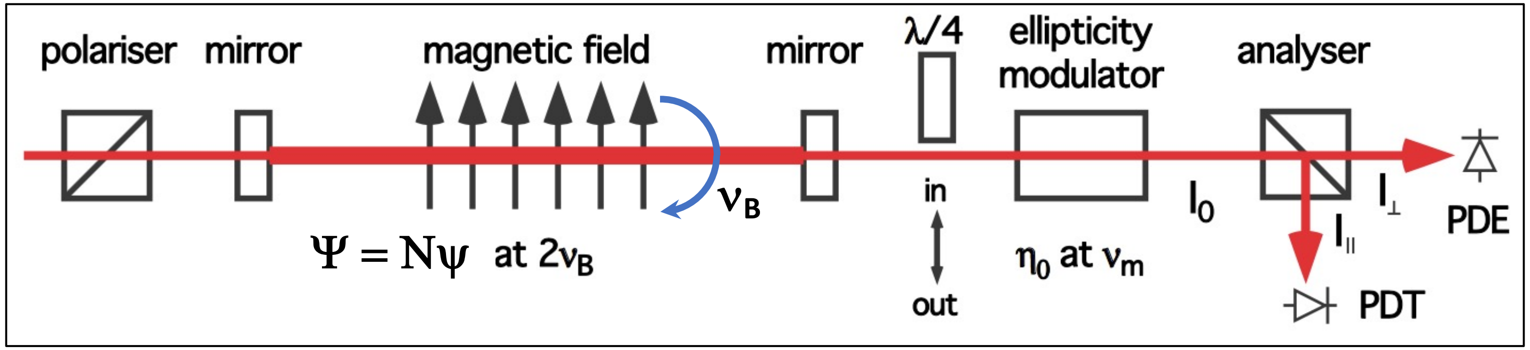
$$A_e = \frac{2}{45\mu_0}\frac{\alpha^2 \lambda\!\!\!{}^{-}_e{}^3}{m_e c^2} = 1.32 \times 10^{-24}\ \text{T}^{-2}$$

$$\Delta n = 3 A_e B_{\text{ext}}^2$$

@ B_{ext} = 2.5 T

$\Delta n = 2.5 \cdot 10^{-23}$

General scheme



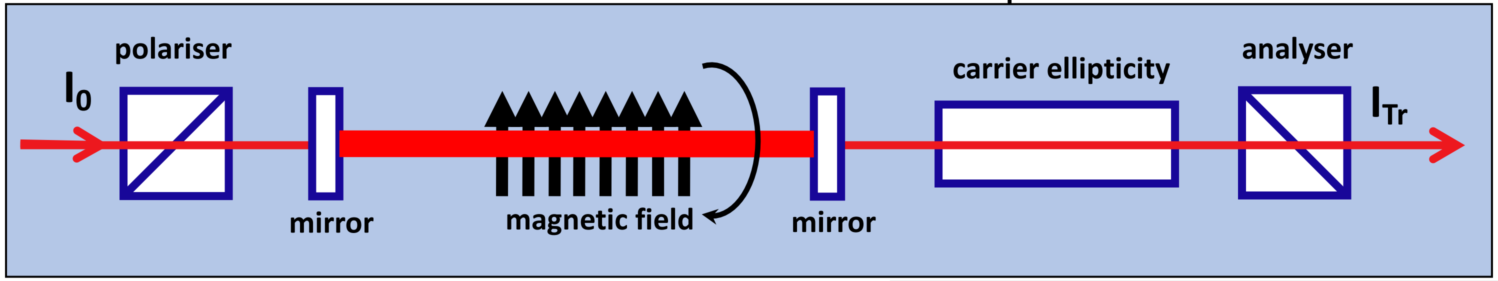
F. Della Valle et al. Eur. Phys. J. C (2016) 76:24
A. Ejlli et al. Physics Reports 871 (2020) 1–74

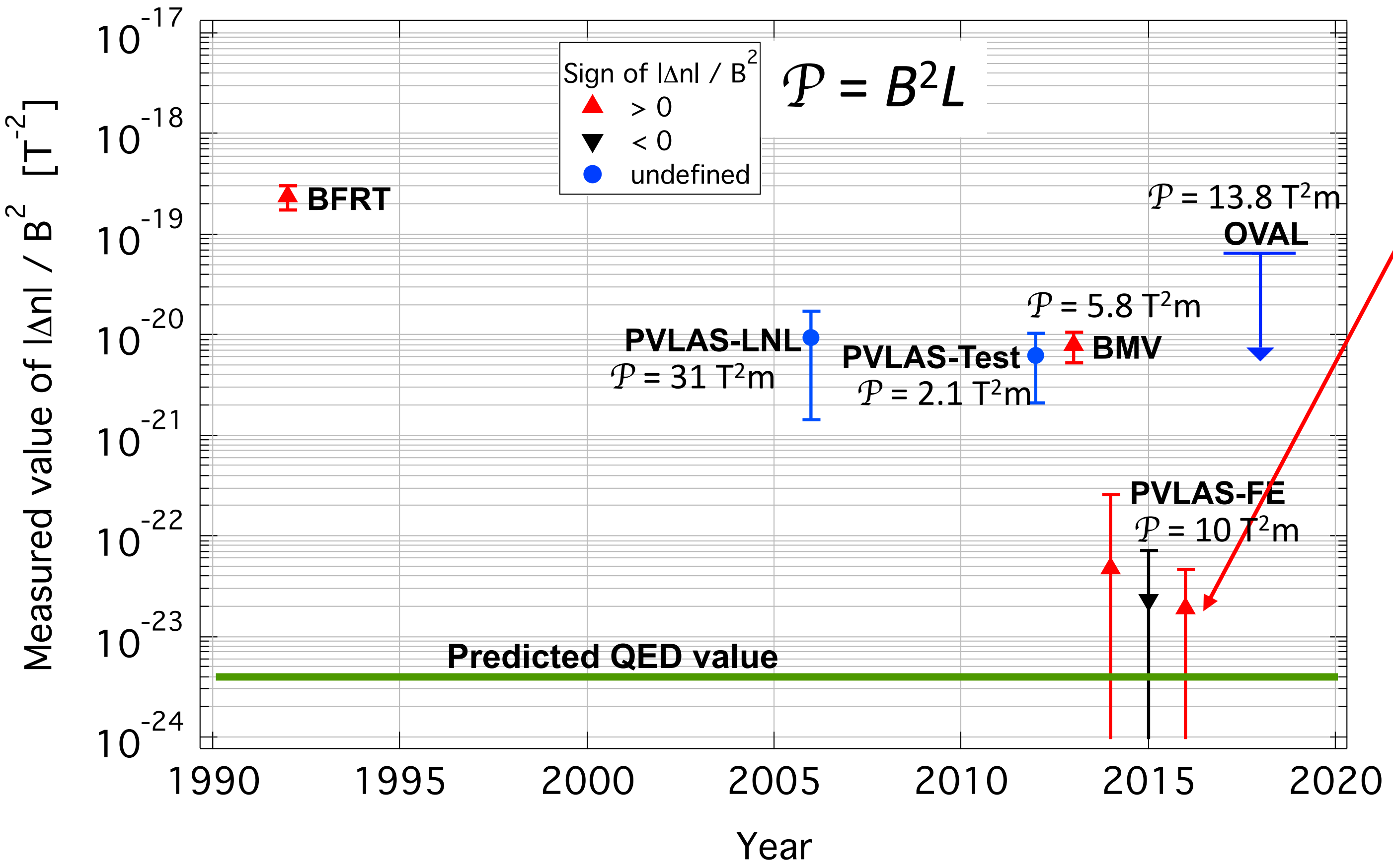
- *L* is the length of the birefringent medium (in our experiment $\Delta n_{\rm B} \propto B^2$)
- Single pass ellipticity: $\psi = \frac{\pi \int \Delta n_{\rm B}\, dL}{\lambda} \sin 2\vartheta(t) = \psi_0 \sin 2\vartheta(t)$. Here ϑ(t) is the angle between the polarisation and the birefringence axis.
- The Fabry-Perot cavity amplifies ψ by a factor $N = 2\mathcal{F}/\pi$. We had $\mathcal{F} = 7 \times 10^5$.
- The ellipticity modulator allows heterodyne detection which linearizes the ellipticity ψ to be measured and allows the distinction between a rotation and an ellipticity. The insertion of the $\lambda/4$ wave plate allows measuring rotations.
- The rotating magnetic field modulates the desired signal due to VMB.

$$\Rightarrow I_{\rm out} \simeq I_0 \left\{\eta^2(t) + 2\eta(t)N\psi(t) + 2\eta(t)\Gamma(t) + \ldots\right\}$$

State of the art

General scheme: modulated or pulsed field





- The PVLAS - FE result remains the most sensitive measurement yet performed: $\Delta n/B^2 = (1.9 \pm 2.7)\cdot 10^{-23}$ T^{-2} with 2.5 T
- Permanent magnets allowed careful debugging of systematics: B^2L = 10 T^2m
- Optical path difference sensitivity: $S_{OPD} = 4\cdot 10^{-19}$ m/√Hz @ ≈ 16 Hz
- Cavity amplification was $N \approx 4.5\cdot 10^5$
- Intrinsic noise from the mirrors limited the sensitivity and the SNR
- Measured noise: x30 shot-noise @ 16 Hz

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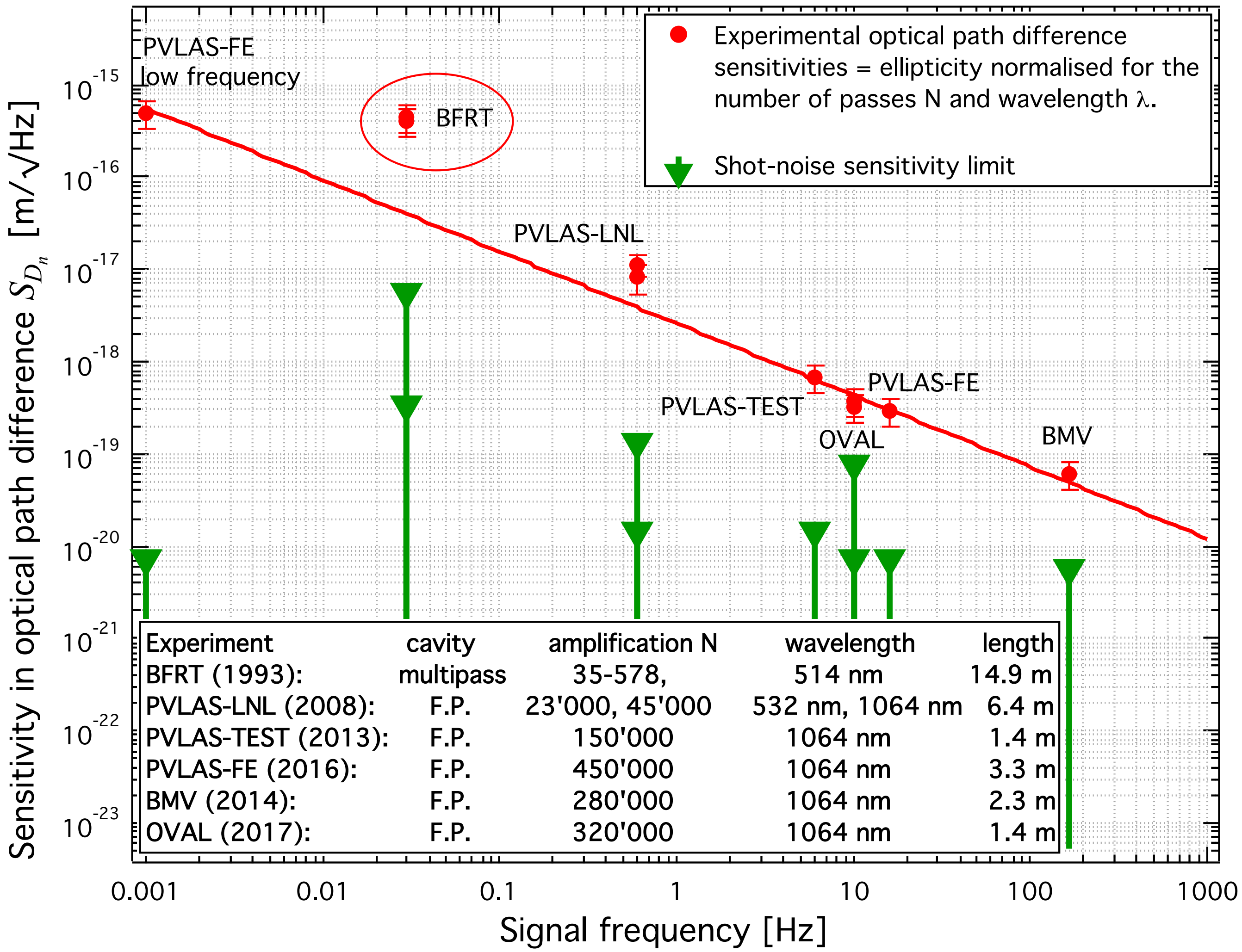
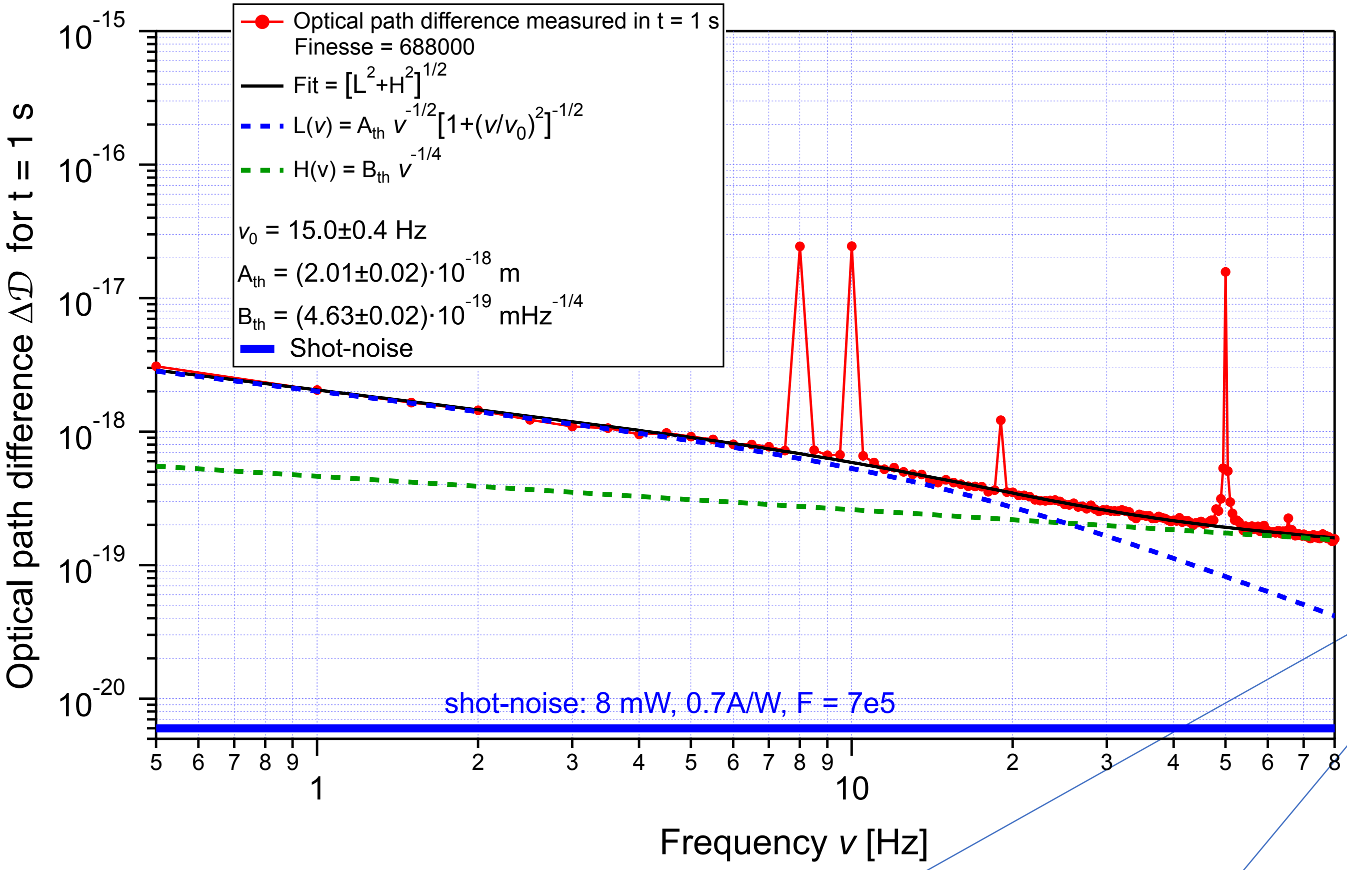
Intrinsic mirror birefringence noise

Limits in the sensitivity of a polarimeter



- No experimental effort has reached shot-noise sensitivity (green) with a high finesse F.P.
- There seems to be a common problem afflicting all experiments
- This noise seems to be an intrinsic property of the cavity mirrors
- With a low finesse cavity one does reach shot-noise. The limit is not the method.

Intrinsic mirror birefringence noise

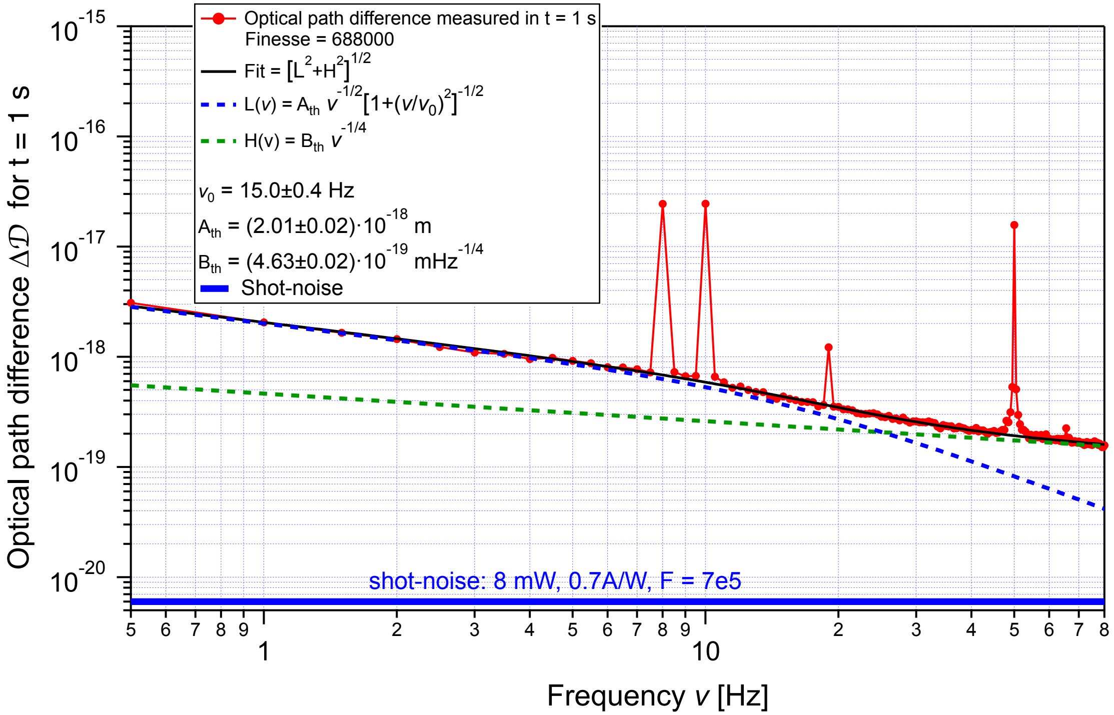


- Typical PVLAS-FE optical path difference noise
- Finesse = 6.88 X 10^5
- Peaks at 8 Hz and 10 Hz represent Cotton-Mouton calibration signals from 850 μbar Argon gas.
- The peak at 19 Hz is generated by a Faraday rotation leakage due to the total cavity static birefringence from the mirrors.
- Brownian? Why the cut-off?
- Thermo-elastic model points to tantala.
- For ET we can measure new coatings. Finesse must be F ≥ 5e4 (R ≥ 99.995%): the amplified mirror noise must be greater than shot-noise.
- Will be testing crystaline GaAs/AlGaAs mirrors.

$$S_{\mathrm{OPD}}(\nu) = \sqrt{\left(\frac{A_{\mathrm{th}}\nu^{-1/2}}{\sqrt{1+(\nu/\nu_0)^2}}\right)^2 + \left(B_{\mathrm{th}}\nu^{-1/4}\right)^2}$$

$$A_{\mathrm{th}} = (2.01 \pm 0.02)\times 10^{-18}\ \mathrm{m}, \quad \nu_0 = (15.0 \pm 0.4)\ \mathrm{Hz}, \quad B_{\mathrm{th}} = (4.63 \pm 0.02)\times 10^{-19}\ \mathrm{m/Hz}^{1/4}$$

Intrinsic mirror birefringence noise



- Estimated the thermoelastic birefringence noise in reflection*
- C_{SO} = stress optic coefficient
- Y = Young's modulus
- α_T = thermal expansion coefficient
- r_0 = beam radius on mirror
- C_T = specific heat capacity
- ρ = density
- λ_T = themal conductivity

Temperature spectral density

$$S_T(\nu) = \sqrt{\frac{8 k_B T^2}{\pi r_0^2 \sqrt{\pi \rho C_T \lambda_T \nu}}} \propto \nu^{-1/4}$$

Optical path difference spectrum

$$S_{\Delta\mathcal{D}} = 2 d_e \sqrt{2} C_{SO} Y \alpha_{\mathrm{T}} S_T(\nu)$$

Fused silica

$$S_{\Delta\mathcal{D}}^{(\mathrm{FS})} \sim 4 \times 10^{-21}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Tantala

$$S_{\Delta\mathcal{D}}^{(\mathrm{Ta})} \sim (1 \div 5) \times 10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}} \quad @ \quad 1\ \mathrm{Hz}$$

Compatible with $B_{\mathrm{th}} = (4.63 \pm 0.02) \times 10^{-19}\ \mathrm{m/Hz}^{1/4}$ from the fit

*Physics Reports 871 (2020) 1–74

HWP defect issues: temperature and alignment

$$\psi(t) = \psi_0 \sin 4\phi(t) + \frac{\alpha_1(t)}{2} \sin 2\phi(t) + \frac{\alpha_2(t)}{2} \sin[2\phi(t) + 2\Delta\phi(t)]$$

Generating 4th harmonic from $\alpha_{1,2}(t)$ in $\psi(t)$: Expansion of the intrinsic HWP defects $\alpha_{1,2}(t)$:

$$\alpha_{1,2}(\phi, \mathrm{T}, r) = \alpha^{(0)}_{1,2}(\mathrm{T}) + \alpha^{(1)}_{1,2}(\mathrm{r(t)}) \cos\phi(\mathrm{t}) + \alpha^{(2)}_{1,2} \cos 2\phi(\mathrm{t}) + \ldots$$

- $\alpha^{(0)}_{1,2}$ (from manufacturer) depends on TEMPERATURE T and appears @ 2^{nd} harmonic in $\psi(t)$
- $\alpha^{(1)}_{1,2}$ depends on WEDGE of wave-plates and their ALIGNMENT: appears @ 1^{st} and 3^{rd} harmonic in $\psi(t)$
- $\alpha^{(2)}_{1,2}$ depends on ALIGNMENT generating 4^{th} harmonic in $\psi(t)$ just like a birefringence signal.
- Time modulation of $\alpha^{(1)}_{1,2}$ due to transverse axis oscillation will also generate a 4^{th} harmonic in $\psi(t)$

$$r(t) = r_0 + \delta r \cos(\phi(t) + \phi_{\delta r})$$

The resulting ellipticity is the combination of the two HWPs.

✓ They can be aligned separately using a frequency doubled laser @ 532 nm