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Extreme K/γ regime in ion channel radiation

A. Frazzitta[1,2,*], M. Yadav[3,*], J. Mann[3], A.R. Rossi[2] and J. B. Rosenzweig[3]
[1] *Department of Physics, University of Rome "La Sapienza", p.le A. Moro, 2-00185 Rome, Italy*
[2] *Department of Physics, INFN—Milan, Via Celoria, 16-20133 Milan, Italy*
[3] *Department of Physics and Astronomy, University of California, Los Angeles, USA*

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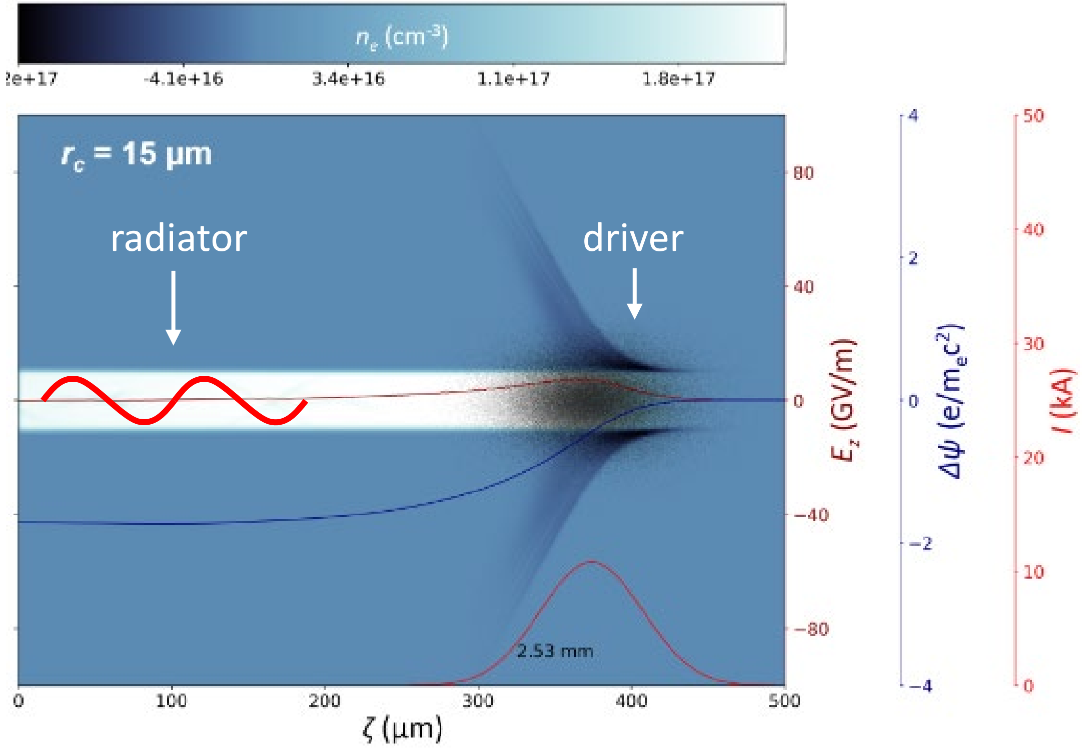
Ion channel: more than blowout

- **IC**: plasma based focusing device
- Ionizing a narrow channel of gas, a strong driver may lead to a **complete electron depletion**, with low longitudinal field
- Nearly **pure electrostatic focusing** ion column: high energy radiation production
- Many concepts coming from undulator theory in radiation modeling: **undulator strength K**

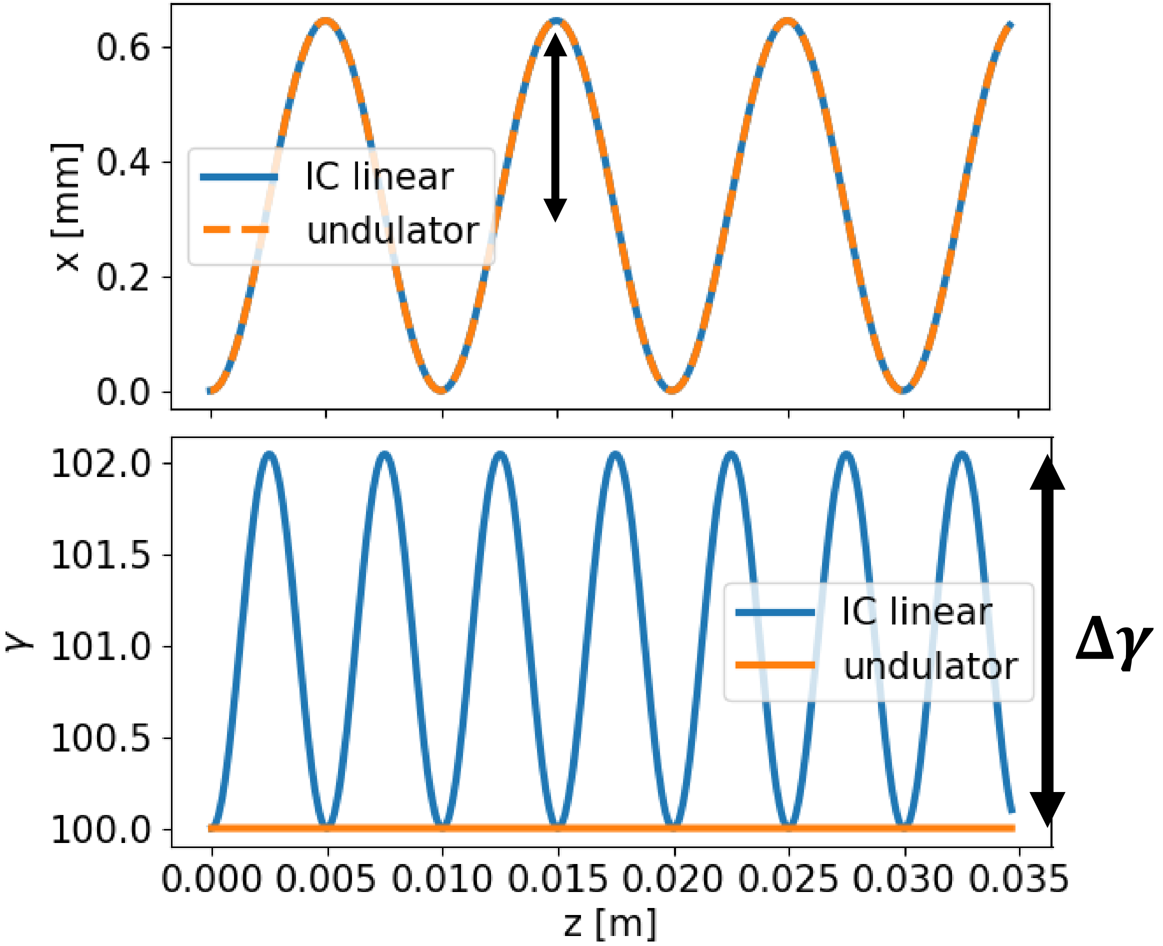


Habib, A. F., et al. **Ultrahigh brightness beams from plasma photoguns**. arXiv. https://doi.org/10.48550/ARXIV.2111.01502

Strength parameter *K*

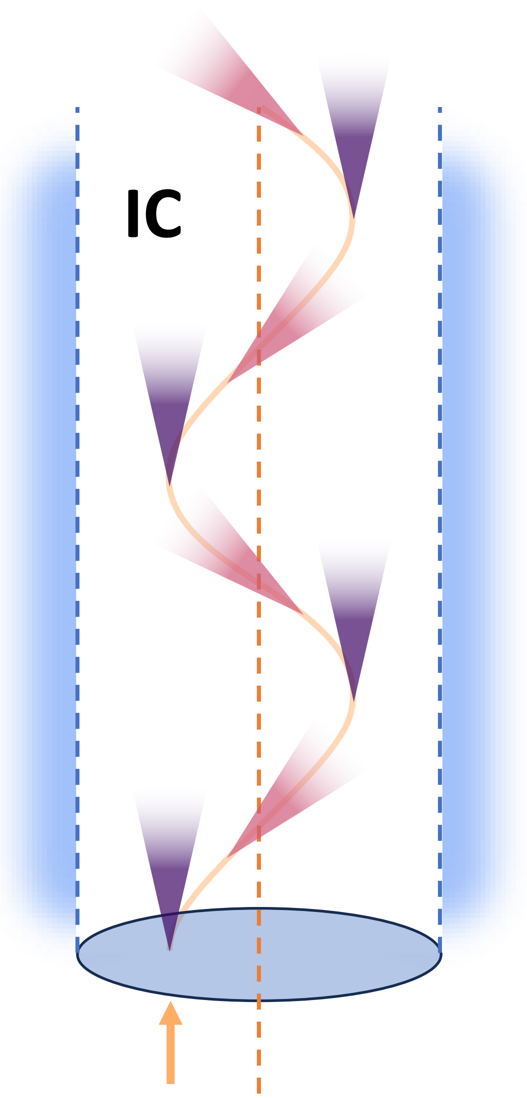
- In undulators ***K* defines the normalized magnetic field strength**, and *gives* the amplitude of the trajectory oscillation
- In IC *K is given* by beam **injection offset** respect to device axis
- For now, if *K* is made the same, trajectories look matched, but they actually differ… in **energy**

$$x_0 = K\gamma k_b$$



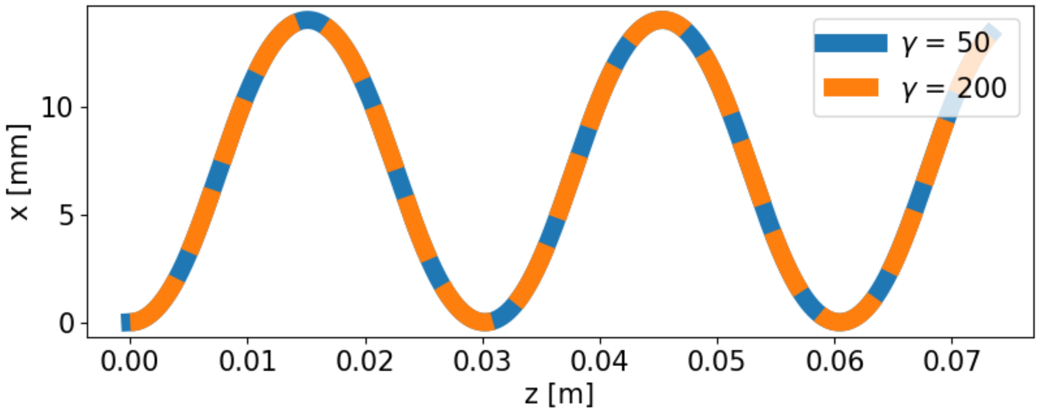
Outline

- $\boldsymbol{K/\gamma}$ looks like the right variable for radiation regimes exploration
- Energy oscillations break linear theory: a fully **nonlinear particle dynamics and radiation** description has been performed
- What did we find out?
  - ➢Possibly interesting **IC radiation regimes**
  - ➢**Finer theory** for known regimes
  - ➢Unexpected **particle dynamics effects**



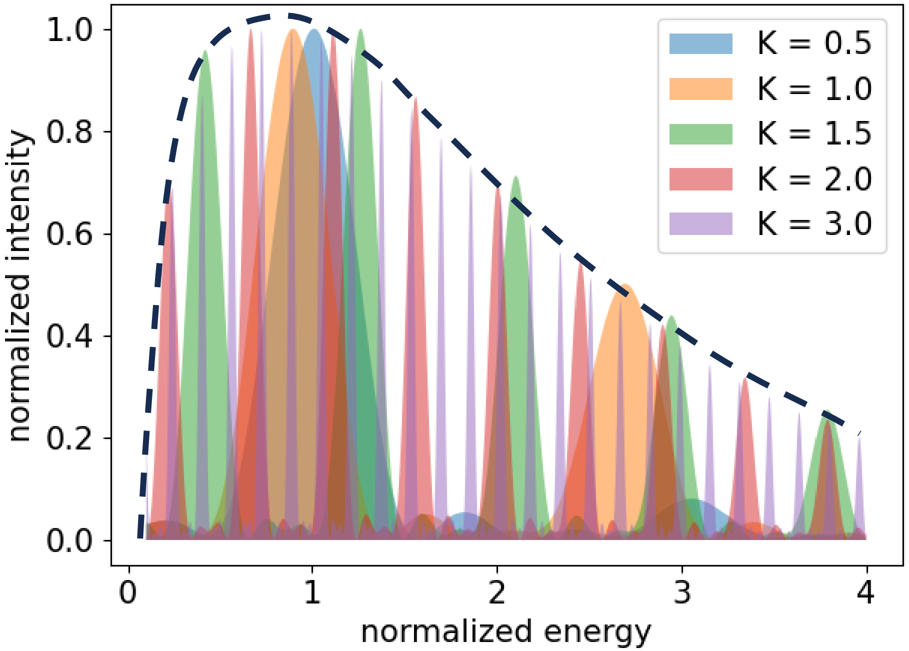
Why K/γ is it the right variable

- Even in highly nonlinear cases, K/γ **is the most general parameter both in undulator and IC:** it defines trajectory geometric properties



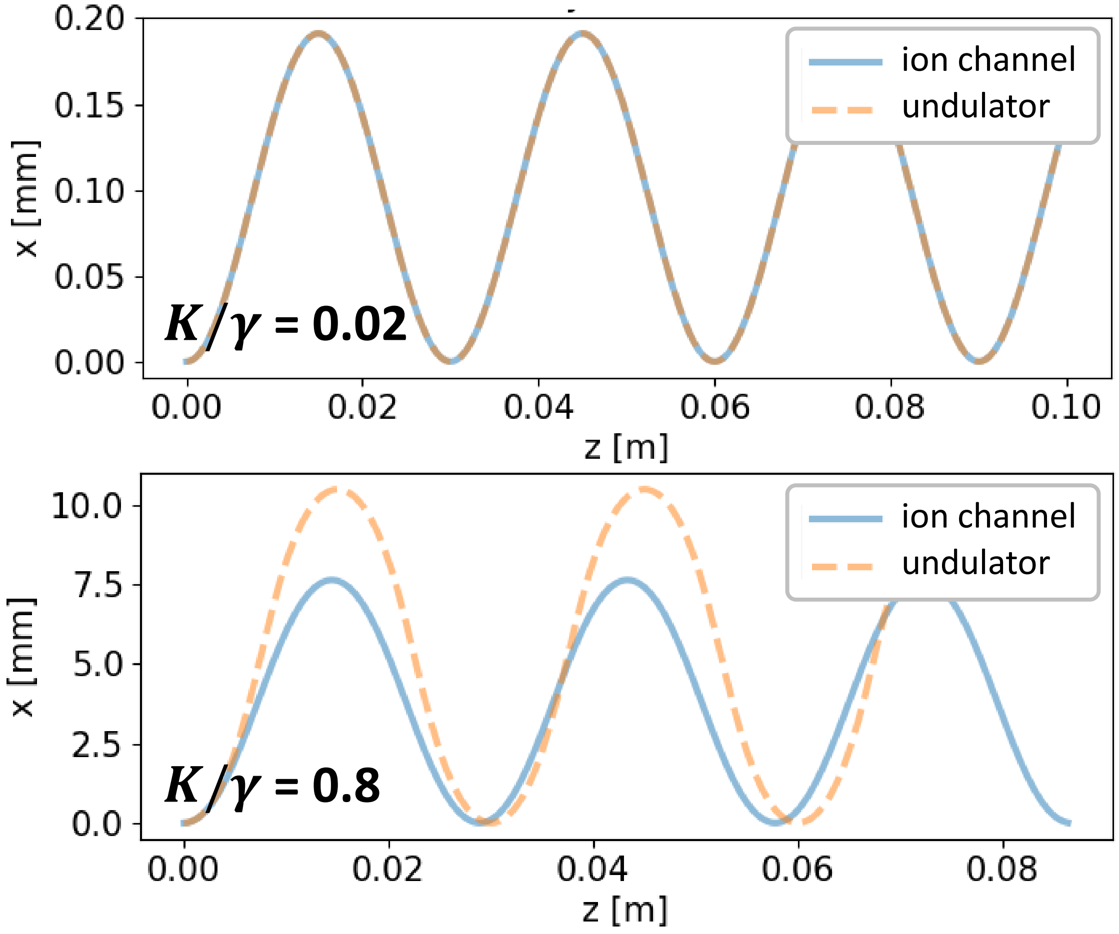
IC, K/γ = 0.9

- Radiation spectrum is **scaled over beam energy**, laying in a K/γ dependent envelope



Comparing IC and undulator: first issues

- For low K/γ **IC and undulator are one and the same**
- For high K/γ **IC amplitude is smaller** and **betatron wavelength is shorter** than expected
  - ➢ This has to do with breaking the linear limit of equations, but how exactly?
- IC amplitude is manually set in the simulation through **undulator equations**: starting point

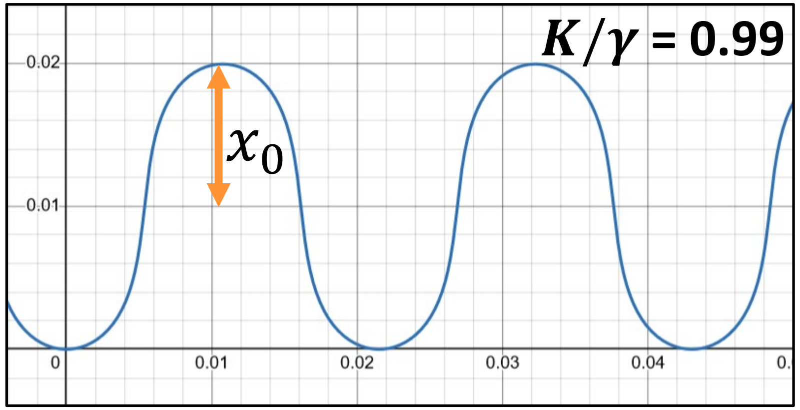


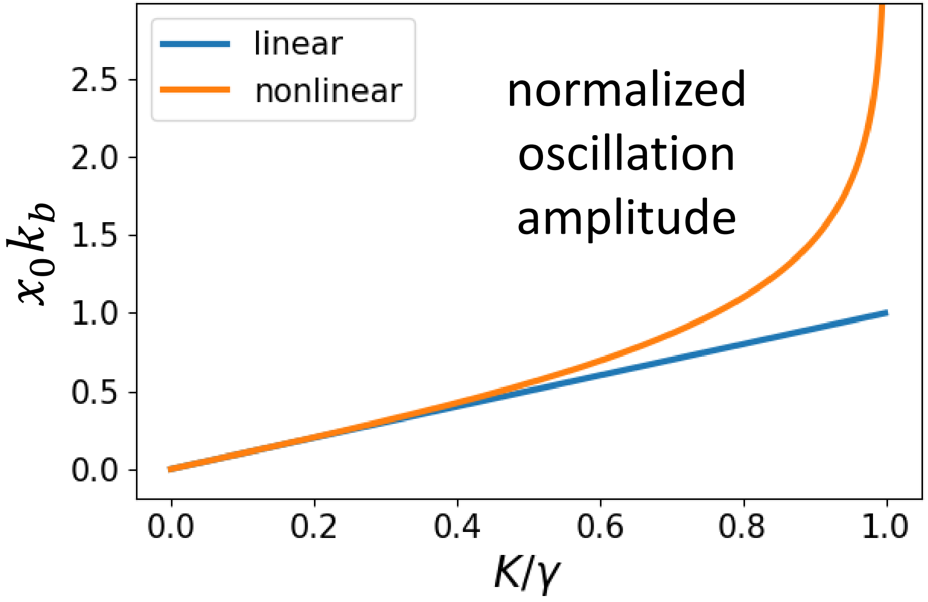
Undulator: amplitude correction

- Ideal undulator field + beam rigidity eq
  → **analytical wiggler trajectories**

$$x_0 = \frac{\operatorname{arctanh} K/\gamma}{k_b} \xrightarrow[K \to 0]{} \frac{K}{\gamma k_b}$$

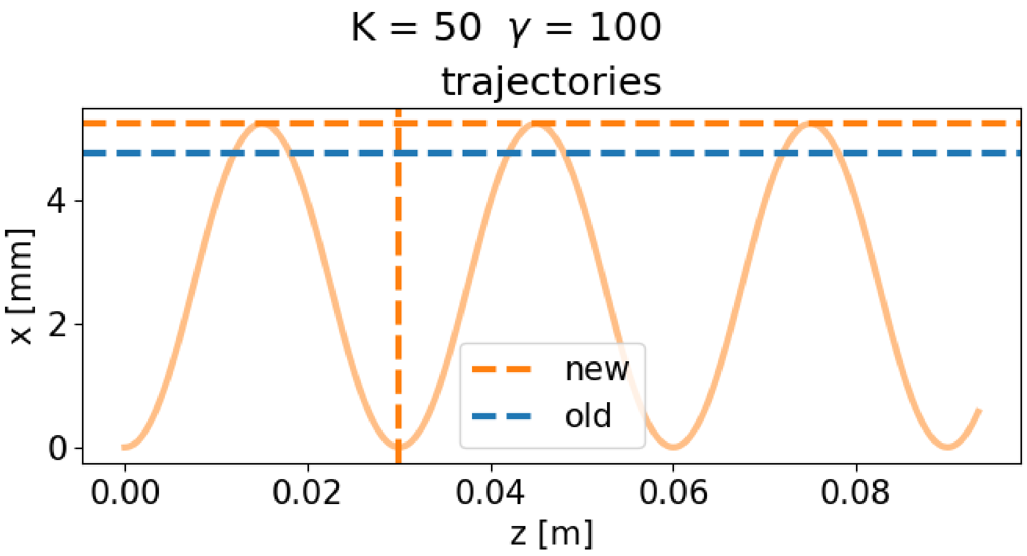
- Amplitude $\boldsymbol{x_0}$ goes linearly as expected for $K/\gamma \to 0$ and **diverges as $\boldsymbol{K/\gamma \to 1}$**
- Ideal limit: after $K/\gamma = 1$, no undulation takes place as **the beam is stuck at first magnetic element**

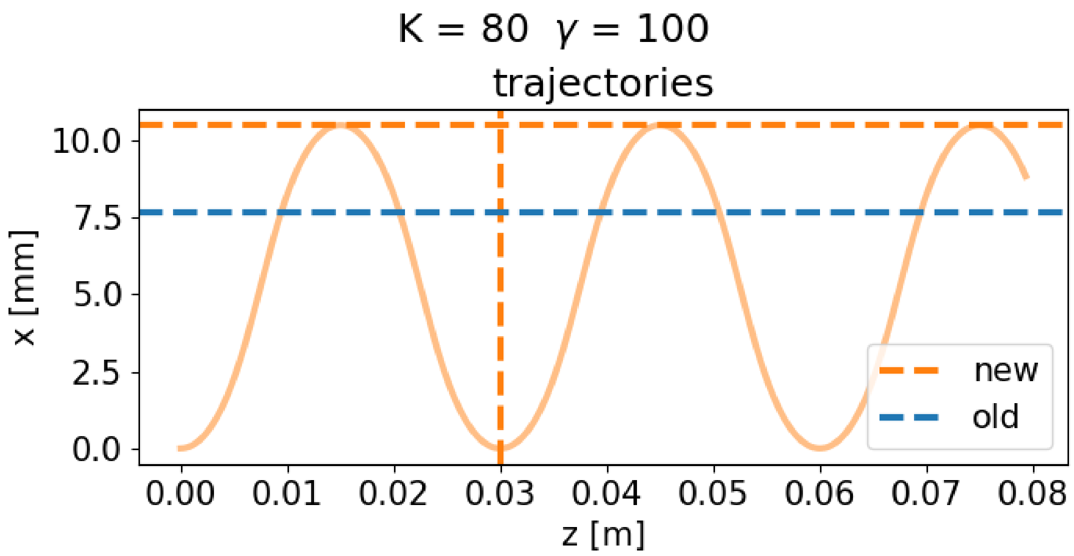




Undulator: amplitude correction

- Numerical checks in agreement with analytical prediction

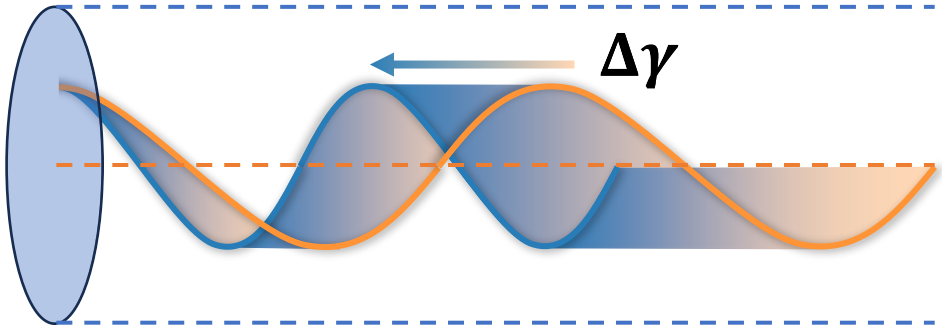


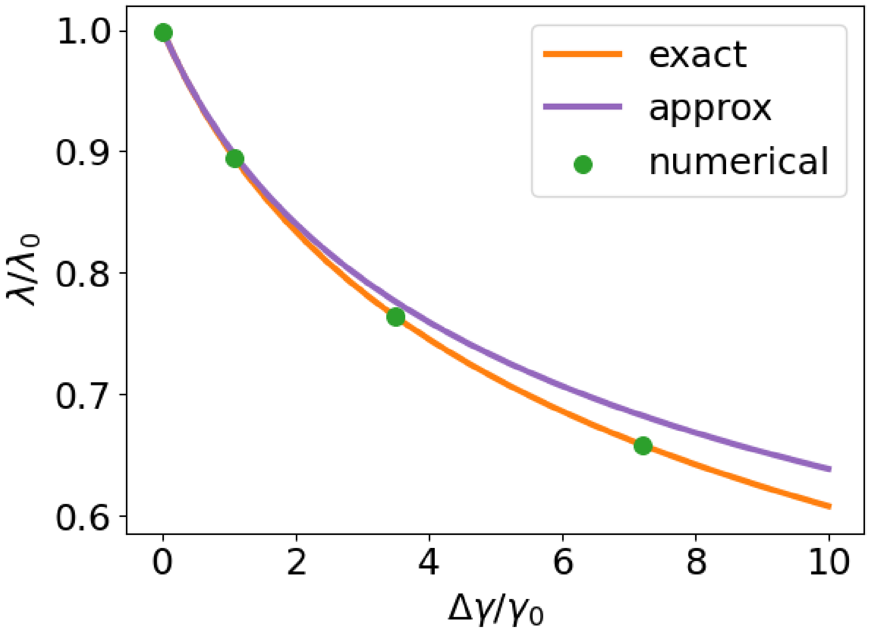


- …great but what about betatron wavelength mismatch? → $\boldsymbol{\Delta\gamma}$

Ion channel: wavelength correction

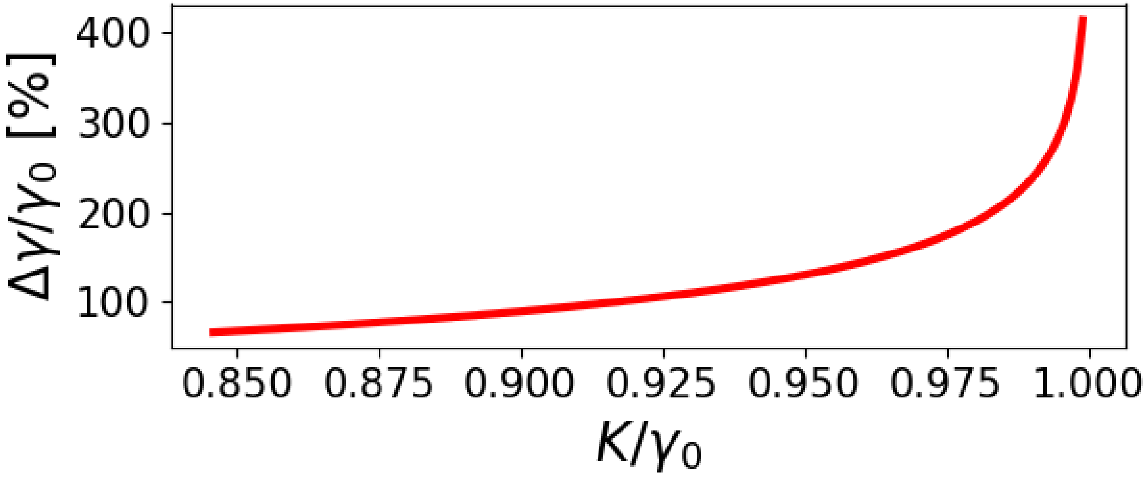
- For **high K/γ :**
  - **greater trajectory angle** due to high transverse momentum, that means **lower average longitudinal speed** and, at a given oscillation period, **shorter wavelength**
  - energy fluctuation $\boldsymbol{\Delta\gamma}$ changes beam rigidity: **greater effective oscillation period**, but not enough to compensate for longitudinal slowdown
- Global effect: nonlinear **wavelength shortening**, analytically described

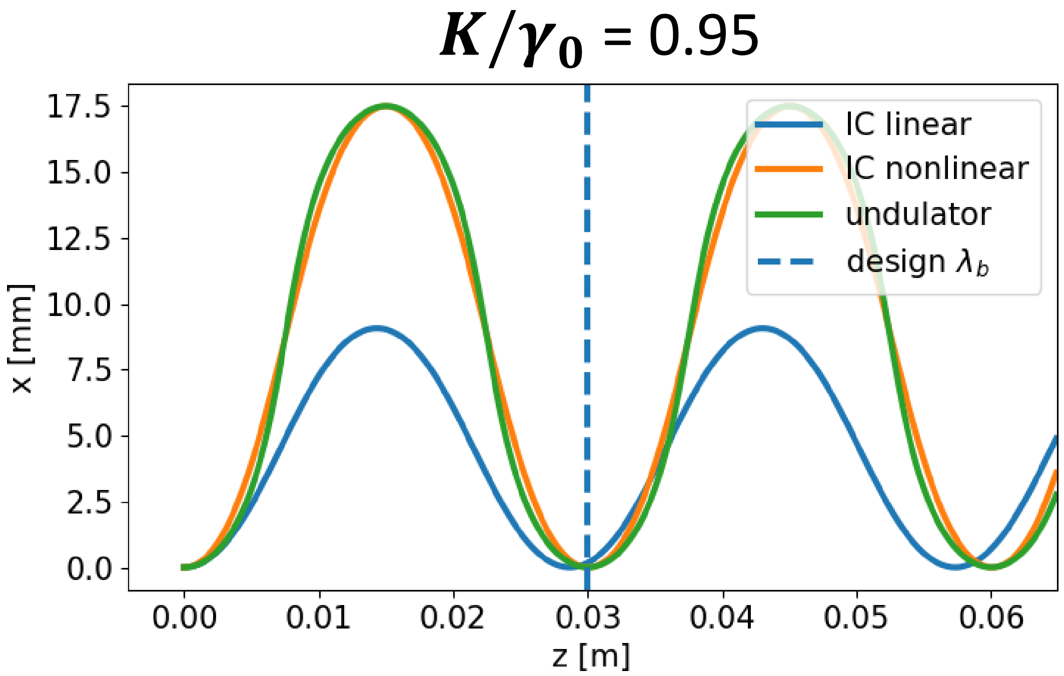




IC vs UND: proper comparison

- Plasma density and K correction give **amplitude and λ_b matching**
- Great **energy oscillations** are found as $K/\gamma \to 1$: big role in **spectral features** definition

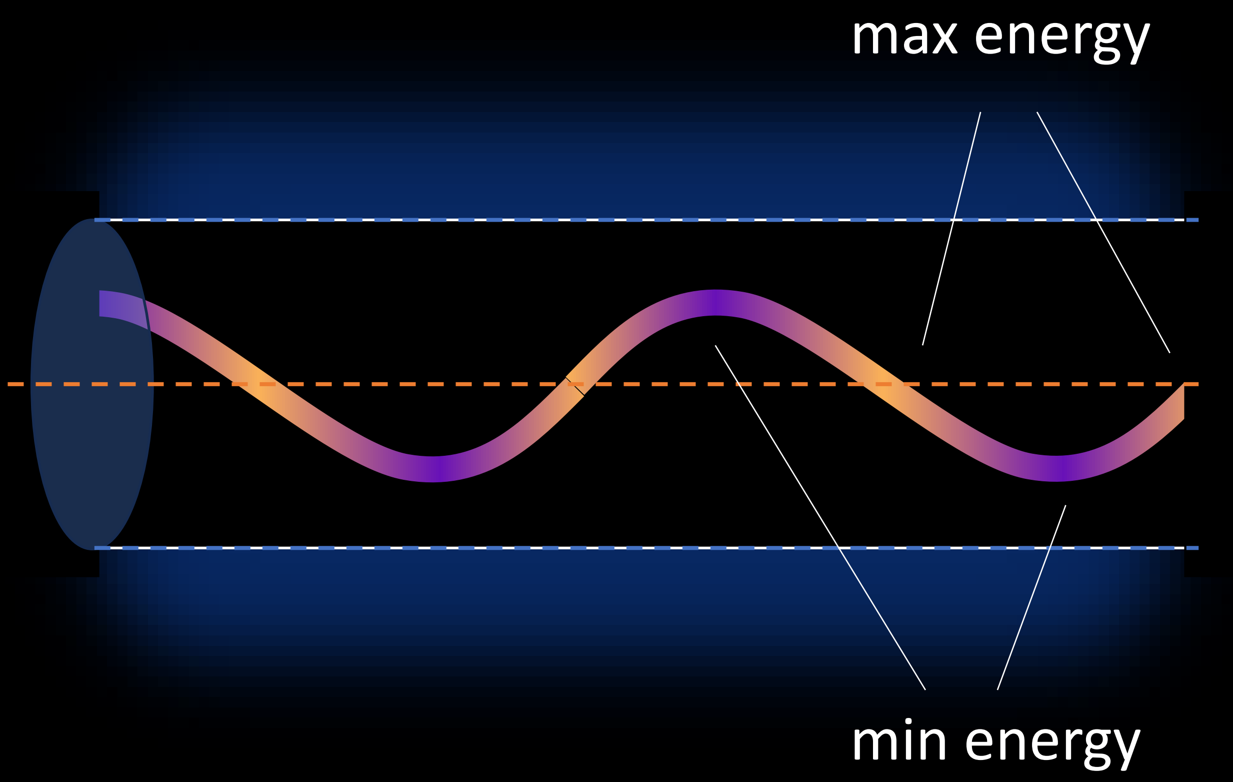


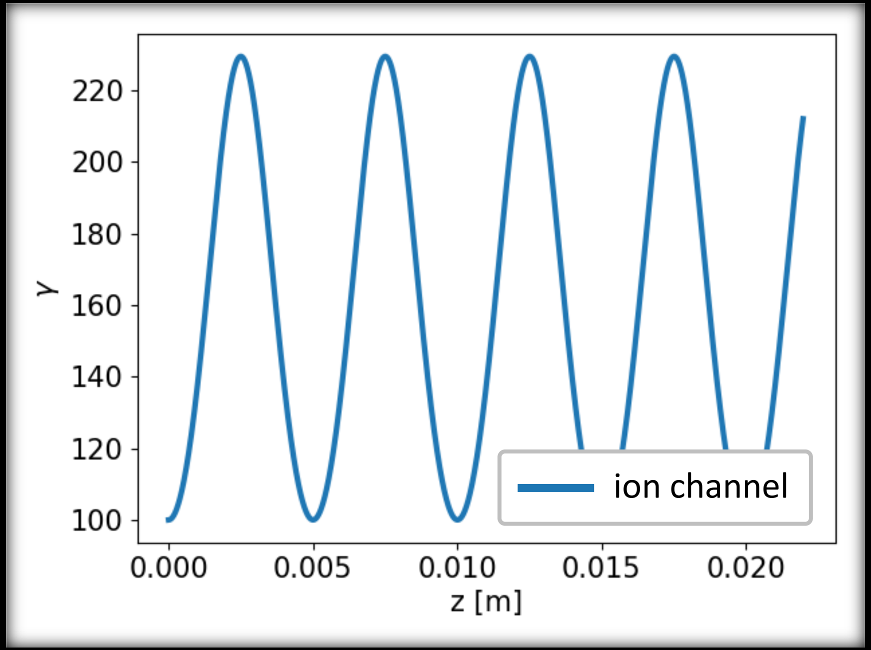


$$x_0 = \frac{\operatorname{arctanh} K/\gamma_0}{k_b}$$

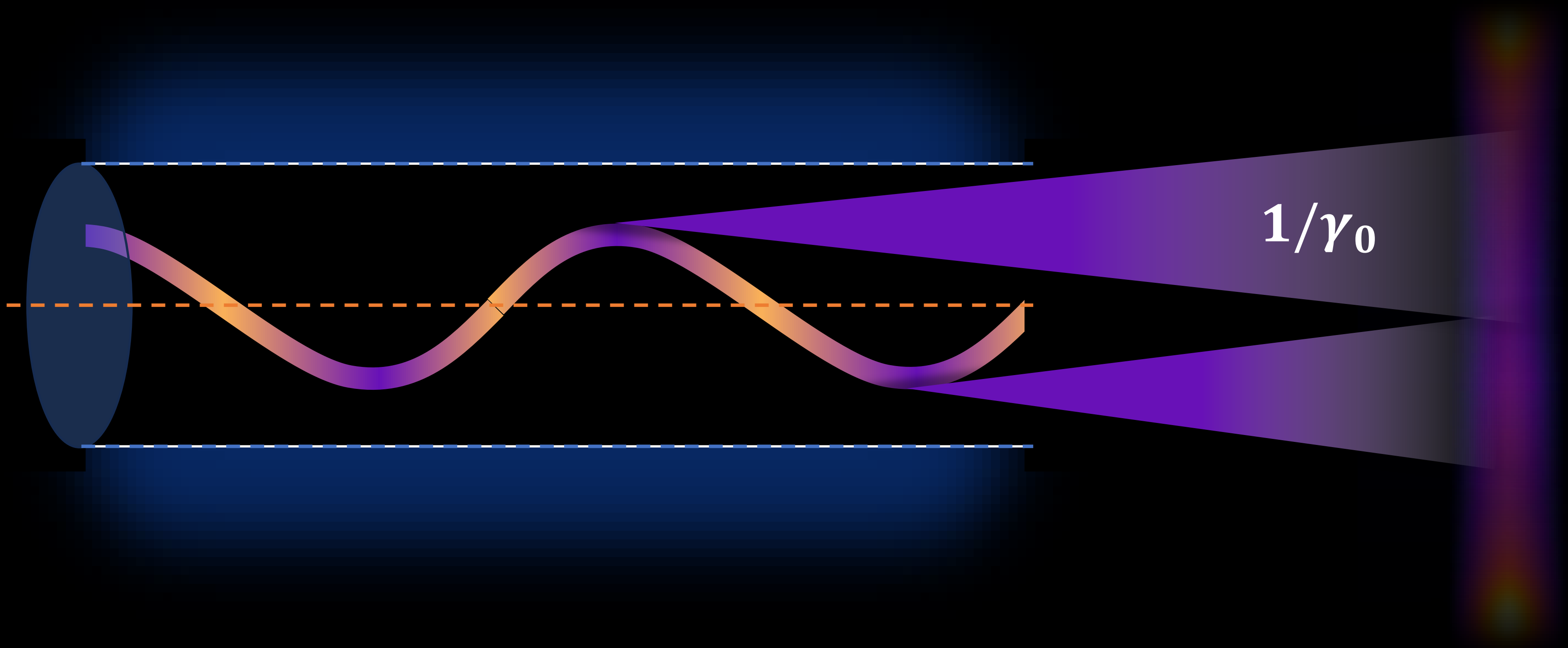
$$n_p \approx n_{p,0}\left(\frac{2}{3} + \left(\frac{k_b x_0}{2}\right)^{4/3}\right)^{-1/2}$$

IC spectrum for high K/γ



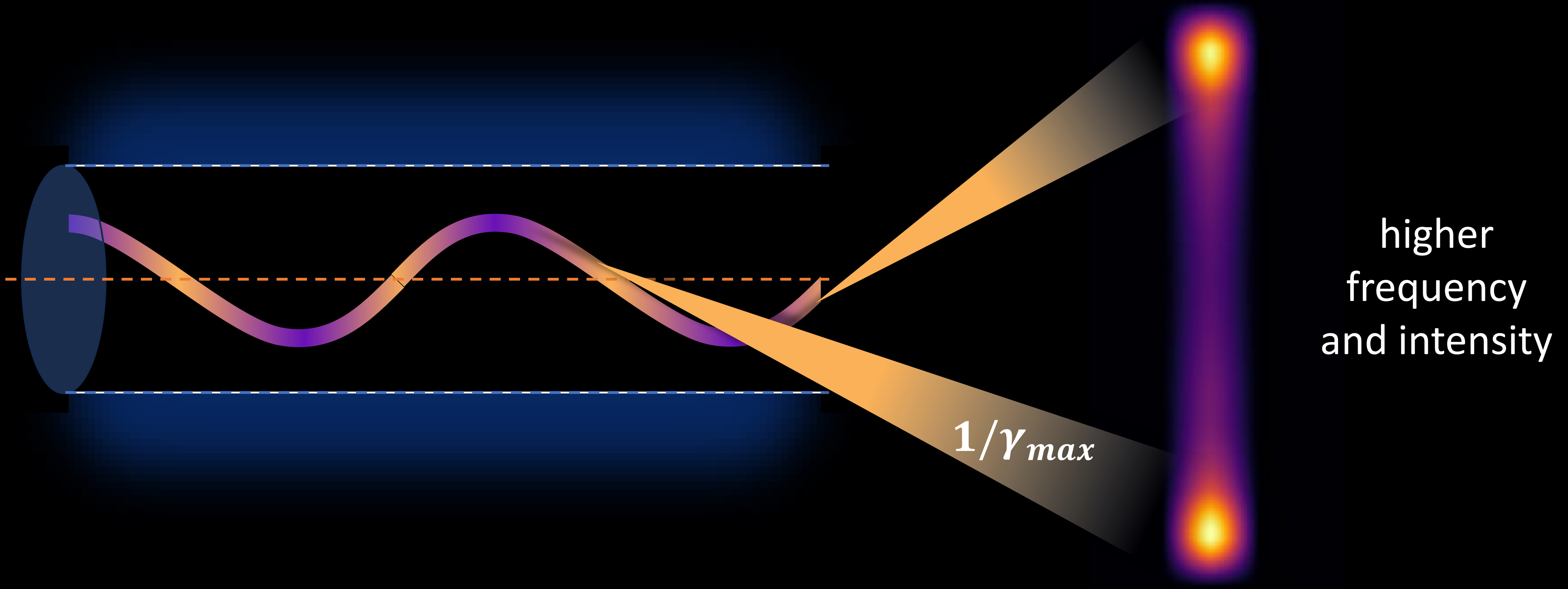


IC spectrum for high *K/γ*



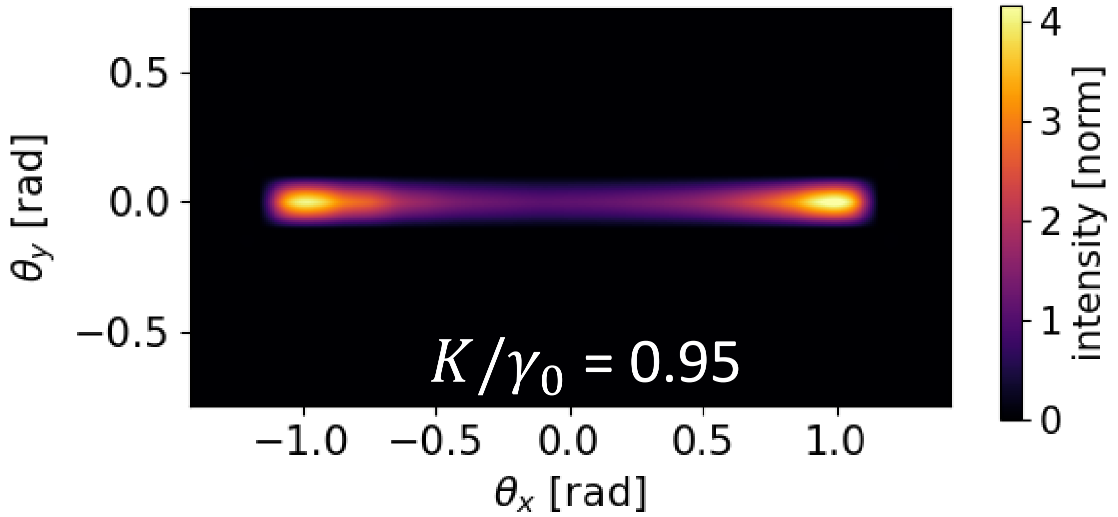
expected frequency and intensity

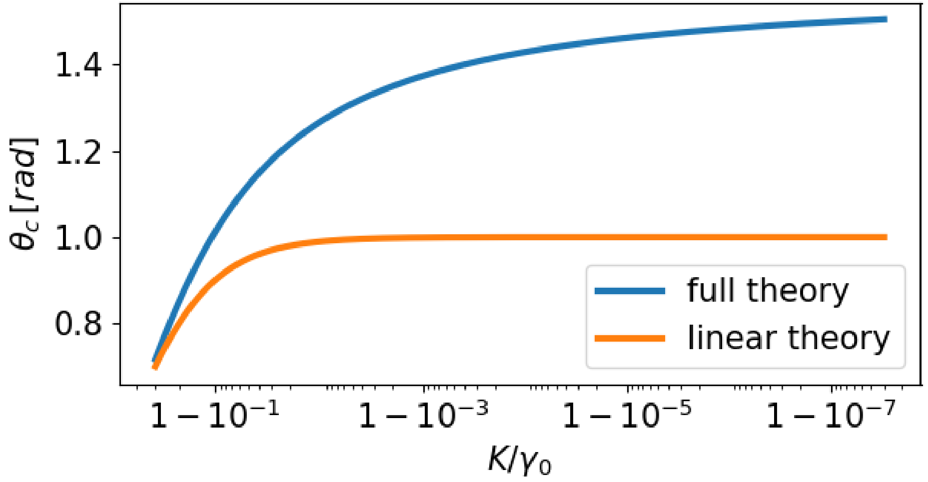
IC spectrum for high K/γ

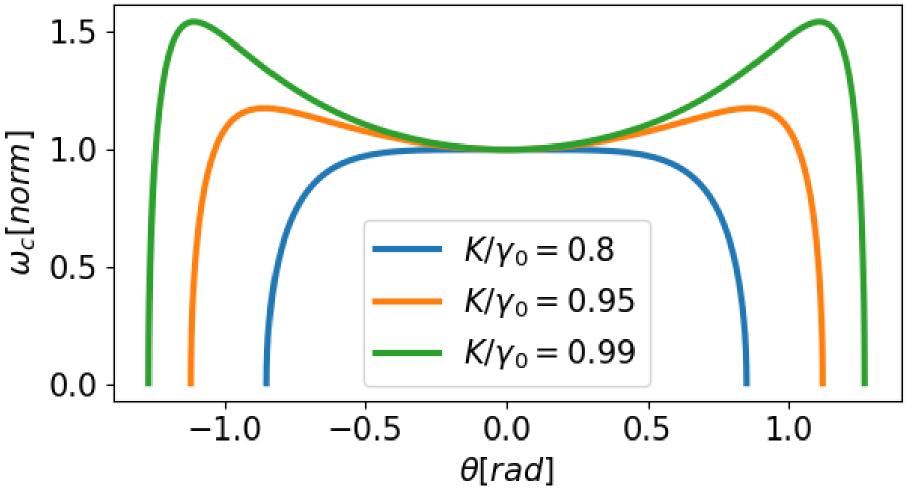


IC spectrum for high *K/γ*

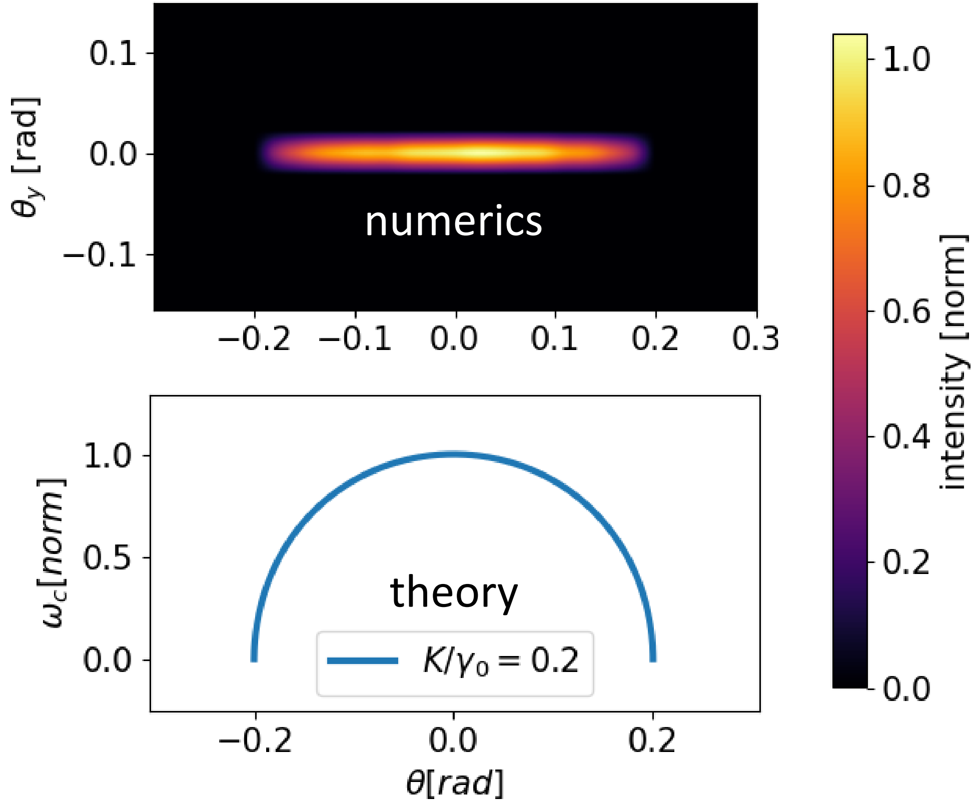
- For growing K/γ, **radiation intensity** and **critical frequency** from planar trajectories gets peaked around two **lobes near critical angle**, that gets values $> \pm$**1 rad**

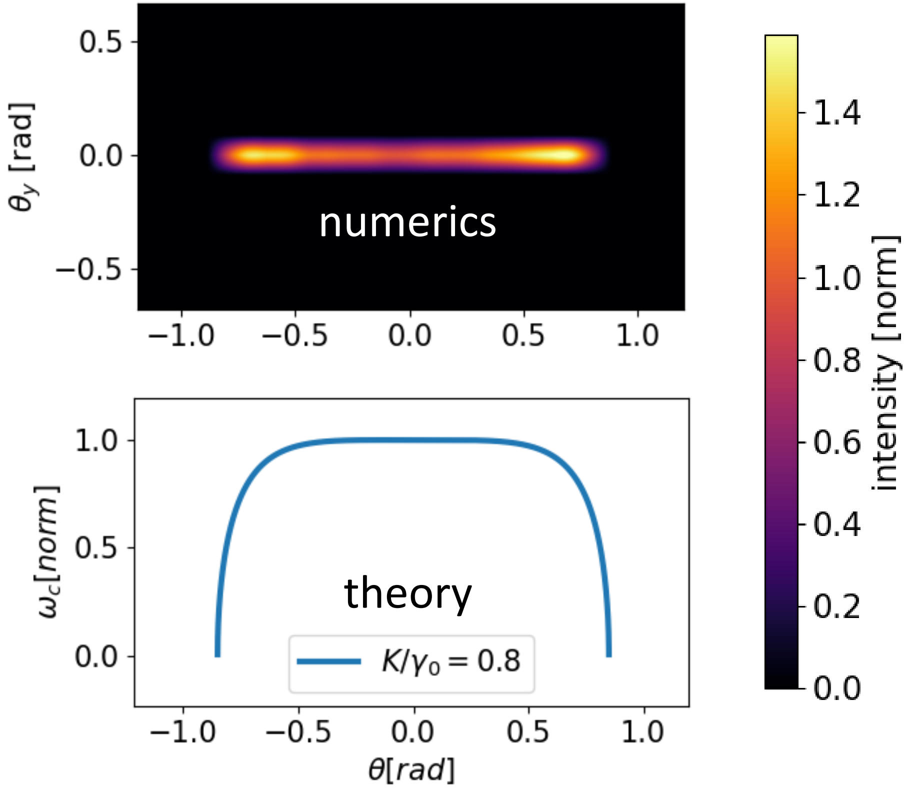






IC single particle radiation spots and critical frequency





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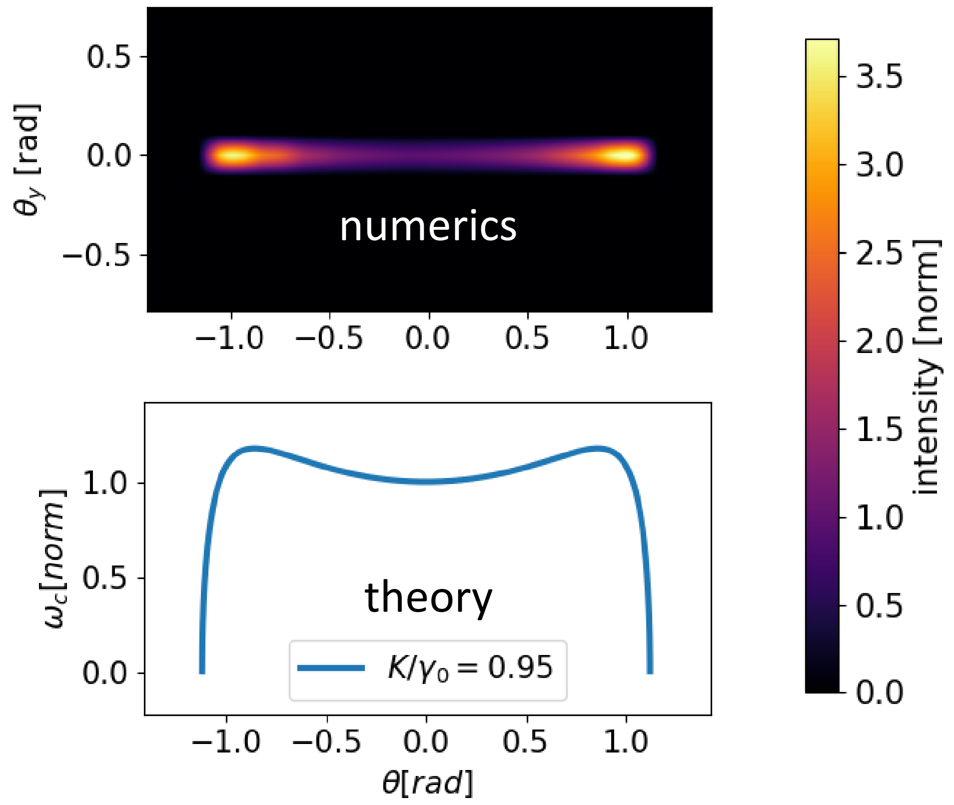
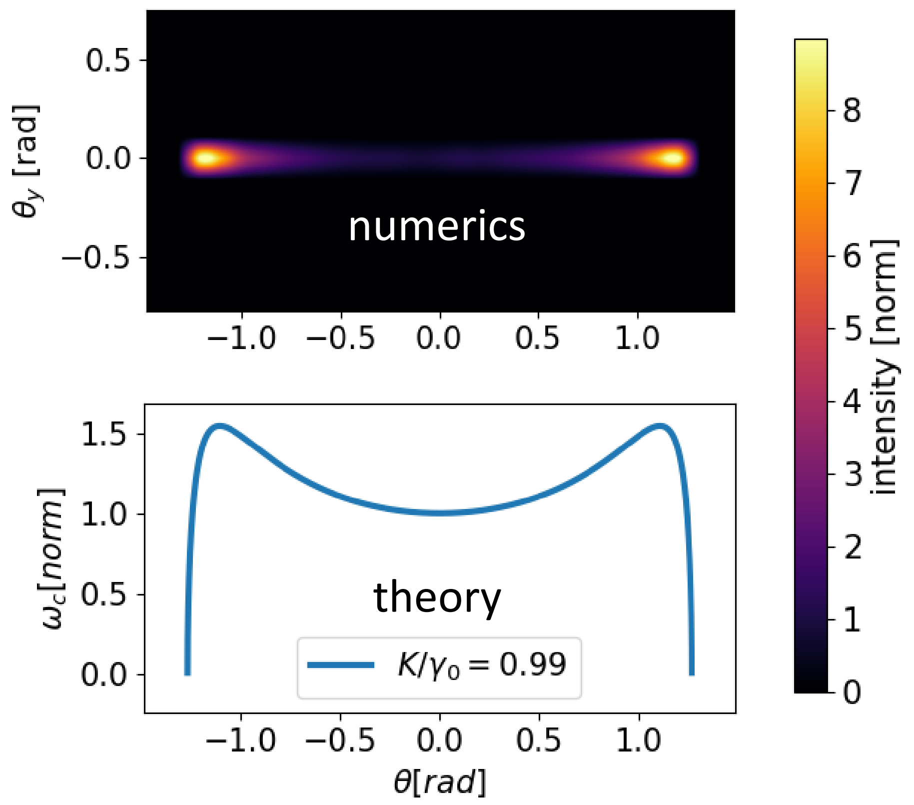
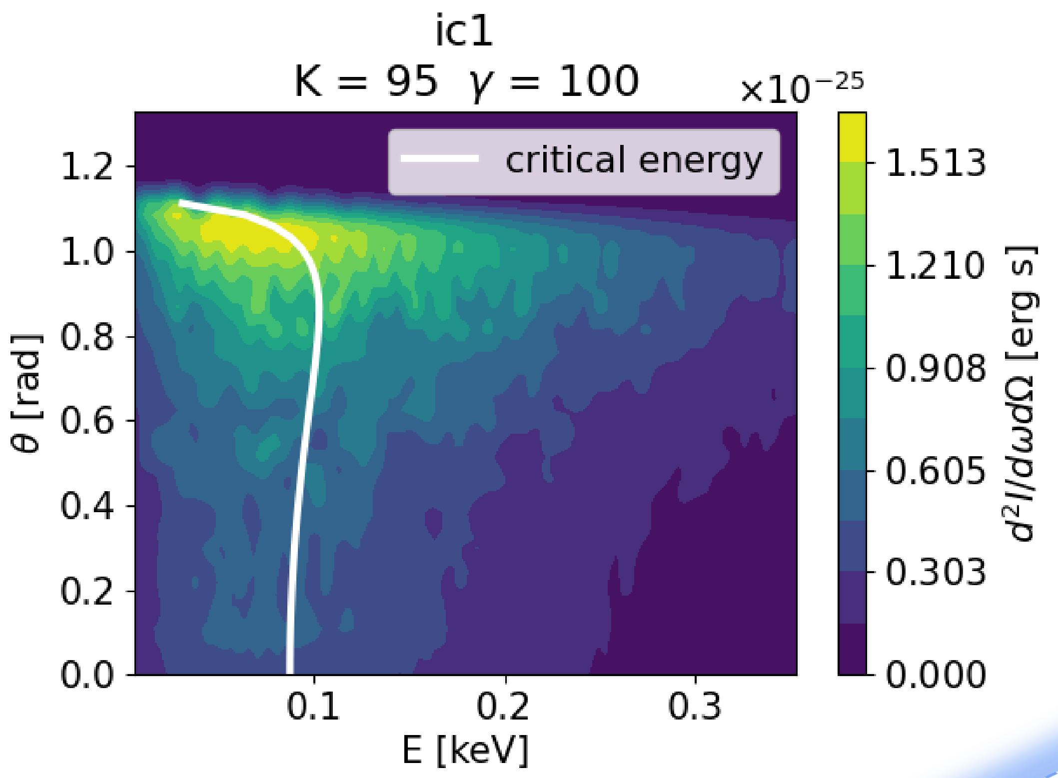
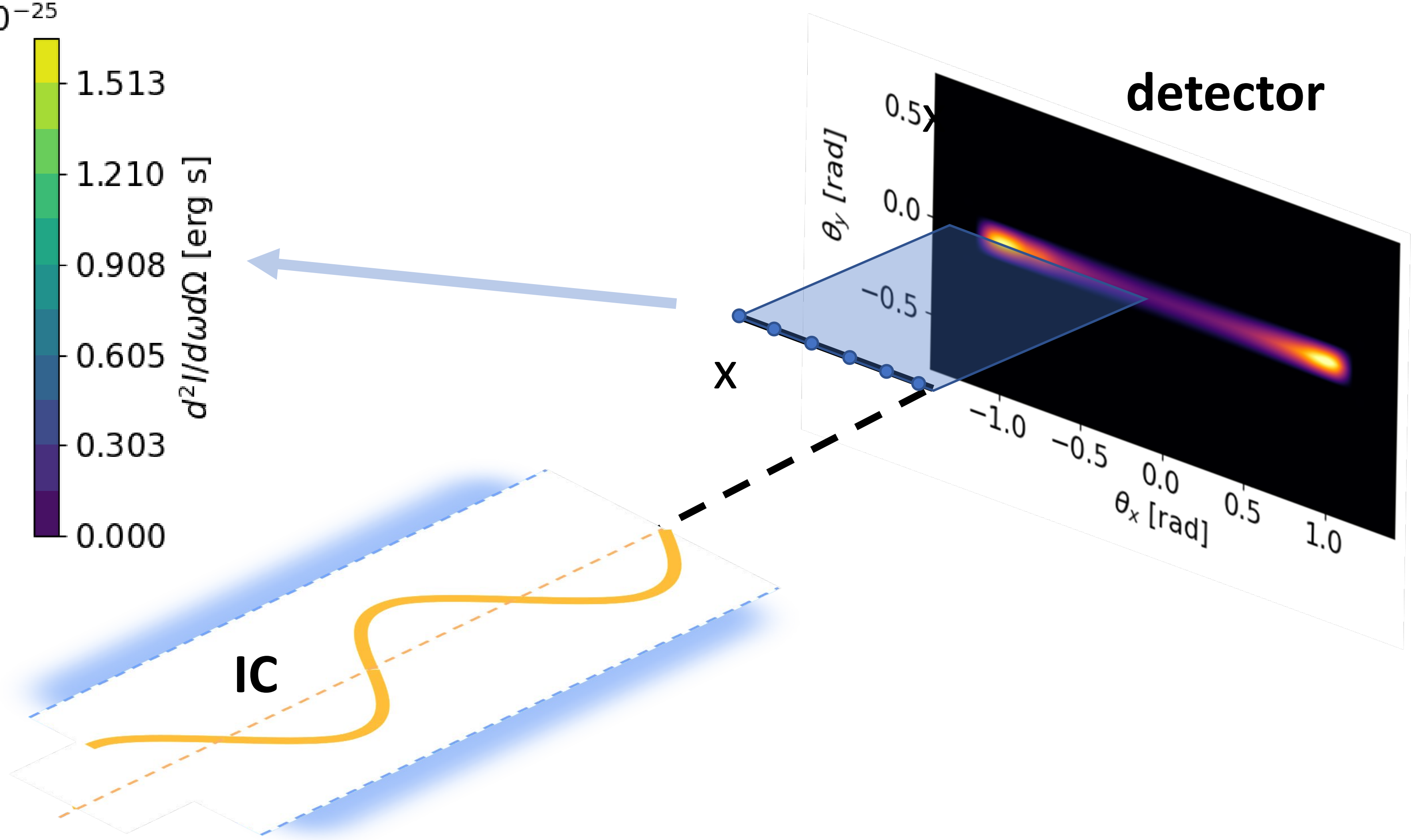




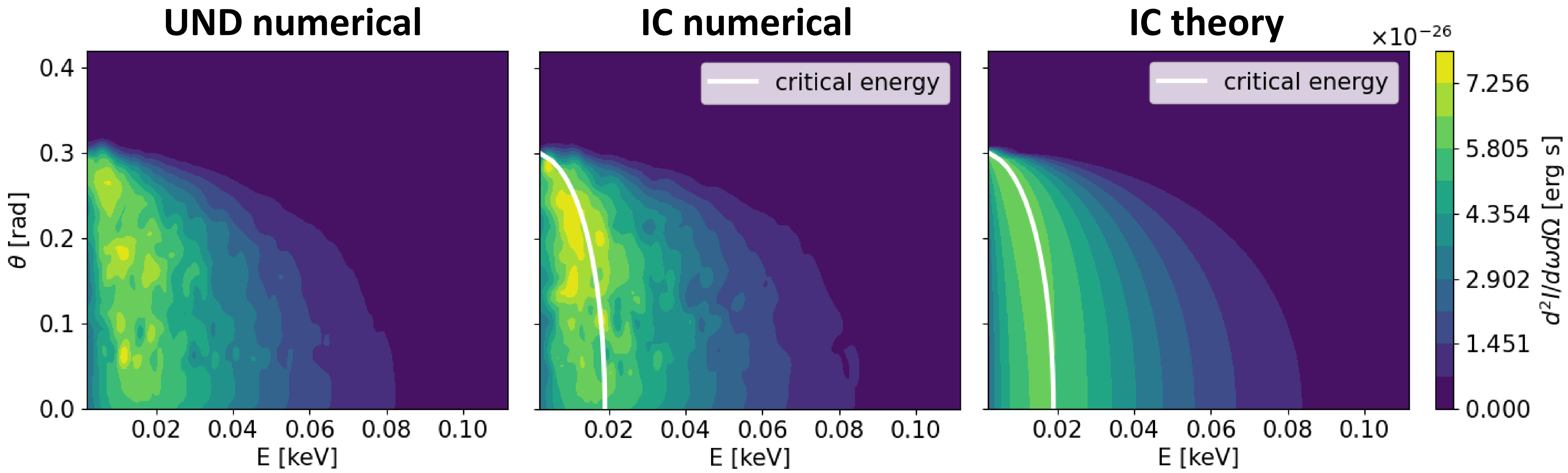




spectrum along oscillation plane

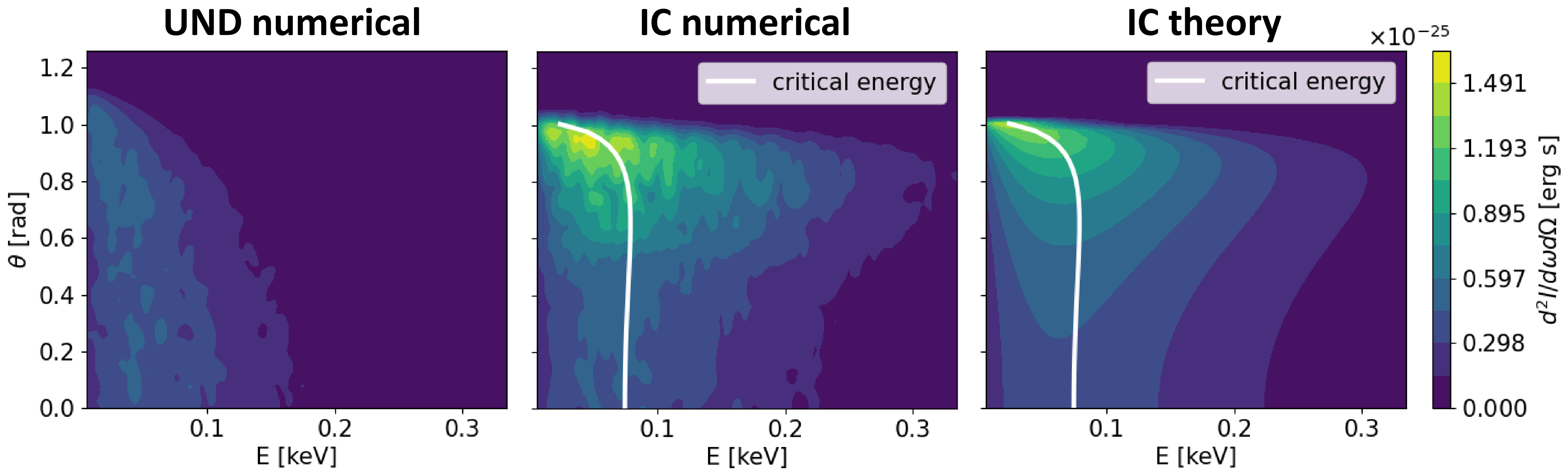
IC vs UND: numerical single particle spectrum comparison

- $K/\gamma = 0.3$ → IC and UND spectra are one and the same



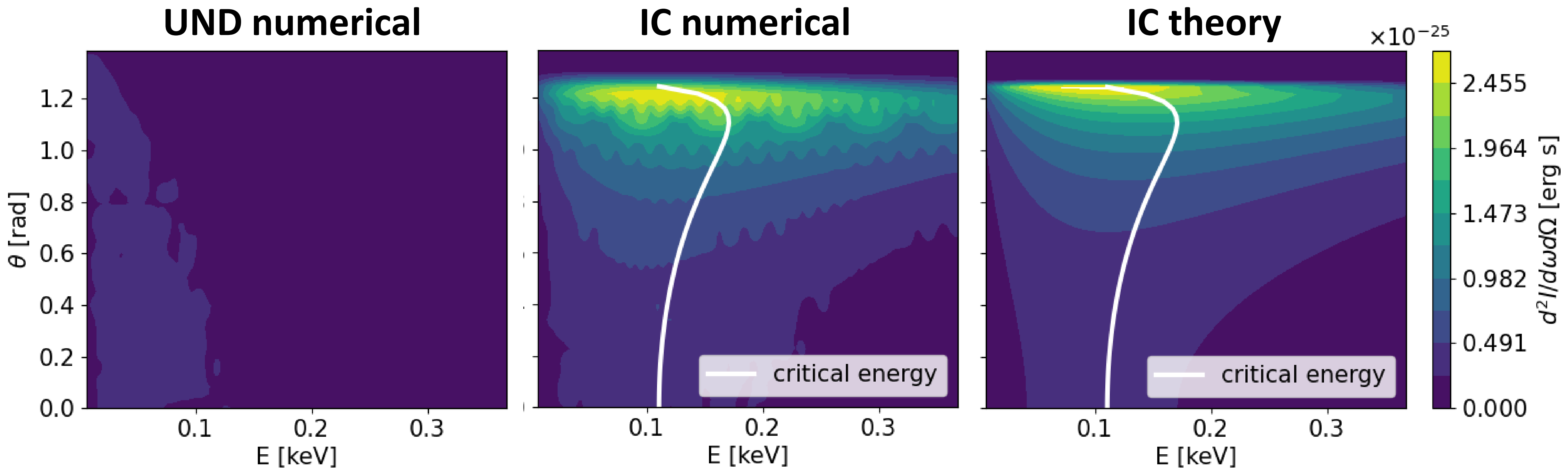
IC vs UND: numerical spectrum comparison

- $\boldsymbol{K/\gamma = 0.9}$ → IC overcomes UND, nearly flat $\omega_c(\theta)$, theory ok



IC vs UND: numerical spectrum comparison

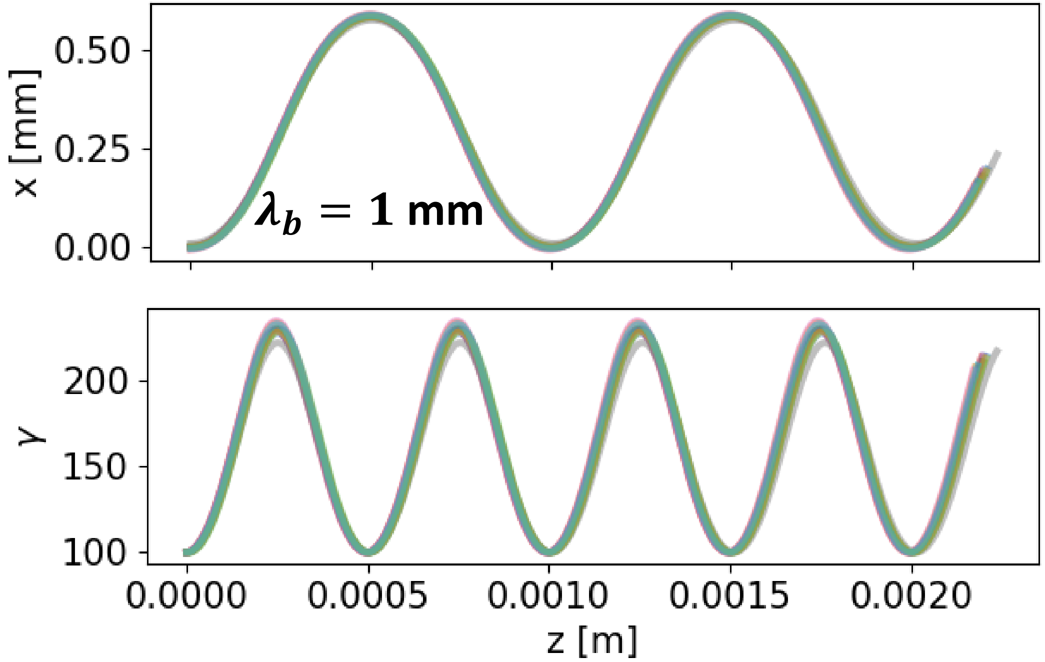
- K/γ = $\mathbf{0.99}$ → ω_c peaked at θ_c, theory still fits



IC spectrum: full beam case study

$K/\gamma_0 = 0.95$	E_0= 50 MeV	n_p= 1.7×10^{17} cm^{-3}	$\epsilon = 1$ mm mrad	x_0 = 290 μm

- Semi-realistic setup, IC may be too wide for such a plasma density
- Theoretical 130% energy oscillation, 5 $\times$ maximal undulator radiation intensity around ± 1 rad at $\approx$ **2 keV**



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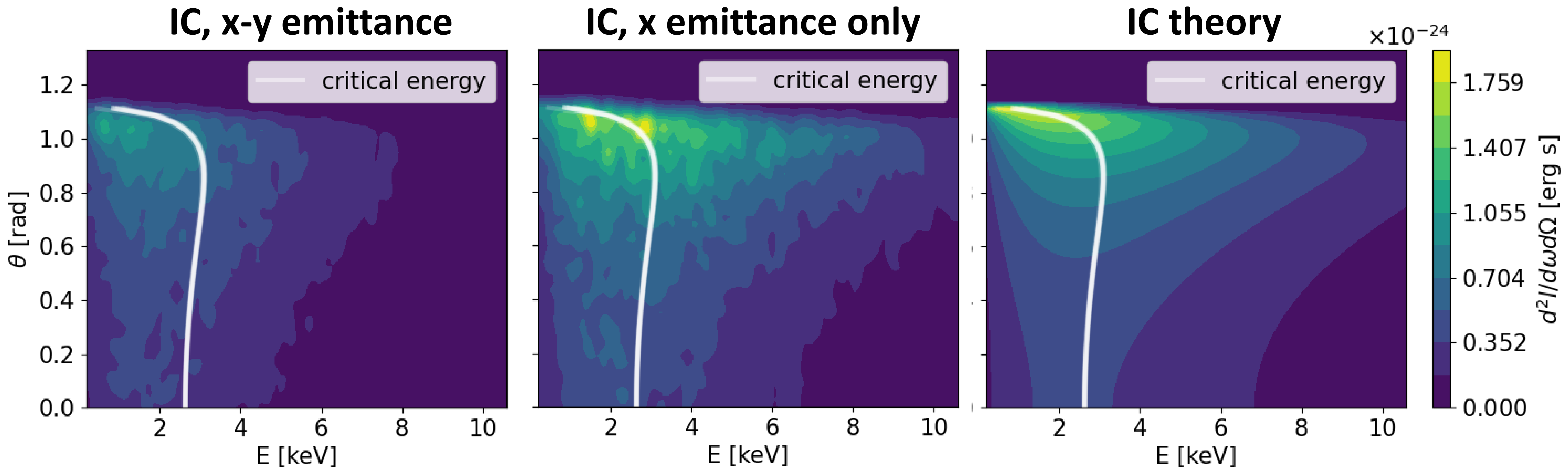
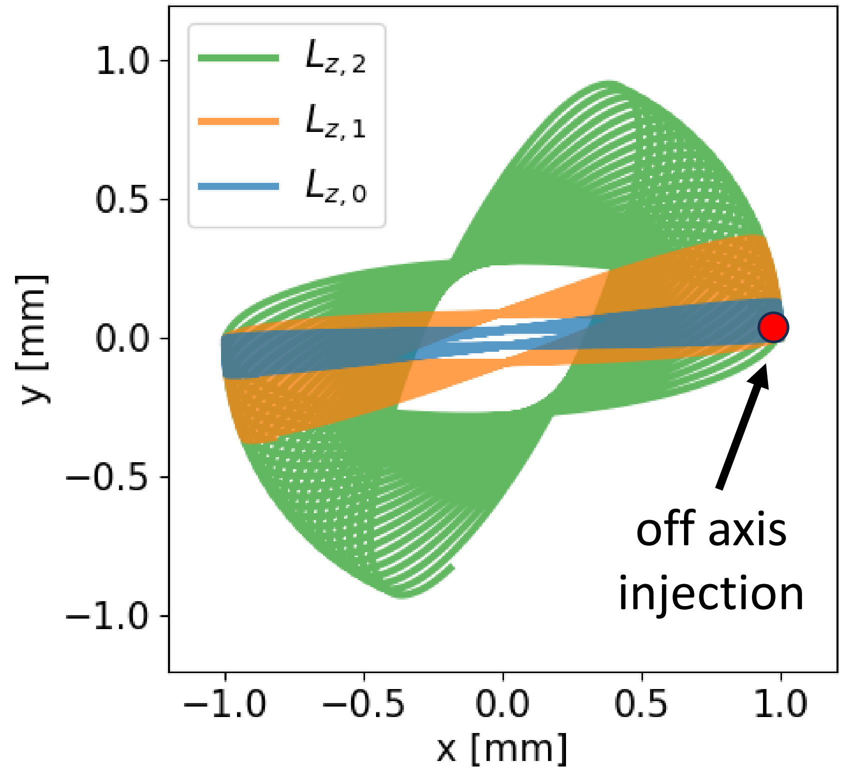


IC high K/γ_0 emittance source: precession

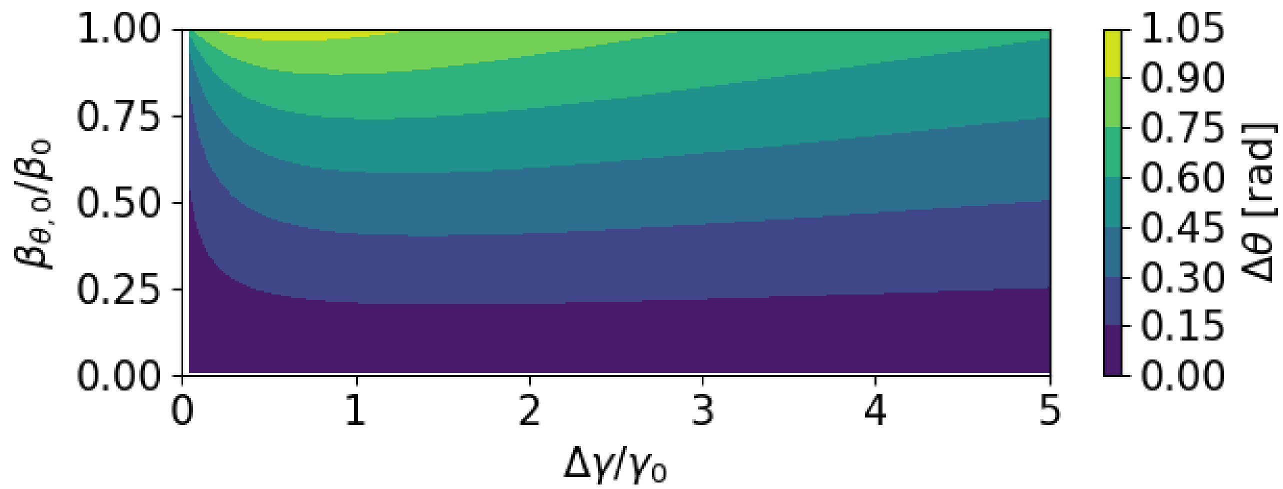
- Transverse orbit analysis reveals an emittance-related **precession**, that **grows with angular momentum** and **focusing strength**
- It is **unrelated to the emission process** and always takes place during electrostatic focusing, strongly enhanced in **high transverse momentum variation** regimes

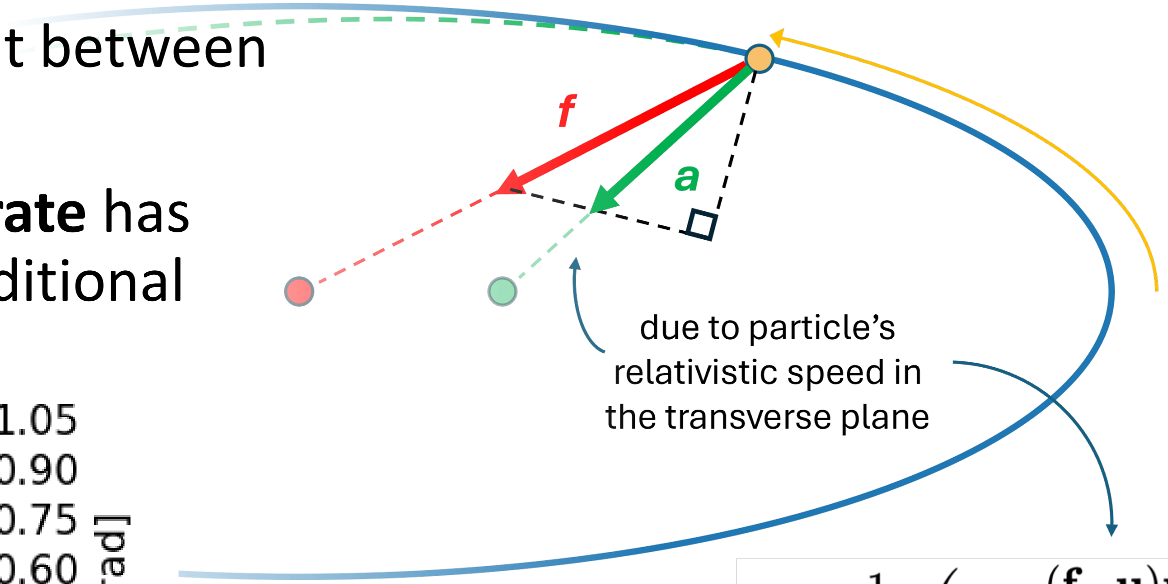
transverse plane



IC high K/γ_0 emittance source: precession

- Precession takes place because of an effective **shift in oscillation center** due to misalignment between force and relativistic acceleration
- An analytical expression for **precession rate** has been calculated and may be used for additional rms emittance term





$$\mathbf{a} = \frac{1}{m\gamma}\left(\mathbf{f} - \frac{(\mathbf{f}\cdot\mathbf{u})\mathbf{u}}{c^2}\right)$$

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Conclusions

- A fully **nonlinear study of particle and radiation dynamics** in focusing **ion channel devices** has been performed, in a comparison with conventional undulators
- **Relativistic dipole emission** has been predicted and numerically observed in IC planar trajectories: may lead to well angular-separated **double spot sources**?
- Unexpected **dynamics-related precession** was observed in strong oscillation regimes, giving an extra emittance growth source that still needs to be evaluated in detail

Thanks for the attention!

paper on arXiv

Extreme radiation emission regime for electron beams in strong focusing ion channels and undulators

A. Frazzitta[1,2,*], M. Yadav[3,*], J. Mann[3], A.R. Rossi[2] and J. B. Rosenzweig[3]
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[2] *Department of Physics, INFN—Milan, Via Celoria, 16-20133 Milan, Italy*
[3] *Department of Physics and Astronomy, University of California, Los Angeles, USA*

(Dated: September 5, 2024)

A fundamental comparison between undulator and ion channel radiation is presented. Conventional theory for both devices fails to describe high K and K/γ regimes accurately, providing an underestimation of particle trajectory amplitude and period. This may lead to incorrect estimation of radiation emission in many setups of practical interest, such as the ion column. A redefinition of plasma density and undulator strength expressions leads to a more reliable prediction of particle behaviour, reproducing the closest possible conditions in the two devices and correctly matching expected betatron oscillation amplitude and wavelength for a wide range of K/γ values. Differences in spectral features of the two devices can then be addressed via numerical simulations of single particle and beam dynamics. In this paper we outline a theoretical framework and compare its results with numerical simulation applied to setups eligible for possible radiation sources.